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The Incidence and Efficiency Costs of Corporate Taxation When
Corporate and Noncorporate Firms Produce the Same Good

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Abstract

One difficulty confronting Harberger's celebrated model of the corporate income tax is how to treat the noncorporate production in primarily corporate sectors and corporate production in primarily noncorporate sectors. This paper presents a two-good model with corporate and noncorporate production of both goods. The incidence of the corporate tax in our Mutual Production Model (MPM) can differ markedly from that in the Harberger Model. The difference between the two models in deadweight loss is also striking, with losses in the MPM over ten times as large as in the Harberger Model.

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Summary

This year marks the 25th anniversary of Arnold Harberger's celebrated model of the corporation income tax. While the model has been enormously useful as an analytical device for studying two-sector economies, its usefulness for understanding the incidence and excess burden of the corporate income tax remains in question.

This paper presents a two-good (sector) model with corporate and noncorporate production of both goods. This mutual production model has three productive factors: capital, labor, and managerial input (entrepreneurial input in the case of noncorporate firms). Those agents who are most productive as entrepreneurs will establish their own firms. Entrepreneurs manage their firms solely by themselves, and their firms may be quite small. In contrast, corporations must operate at greater than a minimum scale. This minimum scale requirement insures that the corporate sector will not disappear in the presence of a corporate income tax.

The incidence of the corporate tax in the mutual production model can differ markedly from that in the Harberger Model. A hallmark of Harberger's corporate tax incidence formula is its dependence on differences across sectors in elasticities of substitution between capital and labor. In contrast, the incidence of the corporate tax in the mutual production model may fall entirely on capital, regardless of sectoral differences in substitution elasticities.

The difference between the two models in the deadweight loss from corporate taxation is even more striking. In one version of the mutual production model, the deadweight loss is over ten times larger than in the Harberger Model. In the Harberger analysis, only the difference in the average corporate tax in the two sectors is distortionary, while the entire tax is distortionary in the mutual production model.

The much greater substitution elasticity in demand for corporate and noncorporate products in the mutual production model appears to explain much of the difference in excess burden between the two models.



I. Introduction

This year marks the 25th anniversary of Arnold Harberger's (1962) celebrated model of the corporation income tax. The Harberger Model, as it has come to be called, has been remarkably influential. It not only vanquished earlier theoretical analyses, but also shifted the debate from theory to the proper measurement of the model's parameters. A voluminous literature now uses the Harberger Model or its extensions to measure the incidence and efficiency costs of corporate taxation.

Confronting all empirical analyses of the Harberger Model is the issue of how to treat noncorporate production in primarily corporate sectors and corporate production in primarily noncorporate sectors. The Harberger Model provides no real guide to this question since it assumes that one good is produced only by corporations and the other good is produced only by noncorporate firms. Stated differently, Harberger models the differential taxation of capital used in the production of different goods, rather than the taxation of capital used by corporations per se. In empirical work the common finesse, initiated by Harberger, is to assume that all firms in a sector are identical and face taxation of capital at a rate equal to the sector's average rate of capital taxation. This assumption is, unfortunately, far from innocuous. In treating each sector as comprising identical firms facing the same tax rate, Harberger ignores the substitution that can arise between corporate and noncorporate producers. Moreover, as Ebrill and Hartman (1982, 1983) and Gravelle (1981) point out, the Harberger Model cannot be easily modified to permit noncorporate production of the corporate good. If there is even a single, equally efficient noncorporate producer of the corporate good, corporate production will entirely disappear in response to the imposition of a tax on corporate income.

This paper presents a two-good (sector) model with corporate and noncorporate production of both goods. This mutual production model has three productive factors: capital, labor, and managerial input (entrepreneurial input in the case of noncorporate firms). Each agent is free to be a corporate manager, an entrepreneur, or a worker. While agents are equally productive as corporate managers or workers, they are not equally productive as entrepreneurs. In equilibrium, those agents who are most productive as entrepreneurs will establish their own firms, with the marginal entrepreneur just indifferent between establishing his own firm and employment as a corporate manager or worker. Entrepreneurs manage their firms solely by themselves, and their firms may be quite small. In contrast, corporations must operate at greater than a minimum scale. This minimum scale requirement insures that the corporate sector will not disappear in the presence of a corporate income tax.

The incidence of the corporate tax in the Mutual Production Model (MPM) can differ markedly from that in the Harberger Model. A hallmark of Harberger's corporate-tax incidence formula is its dependence on differences across sectors in elasticities of substitution between capital and labor. In contrast, the incidence of the corporate tax in

the MPM may fall 100 percent on capital regardless of sectoral differences in substitution elasticities. This result holds for a large class of production functions, including the constant elasticity of substitution (CES) function, if each sector initially has the same capital shares as well as the same corporate share of output.

While one might expect that the two incidence formulae would, in general, differ, the implicit suggestion in the Harberger finesse is that the two formulae will converge as one sector becomes more corporate-intensive and the other less corporate-intensive. Such, however, is not the case. The MPM incidence formula converges to something quite different from that in the Harberger Model. The difference in the two models is further illustrated by assuming CES production functions and using Harberger's (1966) and Shoven's (1976) data to calculate pre- and post-tax equilibria. With these data, there are significant differences in the incidence in the MPM and Harberger Model for a variety of combinations of demand and production elasticities. For example, assuming elasticities of substitution equal to 0.5 and demand elasticity equal to 1, the share of the corporate borne by capital in the MPM is 141 percent, while it is only 82 percent in the Harberger Model.

The difference between the two models in the deadweight loss from corporate taxation is also striking. Using the Harberger-Shoven data and assuming unitary substitution and demand elasticities, the deadweight loss is over ten times larger in the CES version of the MPM than in the Harberger Model. Much of the explanation for this difference is that in the Harberger Model only the difference in the average corporate tax in the two sectors is distortionary, while the entire tax is distortionary in the MPM. Stated differently, if each sector is equally corporate-intensive, the Harberger analysis predicts zero distortion from the corporate tax, whereas the MPM predicts a potentially significant deadweight loss arising from within-sector substitution of noncorporate for corporate production.

The second reason that deadweight loss is so much greater in the MPM than in the Harberger Model involves the elasticity of product demand. A larger elasticity in demand between corporate and noncorporate output appears, *ceteris paribus*, to increase the extent of substitution away from corporate capital and to increase the excess burden. In the MPM the source of excess burden is primarily within-sector substitution of noncorporate for corporate capital, whereas the source of excess burden in the Harberger Model is between-sector substitution; in the MPM the within-sector elasticity of demand for corporate and noncorporate output is infinite. In contrast, the between-sector elasticity of demand, which plays an important role in determining excess burden in the Harberger Model, is thought to be quite small.

The paper proceeds in Section II by pointing out the extent of mutual production in particular industries as well as changes through time in the extent of mutual production. This section indicates that at the two-digit level of aggregation all goods are mutually produced. On

the other hand, at finer levels of aggregation, there appears, for some goods, to be production only by firms subject to the corporate tax. For example, there appear to be no noncorporate manufacturers of televisions. The fact that there is zero noncorporate production of some goods is not, however, a problem for the MPM. The minimum requirement of the MPM is that there be corporate production of both goods, but not necessarily noncorporate production of both goods. There appear to be very few goods that are not produced by firms subject to the corporate tax.

Section III motivates our modeling of the corporate tax as a tax on capital of large firms that are not owned and operated solely by the same individual or solely by a small number of individuals. While this is a multifaceted definition of the base of the corporate tax, such a multifaceted definition appears to be used in practice. Indeed, Regulation 301.7701-2(a)(1) of the Internal Revenue Service code states that "An organization will be taxed as a corporation if its characteristics are such that it more closely resembles a corporation than a partnership or trust."

Section IV presents the MPM. Section V compares the corporate tax incidence formula of the MPM with that of Harberger's. Section VI describes the calculation of no-tax and post-tax equilibria in the MPM and Harberger models, which can be evaluated using the Harberger-Shoven data. Section VII compares the incidence and excess burden of corporate taxation in the two models. Section VIII summarizes the paper and suggests further applications and extensions of the model.

II. The Extent of Mutual Production

Table 1 indicates that there has been and continues to be corporate production in all two-digit industries as well as three-digit industries for which data are available. There is also noncorporate production in virtually all the two-digit and three-digit industries; only four of the 50 listed in Table 1 have solely corporate production. In a large number of industries that Harberger includes in the "corporate sector," the share of noncorporate output has often been quite large. For example, the noncorporate share of output in retail apparel was 38.1 percent in 1957; the 1982 figure is smaller, only 19.6 percent. In retail food, noncorporate production accounted for almost half of output in 1957; more recently it has accounted for over one quarter of output.

There has been considerable change over time in many industries in the corporate share of production. One example is drugstores, whose corporate share of output rose from 38.4 percent in 1957 to 91.4 percent in 1982. Likewise, in agriculture the corporate share of output rose from only 9.2 percent in 1957 to 29.3 percent in 1982. While most industries have become significantly more corporate, several, including mining and motion pictures, have become somewhat noncorporate. These data certainly suggest a substantial degree of within-industry substitution of corporate for noncorporate production over the last three decades.

Table 1. Corporate Share of Output by Industry, 1957-82

	Corporate Percentage of Output		Percentage Change 1957-1982
	1957	1982	
Noncorporate	6.2	20.4	227.3
Agriculture	9.2	29.3	219.7
Production	9.6	22.8	136.5
Services	4.0	62.4	1,463.3
Housing	1.3	2.4	25.0
Crude oil and gas	74.7	84.8	13.6
Corporate	75.8	86.4	13.9
Mining	87.2	84.4	-3.3
Construction	55.8	73.1	31.0
Manufacturing	95.8	97.6	1.9
Food	93.9	97.8	4.1
Tobacco	100.0	100.0	0.0
Textiles	97.0	99.0	2.1
Apparel	85.7	94.3	10.0
Lumber, wood	70.8	85.0	19.9
Furniture	90.5	97.1	7.3
Paper	100.0	100.0	0.0
Printing	89.3	89.3	0.0
Chemicals	98.6	97.4	-1.3
Petroleum	100.0	100.0	0.0
Rubber	100.0	100.0	0.0
Leather	96.9	98.6	1.8
Stone, clay, and glass	92.3	96.8	4.9
Primary metals	98.6	97.3	-1.3
Fabricated metals	94.7	97.8	3.2
Machinery	96.5	98.1	1.6
Electronics	99.1	99.5	0.5
Transportation equipment	99.5	98.7	-0.9
Other	89.7	88.4	-1.5
Transportation, communication and public utilities	92.1	92.3	2.0
Transportation	86.0	81.7	-0.4
Communication, utilities	99.0	98.8	-0.3
Trade	62.7	82.9	32.3
Wholesale	76.2	91.7	20.5
Retail	52.6	74.5	41.6
Food	52.8	75.5	43.1
General merchandise	84.6	96.9	14.5
Apparel	61.9	80.4	29.9
Furniture	46.5	68.0	46.3
Auto, gasoline	50.9	70.7	39.0
Drugstores	38.4	91.4	138.0
Eating and drinking places	28.5	59.0	106.8
Building materials, hardware	51.0	75.5	47.9
Other	38.8	62.9	62.3
Finance, insurance, real estate	60.4	74.1	22.8
Securities dealers	13.2	24.5	85.1
Other finance	90.5	80.9	-10.6
Insurance	80.7	94.7	17.5
Real estate	34.0	36.8	8.2
Services	38.4	61.1	59.2
Hotels	52.6	59.2	12.6
Personal services	34.6	47.6	37.7
Business services	62.1	67.9	9.4
Auto repair	32.6	54.9	68.1
Other repair	33.8	50.3	49.0
Amusements	68.2	73.5	7.8
Motion pictures	89.6	79.5	-11.3
Other	47.0	70.3	49.7
Other services	9.0	53.6	498.4

Source: Output shares for housing are based on estimates of net stocks of residential capital. ("Fixed Capital Stock in the United States: Revised Estimates, 1925-79" and "Fixed Reproducible Tangible Wealth in the United States, 1980-83," both by John C. Musgrave, *Survey of Current Business*, Department of Commerce, February 1981 and August 1984, respectively). Output shares for all other industries are taken from business receipts reported in Department of the Treasury, *Statistics Income, U.S. Business Tax Returns: 1957-58, Partnership Returns, 1978-82, Corporation Income Tax Returns, 1982; Sole Proprietorship Returns, 1982*, *SOI Bulletin* (Summer 1984). Agricultural business receipts for proprietorships, which is no longer included in the proprietorship returns, is reported for 1982 in "Sole Proprietorship Returns, 1984," *SOI Bulletin* (Summer 1986), footnote 4, p. 23. Data on Subchapter S corporations for 1982 is reported only by major industrial division. These receipts were allocated among subcategories based on the distribution of noncorporate output. The table double counts the leasing of residential structures; leasing of residential structures is included both in the real estate industry and the housing industry.

