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**The Intragenerational Redistributive Effects of Unfunded Pension Programs**

Prepared by Luis Cubeddu<sup>1</sup>

Authorized for distribution by Richard Hemming

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**Abstract**

This paper provides a quantitative evaluation of the intracohort redistributive elements of the U.S. social security system in the context of a computable general equilibrium model. It determines how the well-being of individuals that differ by gender, race, and education is affected by the government's social security policy. Differences in life expectancy and labor productivity translate into differences in capital accumulation and labor supply distortions that are responsible for the observed welfare difference between individuals of the same age cohort.

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Author's E-mail Address: lcubeddu@imf.org

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## Summary

Models that quantitatively explore the implications of the government's social security policy have almost exclusively focused on issues of intergenerational redistribution, disregarding altogether the intragenerational transfers that arise from large differences in life expectancy and labor productivity between individuals.

This paper quantifies the extent of intragenerational redistribution in the United States social security system. For this purpose, a general equilibrium, overlapping generations model is developed, where individuals decide how much to work and how much to save for old-age consumption in the context of an unfunded social security program. Individuals belonging to the same age cohort differ in their life expectancy and labor productivity. The model is then calibrated to match many features of the United States economy and social security system.

Differences in mortality risk and labor productivity translate into differences in the magnitudes of capital accumulation and labor supply distortions that are responsible for the observed welfare difference between individuals in the same age cohort. The paper suggests that the United States social security program is lifetime progressive across gender and education, yet lifetime regressive across race. The latter result is very sensitive to the model's calibration.

## I. Introduction

A major question facing policymakers in most western economies is how best to guarantee a minimum level of support to its growing elderly population. Adverse demographic prospects and recent slowdowns in economic growth have motivated the need to reevaluate the viability of public pension programs and the distribution of their financing burden across generations. In addition, as unfunded pension schemes approach maturity, the issue of how this burden is distributed across individuals of the same age cohort has received increased attention. However, models that quantitatively explore the implications of government social security policy have almost exclusively focused on issues of intergenerational redistribution, disregarding altogether the intragenerational transfers that arise from large differences in life expectancy and labor productivity between individuals.

This paper aims to quantify the extent of intragenerational redistribution in the U.S. social security system.<sup>1</sup> In particular, it determines how social security policy affects the well-being of individuals who differ by *gender*, *race*, and *education*. For this purpose, a general equilibrium, overlapping generations economy is developed, where individuals face uncertain lifetimes. Within the same age cohort individuals with different life expectancy and labor productivity coexist. Individuals decide how much to work and how much to save in private assets for old age consumption. Retirement is mandatory and individuals are not altruistic. The return to private saving and wages are determined by profit maximizing firms with standard neoclassical production technology. Government is responsible for administering the social security program. The program is pay-as-you-go and balanced budget, and incorporates many features of the U.S. old-age insurance program.

Related literature includes Auerbach and Kotlikoff (1987), who in an overlapping generations, general equilibrium, simulation model study the short-run and long-run implications of changes in social security policy. Imrohoroglu, Imrohoroglu, and Joines (1995), extend their model by assuming credit and insurance market imperfections and find that unfunded pension schemes may in certain cases enhance the steady-state welfare of a dynamically efficient economy. However, both works disregard ex ante differences in mortality risk and labor productivity between individuals belonging to the same age cohort, and therefore are unable to quantify the extent of intragenerational redistribution inherent in unfunded pension schemes.<sup>2</sup> The paper is closest in spirit to that of Fullerton and Rogers (1993), who quantify the distribution of the burden of the U.S. tax system across 12 different lifetime income groups. Instead, in this paper individuals are categorized into 8 different lifetime groups which differ in terms of their labor productivity

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<sup>1</sup>In this paper, social security is treated purely as an old-age insurance program. Survivor, disability, and hospital insurance features are disregarded.

<sup>2</sup>In Imrohoroglu et al. (1995), since individuals cannot fully insure against unemployment risk, individuals of the same age group may differ ex post not only in their labor income but also in their asset holdings.

and life expectancy. By focusing solely on the U.S. social security system, the model is capable of addressing in more detail certain features of the system.

In dynamically efficient economies, the return to unfunded pension schemes is less than the return to private saving. By essentially forcing individuals to substitute private assets for social security tax contributions, unfunded pension schemes in the presence of perfect insurance markets, are welfare reducing. The magnitude of the loss increases with the expected present value of the difference between the future income that could have been guaranteed by the displaced saving and the social security benefits. Since unfunded pension schemes are designed not to discriminate on the basis of an individual's probability of dying early, the expected rate of return to contributions increases with an individual's life expectancy. In addition, unfunded schemes with progressive tax-benefit links reward individuals with lower-than-average lifetime earnings, at the expense of those with higher-than-average lifetime earnings. The higher the return to social security, the lower the observed welfare loss. However, differences in the expected return to an unfunded pension scheme can explain only part of the observed intracohort variability in welfare.

Differences in workers' productivity-age profiles are also responsible for differences in capital accumulation and labor supply distortions.<sup>3</sup> Assuming a closed economy, the introduction of pay-as-you-go social security crowds out capital formation, causing interest rates to rise and wages to fall. The change in relative factor prices will encourage workers to increase labor supply and saving early in life, so as to enjoy consumption and leisure later in life. Workers with later productivity peaks will not only observe a greater drop in the present value of their labor endowment, but will also find changes in their saving and labor supply behavior more distortionary.

The benchmark economy, which attempts to approximate certain features of the U.S. social security system, has an average replacement rate to labor earnings of 40 percent, a legal retirement age of 65, and a progressive tax-benefit formula. The paper simulates the steady-state effects of eliminating social security on macroeconomic aggregates as well as the lifetime welfare of cohorts that differ in their gender, race, and education. Results indicate that the steady-state welfare gains from eliminating social security are lower for females, whites, and noncollege graduates than for males, nonwhites, and college graduates, respectively. They are on average 40 percent greater for males than females, 4 percent greater for nonwhites than whites, and 9 percent greater for college graduates than noncollege graduates. Findings imply that the current system is lifetime progressive across gender and education, yet lifetime regressive across race.

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<sup>3</sup>Social security is financed through a payroll tax which distorts an individual's labor supply decision. The magnitude of the distortion is a function of both the age-specific net marginal tax rate and the shape of a worker's wage-age profile. Since workers in deciding how much to work perceive no linkage at the margin between social security benefits and taxes, marginal taxes will equal across types for all ages.

The latter result is very sensitive to the model's calibration.

The remainder of the paper proceeds as follows. In Section 2 the model is described. Section 3 details the calibration procedures and Section 4 outlines the algorithm solution. Welfare measures are defined in Section 5. Results of policy experiments and sensitivity analysis are elaborated in Section 6, while Section 7 concludes and suggests extensions for further research.

## II. The Model

The economy is composed of individuals who live a maximum of  $I$  periods each in overlapping generations. In each generation there are  $J$  individual types who differ according to life expectancy and labor productivity. The probability of surviving between age  $i$  and age  $i + 1$ , for a type  $j$  individual is  $s_{ij}$ . Therefore, the unconditional probability of reaching age  $i$  for type  $j$  is  $s^{ij} = \prod_{k=1}^{i-1} s_{kj}$ . The share of age  $i$ , type  $j$  individuals is denoted by  $\mu_{ij}$ . All individual types grow at the exogenous rate  $\lambda_\mu$  and population is to be stable in the sense that the cohort shares for each individual type are time-invariant.<sup>4</sup> This implies that the measure of all different types satisfy the following relationship:

$$\mu_{i+1,j} = \frac{s_{ij}\mu_{ij}}{(1 + \lambda_\mu)} \quad (1)$$

Time subscripts are ignored, as the dynamic feature of the model is captured by the age subscript. Individuals are endowed with one unit of time per period, that must be allocated between work and leisure. One unit of time of an age  $i$ , type  $j$  individual can be transformed into  $\varepsilon_{ij}$  exogenously given units of labor input.

