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The Simple Analytics of Debt-Equity Swaps and Debt Forgiveness

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Abstract

This paper is concerned with debt-equity swaps in which foreign residents are a party to the exchange (i.e., it does not deal with flight capital), and with debt forgiveness. The seemingly unrelated issues of debt-equity swaps and debt forgiveness are jointly treated in this study, because debt forgiveness is in fact a special case of debt-equity swaps. Namely, it is a swap in which a positive amount of debt is exchanged for zero equity. For this reason these two problems have many common features.

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Contents

I. Introduction

The debt crisis of the 1980s has inspired search for innovative solutions to the debt problem. Among the many proposals that have emerged so far, debt forgiveness and debt conversion schemes play a central role. One of the proposed mechanisms for debt forgiveness is to establish an international corporation that will buy back debt of developing countries and forgive part of it (see Kenen (1983)). The proposal has been debated, but so far no action has been taken toward its implementation.

Contrary to the Corporation proposal debt conversion schemes have been implemented in a number of countries, including major debtors such as Argentina, Brazil, Chile, Mexico and the Philippines, and they are being actively considered by a number of other countries, including Honduras, Morocco and Nigeria. A typical debt conversion scheme specifies conditions for the exchange of debt for domestic assets, the entities that may participate in it, and longer-term rights and obligations. In some cases the schemes are designed for foreign creditors or multinational corporations, in others they are designed for domestic residents, where the intention of the latter is to retrieve flight capital. In many cases debt for conversion purposes is acquired on the secondary market. So far only a small share of debt has been converted by means of these schemes--about 2 percent of the debt of countries that have engaged in them--but they may become much more important in the future. When debt is exchanged for equity it is called a debt-equity swap. (See Alexander (1987a) for some facts.)

One central idea behind the Corporation and the debt conversion schemes is to take advantage of the high discounts on debt on the secondary market. It is quite common for debts to be traded at a range of 50 to 60 cents to the dollar, with some debts being traded at even higher discounts. Hence, it is argued, debt forgiveness may be not very costly and the debtor may gain from the conversion of cheap debt into holdings of other domestic assets. In both cases the debt burden is eased.

This paper is concerned with debt-equity swaps in which foreign residents are a party to the exchange (i.e., it does not deal with flight capital), and with debt forgiveness. The seemingly unrelated issues of debt-equity swaps and debt forgiveness are jointly treated in this study, because debt forgiveness is in fact a special case of debt-equity swaps. Namely, it is a swap in which a positive amount of debt is exchanged for zero equity. For this reason these two problems have many common features. I suggest an approach for dealing with them and demonstrate its usefulness by addressing a number of key questions to which it can provide an answer. These questions include the following: What type of resource reallocations between debtor and creditor can be achieved by debt-equity swaps? What are the conditions under which there exist swaps that are beneficial to both parties? What are the special features of these problems? How cheap is debt forgiveness? What is the effect of debt forgiveness on investment in the debtor country? By dealing with these issues one can examine the usefulness of the framework. It should, however, be made clear from the start that I suggest a simple framework that can be extended or modified for particular applications. Here the concern is with the clarification of some fundamental issues rather than with particular applications. For example, I assume the existence of capital controls in the debtor country up to the point at which there are no private capital movements. This is convenient for analytical purposes and it represents a good approximation for some countries (with effective quantative restrictions). One may want to modify it for applications to other countries.

A minimal framework for dealing with some basic issues is developed in the following section. It is based on the assumption that the debtor's real income is a random variable, and that its foreign debt is government owned. As a result of output fluctuations, the government's capacity to service its debt is also a random variable. Consequently, it cannot make the required debt payments in all states of nature. A debt equity swap consists of an exchange of debt for claims to the random output.

A characterization of feasible reallocations of the transfer of resources from debtor to creditor across states of nature by means of debt-equity swaps is provided in Section III. Given the current situation in which swaps are small relative to the stock of debt, the emphasis is on small swaps. It is shown that small Pareto-improving swaps do not always exist, and a necessary and sufficient condition for their existence is derived. The issue of discounts on debt in a swap is taken up in Section IV. It is argued that voluntary swaps cannot involve discounts on the debt's value on the secondary market, and that there may exist Pareto-improving swaps even if they are performed at face value. On the other hand, there exist circumstances in which Pareto-improving swaps require discounts on the face value of the debt.

The implications of the existence of many creditors are explored in Section V. It is shown that due to the fact that debt of the type considered in this paper (i.e., which is fully repaid in some states but only partially repaid in others, with the subset of states of full repayment depending on its size) is priced nonlinearly on international financial markets, there exists a fundamental externality across creditors. This externality implies that the price of debt increases with the size of debt being swapped. Consequently, there may exist small Pareto-improving swaps that will not eventuate. Moreover, this externality increases the cost of debt forgiveness more than proportionately to the extent of forgiveness. These costs are characterized in Section VI, where it is also shown that there are many circumstances where debt forgiveness is very costly, and where only a small share of these costs provides debt relief while the rest is appropriated by the creditors in the form of capital gains. This supports the argument by Dooley (1987).

The model is extended in Section VII in order to deal with share prices, and further extended in Section VII in order to deal with investment. It is shown that debt forgiveness reduces share prices and investment, and that a debt-equity swap raises share prices and investment only if the cost of the swap in terms of equity is sufficiently high. The negative effect of debt forgiveness on investment results from a positive income effect in the second period (the analysis is conducted in a two-period framework). As income increases the demand for equity declines as a result of a decline in desired savings. Consequently, share prices fall and so does investment. Since it is often argued that debt forgiveness should increase investment, because it increases the rate of return on investment as a result of lower future tax rates, the model is subsequently modified in Section IX to take into account the rate of return effect. It is shown that the negative income effect is larger than the positive rate of return effect when the relative measure of risk aversion is larger than one, and the opposite if the relative measure of risk aversion is smaller than one. Hence, debt forgiveness provides an investment stimulus if there is low risk aversion and it dampens investment if there is high risk aversion. Concluding comments are provided in Section X.

II. Minimal Framework

The Debtor country's output is given by $\tilde{\theta}E$, where $\tilde{\theta}$ is a random productivity shock and E is a constant representing the country's activity level in production. In the market interpretation of the model E also represents the number of equities issued by domestic firms. Due to controls on international capital movements E is owned by domestic residents. States of nature are identified with productivity shock levels. Thus, state θ is the state in which the productivity shock obtains the value θ.

The government taxes output at the rate t, so that output owners receive income $(1-t)\theta E$ in state θ. In particular, the owner of one unit of E is entitled to $(1-t)\theta$ units of output in state θ. The government has an external debt D. Required service payments on this debt, which consist of principal plus interest, are RD units of output in every state, where R stands for one plus the interest rate. Tax revenue is used to service the debt. In order to represent the situation of current major debtors who will not be able to repay their debt in some states of nature, it is assumed that there exist realizations of the productivity shock at which tax revenue is insufficient to cover the required debt service payment; i.e.,

$$t\tilde{\theta}E < RD \quad \text{with positive probability.}$$

This implies that there exists a critical value θ_c, defined by

$$\theta_c = RD/(tE), \tag{1'}$$

such that debt is fully repaid in the high-productivity states $\theta>\theta_c$, but cannot be fully repaid in the low-productivity states $\theta<\theta_c$. It is assumed that in the low-productivity states in which tax revenue falls short of debt service payments, creditors receive the entire tax revenue. It is also assumed that t represents the highest possible tax rate, and that the government has no other sources of income (the case in which some domestic firms are government owned will be discussed at a later stage).

It is clear from this specification that apart from states in which tax revenue is insufficient to cover debt repayment ($\theta<\theta_c$) there typically also exist states in which tax revenue exceeds debt repayment ($\theta>\theta_c$). It is therefore necessary to state explicitly what is done in these states with tax revenue in excess of debt repayment. For the purpose of this study it is assumed that it is redistributed to the public as lump-sum transfers. 1/ Under these assumptions state-contingent consumption of Debtor residents is given by

$$c(\theta) = \begin{cases} (1-t)\theta E & \text{for } \theta<\theta_c, \\ (1-t)\theta E + (t\theta E-RD) & \text{for } \theta>\theta_c, \end{cases} \qquad (2')$$

where consumption in low productivity states consists of after-tax output and cunsumption in high productivity states consists of after-tax output plus the lump-sum transfer $t\theta E-RD$. Creditors receive the state-contingent payments

$$d^*(\theta) \begin{cases} t\theta E & \text{for } \theta<\theta_c, \\ RD & \text{for } \theta>\theta_c. \end{cases} \qquad (3')$$

In this setup Debtor residents have no explicit decision problem; they consume their after-tax output plus government transfers. Creditor residents receive full debt repayment in high productivity states and the tax revenue in low productivity states.

