

WP/98/18

INTERNATIONAL MONETARY FUND

Research Department

**Public Debt Indexation and Denomination: The Case of Brazil<sup>1</sup>**

Prepared by Ilan Goldfajn

Authorized for distribution by Donald J. Mathieson

February 1998

**Abstract**

The paper models the optimal debt management strategy of the public sector when issuing nominal, price-level-indexed and foreign-denominated debt securities. The model predicts that the variance of inflation, the size of the public debt, the variance of the real exchange rate, and the correlation of inflation with public expenditures are the main determinants of public debt management. Using this framework, the paper analyzes the Brazilian experience with indexed debt in the last decade. In particular, it explains the large increase of indexed public debt in Brazil prior to the 1994 Real plan and, thereafter, the steady decline in its use.

JEL Classification Numbers: E58, H63

Keywords: Public debt, indexation, Brazil

E-mail: [igoldfajn@imf.org](mailto:igoldfajn@imf.org).

---

<sup>1</sup>This paper was prepared for the Central Bank of Chile's conference on "Indexation, Inflation and Monetary Policy" on August 11-12, 1997. I would like to thank Trevor Alleyne, Márcio Garcia, Fernando Lefort, Klaus Schmidt-Hebbel and, Andrés Velasco for very useful comments. Also, thanks to Ugo Fasano, Patrícia Menandro (Andima), Mário Mesquita, Áureo de Paula, and Evan Tanner for kindly providing me with the data. All errors are my own.

**CONTENTS**

Summary .....	3
I. Introduction .....	4
II. Indexation in Brazil .....	5
III. Model .....	7
A. Commitment Solution .....	10
B. Absence of Commitment .....	11
C. Relevant Assumptions and Extensions .....	14
IV. Empirical Analysis .....	15
A. Data .....	15
B. First Stage: Obtaining the Variance and Covariance Series .....	15
C. Second Stage: OLS Regressions .....	16
B. Results .....	17
V. Conclusions .....	22
Bibliography .....	23
Tables	
1. Determinants of Debt Indexation in Brazil 1980-1995 .....	18
2. Determinants of Debt Maturity in Brazil 1986-1995 .....	19
3. Determinants of Foreign Denominated Debt in Brazil 1980-1995 .....	20
4. Determinants of Debt Composition .....	21
Figures	
1. Inflation and Stabilization Plans .....	25
2. Nominal Debt in Brazil .....	26
3. Real Value of Government Index Bond .....	27
4. Composition of Public Debt .....	28

### SUMMARY

Since the Real plan of 1994, the share of public indexed debt in Brazil has dropped from 70 percent to 30 percent of total debt while both nominal and foreign-denominated debt shares have increased. This paper offers a simple model and preliminary evidence to explain these facts.

To illustrate the main government incentives for issuing different public debt securities, this paper derives a simplified model of debt indexation and denomination. It models a government that wants to minimize the fluctuation of the government budget but also wants to minimize inflation. The smoothing of the budget allows for smaller changes in the tax rate, which given convex distortion costs imply higher utility for a representative consumer.

The main trade-off can be summarized as follows. On the one hand, by issuing indexed securities the government minimizes fluctuations of the real value of its debt resulting from inflation variability and avoids the temptation to inflate away the debt that, in equilibrium, leads to costly higher inflation. On the other hand, if expenditure shocks are positively correlated to inflation shocks, nominal securities serve as implicit contingent debt, reducing the value of debt when the reduction is more needed.

In addition the paper investigates when it is preferable to issue foreign-denominated securities. The latter should be preferred when the real exchange rate does not fluctuate substantially and the correlation between the real exchange rate shocks and government expenditures is negative.

The evidence from ordinary least-squares (OLS) regressions confirms that the variance of inflation, the size of the public debt, and the correlations of inflation with spending are important determinants of public debt indexation in Brazil. However, neither credibility nor the hedging motive is able to explain the proportion issued of foreign-denominated debt.

## I. INTRODUCTION

Since the Real plan of 1994, the Brazilian economy has been in the process of reducing its degree of indexation. For more than three decades, Brazilian wages, rents, financial securities and other contracts were indexed to the price level. At times the frequency of adjustment reached monthly intervals and even daily, as in the case of some financial securities.

An inseparable feature of Brazilian indexation was the association with high rates of inflation. The latter averaged 20 percent per month during the last 3 decades and reached its peak of 82 percent per month during the hyperinflation (or “megainflation”) process just before the Collor Plan (see Figure 1). The association between inflation and indexation has always provoked the debate about whether high and variable inflation induces the indexation of the economy through the breakdown of nominal contracts or, in the opposite direction, indexation perpetuates the inflationary process as argued by the inertial theory of inflation.<sup>1</sup>

Brazilian indexation has relied mainly on price level indexation as opposed to exchange rate indexation, as is common in several other inflationary experiences. This peculiar feature has often been justified by the institutionality of Brazilian indexation mechanisms that dated back to 1964 and concentrated on price level indexation, well before the high inflation process of the late 1980’s and early 1990’s.

This paper investigates Brazilian indexation in the financial sector in the last few decades. In particular, the paper focuses on indexation of the public debt. Government indexed securities were not only a sizable portion of total securities but also took a central role (as reference values and even unit of account) in spreading indexation mechanisms to other contracts in the economy like wages, rents, taxes and other private financial contracts. There are several examples of important government indexed securities like the Obrigacoes Reajustaveis do Tesouro Nacional (ORTN and later OTN) from 1964-1989 or the Bonus do Tesouro Nacional (BTN) from 1989-1991.

The paper will examine the composition of public debt between price-level indexed, nominal and foreign denominated securities. It argues that the volatility of inflation, the level of total debt, the volatility of the real exchange rate and the correlation between expenditures and inflation are important determinants of the composition of public debt.

---

<sup>1</sup>For this debate and the inertial inflation theory, see Arida and Resende (1985) and Simonsen(1983 ). Also see Fischer and Summers (1989) who show how better protection may end up causing more inflation.

In order to illustrate the main government incentives in issuing different public debt securities, this paper derives a simplified model of debt indexation and denomination. It models a government that wants to minimize the fluctuation of the government budget but also wants to minimize inflation. The smoothing of the budget allows for smaller changes in the tax rate, which given convex distortion costs, imply higher utility for a representative consumer.

The main trade-off can be summarized as follows. On the one hand, by issuing indexed securities the government minimizes fluctuations of the real value of its debt from inflation variability and avoids the temptation of inflating away the debt that, in equilibrium, leads to costly higher inflation. On the other hand, if expenditure shocks are positively correlated to inflation shocks, nominal securities serve as implicit contingent debt, reducing the value of debt when it is more needed.

In addition the paper investigates when it is preferable to issue foreign denominated securities. The latter should be preferred when the real exchange rate does not fluctuate substantially and the correlation between the real exchange rate shocks and government expenditures is negative.

The paper is organized as follows. The next section summarizes Brazilian experience with indexation. Section 3 presents the model and section 4 looks at the composition of public debt in Brazil. Finally, section 5 concludes.

