IMF WORKING PAPER

WP/88/20

INTERNATIONAL MONETARY FUND

Research Department

The Strategy of Debt Buybacks: A Theoretical Analysis of the Competitive Case

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February 22, 1988

Abstract

This paper is concerned with analyzing the strategy of a debtor country repurchasing its debt at market prices. The main result, for the case when unpaid interest is rolled over, is that a strategy of announced debt repurchases at market prices, under competitive conditions and rational expectations, will only allow the country to recover debt at par value whenever the market expects the remaining debt to be fully served at some time in the future. When debt holders are myopic with respect to future debt repurchases, a strategy can be devised by which all of the "excess" debt is repurchased at a price equal, in present value, to that one prevailing before the policy was announced.

JEL Classification Numbers:

4310, 4430

* C.E.M.A.(Buenos Aires).This paper was written while the author was a consultant in the Research Department of the International Monetary Fund. I am much indebted to Saul Lizondo,Michael Dooley and members of a seminar at the I.M.F.Research Department for their comments on a previous version.

I. Introduction

The debts of several developing countries are traded well below par in financial markets. To a great extent, this is a reflection of the fact that service payments by debtor countries fall short of contractual interest. Unpaid interest on syndicated loans with commercial banks is rolled over, and the total contractual debt keeps rising over time on this account. This continuous increase in contractual debt, with no improvement in service payments, is bound to lower over time the price at which this debt is traded.

We define "excess" debt as the difference beteen total outstanding debt and that amount of debt that can be fully served. It has been argued that it may be to the advantage of debtor countries to obtain additional funding, other than that devoted to servicing interest, in order to repurchase their outstanding debt at prices well below par. Ideally, the purchases could go up to the point that enough debt is rescued so that actual service payments are sufficient to fully service the remaining debt. At this point the debt problem would have disappeared as the remaining debt would be fully serviced and it would therefore be traded at par. The country would have recovered access to the international financial markets.

In evaluating buybacks, several questions come to mind. First of all, is there a possibility for the debtor country to recover all of the "excess" debt at prices below par?. Second, to the extent that in the absence of buybacks the price of the outstanding debt converges asymptotically to zero, would it be possible to recover the totality of excess debt with just a minimal rate of cash devoted to buybacks per unit of time?.

The answers to the above questions depend fundamentally on the strategies assumed to be played by the participants in this game. In this paper we will be concerned with only one particular strategy that we denote the "competitive" strategy. Basically this strategy assumes that the debtor country has a fixed trade surplus (T) that it devotes fully to the service of the outstanding debt. The country also has an amount of additional cash (A, probably coming from a donation or from sales of domestic assets) per unit of time that is devoted to repurchase outstanding debt at the going market price (i.e. the country bids competitively for her outstanding debt). The creditors are fully competitive and have full information about the debtor country strategy and are also willing to sell debt at the going market price. Since the market price at which transactions are carried depends on actual and expected future developments, it will be assumed that those expectations are formed rationally.

Since we will not be considering uncertainty, the dynamic equilibrium to be derived will be represented by that perfect foresight path that converges to the steady state (a concept that will have to be defined later, particularly for the case of explosive paths where the price of debt falls continuously).

We will analyze two distinct cases depending what is done with the unpaid fraction of contractual interest. As it is presently being done, the unpaid interest is rolled over, and therefore contributes to the increase of the outstanding stock of debt. This point is particularly relevant if the cash buyback strategy aims at repurchasing the totality of outstanding excess debt. In this context it will be shown that such a cash buyback strategy, under rational expectations, requires every period an amount of cash investment in buybacks larger than the amount of unpaid interest. It also turns out that in this case all cash buybacks will have to be made at par. In summary, if the cash buyback strategy is expected to eventually recover all of the excess debt, the country must repurchase all of its excess debt at par whenever the unpaid interest is rolled over. This results, of course, are the only ones consistent with a rational expectations equilibrium. Other assumptions regarding expectations will yield different results.