Now consider a debt-equity swap. Suppose that $\Delta>0$ units of debt are swapped for $\varepsilon>0$ units of equity, where equity is measured in units of E. It is assumed that the creditor cannot take a short position in equity. For the swap to take place the government has to acquire the equity or to provide the Creditor with the resources needed for its acquisition. There are several mechanisms by means of which this can be done; I will discuss some of them in Section VII. At this juncture the reader may find it easiest to assume that the government confiscates the needed

1/ Other alternatives, such as the provision of public goods, are also possible. The important point is to specify a mechanism for the valuation of these resources. It should, however, be clear that the choice of a specification affects some of the results. An example of an alternative tax structure and its implications are presented in Section IX.

equity. After the swap required debt repayments are $R(D-\Delta)$ in every state and the Creditor receives $(1-t)\theta\varepsilon$ in state θ on account of equity holdings. Naturally, for sufficiently small values of Δ there still exist states in which the government cannot fully repay the remaining debt. Assuming that foreign-owned income is also taxed at the rate t, debt is fully repaid in states that satisfy

$$t\theta E > R(D-\Delta),$$

so that the critical value θ_c, which is now a function of Δ, becomes

$$\theta_c(\Delta) = R(D-\Delta)/(tE), \tag{1}$$

i.e., debt is fully repaid in states $\theta>\theta_c(\Delta)$ and the Creditor receives the tax revenue in states $\theta<\theta_c(\Delta)$. In this case Debtor residents' consumption is

$$c(\theta;\Delta,\varepsilon) = \begin{cases} (1-t)\theta(E-\varepsilon) & \text{for } \theta<\theta_c(\Delta), \\ (1-t)\theta(E-\varepsilon) + [t\theta E - R(D-\Delta)] & \text{for } \theta>\theta_c(\Delta), \end{cases} \tag{2}$$

and the Creditor receives payments

$$d^*(\theta;\Delta,\varepsilon) = \begin{cases} (1-t)\theta\varepsilon + t\theta E & \text{for } \theta<\theta_c(\Delta), \\ (1-t)\theta\varepsilon + R(D-\Delta) & \text{for } \theta>\theta_c(\Delta). \end{cases} \tag{3}$$

Thus, the swap reduces the Debtor residents' income from claims to output in all states and it increases their income from government transfers in high productivity states as a result of the easing of the debt service burden. Moreover, it increases the set of states in which debt is fully repaid. These three factors need to be properly weighed in order to evaluate the desirability of the swap from the point of view of the Debtor. The Creditor too has to weigh three factors. The swap increases his income in all states on account of equity holdings, it reduces his income in high productivity states as a result of lower debt service payments, and it increases the set of states in which he receives full repayment on the remaining debt.

In order to evaluate the desirability of swaps, it is assumed that a representative resident of the Debtor has a strictly concave von Neumann-Morgenstern utility function u(c); i.e., the Debtor is risk averse. His subjective probability distribution of states--i.e., productivity shocks--is represented by the cumulative distribution function $G(\theta)$, defined on the interval $[0,+\infty)$. Hence, his expected utility from a given swap, (Δ,ε), is our welfare criterion, and is given by

$$U(\Delta,\varepsilon) = \int_0^\infty u[c(\theta;\Delta,\varepsilon)]dG(\theta). \tag{4}$$

Equations (1), (2) and (4) provide a valuation of every swap from the point of view of the Debtor.

As far as the Creditor is concerned, it is assumed that he has access to financial markets which enable him to hold a well diversified portfolio. Consequently, his marginal utility of state-contingent payments by the Debtor are not affected by the swap. Let $\mu^*(\theta)$ denote his marginal utility of state-θ payments. Then his expected utility of a swap is

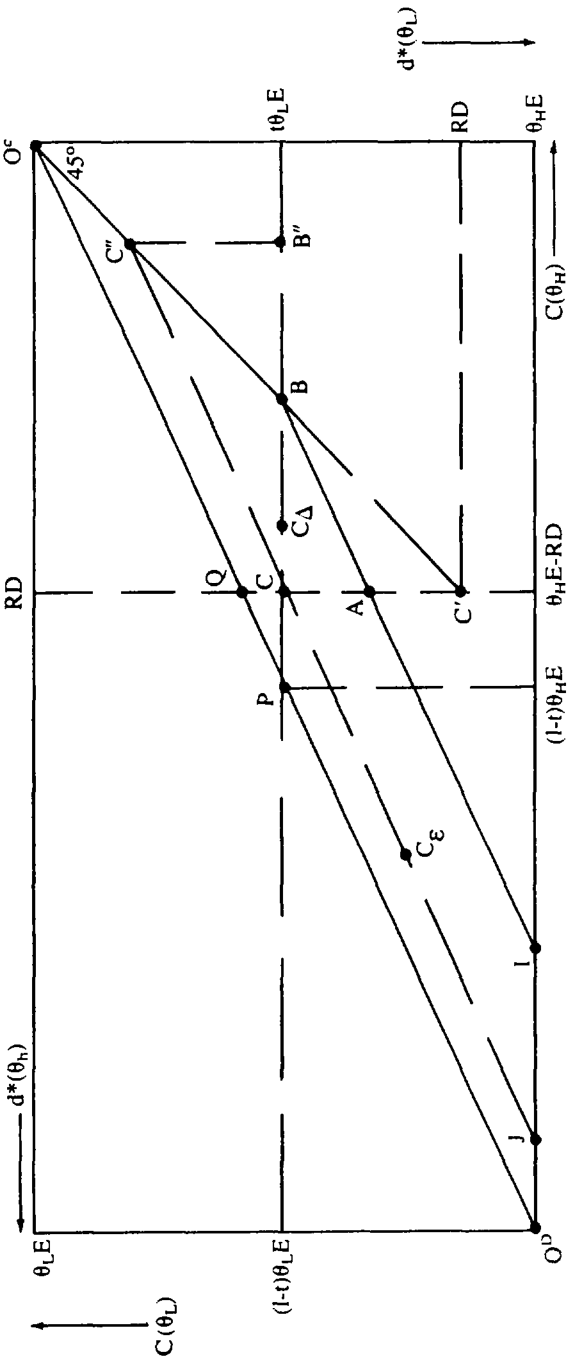
$$U^*(\Delta,\varepsilon) = \int_0^\infty \mu^*(\theta)d^*(\theta;\Delta,\varepsilon)dG^*(\theta), \tag{5}$$

where $G^*(\theta)$ is his subjective probability distribution function. The Creditor's valuation of a swap is represented by (1), (3) and (5).

III. Are Swaps Desirable?

There are several methods of analysis that can be used to answer the question posed in the title of this section. I have chosen to start the discussion with a simple two-state example in order to clarify some basic considerations. Suppose therefore that the productivity shock can obtain only two values, a low value θ_L and a high value θ_H, such that $\theta_L < \theta_c < \theta_H$. In this case the Debtor fully repays his debt in the high-productivity state and repays only partially in the low-productivity state. The resulting allocation of output is described in Figure 1. The box represents available output and its distribution between Debtor and Creditor. High-productivity state allocations are measured horizontally while low-productivity state allocations are measured vertically. The origin of the Debtor allocation is 0^D and the origin of the Creditor allocation is 0^C. Thus, 0^D0^C represents the two-state resource vector available for distribution, and every distribution can be described by a point in the box. If the Debtor were to fully repay his debt in every state, the resulting resource distribution would have been that described by point C', at which the Creditor receives RD in every state and the Debtor receives the residual. However, government resources, which are obtained from tax collection, are the vector $P0^C$, which does not enable full repayment in the low-productivity state. Therefore, the resulting allocation is as described by point C, at which the Debtor receives 0^DC and the creditor receives 0^CC. The Debtor residents' resources consist of 0^DP which they receive from ownership of claims to domestic output plus PC received via government transfers. The excess of tax revenue over debt service payments in the high-productivity state is represented by the length of the line segment PC.

Now, a small positive Δ--small enough to prevent tax revenue in the low productivity state from covering the remaining debt service obligations (i.e., such that $\theta_c(\Delta)>\theta_L$)--shifts the allocation point C to the right, such as to C_Δ, with the rightward movement being larger the larger Δ. This represents also the effect of debt forgiveness of size Δ. On the other hand, a positive ε shifts the allocation point down the broken line CJ (which is parallel to 0^D0^C), such as C_ε, with the downward movement being larger the larger ε. Hence, swaps that satisfy $\theta_c(\Delta)\geq\theta_L$ (i.e., $\Delta<\Delta_L = \overline{CB}/R$, where Δ_L satisfies $\theta_c(\Delta_L) = \theta_L$) can shift the allocation



O^c
45°
$t\theta_L E$
RD
$\theta_H E$
$d^*(\theta_L)$
C''
B''
B
$C(\theta_H)$
$C\Delta$
RD
Q
C
A
C'
$\theta_H E$-RD
P
$(1\text{-}t)\theta_H E$
$C\varepsilon$
I
J
$d^*(\theta_h)$
$\theta_L E$
$(1\text{-}t)\theta_L E$
O^D
$C(\theta_L)$

Figure 1

from C to any point in the parallelogram JCBI. Obviously, allocations that are south west of C are undesirable from the point of view of the Debtor, because they represent lower consumption in every state. Conseqently, if the swap is to be voluntary, it has to shift the allocation into ABC. However, no allocation in ABC improves welfare of both the Debtor and the Creditor if the Creditor's indifference curve is steeper than the Debtor's indifference curve at point C. On the other hand, if the Creditor's indifference curve is flatter than that of the Debtor's at point C, then there exist small debt-equity swaps that raise welfare of both parties. From (4) and (5) these indifference curves are defined by

$$\pi_L u[c(\theta_L)] + \pi_H u[c(\theta_H)] = U(0,0)$$

for the Debtor and

$$\pi_L^* \mu^*(\theta_L) d^*(\theta_L) + \pi_H^* \mu^*(\theta_H) d^*(\theta_H) = U^*(0,0)$$

for the Creditor, where π_i and π_i^*, i=L,H, are subjective probability assessments. The Creditor's indifference curve is linear, and its constant slope is

$$m^* = \pi_H^* \mu^*(\theta_H) / \pi_L^* \mu^*(\theta_L).$$

The slope of the Debtor's indifference curve at C is given by

$$m = \pi_H \mu[c(\theta_H;0,0)] / \pi_L \mu[c(\theta_L;0,0)],$$

where $\mu(c) \equiv u'(c)$ is the marginal utility of consumption. Hence,we have shown that there exist small debt-equity swaps that are beneficial to both parties if and only if $m > m^*$.