### A. Preferences

Preferences are given by the expected discounted utility of a time separable, twice continuously differentiable, strictly concave, utility function of leisure and a consumption good:

$$\sum_{i=1}^I \beta^{i-1} s^{ij} U(c_{ij}, l_{ij}) \quad (2)$$

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<sup>4</sup>A population's steady-state growth rate is determined by its age-specific mortality and fertility rates (assuming these remain constant over time). If different types of individuals have different survival probabilities, as is the case in this paper, then for all types to grow at the same rate, fertility rates must differ. Specifically, individuals with lower life expectancy must have higher birth rates.

where  $\beta$  is the annual discount rate, and  $c_{ij}$  and  $l_{ij}$  are respectively consumption and leisure for an individual age  $i$  and type  $j$ . Every period, earnings are divided between consumption and gross investment. Individuals accumulate assets to smooth consumption over time. In the presence of private annuities individuals can insure against mortality risk. Annuity markets are established to avoid the issue of what to do with the assets of the deceased. Since the ex ante mortality probability of each individual is public information, competitive insurers will offer annuities with different rates of return to individuals with different life expectancies. Individuals of the same age cohort and type, sign a contract in which survivors share assets of the individuals that die. In this manner, next period's asset holdings are this period's saving divided by the probability of surviving. This implies that a type  $j$  individual faces the following budget constraint:

$$\begin{aligned} c_{ij} + y_{ij} &= Ra_{ij} + W(1 - l_{ij})\varepsilon_{ij}(1 - \tau) + b_{ij} \\ y_{ij} &= a'_{i+1,j}s_{ij} \\ a_{1j} &= 0 \\ y_{I+1} &\geq 0 \end{aligned} \tag{3}$$

where  $a_{ij}$  is the accumulated net wealth,  $y_{ij}$  are the gross saving,  $b_{ij}$  are the retirement benefits, and  $a'_{i+1,j}$  is next period's accumulated wealth, of an individual age  $i$  and type  $j$ . The return on asset holdings is  $R$ , the spot price of one unit of labor input in terms of the consumption good is  $W$  and the social security payroll tax is  $\tau$ . Individuals retire at age  $I_R$ , after which they rely on private saving and social security benefits for their old age consumption. Formally,  $\varepsilon_{ij} = 0$ , for  $i \geq I_R$ , and for all type  $j$  individuals. Finally, the model assumes that workers, in deciding how much to work, perceive no linkage at the margin between social security benefits and taxes.

## B. Technology

Firms maximize profits, taking factor and output prices as given. Technology is given by a neoclassical production function,  $f(K, N)$ , where  $K$  is the aggregate capital stock and  $N$  is the aggregate labor input. Capital depreciates at rate  $\delta$ . Firms hire physical capital and effective labor until gross factor prices equal marginal products:

$$\begin{aligned} R &= f_1(K, N) + 1 - \delta \\ W &= f_2(K, N) \end{aligned} \tag{4}$$

## C. Government

Government levies a payroll tax on labor earnings to finance social security pensions. The social security tax rate is the same for all those with labor earnings up to a



maximum level,  $E^{max}$ . The system is pay-as-you-go and the budget is balanced each period, as revenues from payroll taxes equal outlays in the form of social security pensions:

$$\sum_{j=1}^J \sum_{i=1}^I \mu_{ij} b_{ij} = \sum_{j=1}^J \sum_{i=1}^I \mu_{ij} \min\{W(1 - l_{ij})\varepsilon_{ij}; E^{max}\} \tau \quad (5)$$

Social security pension benefits correspond to a fixed proportion of an individual's lifetime average earnings. However, earnings of workers beyond the statutory maximum are not considered when computing an individual's average lifetime earnings. Earnings are indexed to account for labor productivity growth,  $\lambda_y$ . Wages prior to retirement age  $I_R$ , are revalued so that they equal the wages of workers at the time they turned age  $(I_R - 1)$ . Average lifetime indexed earnings for an age  $i$ , type  $j$  individual is given by:

$$m_{ij} = \frac{\lambda_y^{I_R-1-i} \sum_{k=1}^{I_R-1} \min\{W(1 - l_{kj})\varepsilon_{kj}; E^{max}\}}{(I_R - 1)} \quad (6)$$

Social security achieves progressivity not through graduated tax rates, but rather through the structure of benefits. The function relating retirement benefits and average lifetime earnings is highly redistributive, providing a much higher ratio of benefits to preretirement income to retirees with lower earnings history. Retirement benefits are given by:

$$b_{ij} = \begin{cases} 0 & \text{for } i \in [1, \dots, I_R - 1] \\ m_{ij}\eta(m_{ij}) & \text{for } i \in [I_R, \dots, I] \end{cases} \quad (7)$$

where  $\eta(m_{ij})$  is the average earnings replacement rate for an age  $i$ , type  $j$  individual with average lifetime indexed earnings,  $m_{ij}$ . Government announces an average replacement rate  $\eta_{avg}$ , and a benefit formula, and then sets taxes such that the budget is balanced each period. Individuals in the economy are atomistic in that they disregard the effect their labor supply decisions may have on the social security payroll tax.

#### D. Equilibrium

A competitive equilibrium corresponds to a feasible allocation and a set of factor prices, such that the individual's problem is solved for each generation, firms maximize profits, government balances its budget and markets clear. Market clearing conditions for capital and labor markets are given by:

$$\begin{aligned} K &= \sum_{j=1}^J \sum_{i=1}^I \mu_{ij} a_{ij} \\ N &= \sum_{j=1}^J \sum_{i=1}^I \mu_{ij} (1 - l_{ij}) \varepsilon_{ij} \end{aligned} \quad (8)$$

while the goods market clearing condition is:

$$\sum_{j=1}^J \sum_{i=1}^I \mu_{ij}(c_{ij} + y_{ij}) \leq f(K, N) + (1 - \delta)K \quad (9)$$

### III. Model Parametrization and Calibration

#### A. Demographics

Individuals are born into adulthood at age 20 and can live up to age 85, after which death is certain. The population is composed of 8 different lifetime cohorts that differ in gender, race, and education. There are two gender types: male and female; two race types: whites and nonwhites; and two education types: college and noncollege educated. Lifetime cohorts differ in their life expectancy and labor productivity profile. To ensure a stable population, the model assumes that mortality rates remain constant over time and that all lifetime cohorts grow at the same constant exogenous rate. As stated previously, the assumption implies that types with lower life expectancy must also have higher birth rates.<sup>5</sup> In order to match the model's stationary demographic structure with some general features of the current U.S. population, the proportion of individual types at age 20 equals that found in the United States in 1988.<sup>6</sup>

The annual rate of population growth,  $\lambda_\mu$ , is assumed constant at 1.2 percent, which approximately corresponds to the average U.S. rate over the past 25 years. Age-specific survival probabilities across gender and race are taken from the 1988 United States Vital Statistics Mortality Surveys, while mortality differences across education groups come from a study conducted by Elo and Preston (1996). This study indicates that more educated cohorts, through greater access of material and informational resources such as diet, housing, and health care service, live longer on average. Nevertheless, education can only account for about 18 percent of the mortality difference between whites and nonwhites. Figures 3 and 4 show conditional survival probabilities for the different cohorts.

Age, gender, and education specific labor productivities,  $\varepsilon$ , are compiled using CPS March demographic files for 1989-91. The sample includes private sector employees, above the age of 19, not working in the agriculture sector. Data for different years are adjusted using the GDP deflator. For each age cohort and individual type the per annum

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<sup>5</sup>This condition is empirically verified. The data shows that while females, whites and college educated outlive males, nonwhites, and noncollege educated, the latter observe higher birth rates.

<sup>6</sup>While the current U.S. demographic structure is far from being stationary, as the proportion of nonwhites and college educated people in the population has been increasing over time, the assumption allows for the existence of a stable population where different lifetime cohorts coexist.

mean labor earnings and mean hours worked are computed. Mean wages are computed by simply dividing mean earnings by mean hours worked. Wages are then normalized to the average wage in the economy in order to determine the endowment of labor efficiency units. Figures 1 and 2 show wage-age profiles according to gender, race, and education. A polynomial of degree two in age is used to smooth the wage-age profile. The data show that females and noncollege graduates reach their productivity peak before males and college graduates, respectively; early wage growth is considerably higher for college graduates than for noncollege graduates; and, for all ages, whites earn higher wages than nonwhites, even after controlling for gender and education.

Table 2 describes the demographic and economic characteristics of the population. The data show that females outlive males by an average of 5.1 years, yet have on average wages that are 27.1 percent lower than those of males. Whites live on average 3.6 more years and have wages that are 13.6 percent higher than nonwhites. Similarly, college graduates outlive noncollege graduates by 0.8 years, and have wages that are on average 42.1 percent greater than wages of noncollege graduates.

Since household composition changes significantly over the lifecycle due to marriage, divorce, death of spouse, and number of dependents, the model characterizes the lifetime distributional consequences of social security on single individuals rather than on households. This approach is standard in models that study the lifecycle implications of tax policy (see Fullerton and Rogers (1993)). Therefore, men and women are treated as independent decision-making units, and the intrahousehold resource allocation problem is disregarded.<sup>7</sup>

## B. Preferences and Technology

The expected lifetime utility of a type  $j$  individual is given by:

$$\sum_{i=1}^I \beta^{i-1} s_{ij} \frac{(c_{ij}^\theta l_{ij}^{1-\theta})^{1-\gamma}}{1-\gamma} \quad (10)$$

This functional form for preferences implies that leisure is independent of productivity, and has the advantage that the parameters needed for its calibration,  $\beta$ ,  $\theta$ , and  $\gamma$ , have been extensively studied in the literature. In addition, intertemporal separability of utility implies that leisure and consumption at different dates are net substitutes, and that the inverse of the degree of risk aversion equals the degree of intertemporal substitution.

