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Stock Prices, Real Exchange Rates, and Optimal Capital Accumulation

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Abstract

This paper analyzes the dynamics of the real exchange rate and the price of equity for a small open economy using an optimizing model in which the process of capital accumulation entails adjustment costs. The analysis demonstrates that along an adjustment path toward long-run equilibrium, appreciation of the real exchange rate will accompany a decline in the market price of equity, whereas depreciation of the real exchange rate will accompany a rise in the price of equity. This relationship results from the requirement that non-traded inputs are used in the investment process. In the short-run, though, the effects on these variables depend critically on whether disturbances originate in the non-traded sector and on whether disturbances are perceived as temporary or permanent. The disturbances considered include changes in fiscal policies as well as changes in the world interest rate.

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I. Introduction

The 1980s have witnessed large swings in both real exchange rates and equity prices for many countries. These movements have at times been characterized as excessively volatile and have been interpreted by some observers as evidence that exchange rates and equity prices have diverged from market fundamentals.1/ As emphasized by Frenkel and Razin (1986) and Edwards (1987), however, large movements in real exchange rates may instead reflect optimizing behavior by economic actors in the presence of changes in fiscal or trade policies. Indeed, as Khan and Montiel (1987) point out, the real exchange rate is an endogenous variable that will adjust in response to economic shocks in order to reallocate resources efficiently across sectors of the economy.

In this paper, we analyze the dynamics of the real exchange rate and the price of equity for a small open economy using an optimizing model in which the process of capital accumulation entails adjustment costs. Our purpose is to interpret movements in the real exchange rate, the price of equity, and the capital stock as reflecting the outcome of optimal economic decisions by firms and households in response to various economic and fiscal policy shocks. The connection between the exchange rate and the price of equity has been explored in a recent paper by Gavin (1986), which extends the sticky price framework of Dornbusch (1976). Our analysis differs from Gavin's in allowing a role for capital accumulation, leaving aside monetary questions, and using an explicit optimizing model.

Much recent work in open economy macroeconomics has employed optimizing models in order to highlight the important role that real exchange rates play in the capital accumulation process. For example, Bruno (1982), Marion (1984), Razin (1985), and Murphy (1986) use two-period models to study the interaction of the real exchange rate and investment for small open economies. Dooley and Isard (1988) develop an infinite horizon, multi-country model that emphasizes how differences in capital accumulation across countries affects real exchange rates. In all of these papers, the capital stocks adjust to their optimal levels within one period following economic shocks.

In a recent paper, Brock (1987) investigates the dynamics of the real exchange rate and the current account in an infinite horizon optimizing model of a small open economy in which the capital stock adjusts gradually over time. He illustrates the important differences between temporary, permanent and future changes in fiscal policies for macroeconomic adjustment. His formal analysis, however, employs linear adjustment costs in the investment process which requires that the market value of capital (or the price of equity) always equal its replacement cost. This prevents the real exchange rate from changing relative the the price of equity in response to changes in

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1/ See Williamson (1985) for a discussion of real exchange rate misalignment. Borensztein (1987) presents evidence that the appreciation of the U.S. dollar over the period 1980-84 is consistent with the hypothesis of a speculative bubble, although he is also unable to reject alternative explanations. Shiller (1984) describes stock prices as being determined in part by "fads."

tax and government spending policies.1/ In contrast, the model presented in this paper allows the price of equity and the real exchange rate to move independently of each other following economic and policy shocks. This permits a richer treatment of adjustment dynamics.2/

To carry out the analysis, we employ an intertemporal version of the dependent economy framework originated by Salter (1959) and Swan (1960). This adaptation features the role of optimizing consumers and firms in determining the paths of investment, the real exchange rate, and the price of equity for a small open economy that can borrow and lend on world markets.3/ By distinguishing between traded and non-traded goods, the dependent economy setting provides a measure of realism and allows market clearing in the non-traded goods sector to be identified with internal macroeconomic balance. The assumption that consumers and firms are forward-looking in choosing their optimal plans permits careful consideration of how the timing of anticipated changes in policy can influence economic behavior.

The paper finds that along an adjustment path toward long-run equilibrium, the real exchange rate (defined as the relative price of non-traded goods) and the price of equity will move in opposite directions. This relationship arises from the requirement that non-traded inputs are employed in the investment process. The initial short-run effects on these variables, however, depend crucially on whether the shocks are perceived as temporary or permanent and on whether they originate in the home goods sector or elsewhere. For example, a permanent increase in government spending on non-traded output will initially appreciate the real exchange rate and lower the price of equity, whereas a permanent increase in the income tax rate will initially depreciate the real exchange rate and lower the price of equity. In both cases, subsequent adjustment involves real depreciation accompanied by a rising price of equity and a declining capital stock.

The analysis also suggests important qualitative differences in adjustment for a given change in the government's budget deficit depending on the manner in which the change is carried out. For example, a reduction in

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1/ In Brock's model, however, government subsidies to investment activity will drive a wedge between the market value of capital and the real exchange rate. This, though, is a one-time effect with subsequent changes in the market value of capital exactly equal to changes in the real exchange rate.

2/ Bovenberg (1988) also uses an adjustment-cost framework to study capital accumulation in the open economy. His analysis, however, emphasizes the dynamics of the terms of trade rather than the adjustment of the real exchange rate.

3/ See McKenzie (1982) for a similar adaptation of the dependent economy framework. McKenzie assumes that the traded good is the investment good and focuses on the roles of factor-intensity and intertemporal substitution in determining the dynamics of the real exchange rate. The present paper instead follows Brock (1987) in assuming that investment requires inputs of the non-traded good and in focusing on adjustment in response to changes in fiscal policies.

the deficit brought about through a cut in government spending will have quite different effects on the real exchange rate, the price of equity, and investment as compared with an equivalent reduction accomplished by increase in the tax rate. In addition, a given change in government spending itself will have different effects depending on the composition of spending between traded and non-traded goods.1/

The plan of the paper is as follows. Section 2 develops the basic model and provides a graphical solution. Section 3 presents the results of how the economy responds to changes in government fiscal policies and changes in the world interest rate. The paper concludes in Section 4 with a brief summary of its findings and suggestions for future research.

II. The Model

We assume a small open economy producing traded and non-traded goods. The traded good is assumed to be the sole consumption good and has its price determined in world markets. Since the economy is small relative to the rest of the world, residents take the price of the traded good as given. The non-traded good is assumed to be the uninstalled capital good and has its price determined domestically by market clearing. Unlimited borrowing and lending, subject to the usual intertemporal budget constraint, is available to residents of the economy.

1. Production

Traded and non-traded goods are produced using the services of labor and installed capital with constant returns to scale technologies. We assume that the traded sector is capital intensive relative to the non-traded sector, and that factor intensity reversals and specialization in production never occur.2/ Labor is supplied inelastically in amount L and the capital stock is fixed at each point in time but changes over time as a result of investment. Both labor and capital are mobile across production sectors. Under the assumptions of perfect competition, profit maximization, and factor market clearing, the following supply functions can be derived:

1/ See Khan and Lizondo (1987) for a related discussion of how the mix of fiscal policies accompanying a nominal devaluation is of critical importance in determining the extent of real depreciation.