This result can be interpreted as follows. The expected marginal utility of consumption in state i is given by $\pi_i \mu(c_i)$ for the Debtor and by $\pi_i^* \mu^*(\theta_i)$ for the Creditor. These provide marginal valuations of resources in each state. Therefore the result is that there exist small Pareto-improving debt-equity swaps if and only if the debtor values resources in the high productivity state relatively more than the Creditor. It stems from the fact that the potentially beneficial swaps (in the area ABC) bring about a resource transfer from the Creditor to the Debtor in the high-productivity state and from the Debtor to the Creditor in the low-productivity state. Small transfers in the opposite direction are not possible without short equity positions or large swaps (large swaps are discussed at a later stage).

This analysis suggests that there exist reasonable circumstances in which both parties will not be able to agree on a small swap. In order to illustrate this point, consider the case in which the productivity shock in the Debtor country is idiosyncratic; i.e., it is statistically independent of economic conditions in the rest of the world. Then one expects the Creditor's marginal utilities $\mu^*(\theta_i)$ to be the same in both states. If this is indeed the case, and in addition the Creditor and Debtor hold the same subjective probability beliefs, then

$$m < m^* = \pi_H/\pi_L,$$

because as a result of risk aversion the Debtor's marginal utility of consumption is lower in the high-productivity state in which his consumption is higher. Hence, under these circumstances there do not exist small swaps that are beneficial to both parties. It is also clear from this discussion that in the presence of identical probability beliefs the existence of small beneficial swaps requires the Creditor's marginal utility of resources in the high-productivity state to exceed his marginal utility of resources in the low-productivity state. This, however, is a special feature of the two-state case. In the general case swaps bring about a resource transfer from Creditor to Debtor in states close to θ_c, and in the opposite direction in all other--both lower and higher productivity--states (more on this below).

Now consider large swaps, such that $\Delta > \Delta_L$ (which lead to $\theta_c(\Delta) < \theta_L$). Under these circumstances Δ per se shifts the allocation from C to a point on the line segment BO^C (rather then on CB), with the point being closer to the Creditor's origin the larger Δ. This results from the fact the once debt has been reduced by Δ_L, the debtor makes full payments in both states. Therefore every additional debt reduction increases resource transfer from Creditor to Debtor in both states, with the transfer being equal to the forgone debt service payments on the additional debt reduction. Thus, for example, if $\Delta = \overline{CB''}/R$, then the allocation point is shifted to C". As before, a transfer of equity from Debtor to Creditor shifts the allocation point south west, parallel to the diagonal. Hence, if the debt reduction that shifts the allocation to C" is paid for by means of equity, then a suitable relative price can bring the final allocation anywhere on the line segment JC", including C (the fact that the original allocation can be reproduced by means of large swaps is special to the two-state case). More generally, this type of swap can shift the allocation to every point in O^DIBO^C. Allocations on the diagonal are attained by swapping the entire debt for equity. This implies that if there do not exist small swaps that are beneficial to both parties (because $m < m^*$), then there do exist large beneficial swaps. This is seen by observing that $m < m^*$ implies that there exist allocations to the northwest of C which are close to C and which increase both parties' welfare. These allocations can be attained by means of large swaps (such that $\Delta > \overline{CB''}/R$). 1/

1/ One may also consider situations in which the Debtor becomes a creditor. If, for example, the Debtor is risk averse, the Creditor's marginal utility is the same in every state, and both have the same subjective probability distribution, then Pareto-Optima consist of allocations in which the Debtor obtains the same consumption level in every

The general case of many states is easier treated in asset space (Δ,ε). For this reason I present in Figure 2 the asset indifference curves

$$U(\Delta,\varepsilon) = U(0,0) \text{ and } U^*(\Delta,\varepsilon) = U^*(0,0)$$

for the two-state case. They are drawn on the assumption $m > m^*$, which implies that the Creditor's indifference curve is steeper at the origin. Every swap that leads to points above the Debtor's indifference curve and below the Creditor's indifference curve is beneficial to both parties. There is a discontinuity in the slopes of the asset indifference curves at Δ_L because at this point debt changes its nature as an asset, in the sense that the structure of returns on the asset changes. For $\Delta < \Delta_L$ it pays R in the high-productivity state and nothing in the low-productivity state, while for $\Delta > \Delta_L$ a marginal unit pays R in both states. Hence, the shift in the set of states in which debt is fully repaid convexifies the Creditor's preferences over swaps and introduces a nonconvexity in the Debtor's preferences. These effects also exist when the distribution function of productivity shocks has no mass points. It is shown in the Appendix that in the latter case the Creditor's indifference curve is concave and the Debtor's can be concave or convex, in each case ralative to the horizontal axis. Typical asset indifference curves for the case of a smooth distribution are presented in Figure 3, which also represents a case in which small Pareto-improving swaps exist.

It is instructive to understand the reasons for the particular curvatures of the asset indifference curves. As explained in the discussion of the two-state case, shifts in the set of states in which debt is fully repaid change the characteristics of debt as an asset. In the smooth distribution case these changes take place continuously as Δ changes. Thus, every increase in Δ increases the set of states in which the remaining debt is fully repaid and increases repayment per unit debt in the low-productivity states. These changes make remaining debt a higher quality asset. For this reason the larger is Δ, the larger the Creditor's losses from giving up an additional unit of debt. Consequently, he requires a larger marginal compensation in terms of equity in order to maintain a constant expected utility level. This explains the concave shape of the Creditor's indifference curve. Similarly for the Debtor; the larger Δ, the more he stands to gain from a marginal debt reduction. Therefore, at the margin he has to give up more equity per unit debt in order to maintain a constant expected utility level. If he was risk neutral, his asset indifference curve would have been concave, just as the Creditor's. However, risk aversion introduces convexity into his

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1/ (Cont'd from p. 8) state (in the two-state case these are points on a 45 degree line starting at 0^D). These allocations provide perfect insurance, and they can be attained by means of a swap in which the entire debt D plus some bonds issued by the Creditor are exchanged for the entire stock of equities E. Since my interest is mainly in small swaps, this possibility is not considered further.

indifference curve, and so the indifference curve can be concave or convex, depending on his degree of risk aversion.

From equations (1)-(5) one can calculate the slopes of these asset indifference curves

$$\rho(\Delta,\varepsilon) \equiv [-U_{\varepsilon}(\Delta,\varepsilon)/U_{\Delta}(\Delta,\varepsilon)] = \frac{\int_{0}^{\infty} \mu[c(\theta;\Delta,\varepsilon)](1-t)\theta dG(\theta)}{\int_{\theta_c(\Delta)}^{\infty} \mu[c(\theta;\Delta,\varepsilon)]RdG(\theta)}, \tag{6}$$

$$\rho^*(\Delta,\varepsilon) \equiv [-U^*_{\varepsilon}(\Delta,\varepsilon)/U^*_{\Delta}(\Delta,\varepsilon)] = \frac{\int_{0}^{\infty} \mu^*(\theta)(1-t)\theta dG^*(\theta)}{\int_{\theta_c(\Delta)}^{\infty} \mu^*(\theta)RdG^*(\theta)}. \tag{7}$$

For convenience of the following discussions, let $M \equiv \rho(0,0)$ and $M^* \equiv \rho^*(0,0)$ denote the slopes of the asset indifference curves at the origin (it is clear from this representation that in the two-state case $m > m^*$ if and only if $M^* > M$). Then,

<u>Proposition 1</u>. There exist small Pareto-improving debt-equity swaps if and only if $M^* > M$.