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<sup>7</sup>Craig and Batina (1991) use a two-period general equilibrium overlapping generations model to simulate the effect of spouse and retirement insurance on family labor supply. In their specification, they are able to use households as welfare measuring units by assuming that the marital status of the couple does not change over the lifecycle.

All lifetime cohorts are endowed with the same preferences. The discount factor is normalized to account for productivity growth,  $\lambda_y$ , such that  $\beta = \hat{\beta}(1 + \lambda_y)^{\frac{(1-\theta)}{(\gamma-1)}}$ . The true discount factor,  $\hat{\beta}$ , equals 1.011, and is taken from Hurd (1989) study of retired singles, where differences in mortality probabilities across gender and race are accounted for. Consistent with the Becker and Ghez (1975) finding that households allocate approximately one-third of their discretionary time to market activities, the consumption share parameter,  $\theta$ , is 0.33. Given the stability of average hours worked since the Second World War, the elasticity of substitution between consumption and leisure is taken to be 1. The risk aversion parameter,  $\gamma$ , or inverse of the intertemporal elasticity of substitution is 4. The choice represents a compromise between different lifecycle models that explicitly account for leisure, and is consistent with that used in Auerbach and Kotlikoff (1987).<sup>8</sup> Different values for  $\beta$  and  $\gamma$  are chosen as part of a sensitivity analysis.

Since factor shares of income have been fairly constant over time, the paper assumes a Cobb-Douglas production function,  $f(K, N) = K^\alpha N^{1-\alpha}$ . The value of the capital share parameter,  $\alpha$ , depends on how the stock and flow of services from government capital and consumer durables, proprietors' income and inventories, are treated. Since the model contains no household production sector, no government investment, and no explicit treatment of inventories, consistent with Cooley and Prescott (1995), the capital share parameter equals 0.36. The depreciation rate,  $\delta$ , is determined by the ratio of gross investment to capital, which according to National Income Accounts is approximately 0.76. The annual rate of productivity growth,  $\lambda_y$ , is assumed constant at 1.0 percent which approximately corresponds to the average U.S. rate over the past 25 years. After accounting for population and productivity growth, depreciation is 5.4 percent per annum.

### C. Social Security

Social security is treated purely as an old-age insurance program. Spouse, survivor, disability, and health insurance features are disregarded. Social security pension benefits are indexed to labor productivity growth, the benefit structure is progressive and means tested. As dictated by current social security legislation, wages are revalued so that they equal the wages of workers at the time they turned 60. Individuals retire and receive social security benefits starting at age 65.<sup>9</sup>

Monthly social security benefits, known as the Primary Insurance Amount (PIA),

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<sup>8</sup>For the preferences used in this paper, the lower the degree of risk aversion, the less individuals care about consumption smoothing and the more willing they are to substitute labor from periods of low wages to periods of high wages. In the presence of uncertainty, the lower the degree of risk aversion, the smaller the fraction of resources devoted to precautionary savings.

<sup>9</sup>Legislation passed in 1983 calls for a gradual increase in the age at which future retirees are able to receive full benefits. By 2022, the age will be 67.

are a function of the retired individual's Average Index Monthly Earnings (AIME).<sup>10</sup> Since the program achieves its progressivity through the structure of benefits and not through graduated tax rates, benefits are structured so that the PIA increases with the AIME at a decreasing rate. The function relating the PIA to the AIME has three segments with sharply declining ratios of PIA to AIME. These values are calibrated consistent with the legislation specified in the 1993 Social Security Handbook. The first \$401 of AIME entitled the retiree to a primary insurance of 90 cents per dollar of AIME. The next segment covered AIME values up to \$2,420, where each dollar of AIME entitled the retiree to 32 cents of benefits. Above that level, each dollar of AIME produced only 15 percent of primary insurance benefit up to some set maximum. The average replacement rate to income in 1993 was approximately 45 percent and the average personal income close to \$1,920 per month. The replacement rate of an age  $i$ , type  $j$  individual with average annual lifetime earnings,  $m_{i,j}$ , can be summarized as:

$$\eta(m_{ij}) = \begin{cases} 0 & \text{for } i \in [1, I_R - 1] \\ \frac{\sum_{k=1}^4 \pi_k (\min[m_{ij}; m_{k+1}^{bend}] - m_k^{bend})}{m_{ij}} \eta_{avg} & \text{for } i \in [I_R, \dots, I] \end{cases} \quad (11)$$

where  $\eta_{avg}$  is the replacement rate for an individual with average labor earnings corresponding to  $m_{avg}$ . The fraction of primary insurance allowed per unit of AIME, between earnings bend points  $m_k^{bend}$  and  $m_{k+1}^{bend}$ , is defined as  $\pi_k \eta_{avg}$ .<sup>11</sup> Since earnings above \$4,800 per month are not counted in calculating a person's AIME,  $E^{max}$  is set equal 2.5 times the economy's average pretax labor earnings. Figure 5 relates the effective social security replacement rate to an individual's average annual lifetime.

#### IV. Algorithm Description

The solution methodology, the Gauss-Seidel method, is borrowed from Auerbach and Kotlikoff (1987). It involves solving a complicated set of nonlinear equations that specify households' and firms' optimization behavior, and the government's budget constraint. The algorithm starts with guesses for the capital to labor ratio, the age-specific shadow wages and the social security tax rate. When the social security benefit formula is progressive we must also provide a guess for the economy's average labor earnings. The capital to labor guess determines the relative factor prices which when combined with the shadow wage, social security tax rate, and benefit formula solves for the optimal behavior of individuals. The standard procedure in lifecycle models is to go to the last

<sup>10</sup>In computing an individual's AIME, the model considers labor earnings for all ages prior to retirement. Current legislation instead considers the highest 35 years of labor earnings.

<sup>11</sup>The bend points are as follows:  $m_1^{bend} = 0$ ;  $m_2^{bend} = 0.20m_{avg}$ ;  $m_3^{bend} = 1.25m_{avg}$ ;  $m_4^{bend} = 2.5m_{avg}$ . The fraction of PIA allowed per unit of AIME is calculated by multiplying the average replacement by  $\pi$ , where,  $\pi_1 = 2.0$ ;  $\pi_2 = 0.71$ ;  $\pi_3 = 0.33$ ;  $\pi_4 = 0.0$ .

period of an individual's life, where the future is no longer relevant, and solve for the behavior of the individual. In turn, this behavior would describe the nature of the future for individuals of the previous age. The recursive nature of the problem allows for the determination of the behavior for individuals of all ages.

From the derived labor supply decisions, new guesses for shadow wages are obtained. Aggregation of labor supply and saving decisions across all population subgroups in turn provide a new guess for the capital to labor ratio. From the labor supply decisions the earnings of each type of individual are determined, as well as the new social security tax guess which follows from the government's budget constraint. Typically, 10 to 20 iterations are required to achieve convergence to a steady-state equilibrium. The introduction of heterogeneity in age cohort labor productivity and mortality risk only adds to the size and dimension of the problem in hand, but fundamentally does not alter the solution algorithm.

## V. Measures of Welfare

Welfare for type  $j$  individuals, who face a social security policy  $\hat{x}$ , is defined as the expected discounted lifetime utility they derive from optimal consumption and leisure contingency plans:

$$\Psi_j(\hat{x}) = \sum_{i=1}^I \beta^{i-1} s^{ij} (c_{ij}(\hat{x})^\theta l_{ij}(\hat{x})^{1-\theta})^{1-\gamma} \quad (12)$$

The benchmark economy approximates the current social security program, where the average replacement rate to income is 40 percent, legal retirement age is 65, and the benefit formula is progressive. The welfare loss or gain for an individual of type  $j$  of departing from the benchmark economy is defined as the proportional increase or decrease in full lifetime resources required to make an individual of type  $j$  indifferent between the benchmark economy and an *alternative* economy. Because the utility function is homothetic, a change in an individual's lifetime wealth, provided factor prices are fixed, is associated with a proportional change in an individual's lifetime consumption and leisure. Therefore, the resources required to make an individual of type  $j$ , indifferent between the benchmark economy  $\hat{x}$  and the alternative economy  $x^*$  equal:

$$\omega_j(x^*) = \left\{ \frac{\Psi_j(\hat{x})}{\Psi_j(x^*)} \right\}^{\frac{1}{1-\gamma}} - 1 \quad (13)$$

The product of  $\omega_j(x^*)$  and the expected present value of labor endowment in the benchmark economy, represents the additional resources necessary to make individuals of type  $j$  indifferent between the benchmark and alternative economy. The aim of this

exercise is not to make pareto-like statements, but rather statements of the sort: "an individual is better or worse off in economy with social security policy  $x^*$ , than if he or she were to live in an economy with social security policy  $\hat{x}$ ". In order to compare the overall welfare gains or losses associated with alternative social security arrangements, a social welfare function is defined where the lifetime resources of each type of individual is given a weight equivalent to its measure at birth. The increase or decrease in the present value of labor endowment required to make all lifetime cohorts indifferent between the benchmark economy  $\hat{x}$ , and the alternative economy  $x^*$ , is given by:

$$\Omega(x^*) = \frac{\sum_{j=1}^J \mu_{1j} \omega_j(x^*) \sum_{i=1}^I s^{ij} \varepsilon_{ij}}{R(\hat{x})^{i-1}} \quad (14)$$

Tables 4 and 5 show  $\Omega(x^*)$  expressed as both relative to output and relative to the present value of lifetime resources.