## II. INDEXATION IN BRAZIL

Most of Brazilian institutionalization of indexation dates back to 1964 during the first military government of President Castelo Branco.<sup>2</sup> Before that two laws inhibited the emergence of automatic adjustments of contracts. The first, the 1933 law passed by President Vargas simply prohibited contracts from being stipulated in currencies other than the Mil Reis, the Brazilian currency at that time. Besides being a nationalistic reaction in favour of the Brazilian sovereign monetary unit, the law attempted to curb established practice of foreign companies in Brazil to index their prices to the exchange rate in order to neutralize the effect of the devaluation of Mil Reis in their receipts measured in foreign currency. From an economic point of view, the law was based on the fact that exchange rate indexation impaired significantly the correction of balance of payment imbalances.

The second law, the "Lei da Usura" simply established a nominal interest rate ceiling of 12 percent. Since inflation most of the time exceeded this ceiling implying

---

<sup>2</sup>This section draws on Simonsen (1995).

negative real rates, several mechanisms were established to bypass the law. However, long term contracts were nonexistent since they required explicit indexation mechanisms to deal with uncertainty with respect to future inflation rates.

In 1964, the military government decided to reduce the distortions in the economy resulting from the two laws above and introduced the “correção monetária”, a sophisticated indexation mechanism that adjusted taxes, tariffs and some financial contracts but not the exchange rate, wages and bank deposits. The government wanted to avoid the negative effects the absence of indexation had on public receipts and in some financial contracts. The absence of indexation on the exchange rate and wages was justified to allow corrections of external imbalances and to avoid inflationary consequences of wage indexation.

In addition, the government issued the Obrigações Reajustáveis do tesouro Nacional (ORTN). These were public debt securities with maturities from 1 to 20 years that were adjusted automatically for past inflation plus interest rates of 6 percent per year. The objective was to avoid financing the deficit through inflationary monetary expansion. The government believed that indexing the debt would increase the demand for long term securities with fixed real rates and also allow for lower debt service since it expected inflation to decline below market expectations embedded in nominal interest rates paid on nominal securities.

The ORTN rapidly became an important reference value for other contracts. Starting in 1964, mortgage contracts were adjusted by the quarterly variation of the ORTN. This adjustment became known as the UPC (the unit of reference of capital).

Beginning in 1967, during the Costa e Silva government, indexation spread to almost all areas of the economy. Wages could no longer be denied automatic indexation. The exchange rate policy followed a Purchasing Power Parity rule and rents were adjusted every 6 months. In short, the Brazilian economy became highly indexed.

In the next two governments, from 1974 to 1985, both indexation and inflation continued to increase. The Mexican default and the consequent crisis led to higher inflation and renewed efforts to implement restrictive monetary and fiscal policies. With the economy highly indexed to past inflation, all attempts to reduce inflation through traditional policy instruments proved inappropriate.

The inflation process was thought to be inertial and its reduction to require measures that eliminate widespread indexation in the economy. In March 1986, the Cruzado plan, the first of a long series of stabilization plans was implemented. This first attempt was unsuccessful as were the following 4 plans (Bresser, Verão, Collor I and Collor II) as its evident from Figure 1. The widespread indexation of contracts

persisted and the inflation continued to increase.

With high levels of uncertainty with respect to inflation nominal contracts could not survive. During the megainflation process before the Collor plan, the proportion of nominal public debt reached zero. Figure 2 shows the proportion of nominal debt in the hands of the public.

The Real plan of July 1994 succeeded in reducing inflation. It was an ingenious scheme of changing numeraires. In March 1994 nominal prices, wages and other contracts were allowed to be quoted in a unified reference value (URV) that would be replaced by a new currency, the Real, on July, 1994. Since prices were already indexed to several different references, the innovation of the URV was to coordinate a unified unit of account that would substitute for all other indexation mechanism. In the interim period after the introduction of the URV and before its replacement by the real, it was expected that relative prices would converge to their equilibrium value. This was important to the second phase of the conversion, when the URV would be transformed into Real on a one-to-one basis and then pegged to the dollar. This pegging, in fact, caused inflation to plunge from 46 percent in June 1994 to 1.5 percent in September 1994.

With lower inflation and deliberate government legislation to *deindex* the economy, indexation of short term contracts was reduced substantially. In particular, the proportion of indexed public debt diminished substantially. Figure 4 shows the composition of public debt after the Real plan. The most striking fact is that the proportion of nominal debt has been increasing substantially as one should expect from the increasingly stable inflation environment.

### III. MODEL

This section focuses on the trade-off described in the introduction. The government decision to manage its public debt depends on the trade-off between time consistency problems versus tax-smoothing motives. The former generates a higher inflation than optimal which requires, in order to credibly commit to low inflation, a structure based on indexed securities (to either foreign or domestic inflation). Tax smoothing calls for issuing any security whose returns are negatively correlated with the tax needs of the government. This may be the case of both nominal and foreign currency linked debt.

An optimal composition of debt between nominal, indexed and foreign denominated liabilities is derived in a two period example. The objective is to highlight the effects of both tax smoothing and time consistency considerations on the optimal

composition in the simplest possible framework.<sup>3</sup>

The objective of the government is to minimize distortions from taxes ( $\tau$ ) and inflation ( $\pi$ ), both assumed to be quadratic. There are three sources of uncertainty: government spending, real exchange rate and money demand are assumed to be stochastic. The latter introduces uncertainty with respect to the rate of inflation,

$$\text{Min} E \left[ A \frac{\tau^2}{2} + \frac{\pi^2}{2} \right]. \quad (1)$$

In the first period, the government will choose the composition of the debt that it sells to the public and that will mature at the end of period two. There are three financial instruments available: (i) nominal bonds that are sold at a nominal interest rate  $i$  and have a realized rate of return of  $\frac{(1+i)}{(1+\pi)}$  (ii) indexed bonds that pay a real rate of  $r$  plus the realized rate of inflation  $\pi$  and (iii) foreign denominated bond that pay the foreign interest rate  $i^*$  plus the rate of depreciation  $e$  and have a realized real rate of return in domestic currency equal to  $\frac{(1+i^*)(1+e)}{(1+\pi)}$ .

There are no shocks or other financing decisions in the first period. In the second period, given the level of debt and its composition, the government decides about the optimal money growth. After that, the shocks are realized and taxes are levied to balance the budget.<sup>4</sup>

Consumers are in the background, they enter the model as risk neutral debt holders and will demand a nominal interest rate which includes a fixed real interest rate ( $r$ ) plus the expected rate of inflation ( $\pi^e$ ).<sup>5</sup>

$$1 + i = (1 + \pi^e)(1 + r). \quad (2)$$

The assumption of risk neutrality of the investors is not restrictive and is done for simplicity. If I had assumed risk averse investors, they would have demanded an extra premium to hold nominal debt but the main results would still hold.<sup>6</sup>

---

<sup>3</sup>A multiperiod model is developed in Goldfajn (1996). For a related model of public debt indexation see Calvo and Guidotti (1990).

<sup>4</sup>The budget has to balance since it is the last period. In the multiperiod model, taxes and money growth are decided simultaneously and the amount of debt financing adjusts for the shocks.

<sup>5</sup>It is assumed that the real interest rate  $r$  is equivalent to the one demanded on indexed bonds. Even with risk neutrality, equation 2 is an approximation.