It is clear from (6) and (7) that $M^* > M$ can be satisfied even when the Debtor's relative valuation of resources in the high-productivity states is lower than the Creditor's. This stems from the fact that a swap reduces the Debtor's consumption in very high and very low productivity states, and increases his consumption in productivity states around θ_c, as shown in Figure 4. Prior to the swap the Creditor receives OAB, which describes the resource transfer from Debtor to Creditor in every state. The Debtor consumes the difference between OC and OAB, where the slope of OC is E (the stock of equity). Now, given $\Delta > 0$ and $\varepsilon = 0$, the transfer of resources from Debtor to Creditor shifts to $OA_{\Delta}B_{\Delta}$, with the Debtor consuming the difference between OC and $OA_{\Delta}B_{\Delta}$. This describes the effect of debt forgiveness. In the case of a debt-equity swap it is necessary to add to the resource transfer the return on equity $(1-t)\theta\varepsilon$. For sufficiently small values of ε the resulting resource transfer profile becomes OA_SB_S. It is clear that in this case the Creditor loses in states which lie in the interval $(\theta_1,\theta_2)$and gains in all other states (except, of course, θ_1 and θ_2). Since the Debtor receives the difference between OC and the resource transfer profile, he gains in the states in which the Creditor loses and vise versa. Hence, the Debtor gains in states in the

Figure 2

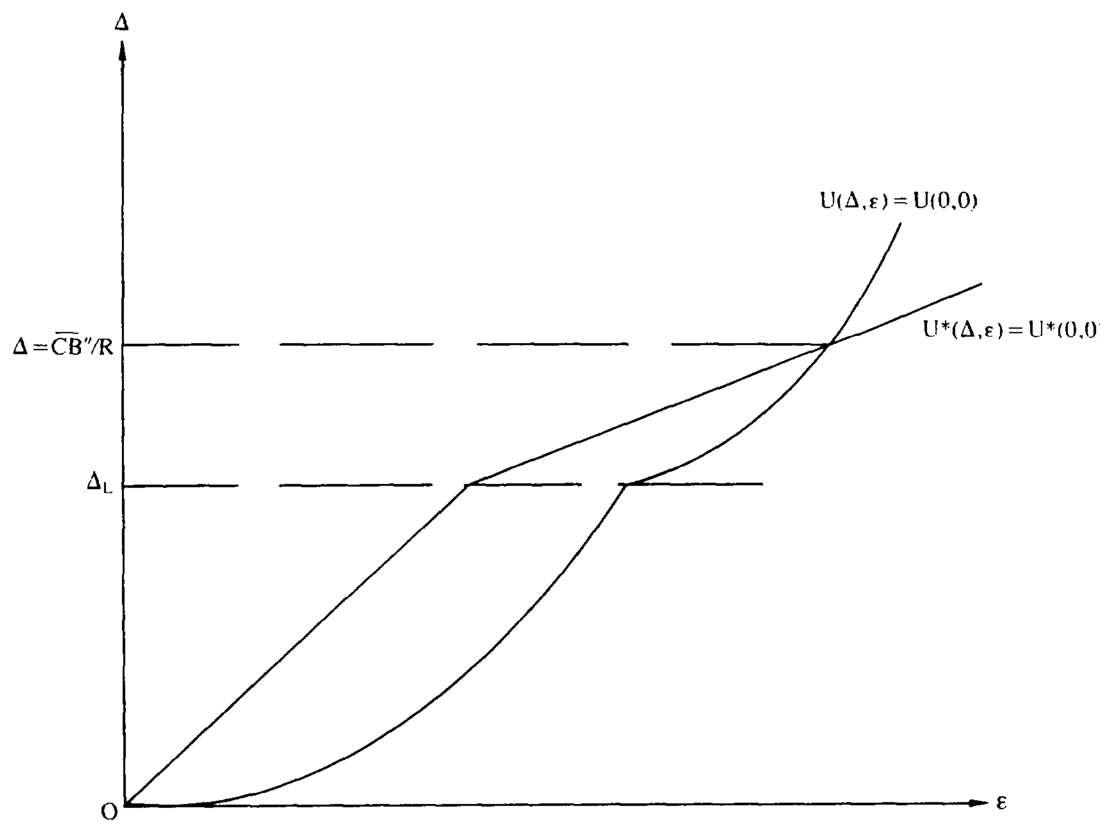


Figure 3

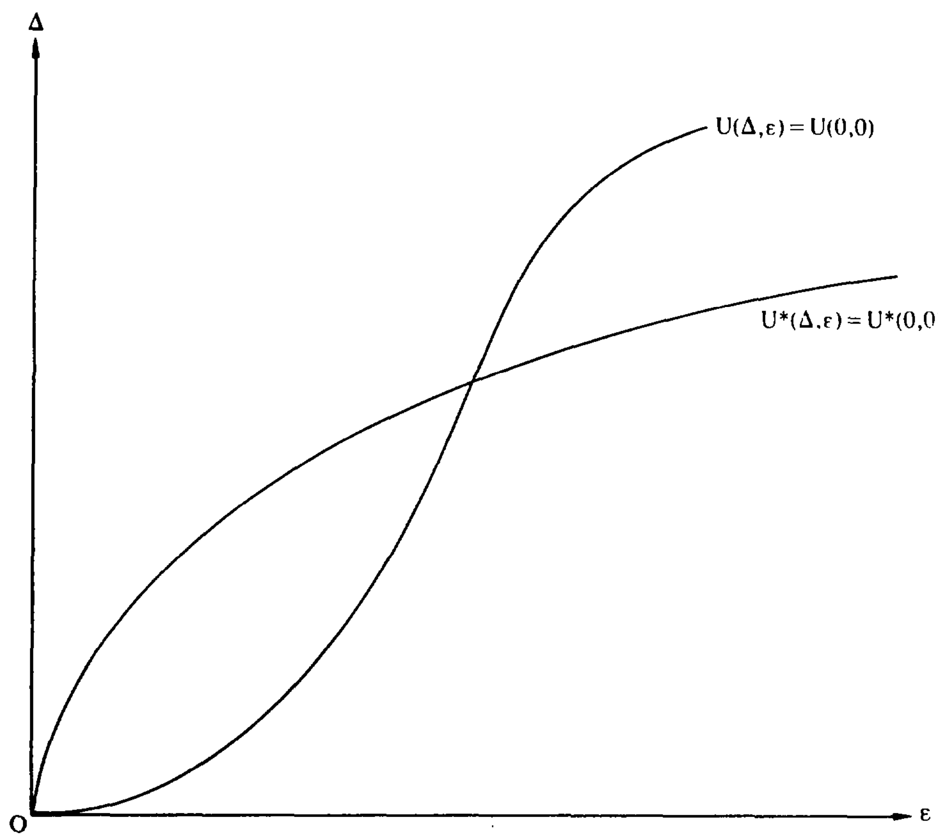
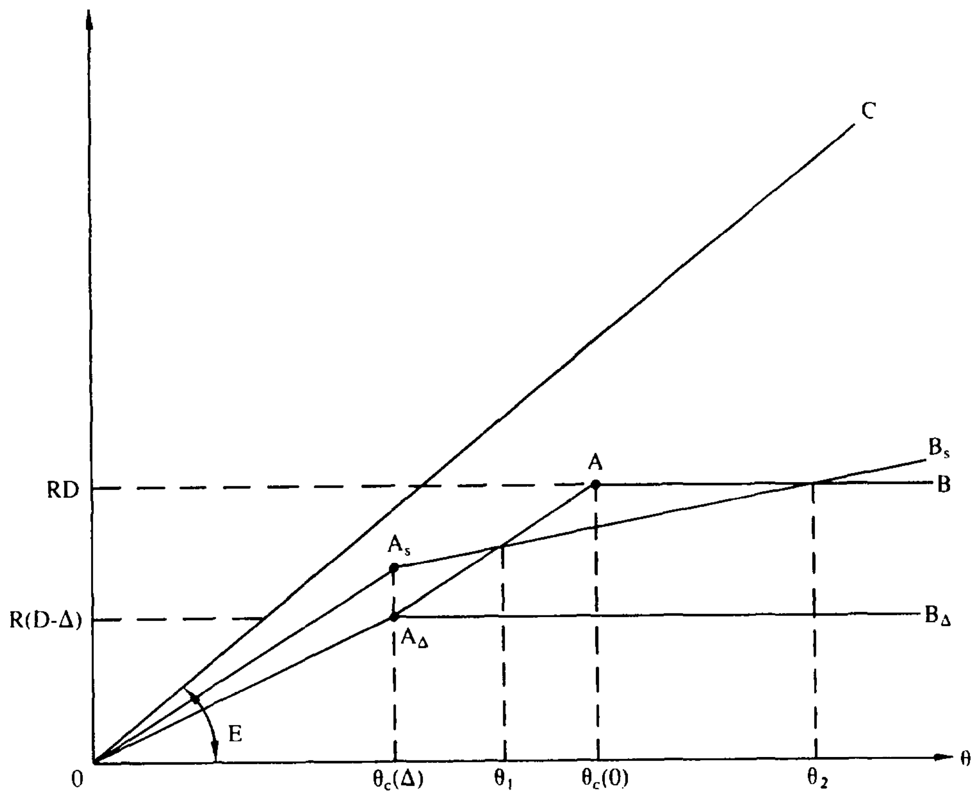


Figure 4


C
A
B_s
RD
B
A_s
R(D-Δ)
B_Δ
A_Δ
E
0
θ
$\theta_c(\Delta)$
θ_1
$\theta_c(0)$
θ_2

interval (θ_1, θ_2) and loses in the other (except for θ_1 and θ_2). Clearly, for sufficiently large values of ε point A is below the post-swap resource transfer profile, which implies that the Creditor gains in all states. These type of swaps are harmful to the Debtor.

IV. Discounts on Debt

Now restrict the discussion to small swaps, and suppose that indeed $M^* > M$, so that there exist debt-equity swaps which are beneficial to both parties. The question addressed in this section is whether a discount on debt is required for such transactions. In order to answer the question it is necessary to define the basis of the discount. Is it the face value of the debt or is it its market value? On the international financial markets the initial valuation of an equity (a unit of ε) is M^* units of debt (with the relative value of equity declining the larger the swap, as one can see from the Creditor's asset indifference curve in Figure 3). Since a small voluntary swap will not take place unless the average rate of exchange between equity and debt, defined by $x = \Delta/\varepsilon$, satisfies

$$M < x < M^*,$$

it is clear that beneficial swaps do not involve a discount on debt relative to its market value. In fact, under these circumstances the equity received by the Creditor is at least as valuable as the debt he has relinquished.

