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Controlling Inflation: The Problem of Non-Indexed Debt

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Abstract

We show that the presence of nominal non-indexed government debt could give rise to more than one equilibrium inflation rate. Conditions for this to occur are discussed in terms of ad hoc and micro-founded models. Solutions to the indeterminacy problem are examined; one solution is shown to be price indexation of debt instruments.

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Summary

The paper studies models in which the monetary and fiscal authorities attempt to maximize the welfare of the "representative" individual. The paper focuses on solutions in which the government is unable to precommit its future policy (or actions).

The paper shows that a policy aimed at controlling inflation may encounter serious difficulties if credit contracts are not fully indexed to the price level. It focuses on the case in which the government issues non-indexed public debt and demonstrates that such a case may lead to more than one equilibrium solution. Since inflation and welfare differ across equilibria, where there are two equilibria, one with relatively low expected inflation and, hence, a low nominal interest rate and the other with relatively high expected inflation and a high nominal interest rate, the one displaying the highest inflation yields the lowest welfare. The paper considers these issues in terms of both ad hoc models and models with firmer microeconomic foundations. Particular attention is given to debt indexation, because it has been tried out in practice and is one of the possible solutions to the above problem.

In the context of the models, the paper is able to show that the equilibrium solution with debt indexation dominates any of the equilibrium solutions attained without indexation. This result has potentially important implications for stabilization programs, given that there seems to be the generalized presumption among economists and policymakers that, contrary to the results of this study, indexation may stand in the way of a successful stabilization program.

I. Introduction

It is probably fair to say that monetary theory has reached a stage where the case of "passive" money (Olivera (1971)) commands the same level of intellectual respect as the standard textbook closed-economy case of active money. To refresh ones memory remember passive money is a situation where the supply of money (or its rate of growth) is an endogenous variable, being, therefore, a function of some of the other variables in the system. The importance of distinguishing between active and passive money systems was sharply brought home by the Mundell-Fleming model (e.g., Dornbusch (1980)), in terms of which one can show that the effects of fiscal and monetary policies with perfect capital mobility depend crucially on whether the economy is under fixed or flexible exchange rates, the former corresponding to passive, and the latter to active money.

The identification of a passive money regime may not be as obvious as the Mundell-Fleming paradigm might suggest. Money, for example, could be active in the short run, but passive in the long run. This would likely be the case in a closed-economy context--where, for simplicity, we assume that population and technical knowledge are constant--if it were not possible to reduce the fiscal deficit below a certain positive fraction of GNP, and if the authorities insisted on trying to stop inflation by refusing to monetize the deficit. In the short run the supply of money would be exogenous, but eventually the accumulation of bonds would tend to make the situation unsustainable and lead to a possible explosive expansion of money supply. 1/

The above delayed-passivity of the supply of money is a fascinating subject because it shows the possibility that politicians implement "bad" but short-run popular policies--a big fiscal deficit with low inflation, for example--without having to suffer the consequences, since inflation might return after they have already left office. 2/ But there are also circumstances when the future upsurge of inflation would have little to do with any kind of "fundamental" mismanagement of the operational part of the fiscal budget; instead, in these cases future inflation arises from the need to pay for the services of the public debt, or to eliminate a generalized bankruptcy situation that occurs if inflation were to be kept relatively low. This slightly unfamiliar theme is the subject matter of this paper.

I became interested in this subject after I noticed the serious straits that economies like Argentina, Chile and Bolivia have been put

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1/ Recent analyses of this case are Sargent and Wallace (1981), McCallum (1984), Liviatan (1984), Drazen (1985), and Calvo (1985).

2/ Of course, the new administration will make every effort to shed the blame on the previous one, but this is always difficult in our macro reality in which random shocks play such an important role that true causal relationships are hidden by a host of spurious correlations.

into as a consequence of not being able to induce a nominal interest rate level compatible with their long-run inflationary targets. This can be seen in Calvo (1986), Corbo, de Melo and Tybout (1986), Edwards and Cox-Edwards (1987), and Sachs (1987)). In all of these experiences, with the possible exception of the ongoing stabilization program in Bolivia, the expost real rate of interest remained at alarmingly high levels, which may have been responsible for the eventual liquefactions and/or socializations of the public and private debt.

A quick helicopter-type look at the above-mentioned experiences, however, may not generate any need to develop new economic theory, since the relatively high nominal interest rates were eventually followed by also relatively high inflation rates (once again, with the momentary exception of Bolivia). Thus, those events would seem to be compatible with the straightforward explanation that "the nominal interest was high, because people realized that inflation was going to flare up in the future due to say, an unduly large fiscal deficit." In fact, I am not going to present a quarrel with this point of view. Instead, I will attempt to go a little further into the economics of the fiscal deficit itself, and explore the possibility that "people expected a relatively high inflation, which brought about relatively high nominal interest rates, which swelled the public debt service, which increased fiscal deficit...." In other words, I am going to study the possibility that the nominal interest rate causes inflation, instead of the other way around. 1/ Notice that the above reasoning would not hold if the public debt was fully indexed to the price level, because in that case the nominal interest rate is determined by the actual, not the expected, rate of inflation. Thus, a central ingredient of our analysis is the existence of non-indexed debt.

The paper is aimed at exploring the above-mentioned relatively novel relationship between the nominal interest rate and inflation. It should be noted from the outset, however, that although the "threat" of high real rates plays an important role for generating "high-inflation" equilibria, our examples are not capable of rationalizing a "transition period" in which the ex post real rate of interest is relatively high due to inflationary expectations (as it appears to be so in the above-mentioned country experiences). 2/

Section 2 presents the central argument in the simplest, almost purely graphical, form. It is shown that in a world where taxation is distorting and the public debt is not indexed to the price level, a benevolent government may choose the level of inflation as a function of

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1/ It is worth mentioning here, however, that other than the reversion of the standard casual relationship between inflation and the nominal interest rate, our analysis will be perfectly consistent with Fisher's equation and the other tenets of orthodox monetary theory.