<sup>6</sup>The results depend on the fact that the government wants to minimize the fluctuations on the real value

The paper assumes uncovered interest rate parity:

$$1 + i = (1 + e^e)(1 + i^*). \quad (3)$$

The money market equation determines the equilibrium inflation rate:

$$\pi = \hat{m} + \tilde{v}, \quad (4)$$

where,  $\hat{m}$  is money supply growth created by the government and  $\tilde{v}$  are velocity shocks that are assumed to be white noise. Here the paper assumes that the government sets the money supply and lets the nominal exchange rate be determined endogenously.<sup>7</sup>

In equilibrium, rational investors will anticipate the money supply growth decision of the government. In the model, this implies that investors anticipate the average inflation generated by the government:

$$\pi^e = E[\pi] = \hat{m}. \quad (5)$$

The budget constraint of the government in period 2 is that taxes have to be equal to spending plus the real value of debt.<sup>8</sup> Using equations 2 and 3 and linearizing the budget constraint I obtain:

$$\tilde{\tau} = \tilde{G} + (1 + r)B[1 - \theta(\tilde{\pi} - \pi^e) - \theta^*(\tilde{q} - q^e)], \quad (6)$$

where  $\theta$  and  $\theta^*$  are the proportions of nominal and foreign denominated debt, respectively,  $B$  the level of total debt, and

$$\tilde{q} = \tilde{\pi} - \tilde{\epsilon}. \quad (7)$$

Unexpected increases in inflation reduce nominal debt and unexpected real appreciations reduce foreign denominated debt (assume for simplicity that foreign inflation is zero).

---

of its debt since they generate costly changes in the tax rates. The degree of risk aversion of the investors does not change this fact.

<sup>7</sup>An alternative and equivalent assumption will be to allow the government to set the rate of depreciation and let money be determined endogenously.

<sup>8</sup>For simplicity revenues from cash balance holdings are ignored.

### A. Commitment Solution

First the commitment solution will be derived. It is assumed that government at 1 can credibly commit the decisions taken by government at 2. Therefore, in period 1 the government will choose both the composition,  $\theta$  and  $\theta^*$ , and the way government 2 will finance itself between taxes and money growth. Minimizing 1 subject to 5 and 6, I obtain the following solution:

$$\hat{m}^* = 0, \quad (8)$$

and the optimal proportions are:

$$\theta = \frac{\sigma_{g\pi}\sigma_q^2 - \sigma_{gq}\sigma_{\pi q}}{B[\sigma_{\pi}^2\sigma_q^2 + (\sigma_{\pi q})^2]} \quad (9)$$

$$\theta^* = \frac{\sigma_{gq}}{B\sigma_q^2} - \frac{\sigma_{\pi q}\theta}{\sigma_q^2}, \quad (10)$$

where  $\sigma$  denotes variance or covariance of the subscripts. It is assumed, for simplicity, that  $r = 0$ .

The optimal money supply growth is zero because there are no benefits from announcing a higher money path since nominal interest rates will increase proportionally to higher inflation rates. Moreover, higher inflation rates imply higher distortion costs, so that governments will optimally commit to a zero inflation path.

The optimal proportion of the debt in nominal terms increases with the covariance of inflation with spending but diminishes with the variance of inflation. The intuition is that shocks to the other components of the budget should be optimally hedged. On one hand, nominal debt is a good hedging device for the government whenever shocks to spending (or any other component of the primary deficit) are positively correlated to inflation shocks and, therefore, negatively correlated to the debt value. This avoids having to raise taxes in bad states of the world.

On the other hand, pure inflation variance only introduces noise to the budget and induces the use of more indexed debt. This will be the case in countries that face large nominal shocks relative to real movements. The real value of their debt would fluctuate without a corresponding change in other budgetary components. This induces the use of a high proportion of indexed debt. In the empirical section of the paper, we will see that this is precisely the case of Brazil.

It is interesting to note the effect of having liabilities in foreign currency. They introduce another instrument for hedging, provided the correlation between the real exchange rate and other domestic budget components (in this case spending) are not zero.

The trade-off with foreign denominated liabilities is that they may introduce more noise to the budget. If the value of foreign currency used to peg part of the debt is not very stable, i.e. , with a high variance of the real exchange rate, taxes will fluctuate more to compensate the movements in the real value of foreign denominated debt.

It is interesting to note that the optimal proportions of nominal and foreign denominated debt derived above imply a specific relationship between them. A larger share of nominal debt is associated with a larger share of foreign denominated debt depending on whether the covariance between inflation and the real exchange rate is positive or negative. If the returns covary negatively, both bonds are complements in the governments portfolio and they should increase proportionally.

### **B. Absence of Commitment**

The assumption of commitment is relaxed.

If the government cannot commit its future behavior, it will face time consistency problems, arising from the fact that it controls the average inflation rate. In period 2, it will be optimal to inflate some of the existing nominal debt. Ex-ante, rational investors will anticipate the future temptation to inflate, adjust expected inflation and demand a higher nominal interest rate. In this equilibrium the inflation rate will be higher than in the previous equilibrium where the government could commit itself.<sup>9</sup>

It is interesting to calculate the effect of future incentives to inflate on the debt composition chosen in period 1.

Solving backwards, government in period two minimizes taxes and inflation costs, but takes as given the level of debt  $B$ , its composition  $\theta$  and  $\theta^*$  and the nominal interest rate  $i$ . Minimizing 1 subject to 5 and 6 in period 2 I obtain the following solution:

$$m^* = AB\theta\Omega, \quad (11)$$

---

<sup>9</sup>The government here cannot systematically affect the real exchange rate,  $q$ .

and

$$\Omega = (\bar{G} + B), \quad (12)$$

where  $\bar{G}$  is expected spending. The first order condition 11 above uses equation 5. It says that the government at 2 equates the expected marginal costs of raising taxes and inflation. The equilibrium values of taxes and inflation, after shocks are realized are:

$$\tilde{\tau} = \tilde{G} + B[1 - \theta\hat{\pi} - \theta^*\hat{q}] \quad (13)$$

$$\pi = m^* + \tilde{v}, \quad (14)$$

where  $\hat{\pi}$  and  $\hat{q}$  are the unexpected shocks to inflation and the real exchange rate.

In comparison to the previous case, the absence of commitment increases the average rate of inflation, creating expected distortion costs. The incentive to inflate and the resulting equilibrium inflation depend on the proportion of nominal debt outstanding,  $\theta$ . Therefore, the government in period one would need to adjust its optimal  $\theta$  to reduce the incentive to inflate. Thus, it will deviate from the optimal proportion of nominal debt for hedging purposes. In other words, the absence of other commitment technologies forces the government to use indexation to satisfy two objectives, hedging and commitment, leaving it worse off.