2/ This assumption about factor intensities is sufficient to ensure that the steady-state equilibrium of the model is a unique saddlepoint. Previous work on capital accumulation in the open economy has employed non-optimizing models in which global stability of the model requires the traded sector to be relatively capital intensive (see, for example, Dornbusch (1980) and Obstfeld and Stockman (1985)). The optimizing model presented in this paper will always be saddlepath stable when the traded sector is capital intensive but will be globally unstable when the traded sector is labor intensive.

$$(1) \qquad y^N = y^N(P,K) \qquad y_1^N > 0,\ y_2^N < 0$$

$$(2) \qquad y^T = y^T(P,K) \qquad y_1^T < 0,\ y_2^T > 0$$

where P is the relative price of the non-traded good in terms of the traded good (which we refer to as the real exchange rate1/), K is the economy-wide capital stock and subscripts denote partial derivatives of the functions. For convenience, the fixed supply of labor and time subscripts are suppressed here and throughout the paper when possible. In equations (1) and (2), an increase in K at a constant real exchange rate will raise output in the relatively capital-intensive traded sector and lower output in the relatively labor-intensive non-traded sector according to the Rybczinski effect. An increase in the real exchange rate, P, will raise output in the non-traded sector and lower output in the traded sector as resources are shifted between sectors.

The assumption that all factors of production are mobile between sectors also implies that the returns to factors are solely functions of the real exchange rate. Accordingly, the rental rate on installed capital can be expressed as:

$$(3) \qquad r^k = r^k(P) \qquad r_1^k < 0$$

where an increase in the real exchange rate will lower the rental rate on capital since the non-traded sector is relatively labor intensive.2/

2. Asset Markets and Investment

Residents of the home country allocate their wealth over two real assets

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1/ An increase in P thus represents a real appreciation. The model presented in this paper focuses exclusively on the real side of the economy in order to study the interaction of relative prices and capital accumulation. In order to determine nominal prices and the nominal exchange rate it would be necessary to add a monetary sector to the economy. This would then raise the important issue of how to model money demand in an optimizing framework; an issue that remains controversial and that is beyond the scope of the present analysis.

2/ As discussed in Section 3, equation (3) implies that the long-run real exchange rate will be determined exclusively by the supply side of the economy. If instead capital were sector-specific rather than mobile between sectors, then rental rates in each sector would depend on the real exchange rate and the sectoral distribution of the capital stock. As noted in Murphy (1988), though, the long-run real exchange rate would continue to be determined exclusively by the supply side provided the two sectors have the same rates of depreciation and adjustment cost functions. The reason is that in the long run rental rates will be equalized across sectors even when capital is sector-specific.

which are assumed to be perfect substitutes for each other. The two assets are an internationally traded bond paying the fixed world interest rate (r) and a domestically traded equity claim on firms in the capital installation industry. Arbitrage in domestic asset markets equates the expected yields on claims to installed capital with the interest rate on bonds:

$$(4)\qquad \dot{V}/V + D/V = r$$

The expected yield on an equity claim to installed capital, given by the left-hand side of equation (4), is composed of the expected capital gain on equity ($\dot{V}/V$) plus the dividend return (D/V). Residents of the economy are assumed to have perfect foresight so that the expected capital gain equals the actual capital gain.

Investment is carried out by firms in the capital installation industry and follows the adjustment cost specification of Hayashi (1981). Firms in the production sectors are assumed to rent capital services from firms in the capital installation industry. To increase the capital stock by J units, firms in the capital installation industry must initially purchase J units of uninstalled capital. This uninstalled capital is then put in place at an installation cost that uses additional non-traded output. Assuming that this installation cost is an increasing function of the amount of capital formation relative to the existing capital stock, total investment expenditure is given by:

$$(5)\qquad I = J[1 + h(J/K)] \qquad h(0) = 0,\ h'(0) = 0,\ h' > 0,\ h'' > 0$$

The assumption that the marginal cost of installation is non-decreasing in J/K ensures determinacy of the capital stock despite constant returns to scale technologies and perfect competition.1/

The representative firm in the capital installation industry is assumed to choose investment so as to maximize its market value. By assuming that all investment expenditure is financed out of retained earnings and that firms have issued no previous debt, the number of equity claims outstanding will be constant and the market value of an individual firm will be proportional to the value of an equity claim.2/ As a result, integration of equation (3), ruling out speculative bubbles by assumption, gives the firm's market value:

1/ The results of this paper will continue to hold if traded goods are used in the investment process provided some amount of non-traded inputs are also required.

2/ The firm is assumed to borrow from residents at the world interest rate when its retained earnings fall short of its investment expenditures. Note that since bonds and equities are perfect substitutes in investor portfolios, the mix of external finance has no effect on the market value of the firm.

(6) $$V(t) = \int_t^{\infty} D(s)\exp^{-r(s-t)}ds$$

where the number of equity shares has been normalized to unity for convenience. Equation (6) defines the market value of a firm in the capital installation industry as equal to the present discounted value of its dividend stream. Dividends are equal to the difference between the firm's after-tax revenue from capital rentals less its expenditures on investment:

(7) $$D = [1-\tau]r^{k}K - PI$$

where r^k is the rental rate for installed capital measured in units of traded goods and τ is the tax rate applied to rental income.1/

The representative firm maximizes equation (6) subject to the following accumulation constraint for the capital stock:

(8) $$\dot{K} = J - \delta K$$

where δ is a constant rate of depreciation. We assume that gross capital formation is always positive, so that the non-negativity condition on gross capital formation (J) need not be explicitly imposed. As shown in the appendix, first order conditions for this problem yield the following set of relationships:

(9) $$q = P[1 + h(J/K) + [J/K]h'(J/K)]$$

(10) $$\dot{q} = [r + \delta]q - [1-\tau]r^{k}(P) - P[J/K]^{2}h'(J/K)$$

Equation (9) sets the marginal value of installed capital equal to its marginal cost, and implicitly defines the rate of capital formation [J/K] as an increasing function of the marginal value of capital relative to its replacement cost:

(11) $$J/K = x(q/P) \qquad x'(q/P) > 0$$

Equation (10) describes the evolution of the marginal value of installed capital over time.

As discussed in Hayashi (1981) and Summers (1981), our assumptions about

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1/ Note that if the firm were allowed to fully deduct its investment expenditures in calculating its taxable income, then changes in the tax rate would have no effect on investment incentives. Although equation (7) assumes zero deductibility, the results of this paper will continue to hold provided investment expenditures are less than fully deductible.

production technologies and installation costs imply that the marginal value of installed capital for this economy will be equal to the average value of installed capital. In other words, the market value of an equity claim on a unit of installed capital will equal the marginal value of an additional unit of installed capital (q). Accordingly, we can identify the variable q as the price of equity measured in terms of traded goods.

By using equation (11) to substitute for the rate of capital formation in equations (5), (8) and (10), we obtain relationships describing investment expenditure, the dynamics of the price of equity, and the dynamics of the capital stock as functions of q, P, and K::

(12) $$I = [1 + h(x(q/P))]x(q/p)K = I(q/P,K)$$

(13) $$\dot{q} = [r + \delta]q - [1-\tau]r^k(P) - P[x(q/P)]^2 h'(x(q/P)) = \phi(q,P,\tau,r)$$

(14) $$\dot{K} = [x(q/P) - \delta]K = \theta(q/P,K)$$

where the partial derivatives of the functions are given as:

$$I_1 = [q/P]Kx' > 0 \qquad \phi_3 = r^k > 0$$

$$I_2 = [1+h]x > 0 \qquad \phi_4 = q > 0$$

$$\phi_1 = [r+\delta-x] = ? \qquad \theta_1 = Kx' > 0$$

$$\phi_2 = -[1-\tau]r_1^k + x[1+h] > 0 \qquad \theta_2 = [x-\delta] = ?$$

The two derivatives with ambiguous signs (ϕ_1 and θ_2) will both be determinate at the steady-state equilibrium for the model where the rate of capital accumulation equals the rate of depreciation ($x=\delta$). For a given path of the real exchange rate and exogenous variables, these equations determine how investment, the capital stock and the price of equity evolve through time.