2/ See Section 6 for a possible extension to cases in which ex post and ex ante real rates of interest are not equal.

the nominal interest rate, which may lead to the existence of multiple self-fulfilling expectations equilibria; each of these equilibria will be chosen depending on the interest rate which is, in turn, determined by the expectations of the private sector. Section 3 defines more specific two-period reduced-form examples to get some further insight into the assumptions that could be made in order to generate these examples, and to make sure that some obvious "second-order" condition be not forgotten. This is further pursued in Section 4, where the analysis begins at a more micro level with some assumptions on the role of money in production. Section 5 discusses extensions to more than two periods, and to setups where there is only private non-indexed debt. Conclusions and more general implications of our analysis are specified in Section 6.

II. A Simple Model

Consider a world of two periods, $t = 0, 1$. In period 0 people form (point) expectations about the rate of inflation between period 0 and period 1, which we denote π^e. We assume that individuals can invest in a risk-free asset with an exogenous one-period real rate of return equal to $r > 0$. Therefore, the co-existence of the latter with non-indexed debt yielding a nominal interest rate i, requires:

$$1 + i = (1 + r)(1 + \pi^e) \tag{1}$$

This is just the Fisher equation.

Let the real stock of public debt at the end of period 0 be denoted by b; if the debt is not indexed, nominal amortization plus interest in period 1 will be:

$$b(1 + i) \tag{1a}$$

Thus, letting π stand for the actual rate of inflation between periods 0 and 1, total real debt service in period 1 would be:

$$b\,\frac{1 + i}{1 + \pi} \tag{2}$$

In period 1 the nominal interest rate, i, is a predetermined variable, so if taxation were socially costly and π could be manipulated by the fiscal or monetary authorities, they may be tempted to set π as large as possible. Notice that changes of π at time 1 do not affect π^e at time 1 because, like i, π^e is a variable which is determined in period 0; thus, increasing π at time 1 would not lead to a reduction in the demand for money if, as we usually do, it is assumed that the latter depends only on expected inflation. Clearly, therefore, if the costs of inflation are related to π^e alone, a government attempting to minimize the social costs of servicing the debt would set $\pi = \infty$, or, alternatively, it would eliminate the present currency. In all other cases, however, the optimal response of government will call for setting π at a finite level, and, in view of equation (2), it is reasonable to expect that π will be an increasing function of i; more formally, we assume that government's optimal response is summarized by the following relationship (Figure 1):

$$\pi = \phi(i), \ \phi' > 0 \tag{3}$$

Let us concentrate on situtations where, in period 0, the public knows that the government in period 1 is going to behave according to equation (3). Since we are abstracting from uncertainty, expectations are accurate (perfect foresight), which means:

$$\pi^e = \pi \tag{4}$$

Combining (1) and (3), we get:

$$1 + i = (1+r)(1+\pi) \tag{5}$$

Curves (3) and (5) are depicted in Figure 1. Both curves are upward sloping and can cross each other more than once. Each crossing depicts an equilibrium. As an example, if the public expects that inflation will be π^0, then the simultaneous existence of nominal and real assets requires that the nominal interest rate be set at i^0 at time 0; when time 1 arrives, and it is the turn for the government to "move," the economy has inherited i^0 and the government will find it optimal to respond by choosing $\pi = \pi^0$, validating expectations. Unfortunately, however, unless we are able to impose further equilibrium conditions, equilibrium will in general not be unique (see Figure 1).

