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Optimal Intertemporal Taxation on Consumption and the Term Structure of Government Debt

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Abstract

This paper addresses the time-consistency problem of optimal policy when intertemporal prices are inflexible. For small, open economies facing given world interest rates, it shows that a consumption tax, rather than a tax on wage income, is time-consistent under a variety of circumstances, even including some cases where the optimal tax rate is not constant over time. This result, when it applies, restores the neutrality of the term structure of government debt, and reaffirms the tax-smoothing theory of debt determination.

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I. Introduction

In the modern literature on the determination of government debt, two distinct strands of investigation can be discerned. The first relates to the size of debt, the second to its term structure. The present paper develops a basic neutrality result concerning the term structure of debt under optimal intertemporal taxation on consumption in an economy with fixed intertemporal prices. As it will become clear, this result has important policy implications for small, open economies in the choice of an optimal, time-consistent form of taxation.

In a world in which Ricardian equivalence does not hold, Zee (1988a) derived a result that relates the optimal size of debt to the optimal level of capital intensity, which may or may not be the level indicated by the golden rule of savings. Suppose, however, for simplicity, one were to abstract from the complexities of the (possible) impact of debt on capital accumulation and growth, then is it still possible to have a normative theory of debt determination? Barro (1979), building on Hall's (1978) interpretation of the permanent-income hypothesis as a way to smooth consumption over time, developed an elegant framework in which the optimal size of debt is derived from a model of tax-smoothing. Barro's analysis leaves open, however, the question of the optimal term structure of debt.

Until recently, a common presumption in economic analyses was that the term structure of debt did not matter, as long as no market imperfections, such as capital rationing, existed. Lucas and Stokey (1983), writing on the time consistency of optimal government policy, are the first to demonstrate that this presumption might not be correct. The time consistency problem, analyzed in a general way by Kydland and Prescott (1977), has to do with the fact that optimal policies, as viewed by governments in different dates, may not coincide with each other, even when there is full information and the governments share a common, unchanging objective function. The reason for the occurrence of time-inconsistent policies in certain classes of models, particularly those dealing with monetary economies and those involving capital accumulation, are by now well-understood. Calvo (1978), for example, has provided a lucid analysis of the incentive a government has in any given period to spring a surprise inflation, the very nature of which cannot be taken into account by the optimization calculus of governments in prior periods; and Fischer (1980), among others, has shown that optimal capital income taxation is time inconsistent because of the durable and supply-inelastic characteristics of the capital already in place.

The most provocative aspects of the Lucas-Stokey contribution involve, however, the finding that the time consistency problem can occur even in a barter economy without capital, and the discovery that the problem can be solved, that is, an optimal policy can be made time consistent, by an appropriately determined term structure of government debt. 1/ These results are pathbreaking, and the solution to the time consistency problem provides debt management with a vitally important policy role hitherto not recognized in the literature.

A crucial mechanism by which the term structure of debt matters in the Lucas-Stokey framework is that intertemporal prices, that is, interest rates, are responsive to changes in the structure of debt maturity. Hence, by restructuring its debt appropriately, a government in any given period can induce the desired changes in intertemporal prices so as to make its optimal policy also appear optimal to its successor. This, however, immediately raises the question as to whether debt maturity is a relevant instrument for ensuring the time consistency of optimal policy in a model in which intertemporal prices are inflexible. In an important paper, Persson and Svensson (1986) provided, unfortunately, a negative answer. They showed that, for a small, open economy that faces given world interest rates, optimal policy cannot be made time consistent by debt restructuring.

Is there no way out, then, of the time consistency quandary for economies that are price takers? The basic contribution of the present paper is to show that these economies can avoid being trapped in the quandary by adopting a tax on consumption rather than on wage income (the common form of taxation assumed in the time consistency literature). The debate on the relative merits of income versus consumption taxation is, of course, itself the subject of a voluminous literature, but the capability of generating time-consistent optimal policy under inflexible intertemporal prices is a property of consumption taxation that the previous literature has failed to recognize. An important implication of this result is that the neutrality of the term structure of debt is restored: it is the size, and not the maturity, of debt that matters under optimal consumption taxation.

The remaining sections of the paper utilize a very simple intertemporal framework to analyze and discuss the time consistency issue of optimal policy and the term structure of debt under consumption taxation. A general proof of the basic proposition of the paper, that is, optimal consumption taxation is time consistent, is contained in the Appendix.