## VI. Findings

### A. Aggregate Welfare Implications

The benchmark economy is one where the average replacement rate to income is 40 percent, legal retirement age is 65, and the benefit formula is progressive. Since the paper is intended to determine the extent of intragenerational redistribution inherent in the U.S. social security system, the model refrains from transition analysis, and focuses solely on the long-run implications of policies that divert from the benchmark.<sup>12</sup> As is well-documented in the literature, increases in pay-as-you-go social security will crowd out capital formation, which in turn will cause pretax wages to fall, interest rates to rise, and ultimately output to fall. Policy aimed at reducing the size of social security will bring about positive long-run macroeconomic effects. The model predicts that eliminating social security will increase steady-state capital by 22.8 percent, aggregate output by 9.8 percent, aggregate consumption by 5.2 percent, and aggregate labor by 3.1 percent. The increase in full lifetime resources required to make all individual types indifferent between the benchmark economy and one where social security is absent equals 2.52 percent of GDP and 2.57 percent of the economy's lifetime labor endowment.<sup>13</sup> The macroeconomic effects of social security policy in steady-state are found in Table 4.

Figures 6 through 9 show assets, net worth, consumption, and hours worked profiles for an average white, male, noncollege graduate. As predicted by standard lifecycle

<sup>12</sup>Obviously, changes in social security for an initial transition period will affect the young and old very differently.

<sup>13</sup>These results are similar to those found in Auerbach and Kotlikoff (1987), who show a replacement rate of 60 percent reduces steady-state capital by 24 percent, and that the welfare loss is about 6 percent of full-time resource.

models, individuals smooth consumption over the lifecycle by borrowing early, accumulating assets over the remainder of their working lives, and dissaving after retirement. In the presence of social security, while individuals need to save less for old-age their net worth in the absence and presence of social security is near equal. The effect on labor supply is explained as follows. A rise in the steady-state return to capital, when introducing an unfunded social security, will encourage individuals to increase work effort and save more early so to reduce work effort and consumption at a later age.

Results indicate that saving and labor effort increase with an individual's life expectancy. Since individuals with higher life expectancy need to finance a longer retirement, they will need to save and work more during their active period of life. Workers with later productivity peaks save less, as their earnings profile bears a closer resemblance to their optimal consumption plan.

## B. Intracohort Welfare Differences

In a pay-as-you-go system, the average return to social security is closely tied to population and labor productivity growth. The program does not discriminate on the basis of an individual's probability of dying early, so the expected rate of return increases with an individual's life expectancy. In addition, the progressive nature of the system will benefit individuals with below average lifetime earnings. The gross expected return to social security of a type- $j$  individual,  $R_{ss,j}$ , is that which equates the present value of expected lifetime contributions to the present value of expected lifetime benefits:

$$\sum_{i=1}^I \frac{s^{ij} \min\{W(1 - l_{ij})\varepsilon_{ij}; E^{max}\}\tau}{R_{ss,j}^{i-1}} = \sum_{i=1}^I \frac{s^{ij}b_{ij}}{R_{ss,j}^{i-1}} \quad (15)$$

Table 3 shows how these returns compare across individuals, population growth, and social security tax policy. Nonwhite male college graduates face the lowest returns, while white female college graduates face the highest. If a proportional tax-benefit formula were in place instead, females, whites, and college graduates would earn higher returns simply because they live longer on average. In the presence of perfect annuities, the expected return to private saving is equal across types of individuals. Since social security essentially forces individuals to hold an annuity, in dynamically efficient economies, a higher return to social security contributions implies a smaller difference between the return to private saving and social security. Therefore, asset accumulation distortions are less severe for those with above average life expectancy and below average earnings.<sup>14</sup>

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<sup>14</sup>The welfare loss increases with the expected present value of the difference between the future income guaranteed by the displaced saving and the social security benefits.



However, differences in the expected returns to social security can help explain only part of the differences in welfare across types of individuals. Social security is financed through a payroll tax which distorts an individual's labor supply decision. Since individuals do not perceive a link between the social security payroll tax and benefits at the margin, the marginal tax rate equals across types for all ages.<sup>15</sup> The variability in capital accumulation and labor supply distortion is then due to differences in workers' age-productivity profiles. Results indicate that workers with later productivity peaks find increases in social security more distortionary than workers with earlier peaks. Unfunded pension schemes crowd out capital formation, cause interest rates to rise, wages to fall, and encourage workers to increase labor effort and saving early in life, so as to enjoy consumption and leisure later in life. Therefore, workers with later productivity peaks will find changes in their capital accumulation and labor supply behavior more distortionary. Finally, workers with later productivity peaks will observe a greater drop in the present value of their labor endowment, as the relative return to capital increases.

Tables 6 and 7 show that, in the long-run, all cohorts experience an increase in their private saving, labor effort, and welfare as social security is eliminated. Interestingly, workers, besides increasing their labor effort, increase the productivity of their work by postponing effort to later in the lifecycle. Results show that individual with higher returns to social security offset to a greater degree increases in mandatory contributions, by reducing their private saving. Therefore, eliminating pay-as-you-go social security is likely to produce a greater increase in the private saving of females, whites and noncollege graduates, since these cohorts on average earn a higher return to social security.

In addition, labor effort is found to be less responsive to changes in social security policy the later a worker's age-productivity peak. Since eliminating social security implies eliminating the payroll tax and reducing the relative return to capital in steady-state, individuals will not only increase their labor supply but also shift their work effort to later in the lifecycle. The later a worker's productivity peak the less labor supply must accommodate to achieve the desired consumption and leisure plan. Females, nonwhites, and noncollege graduates, who on average have earlier age-productivity peaks, experience larger changes in their labor supply.

The extent of intracohort redistribution is quantified by calculating the steady-state relative welfare gain to each cohort of reducing the size of social security. If eliminating social security causes males to enjoy lifetime welfare gains that are greater than

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<sup>15</sup>If workers were to perceive a tax-benefit link, labor supply distortions would be mitigated. Workers with higher life expectancy and lower lifetime earnings would observe lower net marginal taxes and in turn lower labor supply distortions. In addition, since net marginal taxes fall with age, workers would be encouraged to postpone their labor effort. Therefore, those with late productivity peaks will find changes in their labor supply less distortionary. A more elaborate discussion of these issues are found in Feldstein and Samwick (1992), who document social security net marginal tax rates across age, gender, marital status, and income class.

those of females, then social security is said to benefit the latter group at the expense of the former. If the welfare gains from eliminating social security for all groups were equal, we would conclude that the system had no intracohort redistributive elements. Results indicate that welfare gains are greatest for cohorts whose private saving and labor supply are less responsive to changes in the system. Cohorts that have below average life expectancy, above average lifetime earnings, and later productivity peaks stand to gain more from reductions in social security. The results confirm that males, nonwhites, and college graduates experience a greater welfare gain from eliminating social security than females, whites, and noncollege graduates, respectively. These gains are on average: 39.8 percent greater for males than females, 3.8 percent greater for nonwhites than whites, and 9.1 percent greater for college graduates than noncollege graduates. Differences in life expectancy and labor productivity translate into differences in capital accumulation and labor supply distortions, that are in turn responsible for differences in welfare across types of individuals. From these findings, we can infer that the current system is lifetime progressive across gender and education, yet lifetime regressive across race.

## VII. Sensitivity Analysis

This section examines the robustness of the policy experiments. It determines the extent to which results change in the absence of private annuity markets and when considering different values for the risk aversion coefficient,  $\gamma$  and the subjective discount factor,  $\beta$ .