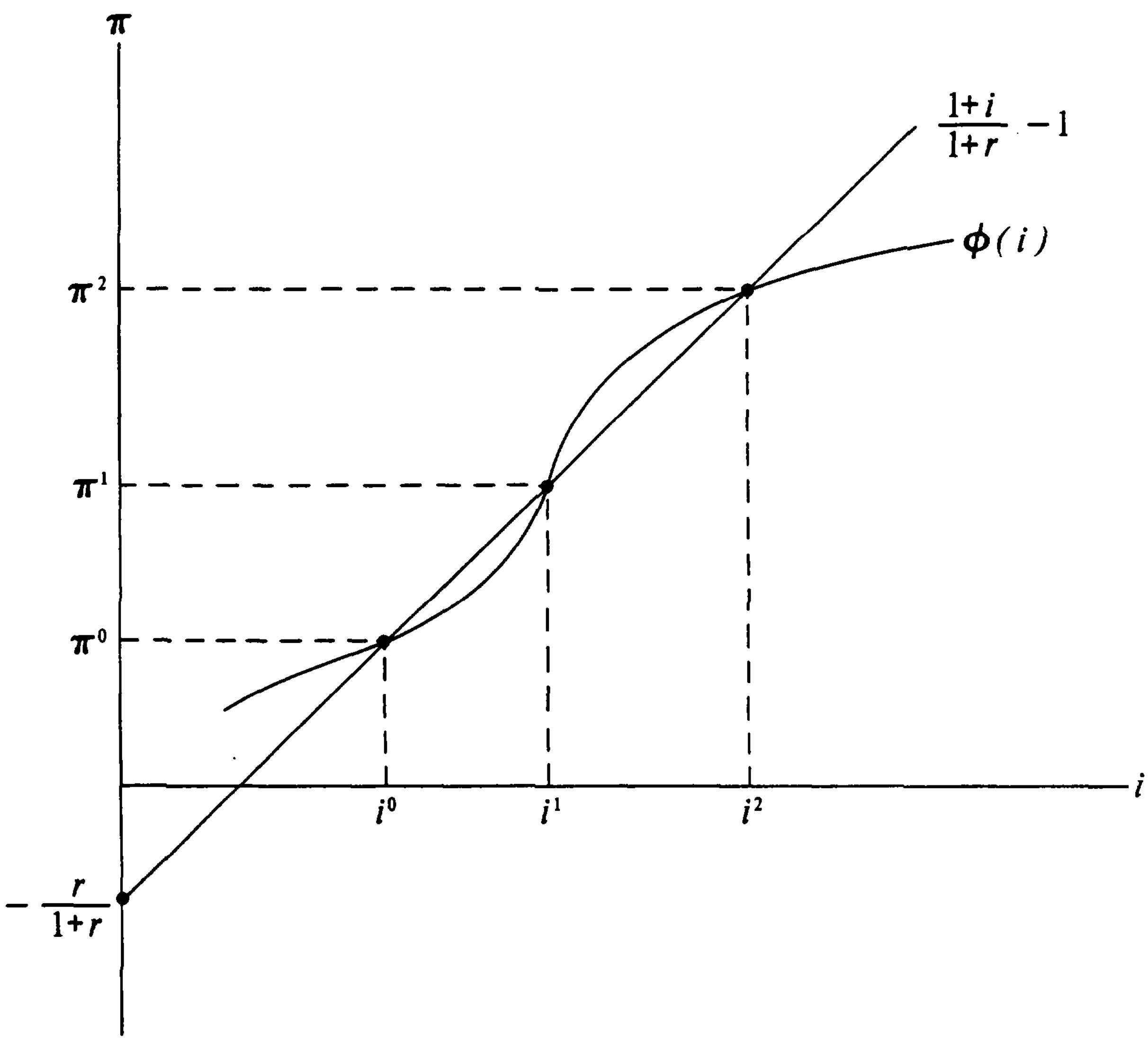
To put some realistic color into the above framework, imagine that after a period of relatively high inflation, the government (at time 0) "puts its house in order" to ensure the existence of a low-inflation equilibrium (π^0, say): if the public believes that inflation will be lowered to π^0 (recall Figure 1), the other "fundamentals" have been arranged to yield an inflation equal to π^0; the stabilization policy is, from that standpoint, "credible." The public knows, however, that if the nominal interest rate settles at i^2 the government will be induced to give up the stabilization program, at least partially. The reason for the latter is that everybody knows that if the nominal interest rate is i^2 it will be unduly costly for the government to keep inflation at π^0; in fact, the government will actually be induced to raise it to π^2, which was the reason why investors required $i = i^2$.

The situation would be quite different if the interest rate was fully indexed to the rate of inflation. Thus, if the real interest rate on bonds is denoted by r_b, then the nominal ex post debt service (including amortization) will be:

$$b(1 + r_b)(1 + \pi)$$

Figure 1

Equilibrium Interest and Inflation Rates



Consequently, the real debt service in period 1 would be:

$$b(1 + r_b) \tag{6}$$

Moreover the Fisher equation with perfect foresight--the equivalent of equation (5) above--now reads:

$$r_b = r \tag{7}$$

implying that the real debt service is simply:

$$b(1 + r) \tag{8}$$

which is independent of both the expected and the actual rate of inflation. Consequently, the optimal response of the government would be *independent of the expected rate of inflation and the nominal* interest rate. To be true, there may still exist other channels through which expected inflation finds its way into the money-printing machines, but indexation of the debt would have removed one of its tentacles.

An important observation is that in our story the government is not pushed to higher inflation because of its "inability to pay." Here the government chooses to generate a higher rate of inflation because it is too costly not to do so. Another observation is that at equilibrium the government has no incentive to surprise the public by departing from the expected policy. It would be wrong, however, to conclude that the role of policy surprises is not important just because unanticipated or surprise inflation is not an attractive option at equilibrium. In the present model equilibrium points themselves are determined by the public taking into account the potential for policy surprises. 1/

In addition to the burden of the debt in period 1, the government normally would face other fiscal obligations, which we denote by g. Since the presence of g makes total government obligations in period 1 equal to:

$$g + b\,\frac{1 + i}{1 + \pi} \tag{9}$$

one would normally expect the optimal inflation response to be an increasing function of g; thus:

$$\pi = \phi(i,g) \quad \phi_i > 0, \ \phi_g > 0 \tag{10}$$

1/ For example, it would be incorrect o assert that the phenomena discussed in this paper are not quantitatively important on the basis of a calculation showing that the potential gain from surprise inflation, say, is a relatively small number. According to our analysis, the potential for advantageous policy surprises may actually be small because the public realizes that otherwise the government would be tempted to surprise them!

Therefore, recalling Figure 1, an increase in government expenditure would shift the set of equilibrium inflation rates. At equilibria like (i^0,π^0) and (i^2,π^2) the rate of inflation would tend to rise (locally), while at equilibria like (i^1,π^1) inflation would tend to fall. Despite the ambiguity, we see that according to this model, government expenditure (not its deficit) affects the rate of inflation.

The predictive power of the model could be improved by dropping some of the above equilibria. One way to do this would be to superimpose the following pseudo-dynamics. We will say that a given equilibrium is stable if (locally) a higher (lower) than equilibrium expected inflation leads the monetary authorities to set $\pi < \pi^e (\pi > \pi^e)$. In other words, an equilibrium is stable if the government's response to wrong inflationary expectations is to set the rate of inflation toward the equilibrium one. According to this criterion, therefore, equilibria (i^0,π^0) and (i^2,π^2) would be stable, while (i^1,π^1) would not. Thus, at a stable equilibrium, a higher government expenditure is always (locally) inflationary.