The increase in the corporate share of output since 1957 may, in part, reflect changes in technology. Undoubtedly a small amount of increase can be attributed to doctors, lawyers, and others in the service sector using pensions and retained earnings to shelter their labor income. But much of the increase in the corporate share of output may reflect changes in corporate versus noncorporate tax treatment of capital income. The general shift toward corporate production coincided with a reduction in the differential taxation of corporate and noncorporate capital income. Gravelle (1987) calculates marginal tax rates on corporate and noncorporate source-capital income, taking into account both personal and corporate taxes. She reports that the total (personal plus corporate) effective tax rate on corporate capital income exceeded that on noncorporate capital income by 0.52 percent in 1957; 0.44 percent in 1962; 0.42 percent in 1971; 0.45 percent in 1975; 0.39 percent in 1982; 0.40 percent in 1986, and 0.32 percent in 1987. ^{1/}

III. What Capital is Subject to the Corporate Tax?

The Internal Revenue Service's (IRS) definition of a corporation as an organization that most closely resembles a corporation becomes a little less circular when we add the IRS' list of corporate characteristics. These include (1) associates; (2) an objective to carry on business and divide the profits; (3) continuity of life; (4) centralization of management; (5) liability for corporate debts limited to corporate property; and (6) free transferability of interests. While these are the corporate characteristics, firms can have these characteristics and still not be subject to the corporate tax. An S corporation is a corporation with 35 or fewer shareholders and is taxed as a partnership. Limited partnerships also fall close to the line with respect to the corporate income tax. IRS Regulation 301.7701-2(a)(2) says that if the limited partnership has "more corporate characteristics than noncorporate characteristics," it will be subject to the corporate tax.

^{1/} These tax rates represent marginal effective (personal plus corporate) tax rates on new investment. For the corporate sector the return (after all taxes) to stockholders and creditors is grossed up by personal taxes on dividends, interest, and capital gains (adjusted for the value of capital gains deferral and the taxation of the inflationary component of capital gains). The marginal tax rate methodology (see Gravelle (1982)) uses a discounted cash flow method to determine the pre-tax real return necessary to pay stockholders and creditors the after-tax return. A similar process is used to measure the noncorporate pre-tax return required to yield the same after-tax return. The differential between these two pre-tax returns is used to measure the corporate-noncorporate tax wedge. This wedge is given by $1 - (1-\mu)/(1-t)$, where μ is the total effective tax rate on corporate capital, and t is the total effective tax rate on noncorporate capital.

One recent response to the rather vague demarcations between corporate and noncorporate enterprises has been the creation of master limited partnerships, some of which have thousands of partners. But it appears, according to The Wall Street Journal of June 30, 1987, that such enterprises have become sufficiently "corporate" that Congress may soon declare them subject to the corporate tax. As Robert McIntyre of Citizens of Tax Justice told The Journal, "If they [the master limited partnerships] want to play with the big boys, they ought to pay taxes with them."

One is likely to come away from the preceding two paragraphs with the sense that defining capital subject to the corporate tax is like trying to define money: "nobody knows precisely what it is, but they know it when they see it." In "seeing" capital that should be subject to corporate tax the government appears to be looking both at the size of the enterprise and the diversity of ownership. Enterprises that are both very large and have many owners appear to be fair game.

But if the criteria for "corporateness" are size and number of owners, why do large firms with multiple owners simply not break up into small firms with a single owner or a small number of owners? The answer is surely that for many products there are some economies, at least for a range, in operating on a large scale. Large-scale production does not necessarily mean integrated ownership; that is, in principle one could imagine different owners of robots or conveyor belts in an automobile plant assembly line. But in the language of Grossman and Hart (1986), integrated ownership provides residual rights of control of physical assets that may be important in settings in which complete ex ante contracting is too costly.

But given that some enterprises are large, why should they have more than a very small number of owners? The answer here appears to involve a number of factors: diversification of risk, the desire to limit liability, information costs of becoming fully informed about all the activities of a large enterprise, and liquidity. These reasons for multiple owners are interrelated. For example, it may be difficult for any one owner to become fully informed about a large firm's activities; but the lack of full information may make investing in a large firm riskier. The limits on full information provide investors with a further interest in reducing their exposure in a particular firm, including limiting their liability.

Granted that many firms are likely to be quite large for technological reasons and that their size induces multiple owners, how is it that proprietorships and partnerships that produce the same good, but are typically small, can compete? Our answer is that there is an offsetting technological advantage to running an enterprise as a partnership or proprietorship and that this advantage involves information and control. Entrepreneurs, with a major stake in their own firm, will have an incentive to stay better informed about their firm's behavior and to control more fully their firm's behavior than will shareholders in large

companies. In short, the offsetting advantage to proprietorships and partnerships is a smaller principal-agent problem than that which arises in the case of large-scale corporations. Nevertheless, this advantage to proprietorships and partnerships dissipates with size. In other words, there are decreasing returns in adding additional factors to the entrepreneurial input.

The MPM presented in the next section is designed to capture, in an admittedly highly stylized setting, the relative advantages both of large-scale production with multiple owners and the typically smaller-scale production by proprietors and partners. The MPM provides a technological advantage to more efficient entrepreneurs that permits them to compete with large corporate firms, but their advantage is subject to decreasing returns. ^{1/} The large corporate firms, on the other hand, have a technical advantage relative to less efficient entrepreneurs. In order for the corporate firms to produce, however, they must produce at greater than a specified minimum scale. Corporate firms will thus coexist with more efficient entrepreneurship both prior to, and after the imposition of, a corporate tax. The difference in corporate and noncorporate technologies is, however, solely with respect to an efficiency coefficient on managerial (entrepreneurial) input; the forms of the corporate and noncorporate production functions within each sector are identical. In particular, corporate and noncorporate firms within each sector exhibit identical substitution elasticities.

IV. The Mutual Production Model

1. Profit maximization

Equation (1) presents corporate output in sector 1, Q_{c1} , as a function of the number of managers, M_{c1} ; the number of workers, L_{c1} ; and the amount of capital, K_{c1} , in sector 1. Equation (2) is the analogous expression for sector 2.

$$Q_{c1} = \begin{cases} H(D_1 M_{c1}, L_{c1}, K_{c1}) & \text{if } M_{c1} \geq \underline{M}_{c1} \\ 0 & \text{if } M_{c1} < \underline{M}_{c1} \end{cases} \quad (1)$$

$$Q_{c2} = \begin{cases} G(D_2 M_{c2}, L_{c2}, K_{c2}) & \text{if } M_{c2} \geq \underline{M}_{c2} \\ 0 & \text{if } M_{c2} < \underline{M}_{c2} \end{cases} \quad (2)$$

^{1/} Williamson (1967), and Calvo and Wellisz (1978) present models explaining the possible limits of entrepreneurial supervision and the attendant loss of control.

The minimum scale constraints in equations (1) and (2) are specified with respect to managerial input. The terms D_1 and D_2 are the respective corporate managerial efficiency coefficients in industries 1 and 2. The functions $H(, ,)$ and $G(, ,)$ are assumed to be linear homogeneous and quasi-concave. Assuming positive production by corporations in both sectors, output per manager of corporations in the respective sectors 1 and 2, q_{c1} and q_{c2} , can be written as:

$$q_{n1} = h(D_1, l_{c1}, k_{c1}) \quad (3)$$

$$q_{c2} = g(D_2, l_{c2}, k_{c2}) \quad (4)$$

The production functions expressing output per entrepreneur in the two sectors are identical, respectively, to those in (3) and (4) except with respect to the managerial (entrepreneurial) efficiency coefficient. Equations (5) and (6) express output per entrepreneur in the two sectors for entrepreneurs with respective efficiency coefficients A and B:

$$q_{n1}(A) = h(A, l_{n1}, k_{n1}) \quad (5)$$

$$q_{n2}(B) = g(B, l_{n2}, k_{n2}) \quad (6)$$

The symbol n stands for noncorporate.

Each agent in the economy can potentially become an entrepreneur in one of the two sectors; alternatively, the agent can be a manager or worker. While all agents are equally productive as managers or workers, as entrepreneurs their productivity depends on their efficiency coefficients. ^{1/} Each agent has a pair of coefficients A and B. The number of agents with the pair of coefficients A and B is given by the joint density function $f(A,B)$ times the labor force, \bar{L} .

Profit maximization by corporations in sector 1 implies the following first-order conditions:

$$\delta h(D_1, l_{c1}, k_{c1}) / \delta l_{c1} = W/P_1 \quad (7)$$

$$\delta h(D_1, l_{c1}, k_{c1}) / \delta k_{c1} = R(1 + \tau)/P_1 \quad (8)$$

$$\pi_{c1} = P_1 h(D_1, l_{c1}, k_{c1}) - W l_{c1} - R(1 + \tau) k_{c1} = W \quad (9)$$

^{1/} After developing our model we became aware of Lucas' (1978) paper that also models entrepreneurs as managers with differing abilities, but demonstrates how such a model can explain secular changes in firm size. Chamley (1983) is another example of an early analysis of differing entrepreneurial abilities and the choice of occupation.

where P_1 is the price of good 1, τ is the corporate tax rate, π_{c1} is corporate profit per manager in sector 1, W is the wage rate, and R is the net return to capital. Since agents are equally productive as workers or managers, they must receive the same wage in either occupation; equations (7) and (9) express this point. The analogous profit maximizing conditions for sector 2 are given below.

$$\delta g(D_2, l_{c2}, k_{c2}) / \delta l_{c2} = W/P_2 \quad (10)$$

$$\delta g(D_2, l_{c2}, k_{c2}) / \delta k_{c2} = R(1 + \tau)/P_2 \quad (11)$$

$$\pi_{c2} = P_2 h(D_2, l_{c2}, k_{c2}) - W l_{c2} - R(1 + \tau) k_{c2} = W \quad (12)$$

Using the fact that the partial derivatives of $F(, ,)$ and $G(, ,)$ are homogeneous of degree zero, equations (7) and (8) can be re-expressed as:

$$l_{c1} = D_1 m_1(W/P_1, R(1 + \tau)/P_1) \quad (13)$$

$$k_{c1} = D_1 n_1(W/P_1, R(1 + \tau)/P_1) \quad (14)$$

And equations (10) and (11) may be written as:

$$l_{c2} = D_2 m_2(W/P_2, R(1 + \tau)/P_2) \quad (15)$$

$$k_{c2} = D_2 n_2(W/P_2, R(1 + \tau)/P_2) \quad (16)$$

Substituting (13) and (14) into (9) indicates that corporate profits per manager in sector 1, π_{c1} , can be written as D_1 times a function x_1 of W , $R(1 + \tau)$, and P_1 ; that is,

$$\pi_{c1} = D_1 x_1(W, R(1 + \tau), P_1) \quad (9')$$

Analogously,

$$\pi_{c2} = D_2 x_2(W, R(1 + \tau), P_2) \quad (12')$$

Since entrepreneurs in sectors 1 and 2 have the same production function as corporations in their respective sectors, except for the efficiency coefficient, their factor demands can be written as:

$$l_{n1}(A) = A m_1(W/P_1, R/P_1) \quad (17)$$

$$k_{n1}(A) = An_1(W/P_1, R/P_1) \quad (18)$$

$$l_{n2}(B) = Bm_2(W/P_2, R/P_2) \quad (19)$$

$$k_{n2}(B) = Bn_2(W/P_2, R/P_2) \quad (20)$$

Note that these expressions do not include the corporate tax rate, τ .

Profits per entrepreneur in sectors 1 and 2, $\pi_{n1}(A)$ and $\pi_{n2}(B)$, can be written using the $x_1(, ,)$ and $x_2(, ,)$ functions as:

$$\pi_{n1}(A) = Ax_1(W, R, P_1) \quad (21)$$

$$\pi_{n2}(B) = Bx_2(W, R, P_2) \quad (22)$$

2. Choice of occupation

In deciding whether to be an entrepreneur or to be a worker or manager, each agent considers the profits he would make as an entrepreneur in either industry A or B as well as the wage paid to workers and managers. An agent who is just indifferent between becoming an entrepreneur in industry A and working as either a manager or worker satisfies the following:

$$\pi_{n1}(\underline{A}) = \underline{A}x_1(W, R, P_1) = D_1x_1(W, R(1+\tau), P_1) = W \quad (23)$$

where \underline{A} is the efficiency coefficient that would make an agent just indifferent between the three occupations. The corresponding minimum efficiency coefficient \underline{B} is defined by:

$$\pi_{n2}(\underline{B}) = \underline{B}x_2(W, R, P_1) = D_2x_2(W, R(1+\tau), P_2) = W \quad (24)$$

Combining (21) with (23) and (22) with (24) implies:

$$\pi_{n1}(A) = (A/\underline{A})W \quad (25)$$

$$\pi_{n2}(B) = (B/\underline{B})W \quad (26)$$

Agents who choose to be entrepreneurs in sector 1 must earn profits at least as large as W , but their profits as entrepreneurs in sector 1 must also be at least as large as what they can earn as entrepreneurs in sector 2. Agents who are just indifferent between being entrepreneurs in the two sectors satisfy:

$$\pi_{n1}(A) = \pi_{n2}(B) \quad \text{or} \quad A/\underline{A} = B/\underline{B} \quad (27)$$

Agents with values of A and B less than \bar{A} and \bar{B} , respectively, will be workers or managers. Those with $A > \bar{A}$ and $B < \bar{A}B/\bar{A}$ will be entrepreneurs in sector 1; those with $B > \bar{B}$ and $A < \bar{B}A/\bar{B}$ will be entrepreneurs in sector 2. The terms \bar{A} and \bar{B} are maximum values of A and B, respectively.