3. Consumption

The infinitely-lived representative household chooses consumption so as to maximize the discounted sum of instantaneous utility:

(15) $$U = \int_t^\infty u(c(s))\exp^{-\gamma(s-t)}ds$$

where instantaneous utility is a function of consumption of the traded good

(c) and γ is the constant rate of time preference.1/ Equation (15) is maximized subject to an accumulation constraint that sets the change in household bond holdings (B^h) equal to the excess of income over consumption expenditure:

$$(16) \qquad \dot{B}^h = rB^h + D + [1-\tau]wL + R - c$$

where income consists of interest earnings on bond holdings (rB^h), dividend payments on equity holdings (D), after-tax labor income ($[1-\tau]wL$), and lump-sum transfers payments (R).2/ As shown in the appendix, the first order conditions for the household's problem are:

$$(17) \qquad u'(c) = \lambda$$

$$(18) \qquad \dot{\lambda} = \lambda[\gamma - r]$$

where λ is the marginal utility of consumption. Note that with a constant world interest rate and a constant rate of time preference we must impose the condition $r=\gamma$ to ensure the existence of a steady state with a non-zero but finite level of consumption. It follows, therefore, from equations (17) and (18) that residents will smooth consumption perfectly over time yielding a constant level of consumption for any given path of lifetime income.

4. The Government

The government is assumed to finance transfer payments and its purchases of consumption goods and uninstalled capital goods through a tax on incomes from labor and capital and by borrowing on international markets. We assume that the government obeys its intertemporal budget constraint so that the present value of future receipts minus the present value of future expenditures exactly offsets the initial stock of government debt. The accumulation constraint for government debt is given as:

1/ By assuming that only the traded good is consumed, the analysis leaves aside important issues concerning how the time profile of consumption is related to the domestic real interest rate. Incorporating non-traded consumption, though, would complicate the analysis and possibly obscure the specific mechanisms highlighted here. See Dornbusch (1983) for a model of optimal borrowing in which both traded and non-traded goods are consumed.

2/ The household also receives income in the form of capital gains on equity. Since the number of outstanding equity claims is assumed to be constant (recall that firms finance investment expenditure out of retained earnings or by borrowing), the capital gain exactly equals the increase in the value of equity holdings. As a result, identical capital gain terms would show up on both sides of the accumulation equation for household wealth and would therefore cancel, yielding equation (16).

$$\dot{B}^g = rB^g + PG^N + G^T + R - \tau[r^k K + wL] \quad (19)$$

where B^g is the stock of government debt. Government purchases of goods are assumed to have no direct effect on the utility of residents in the economy. 1/

5. Equilibrium

Equilibrium in the economy occurs when the market for the non-traded capital good clears. This condition must hold at each point in time and determines the equilibrium path for the real exchange rate:

$$y^N(P,K) = I(q/P,K) + G^N \quad (20)$$

Equation (20) determines the equilibrium real exchange rate for given values of the capital stock, price of equity, and the level of government spending on the non-traded good. Solving for the real exchange rate yields:

$$P = \vartheta(q,K,G^N) \qquad \vartheta_1 > 0,\ \vartheta_2 > 0,\ \vartheta_3 > 0 \quad (21)$$

where the signs of the partial derivatives follow from equations (1), (11), and (12). An increase in q raises investment, thereby bidding up the price of the non-traded, uninstalled capital good. A rise in the capital stock will both raise the level of investment and lower the output of the labor-intensive non-traded sector, thus requiring an increase in the price of the non-traded good to clear the market. Finally, an increase in government purchases of the non-traded good will yield an increase in the price of the non-traded good.

6. Solution of the Model

By using equation (21) to substitute for the real exchange rate in equations (13) and (14), we obtain two relationships describing the evolution of the capital stock and price of equity as functions of q, K, G^N, τ, and r:

$$\dot{q} = H(q,K,G^N,\tau,r) \quad (22)$$

$$\dot{K} = F(q,K,G^N) \quad (23)$$

To solve the model, these equations are linearized around the point where $\dot{q}=0$ and $\dot{K}=0$ to obtain:

1/ The results of this paper would continue to hold if government purchases were incorporated as a separable term in the utility function of the representative household.

$$(24)\quad \begin{bmatrix} \dot{q} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ F_1 & F_2 \end{bmatrix} \begin{bmatrix} dq \\ dK \end{bmatrix} + \begin{bmatrix} H_3 & H_4 & H_5 \\ F_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} dG^N \\ d\tau \\ dr \end{bmatrix}$$

where the derivatives, evaluated at the steady-state values of q and K, are given as:

$H_1 = \phi_1+\phi_2\vartheta_1 > 0$

$H_2 = \phi_2\vartheta_2 > 0$

$H_3 = \phi_2\vartheta_3 > 0$

$H_4 = \phi_3 > 0$

$H_5 = \phi_4 > 0$

$F_1 = \theta_1[P-q\vartheta_1]/P^2 > 0$

$F_2 = [\theta_2-\theta_1 q\vartheta_2/P^2] < 0$

$F_3 = -\theta_1 q\vartheta_3/P^2 < 0$

and the changes (dz) are expressed as deviations from steady-state values. The determinant of this differential equation system is equal to $[H_1F_2-H_2F_1]$, which is negative thereby ensuring that the equilibrium where $\dot{q}=0$ and $\dot{K}=0$ is a saddlepoint. For constant levels of the exogenous variables, the solution for q and K along the stable trajectory is given in general form as:

$$(26)\qquad dq = \omega_1 Z\exp^{\mu t}$$

$$(27)\qquad dK = \omega_2 Z\exp^{\mu t}$$

where Z is an arbitrary constant determined by the initial value of K, μ is the stable (negative) eigenvalue of the transition matrix, and the ω_i are the elements of the eigenvector associated with μ. By using equations (26) and (27), the relationship between q and K along the stable path can be expressed as:

$$(28)\qquad dq = [\omega_1/\omega_2]dK$$

where $[\omega_1/\omega_2] = -H_2/[H_1-\mu] < 0$. As a result, the price of equity and the capital stock will move in opposite directions along the stable saddlepath.

To calculate how the real exchange rate evolves over time, we first linearize equation (21) around the steady state and then substitute for dq using equation (28) to obtain:

(29) $dP = \{\vartheta_1[\omega_1/\omega_2] + \vartheta_2\}dK$

By using the definition of ω_1/ω_2, the term in curly brackets can be rewritten as:

(30) $\vartheta_2\{1-\alpha\vartheta_1/[r+\alpha\vartheta_1-\mu]\} > 0$

where $\alpha = -[1-\tau]r_1^k + x[1+h]$. This term is always positive since μ is negative, implying that the real exchange rate and the capital stock will move in the same direction along a stable adjustment path. Hence, the model predicts real appreciation accompanying capital accumulation as the economy adjusts toward long-run equilibrium along the stable saddlepath, in contrast to Brock (1987) where real depreciation is associated with capital accumulation along the stable saddlepath. The reason for this difference is that the present analysis models adjustment costs as an increasing rather than linear function of the rate of capital formation, thereby permitting the value of installed capital (the price of equity) to diverge from the cost of uninstalled capital (the real exchange rate).