If debt holders are myopic with respect to future debt repurchases and consider each repurchase as the last one, we show there is a strategy of continuous debt repurchases at a minimum feasible rate that amounts, in present value, to repurchasing the totality of the excess debt at the current market price prevailing the instant before the repurchases are announced.

The alternative to the rollover of the interest is that unpaid interest is forgiven, provided the stipulated amount T per period is devoted to service the debt. It is assumed that T is the maximum amount the country can commit to the service of interest.

It is obvious that if the debtor can only serve for ever, and with certainty, a fraction of the contractual interest, then the debtor must be insolvent. Under those circumstances, bankruptcy laws in most countries require a freeze on interest charges with consequences similar to those we assume about the forgiving of the unpaid fraction of contractual interest. In this case we will show that any amount in excess of T that can be devoted to cash buybacks at market prices, will eventually be successful in repurchasing the total amount of excess debt. Furthermore, the debt repurchases will be done at a price below par along the rational expectations path. A lower rate of cash purchases per unit of time will be associated with a lower purchase price for the debt in spite of which it will require a longer time to retire the entire amount of excess debt. However, it can be shown that the present value of all cash payments will also fall. The debtor country, therefore, faces a negative trade off between the present value of the cost of retiring its excess debt and the length of time required to do it. This case is interesting to analyse because the present value of all cash payments for debt repurchase could be taken as a measure of the amount that debtors would accept as a once and for all exchange of cash for excess debt.

II. Cash Buybacks with Rollover of Unpaid Interest

In order to simplify the analysis, we will make only those assumptions that are absolutely essential for the purpose of describing the dynamic consequences of debt buybacks. We will therefore assume that the debt of the country is structured as a perpetuity, paying a contractual interest equal to the competitive rate, that we denote by i. The total face value of outstanding debt is denoted by B. Per unit of time the debtor only has available for interest service the fixed sum T, that is less than contractual interest service due $(i.B)$. The amount T is distributed proportionally among all existing titles. Each title, therefore is paid an amount equal to T/B. In addition, the unpaid interest is documented as a new issue of debt in an amount of $(i - T/B)$ per existing title of \$1.

The model we will analyze is determined by two equations. The first one describes the trajectory of the equilibrium market price for the outstanding debt, while the second describes the trajectory of the outstanding debt as a function of unpaid interest and the rate of debt repurchases.

At any moment, the holder of one title of debt promising to pay an interest rate of i in perpetuity faces the following alternatives:

(1) Sell his title at a market price of p dollars per dollar of face value of the debt and reinvest the proceedings at the rate i. The return is $i.p$

(2) Keep the title and collect (T/B) in cash and an additional amount of debt titles equal to $(i-T/B)$ on account of unpaid interest;this last payment amounts to the roll over of the unpaid interest and has a market value of $p.(i-T/B)$. In addition, by keeping his debt title, the holder is entitled to the expected rate of capital gain Dpe. The total return of keeping the debt title is therefore : $(T/B) + p.(i-T/B) + Dpe$. Since we are assuming rational expectations with perfect foresight, the expected change in the price is equal to the actual change, $Dpe = Dp$. Throughout the paper we will use the notation $Dx= dx/dt$.

Market equilibrium requires that the time path of p be such that the holder be indifferent between keeping the debt title or selling it. This implies that the return be identical under both alternatives:

$$(1) \quad i.p = (T/B) + p.(i-T/B) + Dp .$$

Equation (1) is fully equivalent to the following expression showing the actual market price as the present discounted value of all future expected returns on the asset:

$$(2)\ p(t) = \int_t^{\infty} \frac{\{T + p(s).[i.B(s)-T]\}}{B(s)} .\exp[-i.(s-t)].ds$$

From equation (1) we obtain the following differential equation describing the equilibrium trajectory of the market price, p :

$$(3)\ Dp = (T/B).(p-1)\ .$$

The differential equation (3) describes the path of p given the initial value (still to be determined) and the trajectory of B.