3. General equilibrium conditions

The conditions that the supplies of labor and capital equal their respective demands are given in equations (28) and (29).

$$\bar{L} = M_{c1} + L_{c1} + M_{c2} + L_{c2} + L_{n1} + L_{n2} + E_1 + E_2 \quad (28)$$

$$\bar{K} = K_{c1} + K_{c2} + K_{n1} + K_{n2} \quad (29)$$

The terms \bar{L} and \bar{K} stand for the total supplies of labor and capital. L_{n1} and L_{n2} are total noncorporate labor demands in sectors 1 and 2, while K_{n1} and K_{n2} are total noncorporate capital demands in sectors 1 and 2. Recall that:

$$L_{c1} = M_{c1} D_1 m_1 (W/P_1, R(1+\tau)/P_1) \quad (30)$$

$$L_{c2} = M_{c2} D_2 m_2 (W/P_2, R(1+\tau)/P_2) \quad (31)$$

$$K_{c1} = M_{c1} D_1 n_1 (W/P_1, R(1+\tau)/P_1) \quad (32)$$

$$K_{c2} = M_{c2} D_2 n_2 (W/P_2, R(1+\tau)/P_2) \quad (33)$$

The noncorporate demands for capital and labor are given by:

$$L_{n1} = \bar{L} \int_{\bar{A}}^{\bar{A}} \int_0^{\bar{A}B/\bar{A}} l_{n1}(A) f(A, B) dA dB \quad (34)$$

$$L_{n2} = \bar{L} \int_{\bar{B}}^{\bar{B}} \int_0^{\bar{B}A/\bar{B}} l_{n2}(B) f(A, B) dA dB \quad (35)$$

$$K_{n1} = \bar{L} \int_{\bar{A}}^{\bar{A}} \int_0^{\bar{A}B/\bar{A}} k_{n1}(A) f(A, B) dA dB \quad (36)$$

$$K_{n2} = \bar{L} \int_{\underline{B}}^{\bar{B}} \int_0^{\underline{BA/B}} k_{n2}(B) f(A,B) dAdB \quad (37)$$

The limits of integrating in, for example, equation (34) may be explained in the following way. For an agent to choose to be an entrepreneur in sector 1, the agent's A must be at least as large as \underline{A} , and his B must be less than $\underline{AB/A}$. Equation (34) sums the labor demands of all entrepreneurs whose values of A and B satisfy these conditions. The remaining terms in (28) to be defined, E_1 and E_2 , stand for number of entrepreneurs in sectors 1 and 2, respectively. E_1 and E_2 satisfy:

$$E_1 = \bar{L} \int_{\underline{A}}^{\bar{A}} \int_0^{\underline{AB/A}} f(A,B) dAdB \quad (38)$$

$$E_2 = \bar{L} \int_{\underline{B}}^{\bar{B}} \int_0^{\underline{BA/B}} f(A,B) dAdB \quad (39)$$

Following Harberger, equation (40) specifies that the relative aggregate demand for the two goods depends only on their relative price:

$$Q_1/Q_2 = V(P_1/P_2) \quad (40)$$

The terms Q_1 and Q_2 are the respective total supplies of goods 1 and 2, that is,

$$Q_1 = Q_{c1} + Q_{n1} = Q_{c1} + \bar{L} \int_{\underline{A}}^{\bar{A}} \int_0^{\underline{AB/A}} q_{n1}(A) f(A,B) dAdB \quad (41)$$

$$Q_2 = Q_{c2} + Q_{n2} = Q_{c2} + \bar{L} \int_{\underline{B}}^{\bar{B}} \int_0^{\underline{BA/B}} q_{n2}(B) f(A,B) dAdB \quad (42)$$

Finally, equation (43) states that the value of output equals the value of national income I, which is taken as the model's numeraire.

$$P_1 Q_1 + P_2 Q_2 = I \quad (43)$$

4. Solving the model and comparisons with the Harberger Model

The model's solution can be determined as follows. From equations (23) and (24) one can write \underline{A} as a function of W and R and P_1 , and \underline{B} as a function of W and R and P_2 . These relationships plus (17)

through (20) imply that L_{n1} , L_{n2} , K_{n1} , and K_{n2} as well as E_1 and E_2 can each be written as functions of W and R and P_1 and P_2 . Substituting these relationships into equations (28) and (29) and also using (13) through (16) gives two equations in W , R , M_{c1} , M_{c2} , P_1 , and P_2 .

Substituting using (3) through (6) as well as (13) through (20) into equations (40) and (43) gives two additional equations in these six variables. The final two equations needed to solve for these six variables arise from equating (7) and (9), as well as (10) and (12), and substituting in from (13) through (16).

The differentials of these last two equations, presented in (44) and (45), as well as the differentials of (23) and (24), presented in (46) and (47), are important for understanding how the MPM differs from the Harberger Model.

$$\hat{P}_1 = (1 - \beta_1)\hat{W} + \beta_1(\hat{R} + \hat{\tau}) \quad (44)$$

$$\hat{P}_2 = (1 - \beta_2)\hat{W} + \beta_2(\hat{R} + \hat{\tau}) \quad (45)$$

$$\hat{A} = \frac{(1 - \beta_1)\hat{W} + \beta_1\hat{R} - \hat{P}_1}{(1 - \alpha_1 - \beta_1)} \quad (46)$$

$$\hat{B} = \frac{(1 - \beta_2)\hat{W} + \beta_2\hat{R} - \hat{P}_2}{(1 - \alpha_2 - \beta_2)} \quad (47)$$

In the above equations $\hat{}$ stands for percentage change. The terms β_1 and β_2 are the respective income shares of capital in sectors 1 and 2 and in the pre-tax equilibrium; and α_1 and α_2 are the respective income shares of workers in corporate sectors 1 and 2 in the pre-tax equilibrium. Note that in the pre-tax equilibrium the income shares of corporate and noncorporate firms within a sector are identical; in the pre-tax equilibrium each noncorporate firm looks just like its corporate counterpart except for the scale of its inputs and production.

Equations (44) and (45) indicate that if each sector's initial income shares are identical, P_1 and P_2 will change by the same percentage. This result is quite different from that in the Harberger Model in which the relative price of the two goods always changes regardless of initial income shares. Intuitively, since corporations in each sector in the MPM will still be producing after the tax is imposed, they will both experience the same percentage increase in marginal cost (which equals the price of output) if their initial factor shares are

identical. This property, that relative output prices are not necessarily affected by the corporate tax, holds regardless of the relative corporateness of the two sectors, provided there is non-zero corporate production in each sector.

Equations (46) and (47) indicate how the minimum efficiency coefficients \underline{A} and \underline{B} respond to changes in factor and output prices when the corporate tax is imposed. Combining (44) with (46) as well as (45) with (47) indicates that both minimum efficiency coefficients fall in response to the corporate tax; hence, the tax leads to an increase in the number of noncorporate firms.

V. Corporate Tax Incidence in the Mutual Production Model

1. Capital's share of the tax burden

The Appendix derives the formula, presented in equation (48), for the incidence of the corporate tax in the MPM.

$$\begin{aligned}
 \hat{R} / \hat{\tau} = & \\
 & + \phi(\beta_2 - \beta_1)^2 K_1 K_2 / (\beta_1 K_2 + \beta_2 K_1) \\
 & + K_{n2} [(\beta_2 \eta_{kw} + \theta_1 (\beta_2 - \beta_1) \eta_{kr})(1 - \beta_2) - (\beta_2 \eta_{lw} + \theta_1 (\beta_2 - \beta_1) \eta_{lr}) \alpha_2] \\
 & + K_{n1} [(\beta_1 \epsilon_{kw} + \theta_2 (\beta_1 - \beta_2) \epsilon_{kr})(1 - \beta_1) - (\beta_1 \epsilon_{lw} + \theta_2 (\beta_1 - \beta_2) \epsilon_{lr}) \alpha_1] \\
 & - \frac{K_{c1} [(1 - \beta_1)^2 \epsilon_{kr} - 2\beta_1 (1 - \beta_1) \epsilon_{kw} + \alpha_1 \beta_1 \epsilon_{lw}] - K_{c2} [(1 - \beta_2)^2 \eta_{kr} - 2\beta_2 (1 - \beta_2) \eta_{kw} + \alpha_2 \beta_2 \eta_{lw}]}{K_1 [(1 - \beta_1)^2 \epsilon_{kr} - 2\beta_1 (1 - \beta_1) \epsilon_{kw} + \alpha_1 \beta_1 \epsilon_{lw}] + K_2 [(1 - \beta_2)^2 \eta_{kr} - 2\beta_2 (1 - \beta_2) \eta_{kw} + \alpha_2 \beta_2 \eta_{lw}]} \\
 & - \phi(\beta_2 - \beta_1)^2 K_1 K_2 / (\beta_1 K_2 + \beta_2 K_1) \tag{48}
 \end{aligned}$$

In (48) all terms above the line are in the numerator, and all terms below the line are in the denominator. ϵ_{ij} and η_{ij} are the elasticities of demand of factor i in response to a change in the input price of factor j for sectors 1 and 2, respectively. The term ϕ is the elasticity of substitution in demand of good 1 for good 2 in response to a change in the relative price of the two goods. The term $\theta_1 = 1 - \theta_2$ is the share of total national income spent on good 1. Note that the formula is general with respect to the extent of noncorporate production; that is, zero noncorporate production in either one or both industries can be considered simply by specifying that K_{n1} and/or K_{n2} are zero.

While the incidence formula seems formidable, it simplifies considerably in the case that $\beta_1 = \beta_2$. In this case the terms involving the demand elasticity, ϕ , drop out, which is to be expected given (44) and (45) which indicate that there is no change in the relative price of the two goods in the case of equal initial shares. Alternatively, a simpler expression results if one assumes that:

$$\begin{aligned}\alpha_1 \varepsilon_{lw} &= (1-\beta_1) \varepsilon_{kw} , \\ \alpha_2 \eta_{lw} &= (1-\beta_2) \eta_{kw} , \\ \beta_1 \varepsilon_{kr} &= (1-\alpha_1) \varepsilon_{lr} , \\ \beta_2 \eta_{kr} &= (1-\alpha_2) \eta_{lr}\end{aligned}\tag{49}$$

The equalities in (49) hold for the CES family of production functions given in (50), where D_i is the managerial efficiency coefficient, and H_i , a_i , b_i , and ρ_i ($i=1,2$) are production function parameters. They may hold for other production functions as well, at least locally.

$$Q_i = H_i [(1-a_i-b_i)(D_i M_i)^{-\rho_i} + a_i L_i^{-\rho_i} + b_i K_i^{-\rho_i}]^{-1/\rho_i} \quad i=1,2 \tag{50}$$

With the equalities in (49), the incidence formula simplifies to:

$$\frac{-\hat{R}}{\hat{\tau}} = \frac{[(1-\beta_2)+(\beta_2-\beta_1)\theta_1][\sigma_1 K_{c1}+\sigma_2 K_{c2}] + (\beta_2-\beta_1)[\theta_2(\sigma_1-\phi)K_1-\theta_1(\sigma_2-\phi)K_2]}{[(1-\beta_2)+(\beta_2-\beta_1)\theta_1][\sigma_1 K_1+\sigma_2 K_2] + (\beta_2-\beta_1)[\theta_2(\sigma_1-\phi)K_1-\theta_1(\sigma_2-\phi)K_2]}\tag{51}$$

According to (51), as in Harberger's model, if all elasticities are equal, the burden of the tax falls 100 percent on capital. Alternatively, if the capital shares are equal and if $\sigma_1 = \sigma_2$, the incidence will also be 100 percent on capital. Unlike Harberger's model, however, the incidence of the corporate tax can be 100 percent on capital regardless of the elasticities of substitution in production. In (51) if $\beta_1 = \beta_2$, the case of equal capital shares, and if $K_{c1}/K_1 = K_{c2}/K_2$, the incidence on capital is 100 percent regardless of the values of σ_1 and σ_2 .