The government in period one minimizes the loss function in period two choosing the appropriate indexation of debt. For this purpose, it is possible to explicitly derive the loss function that the government faces in period one (dropping irrelevant terms and using the values from equations (11) to (14):

$$Loss = A^2 B^2 \theta^2 \Omega^2 - 2\theta B \sigma_{g\pi} - 2\theta^* B \sigma_{gq} + B^2 (\theta^2 \sigma_{\pi}^2 + \theta^{*2} \sigma_q^2 + 2\theta\theta^* \sigma_{\pi q}). \quad (15)$$

The first term in the right hand side of the equation is the price for not being able to commit to not inflate the nominal debt. It is the traditional inflationary bias of time consistent solutions. Indexing the debt is one of the commitment technologies. The government will now take into account this term when choosing its optimal proportion of nominal debt. It is clear from here that setting  $\theta = 0$  will completely avoid this type of cost. The second and third term are the hedging role of nominal debt and foreign denominated debt. If inflation (real exchange rate) and government spending are positively correlated, tax rates will fluctuate less and governments can reduce distortions in the economy. The point is that it is optimal to have contingent debt that reduces the real value of debt when financing needs are higher. Nominal and foreign denominated debt are implicitly contingent debt. The last term in the right works against issuing both

nominal and foreign debt. If the variance of inflation or real exchange rate are high it will imply large changes in real value of debt which will increase the variance of tax rates. Setting  $\theta = 0$  and  $\theta^* = 0$ , completely indexing the debt, eliminates this last effect.

Government in period one minimizes the loss function and obtains the optimal proportion of debt in nominal and foreign currency:

$$\theta = \frac{\sigma_{g\pi}\sigma_q^2 - \sigma_{gq}\sigma_{\pi q}}{B[(A^2\Omega^2 + \sigma_\pi^2)\sigma_q^2 + \sigma_{\pi q}^2]} \quad (16)$$

$$\theta^* = \frac{\sigma_{gq}}{B\sigma_q^2} - \frac{\sigma_{\pi q}\theta}{\sigma_q^2}. \quad (17)$$

This optimal value has an additional term  $\Omega^2 A$  when compared to the commitment case. The government now has to take into account the marginal cost, in terms of higher average inflation, of not indexing one extra unit of debt. This additional cost will reduce the optimal proportion of nominal debt.

The higher the term  $\Omega^2 A$  the more tempted the government appears to want to inflate the debt. Thus, to credibly commit itself to not inflate, the government will use a higher proportion of indexed debt. This will reduce the amount of nominal debt available for hedging purposes, deviating more from the commitment solution.

It is interesting to note that the  $\Omega^2 A$  term depends on both the level of total debt and of the average spending. If the total debt,  $B$ , is larger, the more tempted the government is to inflate the debt. Therefore, the higher the debt, the lower the optimal proportion of nominal debt. This effect is similar to the one derived in Blanchard and Missale (1994).

The effect of the level of debt on foreign denominated debt is more ambiguous. First, there is the direct effect of the level of debt on the composition through the hedging purposes. For given variances and covariances and the level of shocks to spending, the higher the total debt the lower the proportion of foreign denominated debt needed to obtain the same amount of hedging. Second, the impact of the term  $\Omega^2 A$  (through  $\theta$  on equation (17)) depends whether the covariance between inflation and real exchange rate is positive or negative. The latter will also define if foreign denominated debt and nominal debt are complementaries or substitutes in the governments portfolio.

I could restrict the values of  $\theta$  and  $\theta^*$  to be between 0 and 1, which means that there is no "negative debt" in any component of the debt, i.e, governments do not hold net positive claims with the private sector. In this case corner solutions may arise.

### C. Relevant Assumptions and Extensions

Before I proceed to the empirical analysis, it is relevant to discuss a few issues on debt management that the model has not stressed. First, in reality, governments claim that they manage debt to “minimize the borrowing cost”. However, if markets work efficiently and there is no free lunch, any gains from shifting to cheaper securities should imply higher risks to the government.<sup>10</sup> Since higher risks to the government imply, ultimately, higher risks to the society (for example with a higher probability of raising taxes to close the budget), it is not clear that there are any gains from this strategy. In fact, taking into account both debt holders and taxpayers, the literature on debt management has showed that in the absence of tax or inflation distortions debt management is irrelevant (see Agell et al. (1992)). In the model, I have simplified the calculations by omitting the objective of “minimizing the borrowing cost” and assuming risk neutral investors and equivalent real returns for all the securities. The risk neutrality assumption becomes relevant when distribution issues are involved - for example, when the government wants to distribute income from debt holders to taxpayers.

Second, in principle governments can smooth shocks intertemporally by raising debt in bad moments and repaying debt in good times. In this case, the nature of the shock is irrelevant and debt composition is indeterminate. However, in a previous paper, I have shown that even if governments smooth taxes through time there is a role to smooth taxes through “states of nature,” the hedging argument described above (see Goldfajn (1996)). The relative importance of the hedging argument is an empirical matter. The following section evaluates the importance of hedging and credibility arguments in the case of Brazil.

---

<sup>10</sup>For example, when the Mexican central bank shifted from domestic denominated CETES to dollar denominated Tesobonos and implicitly assumed the exchange rate risk.

#### IV. EMPIRICAL ANALYSIS

The implications of the model can be summarized as follows:

- The proportion of nominal debt should decrease with the level of total debt.
- Nominal debt should be negatively related to the variance of inflation.
- Nominal debt should increase when the covariance of inflation and spending is larger.
- Foreign denominated debt should increase when the covariance between the real exchange rate and spending is larger.
- Foreign denominated debt should be negatively related to the variance of the real exchange rate.

The purpose of this section is to test the main determinants of public debt indexation and denomination. The analysis will focus on the joint behavior of the macroeconomic series that determine the budget constraint of the government (government spending and inflation), the level of the debt and its composition for Brazil.

##### A. Data

The paper uses monthly public debt data obtained directly from the Central Bank of Brazil and from the historical series published in Andima (1993) from 1980 to 1997. The data uses disaggregated data on the composition of public debt between indexed and nominal debt held by the public (net of the holdings of the Central Bank).

In addition, the paper uses monthly data on real exchange rate, inflation and spending. Inflation is measured by the wholesale price index changes and spending is the total disbursement of the government net of interest rate payments. The original source of the series is the Central Bank of Brazil. The series of real exchange rate is multilateral based on wholesale price index and was calculated in Goldfajn and Valdés (1996).

##### B. First Stage: Obtaining the Variance and Covariance Series

The paper first obtains the 4 variance and covariance series for Brazil: (i) the variance of inflation (ii) the covariance of inflation with government spending (iii) the

variance of the real exchange rate and (iv) the covariance of the real exchange rate with spending.

The variance-covariance structure of innovations in government spending, real exchange rate and inflation was obtained for Brazil by performing a VAR of these variables. The variances were obtained from the residuals of the VAR. In the same manner, the covariances were obtained from the off diagonal terms covariance matrix of the residuals.

The series of variance/covariance were estimated with a series of rolling VAR estimations. For each year a new VAR was estimated with a different data set available to estimate the covariance matrix. For each estimation, the last 40 observations were used and two lags used. In this way a time series of estimates of the 4 relevant variance/covariances were obtained. The series start in January 1980 and end in December 1995.

There are three series of correlations that are relevant to discuss. First, the correlation between spending and real exchange rate innovations in Brazil tends to be positive in most of the sample giving a hedging role for the foreign denominated debt. This is an expected result since more spending increases demand for nontradables which appreciates the currency. Second, the correlation between spending and inflation is positive at the beginning of the sample and at the very end (1980-1985 and 1994-96) but becomes negative at the end of the 1980's when inflation reached hiperinflationary levels. The reason is that for low and medium levels of inflation, government spending created pressures on prices or, in the reverse causality, increases in prices created more spending for the government. In this case, there was a role for nominal debt as hedging device. However, when inflation rates became very high, shocks to inflation tended to reduce government spending since wages and payments in the public sector were not perfectly indexed to the price level. Finally, the correlation between inflation and real exchange rate is positive in the whole sample. With less than perfect flexible prices or exchange rates, positive shocks to inflation did not always translate into proportional shocks to the nominal devaluation rates leading to real appreciations of the currency. This last fact makes nominal and foreign denominated debt be substitute in the government portfolio.