### A. The Absence of Annuities

Empirical evidence suggests the near absence of private formal or informal markets to insure against uncertain longevity. In what follows, the robustness of the model when private markets to insure against the event of death do not exist is tested.<sup>16</sup> In the absence of private annuities, the individual's problem is slightly different. Individuals still accumulate assets for lifecycle reasons, but now precautionary motives become relevant as uncertainty about an individual's life expectancy induce saving to cover consumption in the event he or she lives longer than expected.<sup>17</sup> In contrast to equation 3, gross saving is now given by:

$$y_{ij} = a'_{i+1,j} + \phi_{ij} \quad (16)$$

where  $\phi_{ij}$  represents the lump-sum transfer of accidental bequests for an age  $i$ , type  $j$  individual. Government is now responsible for collecting and distributing the accidental

<sup>16</sup>Engen and Gale (1993), show that only about 2 percent of the elderly own individual annuities.

<sup>17</sup>Precautionary saving in response to risk is associated with convexity of the marginal utility function or a positive third derivative. The model's preferences guarantee a positive precautionary saving motive. See Kimball (1990), for more on this issue.

bequests. Unintended bequests are assumed to be taxed at 100 percent and returned in a lump-sum fashion to survivors of all ages.

$$\phi = \frac{R \sum_{j=1}^J \sum_{i=1}^I \mu_{ij} (1 - s_{ij}) a_{ij}}{\sum_{j=1}^J \sum_{i=1}^I \mu_{ij}} \quad (17)$$

In the presence of uninsurable mortality risk individuals discount the future more heavily and consume earlier in life than they would otherwise. Social security, by partially substituting for private annuities, improves an individual's consumption allocation and reduces accidental bequests. The computational algorithm for the no-annuity-case is slightly different. Besides providing starting guesses for the capital to labor ratio, the age-specific shadow wages, the social security tax rate and the economy's average labor earnings; a guess for the lump-sum transfer of accidental bequest must also be specified.

Results indicate that benefits of social security in the form of insurance provision for uncertain longevity are outweighed by the cost of social security in the form of a lower capital stock, output, labor, and consumption.<sup>18</sup> Table 5 shows that, in an economy without annuities, the proportional increase in full lifetime resources required to make an individual indifferent between the benchmark economy and one where social security is absent equals 2.36 percent of output. In the presence of annuities the welfare gain of eliminating social security is only 0.16 percentage points greater.<sup>19</sup>

Next, we compare the difference in intragenerational well-being between an economy where annuities markets are present and one where they are absent. Since all individual types have the same degree of risk aversion, in the absence of annuities those with greater mortality risk have more to gain from insurance provision than those with lower

<sup>18</sup>While recent work by Imrohoroglu et al. (1995), and Valdivia (1997) show that under certain conditions the gains of social security can outstrip the costs; their economies differ in some very important dimensions to that of this paper. Imrohoroglu et al. (1995), assume that individuals also face uninsured unemployment risk. The introduction of an additional source of uninsured risk increases the precautionary motives for saving, and increases the gains of introducing social security by reducing the size of unintended bequests. Valdivia (1997), assumes that bequests are operational and that preferences are of constant relative risk aversion. In this framework the costs of living longer than expected are greater, partly because precautionary motives are absent and partly because reduced bequests affect the welfare of future generations. In addition, both papers restrict individual labor supply decisions and hence underestimate the potential distortionary effects of a wage-tax financed social security system. Finally, the comparison of welfare results across these models is complicated because, unlike in this paper, different employment (Imrohoroglu et al. (1995)) and mortality (Valdivia (1997)) histories translate into intra-age wealth heterogeneity. Further research and sensitivity analysis on the subject is warranted, yet outside the scope of this paper.

<sup>19</sup>The expected difference between the return to private capital and the return to social pensions is smaller in the absence of private annuities, hence capital accumulation distortions are less severe. In addition, since the resulting marginal taxes are lower, so will the labor supply distortions in the absence of these markets.

mortality risk. Results suggest that while, in the absence of annuities, social security continues to benefit females, whites, and noncollege educated at the expense of males, nonwhites, and college educated, the welfare difference between cohorts is smaller. The long-run welfare gains of eliminating social security are on average: 52.2 percent greater for males than females, 0.8 percent greater for nonwhites than whites, and 6.3 percent greater for college than noncollege graduates (see Tables 8 and 9).<sup>20</sup>

## B. Changes in the Degree of Risk Aversion

A fall in the risk aversion parameter  $\gamma$ , is equivalent to an increase in the intertemporal elasticity of substitution, and hence an increase in the desire to postpone consumption. The model confirms that lowering the risk aversion parameter, from  $\gamma = 4$  to  $\gamma = 3$ , will increase the capital to output ratio in the benchmark economy by 12 percent.

Results indicate that capital accumulation is less responsive to changes in social security the lower the degree of risk aversion. Eliminating social security will cause aggregate capital to fall by 21.3 percent when  $\gamma = 3$ , yet by 22.8 percent when  $\gamma = 4$ . The degree of risk aversion will affect capital accumulation distortions in two ways. First, the inefficiency associated with intertemporal distortions increases with the degree of risk aversion. In addition, since capital accumulation falls with the degree of risk aversion, the difference between the steady-state return to capital and the return to social security increases. These facts imply that the welfare gain of eliminating social security increases with the value of the risk aversion parameter. Table 5 details aggregate implications of changing parameter values for the discount factor and risk aversion parameter.

Finally, a reduction in the risk aversion parameter will also increase the intracohort difference in well-being. The welfare gains of eliminating social security are now 52.3 percent greater for males than females, 7.6 percent greater for nonwhites than whites, and 13.6 percent greater for college than noncollege educated. For more details, see Tables 8 and 9.

## C. Changes in the Discount Factor

In overlapping generations economies, the market rate of discount exceeds the rate at which individuals discount the future. Therefore, the lower the discount factor, the weaker are the incentives to postpone consumption and consequently the lower the economy's stock of capital. As predicted, lowering the discount factor from  $\beta = 1.011$

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<sup>20</sup>In the absence of annuities, the expected return to private assets increases with an individual's survival probability. Since the return to social security contributions increase with life expectancy, the expected difference between the return to private saving and social security need not fall with life expectancy, as is true in the case when private annuities are present.

to  $\beta = 0.98$ , will reduce the capital to output ratio in the benchmark economy by 30 percent, from 3.40 to 2.63.

In addition, reducing the discount factor will cause capital accumulation to be less responsive to changes in social security policy. Eliminating social security will increase aggregate capital by 22.8 percent and 15.2 percent, for  $\beta = 1.011$  and  $\beta = 0.98$ , respectively. Yet, the welfare gain associated with social security, as a percentage of the present value of lifetime resource, is 0.64 percentage points greater for  $\beta = 0.98$ .<sup>21</sup> While, on the one hand, a lower discount factor reduces saving incentives and hence saving distortions, on the other hand, it increases saving and labor supply distortion, by increasing the difference between the steady-state return to capital and the return to social security. Simulation results indicate the latter effect dominates the former (see Table 5).

Next, we evaluate how changes in the subjective discount factor might affect the magnitude of intragenerational redistribution. Results indicate that increases in the rate of time preference will reduce the welfare difference between our lifetime cohorts. The welfare gains of eliminating social security are now 20.5 percent and 4.2 percent greater for males and college educated, respectively. However, in contrast to previous results, whites stand to gain slightly more from reductions in social security than nonwhites. While whites observe on average higher returns to social security contributions, they also observe on average later labor productivity peaks. As the desire to postpone consumption lessens, the positive effect of observing higher returns is outweighed by the negative effect of having later productivity peaks. Hence, whether social security is lifetime regressive or progressive across race, is very sensitive to changes in the discount factor.

## VIII. Conclusion

This paper provides a quantitative evaluation of the intragenerational redistributive elements of the U.S. social security system, in the context of a general equilibrium model. Differences in life expectancy and labor productivity translate into differences in capital accumulation and labor supply distortions, that are in turn responsible for differences in the welfare of individuals that differ by gender, race, and education. Results suggests that the current old-age insurance scheme is lifetime progressive across gender and education, yet lifetime regressive across race. The latter result is sensitive to parameter calibration.

However, this paper has important shortcomings. It studies the lifecycle behavior of single men and women, and treats social security purely as an old-age insurance program. How then do we reconcile the finding that social security benefits females at the

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<sup>21</sup>For smaller discount factors, the increase in full lifetime resources required to make all individuals indifferent between the benchmark economy and one where social security is absent is larger relative to the economy's present value of labor endowment.

expense of males when in reality men and women, as husbands and wives, make joint economic decisions? In addition, social security provides not only retirement insurance but it also plays an important role in the provision of life insurance to dependent spouses and survivors.<sup>22</sup>

A natural extension of this paper would be to re-evaluate the intracohort redistribution of social security (including spouse and survivor insurance) at the household level rather than at the individual level. In the proposed framework, a household would be characterized not only by the age, gender, and race of the head and corresponding spouse (if married), but also by its marital status and number of dependents.<sup>23</sup> Survival benefits are likely to be greater for groups with higher fertility rates, and higher probabilities of death of a household head. In addition, transfers to dependent spouses will differ according to the degree of household specialization, determined by the relative productivity (education) differences of the spouses. Finally, adding marital status to the model allows us to address life insurance ownership questions, and the extent to which it is affected by public provision.