In the case of equal capital shares, but unequal elasticities of substitution and unequal corporate intensiveness, the incidence formula, expressed as capital's share of the tax burden, simplifies to:

$$-\frac{\hat{R}K}{\hat{\tau}K_c} = \frac{[\sigma_1 K_{c1}/K_c + \sigma_2 K_{c2}/K_c]}{[\sigma_1 K_1/K + \sigma_2 K_2/K]} \quad (52)$$

In (52) K_c stands for total corporate capital. Note that the incidence on corporate capital equals the elasticities of substitution in each sector weighted by each sector's share of corporate capital divided by the elasticities of substitution weighted by each sector's share of total capital. In the extreme cases in which σ_1 (σ_2) equals infinity, the reduction in the after-tax return to capital is the same that would arise if there was only sector 1 (2).

It may be useful to compare (52) with the corresponding Harberger incidence formula in the case of equal capital shares. This formula (see Kotlikoff and Summers (1987)) is given in (53), where sector 1 is assumed to be the corporate sector.

$$-\frac{\hat{R}}{\hat{\tau}} = \frac{K_1}{K} \left[\lambda_k + \frac{(1-\lambda_k)}{[\sigma_1 K_1/K + \sigma_2 K_2/K]} \right] \quad (53)$$

In (53) λ_k is capital's share of national income.

In contrast to the MPM formula (52), in which the incidence can be 100 percent on capital regardless of the values of σ_1 and σ_2 , in the Harberger formula (53) the incidence will generally differ from 100 percent in the case of unequal elasticities of substitution. For example, if $\sigma_1 = 0$, capital and labor are used in fixed proportions in industry 1. Hence, in the Harberger Model taxing capital in industry 1 is equivalent to taxing both factors at the same rate, and the incidence on capital equals capital's share of national income, λ_k . In the MPM, on the other hand, capital's share of the tax burden when $\sigma_1 = 0$ is $(K_{c2}/K_c)/(K_2/K)$. This term can be greater than 100 percent or close to zero depending on whether sector 2's share of corporate capital exceeds or is less than its share of total capital. The same results for the two models also hold when $\sigma_2 = \infty$. Suppose next that $\sigma_1 = \infty$ or $\sigma_2 = 0$. In this case, Harberger's formula predicts that more than 100 percent of the tax burden will fall on capital. In contrast, capital's share of the tax burden in the MPM is $(K_{c1}/K_c)/(K_1/K)$.

The limiting case in the MPM when K_{c2} approaches zero and K_{c1} approaches K_1 , is particularly instructive. Here the MPM incidence on capital approaches $\sigma_1 K/(\sigma_1 K_1 + \sigma_2 K_2)$, which is different from (53), despite the fact that the economy looks increasingly Harbergian in that virtually all corporate capital is in sector 1.

While the assumptions given in (49) which lead to (51) are satisfied by a wide class of functions, there are other quite plausible functional forms that do not satisfy (49) and imply a different incidence outcome. For example, equation (54) presents a production function which is Cobb-Douglas between managerial (entrepreneurial) input and a CES function of labor and capital.

$$Q_i = H_i (D_i M_i)^{(1-\alpha_i-\beta_i)} [\Omega_i L_i^{-\rho_i} + (1-\Omega) K_i^{-\rho_i}]^{-(\alpha_i+\beta_i)/\rho_i} \quad i=1,2 \quad (54)$$

Equation (55) gives the incidence formula in the case of equal capital shares ($\beta_1 = \beta_2 = \beta$) that results from assuming that each sector's corporate and noncorporate production functions are of the form given in (54). This equation is clearly quite different from equation (52).

$$\frac{\hat{R}}{\hat{\tau}} = \frac{\frac{(1-\beta)[(\alpha_1 \sigma_1 + \beta) K_{c1}]}{\alpha_1 + \beta} + \frac{(\alpha_2 \sigma_2 + \beta) K_{c2}}{\alpha_2 + \beta} - \frac{\beta[\alpha_1(1-\sigma_1) K_1]}{\alpha_1 + \beta} + \frac{\alpha_2(1-\sigma_2) K_2}{\alpha_2 + \beta}}{\frac{(1-\beta)[(\alpha_1 \sigma_1 + \beta) K_1]}{\alpha_1 + \beta} + \frac{(\alpha_2 \sigma_2 + \beta) K_2}{\alpha_2 + \beta} - \frac{\beta[\alpha_1(1-\sigma_1) K_1]}{\alpha_1 + \beta} + \frac{\alpha_2(1-\sigma_2) K_2}{\alpha_2 + \beta}} \quad (55)$$

2. Incidence of workers, managers, and entrepreneurs

In the MPM the wage rate is likely to fall even in cases in which capital bears more than 100 percent of the tax. In contrast to workers, managers, and capital owners, all of whom are likely to be made worse off by the corporate tax, entrepreneurs who were producing prior to the tax are made better off. The reason is that in the presence of the tax they become a relatively scarce factor input; their productive input is equivalent to that of a larger number of managers, but their productive output is untaxed. In other words, they are the sine qua non of nontaxed noncorporate production.

The formula for the change in the wage rate is determined by combining equations (44), (45), and Appendix equation (A1.7):

$$\hat{W} = - \frac{(\theta_1 \beta_1 + \theta_2 \beta_2)}{\theta_1(1-\beta_1) + \theta_2(1-\beta_2)} (\hat{R} + \hat{\tau}) \quad (56)$$

According to (56), the wage falls unless the rental rate on capital falls by more than the increase in the tax rate, that is, unless the pre-tax cost of capital falls in the corporate sector. In the CES case with equal capital income shares the wage unambiguously falls. The formula for this case is given by:

$$\hat{W} = - \frac{1}{(1-\beta)} \frac{(\alpha_1 K_{n1} + \sigma_2 K_{n2})}{(\sigma_1 K_1 + \sigma_2 K_2)} \hat{\tau} \quad (57)$$

Using (57), if both sectors are equally corporate-intensive, one can show that the decline in labor income as a share of tax revenue equals the ratio $\alpha/(1-\beta)$ times the ratio of noncorporate to corporate capital.

VI. Calculating No-Tax and Post-Tax Equilibria in the MPM and Harberger Models

1. Method of initializing model

In measuring the incidence and efficiency cost of corporate taxation in the MPM we follow Harberger and Shoven using observed average corporate tax rate. We acknowledge that the effective marginal corporate income tax may differ from the observed average corporate tax because of debt finance, as discussed by Stiglitz (1974) and Gordon Malkiel (1981). Our purpose, however, is not to question Harberger's and Shoven's choice of data, but simply to illustrate using their data and procedures how the predictions of the MPM can differ from those of the Harberger Model.

We follow Shoven in calculating no-tax and post-tax equilibria rather than simply evaluating derivative formulae, such as (51); such formulae are valid only for small tax changes and must be evaluated with data on the pre-tax equilibrium which, unfortunately, is unobservable. In calculating no-tax and post-tax equilibria we use the CES production function, given in (50), and assume the following CES utility function:

$$U = [dQ_1^{-\gamma} + (1-d)Q_2^{-\gamma}]^{-z/\gamma} \quad (58)$$

We also assume that the joint density function, $f(A,B)$, is the product of two independent exponential functions, $\phi \exp(-\phi A)$ and $\Gamma \exp(-\Gamma B)$. The values of the two parameters ϕ and Γ were chosen to produce the observed post-tax ratio of corporate to noncorporate capital in each sector. To test the sensitivity of the results to the choice of this joint density function, we also calculated no-tax and post-tax equilibria assuming a fixed number of entrepreneurs in each sector.

The observed post-tax equilibria provide us with parameter values that are used in computing the no-tax equilibria. We measure factors and goods in units such that R , W , P_1 , and P_2 equal unity in the post-tax equilibrium. We also set D_1 and D_2 equal to unity. These conventions, together with information on factor and output shares; information on average corporate tax rates in each sector; a specified level of national income, I ; the choice of σ_1 , σ_2 , and ϕ ; and the

calibrated values of Φ and Γ , permit us to solve for the values of \bar{K} and \bar{L} . Given these parameter values, solving for the no-tax equilibrium is straightforward.

To make these statements precise, Table 2 lists the equations of the MPM in the post-tax equilibrium for the case of CES production and utility functions. In Table 2 the tax rate t levied on pre-tax capital income is related to the tax rate τ on post-tax capital income according to $t = \tau / (1 + \tau)$. We express these formulae in terms of t since the U.S. corporate tax is levied on pre-tax capital income. The parameters α_{c1} and α_{c2} are the income shares of workers in the two industries in both corporate and noncorporate firms in the post-tax equilibrium. While we subscript α here by c , the corporate and noncorporate post-tax labor shares within a sector are identical. Note also that the α_{ci} terms here are post-tax shares, whereas the α_i s used above and in the Appendix are no-tax shares. The parameters β_{c1} and β_{c2} equal the corporate ratios in industries 1 and 2 of pre-tax capital income to the value of output.

The parameters α_{ci} and β_{ci} ($i=1,2$) are related to the underlying CES production function parameters as well as the tax rate t according to the formulae:

$$\alpha_{ci} = a_i^{1/(1+\rho_i)} H_i^{-\rho_i/(1+\rho_i)}, \quad \beta_{ci} = b_i^{1/(1+\rho_i)} [H_i(1-t)]^{-\rho_i/(1+\rho_i)}$$

Given the values of α_{c1} , α_{c2} , β_{c1} , β_{c2} , t , d , γ , I , ρ_1 , and ρ_2 as well as the parameters Φ and Γ , (59)-(76) can readily be solved for the values of \bar{K} and \bar{L} . Once we have these total factor supplies, we can solve for the pre-tax equilibrium using the equations listed in Table 3. These equations correspond to those of Table 2 except that the output and factor prices are now endogenous. In Table 3 there are 16 equations in the 16 unknowns K_{c1} , K_{c2} , K_{n1} , K_{n2} , L_{c1} , L_{c2} , L_{n1} , L_{n2} , R , W , P_1 , P_2 , M_{c1} , M_{c2} , A , and B . The two outputs Q_1 and Q_2 are not additional unknowns since they can be expressed via the production functions in terms of the factor inputs.

2. Parameterization of the model

Table 4 lists the values of the parameters appearing in Tables 3 and 4 that are used in calculating pre- and post-tax equilibria. In the 1957 data used by Shoven (1976), I is \$296 billion, with the capital income share equal to 0.60 in the "noncorporate" sector and 0.20 in the "corporate" sector. In analyzing the MPM we let sector 1 correspond to the "noncorporate" sector and sector 2 correspond to the "corporate" sector. The 1957 post-tax share of total national income of sector 1,

Table 2. Equations of Post-Tax Equilibrium

$$R = 1 \quad (59)$$

$$P_1 = 1 \quad (60)$$

$$W = 1 \quad (61)$$

$$P_2 = 1 \quad (62)$$

$$\underline{A} = \left| \frac{1 - \alpha_{c1} - \beta_{c1} (1-t)^{\rho_1 / (1+\rho_1)}}{(1 - \alpha_{c1} - \beta_{c1})} \right|^{-(1+\rho_1) / \rho_1} \quad (63)$$

$$\underline{B} = \left| \frac{1 - \alpha_{c2} - \beta_{c2} (1-t)^{\rho_2 / (1+\rho_2)}}{(1 - \alpha_{c2} - \beta_{c2})} \right|^{-(1+\rho_2) / \rho_2} \quad (64)$$

$$K_{c1} + K_{c2} + K_{n1} + K_{n2} = \bar{K} \quad (65)$$

$$L_{c1} + L_{c2} + L_{n1} + L_{n2} + M_{c1} + M_{c2} + E_1 + E_2 = \bar{L} \quad (66)$$

$$L_{c1} / M_{c1} = \alpha_{c1} (1 - \alpha_{c1} - \beta_{c1}) \quad (67)$$

$$L_{c2} / M_{c2} = \alpha_{c2} (1 - \alpha_{c2} - \beta_{c2}) \quad (68)$$

$$\frac{K_{c1}}{M_{c1}} = \frac{\beta_{c1} (1-t)}{(1 - \alpha_{c1} - \beta_{c1})} \quad (69)$$

$$\frac{K_{c2}}{M_{c2}} = \frac{\beta_{c2} (1-t)}{(1 - \alpha_{c2} - \beta_{c2})} \quad (70)$$

$$K_{n1} = \frac{\beta_{c1} (1-t)^{\rho_1 / (1+\rho_1)} [1 - \alpha_{c1} - \beta_{c1} (1-t)^{\rho_1 / (1+\rho_1)}]^{1/\rho_1}}{(1 - \alpha_{c1} - \beta_{c1})^{(1+\rho_1) / \rho_1}} \bar{L} \int_{\underline{A}}^{\infty} \int_0^{\underline{AB}/\underline{A}} \frac{\underline{AB}/\underline{A}}{\underline{A} \Phi} e^{-(\Phi \underline{A} + \Gamma \underline{B})} d\text{AdB} \quad (71)$$