### **C. Second Stage: OLS Regressions**

The second stage uses the estimated variance/covariances obtained in stage 1 to estimate their influence in the choice of  $\theta$  and  $\theta^*$ , in an OLS regression. The period of estimation is 1980-1995 and the frequency is monthly.

The proportion of nominal debt is regressed against the variance of inflation, the covariance of spending and inflation, the level of public debt as a proportion of GDP, a constant, a time trend and dummy variables to control for both the Collor and Real plan.<sup>11</sup> The latter are included because both plans had important structural breaks with potential important influence in the composition of public debt. The Collor plan of March 1990 froze a good part of the financial securities available, including public debt securities. The Real plan represented a structural break from previous inflationary environment and, therefore, with potentially important effects on the composition of public debt.

Indexed debt in Brazil was not a guaranteed positive real return to its holders. Figure 3 shows the real value of an indexed bond from 1976 to 1991. The picture makes clear that indexation in Brazilian indexed securities was far from perfect. The indexation mechanism lagged behind actual inflation and did not protect the real value of this type of debt. An interesting way to test the effect of imperfect indexation is to observe the behavior of the average maturity of all public debt, including indexed debt. If indexation is imperfect, agents will require shorter maturities, even in indexed debt, to avoid large fluctuations in the real value of debt. Therefore, I can repeat the OLS regressions above using the average maturity as the dependent variable since the predictions of the model can be equally applied to the average maturity of debt. The effects of the variance of inflation, the level of public debt and the covariance of inflation and spending on the real value of debt should be correlated to the average maturity of debt.

Finally, the proportion of foreign denominated debt is regressed against the variance of the real exchange rate, the covariance of spending and the real exchange rate, the level of public debt as a proportion of GDP, a constant, a time trend and dummy variables to control for both the Collor and Real plan.

#### **D. Results**

The results of the regression are presented in Tables 1-3. The expected signs are presented in the last row.

In Table 1, the level of debt seems to have a significant effect and in the direction predicted by the model. A higher level of debt reduces the long nominal proportion of debt. This supports the model since having a higher total debt affects the incentives to inflate and influences the decision of having nominal debt.

---

<sup>11</sup>Neto (1996) runs similar regression to test the model in Goldfajn (1996). However his specification and results differ somewhat from the ones obtained below.

The variance of inflation has a negative effect on the proportion of nominal debt, as inflation uncertainty tend to reduce maturities. As predicted by the model, the higher variance of inflation should reduce the amount of nominal debt. This phenomenon was an important feature of the Brazilian economy where the deepening of financial indexation was positively correlated to the uncertainties brought with high inflation rates.

The covariance with spending also fit the predictions of the model. The coefficient on the covariance of inflation/spending have the correct sign. It is positive and significant at the 5 percent level. It confirms the prediction of the model that the higher this covariance, the higher the proportion of nominal debt.

In addition, the results show that both the Collor and Real plans increased the proportion of nominal public debt. The former through a compulsory freezing of financial assets while the latter through a structural change in the inflationary regime.

In Table 2, the regressions with the average maturity of public debt are presented. The coefficients on debt-GDP ratio, variance of inflation, the covariance of inflation with spending and the Collor plan have the same signs as predicted by the model and are equivalent to the ones obtained in the regressions presented in Table 1.

Table 1 - Determinants of Debt Indexation in Brazil - 1980-1995  
 Dependent Variable is the Proportion of Nominal Debt  
 (t-statistics in parentheses)

	$D/Y$	Trend	$\sigma_{\pi}$	Collor I	Real	$\sigma_{\pi g}$	$\bar{R}^2$
1	-0.05 (-3.8)	0.26 (4.93)	-0.14 (-2.38)				.30
2	-0.06 (-3.86)	0.27 (5.01)	-0.21 (-3.39)	20.3 (1.7)			.34
3	-0.07 (-4.93)	0.24 (3.48)	-0.20 (-3.09)	20.1 (1.7)	19.1 (2.85)		.40
4	-0.06 (-4.78)	0.23 (3.41)	-0.11 (-1.54)	18.5 (1.56)	20.85 (2.9)	1.04 (2.64)	.44
	-		-	+	+	+	

The last row contains the expected signs predicted by the model

A constant was included in each regression

T-statistics were corrected by Newey-West consistent matrix

Table 2 - Determinants of Debt Maturity in Brazil - 1986-1995  
 Dependent Variable is the Average Maturity  
 (t-statistics in parentheses)

	$D/Y$	Trend	$\sigma_{\pi}$	Collor I	Real	$\sigma_{\pi g}$	$\bar{R}^2$
1	-0.03 (-6.6)	0.12 (6.99)	-0.07 (-5.02)				.62
2	-0.03 (-6.9)	0.12 (7.14)	-0.09 (-5.73)	5.43 (3.79)			.65
3	-0.03 (-5.83)	0.13 (4.41)	-0.09 (-6.34)	5.46 (3.88)	-1.94 (-0.59)		.65
4	-0.03 (-6.17)	0.11 (3.94)	-0.06 (-4.43)	4.98 (3.59)	0.18 (0.06)	0.27 (2.57)	.65
	-		-	+	+	+	

The last row contains the expected signs predicted by the model

A constant was included in each regression

T-statistics were corrected by Newey-West consistent matrix

In Table 3, the results of the regressions with foreign denominated debt are presented. First, The coefficients on the variance of the real exchange rate and covariance of real exchange rate with spending have the right signs but are insignificant. Second, the level of debt has a negative effect on the proportion of foreign denominated debt. Credibility arguments should imply a positive sign.<sup>12</sup>

<sup>12</sup>The regressions in Table 1 and 3 were repeated using instead the *amount* of nominal or foreign denominated debt as the independent variable. This isolates the direct negative effect of total debt from the credibility effect that we want to stress. The results are qualitatively identical.

Table 3 - Determinants of Foreign Denominated Debt in Brazil - 1980-1995  
 Dependent Variable is the Proportion of Foreign Denominated Debt  
 (t-statistics in parentheses)

	$D/Y$	Trend	$\sigma_{RER}$	Collor I	Real	$\sigma_{RERg}$	$\bar{R}^2$
1	-0.01 (-2.87)	0.05 (4.81)					.38
2	-0.02 (-2.42)	0.10 (5.13)	-0.27 (-1.12)				.52
3	-0.02 (-2.96)	0.10 (4.12)	-0.23 (-0.88)	-2.34 (-0.87)	1.72 (0.84)		.52
4	-0.02 (-2.95)	0.10 (4.16)	-0.21 (-0.84)	-2.36 (-0.87)	1.66 (0.80)	0.11 (0.88)	.44
			-			+	

The last row contains the expected signs predicted by the model

A constant was included in each regression

T-statistics were corrected by Newey-West consistent matrix

In order to take into account the covariance between the residuals of the regressions that underlie the results in Tables 1 and 3, a Seemingly Unrelated Regression (SUR) system was estimated. As shown in Table 4, the results are similar to the ones obtained in the previous tables.