This point is more general. Since the Creditor's indifference curve curve in Figure 3 can be interpreted as the market's valuation of the Debtor's equity in terms of debt, it is clear that any debt-equity swap that is beneficial to the Creditor involves no discount on debt relative to its market value. Due to the assumed restrictions on international capital flows in the Debtor country, the Creditor's access to the Debtor's stock market by means of the swap provides him with a privilege that can generate benefits in excess of the forgone debt, even when the Debtor gains. Hence,

Proposition 2. Voluntary swaps do not involve a discount on debt relative to its market value.

On the other hand, Pareto-improving swaps may or may not require a discount on the debt's face value. As explained above, access to the Debtor's stock market is valuable, because by swapping debt for equity at the debt's face value, which is larger than its market value, the Creditor may still make a profit. If, for example, on the Debtor's stock market one equity is worth x units of debt at its face value, and $M < x < M^*$, then a debt-equity swap at market prices of equity and the debt's face value is beneficial to both parties. Moreover, in this case there exist $0 < \alpha < 1$ such that the Creditor will also agree to swaps in which a unit face value of the debt is exchanged for α dollars worth of equity. But if an equity on the stock market is worth $x' < M < M^*$ units of debt at face value, then the Debtor will refuse every debt-equity swap that does not entail discounting of the debt. In fact, the Debtor will refuse every deal in which he has to pay more than $\alpha' = x'/M$ on a dollar of debt, while the Creditor will refuse every deal in which he receives less than $\alpha'' = x'/M^*$ on a dollar of debt. However, both stand to gain from a small swap if a unit of debt is traded for α per dollar, where $\alpha'' < \alpha < \alpha'$. In this case the effective price of an equity is $x = x'/\alpha$, and it satisfies $M < x < M^*$. Hence, we have shown:

Proposition 3. If $M < M^*$, then there exist small Pareto-improving swaps without discounts on the face value of debt, unless the market price of equity is below M face value units of debt.

A global analysis of this issue, by means of the asset indifference curves in Figure 3, is now rather obvious.

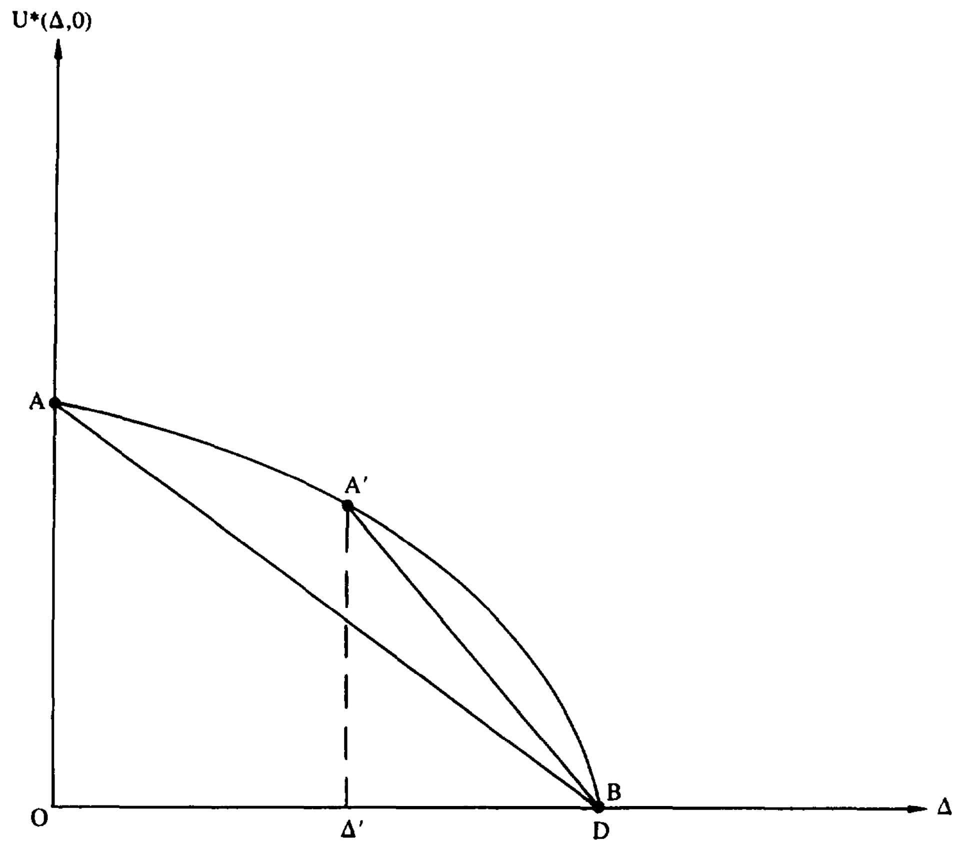
V. Many Creditors

So far our discussion was confined to the case of a single Creditor. In the presence of many creditors the indifference curve $U^*(\cdot)$ does not represent the amount of equity that the market will request in exchange for debt. This is so because the indifference curve is concave, reflecting the nonlinearity in the international financial markets' valuation of debt. This nonlinearity introduces an externality across creditors. The externality stems from the fact that if a single creditor reaches a swap agreement, the remaining creditors make a capital gain on their debt. This is most easily seen in Figure 5, where $U^*(\Delta,0)$ represents the international finanancial markets' valuation of the remaining debt as a function of Δ. 1/ This function is concave and declining, as one can see from the denominator on the far right hand side of (7), which is equal to $-U^*_\Delta(\Delta,0)$; i.e., the slope of the function. It is clear from the figure that the value of a unit of remaining debt is equal to the slope of AB in the absence of a swap and to the slope of A'B in the presence of a swap of debt of size Δ', with the latter being larger, the larger Δ'. Hence, swaps increase the unit value of remaining debt, bringing about a capital gain to creditors that do not participate in the swap. 2/

1/ Since $U^*(\Delta,\varepsilon) \equiv U^*(\Delta,0) + U^*(0,\varepsilon)$, the valuation of debt is separate valuation of equity. Therefore the first term on the right hand side represents the value of remaining debt.

2/ Bulow and Rogoff (1986) and Alexander (1987b) also argue that the value of marginal debt is not equal to the average value of the existing stock.

Figure 5



The externality that has been identified above is quite general, because it stems from the fact that international financial markets do not price debt linearly. It is relevant in every case in which valuation of debt plays an essential role, such as the analysis of debt forgiveness (see the discussion in the following section). In fact, the free rider problem that has been discussed by Krugman (1985) can also be cast in similar terms.

The implication of this discussion is that if--in the presence of a competitive fringe of creditors and perfect information--the Debtor wants to swap debt for equity, then the price per unit debt that he will pay must equal the post-swap value of a unit of remaining debt. For suppose it is higher; then every remaining creditor will agree to swap his debt at a lower price. And if it is lower, every creditor will refuse to swap, because the resale value of his asset is higher than the offered swap-price. Therefore, the equilibrium swap exchange rate is:

$$x(\Delta) = \frac{U_{\varepsilon}^{*}(0,0)}{U^{*}(\Delta,0)/(D-\Delta)}, \tag{8}$$

where the denominator on the right hand side is the equilibrium price of debt and the numerator is the price of equity (the price of equity is independent of the swap; i.e., $U_{\varepsilon}^{*}(\Delta,\varepsilon) = U_{\varepsilon}^{*}(0,0)$, and it is represented by the numerator on the far right hand side of (7)). Since $x(\cdot)$ represents face value debt per equity and it is a declining function, (8) shows that

Proposition 4. The larger the debt to be swapped, the lower the price the Debtor will receive for his equity.

The Debtor's optimal decision is presented in Figure 6. The curve $\varepsilon = \Delta/x(\Delta)$ represents equilibrium market opportunities. Its slope is flatter than the slope of the indifference curve $U^{*}(\cdot)$, and is located to its right. This location reflects the externality discussed above. Taking advantage of market opportunities, the Debtor's optimal policy is to swap Δ_0 units of debt for equity. This will raise his expected utility to U_0.

Finally, now the existence of small beneficial swaps to the Debtor requires $M < x(0)$, where $x(0)$ is the slope of the $\varepsilon = \Delta/x(\Delta)$ curve at the origin. Clearly, since $x(0) < M^{*}$, it might happen that $x(0) < M < M^{*}$. In this case small Pareto-improving swaps exist, but they will not eventuate. Hence,

Proposition 5. In the presence of a competitive fringe of creditors there exist circumstances in which no small swap will take place despite the availability of Pareto-improving small swaps.

VI. Buy-Backs and Debt Forgiveness

There exist several proposals for debt forgiveness (see, for example, Cline (1987)). One of them concerns the establishment of an international corporation that will buy back debt and forgive part of it to the debtors (the Kenen proposal). As pointed out by Dooley (1987), existing market discounts cannot be used to assess the cost of debt forgiveness, because the anticipation of debt forgiveness raises the market price of debt. He used a particular procedure to calculate the cost of debt forgiveness. In what follows I use our model to shed light on this issue.