Table 2 (concluded). Equations of Post-Tax Equilibrium

$$K_{n2} = \frac{\beta_{c2}(1-t)^{\rho_2/(1+\rho_2)}[1-\alpha_2-\beta_{c2}(1-t)^{\rho_2/(1+\rho_2)}]^{1/\rho_2}}{(1-\alpha_{c2}-\beta_{c2})^{(1+\rho_2)/\rho_2}} \bar{L} \int_A^\infty \int_0^\infty \frac{BA/B}{B\Phi} e^{-(\Phi A + \Gamma B)} dAdB \quad (72)$$

$$\frac{K_{n1}}{L_{n1}} = \frac{\beta_{c1}(1-t)^{\rho_1/(1+\rho_1)}}{\alpha_{c1}} \quad (73)$$

$$\frac{K_{n2}}{L_{n2}} = \frac{\beta_{c2}(1-t)^{\rho_2/(1+\rho_2)}}{\alpha_{c2}} \quad (74)$$

$$Q_1 + Q_2 = I \quad (75)$$

$$\frac{Q_1}{Q_2} = \left[\frac{1}{(1-d)} \right]^{1/(1+\gamma)} \quad (76)$$

Table 3. Equations of No-Tax Equilibrium

$$K_{c1} + K_{c2} + K_{n1} + K_{n2} = \bar{K} \quad (77)$$

$$L_{c1} + L_{c2} + L_{n1} + L_{n2} + M_{c1} + M_{c2} + E_1 + E_2 = \bar{L} \quad (78)$$

$$\frac{L_{c1}}{L_{c1}} = \frac{\alpha_{c1}}{(1-\alpha_{c1}-\beta_{c1})} \quad (79)$$

$$\frac{L_{c2}}{M_{c2}} = \frac{\alpha_{c2}}{(1-\alpha_{c2}-\beta_{c2})} \quad (80)$$

$$\frac{K_{c1}}{M_{c1}} = \frac{\beta_{c1}(1-t)^{\rho_1/(1+\rho_1)}}{(1-\alpha_{c1}-\beta_{c1})} \left(\frac{R}{W}\right)^{-1/(1+\rho_1)} \quad (81)$$

$$\frac{K_{c2}}{M_{c2}} = \frac{\beta_{c2}(1-t)^{\rho_2/(1+\rho_2)}}{(1-\alpha_{c2}-\beta_{c2})} \left(\frac{R}{W}\right)^{-1/(1+\rho_2)} \quad (82)$$

$$P_1 = [(1-\beta_{c1}) W^{\rho_1/(1+\rho_1)} + \beta_{c1}(R(1-t))^{\rho_1/(1+\rho_1)}]^{(1+\rho_1)/\rho_1} \quad (83)$$

$$P_2 = [(1-\beta_{c2}) W^{\rho_2/(1+\rho_2)} + \beta_{c2}(R(1-t))^{\rho_2/(1+\rho_2)}]^{(1+\rho_2)/\rho_2} \quad (84)$$

$$K_{n1} = \frac{\beta_{c1}(1-t)^{\rho_1/(1+\rho_1)}}{(1-\alpha_{c1}-\beta_{c1})} \left(\frac{R}{W}\right)^{-1/(1+\rho_1)} \bar{L} \int_0^\infty \int_{\underline{A}}^{\underline{AB}/\underline{A}} \underline{A}^\phi \Gamma e^{-(\phi A + \Gamma B)} dA dB \quad (85)$$

$$K_{n2} = \frac{\beta_{c2}(1-t)^{\rho_2/(1+\rho_2)}}{(1-\alpha_{c2}-\beta_{c2})} \left(\frac{R}{W}\right)^{-1/(1+\rho_2)} \bar{L} \int_{\underline{B}}^\infty \int_0^{\underline{BA}/\underline{B}} \underline{B}^\phi \Gamma e^{-(\phi A + \Gamma B)} dA dB \quad (86)$$

Table 3 (concluded). Equations of No-Tax Equilibrium

$$\frac{K_{n1}}{L_{n1}} = \frac{\beta_{c1}(1-t)^{\rho_1/(1+\rho_1)}}{\alpha_{c1}} \left(\frac{R}{W}\right)^{-1/(1+\rho_1)} \quad (87)$$

$$\frac{K_{n2}}{L_{n2}} = \frac{\beta_{c2}(1-t)^{\rho_2/(1+\rho_2)}}{\alpha_{c2}} \left(\frac{R}{W}\right)^{-1/(1+\rho_2)} \quad (88)$$

$$P_1 Q_1 + P_2 Q_2 = I \quad (89)$$

$$\frac{Q_1}{Q_2} = \left[\frac{d}{(1-d)}\right]^{1/(1+\gamma)} \left[\frac{P_2}{P_1}\right]^{1/(1+\gamma)} \quad (90)$$

$$\underline{A} = 1 \quad (91)$$

$$\underline{B} = 1 \quad (92)$$

Table 4. Parameter Values for the Mutual Production
and Harberger Models

\bar{L} (described in text)

\bar{K} = \$59.5 billion

σ_1 = 0.5, 0.75, 1, 2

σ_2 = 0.5, 0.75, 1, 2

ϕ = 0.5, 1

θ = 0.15

α_{c1} = 0.19

α_{c2} = 0.57

$f(A,B) = \phi \Gamma \exp-(\phi A + \Gamma B)$

ϕ, Γ (described in text)

$$\beta_{c1} = \frac{0.60 [0.017 + 0.983(1-t)^{\sigma_1}]}{0.017 + 0.983(1-t)}$$

$$\beta_{c2} = \frac{0.20 [0.630 + 0.370(1-t)^{\sigma_2}]}{0.630 + 0.370(1-t)}$$

t = 0.45

I = \$296 billion

$$\gamma = \frac{1 - \phi}{\phi}$$

$$\left(\frac{d}{1-d}\right)^{1/(1+\gamma)} = \frac{\theta}{1-\theta}$$

$$\rho_i = \frac{1 - \sigma_i}{\sigma_i} \quad (i = 1, 2)$$

θ , equals 0.15 (Shoven (1976)). The MPM requires knowledge of the post-tax shares of income paid to workers in the two sectors, α_{c1} and α_{c2} . Note that for the CES model these shares are identical for corporate and noncorporate firms within each sector. The values of α_{c1} and α_{c2} were determined using the 1959 proprietorship tax returns that report labor payments for the industries in the two sectors. ^{1/}

Determining the values of β_{c1} and β_{c2} is a bit more involved. To calculate the values of β_{c1} and β_{c2} we used the following relationships:

$$\beta_{ci} = \lambda_i \left| \frac{(K_{ci}/K_i) + (K_{ni}/K_i)(1-t)^{\sigma_i}}{(K_{ci}/K_i) + (K_{ni}/K_i)(1-t)} \right| \quad \text{for } i = 1, 2 \quad (93)$$

where λ_i stands for the sector i 's share of capital income (0.60 for sector 1 and 0.20 for sector 2). This equation simply relates capital's (unobserved) net of tax income share for corporate firms to the overall pre-tax share of capital income in the sector (which is observed). Not surprisingly, the equation involves the corporate and noncorporate shares of capital in the sector as well as the tax rate t . It also uses the following relationship between noncorporate and corporate net-of-tax capital shares:

$$\beta_{ni} = \beta_{ci}(1-t)^{\rho_i/(1+\rho_i)}.$$

The value of t , the average corporate tax rate, for 1957 is 0.45 according to data reported in The Economic Report of the President, 1987. ^{2/} To solve for the post-tax corporate and noncorporate shares of

^{1/} Harberger and Shoven measure capital income as the sum of interest, profits, rents, and property taxes. These items, except for property taxes, appear on the proprietorship tax returns. Property taxes were imputed based on their fraction of capital income as reported in Rosenberg (1969). Labor payments as a share of total factor income were weighted by industry. Lessors of real estate were used to determine values of housing. For this industry there appears to be an error in the 1959 data (the first year for which the necessary detail is available). For this reason the ratio of labor income to total factor income for the next year available, 1962, was used.

^{2/} The revenue base for measuring this tax rate is corporate profits plus interest. Note that the 0.45 tax rate, which does not consider personal taxation, is smaller than the 0.52 tax rate cited in Section II which corresponds to the 1957 differential tax on corporate versus noncorporate capital income taking into account both personal and corporate taxation. Hence, considering personal taxes would increase the estimates of excess burden in the MPM reported in Section VII.

capital within each sector entering (93), we used the following relationships:

$$\frac{K_{ci}}{K_i} = \frac{t_i(1-t)}{t(1-t_i)}, \quad \frac{K_{ni}}{K_i} = \frac{t - t_i}{t(1-t_i)} \quad i = 1, 2 \quad (94)$$

where t_i is the average corporate tax rate reported in 1957 in sector i . The specific values determined from Rosenberg's (1969) data are $t_1 = 0.014$ and $t_2 = 0.340$. Equation (94) simply exploits the idea that if the tax t is levied only on corporate firms and one observes the average tax rate t_i in sector i , where the average is computed using total sector i capital, one can infer the corporate share of the sector's capital, that is, the share of the capital that is subject to the tax t . The calculated values of K_{c1}/K_1 and K_{c2}/K_2 are 0.017 and 0.630, respectively. These values as well as the values for β_i appear in equation (93) and Table 4's formulae for β_{c1} and β_{c2} . Note that for a given value of t , β_{ci} depends on the value of σ_i .

In addition to observing indirectly the ratios of corporate to noncorporate capital in each sector, we can also indirectly observe each sector's post-tax share of total capital. ^{1/} Taken together, these ratios determine the post-tax ratios K_{n1}/\bar{K} , and K_{n2}/\bar{K} as well as the ratios K_{c1}/\bar{K} and K_{c2}/\bar{K} . These four ratios can be used to determine \bar{K} in the following manner: the four ratios plus equations (67)-(70) and (73)-(74) can be substituted using the production function into equation (75) yielding one equation in the unknown \bar{K} . Given this value we can determine the levels of K_{n1} , K_{n2} , K_{c1} , and K_{c2} . Next these capital values can be used together with equations (67)-(70) and (73)-(74) to determine L_{c1} , L_{c2} , M_{c1} , M_{c2} , L_{n1} , and L_{n2} . Plugging these six values into equation (66) and using the formulae (38) and (39) for E_1 and E_2 yields a single equation in the remaining unknown parameters \bar{L} , Φ , and Γ . Equations (71) and (72), given the values of K_{n1} and K_{n2} , represent the other two equations needed to solve for these remaining parameters. Note that these three equations involve ρ_1 and ρ_2 ; hence, the values of \bar{L} , Φ , and Γ will differ with each choice of the two elasticities of substitution in production.

^{1/} $K_1/K_2 = \theta_1 l_1(1-t_1)/\theta_2 l_2(1-t_2)$.

VII. A Comparison of Tax Incidence and Excess Burden
in the MPM and Harberger Models

1. Tax incidence

Table 5 presents the share of the tax burden borne by capital for different combinations of demand and production substitution elasticities for the MPM CES example and the Harberger Model. For many combinations of elasticities the incidence in the two models is quite different. For example, if the demand elasticity is unity and both production elasticities equal 0.5, capital bears only 82 percent of the tax burden according to the Harberger Model, but it bears 141 percent of the tax burden according to the MPM. The difference in the predicted incidence in this case is over half the total tax.

Capital's share of the tax burden can be both larger and smaller in the Harberger Model compared with the MPM; in the case that the corporate and noncorporate substitution elasticities are 0.5 and 2, respectively, and the demand elasticity is unity, capital's share of the tax burden is 61 percent in the Harberger Model, but only 27 percent in the MPM. Another difference in the incidence results is that, *ceteris paribus*, a higher demand elasticity raises capital's share of the tax burden in the MPM, but lowers capital's share in the Harberger Model.

The incidence in the MPM is a bit different for some parameters if one assumes that the number of entrepreneurs is fixed in each sector. For example, in the case that both corporate and noncorporate substitution elasticities equal 0.5 and the demand elasticity equals unity, capital's share of the tax burden is 131 percent of the revenue (compared with 141 percent in the case of a variable number of entrepreneurs). Another example is the case of a unitary demand elasticity and respective corporate and noncorporate substitution elasticities of 2 and 1. In this case the incidence on capital is 129 percent of the tax revenue when entrepreneurs are in variable supply, but 134 percent of revenue when they are in fixed supply.