Integrating spouse and survivor insurance in a modeling context where men and women make joint economic decisions, and face changes in their marital status is likely to open a realm of unexplored policy issues.

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<sup>22</sup>One quarter of all Old Age and Survivor Insurance (OASI) payments goes to survivors.

<sup>23</sup>Cubeddu and Ríos-Rull (1997), in a similar framework, study how changes in the patterns of household formation and dissolution affect saving decisions at the household and aggregate level.

Table 1: Wage Index by Age, Sex and Education  
Current Population Survey: 1989-1991

	White				Nonwhite			
	College		Noncollege		College		Noncollege	
Age	Male	Female	Male	Female	Male	Female	Male	Female
<i>20-24</i>	0.936	0.860	0.670	0.599	0.901	0.878	0.615	0.566
<i>25-29</i>	1.272	1.118	0.898	0.718	1.146	1.020	0.798	0.639
<i>30-34</i>	1.548	1.277	1.047	0.764	1.389	1.171	0.863	0.696
<i>35-39</i>	1.769	1.297	1.149	0.803	1.656	1.315	0.970	0.751
<i>40-44</i>	1.895	1.279	1.250	0.831	1.683	1.359	1.021	0.788
<i>45-49</i>	1.984	1.178	1.297	0.813	1.782	1.294	1.010	0.769
<i>50-54</i>	2.044	1.129	1.263	0.778	1.544	1.111	0.949	0.664
<i>55-59</i>	2.084	1.137	1.233	0.757	1.542	1.013	0.953	0.641
<i>60-64</i>	1.975	1.066	1.194	0.734	1.184	1.040	0.941	0.568
<i>65 +</i>	1.643	0.902	0.906	0.643	1.631	1.094	0.665	0.493

Table 2: Demographic and Economic Characteristics of Population

Population Type			Life Expectancy	Average Productivity	Percent of Population
Overall			73.70	1.000	100.00
Male	White	Noncollege	71.61	1.018	28.35
		College	72.09	1.659	9.52
	Nonwhite	Noncollege	66.65	0.991	7.66
		College	67.04	0.845	2.57
Female	White	Noncollege	76.87	0.725	30.37
		College	77.47	1.157	10.20
	Nonwhite	Noncollege	73.37	0.671	8.46
		College	73.90	1.159	2.87

Table 3: Social Security Expected Rate of Return (percent)

			Progressive Tax		Flat Tax
Population Type			$\lambda_\mu = 1.2$	$\lambda_\mu = 2.0$	$\lambda_\mu = 1.2$
Male	White	Noncollege	1.18	2.09	1.21
		College	0.53	1.53	1.31
	Nonwhite	Noncollege	0.82	1.69	0.64
		College	0.36	1.27	0.73
Female	White	Noncollege	2.18	2.98	1.87
		College	1.87	2.67	1.92
	Nonwhite	Noncollege	1.91	2.70	1.49
		College	1.55	2.36	1.59



Table 4: Social Security and Economic Aggregates

Benchmark Economy:  $\eta = 40\%$ ,  $I^R = 65$ , Progressive Benefit Formula.

<i>Perfect Annuity Markets: <math>\beta = 1.011</math>, <math>\gamma = 4</math>, <math>\theta = 0.33</math></i>								
Policy	$\tau$ (%)	<b>K/Y</b>	<b>K</b>	<b>N</b>	<b>C</b>	<b>Y</b>	$\Omega/PVLE$	$\Omega/Y$
<i>Benchmark</i>	9.40	3.400	1.916	0.283	0.418	0.564	0.00	0.00
$\eta = 0\%$	0.00	3.803	2.354	0.292	0.440	0.619	2.57	2.52

Table 5: Sensitivity Analysis: Social Security and Economic Aggregates

Benchmark Policy:  $\eta = 40\%$ ,  $I^R = 65$ , Progressive Benefit Formula.

<b>No Annuity Markets: <math>\beta = 1.011</math>, <math>\gamma = 4</math>, <math>\theta = 0.33</math></b>								
Policy	$\tau$ (%)	<b>K/Y</b>	<b>K</b>	<b>N</b>	<b>C</b>	<b>Y</b>	$\Omega/PVLE$	$\Omega/Y$
<i>Benchmark</i>	9.33	3.351	1.838	0.278	0.403	0.549	0.00	0.00
$\eta = 0\%$	0.00	3.834	2.330	0.285	0.431	0.608	2.36	2.44
<i>Perfect Annuity Markets: <math>\beta = 1.011</math>, <math>\gamma = 3</math>, <math>\theta = 0.33</math></i>								
Policy	$\tau$ (%)	<b>K/Y</b>	<b>K</b>	<b>N</b>	<b>C</b>	<b>Y</b>	$\Omega/PVLE$	$\Omega/Y$
<i>Benchmark</i>	10.04	3.813	2.393	0.296	0.445	0.628	0.00	0.00
$\eta = 0\%$	0.00	4.248	2.903	0.303	0.462	0.683	1.99	2.27
<i>Perfect Annuity Markets: <math>\beta = 0.980</math>, <math>\gamma = 4</math>, <math>\theta = 0.33</math></i>								
Policy	$\tau$ (%)	<b>K/Y</b>	<b>K</b>	<b>L</b>	<b>C</b>	<b>Y</b>	$\Omega/PVLE$	$\Omega/Y$
<i>Benchmark</i>	8.09	2.631	1.207	0.266	0.367	0.459	0.00	0.00
$\eta = 0\%$	0.00	2.825	1.391	0.275	0.386	0.492	3.20	2.20

Table 6: Intracohort Welfare and Social Security (Part A)  
Benchmark Policy:  $\eta = 40\%$ ,  $I^R = 65$ , Progressive Benefit Formula.

<i>Perfect Annuity Markets: <math>\beta = 1.011, \gamma = 4, \theta = 0.33</math></i>				
	Percent Change			
Population	Savings	Hours	Eff Hours	$\omega_j$
Male	22.22	1.66	2.45	3.21
Female	23.18	3.68	3.96	1.94
White	23.26	2.55	2.99	2.55
Nonwhite	20.83	2.94	3.31	2.65
College	20.32	2.42	2.93	2.76
Noncollege	23.93	2.71	3.12	2.51

Table 7: Intracohort Welfare and Social Security (Part B)  
Benchmark Policy:  $\eta = 40\%$ ,  $I^R = 65$ , Progressive Benefit Formula.

<i>Perfect Annuity Markets: <math>\beta = 1.011, \gamma = 4, \theta = 0.33</math></i>						
			Percent Change			
Population			Savings	Hours	Eff. Hours	$\omega_j$
Male	White	Noncollege	24.29	1.71	2.53	3.10
		College	20.18	0.88	2.03	3.55
	Nonwhite	Noncollege	20.52	2.26	2.88	3.15
		College	16.19	1.93	2.73	3.37
Female	White	Noncollege	24.53	3.58	3.88	1.89
		College	21.32	3.89	4.12	1.93
	Nonwhite	Noncollege	23.44	3.82	3.99	2.04
		College	20.20	3.55	3.89	2.26

Table 8: Sensitivity Analysis: Intracohort Welfare and Social Security (Part A)

<i>No Annuity Markets: <math>\beta = 1.011, \gamma = 4, \theta = 0.33</math></i>				
	Percent Change			
<b>Population</b>	Savings	Hours	Eff. Hours	$\omega_j$
Male	31.40	0.97	1.94	2.95
Female	23.47	3.32	3.72	1.93
White	26.87	2.10	2.67	2.43
Nonwhite	27.07	2.11	2.56	2.45
College	23.92	2.16	2.75	2.56
Noncollege	28.38	2.09	2.59	2.40
<i>Perfect Annuity Markets: <math>\beta = 1.011, \gamma = 3, \theta = 0.33</math></i>				
	Percent Change			
<b>Population</b>	Savings	Hours	Eff. Hours	$\omega_j$
Male	20.04	1.05	1.95	2.70
Female	22.27	2.92	3.21	1.29
White	21.67	1.89	2.41	1.96
Nonwhite	19.63	2.27	2.67	2.12
College	18.78	1.74	2.34	2.21
Noncollege	22.46	2.05	2.52	1.92
<i>Perfect Annuity Markets: <math>\beta = 0.980, \gamma = 4, \theta = 0.33</math></i>				
	Percent Change			
<b>Population</b>	Savings	Hours	Eff. Hours	$\omega_j$
Male	14.39	1.81	2.51	3.56
Female	15.68	3.57	3.86	2.83
White	15.67	2.55	2.96	3.20
Nonwhite	13.28	2.96	3.34	3.18
College	12.72	2.49	2.95	3.30
Noncollege	16.31	2.69	3.08	3.16