1/ Persson, Persson, and Svensson (1987) have recently suggested that the time consistency problem in a monetary economy can likewise be solved by an appropriately determined term-structure of nominal government debt. However, the validity of this proposed solution has been called into question by Calvo and Obstfeld (1988).

II. Time-Consistent Taxation on Consumption

In order to bring the essentials of the time consistency problem into focus, the simplest possible intertemporal framework that permits the derivation of closed-form solutions of the term structure of government debt will be adopted. 1/ Consider a three-period ($t = 0,1,2$) model of an economy endowed with a given and equal level of income (x) in each period and facing an exogenous path of interest rate (r_t). Income in any period t is allocated between net-of-tax private consumption (c_t) and government expenditure (g), which is also assumed to be given and constant over time. Economy-wide equilibrium in each period must therefore satisfy

$$x = c_t + g, \qquad t = 0,1,2. \tag{1}$$

The government finances its budget each period by an ad valorem tax on private consumption at the rate τ_t and by issuing debt, which can be in the form of either one-period or two-period bonds carrying the same rate of return as the given rate of interest in that period. As a notational convention, one-period and two-period bonds issued in period t will be denoted, respectively, by ${}_tb_{t+1}$ and ${}_tb_{t+2}$. The government in period 0 has a budget constraint in each of the three periods given by, respectively,

$$\tau_0 c_0 + {}_0b_1 + {}_0b_2 = g, \tag{2.1}$$

$$\tau_1 c_1 = g + r_1({}_0b_1 + {}_0b_2) + {}_0b_1, \text{ and} \tag{2.2}$$

$$\tau_2 c_2 = g + (1 + r_2)\,{}_0b_2. \tag{2.3}$$

Let ρ_t be the present value in period 0 of quantities in period t, that is, $\rho_0 \equiv 1$ and $\rho_t \equiv \Pi_{i=1}^{t}(1+r_i)^{-1}$, $t = 1,2$. Then equations (2.1)-(2.3) can be consolidated into an intertemporal budget constraint of the familiar form:

$$gR = \Sigma_{t=0}^{2}\rho_t \tau_t c_t, \tag{3}$$

where $R \equiv \Sigma_{t=0}^{2}\rho_t$. Implicit in the above formulation is that the government in period 0 does not inherit any outstanding debt from its predecessor. 2/ As bonds do not appear in equation (3), the period-wise

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1/ The Appendix contains a general proof of the results of this section.

2/ The interested reader may wish to verify that this assumption is inconsequential to the analysis that follows.

budget constraints of equations (2.1)-(2.3) must be used eventually to solve for the optimal term structure of government debt.

The problem facing the representative consumer is to allocate his consumption intertemporally, for given tax rates τ_t over the three periods, so as to maximize his utility

$$U = \Sigma_{t=0}^{2}\beta^{t}\ln c_t, \qquad 1 \geq \beta \equiv (1+\eta)^{-1} > 0, \quad (4)$$

subject to the intertemporal budget constraint

$$xR = \Sigma_{t=0}^{2} \rho_t(1+\tau_t)c_t. \quad (5)$$

The constant η represents the rate of his pure time preference per period. Equation (5) states that the present value of the consumer's endowed income stream must be equal to the present value of his lifetime consumption gross of tax.

The first-order conditions of the above maximization problem are

$$\frac{\partial U}{\partial c_t} \Big/ \frac{\partial U}{\partial c_{t+1}} = \beta(1+r_{t+1})(1+\tau_t)/(1+\tau_{t+1}), \qquad t = 0,1, \quad (6)$$

which, together with equation (5), can be used to solve for the consumption functions

$$c_t = \beta^{t}xR/[\rho_t(1+\tau_t)B], \qquad t = 0,1,2, \quad (7)$$

where $B \equiv \Sigma_{t=0}^{2}\beta^{t}$. Substituting equation (7) into equation (4) yields the indirect utility function

$$V_0 = K_0 - \Sigma_{t=0}^{2}\beta^{t}\ln(1+\tau_t), \quad (8)$$

where K_0 is some constant. The optimal taxation problem for the government in period 0 is, therefore, to maximize equation (8) subject to equation (3), whose first-order conditions turn out to have the simple form of

$$(1 + \tau_t) = -\lambda_0 xR/B, \qquad t = 0,1,2, \tag{9}$$

where λ_0 is the relevant Lagrangean multiplier. Since the right-hand side of equation (9) is a constant, it implies that

$$\tau_0^* = \tau_1^* = \tau_2^* = \tau^*, \tag{10}$$

that is, the optimal tax rate should be constant over time. Substituting equation (10) into equations (3) and (5) yields the explicit solution for the optimal tax rate as

$$\tau^* = g/(x - g), \tag{11}$$

which is positive on the reasonable assumption that $x > g$.