A graphical solution of the model is presented in Figure 1 where the steady-state values of q and K are given by the intersection of the $\dot{q}=0$ schedule with the $\dot{K}=0$ schedule. The $\dot{q}=0$ schedule is downward sloping because an increase in K (accounting for general equilibrium effects on the real exchange rate) lowers the return to capital (and thus dividends), thereby requiring a fall in the price of equity so that the yield on equity with zero expected capital gains equals the world real interest rate. The $\dot{K}=0$ schedule is upward sloping because an increase in K induces a real appreciation (through general equilibrium effects on the non-traded goods market), which raises the cost of investment and requires an increase in q to keep investment from falling. As shown in Figure 1, the stable saddlepath (SS), described by equation (28), is downward sloping and follows from the pattern of adjustment for points off of the $\dot{q}=0$ and $\dot{K}=0$ schedules.

7. <u>The Current Account</u>

A relationship for the current account surplus can be derived by subtracting the accumulation equation for government debt (19) from the accumulation equation for household bond holdings (16) to yield:

(31) $\dot{B} = rB + r^kK + wL - PI - PG^N - G^T - c$

Since total income must equal the value of output, this equation can be rewritten as:

$$(32) \qquad \dot{B} = rB + y^T - G^T - c$$

where the market-clearing condition for the non-traded good has been used. As mentioned above, residents of this economy smooth consumption perfectly. This implies that consumption can be written as the annuity value of lifetime income:

$$(33) \qquad c = rB^h + r\int_t^\infty \{[1-\tau][r^k K + wL] - PI + R\}\exp^{-r[s-t]}ds$$

In addition, residents in this economy fully anticipate that the government budget will balance intertemporally:

$$(34) \qquad B^g = \int_t^\infty \{\tau[r^k K + wL] - R - PG^N - G^T\}\exp^{-r[s-t]}ds$$

where the government is assumed to adjust transfers (R) whenever a change in spending or tax revenue occurs so as to ensure that equation (34) holds. By substituting for transfer payments in equation (33) using equation (34), consumption can be rewritten as:

$$(35) \qquad c = rB + r\int_t^\infty [y^T(P,K) - G^T]\exp^{-r[s-t]}ds$$

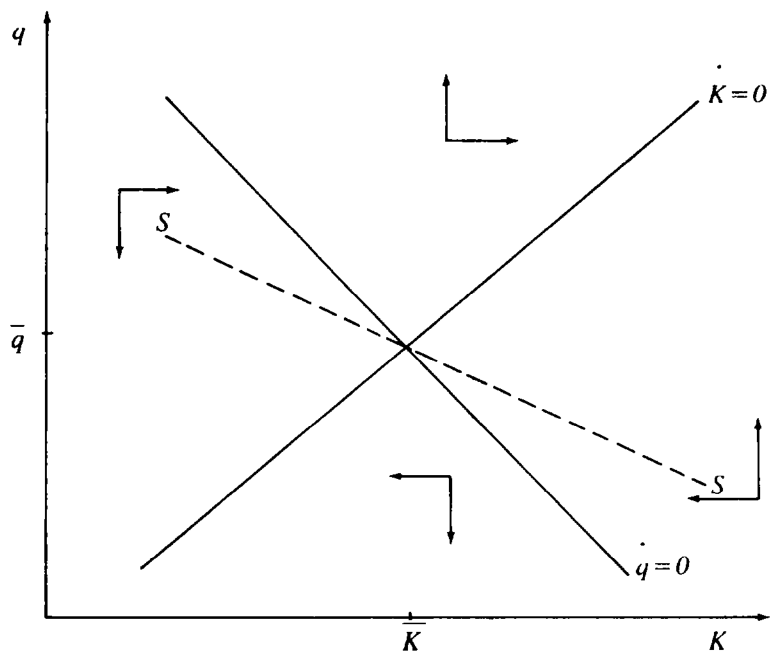
where again the non-traded goods market clearing condition and the equality between total incomes and the value of output have been used to simplify the expression. By employing equation (35) to substitute for consumption in equation (32), the current account surplus can be expressed as:

$$(36) \qquad \dot{B} = y^T(P,K) - G^T - r\int_t^\infty [y^T(P,K) - G^T]\exp^{-r[s-t]}ds$$

This equation simply states that the current account will be in surplus or deficit, respectively, depending on whether the quantity $[y^T - G^T]$ is falling or rising through time.

Along the equilibrium saddlepath the traded sector may either expand or contract as capital accumulates, depending upon parameter values. Although capital accumulation at a constant real exchange rate will expand the relatively capital-intensive traded sector, the accompanying real appreciation draws resources out of the traded sector and into the non-traded sector. If

FIGURE 1
STEADY-STATE EQUILIBRIUM



the latter effect dominates, then the traded sector will contract.1/ The next section assumes that the relative price effect does not dominate the Rybczinski effect so that the traded sector expands as capital accumulates. Accordingly, for a given level of government spending on traded output, the current account will be in deficit during a phase of capital accumulation and in surplus during a phase of capital decumulation.

III. Adjustment to Economic and Policy Shocks

This section considers how the real exchange rate, the price of equity, and the capital stock adjust in response to changes in fiscal policies and the world real interest rate. Both temporary and permanent changes are examined.

1. A Reduction in the Income Tax Rate

A permanent reduction in the income tax rate (τ) that is financed by government borrowing will initially raise the after-tax return to equity and thereby increase the demand for equity claims.2/ This increased demand for equities leads to an immediate rise in the price of equity in order to restore equilibrium in the equity market. As noted earlier, the assumed specification of adjustment costs in the model imply that the average and marginal values of installed capital are equivalent. Accordingly, an increase in the price of equity also represents an increase in the marginal value of installed capital. Given the initial real exchange rate (which represents the cost of uninstalled capital), the higher price of equity induces an increase in the rate of investment by firms in the capital installation industry.

1/ From equation (29), it is clear that a real appreciation must accompany capital accumulation along the saddlepath. Whether this is associated with an expanding or contracting level of investment expenditure (I) (and thus an expansion or contraction in the non-traded sector) depends on whether or not the rate of increase in investment demand in equation (20) is greater than the rate of decline in the supply of the non-traded capital good. If investment demand rises more quickly than the supply falls, then the non-traded sector will expand and the traded sector will contract as the capital stock grows. To simplify the analysis of the current account in Section 3, we rule out this possibility by assuming that capital accumulation along the saddlepath is accompanied by a falling level of investment expenditure and thus an expanding traded sector.

2/ The future level of lump-sum transfers to households is assumed to be the revenue variable that the government adjusts in order to ensure that the government's budget is balanced intertemporally. Because households fully anticipate this future adjustment and because transfers are non-distortionary, the exact timing of the adjustment in transfers is irrelevant for the impact of the policy change on the economy. If instead the government adjusts a distortionary tax in order to ensure that its budget constraint is satisfied, then the timing of the adjustment would be of critical importance in determining the effect of the initial tax cut on the economy.