The outstanding stock of debt increases over time on account of the interest unpaid and falls on account of current cash buybacks (A/p):

$$(4)\ DB = i.B - T - A/p\ .$$

Cash buybacks are supposed to continue, i.e. $A>0$, until all excess debt is repurchased. We define the sustainable level of debt as that level that can be fully served with the current trade surplus,

$$(5)\ B^{*}= T/i.$$

Using the concept of sustainable debt, we define excess debt as B - B*. If $B=B^{*}$, it is assumed that $A=0$ and by (4), B will remain unchanged at the level B*. Given the values of A, T, i, p(0) and B(0), equations (3) and (4) describe the time path of the stock of outstanding debt and its equilibrium market price.

The initial level p(0), under rational expectations, is determined by the condition that the system converge to its stationary state. Whenever *such stationary state exists, we will show that the initial value p(0) is* unique. There are cases, however, in which a stationary state for p and B is not feasible. For those cases, however, there exists a balanced growth path, along which the market value of the outstanding debt, $Z=p.B$, remains constant. We assume that rational traders will choose this balanced growth path as the alternative to the non-existent stationary state;also in this case we show that the initial value of p(0) is uniquely determined.

Steady State With A=0

Let us first analyze the case when there are no debt repurchases, i.e. when A=0 at all times.

The behaviour of p and B is described in this case by equations (3) and (6):

(3) $Dp = (T/B).(p-1)$

(6) $DB = i.B - T \quad , B > T/i=B^{*}$

The only steady state is for p=1 and B=B* . However, this steady state is not attainable since by assumption the initial stock of outstanding debt is B > B* and therefore, by (6) , B will grow without bounds. We will show, however, that there is an initial value of p(0) consistent with a constant market value of outstanding debt equal to the present value of cash payments being made by the debtor country, i.e. $Z^{*}=T/i=p(t).B(t)$ for all t. To show this , we use (3) and (6) to obtain the differential equation describing the behaviour of Z(t):

(7) $$DZ/Z = Dp/p + DB/B =$$

$$= (T/Z).(p-1) + i- p.T/Z =$$

$$DZ/Z = i - T/Z .$$

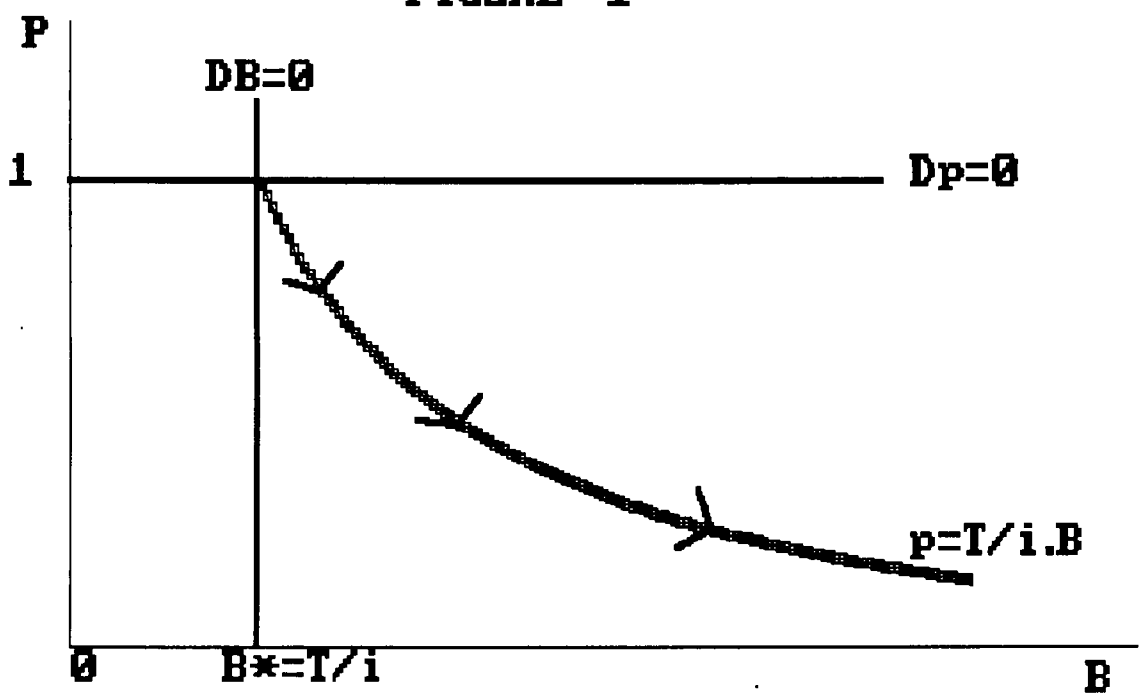
Equation (7) has a stationary state at $Z^{*} = T/i$ and it is dynamically unstable. Under rational expectations this means that only the value $Z(t) = Z^{*}$ for all t is consistent with market equilibrium. This means that at all times it will be $p(t)=Z^{*}/B(t)$. In particular, at t=0, the initial value of p(0) is determined by $p(0) = Z^{*}/B(0) = T/i.B(0)$.