Table 6 presents the incidence of the corporate tax in the MPM on workers and managers as a group and on entrepreneurs. The results are very sensitive to the choice of elasticities. For the case of unitary elasticities the income of workers and managers falls by 28 percent of the tax revenue in response to the corporate tax, while the profits of entrepreneurs rise by 28 percent. For the case that the corporate and noncorporate substitution elasticities are 1 and 2, respectively, the loss to workers exceeds 60 percent of the tax revenue, while the gain to entrepreneurs is almost a quarter of tax revenue. Finally, if the corporate and noncorporate elasticities are 2 and 0.5, respectively, the income of workers and managers rises by over one third of revenue, while that of entrepreneurs rises by almost one fifth of revenue.

Table 5. Comparison of the Incidence of the Corporate
Income Tax in the MPM and Harberger Models

Elasticity of Substitution in Production		Share of Tax Burden Falling on Capital (by Demand Elasticity)			
		Mutual		Harberger Model	
		Production Model			
Corporate	Noncorporate	$\phi = 0.5$	$\phi = 1$	$\phi = 0.5$	$\phi = 1$
1.00	1.00	0.94	1.00	1.08	1.00
2.00	2.00	1.02	1.03	1.15	1.12
0.75	0.75	1.00	1.10	1.04	0.93
0.50	0.50	1.23	1.41	0.97	0.82
1.00	2.00	0.60	0.63	0.93	0.87
1.00	0.75	1.11	1.20	1.13	1.04
1.00	0.50	1.37	1.47	1.19	1.08
2.00	1.00	1.26	1.29	1.27	1.22
0.75	1.00	0.79	0.88	0.99	0.89
0.50	1.00	0.58	0.71	0.84	0.73
2.00	0.50	1.53	1.57	1.34	1.27
0.50	2.00	0.21	0.27	0.68	0.61

Table 6. Incidence of the Corporate Income Tax on
Workers and Managers and on Entrepreneurs

Elasticity of Substitution in Production		Incidence as Share of Tax Burden			
Corporate	Noncorporate	Workers and Managers		Entrepreneurs	
		$\phi = 0.5$	$\phi = 1$	$\phi = 0.5$	$\phi = 1$
1.00	1.00	0.34	0.28	-0.28	-0.28
2.00	2.00	0.12	0.12	-0.14	-0.15
0.75	0.75	0.32	0.22	-0.32	-0.33
0.50	0.50	0.16	0.00	-0.39	-0.41
1.00	2.00	0.63	0.61	-0.23	-0.24
1.00	0.75	0.18	0.10	-0.29	-0.31
1.00	0.50	-0.05	-0.14	-0.32	-0.33
2.00	1.00	-0.10	-0.12	-0.16	-0.17
0.75	1.00	0.51	0.43	-0.30	-0.31
0.50	1.00	0.73	0.61	-0.31	-0.32
2.00	0.50	-0.34	-0.38	-0.19	-0.19
0.50	2.00	1.04	1.00	-0.25	-0.27

2. Excess burden

a. Estimates

Table 7 compares the deadweight loss in the two models. Our excess burden measure is based on a compensating variation. We determine the amount of additional income needed in the post-tax equilibrium to regain the no-tax level of utility. Measuring the distortion based on an equivalent variation, that determines the amount of pre-tax income that needs to be taken away to achieve the post-tax utility level, yielded quite similar results.

The excess burden measured as a fraction of tax revenue is more than ten times larger in the MPM than in the Harberger Model for all elasticity combinations. For most of the combinations, the deadweight loss in the MPM exceeds the tax revenue. Consider the case of unitary production and demand elasticities. In this case the excess burden in the Harberger Model is only 8 percent of revenue, while it is 123 percent of revenue in the MPM. Even if all elasticities are smaller, for example, 0.5, the MPM predicts a sizable distortion, 129 percent of revenue, while the Harberger Model's predicted distortion is only 4 percent of revenue.

The excess burden in the MPM is considerably smaller if one assumes that entrepreneurs are in fixed supply. For example, in the case that all elasticities are unity, holding fixed the number of entrepreneurs reduces the excess burden from 123 percent to 74 percent of revenue. If all elasticities equal 0.5, the excess burden is reduced from 129 percent of revenue to 66 percent of revenue. But even with the supply of entrepreneurs fixed, the excess burden in the MPM is still at least seven times that in the Harberger Model.

b. Understanding the differences in excess burden

One reason that the excess burden in the MPM is so much greater than in the Harberger Model involves the size of the distortionary tax rates in the two analyses. Although the MPM results are based on the same tax data, including the same tax revenue, the effective distortionary wedge in the MPM is 82 percent, while it is only 50 percent in the Harberger-Shoven procedure. Since excess burden roughly rises with the square of the tax rate, the difference in effective distortionary taxes can, by itself, account for an MPM excess burden that is 2.6 times the Harberger Model's excess burden.

To understand these differences note that in the MPM the economy-wide average corporate tax rate, calculated as total corporate revenues divided by total corporate income, is 0.45. In terms of the model's tax variable τ , this value of 0.45 for t corresponds to a value of τ of 0.82, since $\tau = t/(1-t)$. With such a large distortionary tax, the considerable size of the distortion in the MPM is not surprising. In contrast, in the Harberger-Shoven analysis the distortionary corporate

Table 7. Comparison of the Excess Burden of the Corporate Income Tax in the MPM and Harberger Models

Elasticity of Substitution in Production		Excess Burden Divided by Tax Revenue (by Demand Elasticity)			
		Mutual		Harberger Model	
		Production Model			
Corporate	Noncorporate	$\phi = 0.5$	$\phi = 1$	$\phi = 0.5$	$\phi = 1$
1.00	1.00	1.22	1.23	0.07	0.08
2.00	2.00	1.22	1.22	0.11	0.13
0.75	0.75	1.24	1.26	0.05	0.07
0.50	0.50	1.29	1.30	0.04	0.05
1.00	2.00	0.99	1.00	0.08	0.10
1.00	0.75	1.29	1.30	0.06	0.08
1.00	0.50	1.37	1.38	0.06	0.07
2.00	1.00	1.39	1.40	0.07	0.10
0.75	1.00	1.16	1.18	0.06	0.07
0.50	1.00	1.10	1.13	0.05	0.06
2.00	0.50	1.50	1.51	0.11	0.13
0.50	2.00	0.84	0.85	0.06	0.06

tax is the difference between the average corporate tax rates in the two sectors. But this average tax in each sector is computed based on total sector capital income, not simply the corporate income in the sector. By averaging over noncorporate as well as corporate capital in determining the tax rates in each sector, Harberger and Shoven dilute the effective distortionary corporate tax. Since $t_1 = 0.014$ and $t_2 = 0.340$, the effective distortionary tax in the Harberger-Shoven procedure is only 0.50, which corresponds to $(0.340 - 0.014)/[(1 - 0.340)(1 - 0.014)]$.

The second reason that the excess burden is so much larger in the MPM than in the Harberger Model involves differences in the two models in the source of inefficiency in conjunction with differences in within-sector and between-sector demand elasticities. To understand this point first note that the approximation formula for excess burden is the same in both models, namely $0.5\tau^2 \partial K_c / \partial \tau$, where K_c stands for total corporate capital. But the change in corporate capital in the MPM is due, ultimately, to within-sector substitution of noncorporate capital as well as other factors for corporate capital. Indeed, were there no within-sector substitution, that is, were there no noncorporate production either before or after the tax, $\partial K_c / \partial \tau$ in the MPM would be zero. In contrast to the MPM, in the Harberger Model $\partial K_c / \partial \tau$ is negative because of between-sector substitution of capital.

The fact that the MPM's ultimate source of the inefficiency is within-sector rather than between-sector changes in capital does not by itself suggest that excess burden is larger in the MPM. But one needs to consider these differences in the source of excess burden in light of differences in the within- versus between-sector elasticities of demands for corporate and noncorporate goods. In the MPM the within-sector demand elasticity between corporate and noncorporate output is infinite. In contrast, in both models, the between-sector demand elasticity between corporate and noncorporate goods is assumed to be small, typically unity or less. To appreciate how this difference in demand elasticities may affect the reduction in corporate capital and, thus, excess burden, consider how excess burden in the Harberger Model changes as the between-sector demand elasticity increases. Assuming unitary elasticities of substitution in production, raising the demand elasticity in the Harberger Model from unity to 10 increases the excess burden by a factor of 3. Together with the 2.6 factor arising from differences in effective tax wedges, this factor of 3 suggests an excess burden in the MPM that could easily exceed that in the Harberger Model by a factor of 7.

c. Does the method of aggregation affect the MPM results?

Harberger allocated U.S. industries to "corporate" and "noncorporate" sectors based on the size of their average tax. While this was appropriate for Harberger's purposes, his two-sector division is not necessarily the most appropriate two-sector division from the

perspective of the MPM. Hence, it is important to understand how the MPM results would differ if the two sectors were chosen differently.

If all production functions of the underlying products are locally identical (all own- and cross-factor demand elasticities as well as factor shares are the same for all goods) the method of aggregation affects neither incidence nor excess burden in the MPM. To see this, consider the general formula of tax incidence, equation (48). In the case that sectors 1 and 2 have the same technology locally, the incidence on capital is independent of both the ratio of K_1 and K_2 and the degree of corporate intensity in the two sectors. Next consider the triangle approximation formula for excess burden in the MPM. As in the Harberger Model, this formula is given by $0.5\tau^2 dK_c/d\tau$. Some additional differentiation of the MPM leads to the following formula for $dK_c/d\tau$:

$$\begin{aligned} \frac{dK_c}{d\tau} = & \hat{k}_{n1} K_{n1} + \hat{k}_{n2} K_{n2} + \left(\frac{\beta_1}{1-\alpha_1-\beta_1} \right)^2 \underline{wA} \int_0^B f(\underline{A}, B) dB \\ & + \left(\frac{\beta_2}{1-\alpha_2-\beta_2} \right)^2 \underline{wB} \int_0^A f(A, \underline{B}) dA \\ & + \left(\frac{\beta_2}{1-\alpha_1-\beta_1} - \frac{\beta_2}{1-\alpha_2-\beta_2} \right)^2 \underline{w} \frac{A}{B^2} \int_{\underline{B}}^{\bar{B}} B^2 f\left(\frac{A}{B}, B\right) dB \end{aligned} \quad (95)$$

When sectors 1 and 2 have the same technology locally, the percentage change in k_{n1} equals the percentage change in k_{n2} . Hence, equation (95) indicates that when the two technologies are locally identical, $dK_c/d\tau$ is independent of the ratio of K_{n1} to K_{n2} . In this case the MPM's excess burden is the same regardless of whether one allocates all noncorporate production to one sector or spreads the noncorporate output between the two sectors.

This last statement may seem surprising in light of the above argument that it is the infinite within-sector corporate-noncorporate demand elasticity that explains the large excess burden. How can the MPM excess burden be large if the within-sector demand elasticity is not relevant for one of the sectors because the sector is totally corporate? The answer is that, in the case of identical local technologies, all that is needed is one sector in which the demand elasticity between corporate and noncorporate goods is infinite, provided that sector has all noncorporate capital.

From equation (95) and some additional calculations with the model it appears that excess burden in the MPM is larger if one chooses the two sectors to maximize the difference between capital income shares. In other words, aggregations that do not maximize the differences between capital income shares will understate the excess burden. Oddly enough, Harberger's division of output into two sectors produces two sectors with quite different capital income shares. Hence, we believe the excess burden reported above for the MPM would be only slightly larger for other choices of the two sectors with even more divergent capital income shares. This discussion and our additional calculations also suggest that disaggregating the MPM into more than two sectors would also increase the excess burden.

VIII. Summary and Suggestions for Additional Research

The model developed in this paper exhibits mutual production of each good by corporate and noncorporate firms. Noncorporate firms arise endogenously; those individuals who choose to become entrepreneurs are more efficient than corporate managers, but since their managerial input is fixed, their output is subject to diminishing returns. In addition, entrepreneurs can only produce by themselves. Corporate firms have a technological advantage relative to less efficient entrepreneurs in producing at a large scale. Hence, corporate firms co-exist with the limited number of more efficient entrepreneurs both prior to and after the introduction of a corporate income tax, although the tax induces production by less efficient entrepreneurs. While the efficiency of their managerial input differs, both corporate and noncorporate firms within each sector have identical production functions.

In contrast to the Mutual Production Model, Harberger's model of the corporate tax collapses in the presence of corporate and noncorporate production of the same good, with the corporate sector disappearing. Harberger's model is really not a model of a tax on income of corporations per se. Instead of analyzing a tax on the corporate form, Harberger models the differential taxation of capital used in the production of different goods. The U.S. corporate income tax, however, does not apply differentially to producers of different commodities; rather, it is levied on corporate as opposed to noncorporate firms. By ignoring noncorporate production in primarily corporate sectors and corporate production in primarily noncorporate sectors, Harberger's model ignores the potential substitution of noncorporate for corporate production within each sector.