Table 9: Sensitivity Analysis: Intracohort Welfare and Social Security (Part B)

No Annuity Markets: $\beta = 1.011, \gamma = 4, \theta = 0.33$						
			Percent Change			
Population			Savings	Hours	Eff. Hours	$\omega_j$
Male	White	Noncollege	33.13	1.09	2.04	2.84
		College	27.92	0.42	1.78	3.32
	Nonwhite	Noncollege	32.86	1.13	1.83	2.84
		College	26.40	1.16	2.05	3.02
Female	White	Noncollege	24.29	3.16	3.54	1.14
		College	21.29	3.99	4.28	1.80
	Nonwhite	Noncollege	25.51	3.10	3.30	1.99
		College	22.33	3.35	3.76	2.04
Perfect Annuity Markets: $\beta = 1.011, \gamma = 3, \theta = 0.33$						
			Percent Change			
Population			Savings	Hours	Eff. Hours	$\omega_j$
Male	White	Noncollege	22.08	1.12	2.03	2.56
		College	17.50	0.27	1.57	3.06
	Nonwhite	Noncollege	18.98	1.64	2.32	2.68
		College	14.73	1.28	2.16	2.95
Female	White	Noncollege	23.60	2.83	3.14	1.21
		College	20.45	3.10	3.34	1.30
	Nonwhite	Noncollege	22.53	3.09	3.26	1.41
		College	19.30	2.78	3.14	1.70
Perfect Annuity Markets: $\beta = 0.980, \gamma = 4, \theta = 0.33$						
			Percent Change			
Population			Savings	Hours	Eff. Hours	$\omega_j$
Male	White	Noncollege	16.76	1.82	2.55	3.52
		College	11.46	1.16	2.14	3.82
	Nonwhite	Noncollege	12.73	2.39	2.96	3.44
		College	8.14	2.21	2.93	3.54
Female	White	Noncollege	16.84	3.46	3.77	2.81
		College	14.15	3.77	4.02	2.82
	Nonwhite	Noncollege	15.85	3.68	3.86	2.86
		College	13.00	3.56	3.90	2.95