Over time B grows at the rate $DB/B = i-T/B$ and p grows at the rate $Dp/p= (T/B).(p-1)$. Since at all times it is $p.B = T/i$, it is easy to see that the sum of both growth rates is identically equal to zero at all times. The stationary state solution $Z=Z^{*}$ is therefore feasible and consistent with the equations describing the behaviour of p and B.

We conclude that in the absence of debt repurchases , the equilibrium market price of outstanding debt is determined by the condition that the market value of the outstanding debt be equal to present value of all future cash payments by the debtor country. If those payments are not enough to pay the full contractual interest, the outstanding stock of debt will be growing over time and p will be falling at exactly the negative of the growth rate in B.

Figure 1 describes the equilibrium when A=0. The locus DB=0 is represented by the vertical line at B=B*=T/i and the locus Dp=0 is the horizontal line at p=1. The arrows indicate the direction of the motion of p and B that follows from the differential equations (3) and (6). The rectangular hyperbola p=T/i.B represents the unique balanced growth path consistent with the rational expectations trajectory for p and B.

FIGURE 1



Equilibrium Solution for A>0

Assume now the debtor country is granted a gift of A>0 per unit of time to be applied exclusively for debt repurchases at market prices. The gift is supposed to remain in effect as long as B > B* . The dynamic behaviour of p and B is now described by equations (3) and (4):