This within-sector substitution permits a source of inefficiency not included in Harberger's formulation, namely the substitution of less efficient noncorporate production for more efficient corporate production. The within-sector substitution explains why the deadweight loss from corporate taxation is many times larger in the MPM than in the Harberger Model. Indeed, the excess burden in the CES illustration of the MPM is typically larger than the corporate tax revenue.

The within-sector substitution also obviates much of the source of relative price changes arising in the Harberger Model and alters the effects of sectoral differences in substitution elasticities. These are the major reasons why the incidence of the corporate income tax can differ so greatly in the MPM and Harberger Model. In contrast to the Harberger incidence formula, demand effects drop out of the MPM incidence formula if both sectors have CES production functions and also have the same initial shares of capital income. If, in addition, each sector is equally corporate-intensive, the MPM's tax incidence is 100 percent on capital regardless of differences in substitution elasticities between the two sectors.

The CES results for 1957, while striking, must be viewed in perspective. First, we have followed Harberger and Shoven in using average, rather than effective marginal, tax rates in the calculations and in ignoring personal taxes. Using Gravelle's (1987) estimates of the total (corporate plus personal) marginal tax wedge would raise our estimate of excess burden for 1957. Parenthetically, Gravelle's estimates indicate a much lower excess burden for 1987 and a much larger efficiency gain from the 1986 tax reform act than has previously been suggested. Second, like Harberger and Shoven, we have ignored depreciation; proper adjustment for depreciation would reduce the deadweight loss estimates. Third, if marginal debt-equity ratios differ from average debt-equity ratios, the marginal effective tax wedge would differ from the average corporate tax rate considered in this paper as well as the total (personal plus corporate) tax wedge calculated by Gravelle (1987). Fourth, the results might differ if the model were disaggregated, as in Shoven (1976) and Ballard, et. al. (1985), to include many sectors. Fifth, the joint density function determining entrepreneurs' abilities should be estimated empirically. Sixth, the model could be expanded to treat managerial and entrepreneurial input, on the one hand, and labor, on the other hand, as distinct production factors. And seventh, the assumption that corporate and noncorporate production functions are identical within a sector needs to be tested and potentially relaxed. These issues provide ample scope for additional research.

In conclusion, perhaps the best celebration of the 25th anniversary of Harberger's remarkably influential model would be a rebirth of analytical attention to the questions of what constitutes a corporation and what precisely does the corporation income tax tax.

Appendix

The $\hat{}$ symbol refers below to the percentage change in a variable. All prices initially equal 1.

The differential of the demand equation (40) can be expressed as:

$$\hat{Q}_1 - \hat{Q}_2 = \phi(\hat{P}_2 - \hat{P}_1) \quad (A1)$$

where ϕ is the elasticity of substitution between the two products.

The differential of output in industry 1 is:

$$\begin{aligned} \hat{Q}_1 &= (1-\alpha_1\beta_1)\hat{M}_{c1} \frac{Q_{c1}}{Q_1} + \beta_1\hat{K}_{c1} \frac{Q_{c1}}{Q_1} + \alpha_1\hat{L}_{c1} \frac{Q_{c1}}{Q_1} \\ &+ \frac{\bar{L}}{Q_1} = \int_{\underline{A}}^{\bar{A}} \int_0^{\frac{BA}{\underline{A}}} [dl_{n1}(A) + dK_{n1}(A)] f(A,B) dBdA \\ &- \bar{L} \frac{dA}{Q_1} \int_0^B [1+l_{n1}(\underline{A}) + k_{n1}(\underline{A})] f(\underline{A},B) dB \\ &- \bar{L} \frac{\left| \frac{dA}{\underline{A}^2} \frac{B}{\underline{A}} - \frac{dB}{\underline{A}} \right|}{Q_1} \int_{\underline{A}}^{\bar{A}} \left[\frac{\underline{A}}{\underline{A}} + l_{n1}(A) + k_{n1}(A) \right] f\left(A, \frac{B,A}{\underline{A}}\right) AdA \quad (A2) \end{aligned}$$

The change in output in industry 2 can similarly be written as:

$$\begin{aligned} \hat{Q}_2 &= (1-\alpha_2\beta_2)\hat{M}_{c2} \frac{Q_{c2}}{Q_2} + \beta_2\hat{K}_{c2} \frac{Q_{c2}}{Q_2} + \alpha_2\hat{L}_{c2} \frac{Q_{c2}}{Q_2} \\ &+ \frac{\bar{L}}{Q_2} \int_{\underline{B}}^{\bar{B}} \int_0^{\frac{A}{\underline{B}}} [dl_{n2}(B) + dk_{n2}(B)] f(A,B) dAdB \end{aligned}$$

$$\begin{aligned}
 & - \frac{\bar{L}}{Q_2} \int_0^{\underline{A}} [1 + l_{n2}(\underline{B}) + k_{n2}(\underline{B})] f(\underline{A}, \underline{B}) d\underline{A} \\
 & - \frac{\left| \frac{d\underline{B}}{\underline{B}^2} \frac{\underline{A}}{\underline{B}} - \frac{d\underline{A}}{\underline{B}} \right| \bar{L}}{Q_2} \int_{\underline{B}}^{\bar{B}} \left[\frac{\underline{B}}{\underline{B}} + l_{n2}(\underline{B}) + k_{n2}(\underline{B}) \right] f\left(\frac{\underline{AB}}{\underline{B}}, \underline{B}\right) B d\underline{B} \quad (A3)
 \end{aligned}$$

We combine (A1), (A2), and (A3) noting that: $\hat{K}_{c1} = \hat{M}_{c1} + \hat{k}_{c1}$,
 $\hat{L}_{c1} = \hat{M}_{c1} + \hat{l}_{c1}$, $\hat{K}_{c2} = \hat{M}_{c2} + \hat{k}_{c2}$, $\hat{L}_{c2} = \hat{M}_{c2} + \hat{l}_{c2}$, $Q_{c1}/Q_1 = K_{c1}/K_1$,
 and $Q_{c2}/Q_2 = K_{c2}/K_2$.

$$\begin{aligned}
 & [\hat{M}_{c1} + \beta_1 \hat{k}_{c1} + \alpha_1 \hat{l}_{c1}] \frac{K_{c1}}{K_1} + \frac{\bar{L}}{Q_1} \int_{\underline{A}}^{\bar{A}} \int_0^{\frac{\underline{BA}}{\underline{A}}} [dl_{n1}(\underline{A}) + dk_{n1}(\underline{A})] f(\underline{A}, \underline{B}) d\underline{B} d\underline{A} \\
 & - \frac{\bar{L} d\underline{A}}{Q_1} \int_0^{\underline{B}} [1 + l_{n1}(\underline{A}) + k_{n1}(\underline{A})] f(\underline{A}, \underline{B}) d\underline{B} - \frac{\left| \frac{d\underline{AB}}{\underline{A}^2} \frac{d\underline{B}}{\underline{A}} \right| \bar{L}}{Q_1} \int_{\underline{A}}^{\bar{A}} \left(\frac{\underline{A}}{\underline{A}} + l_{n1}(\underline{A}) + k_{n1}(\underline{A}) \right) \\
 & \quad \times \bar{L} f\left(\underline{A}, \frac{\underline{BA}}{\underline{A}}\right) d\underline{A} \\
 & - (\hat{M}_{c2} + \beta_2 \hat{k}_{c2} + \alpha_2 \hat{l}_{c2}) \frac{K_{c2}}{K_2} - \frac{\bar{L}}{Q_2} \int_{\underline{B}}^{\bar{B}} \int_0^{\frac{\underline{BA}}{\underline{B}}} [dl_{n2}(\underline{B}) + dk_{n2}(\underline{B})] f(\underline{A}, \underline{B}) d\underline{B} d\underline{A} \\
 & + \frac{\bar{L}}{Q_2} \int_0^{\underline{A}} [1 + l_{n2}(\underline{B}) + k_{n2}(\underline{B})] f(\underline{A}, \underline{B}) d\underline{A} + \left(\frac{d\underline{BA}}{\underline{B}^2} - \frac{d\underline{A}}{\underline{B}} \right) \\
 & \quad \times \bar{L} \int_{\underline{B}}^{\bar{B}} \left(\frac{\underline{B}}{\underline{B}} + l_{n2}(\underline{B}) + k_{n2}(\underline{B}) \right) f\left(\frac{\underline{AB}}{\underline{B}}, \underline{B}\right) B d\underline{B} = \phi(\hat{P}_2 - \hat{P}_1) \quad (A1')
 \end{aligned}$$

We now differentiate (29) and divide by $M_{c1}k_{c1}$ (that is, by K_{c1}) which yields:

$$\begin{aligned}
 0 = & \hat{M}_{c1} + \hat{k}_{c1} + \hat{M}_{c2} \frac{M_{c2} k_{c2}}{M_{c1} k_{c1}} + \frac{M_{c2}}{M_{c1}} \frac{k_{c2}}{k_{c1}} \hat{k}_{c2} \\
 & + \frac{\bar{L}}{M_{c1} k_{c1}} \int_{\underline{A}}^{\bar{A}} \int_0^{\frac{\bar{BA}}{\underline{A}}} dk_{n1}(\underline{A}) f(\underline{A}, B) d\bar{B} d\underline{A} - \frac{\bar{L} d\underline{A}}{M_{c1} k_{c1}} \int_0^{\bar{B}} k_{n1}(\underline{A}) f(\underline{A}, B) dB \\
 & - \frac{\bar{L}}{M_{c1} k_{c1}} \left(\frac{d\bar{AB}}{A^2} - \frac{dB}{\underline{A}} \right) \int_{\underline{A}}^{\bar{A}} k_{n1}(\underline{A}) f\left(\underline{A}, \frac{\bar{BA}}{\underline{A}}\right) d\underline{A} \\
 & + \frac{\bar{L}}{M_{c1} k_{c1}} \int_{\underline{B}}^{\bar{B}} \int_0^{\frac{\bar{AB}}{\underline{B}}} dk_{n2}(\underline{B}) f(\underline{A}, B) d\bar{A} d\underline{B} - \frac{\bar{L} d\underline{B}}{M_{c1} k_{c1}} \int_0^{\bar{A}} k_{n2}(\underline{B}) f(\underline{A}, \underline{B}) d\underline{A} \\
 & - \frac{\bar{L}}{M_{c1} k_{c1}} \left(\frac{d\bar{BA}}{\underline{B}^2} - \frac{d\underline{A}}{\underline{B}} \right) \int_{\underline{B}}^{\bar{B}} k_{n2}(\underline{B}) f\left(\frac{\bar{AB}}{\underline{B}}, \underline{B}\right) d\underline{B} \quad (A4)
 \end{aligned}$$

The first four terms represent the changes in corporate capital in the two sectors. The next three terms are the changes in capital for noncorporate production in industry 1 due, respectively, to changes in capital in existing firms, changes due to new firms established by former corporate managers and workers, and by new firms established by entrepreneurs who previously had firms in industry 2.