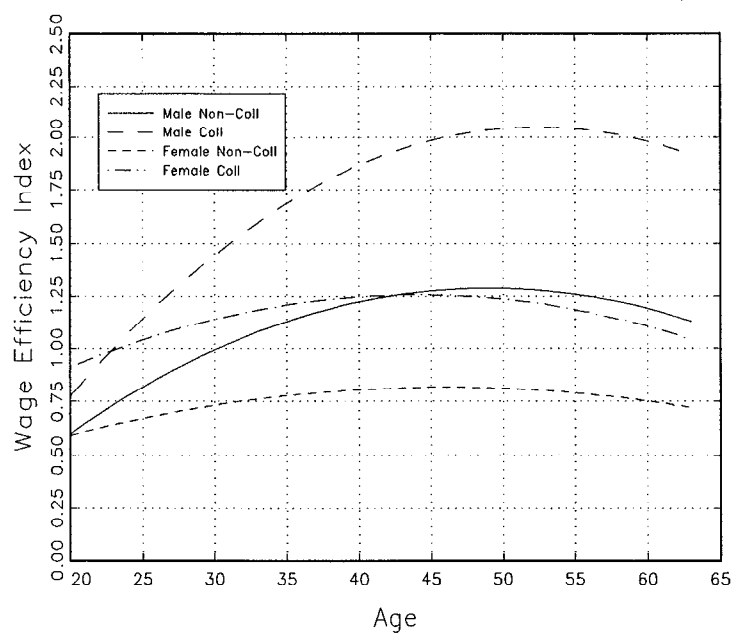


Figure 1: Wage Efficiency Index: Whites

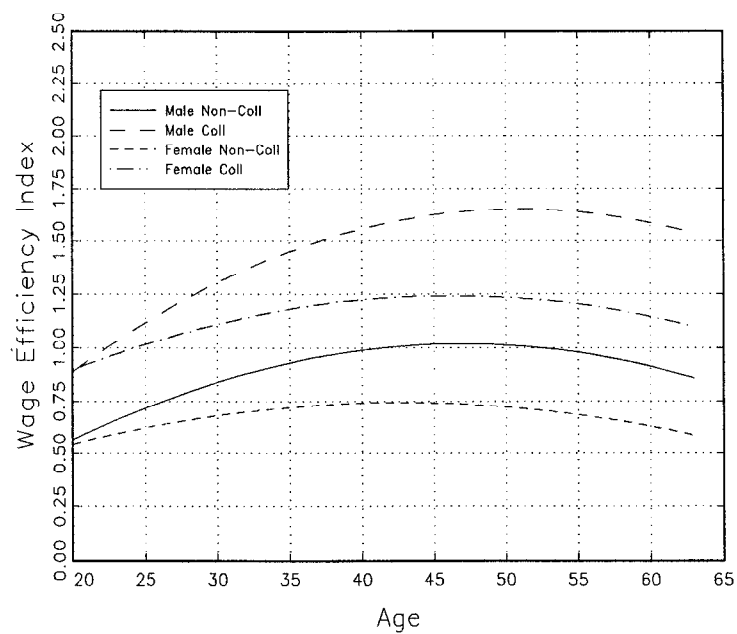


Figure 2: Wage Efficiency Index: Nonwhites

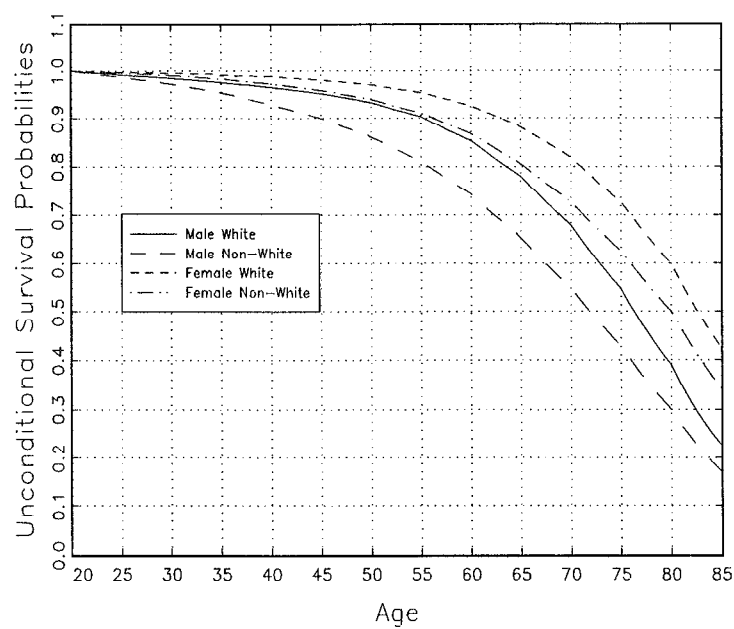


Figure 3: Unconditional Survival Probabilities: College Educated

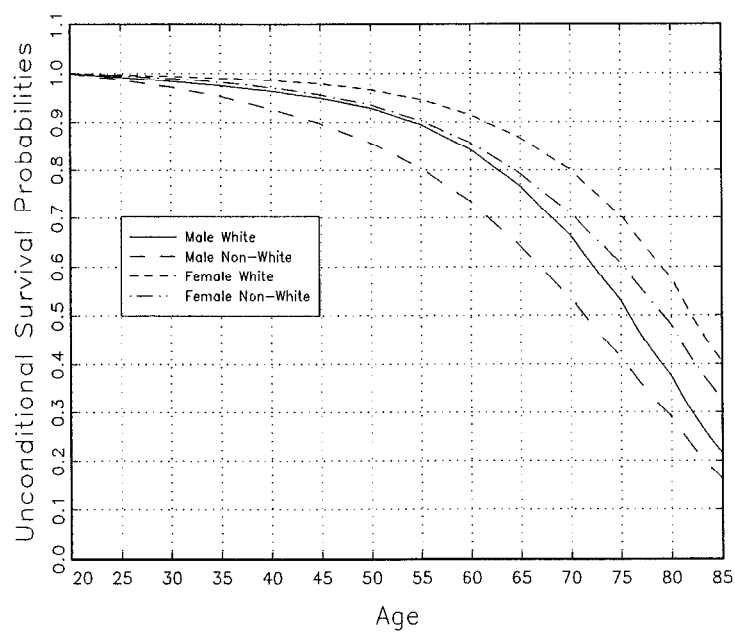


Figure 4: Unconditional Survival Probabilities: Noncollege Educated

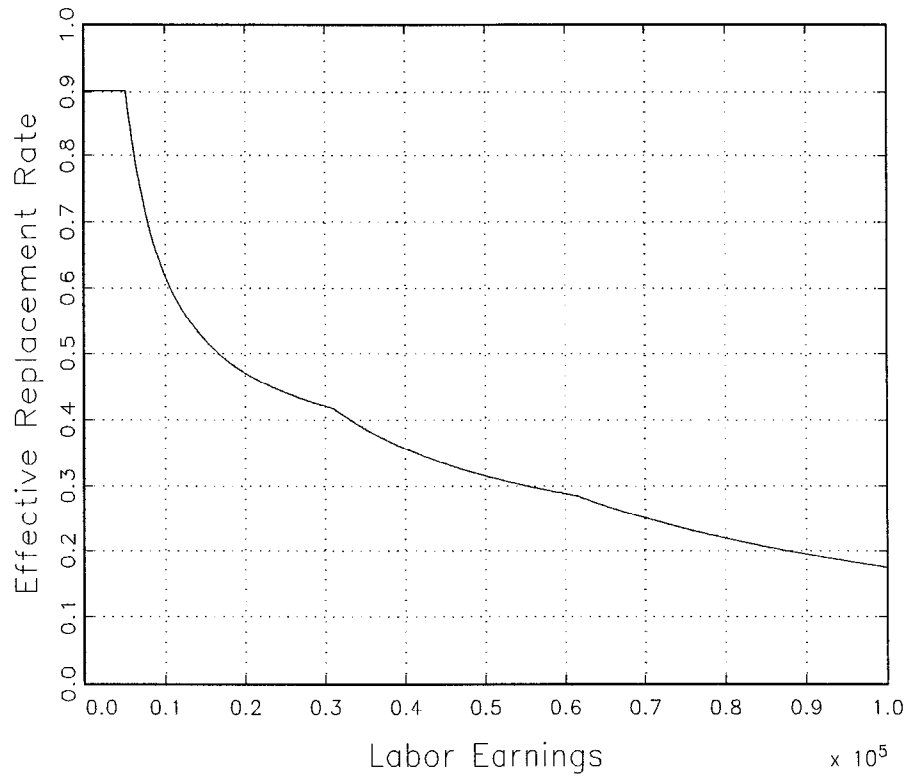


Figure 5: Effective Replacement Rate by Income

Social Security Handbook: 1993

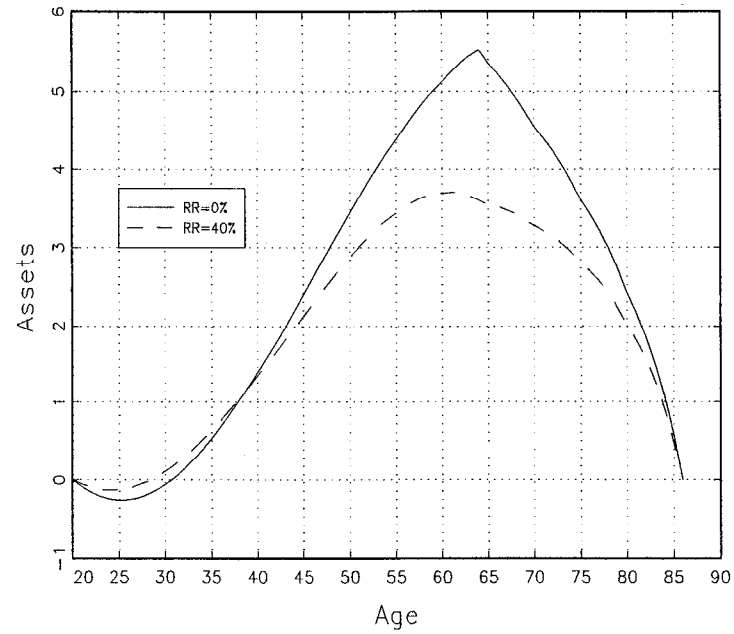


Figure 6: Assets Age Profile and Social Security Policy

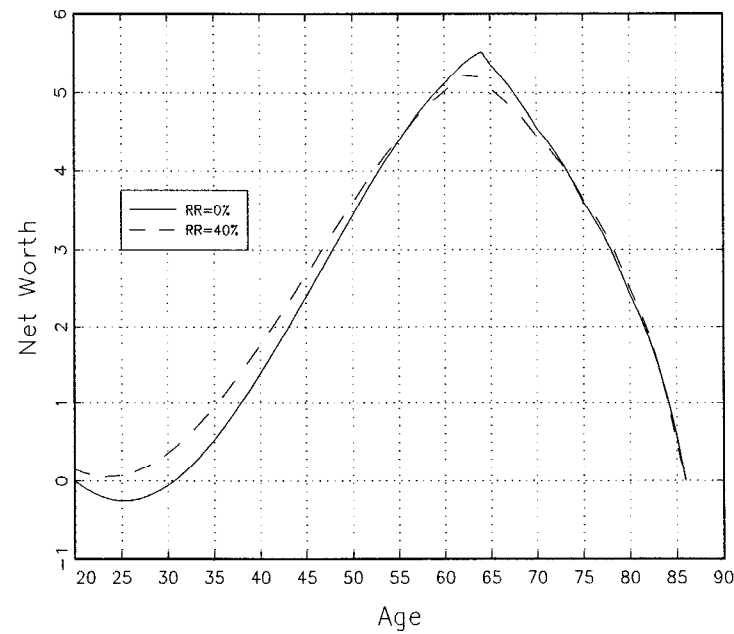


Figure 7: Net Worth Age Profile and Social Security Policy

(Male, White, Noncollege)



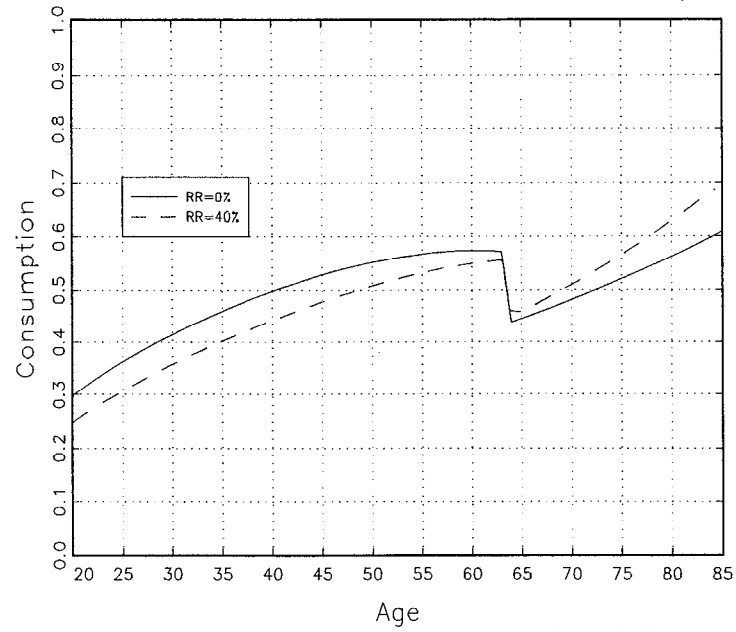


Figure 8: Consumption Age Profile and Social Security Policy

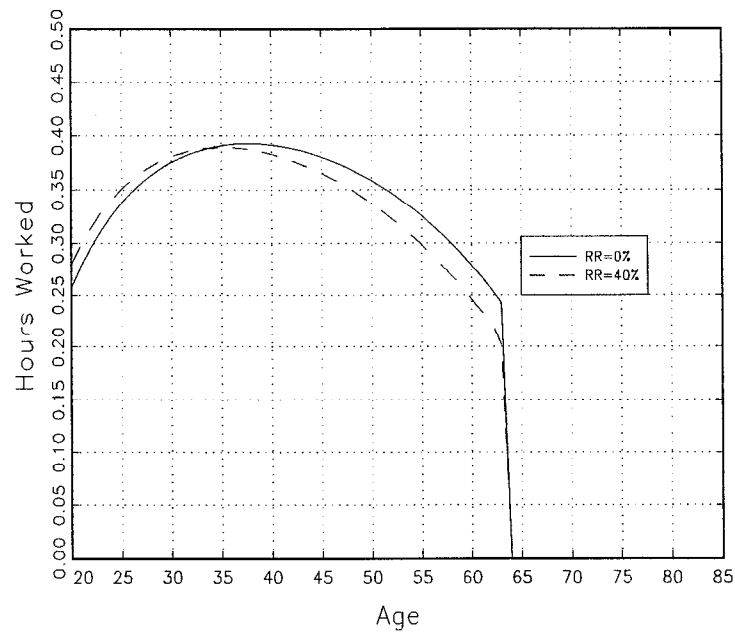


Figure 9: Hours Worked Age Profile and Social Security Policy

(Male, White, Noncollege)

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