Similar conclusions could be reached in relation to the stock of bonds; one could, for instance, show that at stable equilibria the higher the debt outstanding, the higher will also be (locally) the rate of inflation. There is also the interesting possibility of "loosing" some of the equilibria as curve ϕ in Figure 1 shifts up or down; but at this stage our discussion will benefit from further parameterization.

III. An Ad Hoc Model of the Inflation/Taxation Costs

In this section we will examine more detailed specifications of the social costs involved in taxation and inflation. If total government expenditure is given by equation (9), then the required taxes, x, must satisfy: 1/

$$x = g + b\,\frac{1 + i}{1 + \pi} \tag{11}$$

We denote the deadweight loss of taxation by z(x), and we assume:

$$z(x) \geqq 0 \text{ for all } x \tag{12a}$$

$$z''(x) > 0 \tag{12b}$$

Condition (12a) requires no discussion, while (12b)--strict convexity--is made to ensure the existence of a global optimum.

1/ For simplicity, and without loss of generality, we will hereafter abstract from the inflation tax on non interest-bearing money. Notice, incidentally, that its inclusion would amount to just adding some function of (*) to the right-hand side of equation (11).

More controversial and less obvious are the costs of actual (as against expected inflation), $f(\pi)$. We will examine the following formulation:

$$f(\pi) = \frac{\alpha}{2}\left(\frac{\pi}{1+\pi}\right)^2 + \frac{\beta}{2}\pi^2, \quad \alpha > 0, \ \beta > 0 \tag{13}$$

for $\pi > -1$. Most of the work in this area has assumed a form akin to the second term in equation (13) (Barro and Gordon (1983), and Bohn (1987)), by which the marginal cost of inflation increases as the economy moves away from some (unconstrained) optimal level ($\pi = 0$ in the present case). The first term is, on the other hand, somewhat unusual (see, however, Calvo (1987)); it shares the above-mentioned property of π^2 for $\pi \leq 0.5$, but the implied marginal cost of inflation declines monotonically for $\pi \geq 0.5$. In fact the first term of equation (13) converges to $(\alpha/2)$ as $\pi \to \infty$. The two terms together, therefore, allow us to capture a situation where the cost of inflation rises steeply for relatively low inflation, reaches some kind of a plateau, and eventually rises without bound for large inflation. What I have in mind is a situation where going from zero to 20 percent annual inflation raises costs considerably, the marginal cost of going from 20 percent to 80 percent is positive and relatively small, but marginal cost rises sharply, once again, when inflation exceeds 2,000 percent per annum. Taking equations (11) and (13) into account, total cost, v, satisfies:

$$v = z\left(g + b\,\frac{1+i}{1+\pi}\right) + \frac{\alpha}{2}\left(\frac{\pi}{1+\pi}\right)^2 + \frac{\beta}{2}\pi^2 \tag{14}$$

Consider now the cost-minimization problem faced by the government in period 1. Recalling our previous discussion, we take as given the nominal interest rate, i, and minimize equation (14) with respect to π. The first-order condition for this problem is:

$$\partial v/\partial\pi = 0 = -z'(x)b\,\frac{1+i}{(1+\pi)^2} + \alpha\,\frac{\pi}{(1+\pi)^3} + \beta\pi \tag{15}$$

Without loss of generality, we will constraint our attention to the case where $i > -1$; furthermore, we will assume throughout that $g \geq 0$ and $b > 0$. Thus, under these conditions, it follows, by equation (11), that $x > 0$, which, by equations (12) and (15), implies that $dv/d\pi < 0$ for $\pi \leq 0$. Hence, the minimum of v is attained at $\pi > 0$, positive inflation. Moreover, at a point where equation (15) is satisfied one can check (recalling that π is necessarily positive for $dv/d\pi = 0$) that the second-order condition for a minimum also holds (i.e., $d^2v/d\pi^2 > 0$). This can readily be used--noticing that $v \to \infty$ as $\pi \to \infty$--to argue that, given i, there exists a unique value of π that minimizes total cost, v. Thus, we have just proved the existence of an optimum response function like $\phi(\cdot)$ in the previous section.

As in previous section, we define equilibrium as a point where curve $\phi(\cdot)$ crosses the one corresponding to the Fisher equation under perfect foresight (equation (5)). Thus, the set of equilibrium solutions can be formally found by using equation (5) into (15), which, recalling (11), yields:

$$-z'(g+(1+r)b)b\,\frac{1+r}{1+\pi} + \alpha\,\frac{\pi}{(1+\pi)^3} + \beta\pi = 0 \tag{16}$$

or, equivalently:

$$\Omega(\pi) \equiv \alpha\,\frac{\pi}{(1+\pi)^2} + \beta\pi(1+\pi) = z'(\bar{x})b(1+r) \tag{17}$$