As illustrated in Figure 2, a permanent tax cut shifts the $\dot{q}=0$ schedule to the right. Since the capital stock is predetermined at each point in time, the immediate impact is a jump in the price of equity sufficient to place the economy on the saddlepath.1/ The higher rate of investment then drives the dynamics as capital accumulates and the price of equity declines along the saddlepath. Both the capital stock and price of equity increase in the new long-run equilibrium, although in the short run the price of equity overshoots its long-run value.

The rise in investment produces an initial appreciation of the real exchange rate that shifts resources into production of uninstalled capital goods. This appreciation, however, is proportionately less than the rise in the price of equity since, according to equation (20), a proportionate increase in the real exchange rate would produce an excess supply of non-traded output. As shown by equation (29), though, real appreciation must accompany capital accumulation along a saddlepath. As a result, the short-run effect is an undershooting of the long-run real exchange rate.

The tax cut raises the lifetime income of households because it increases the future level of traded output.2/ This induces an immediate increase in the level of consumption, as detailed in equation (35). The initial real appreciation reduces the size of the traded sector, which with a higher level consumption results in a current account deficit. As capital accumulates along the adjustment path, the traded sector gradually expands. From equation (36), the economy, therefore, runs a slowly diminishing current account deficit as time passes.

A temporary reduction in the income tax rate is illustrated in Figure 3. As in the case of a permanent reduction, the initial effect is an upward jump in the price of equity that is accompanied by a less than proportionate real appreciation. The jump in the price of equity and the extent of real appreciation will be greater the longer is the period of time during which the tax cut is in effect. In all cases, though, the jump in the price of equity must be smaller than for a permanent tax change so as to yield dynamics that place the economy on the original saddlepath (SS) at the time the tax cut is rescinded. The price of equity declines during the period preceding the tax

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1/ If the increase in the price of equity is either less than or greater than the amount needed to place the economy on the stable saddlepath, then the economy will never reach the steady-state equilibrium given by the intersection of the $\dot{q}=0$ and $\dot{K}=0$ schedules. Although all divergent paths can not be ruled out a priori, this paper makes the usual assumption that the dynamic adjustment of the economy always attains a non-degenerate steady-state equilibrium. See Obstfeld and Rogoff (1983) for a model in which some divergent paths can be ruled out.

2/ As mentioned in Section 2, the analysis of the current account here and throughout the remainder of the paper assumes that the traded sector expands as capital accumulates along the saddlepath. If instead the traded sector were to contract as capital accumulates, then a current account surplus would accompany capital accumulation along the saddlepath.

FIGURE 2
A PERMANENT TAX CUT

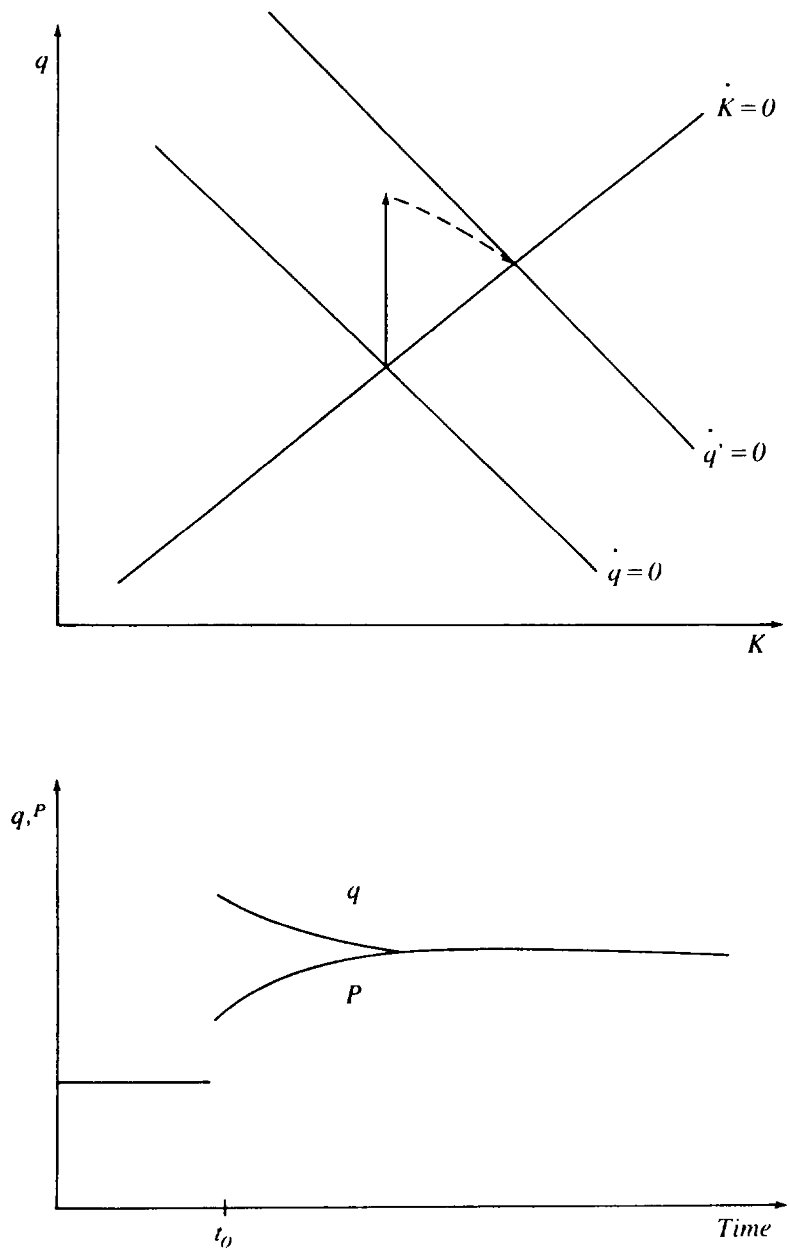
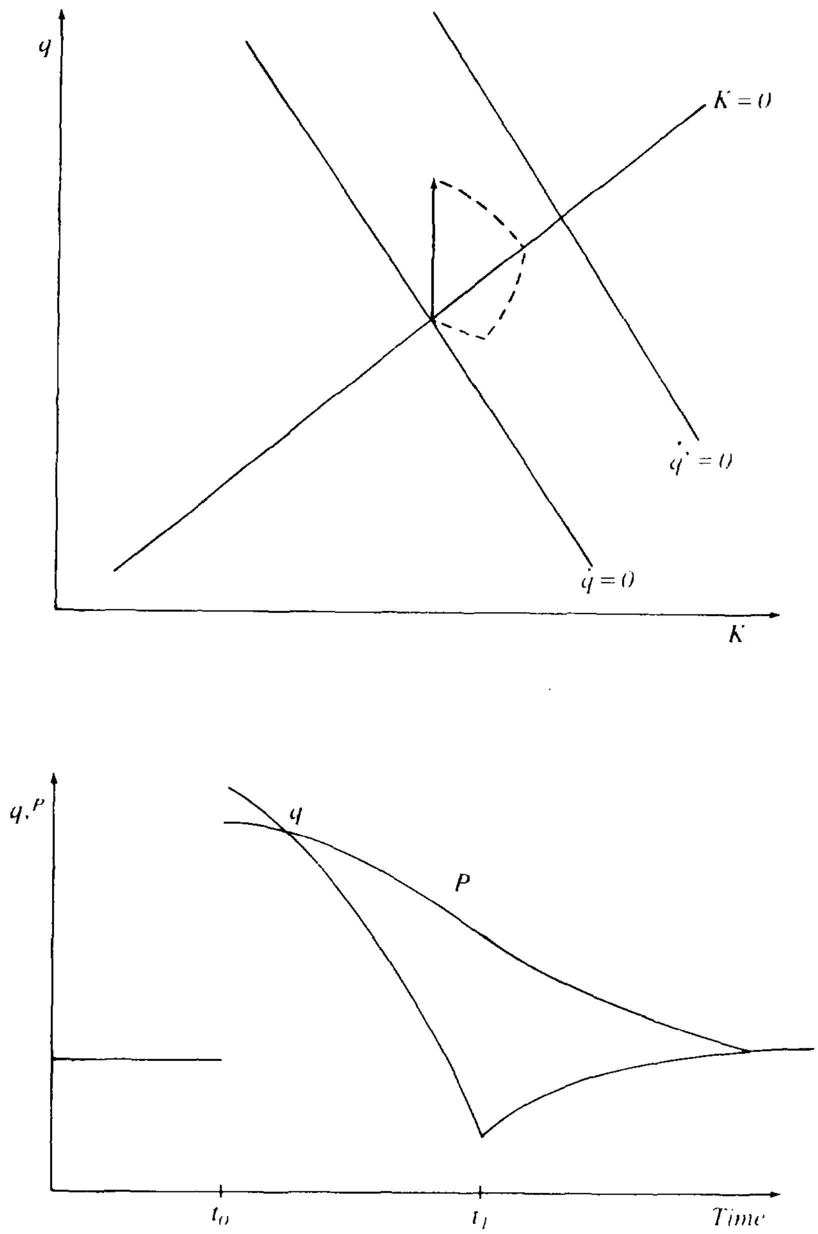