(3) $Dp = (T/B).(p-1)$

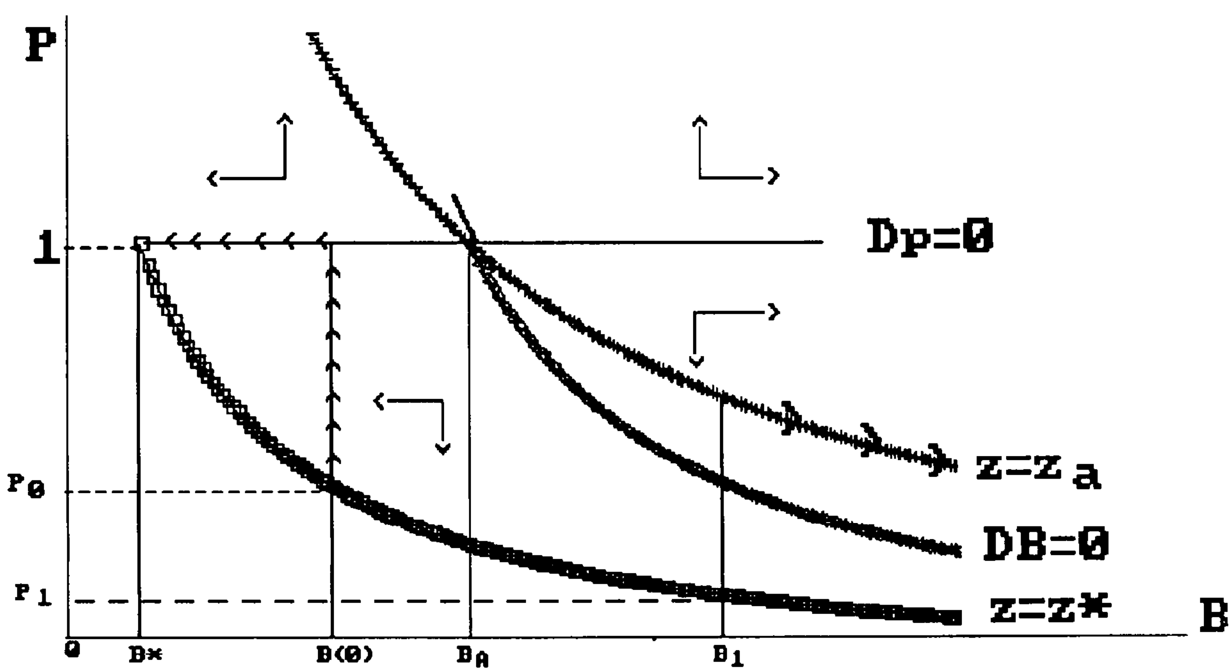
(4) $DB = i.B - T - A/p$

To the extent that A>0 there is a stationary state at the values p=1 and Ba= (T+A)/i > B* . Figure 2 shows the face diagram for the case A>0. It is easy to see that the stationary state at (1,Ba) is unstable, meaning that there is no initial pair of p(0) and B(0) that will converge to it other than the pair (1,Ba). Such stationary state is consistent with rational expectations only if by chance B(0)= Ba.

Notice that B(0)= Ba implies i.B(0)-T=A and therefore a stationary state at (1,Ba) will only be feasible if the amount of cash devoted to buybacks is exactly identical to the initial amount of unpaid interest.

If B* < B(0) < Ba , there is a unique rational expectations solution for which p(t) = 1 for all t. For B(0) as drawn in Fig.2, any p(0) different from unity will lead to p rising or falling without bound or, alternatively, to the stock of debt reaching B* at a time when p differs from unity. These trajectories are ruled out under rational expectations because as B reaches B* it is required that p=1. If at that moment p differs from one, it would have to jump up to that level and this is ruled out under the assumption of rational expectations since it would imply an anticipated infinite rate of return for that instant.

FIGURE 2



The equilibrium trajectory, therefore, requires p=1 and B gradually falling along the Dp=0 line. As B reaches B*, A becomes zero and the DB=0 schedule reverses to the vertical line at B=B*. The steady state for the new system is at (1 , B*) which is exactly where both p and B happen to be at that time. As required for a rational expectations path, there are no jumps in the price of debt at the time that the cash buybacks end.

Assume now that A is less than the amount of unpaid interest at time

0. In that case initial debt must be greater than Ba. Such a level is indicated as B1 in Figure 2. In the absence of buybacks, the market price of B1 would have been $p1= T/i.B1$, along the $Z=Z^*$ rectangular hyperbola. It can be seen that there is no initial value of p(0) such that it will converge to unity in the exact time that B1 converges to B*. This means that for the actual levels of A and B1, the debt repurchase strategy is not feasible, in the sense that debt cannot be reduced to the self-sustainable level B* along a rational expectations path. If, however, A is to remain at its present level for ever(of course one could wonder why should it?), the rational expectations equilibrium would imply that p and B move along the rectangular hyperbola described by the locus $Z = Za = (T+A)/i$. This means that p will jump from the level $Z^*/D1$ to the higher level $Za/D1$. The absolute increase in p as a consequence of the announcement of the $A>0$ policy is then $A/i.D1$; that is, p rises by the capitalized amount of the new money available for buybacks. After the initial rise, p starts falling as B rises , both moving along the $Z = Za$ locus. Eventually p will again reach the level it had before the announcement of the buyback policy.

We conclude that the only buyback policy able to bring debt to the sustainable level, under rational expectations, is the one where the amount of money devoted every period for debt repurchase is larger than the amount of interest being rolled over. It is also the case that under this policy all debt repurchases will have to be made at par. If the unpaid interest is rolled over, there is no debt forgiveness whatsoever in the fact that countries are allowed to repurchase their excess debt at market prices. If debt is finally reduced to the sustainable level, the excess debt would have been bought at par. If the amount of cash buybacks is not sufficient to bring debt back to the sustainable level, the market price of debt will rise in response to the capitalized value of the cash devoted to the buyback, but it will eventually fall below the pre-buyback level as debt keeps accumulating for ever.