We similarly differentiate equation (28) for a fixed stock of labor and divide by $M_{c1} l_{c1}$ (which equals L_{c1}).

$$\begin{aligned}
 0 = & \hat{M}_{c1} \frac{[1+l_{c1}]}{l_{c1}} + \hat{l}_{c1} + \frac{\hat{M}_{c2} M_{c2} (1+l_{c2})}{M_{c1} l_{c1}} + \frac{M_{c2} l_{c2} \hat{l}_{c2}}{M_{c1} l_{c1}} \\
 & + \frac{\bar{L}}{M_{c1} l_{c1}} \int_{\underline{A}}^{\bar{A}} \int_0^{\frac{\bar{BA}}{\underline{A}}} dl_{n1}(\underline{A}) f(\underline{A}, B) d\bar{B} d\underline{A} - \frac{\bar{L} d\underline{A}}{M_{c1} l_{c1}} \int_0^{\bar{B}} [1+l_{n1}(\underline{A})] f(\underline{A}, B) dB \\
 & - \frac{\bar{L}}{M_{c1} l_{c1}} \left(\frac{d\underline{A} B}{A^2} - \frac{dB}{\underline{A}} \right) \int_{\underline{A}}^{\bar{A}} [1+l_{n1}(\underline{A})] f\left(\underline{A}, \frac{\bar{BA}}{\underline{A}}\right) d\underline{A}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{\bar{L}}{M_{c1} l_{c1}} \int_{\underline{B}}^{\bar{B}} \int_0^{\frac{AB}{\underline{B}}} d l_{n2}(\underline{B}) f(\underline{A}, \underline{B}) d \underline{A} d \underline{B} - \frac{d \underline{B}}{M_{c1} l_{c1}} \frac{\bar{L}}{c1} \int_0^{\bar{A}} [1 + l_{n2}(\underline{B})] f(\underline{A}, \underline{B}) d \underline{A} \\
 & - \frac{\bar{L}}{M_{c1} l_{c1}} \left(\frac{d \underline{B}}{B^2} \frac{\underline{A}}{B} - \frac{d \underline{A}}{\underline{B}} \int_{\underline{B}}^{\bar{B}} [1 + l_{n2}(\underline{B})] f\left(\frac{AB}{\underline{B}}, \underline{B}\right) \right) \Big|_{\underline{B}} d \underline{B} \quad (A5)
 \end{aligned}$$

To solve the model, we solve equation (A1.4) for \hat{M}_{c1} and substitute into (A1') and into (A5). Then the new equation for (A5) is solved for \hat{M}_{c2} . The resulting expression for \hat{M}_{c2} is then substituted into the final version of (A1'). In the course of this substitution several of the terms with integrals will cancel out. When equation (A1.4) is substituted into (A5'), the term

$$\frac{\bar{L}}{M_{c1} k_{c1}} \frac{d \underline{A}}{c1} \int_0^{\underline{B}} k_{n1}(\underline{A}) f(\underline{A}, \underline{B}) d \underline{B}$$

which is now multiplied by $\frac{(1 + l_{c1})}{l_{c1}}$ will cancel the term

$$\frac{\bar{L}}{M_{c1} l_{c1}} \frac{d \underline{A}}{c1} \int_0^{\underline{B}} [1 + l_{n1}(\underline{A})] f(\underline{A}, \underline{B}) d \underline{B}$$

since $l_{n1}(\underline{A}) = l_{c1}$, and $k_{n1}(\underline{A}) = k_{c1}$. When equation (A1.4) is substituted into equation (A1.1'), the term

$$\frac{\bar{L}}{M_{c1} k_{c1}} \frac{d \underline{A}}{c1} \int_0^{\underline{B}} k_{n1}(\underline{A}) f(\underline{A}, \underline{B}) d \underline{B}$$

which is now multiplied by K_{c1}/K_1 (recall that $[M_{c1} k_{c1} = K_{c1}]$) will cancel

$$\frac{\bar{L}}{Q_1} \frac{d \underline{A}}{c1} \int_0^{\underline{B}} [1 + l_{n1}(\underline{A}) + k_{n1}(\underline{B})] f(\underline{A}, \underline{B}) d \underline{B}$$

by virtue of the fact that

$$\frac{1+l_{n1}(\underline{A}) + k_{n1}(\underline{A})}{Q_1} = \frac{k_{n1}(\underline{A})}{K_1}.$$

By the same reasoning the term

$$\frac{\bar{L}}{M_{c1} k_{c1}} \left(\frac{dA}{\underline{A}^2} \underline{B} - \frac{dB}{\underline{A}} \right) \int_{\underline{A}}^{\bar{A}} k_{n1}(A) f\left(A, \frac{BA}{\underline{A}}\right) AdA$$

will cancel the term

$$\frac{\bar{L}}{Q_1} \left(\frac{dA}{\underline{A}^2} \underline{B} - \frac{dB}{\underline{A}} \right) \int_{\underline{A}}^{\bar{A}} \left(\frac{A}{\underline{A}} + l_{n1}(A) + k_{n1}(A) \right) f\left(A, \frac{BA}{\underline{A}}\right) AdA.$$

The same reasoning will also result in terms of canceling out when the new (A1.5) is substituted into (A1').

The only integrals which do not cancel out through this process are those relating to the switching of existing entrepreneurs from one industry to another. These terms, which are each multiplied by

$$\left(\frac{1}{k_{c1}} + \frac{1}{k_{c2}} \right) \left(\frac{1}{(1+l_{c1})k_{c2}} - \frac{1}{(1+l_{c2})k_{c1}} \right), \text{ are given by:}$$

$$\begin{aligned} & - L(1+l_{c2})k_{c1} \left(\frac{dB}{\underline{B}^2} \underline{A} - \frac{dA}{\underline{B}} \right) \int_{\underline{B}}^{\bar{B}} k_{n2}(B) f\left(\frac{A,B}{\underline{B}}, B\right) BdB \\ & + \bar{L} k_{c1} k_{c2} \left(\frac{dB}{\underline{B}^2} \underline{A} - \frac{dA}{\underline{B}} \right) \int_{\underline{B}}^{\bar{B}} [1+l_{n2}(B)] f\left(\frac{A,B}{\underline{B}}, B\right) BdB \\ & - \bar{L}(1+l_{c1})k_{c2} \left(\frac{dA}{\underline{A}^2} \underline{B} - \frac{dB}{\underline{A}} \right) \int_{\underline{A}}^{\bar{A}} k_{n1}(A) f\left(A, \frac{B,A}{\underline{A}}\right) AdA \\ & + \bar{L} k_{c1} k_{c2} \left(\frac{dA}{\underline{A}^2} \underline{B} - \frac{dB}{\underline{A}} \right) \int_{\underline{A}}^{\bar{A}} [1+l_{n1}(A)] f\left(A, \frac{B,A}{\underline{A}}\right) AdA \end{aligned}$$

Note, however, that if we bring l_{c2} with the first term inside the integral it becomes $l_{c2}B$ which is $l_{n2}(B)B$, and if we bring k_{c2} in the second term into the integral it becomes $k_{c2}B$ which is $k_{n2}(B)B$. Thus these parts of the integral will cancel, and similarly, parts of the second two terms will cancel. Moreover, since the number of entrepreneurs switching must net to zero, the terms with "1" cancel.

We are left with:

$$\begin{aligned} & \bar{l}k_{c1} \left(\frac{dB}{B^2} \frac{A}{B} - \frac{dA}{B} \right) \int_{\underline{B}}^{\bar{B}} k_{n2}(B) f\left(\frac{A}{B}, B\right) B dB \\ & - \bar{l}k_{c2} \left(\frac{dA}{A^2} \frac{B}{A} - \frac{dB}{A} \right) \int_{\underline{A}}^{\bar{A}} k_{n1}(A) f\left(A, \frac{B}{A}\right) A dA \end{aligned}$$

By substituting $k_{n2}(B) = k_{c2} \frac{B}{\underline{B}}$ and $k_{n1}(A) = k_{c1} \frac{A}{\underline{A}}$, and expressing

$\frac{dB}{B}$ as $\hat{\underline{B}}$ and $\frac{dA}{A}$ as $\hat{\underline{A}}$:

$$\begin{aligned} & - \bar{l}k_{c1}k_{c2} \frac{A}{\underline{B}^2} [\hat{\underline{B}} - \hat{\underline{A}}] \int_{\underline{B}}^{\bar{B}} B^2 f\left(\frac{AB}{\underline{B}}, B\right) dB \\ & + \bar{l}k_{c1}k_{c2} \frac{B}{\underline{A}^2} [\hat{\underline{B}} - \hat{\underline{A}}] \int_{\underline{A}}^{\bar{A}} A^2 f\left(A, \frac{BA}{\underline{A}}\right) dA \end{aligned}$$

By using the relationship $A = \frac{A}{\underline{B}} B$, which holds along the path of integration, these terms also cancel.

We note that $\frac{1}{(1+l_{c1})k_{c2} - (1+l_{c2})k_{c1}}$ which appears in the final

equation is equal to $(\beta_2 - \beta_1)q_{cq}q_{c2}$, where the q terms stand for output per corporate manager. The terms \hat{k}_{n1} and \hat{k}_{n2} , and so forth refer to changes in noncorporate capital and refer to the solution to the integrals which reflect changes in capital and labor of existing noncorporate firms. The final equation is:

$$\begin{aligned} & [(K_{c1}\hat{k}_{c1} + K_{n1}\hat{k}_{n1})(1-\beta_1) + (K_{c2}\hat{k}_{c2} + K_{n2}\hat{k}_{n2})(1-\beta_2) \\ & - (K_{c1}\hat{l}_{c1} + K_{n1}\hat{l}_{n1})\alpha_1 - (K_{c2}\hat{l}_{c2} + K_{n2}\hat{l}_{n2})\alpha_2] \frac{1}{\beta_2 - \beta_1} \left(\frac{\beta_1}{K_1} + \frac{\beta_2}{K_2} \right) = \phi(\hat{P}_2 - \hat{P}_1) \end{aligned} \quad (A1'')$$

We now substitute into (A1.1'') from equations (A1.6). In this substitution we use the facts that $(\alpha_1/\beta_1)\epsilon_{1r} = \epsilon_{kw}$ and $(\alpha_2/\beta_2)\eta_{1r} = \eta_{kw}$.

$$\begin{aligned} \hat{l}_{c1} &= \epsilon_{1w}(\hat{W} - \hat{P}_1) + \epsilon_{1r}(\hat{R} + \hat{\tau} - \hat{P}_1) \\ \hat{k}_{c1} &= \epsilon_{kw}(\hat{W} - \hat{P}_1) + \epsilon_{kr}(\hat{R} + \hat{\tau} - \hat{P}_1) \\ \hat{l}_{n1} &= \epsilon_{1w}(\hat{W} - \hat{P}_1) + \epsilon_{1r}(\hat{R} - \hat{P}_1) \\ \hat{k}_{n1} &= \epsilon_{kw}(\hat{W} - \hat{P}_1) + \epsilon_{kr}(\hat{R} - \hat{P}_1) \\ \hat{l}_{c2} &= \eta_{1w}(\hat{W} - \hat{P}_2) + \eta_{1r}(\hat{R} + \hat{\tau} - \hat{P}_2) \\ \hat{k}_{c2} &= \eta_{kw}(\hat{W} - \hat{P}_2) + \eta_{kr}(\hat{R} + \hat{\tau} - \hat{P}_2) \\ \hat{l}_{n2} &= \eta_{1w}(\hat{W} - \hat{P}_2) + \eta_{1r}(\hat{R} - \hat{P}_2) \\ \hat{k}_{n2} &= \eta_{kw}(\hat{W} - \hat{P}_2) + \eta_{kr}(\hat{R} - \hat{P}_2) \end{aligned} \quad (A6)$$

The incidence formula (48) results from the above substitution and the relationships (A1.7) - (A1.11) given below:

$$\hat{P}_1\theta_1 + \hat{P}_2\theta_2 = 0, \quad (A7)$$

which is the result of differentiating (43).

$$\hat{W} - \hat{P}_1 = - \frac{\beta_1(\hat{R} + \hat{\tau})}{\text{Den.}} \quad (A8)$$

$$\hat{W} - \hat{P}_2 = - \frac{\beta_2(\hat{R} + \hat{\tau})}{\text{Den.}} \quad (\text{A9})$$

$$(\hat{R} - \hat{P}_1) = \frac{(1 - \beta_1)\hat{R} - (\theta_2)(\beta_1 - \beta_2)\hat{\tau}}{\text{Den.}} \quad (\text{A10})$$

$$(\hat{R} - \hat{P}_2) = \frac{(1 - \beta_2)\hat{R} - (\theta_1)(\beta_2 - \beta_1)\hat{\tau}}{\text{Den.}} \quad (\text{A11})$$

In (A8) - (A11) the term $\text{Den} = \theta_1(1 - \beta_1) + \theta_2(1 - \beta_2)$.

For the CES function given in (50) it is easy to show that the factor demands for capital per manager and labor per manager satisfy:

$$\hat{k}_{ci} = \frac{\sigma_i[(1 - \alpha_i)(\hat{R} + \hat{\tau} - \hat{P}_i) + \alpha_i(\hat{W} - \hat{P}_i)]}{(1 - \alpha_i - \beta_i)} \quad i=1,2 \quad (\text{A12})$$

$$\hat{l}_{ci} = \frac{\sigma_i[(1 - \beta_i)(\hat{W} - \hat{P}_i) + \beta_i(\hat{R} + \hat{\tau} - \hat{P}_i)]}{(1 - \alpha_i - \beta_i)} \quad i=1,2 \quad (\text{A13})$$

In the case of the Cobb/Douglas/CES function capital per manager and labor per manager satisfy for $i = 1, 2$:

$$\hat{k}_{ci} = - \frac{\beta_i(\hat{R} + \hat{\tau} - \hat{P}_i) + \alpha_i(\hat{W} - \hat{P}_i) + \alpha_i\sigma_i(1 - \alpha_i - \beta_i)(\hat{R} + \hat{\tau} - \hat{W})}{(1 - \alpha_i - \beta_i)(\alpha_i + \beta_i)} \quad (\text{A14})$$

$$\hat{l}_{ci} = - \frac{\beta_i(\hat{R} + \hat{\tau} - \hat{P}_i) + \alpha_i(\hat{W} - \hat{P}_i) + \beta_i\sigma_i(1 - \alpha_i - \beta_i)(\hat{R} + \hat{\tau} - \hat{W})}{(1 - \alpha_i - \beta_i)(\alpha_i + \beta_i)} \quad (\text{A15})$$

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