FIGURE 3
A TEMPORARY TAX CUT



increase while the capital stock at first rises and then declines. After the increase in the tax rate, the economy adjusts along the original saddlepath, with the price of equity rising and the capital stock falling.

The bottom panel of Figure 3 shows the adjustment pattern of q and P. The real exchange rate appreciates on impact by proportionately less than the rise in the price of equity. Since capital accumulates for a period of time, it is possible for the real exchange rate to continue to appreciate.1/ Eventually, though, the falling price of equity dominates, lowering investment demand and inducing real depreciation. The rate of real depreciation rises until the tax rate increases at time t_1, after which the rate of real depreciation gradually declines. In terms of Figure 3, the capital stock begins to decline at some time prior to the actual increase in the tax rate, when the ratio q/P momentarily equals its steady-state value. Unlike the case of a permanent tax cut, a temporary tax cut thus yields an episode during which the real exchange rate and the price of equity adjust over time in the same direction.

The current account initially moves into deficit following a temporary tax cut since the immediate real appreciation draws resources out of the traded goods sector while at the same time the increase in lifetime income (due to the temporary future increase in the capital stock) raises consumption demand.2/ The initial deficit will be somewhat smaller than in the case of a permanent tax cut because both the real appreciation and the increase in lifetime income are smaller. From the analysis of adjustment along the stable saddlepath, however, it must be the case that the current account is in surplus as the capital stock and the level of traded output decline once the tax rate is increased back to its initial level. Accordingly, the current account moves from deficit into surplus at some time prior to the increase in the tax rate and then the surplus gradually declines following the increase in the tax rate.

2. An Increase in Government Spending

An increase in government spending financed through borrowing will have very different effects on the economy depending upon whether the government purchases traded or non-traded output. The major difference between these two types of expenditures is that purchases of non-traded output will directly

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1/ Continued real appreciation is more likely the longer is the time during which the tax cut is in effect. What is needed is for the capital stock to be rising at a sufficient rate relative to the rate at which the price of equity is falling so that the change in the real exchange rate, as given by equation (29), is positive.

2/ Again, the analysis assumes that the traded sector expands as capital accumulates along the saddlepath. Note that the non-monotonic adjustment of the current account, described as stages in the balance of payments by Fischer and Frenkel (1972), arises here because the dynamics of the economy off of the saddlepath are driven by both eigenvalues of the system.

affect conditions in the non-traded goods market, thereby requiring adjustment of the real exchange rate and the price of equity as private investment is crowded out. In contrast, spending on traded goods will have no direct impact on conditions in the markets for non-traded output or equity.

An increase in government spending on traded output will have no effect on the price of equity, the capital stock, or the real exchange rate.1/ The only effect will come as households react to the increase in spending by smoothing the drop in lifetime income represented by the future lump-sum tax burden. When the increase in spending on traded output is permanent, there will be no effect on the current account since consumption falls by the full amount of the increase in spending. For a temporary increase in spending on traded output, the current account will move into deficit as consumption declines by less than the increase in spending.

As shown in Figure 4, a permanent increase in government spending on non-traded output will shift the $\dot{q}=0$ and $\dot{K}=0$ schedules to the left by equal amounts so that in the new steady state, the price of equity remains unchanged. This occurs because the steady-state value of the return to capital in this model is determined exclusively by the real exchange rate. Since government spending on non-traded goods does not directly affect the return to installed capital, in the new steady state the return to capital, the real exchange rate, and hence the price of equity will remain unchanged from their values at the initial steady state. Government spending on non-traded goods effectively crowds out investment and thereby reduces the steady-state level of the capital stock.

The immediate effect of a permanent increase in spending on non-traded output is a real appreciation, which makes uninstalled capital more expensive and lowers the return to installed capital. The equity market reacts to the decline in the return to capital by immediately bidding down the price of equity so as to reestablish equilibrium. With the price of equity now lower and the cost of uninstalled capital higher, the ratio q/P has clearly fallen so that the rate of investment spending drops below the rate of depreciation. Along the adjustment path the capital stock declines as the price of equity rises and the real exchange rate depreciates. The economy runs a gradually diminishing current account surplus as traded output declines along the adjustment path.2/

A temporary increase in government spending on non-traded output is illustrated in Figure 5. The initial effects on the price of equity and the real exchange rate are similar to the case of a permanent increase. The

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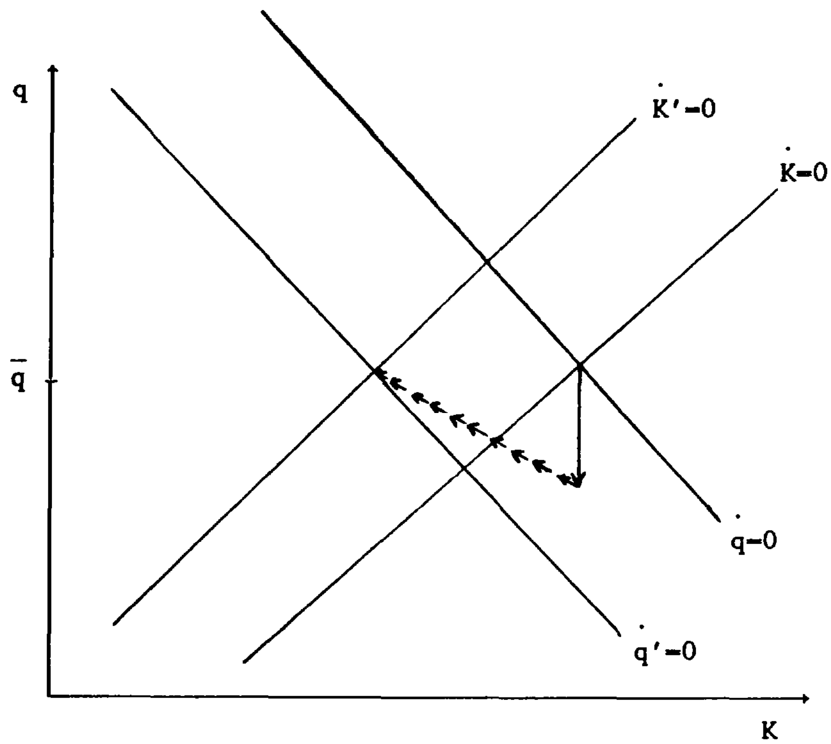
1/ A change in government spending on traded goods, however, would have an effect on the price of equity, the real exchange rate and investment if consumption included non-traded goods. The channel here would be through the change in non-traded consumption induced by the change in the future tax burden associated with the change in government spending.