where,

$$\bar{x} = g + (1+r)b \tag{18}$$

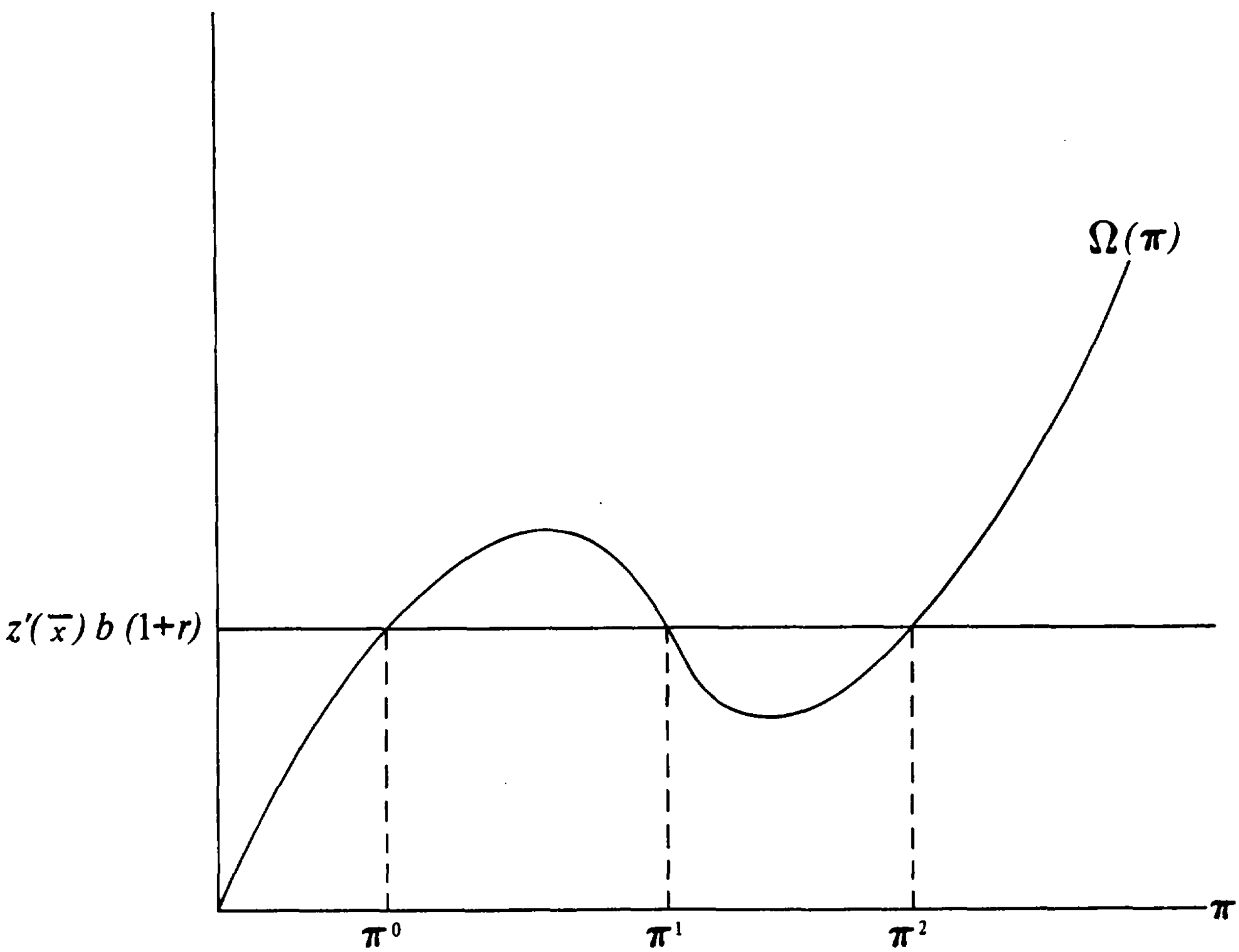
Notice that equilibrium taxes, $\bar{x}$, are independent of monetary factors. Condition (17) is depicted in Figure 2.

Figure 2 shows immediately that it is possible to have (at most) three equilibrium solutions, confirming the findings of the previous section. Furthermore, we can use Figure 1 to depict the $\phi(\cdot)$ function associated with equation (15) in the three-solutions case. Clearly, the low- and the high-inflation solutions are stable; stability, however, cannot rule out the existence of multiple solutions. Finally, as can be easily checked by looking at Figure 2, there are values of equilibrium taxes, $\bar{x}$, above (below) which the economy displays a unique equilibrium; the important point is, however, that as equilibrium taxes go up (down), we tend to lose the low-inflation (high-inflation) equilibria. A sudden rise (fall) in government expenditure, g, or the stock of debt, or the real interest rate, r, may result in moving catastrophically (in the technical sense of the word) from the low-inflation (high-inflation) to the high-inflation (low-inflation) equilibrium.

In order to understand the economics behind the existence of multiple equilibria, it is useful to go back to our discussion of equation (13). In the first place notice that when there are three equilibrium solutions, equation (16) implies a ϕ function like the one in Figure 1. We see in Figure 1 that (starting at the left) the first section of the ϕ - curve is relatively flat; thus, there is a sluggish response of π to a change in the nominal interest rate, i; this is so because the policymaker is conscious of the fact that the cost of inflation is a relatively steep function of inflation when inflation is relatively low. As we traverse the first equilibrium (i^0,π^0), the marginal cost of inflation rises but less steeply, so the optimal response of government becomes more sensitive to the rate of interest

Figure 2

Equilibrium Rates of Inflation



(i.e., ϕ becomes steeper); after the second equilibrium (i^1, π^1), however, the marginal cost becomes, once again, a progressively steeper function of the rate of inflation, which leads the government to be less responsive to changes in the nominal interest rate--thus, curve ϕ becomes flatter once again, and a new equilibrium evolves.

The main contribution of this section is to show that multiple equilibria arises even though government is always reacting in a globally optimal manner, and that there may be a sudden outburst of escalation or slowdown of inflation in response to relatively small changes in equilibrium taxes and government expenditure; the latter is shown to be the case even if the economy has a tendency to move to neighboring equilibria in response to shocks, and it is related to the catastrophic loss of low-inflation (high-inflation) equilibria as $\bar{x}$ rises (falls).

IV. Some Microfoundations

In this section we will intern ourselves a little more into the foundations of the earlier type of argument in order to provide a more solid and intuitive justification for multiple equilibria.