Table 4  
 Determinants of Debt Composition  
 System Estimation - SUR  
 (t-statistics in parentheses)

Variable	Nominal	Foreign Denominated
$D/Y$	-0.05 (-6.7)	-0.01 (-6.2)
Trend	0.25 (8.9)	0.11 (7.8)
$\sigma_{\pi}$	-0.09 (-2.04)	0.001 (0.07)
$\sigma_{\pi g}$	1.4 (2.87)	0.36 (3.68)
$\sigma_{RER}$	0.13 (0.42)	-0.10 (-0.97)
$\sigma_{RERg}$	-1.57 (-3.63)	-0.02 (-0.11)
Collor I	16.9 (3.22)	-0.64 (-0.38)
Real	21.6 (5.27)	1.81 (1.36)
$\bar{R}^2$	0.47	0.57

A constant was included in the regressions

## V. CONCLUSIONS

Since the Real plan the share of public indexed debt has dropped from 70 to 30 percent of total debt while both nominal and foreign denominated debt shares have increased. This paper offers a simple model and preliminary evidence to explain these facts.

The paper develops a model of public debt management that concentrates on hedging and credibility motives. The model highlights that indexed debt should be issued to enhance the credibility of the government or to avoid unnecessary fluctuations in the real value of debt from variable inflation rates. In contrast, nominal debt serves the purpose of hedging shocks to the budget when inflation is positively correlated to spending. In addition, foreign denominated debt may also serve as a hedging device if the real exchange rate is positively correlated with spending and does not fluctuate substantially.

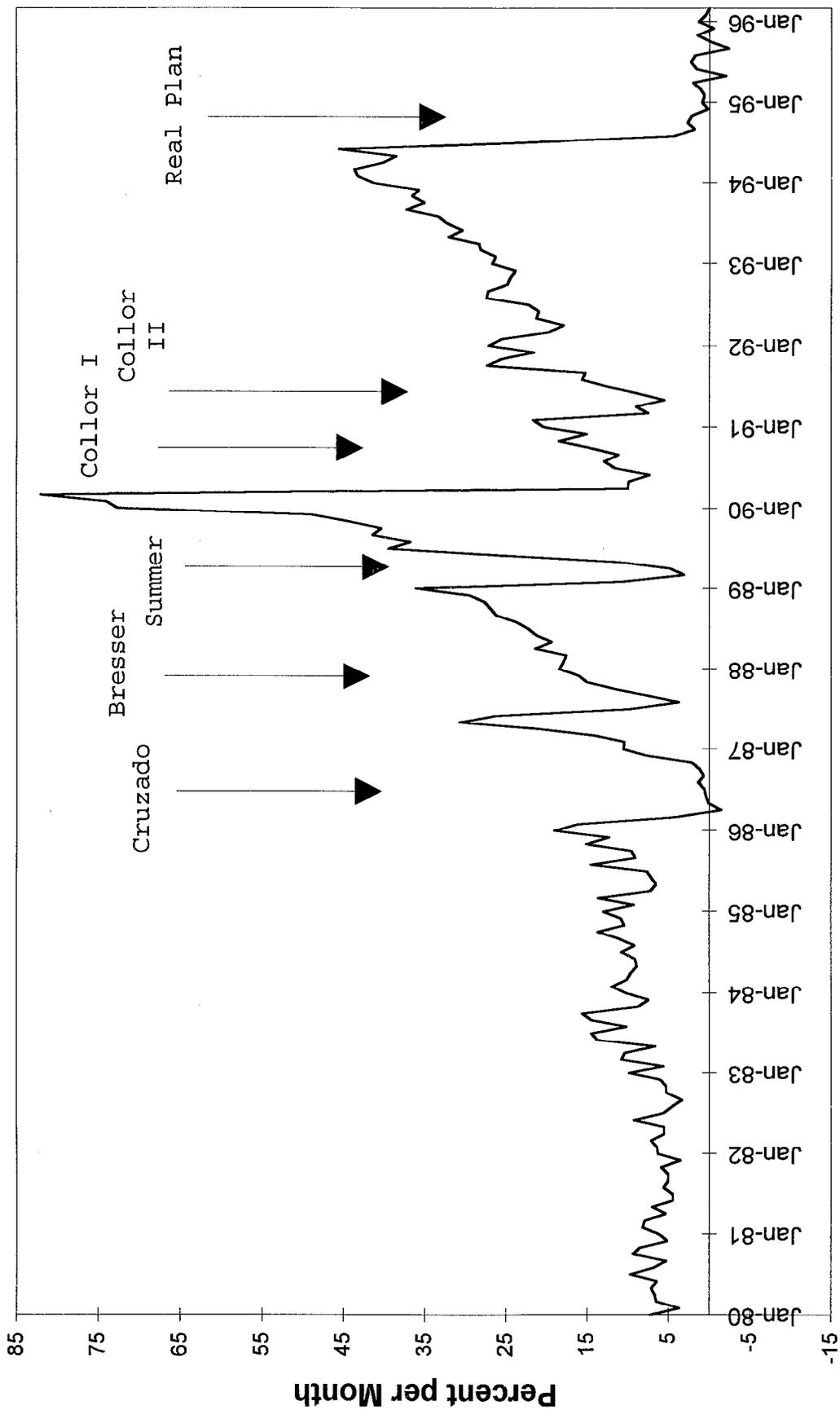
The evidence from OLS regressions confirms that the variance of inflation, the size of the public debt and the correlations of inflation with spending are important determinants of public debt indexation in Brazil.

**BIBLIOGRAPHY**

- J. Agell, M. Persson and B.M. Friedman, 1992, *Does Debt Management Matter?* (Oxford: Oxford University Press).
- Andima, 1992, *Séries Históricas - Dívida Pública*, (Rio de Janeiro: Andima).
- P. Arida and A.L. Resende, "Inertial Inflation and Monetary Reform in Brazil," in J. Williamson, editor, 1985, *Inflation and Indexation*, (Washington: IIE).
- R. Barro, 1997, "Optimal Debt Management of Indexed and Nominal Debt," NBER WP No. 6197, (Cambridge, Massachusetts: NBER).
- O. Blanchard and A. Missale, 1994, "The Debt Burden and Debt Maturity," *American Economic Review*, Vol. 84(March), pp. 309–319.
- H. Bohn, 1988, "Why Do We have Nominal Government Debt?" *Journal of Monetary Economics*, Vol. 21(January), pp. 127–40.
- H. Bohn, 1990, "A Positive Theory of Foreign Currency Debt," *Journal of International Economics*, Vol. 29, pp. 273–92.
- H. Bohn, 1990, "Tax Smoothing with Financial Instruments," *American Economic Review*, Vol. 80 (December), pp. 1217–30.
- G.A. Calvo, 1988, "Servicing the Public Debt : the Role of Expectations." *American Economic Review*, Vol. 78 (September), pp. 647–61)
- G.A. Calvo and P. Guidotti, 1990, "Indexation and Maturity of the Government Bonds: An exploratory model," in R. Dornbusch and M. Draghi, editors, *Public Debt Management: Theory and History*, pp. 52–93, (Cambridge: Cambridge University Press).
- G.A. Calvo and P. Guidotti, 1992, "Optimal Maturity of Nominal Government Debt: An Infinite-horizon Model," *International Economic Review*, Vol. 33(November), pp. 895–919.
- G. Calvo and M. Obstfeld, 1990, "Time Consistency of Fiscal and Monetary Policy: A Comment," *Econometrica*, Vol. 58(September), pp. 1245–47.
- R. Dornbusch and M. Draghi, 1990, *Public Debt Management: Theory and History*, (Cambridge: Cambridge University Press).