2/ As before, this section assumes that the traded sector expands (contracts) as capital accumulates (decumulates) along a saddlepath.

Figure 4

A Permanent Increase in Government Purchases of Non-traded Goods



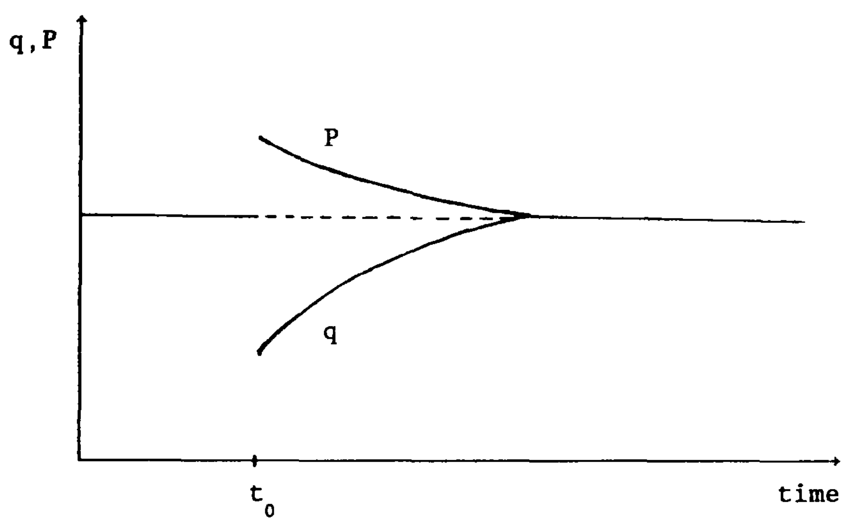
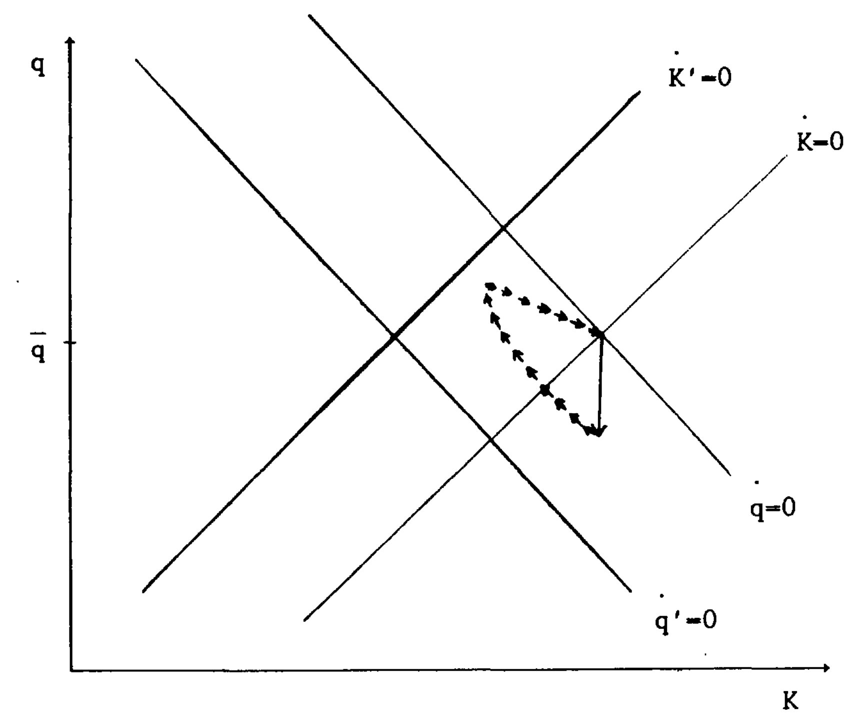
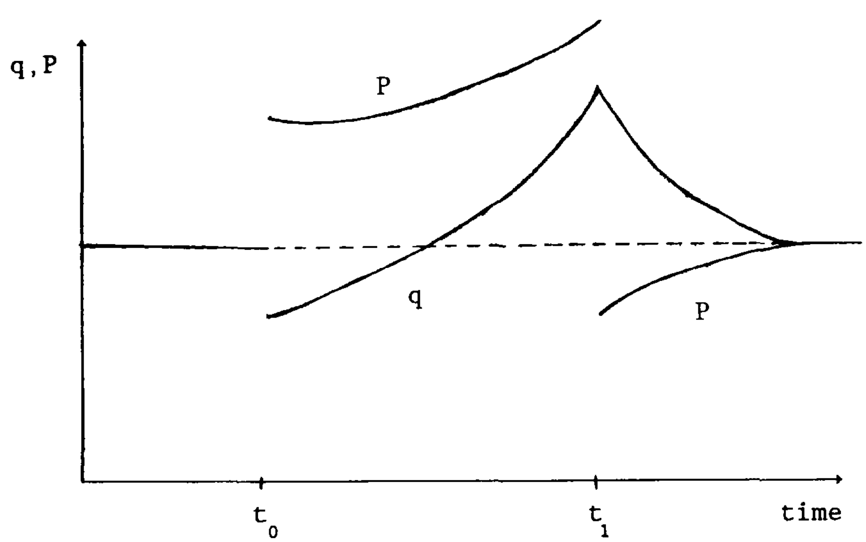


Figure 5

A Temporary Increase in Government Purchases of Non-traded Goods





decline in the price of equity is less than for a permanent increase since the resulting dynamics must move the economy back to the saddlepath SS associated with the initial equilibrium. This implies from equation (21) that for a given increase in government spending, a temporary increase will produce a greater initial real appreciation. Since the ratio q/P has fallen, the rate of investment falls and the capital stock begins to decline.

Along the adjustment path, the price of equity will rise at an increasing rate until the level of government spending is reduced. As shown in the bottom panel of Figure 5, the real exchange rate may decline for a while, but will eventually begin to rise as the increasing price of equity dominates the declining capital stock in determining the real exchange rate (equation 21). When the level of spending is finally reduced, the real exchange rate immediately depreciates, thereby lowering the cost of uninstalled capital relative to the price of equity so that investment rises.1/ The final phase of adjustment is characterized by a gradually declining price of equity, a rising capital stock, and an appreciating real exchange rate. As in the case of a temporary change in the tax rate, a temporary change in government spending on non-traded goods will be characterized by a period of time during which the real exchange rate and the price of equity move in the same direction prior to the final phase of adjustment along the saddlepath.