Most of our time will be spent on justifying the first term in equation (13) since it is essential to generate multiplicity of equilibria (recall equation (17)). We will assume that output is a function of real monetary balances, m, and satisfies the following functional form:

$$F(m) = y - \frac{\gamma}{2}(m - \bar{m})^2 \tag{19}$$

where y, γ, and $\bar{m}$ are positive constants; y is, obviously, the maximum attainable output, and $\bar{m}$ is the unconstrained, or Friedman's, optimum quantity of money. The assumption that liquidity shortages (i.e., $m < \bar{m}$) cause lower output, has an intuitive appeal which is probably not shared by the assumption that excess liquidity (i.e., $m > \bar{m}$) interferes with production. Fortunately, however, for the relevant case where the equilibrium nominal interest rate is positive, the economy operates in the liquidity-shortage region, so that the possible objection is of no concern to us. The quadratic form is adopted for simplicity.

We assume that in period 0 (competitive) firms choose the level of nominal monetary balances that they are going to use in period 1, M_1, taking into account the expected price level in period 1, P_1^e. Thus nominal expected profits in period 1 are given by:

$$P_1^e F(M_1/P_1^e) - (1 + i)M_1 \tag{20}$$

Thus, at optimum

$$F'(m^e) = 1 + i \tag{21}$$

where $m^e = M_1/P_1^e$. Hence, by equations (19) and (21), we get:

$$m^e = \bar{m} - \frac{1}{\gamma}(1 + i) \tag{22}$$

This defines the demand for money. 1/

The stock of real monetary balances in period 1,

$$m = M_1/P_1 \tag{23}$$

may differ from m^e to the extent that $P_1 \neq P_1^e$. Let us define:

$$1 + \pi^e = P_1^e/P_0 \tag{24}$$

where P_0 is the price level in period 0. Then, Fisher equation (1) and (24) imply:

$$P_1^e/P_0 = \frac{1 + i}{1 + r} \tag{25}$$

Thus, denoting, once again:

$$1 + \pi = P_1/P_0 \tag{26}$$

we have, recalling equations (23) and (24):

$$m = (M_1/P_1^e)(P_1^e/P_0)(P_0/P_1) = m^e \frac{1 + i}{1 + r}\frac{1}{1 + \pi} \tag{27}$$

Consequently, by equations (22) and (27), we get:

$$m - \bar{m} = \left[\bar{m} - \frac{1}{\gamma}(1+i)\right]\frac{1 + i}{(1 + r)(1 + \pi)} - \bar{m} \tag{28}$$

The term in square brackets equals $m^e > 0$ (by assumption). Thus, given i, an increase in actual inflation, π, unambiguously lowers real monetary balances, m; furthermore, since at equilibrium $(1+i) = (1+r)(1+\pi)$ and, by equation (22), $m < \bar{m}$, an increase in π depresses output. This represents the "cost side" of inflation in the present model.

1/ We will constrain our attention to regions in which $m^e > 0$.

Net output, which will be our measure of social welfare, W, is the difference between total output and the "deadweight" loss from taxation; thus, recalling equations (11) and (12), net output is given by:

$$W = F(m) - z(x) \tag{29}$$

In line with our previous analysis, we assume that in period 1 the government tries to maximize W taking the nominal interest i as given. This will allow us to derive the corresponding optimal response function, ϕ. The first-order condition for this problem is, recalling equation (28);

$$\partial W/\partial\pi = -\gamma\{\bar{m}-[\bar{m}-(1+i)/\gamma]\frac{1+i}{(1+r)(1+\pi)}\}[\bar{m}-(1+i)/\gamma]\frac{1+i}{(1+r)(1+\pi)^2}$$

$$+ z'(x)b\frac{1+i}{(1+\pi)^2} = 0 \tag{30}$$

The last term is unambiguously positive and, as in the model of Section 3, captures the tax-collection costs which are saved by the inflation-provoked reduction in total debt.

Notice that if $(1+i) > 0$ and the associated $m^e > 0$ (recall equation (22))--the relevant region--then $\partial^2 W/\partial\pi^2 < 0$ at points where equation (30) holds. This shows that if equation (30) holds for a given π and i, then π is the optimal response to i, it corresponds to the unique global maximum given i; thus, the associated $\pi = \phi(i)$.

For a full characterization of the equilibrium solutions all that we have to do now is to use the Fisher equation with perfect foresight (5) in (30) which, recalling (18), yields:

$$(1+i)[\bar{m} - (1+i)/\gamma] = z'(\bar{x})b(1+r) \tag{31}$$

Except in a borderline case, equation (31) has either two distinct solutions in $i \geq -1$, or no solution at all. The no-solution case is the relevant one when $\bar{x}$ is larger than a well-defined critical level. Recalling equation (17), we see that this configuration of solutions is exactly what we would get in the ad hoc model of previous section if $\beta = 0$, i.e., if the second term in equation (13) was eliminated.

In sum, the previous microeconomic story shows that the change in the curvature of the cost-of-unanticipated-inflation function ($f(\pi)$ in equation (13)) can be obtained in a context where the underlying production and utility functions have the standard curvatures.

Finally, by the analysis of Section 3, three or more roots can be obtained if the cost of inflation becomes unbounded as $\pi\to\infty$. This would be relatively easy to engineer in the present context, for instance, by assuming a nonlinear utility function (not a linear one like above) such that marginal utility becomes infinitely large as output (or net output) hits a critical low level.

V. Extensions: Several Periods, Private Credit Markets

1. Several periods

Thus far, our models assume that the government has to raise taxes to finance its fiscal expenditures at time 1. It is interesting to examine the more realistic situation in which it is possible to finance the deficit via bonds. In order to consider that case, however, it is necessary to allow for at least one more period.