Suppose the corporation buys and forgives Δ of debt (it may buy more, but the following analysis depends only on the amount forgiven). Then our analysis suggests that the unit value of remaining debt will be

$$p_D(\Delta) = U^*(\Delta,0)/(D-\Delta). \tag{9}$$

In the presence of a competitive fringe of creditors this is also the price the corporation will pay, so that the cost of debt forgiveness is $\Delta p_D(\Delta)$. Now assume for simplicity that $\mu^*(\theta) = 1/R$ for every θ; i.e., the valuation of resources on international financial markets does not depend on the Debtor's productivity shock and it equals one plus the interest rate. Then the expected value of the payment of R in every state is equal to one, and (3), (5) and (9) imply

$$p_D(\Delta) = \Pi(\Delta) + [1-\Pi(\Delta)]\xi[t\tilde{\theta}E/R(D-\Delta) \mid \tilde{\theta} < \theta_c(\Delta)], \tag{9a}$$

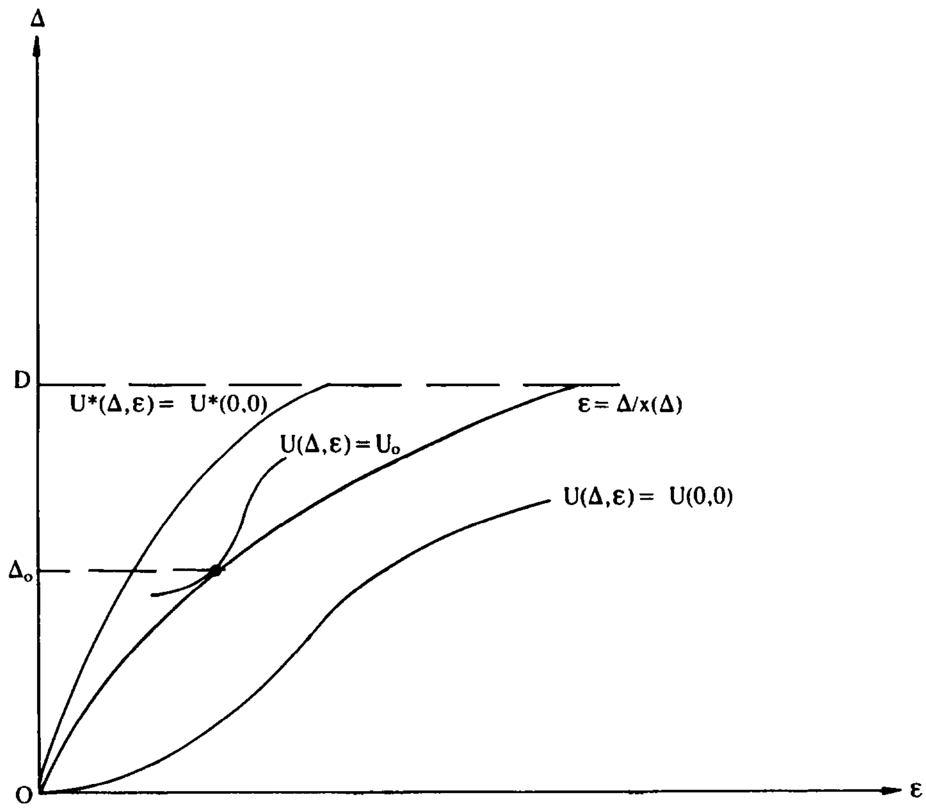
where $\Pi(\Delta) \equiv 1-G^*[\theta_c(\Delta)]$ is the probability that $\tilde{\theta} > \theta_c(\Delta)$ and $\xi[\cdot]$ is the expected value of the repayment share $t\tilde{\theta}E/R(D-\Delta)$ conditional on the productivity shock being smaller than $\theta_c(\Delta)$. Hence, the equilibrium price--which is the mean repayment share--is a weighted average of one (full repayment) and the mean repayment share in the low productivity states in which debt is only partially repaid. This formula enables one to calculate the effect of debt forgiveness on the price of debt.

For example, in the case where the productivity shock is distributed uniformly on the interval [0,1], we obtain

$$p_D(\Delta) = 1 - \frac{1}{2}\theta_c(\Delta) = 1 - \frac{1}{2}\theta_c(0)(1 - \frac{\Delta}{D}). \tag{9b}$$

Hence, if $\theta_c(0) = .8$ (the probability of full repayment is initially 20 percent), debt is valued at 60 cents to the dollar. And if the corporation comes into being with the intention of forgiving 20 percent of the debt (i.e., $\Delta/D = .2$), then the price of debt goes up to 68 cents to the dollar. Thus, a 20 percent forgiveness increases prices by close to 14 percent. This calculation suggests that a great deal of the corporation's resources will go to the creditors, despite its explicit intention to help the Debtor. This is in line with Dooley's argument. Thus, if debt is $100 billion and the corporation buys back 20 percent of it, it

Figure 6



spends \$13.6 billion. The remaining claims of the creditors are worth \$54.4 billion. Therefore, in order to reduce the value of claims by \$5.6 billion the corporation has spent \$13.6 billion. This means that \$8 billion--more than half the corporation's resources--go to the creditors, which is exactly the amount of the capital gain on the initial debt.

More generally, (9a) implies $p_D(D) = 1$; i.e., total debt forgiveness raises the price to its face value. Therefore, depending on the degree of debt forgiveness the price can end up anywhere between $p_D(0)$ and 1. Hence, if initially debt is traded at a high discount, say 20 cents to the dollar (as some of Peru's debt was traded), then a sufficiently high degree of debt forgiveness will bring about huge capital gains to the creditors with relatively little debt relief. For example, a buy-back and debt forgiveness of the entire debt provides debt relief of 20 cents per dollar costs, with the remaining 80 cents going to the creditors. Hence,

Proposition 6. A buy-back and debt forgiveness may reduce the value of debt by only a small share of its cost, with the remaining share being appropriated by creditors.

VII. Share Prices

We now consider the effect of debt-equity swaps and debt forgiveness on share prices in the Debtor country. In the process of this analysis I clarify some taxation issues and the role of government ownership of domestic companies. The current interest in share prices is mainly motivated by the desire to understand the effects on investment, an issue that is addressed in the next section.

In order to deal with share prices it is necessary to somewhat extend the model, because in the previous one-period formulation no suitable numeraire exists for their measurement. Assume therefore that there are two periods. The discussion in the previous sections applies to the second period, except for the debt-equity swap that takes place in the first period. Residents of the Debtor country choose in the first period first-period consumption c_0 and the amount of domestic equity they wish to hold e. Because of the existing restrictions on international capital movements they cannot hold foreign assets or borrow abroad, so that equity provides the only instrument by means of which they can transfer purchasing power from the first to the second period. 1/

1/ It is easy to add a domestic bond market to the model. However, in the absence of capital movements this market has to clear at zero indebtedness. Consequently, the following analysis would not be affected by this modification. In fact, one can calculate from what follows the equilibrium interest rate on this bond market.

Observe that from (2) second period consumption can be written as

$$c(\theta;\Delta,e) = (1-t)\theta e + T(\theta,\Delta),$$

where $e = E-\varepsilon$ represents Debtor residents' holdings of domestic equity and $T(\theta,\Delta)$ represents government transfers, with

$$T(\theta,\Delta) = \begin{cases} 0 & \text{for } \theta < \theta_c(\Delta), \\ t\theta E - R(D-\Delta) & \text{for } \theta \geq \theta_c(\Delta). \end{cases} \tag{10}$$

The size of Δ is exogeneous from their point of view, but in a market economy they can choose e.

Now the representative individual's preferences over first-period consumption and equity holdings can be written as

$$V(c_0,e;\Delta) \equiv u(c_0) + \delta\int_0^\infty u[(1-t)\theta e + T(\theta,\Delta)]dG(\theta), \tag{11}$$

where the right hand side is equal to the utility from first-period consumption plus the discounted expected utility from second-period consumption, with δ being the subjective discount factor. The individual's budget constraint is

$$c_0 + qe \leq y + qE - qC(\Delta), \tag{12}$$

where q is the price of equity, y is first-period output, and $C(\Delta)$ is the cost of the swap in terms of equity, with $qC(\Delta)$ being the tax imposed by the government in the first period in order to acquire the resources needed for the swap. In the case of debt forgiveness $C(\Delta) \equiv 0$. Thus, the left hand side represents spending on consumption and equity, while the right hand side represents resources available to the private sector, which consist of output plus the market value of initial equity holdings minus taxes.

The individual chooses c_0 and e so as to maximize (11) subject to (12). Denoting by $s(c_0,e;\Delta)/V_{c_0}(c_0,e;\Delta)$ his marginal rate of substitution between equity and consumption, the first order conditions of this problem yield

$$q = s(c_0,e;\Delta). \tag{13'}$$

However, due to the restrictions on capital movements, the clearing of the first-period commodity market requires $c_0 = y$ and the clearing of the equity market requires $e = E-C(\Delta)$. Therefore, (13') and the market clearing conditions imply that the equilibium price of equity as a function of the size of the swap is

$$q(\Delta) \equiv s[y,E-C(\Delta);\Delta]. \tag{13}$$

The right hand side of (13) represents the demand price for equity. Once $C(\Delta)$--the cost of the swap in terms of equity--is specified, this formula enables one to calculate the response of share prices to swaps. If there exists a single creditor, this cost may be determined by a solution to a bargaining problem, and if there exists a fringe of competitive creditors, then $C(\Delta) = \Delta/x(\Delta)$.