Our conclusion that cash buybacks of outstanding debt may not represent any long run advantage to the debtor country has to be taken within the model in which it was derived. Not only we have assumed perfect foresight and rational expectations but we also assumed that the buyback policy is openly announced. This implies that title owners can discount the fact that all of their titles, actual and forthcoming, will be quoted at par eventually, if the policy is expected to be successful in bringing debt back to the sustainable level. This leads to the only feasible alternative under rational expectations implying that all debt titles start being quoted at par from the very moment the buyback policy is announced.

The Case of Myopic Expectations With Respect to Buybacks

An alternative view is that all buybacks are considered to be the last one by titleholders. In the limit, if the market is totally myopic with respect to the buyback policy, they will not consider that $A>0$ will remain in the future and in consequence will sell their assets along the $Z=Z^*$ curve in Figure 2. The market price of outstanding debt will only rise gradually as the stock of debt is reduced by the continued purchases. In this limiting case, the total amount of cash payments necessary to recover the excess debt will be the area under the $Z=Z^*$ curve between B^* and $B(0)$ plus the value of the accumulated amount of rolled over interest during the period.The length of the period required to repurchase the debt in turn, depends on the rate of cash buybacks and the initial stock of excess debt.

The length of the period required to repurchase the excess debt under the myopic assumption can be calculated in the following manner.

First solve for $B(t)$ from (4) after substituting $p(t)=T/i.B(t)$. Defining $x = (A/T)-1$, the solution for $B(t)$ is:

$$(8) \quad B(t) = -(B^*/x) + \{B(0) + B^*/x\}.\exp(-ixt)$$

Equating $B(t)$ in (8) to B^* we can compute the length of time required to complete the purchase of the excess debt, t^* :

$$(9) \quad t^* = (1/ix).\mathrm{Ln}.\{(1+x)/(1+x.B(0)/B^*)\}$$

There will be a positive solution for t^* only if $1 + x.B(0)/B^* >0$, a condition that implies that debt repurchases are larger than unpaid interest.

The present value of cash expenditures is therefore:

$$(10) \quad PV = (A/i).\{1 - \exp(-it^*)\} = (1+x).B^*.\{1-\exp(-it^*)\}$$

Consider now the limiting case where $x= -B^*/B(0)$ for which t^* tends to infinity. This is the case where the debt that can be bought by A , at the going market price of $p= T/i.B(0)$, is exactly equal to the amount of interest being rolled over. With fully rational expectations, p will jump to one and the debt will remain for ever at $B(0)$. With myopic expectations regarding the $A>0$ policy, as A tends to this level, the stock of debt also tends to B^* as t^* tends to infinity and the present value of cash payments tends to :

$$(11) \quad PV \longrightarrow PV^*= (1-B^*/B(0)).B^* = \{B(0)-B^*\}.\{B^*/B(0)\}$$

The present value of cash payments per unit of debt bought, PV*(p), is therefore the ratio of PV* to (B(0)-B*):

PV*(p) = B*/B(0) ,   which is identical to the initial price

prevailing just before the debt repurchase plan was to be announced.

The above means that with myopic expectations regarding future cash buybacks, a policy of the minimum feasible cash buyback will in fact repurchase all the excess debt(in infinite time) at an actual net present value identical to the market value of the existing excess debt prevailing the instant before the buyback policy is announced. It would be fully equivalent (in present value terms) to an agreement to purchase all the excess debt at the current market price (equal to B*/B(0)).

The former transaction, however, could never be duplicated in practice with a single repurchase offer for the total stock of excess debt at the current price since the price will immediately jump to unity. Such an offer would imply for the debtor to lose the informational advantage it had thanks to the myopic expectations assumed on the part of the holders of debt.