We, assume that there are three periods: 0, 1 and 2. In period 0, b_0 is predetermined as in the previous models; however, at the end of period 1, the stock of bonds, b_1, can be non-zero, but it must satisfy the budget constraint:

$$b_1 = g_1 + b_0 (1+i_0)/(1+\pi_1) - x_1 \tag{32}$$

where:

$$1 + \pi_t = P_t/P_{t-1}, \; t = 1,2 \tag{33}$$

and the subindexes in the other familiar variables indicate the period to which they correspond, or at which they are determined. Period 2 is the last one, and thus taxes, x_2, must be raised to cover all the expenses, resulting in:

$$x_2 = g_2 + b_1(1+i_1)/(1+\pi_2) \tag{34}$$

We assume that, as before, government attempts to minimize social cost; however, in the present context we have to be more careful, because what is optimal to announce from the perspective of period 1, may not be optimal to implement in period 2 (i.e., "time inconsistency" may arise). We are going to be interested mostly in time-consistent paths in which government at time 1 takes into account its own optimal actions from the perspective of period 2.

The (present discounted) cost as seen from period 1, V_1, is:

$$V_1 = z(x_1) + f(\pi_1) + \frac{1}{1+\delta}[z(x_2) + f(\pi_2)] \tag{35}$$

where $\delta \geq 0$ is the planner's rate of discount, whereas the cost in period 2, V_2, is simply:

$$V_2 = z(x_2)+f(\pi_2) \tag{36}$$

The minimization of V_2 with respect to π_2 given i--which is the problem faced by the government in period 2--is identical to the one we examined in the previous sections. The minimization of V_1, however, offers some new vistas.

In order to get a better appreciation of the problem of minimizing V_1, let us use equations (32) and (34) in (35), so:

$$V_1 = z(g_1 - b_1 + \frac{1+i_0}{1+\pi_1} b_0) + f(\pi_1) + \frac{1}{1+\delta}[z(g_2 + \frac{1+i_1}{1+\pi_2} b_1) + f(\pi_2)] \quad (37)$$

The government in period 1 is assumed to minimize V_2 given i_0, b_0 and the exogenous path of g, by choosing x_1, b_1 and π_1 (the variables under its direct control) subject to equation (32), taking into account that in the next period it will attempt to minimize V_2 taking b_1 as given (which is a direct consequence of actions taken in period 1), and i_1 which is, in equilibrium, determined in the same manner as in the previous sections.

One can achieve an even better understanding of the maximization problem in period 1 by splitting it into partial ones at each point in time. Let us define $C(g_t - b_t; i_{t-1}, b_{t-1})$ as the minimum with respect to π_t of:

$$z(x_t) + f(\pi_t) \quad (38)$$

subject to:

$$x_t = g_t - b_t + \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} \quad (39)$$

taking as given $(g_t - b_t)$, b_{t-1}, and i_{t-1}. This is essentially the minimum cost problem studied in previous sections.

It is thus clear that the minimum of V_2 is $C(g_2; i_1, b_1)$. By our previous analysis there may be several values of i_1 which are consistent with perfect foresight. Let us define the following set:

$$I(g_1 - b_1) = \{i_1 : (1+i_1) = (1+r)(1+\pi), \text{ and } \pi \text{ solves}$$
$$(38), (39) \text{ at } t = 2, \text{ given } b_0 \text{ and } i_1\} \quad (40)$$

Clearly, this is the set of nominal interest rates between periods 1 and 2 for which the perfect-foresight Fisher equation (5) would hold. Therefore, the present discounted cost of choosing b_1 in period 1, given g_1, i_0 and b_0 is:

$$C(g_1 - b_1; i_0, b_0) + \frac{1}{1+\delta} C(g_2; i_1, b_1) \quad (41)$$

where:

$i_1 \varepsilon\ I(g_1 - b_1)$.

The first interesting problem that we encounter in trying to minimize equation (41) with respect to b_1 is that, due to the multiplicity of equilibria discussed in the previous sections, equation (41) is, in general, a correspondence, not just a regular function. Consequently, the last-period multiplicity of equilibria presents government with the serious dilemma of deciding which one of the various equilibria the public would expect. The public, in turn, needs to know government's policy about b_1 before being able to figure out the set of equilibrium interest rates--i.e., the set $I(\cdot)$. Therefore, we now see that by extending the horizon we have compounded the problem from being one of multiplicity of equilibria, to one where, in addition, there exists a fundamental uncertainty about the optimal response and behavior.

A formal solution to the above problem would be to assume, for example, that government is "optimistic" and expects the public to anticipate the lowest equilibrium inflation. This is equivalent to saying that in equation (41) i_1 is the minimum of the set $I(g_1 - b_1)$; under normal circumstances there will be a well-defined minimum of equation (41) with respect to b_1 and, quite possibly, such an attitude on the part of government will lead to higher values of b_1 (i.e., more debt in the second period) than if a higher $i_1 \varepsilon I(g_1 - b_1)$ were expected.

The point of the matter is, however, that government would not be optimizing if the public's expectations differed from the government's forecast, and it is not clear what is optimal for the government to do in the absence of some prior distribution over the set of equilibrium i_1's.

In sum, the addition of more periods to the previous story not only does not help in reducing the number of equilibria, but also reveals the fundamental uncertainty that a policymaker must face when the result of his present actions depends on people's expectations about the future, in a world with more than one equilibrium future path.

2. Private credit markets

The existence of nominal contracts may be problematic even in the absence of government's debt, or in a world where all the public debt is indexed to the price level. For example, Argentina during 1977-1982 appears to be an instance in which the government took measures to reduce the real value of private debt by provoking a significant increase in the rate of inflation (Fernandez (1983, and Baliño (1987)). Leaving aside the specifics of this episode, it appears that, for an extended period of time, the private sector expected a sudden devaluation of the currency. It turned out that this did not happen (the "peso" problem); as a consequence, during that period the ex post real rate of interest tended to be substantially high, a situation that induced a sizable and unplanned--and apparently undesirable--redistribution of wealth within the private sector. The simple solution to the problem was to provoke a maxi-devaluation.