Consider the latter case for concreteness. In this case the equity cost of a swap increases with Δ, and differentiation of (13) yields

$$q' = -s_e C' + s_\Delta. \tag{14}$$

This formula shows that share prices are affected by two considerations. First, the larger the swapped debt the more equity has to be relinquished, which leaves domestic stockholders with less equity holdings. Since the demand price $s(\cdot)$ is declining in equity holdings, this element brings about an increase in share prices. Second, the swap has a direct effect on the demand price for equity, which is represented by the second term on the right hand side of this formula. This effect stems from government transfers. The larger Δ, the larger the transfers that the individual receives in the second period, and therefore the less he values equity which is used to transfer purchasing power from the first to the second period (i.e., s_Δ is negative). This reduces the demand price for equity. Consequently, the change in share prices depends on which one of these considerations extracts a stronger influence. If the former is stronger, the stock market goes up. And if the latter is stronger, the stock market goes down. The former is zero in the case of debt forgiveness.

Figure 7 provides a diagramatic representation of these considerations. Initially the economy is at point A, where first-period consumption is y and equity holdings are E. There is a family of indifference curves representing $V(c_0,e;0)$, with the indifference curve I(0) passing through A. The slope of this indifference curve represents the equilibrium equity price q(0). The line with slope q(0) that passes through A represents the relevant budget line for the individual decision problem. Naturally, he chooses point A.

Now, for a positive Δ the indifference curves of $V(c_0,e;\Delta)$ are flatter than the indifference curves of $V(c_0,e;0)$, with the slope being smaller the larger Δ. Therefore, if the individual was faced with the budget line of Figure 7 but with a positive Δ, he would choose B, at which the indifference curve $I(\Delta)$ is tangent to the budget line. The shift from A to B reflects the decline in the demand for equity as a result of the increase in second-period government transfers. In addition, the government imposes in the first period taxes $qC(\Delta)$. These taxes reduce the budget line. The leftward shift of the budget line is just equal to

$C(\Delta)$, which is the number of equities swapped (equal to ε in our previous notation). Hence, if ICC is the income expansion curve along which the marginal rate of substitution is constant at the level $q(0)$, then if $C(\Delta) < \overline{A'A}$ the demand point at constant share prices will be on ICC above A', and if $C(\Delta) > \overline{A'A}$ the demand point at constant share prices will be on ICC below A'. In the former case aggregate demand for equity--by domestic residents and foreign creditors--declines, leading to a decline in share prices. In the latter case aggregate demand increases, leading to a rise in share prices. This proves the following global result:

Proposition 7. A debt-equity swap raises share prices if and only if the equity cost of the swap is sufficiently high.

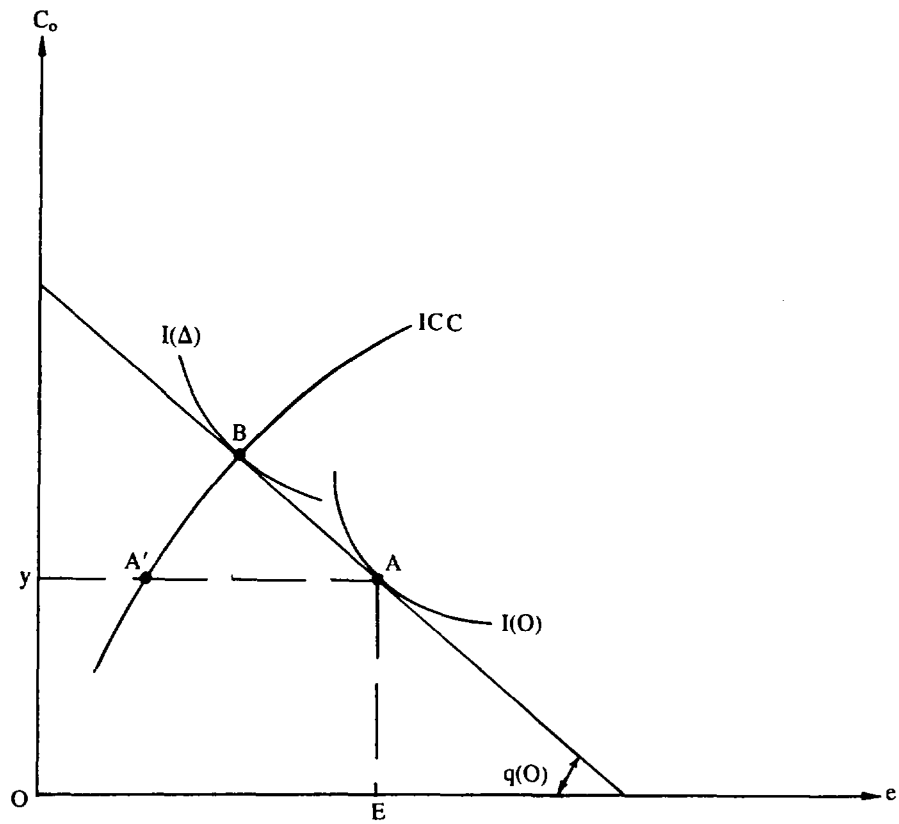
Observe that this proposition applies to both the case of a single creditor and the case of many creditors. An immediate implication is that in the case of debt forgiveness--viewed as a swap with $C(\Delta) \equiv 0$--share prices have to fall.

Proposition 8. Debt forgiveness brings about a decline of share prices.

By applying the Envelope Theorem to the individual's decision problem it is now easy to see that the condition for beneficial swaps for the Debtor are the same as before. This is no accident. The extended model is virtually identical to the one-period model, because in equilibium $c_0 = y$, which is a constant. Hence, by disregarding the constant first-period consumption level one does not affect the nature of the problem. However, its introduction provides natural units in which share prices can be measured; that is, first-period consumption.

It is also clear from this analysis that a swap involves a substitution of second-period for first-period taxes. The government increases taxes in the first period (in which the swap is performed) and reduces net taxes in the second period (in some states). First-period taxes are needed in order to obtain the resources required for the swap. These can be imposed directly, or indirectly by means of printing money (inflation tax). The current model cannot deal with the monetary aspects of the problem. But it should be clear from the analysis that resources for the swap have to be extracted one way or another. There is an alternative to taxation if the government owns domestic companies. Suppose, for example, that there are no taxes in the second period, but instead the government owns a proportion t of the domestic companies. In this case a share provides θ units of output in state θ. Now suppose that the government swaps its own equity for debt. Then the foregoing analysis goes through with every share in the previous case replaced by $(1-t)$ shares in the current case, provided $tE > (1-t)\varepsilon$. The last condition states that the government has enough shares to perform the swap. Combinations of partial ownership and partial taxation are also possible. The essential point is that if the government owns equity, then it redistributes the excess of

Figure 7



its income from equity over debt repayment to the private sector. The public is indirectly the owner of government companies and its debt. (This equivalence breaks down in the presence of investment.)

VIII. Investment

In this section the two-period model is extended to allow for investment. This is done as follows. Let the activity level E be a function of first-period investment I. Namely, the larger the investment level the more equities there are (these are real equities) and the proportionately larger is output in each state. Given an equity price q, the net value of firms is

$$qE(I) - I.$$

It is assumed that $E(\cdot)$ is a concave function and that firms choose the investment level so as to maximize their net value. Their equilibrium condition is

$$qE'(I) = 1. \tag{15}$$

This condition describes demand for investment as a function of share prices, or alternatively, the supply price of equity as a function of the investment level.

Now the individual's budget constraint contains the net value of firms rather than gross value, so that (12) is replaced by

$$c_0 + qe \leq y + qE(I) - I - qC(\Delta,I), \tag{12'}$$

where this time the cost may also depend on the investment level (it is, for example, easy to see that in the presence of a competitive fringe of creditors $x(\cdot)$ is a function of I, because it depends on E). He maximizes (11) subject to (12'), taking the investment level as given. However, the equilibrium condition in commodity markets is now

$$c_0 + I = y.$$

Therefore (13) is replaced by

$$q = s[y - I, E(I) - C(\Delta,I);\Delta], \tag{16}$$

which describes the demand price for equity as a function of investment and swap size.

For every Δ conditions (15) and (16) determine equilibrium share prices and investment. The equilibrium determination of these variables is described in Figure 8. Curve S describes the supply price of equity while curve D describes the demand price in the absence of swaps. The intersection point A describes equilibrium investment and share prices. Now suppose that a swap takes place. It does not affect the supply curve. However, the demand curve at point A increases if and only if the condition for a share price increase that was derived in the previous section applies, because the previous analysis applies to the case of a fixed investment level. If it increases, investment and share prices go up, and if it declines, investment and share prices decline. Therefore, using Propositions 7 and 8, we have shown:

Proposition 9. A debt-equity swap raises investment and share prices if and only if the equity cost of the swap is sufficiently high.

Proposition 10. Debt forgiveness reduces investment and share prices.

The last proposition has important implications. It shows that debt forgiveness brings about a reduction in the capacity to repay debt. The decline in investment reduces the set of states in which debt is fully repaid and payments in states in which it is only partially repaid. For this reason the size of a buy-back underestimates the extent of debt forgiveness. By the same token our calculation of the capital gain resulting from debt forgiveness overestimates it. A proper calculation needs to take into account the investment effect, which is rather easy to do on the basis of our analysis. A key question is whether this proposition is robust.