The evolution of the current account can be determined by using equation (36). The current account may initially move into either surplus or deficit since the initial real appreciation leads to a contraction in traded output that may either be less than or greater than the reduction in consumption due to the smoothing of the loss in lifetime income. Lifetime income falls because households anticipate the future lump-sum tax burden associated with the temporary increase in government purchases of non-traded goods. For a sufficiently temporary increase in government spending, the reduction in consumption will be negligible so that a deficit will emerge. On the other hand, if the increased level of spending is in effect for a long period of time, then the reduction in consumption may dominate the initial contraction in traded output so that a current account surplus results.

Regardless of the initial effects, though, a growing current account deficit will eventually emerge as the traded sector continues to contract prior to the reduction in spending. When spending is finally reduced, the real depreciation shifts resources into the traded sector, thus expanding the traded sector and reducing the current account deficit. This shift of resources into the traded sector can not produce a current account surplus because the final phase of adjustment along the stable saddlepath is characterized by an expanding traded sector which, from equation (36), can only be consistent with a gradually declining current account deficit.

1/ In Figure 5 it is possible that the adjustment trajectory actually crosses into the region where the capital stock begins to rise prior to the actual reduction in spending. This will occur if the price of equity increases sufficiently relative to the real exchange rate so as to actually raise the ratio q/P above its steady-state value. In this situation, the capital stock will begin to rise before the reduction in spending occurs.

3. A Reduction in the World Interest Rate

To consider a permanent reduction in the world interest rate (r), it is necessary to assume that the rate of household time preference (γ) also falls in order to ensure a determinate steady state with $r=\gamma$. Under this assumption, a reduction in r will shift the $\dot{q}=0$ schedule to the right. The immediate effect is an increase in the price of equity as residents attempt to shift their asset holdings from the less attractive international bond into equity. The real exchange rate appreciates by proportionately less than the rise in the price of equity for reasons analagous to the case of a tax cut described above. Adjustment proceeds with capital accumulation accompanied by a falling price of equity and real appreciation in a manner equivalent to the case of a permanent reduction in the tax rate. Assuming that the traded sector expands as capital accumulates, the current account exhibits a gradually declining deficit along the adjustment path.

The only important difference from the case of a tax cut concerns the level of consumption and hence the trade account. Depending on whether the economy has a positive or negative initial net asset position, the decline in the world interest rate will either lower or raise the level of lifetime income and thus consumption. For example, if the economy has a positive initial net asset position, the trade account will be in smaller deficit along the adjustment path (accompanied by a smaller service account surplus), and the final steady state will exhibit a smaller trade deficit, relative to the case of a tax cut that yields an equivalent shift in the $\dot{q}=0$ schedule.

A temporary reduction in the world interest rate can be analyzed in the same way as a temporary cut in the tax rate. Again, the only important difference concerns the effect on lifetime income and consumption resulting from the initial net asset position of the economy.

IV. Summary

This paper has considered how the price of equity, the real exchange rate and the capital stock for a small open economy adjust following changes in government fiscal policies and changes in the world interest rate. The results suggest important qualitative differences in adjustment for a given change in the government's budgetary stance depending on how the change is brought about. For example, an expansion in the government's budget deficit will have quite different effects on the price of equity, the real exchange rate, and the capital stock depending on whether the expansion is achieved through cutting taxes or raising government spending. Futhermore, a given change in government spending will have different effects depending on the breakdown of spending between traded and non-traded output. This serves to reinforce the point made by Frenkel and Razin (1986), that the composition of a change in budgetary policy is of critical importance in understanding its likely impacts on the economy.

The analysis also demonstrates how an optimizing perfect foresight framework in which all markets clear can generate significant movements in the equilibrium levels of the real exchange rate and the price of equity in response to economic and policy shocks simply because investment requires scarce resources. Interpreting such movements as evidence of market inefficiencies or as deviations from fundamental equilibrium values would be incorrect since adjustment in this framework is driven entirely by equilibrium in asset and goods markets. The results, however, do indicate that a stable and consistent set of fiscal policies can play an important role in preventing unnecessary volatility in real exchange rates and equity prices. Such volatility can generate important externalities in the actual economy where individuals are likely to be risk averse.

One general feature of the analysis emerges in comparing the effects of temporary changes in fiscal policies with the effects of permanent changes in these policies. During episodes when a temporary policy change is in place, the real exchange rate and the price of equity will move in the same direction. On the other hand, changes in policy that are expected to be permanent will lead to opposite movements in the real exchange rate and the price of equity as the economy adjusts toward long-run equilibrium. For example, following the initial impact effects, a temporary increase in government spending on non-traded goods will yield a rising price of equity accompanied by real appreciation during the period of time that the increased spending is in effect, whereas a permanent increase in spending on non-traded goods is followed by a rising price of equity and real depreciation. This suggests that historical episodes of real appreciation accompanied by surging equity markets may be associated with fiscal policies that are expected to be short-lived.

As with any simple framework, this paper has chosen to emphasize certain aspects of adjustment in the open economy while leaving aside or ignoring additional important issues. The purpose here was to provide an illustrative framework for considering a possible mechanism centered on the investment process through which economic and policy shocks may be transmitted to the real exchange rate, the price of equity, and the current account. Although this emphasis hopefully provides some insight into the issues addressed, it can not answer many important questions. For example, it would be useful to incorporate money into the analysis in order to study the nominal exchange rate and monetary policies. In addition, the framework could be extended to consider the effects of changes in the terms of trade. Finally, the present model could be easily adapted to analyze how productivity shocks in different sectors affect the real exchange rate and the price of equity.

Appendix

The maximization problem facing a representative firm in the capital installation industry is solved by setting up the present-value Hamiltonian:

$$\text{(A.1)} \qquad H = \{[1-\tau]r^k K - P[J[1+h(J/K)]] + q[J-\delta K]\}\exp^{-rt}$$

and deriving the following first-order conditions:

$$\text{(A.2)} \qquad \partial H/\partial J = \{q - P[1 + h + [J/K]h']\}\exp^{-rt} = 0$$

$$\text{(A.3)} \qquad \partial H/\partial K + d[q \exp^{-rt}]/dt = \{[1-\tau]r^k + P[J/K]^2 h' - q\delta + \dot{q} - rq\}\exp^{-rt} = 0$$

where q is the multiplier on the firms's capital accumulation constraint. The firm is assumed to take P and r^k as given since it is small relative to the market. Equations (9) and (10) of the text are obtained by rearranging (A.2) and (A.3).

The maximization problem facing the representative household is solved by setting up the following present value Hamiltonian:

$$\text{(A.4)} \qquad H = \{u(c) + \lambda[rB^H + D + [1-\tau]wL + R - c]\}\exp^{-\gamma t}$$

and taking first-order conditions:

$$\text{(A.5)} \qquad \partial H/\partial c = \{u' - \lambda\}\exp^{-\gamma t} = 0$$

$$\text{(A.6)} \qquad \partial H/\partial B^H + d[\lambda \exp^{-\lambda t}]/dt = \{\lambda r + \dot{\lambda} - \gamma\lambda\}\exp^{-\gamma t} = 0$$

where λ is the multiplier on the household's accumulation constraint. Equations (17) and (18) of the text follow from (A.5) and (A.6).

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