Some aspects of the above-mentioned scenario are covered by the model in Section 2, because there is nothing there that really requires the existence of government debt; we referred to government debt in that section in order to motivate the existence of an upward-sloping optimal response function on the part of the government, $\phi(\cdot)$. However, if the government were concerned about wealth distribution between lenders and borrowers, it would also pay close attention to the following variable:

$$\frac{1 + i}{1 + \pi} \tag{42}$$

and, hence, optimal π would be, once again, related to i, and a similar multiple-equilibrium story could be told without assuming the existence of public debt. Thus, with this interpretation in mind, the model can readily be used to explain the inflationary explosion that followed the peso-problem period in Argentina, for example, as a consequence of prior high nominal interest rates. 1/

In the main sections of this paper we chose not to focus on the non-indexation of private credit transactions mainly because we do not seem to have a very good explanation for the lack of indexation of private contracts (see Fischer(1983)), and, consequently, we have very little idea of how sensitive nominal contracts are to policy changes. Of course, we also do not have a complete theory of why governments fail to index their debt, but there are at least nationalistic and other kind of atavistic considerations that could help to explain it.

VI. Summary and Conclusions

The central point of this paper is that a policy aimed at controlling the price level may encounter serious difficulties if credit contracts are not fully indexed to the price level. To make this point, we focused on the realistic case in which the government issues non-indexed debt, and showed that there is a strong theoretical argument supporting the view that the economy may exhibit more than one equilibrium solution.

1/ It should be noted, however, that the perfect-foresight assumption rules out the existence of equilibria with real rates of interest higher than r. Hence, the model is not capable of explaining the incredibly high real interest rates that prevailed during the peso-problem period, unless one is prepared to argue that the latter reflected an exogenous increase in r.

In our story, the government tries at all times to maximize social welfare; since taxes induce dead-weight losses, the existence of nominal debt is a constant temptation to shrink it by means of inflation. When the expected inflation is relatively low, the nominal interest will also be low and, thus, the attractiveness of inflation for reducing the size of public debt will be relatively small. This shows the possibility of a low-inflation equilibrium. On the other hand, if inflation was expected to be relatively high, the nominal interest rate will tend to reflect it point by point, and hence the temptation to liquefy the debt will be enhanced, which, in our examples, gives rise to the high-inflation equilibrium.

An obvious implication of the analysis is that, contrary to the view expressed in connection with the recent Austral and Cruzado Plans, de-indexation of the public debt may jeopardize, not help, the success of a stabilization program.

It would be too premature, however, to conclude that full indexation is optimal for a "real world" situation in which there are a variety of random shocks, and fully contingent contracts are prohibitively costly. In fact, history teaches us that inflation/devaluation processes have provided handy and, perhaps, relatively cheap ways of reducing the public debt, 1/ given that they did not require approval by Congress and the government could not be sued for breach of contract. 2/ On the other hand, if the public debt had been fully indexed to the price level, countries may have been forced to engage in open default, possibly a very costly solution, as testified by our present experience with international debt.

An interesting alternative to debt indexation is to attempt to put bounds on the nominal interest rate. The government, for example, may refuse to sell Treasury bills below a certain price. In the context of the models studied in this paper, for instance, the government could lock the economy into the low-inflation equilibrium by refusing to sell bonds at (implicit) interest rates higher than the smallest (equilibrium) one. In practice, however, this policy may run into two types of difficulties: (a) It is hard to have an accurate estimate of the relevant parameters (particularly, in the short run); and (b) the private sector may also be engaging in non-indexed credit transactions, giving rise to an independent source of multiplicity of equilibrium (see Section V.2). Problem (a) can be partially resolved by enlarging the band of interest rates acceptable to government, but still ruling out the ones that would obviously be unsustainable in equilibrium without an eventual inflationary surge. Problem (b) is more difficult, because it may be very hard for the government to regulate private credit. At

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1/ Keynes (1971) has an interesting discussion of these issues in the aftermath of WW1.

2/ This does not mean, of course, that there were no costs for the politicians in charge of the operation.

best, the monetary authorities could regulate the interest rate at financial institutions, but my feeling is that large borrowers and lenders have no major difficulty in bypassing that kind of regulation, and in the final analysis such a policy may result in only hurting the "small guy." An alternative, which should be more thoroughly analyzed, would be to use differential taxes favoring indexed over nominal credit contracts.

The existence of multiple equilibrium solutions which could be Pareto ranked (like in our examples) implies that the government could bring about a Pareto improvement if it just had the means to change expectations from one equilibrium solution to another. The policymaker who succeeds in such a task will assure himself a place in history and in the hearts of his people as the architect of price stability with no perceivable social cost. I feel, however, that to bring about such a wholesale change of expectations is not a trivial problem unless it involves some kind of pre-commitment on the part of the government.

It is interesting to note that the imposition of price controls--like in the recent "heterodox" stabilization programs of Argentina and Israel--would not necessarily solve the indeterminacy problem in the present context, because the controls themselves would not be credible. Unless there exists some way to control the controllers, the public would know that if today's rates of interest correspond to the high-inflation equilibrium, then price controls will be relaxed in the future to generate the then optimal rate of inflation.

In closing, I would like to emphasize, once again, that our examples still miss an interesting transition period in which the ex post real interest rate exceeds the ex-ante one--as it appears to be the case in the above-mentioned experiences of Bolivia, Chile and Argentina. This is, however, relatively straightforward to remedy by introducing uncertainty about some taste parameter or government expenditure, for example, and it looks like a promising line for future research.

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