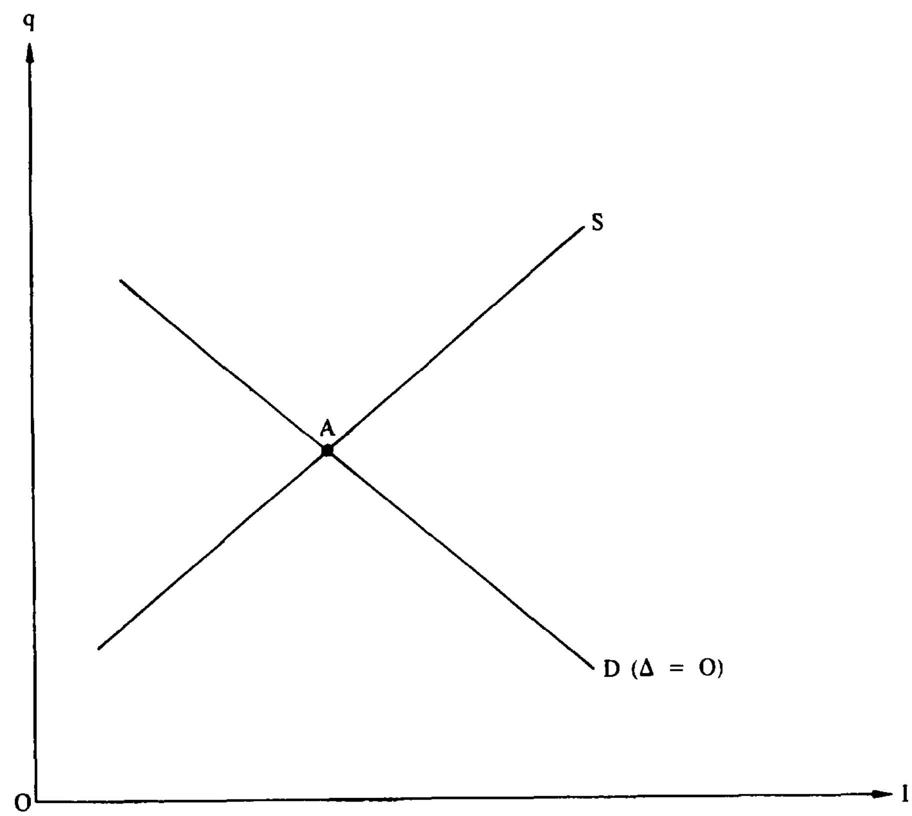
IX. More on Debt Forgiveness

The result that debt forgiveness reduces investment is controversial. The proper interpretation of my finding is that there exists an intrinsic feature of the problem that has a negative affect on investment, this feature being the positive income effect in the second period. Other features, which are absent from my model, may have opposite effects. If they exist, the effect of debt forgiveness on investment becomes an empirical issue.

A popular argument states that debt forgiveness leads to a reduction of taxes in the debtor country, thereby increasing the rate of return on investment and stimulating investment spending. I wish to deal explicitly with this possibility, and compare its strength with the income effect. This requires a modification of the structure of taxes and transfers.

In the previous specification debt relief leads to higher second-period income of the private sector via government transfers. If these transfers where instead redistributed to firms proportionately to output, they would raise the rate of return on investment. In fact, this procedure provides the strongest possible investment stimulus. Hence, if under these circumstances we were to compare the income effect with the

Figure 8



rate of return effect, we would obtain a lower bound on the strength of the income effect relative to the rate of return effect. This is the case treated below. It is, however, helpful to observe that this procedure has an equivalent tax structure representation of the following type. The government imposes state contingent tax rates on output $\tau(\theta) \leq t$, where t is the ceiling on possible tax rates. This implies that in all states $\theta < \theta_c(\Delta,I)$ it cannot collect enough revenue to cover debt service payments, where $\theta_c(\Delta,I)$ is defined in (1) as before and E depends on investment. Hence, in these low productivity states it applies the tax rate t and creditors receive the tax revenue, exactly as before. On the other hand, in all states $\theta \geq \theta_c(\Delta,I)$ the tax rate is adjusted to ensures a tax revenue equal to the required debt service payments RD. There are no lump-sum transfers in this system. The result is that the distribution of returns on a unit of debt does not change, but the state contingent return on an equity (a unit of E) becomes

$$\eta(\theta;\Delta,I) = \begin{cases} (1-t)\theta & \text{for } \theta \leq \theta_c(\Delta,I), \\ \theta - R(D-\Delta)/E(I) & \text{for } \theta \geq \theta_c(\Delta,I). \end{cases} \quad (17)$$

The return on equity holdings is the same as before in low-productivity states and higher in high-productivity states. 1/ The preference ordering (11) is replaced by

$$V(c_0,e;\Delta,I) = u(c_0) + \delta\int_0^\infty u[\eta(\theta;\Delta,I)e]dG(\theta). \quad (11')$$

The representative individual maximizes (11') subject ot (12'). His first order conditions imply

$$q = \hat{s}(c_0,e;\Delta,I),$$

where $\hat{s}(\cdot) \equiv V_e(\cdot)/V_{c_0}(\cdot)$ is the marginal rate of substitution between equity and first-period consumption.

Now define

$$S(c_0,e;\Delta_1,\Delta_2,I) \equiv \delta\int_0^\infty \mu[\eta(\theta;\Delta_1,I)e]\eta(\theta;\Delta_2)dG(\theta)/\mu(c_0), \quad (18)$$

where $\mu(\cdot)$ is (as before) the marginal utility of consumption. Then we have

$$\hat{s}(c_0,e;\Delta,I) \equiv S(c_0,e;\Delta,\Delta,I).$$

1/ It is possible to describe feasible reallocations under this tax system in the same way as before. For example, in the two-state case point C in Figure 1 remains the initial point. A reduction of Δ shifts it to the right, say to C_Δ. Now, however, an addition of ε does not shift it to C_ε, but rather south-west along the ray O^DC.

Namely, when $\Delta_1 = \Delta_2 = \Delta$, $S(\bullet)$ represents the marginal rate of substitution. The distinction between Δ_1 and Δ_2 helps to separate the income from the rate of return effect; a change in Δ_1 represents the income effect while a change in Δ_2 represents the rate of return effect. An increase in Δ_1 raises income from given equity holdings, thereby reducing the marginal utility of consumption in the second period and the demand price for equity. An increase in Δ_2 increases the return on equity holdings without changing second-period consumption, thereby raising the demand price for equity. The net effect of debt forgiveness is represented by

$$\hat{s}_\Delta(c_0,e;\Delta,I) = S_{\Delta_1}(c_0,e;\Delta,\Delta,I) + S_{\Delta_2}(c_0,e;\Delta,\Delta,I).$$

It is now straightforward to see from (18) that for $\Delta_1 = \Delta_2 = \Delta$ the sum of these effects is negative if the Arrow-Pratt measure of relative risk aversion $r(c) \equiv -\mu'(c)c/\mu(c)$ is larger than one for all relevant consumption levels, and the sum of these effects is positive if $r(c)$ is smaller than one for all relevant consumption levels.

The equilibrium condition (16)--with $C(\Delta,I) \equiv 0$--is now replaced by

$$q = \hat{s}[y-I,E(I);\Delta,I], \tag{16'}$$

with $\hat{s}_I(\bullet) = \hat{s}_\Delta(\bullet)E'(I)(D-\Delta)/E(I)$ (see Appendix). Hence, if $r(c) > 1$ for all consumption levels, $d\hat{s}/dI < 0$, and (16') can be represented by the demand curve in Figure 8. The supply curve, representing (15), does not change. In this case debt forgiveness shifts down the demand curve, thereby reducing investment and share prices. This proves,

Proposition 11. If the relative degree of risk aversion is larger than one, the income effect dominates the rate of return effect, and debt forgiveness reduces investment and share prices.

If the relative degree of risk aversion is smaller than one, the demand curve may be upward sloping. If it does, assume that it is flatter than the supply curve. Then we have,

Proposition 12. If the relative degree of risk aversion is smaller than one, the rate of return effect dominates the income effect, and debt forgiveness raises investment and share prices.

The implication is that in the presence of a rate of return effect, debt forgiveness reduces investment when risk aversion is high and increases investment when risk aversion is low.

X. Conclusions

Debt-equity swaps and debt forgiveness are practical issues that require careful analysis. The results of this paper demonstrate that there do not exist simple and clear-cut answers to a number of major questions, but they also show how to identify relevant considerations. Some of the practical conclusions are:

(1) Small debt-equity swaps can be beneficial to both parties, but this is not always the case.

(2) In the presence of many creditors there is a unique price at which a swap of a given size can be performed, with the price being higher the larger the swap.

(3) Under these circumstances small swaps may fail to take place despite the existence of Pareto-improving deals.

(4) Voluntary swaps will not take place with discounts on the debt's market value, and voluntary swaps may or may not require discounts on the debt's face value.

(5) A buy-back and debt forgiveness may be very costly, with the major part of the benefits accruing to the creditors (rather than the debtor).

(6) Debt forgiveness may reduce investment in the debtor country, thereby imposing a secondary cost via a reduction of debt service payments.

(7) A debt-equity swap may also increase or reduce investment. In all cases with ambiguous answers, we have identified the conflicting elements that have to be assessed empirically.

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