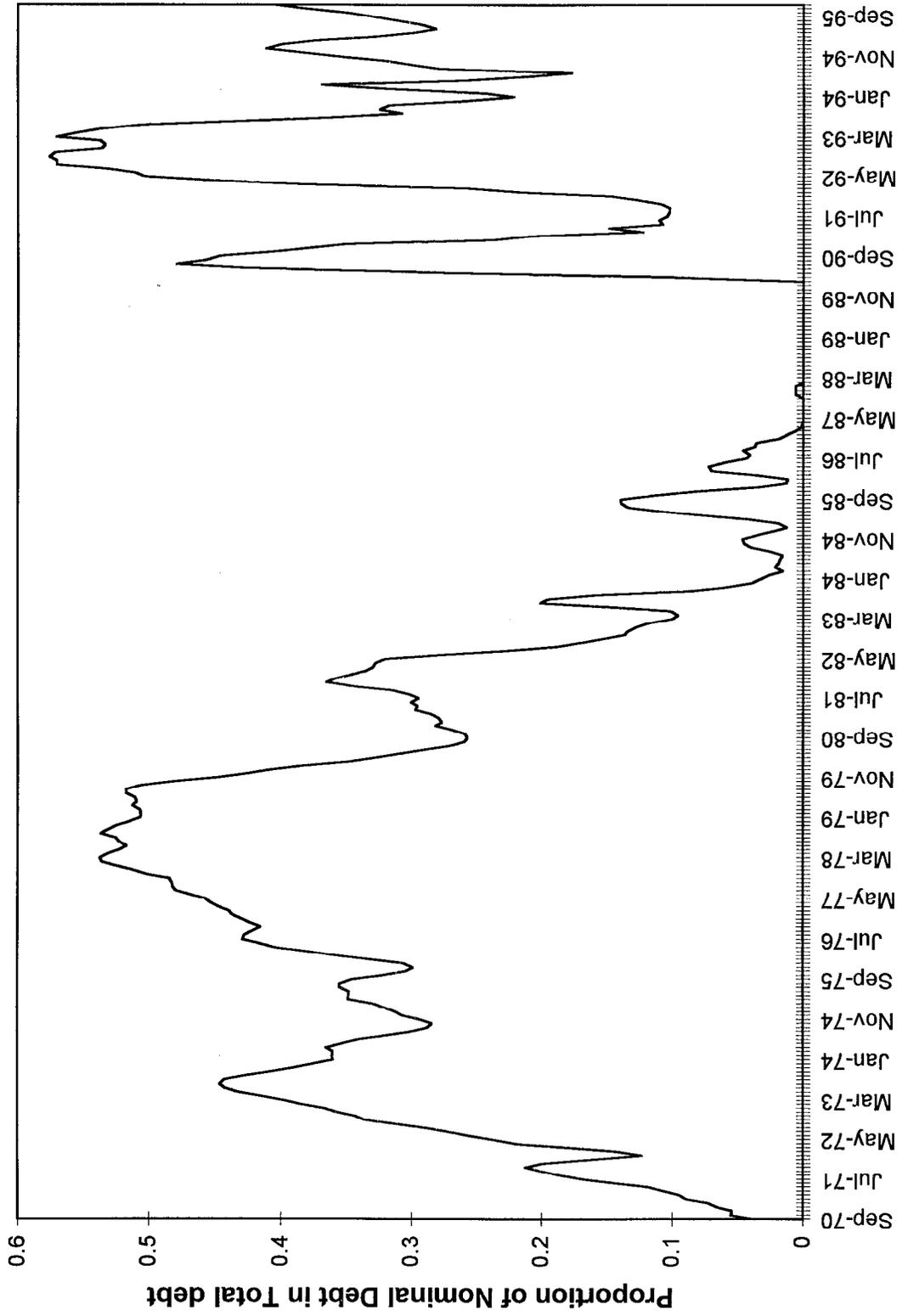
- S. Fischer, 1983, "Welfare Aspects of Government Issue of Indexed Bonds," in R. Dornbusch and M.H. Simonsen, editors, *Inflation, Debt and Indexation*, (Cambridge: MIT Press).
- S. Fischer and L. Summers, 1989, "Should Governments Learn to Live with Inflation?" *American Economic Review*, 79, pp. 382-87.
- M. Garcia, 1996, "Avoiding some Costs of Inflation and Crawling toward Hyperinflation: the Case of the Brazilian Domestic Currency Substitute" *Journal of Development Economics*.
- I. Goldfajn, 1996, "On Public Debt Indexation and Denomination," Brandeis University, WP 345, (Waltham, Massachusetts: Brandeis University).
- I. Goldfajn and R. Valdés, 1996, "The Aftermath of Appreciations," NBER WP No. 5650, (Cambridge, Massachusetts: NBER).
- V. Miller, 1988, "The optimal Currency Denomination of Government Debt and Central Bank Independence: An Exploratory Approach" (Mimeo, Université du Québec a Montreal).
- A. Missale, 1994, *Public Debt Management*, (PhD thesis, MIT).
- A.P. Neto, 1996, "Considerações Sobre a Gestão de Dívida Pública no Brasil." *Mimeo, PUC Rio de Janeiro*.
- M.H. Simonsen, 1983, "Indexation: Current Theory and the Brazilian Experience" In R. Dornbusch and M.H. Simonsen, editors, *Inflation, Debt and Indexation* (Cambridge, Massachusetts: MIT Press).
- M.H. Simonsen, 1995, *30 Anos de Indexação*, (Rio de Janeiro: Fundação Getulio Vargas).
- E Tanner, 1995, "Intertemporal Solvency and Indexed Debt: Evidence from Brazil," *Journal of International Money and Finance*, Vol. 14(4), pp. 549-73.