III. Cash Buybacks Without Rollover of Unpaid Interest

In what follows we will assume that unpaid interest is simply forgiven instead of being rolled over. This would be a natural outcome under bankrupcy laws of most countries if the debtor is actually broke. It may also come about as a consequence of an agreement with the country's creditors whereby they accept a settlement freezing the level of the debt in exchange for a fixed payment per period. To make the analysis relevant we assume that the fixed payment per period (T) is not enough to serve the existing stock of debt (Bo) at competitive interest (i).

It is assumed that the country, in addition to the fixed payment of T dollars per unit of time can also devote A dollars per unit of time to debt repurchase at market prices. As before we will assume that A>0 whenever i.B> T (and A=0 otherwise).

The following equations describe the dynamic behaviour of the market price and the outstanding stock of debt:

(12) Dp = i.p - (T/B) (derived from (1) setting the second term in the RHS equal to 0, since there in no rollover of interest)

(13) DB = -(A/p)

If cash buybacks are zero, eqn.(13) is substituted for DB=0. With

A=0, the stock of debt remains at B(t) = B(0) and the rational expectations solution for p is p(t) = T/i.B(0) at all times.

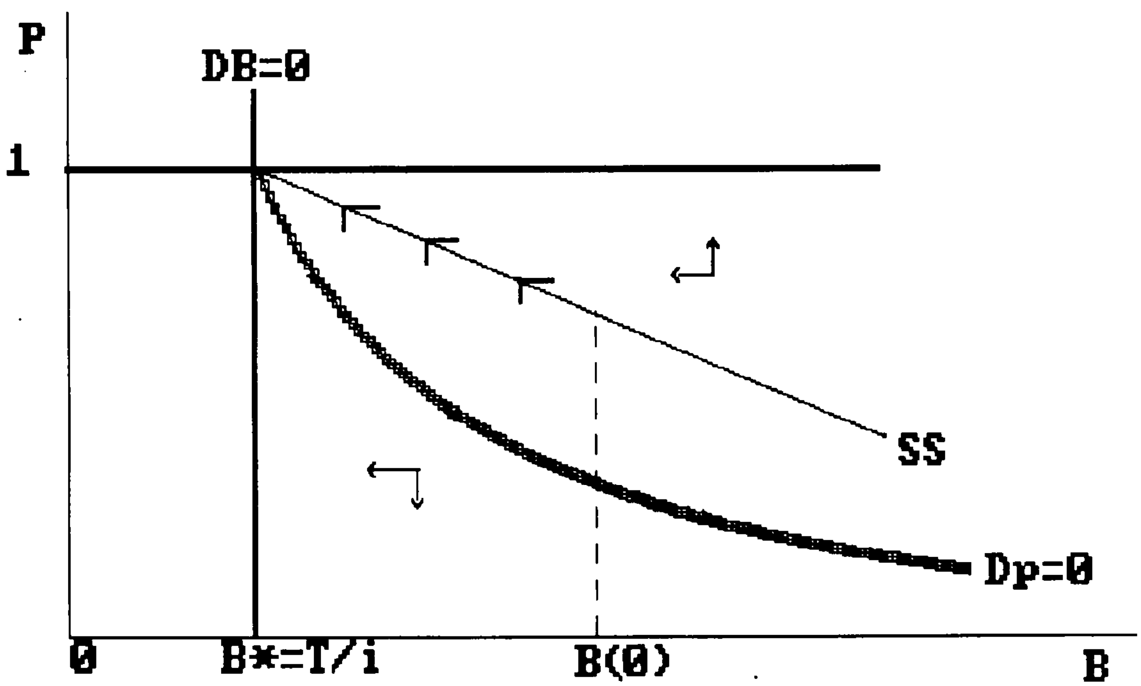
Figure 3 shows the dynamic configuration described by equations (12) and (13) for A>0 . There is only one steady state at B=B*=T/i and p=1 .The system has saddle path stability and the rational expectation solution lies along the saddle path, denoted by the dotted curve SS in Figure 3.

Before the announcement of the buyback policy, the initial equilibrium was at the intersection of the Dp=0 line and the initial stock of debt B(0). Also, DB was equal to zero at all times.

With the announcement of the A>0 policy, DB becomes negative and the unique solution under rational expectations requires a jump in p to the corresponding level along the saddle path. After the initial jump in p, both p and B move along the saddle path towards the long run equilibrium at (1,B*).