Figure 1. Inflation and Stabilization Plans



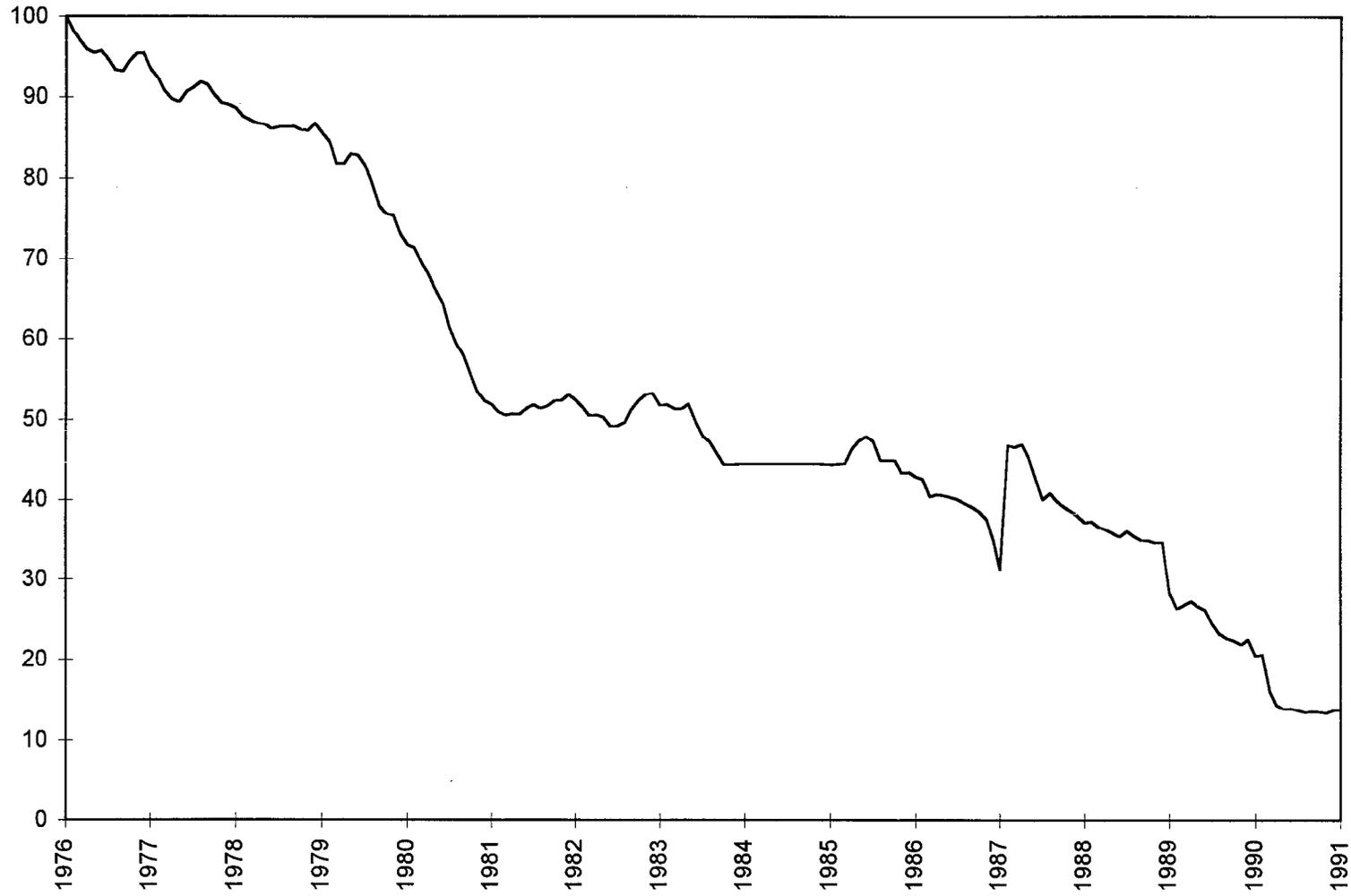
Source: Conjuntura Economica (IPA/DI)

Figure 2. Nominal Debt in Brazil



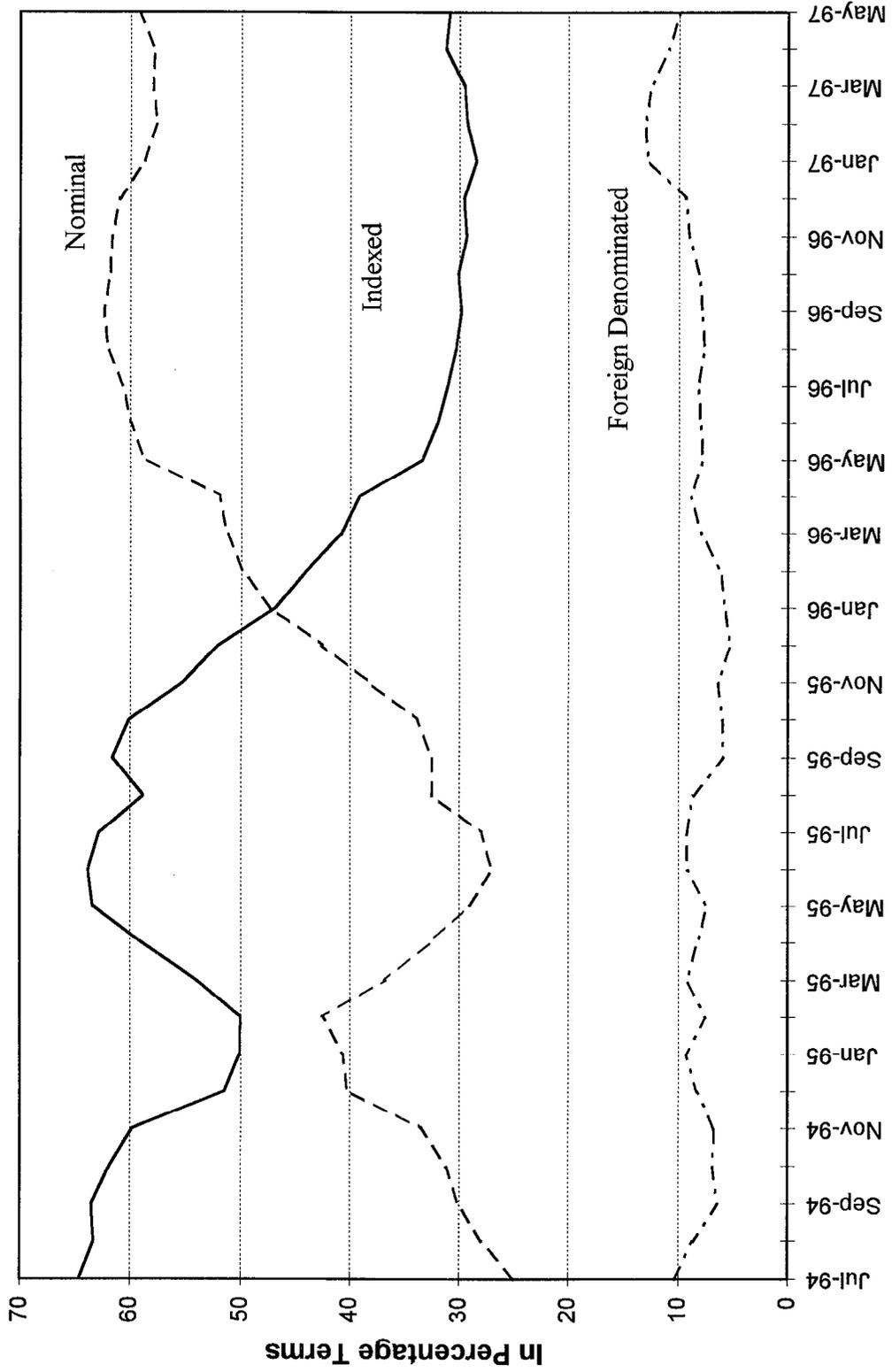
Source: Andima

**Figure 3. Real value of Government Index Bond  
(Jan1976=100)**



Source: Tanner (1995)

Figure 4. Composition of Public Debt



Source: Banco Central do Brasil