The lower the rate of debt repurchase, A, the closer will the saddle path be to the Dp=0 line and therefore the smaller will be the initial jump in p at the moment the policy is announced. Nevertheless, for any A>0, both p and B eventually converge to the long run equilibrium (1,B*).

FIGURE 3



In the limit, as A grows smaller, the saddle path approaches the DP=0 line and all purchases are made at the price indicated by this line (or infinitely close to it). The undiscounted sum of cash disbursements will approach the area under the Dp=0 curve; this area can be calculated as follows:

Denote the area under the Dp=0 curve between B* and B(0) by Cmin.. We can see by integration of p=T/iB= B*/B that :

$$Cmin. = B^*.Ln.(B(0)/B^*)$$

Consider a situation in which B(0) = 2B* so that initially p was equal to .50.In this case the Cmin. equals 0.693*(B(0)-B*). This means that as the rate of cash buybacks tends to zero,the excess debt can be retired for a total cash expenditure that tends to 69.3% of its face value. The actual present value of disbursements, however, will also depend on how long does it actually take to do the operation. As A grows smaller, not only does the undiscounted sum of required cash disbursements fall but also the required time over which that sum is uniformly spent rises. On both counts, the net present value of the cash disbursements tends to fall.

It can be formally shown that as A tends to zero, the net present value of the disbursement also tends to zero. This means that in the limit, all of the excess debt could be retired for a zero investment in terms of its net present value. Intuitively, this result can be ilustrated as follows:

The Present Value of a constant rate of purchases A during a period of time t, is:

$$PV = (A/i).\{1-exp(-it)\}$$

Multiplying and dividing the RHS by t, and defining C=A.t as the undiscounted sum of disbursements, PV becomes:

$$PV = (C/i).\{1-exp(-it)\}/t \ .$$

We know that as A becomes smaller, C falls towards Cmin. and t tends to infinity. It is easy to see, therefore, that as A goes to zero, also does PV. 1/

The country, therefore, faces a trade off between the present value of the cost of retiring its excess debt and the time required to do it.

1/ From the integration of (12) and (13) and the condition that at the time t^* when $B(t^*) = B^*$ it must be $p(t^*)=1$, we can derive the saddle path, as well at the value of t^* and of PV. In particular, the present value of any rate of purchases A over the amount of time required to retire all the excess debt is given by:

$$PV = B^*.q.\{1 - (1+q).[q + (B(0)/B^*)^{((1+q)/q)}]^{(-1)}\},$$

where $q = (A/T)$. It can be seen that as q (and A) tends to zero, PV also tends to zero. Similarly, as q tends to infinity, implying that all the excess debt is rescued instantaneously, PV tends to the value $B(0)-B^*$, meaning that all the debt is rescued at par value.

V. Conclusions

The main result of this paper, in the case when unpaid interest is rolled over, is that a strategy of announced debt repurchases at market prices, under competitive conditions and rational expectations, is not bound to help debtor countries to recover all of their excess debt at prices below par value. Only when debt holders are myopic with respect to future debt repurchases, can a strategy be devised by which all of the excess debt is repurchased at a price equal, in present value, to that one prevailing before the policy was announced.

If unpaid interest is forgiven, a case that may happen if an agreement is reached by which debtor countries pay only a stipulated fixed amount per unit of time (e.g. a fraction of the trade surplus or exports), any constant rate of debt repurchases will succeed in bringing debt down to that sustainable level where the fixed payment equals the service at competitve interest. However, the present value of those purchases falls as the rate of purchase decreases. There is therefore a trade off between the present value of the cost of repurchasing the excess debt and the time required for the debt to be quoted at par.

We have only analyzed the "competitive" case, when creditors are not faced with non competitive offers of the "take it or leave it" type. All bids for debt repurchase are assumed to be competitively made and openly announced.

Many strategies of debt repurchase other than those presented here can surely be found. This paper offers what can be considered to be the competitive frame of reference, within which the other alternatives can be compared and evaluated.