The above solution to the optimal taxation problem implies an optimal term structure of government debt. If the tax rate is set according to equation (11), then, from equation (2.1), the total debt (both one-period and two-period bonds) issued by the government in period 0 is

$$\Sigma_{t=1}^{2}\ {}_0b_t = g(1 - R/B), \tag{12}$$

which indicates that whether the government budget should be in surplus or deficit in period 0 would depend on the magnitude of the sum of discounting factors for present value purposes relative to that of the sum of discounting factors for time preference purposes. To get an explicit breakdown between one-period and two-period bonds that the government should issue, first substitute equation (12) into equation (2.2) to obtain

$${}_0b_1 = g[(\beta/\rho_1 + r_1)R/B - 1/\rho_1]. \tag{13}$$

Subtracting equation (13) from equation (12) then yields 1/

$${}_0b_2 = g\{1 - [(1 + \beta)R/B - 1]/\rho_1\}. \tag{14}$$

1/ It is, of course, also possible to solve for ${}_0b_2$ directly from equation (2.3).

As an illustration of the implication of equations (12)-(14) for the term structure of government debt, consider the special case in which R = B. Then it is clear from equation (12) that the government budget should be balanced in period 0, although this does not necessarily suggest that no debt should be issued in that period. Equation (13) reveals that ${}_0b_1 \gtrless 0$ as $\beta \gtrless \rho_1$, that is, a positive (negative) amount of one-period bonds should be issued in period 0 if the interest rate in period 1 is greater (less) than the rate of time preference. Because of the constraint imposed by equation (12), the amount of two-period bonds to be issued under this circumstance must exactly offset the amount of one-period bonds, a conclusion which is confirmed by equation (14).

In one respect, the theory of debt determination outlined above is conceptually analogous to that of Barro's (1979). Although Barro did not address the issue of the term structure of debt (his model has only one-period bonds), his central proposition was that debt be used as an instrument to counter fluctuations in income relative to some trend value. 1/ In the present paper, debt determination has to do with the desire to counter fluctuations in the rate of interest relative to that of time preference. Hence, in both instances, the fundamental reason for debt issuance is for tax-smoothing purposes.

An important aspect of the optimal tax solution--given in equation (11)--to be investigated is whether it is time consistent; that is, whether the government in period 1, in solving its own maximization problem, will still find the solution stated in equation (11) to be optimal (when nothing else in the model is changed). On the assumption that the government in period 0 did, in fact, leave to its successor a debt structure according to equations (13) and (14), the budget constraints for periods 1 and 2 facing the government in period 1 are, respectively,

$$\tau_1 c_1 + {}_1b_2 = g + r_1({}_0b_1 + {}_0b_2) + {}_0b_1, \quad \text{and} \tag{15}$$

$$\tau_2 c_2 = g + (1 + r_2)\,{}_0b_2 + (1 + r_2)\,{}_1b_2. \tag{16}$$

Notice that the government in period 1 is allowed to issue, if it wishes to, only one-period bonds. This is simply an artifact of a three-period

1/ This stems directly from the fact that, in his model, the excess burden of taxation is specified to be a homogeneous function of income.

model, which is immaterial to the problem at hand. Consolidating equations (15) and (16) yields the intertemporal budget constraint

$$\Sigma_{t=1}^{2}\ {}_0b_t + g(R - 1) = \Sigma_{t=1}^{2}\rho_t\tau_t c_t. \tag{17}$$

The optimal taxation problem facing the government in period 1 is similar to that which faced the government in period 0; that is, to maximize, subject to equation (17), the indirect utility function

$$V_1 = K_1 - \Sigma_{t=1}^{2}\beta^t \ln(1 + \tau_t), \tag{18}$$

where K_1 is some constant. As it turns out, the first-order conditions of this maximization problem have similar forms to those of the previous government:

$$(1 + \tau_t) = -\lambda_1 xR/B, \qquad t = 1,2, \tag{19}$$

where λ_1 is, again, the relevant Lagrangean multiplier. Equation (19) makes it clear that the government in period 1 would still find it optimal to have a constant tax rate over its planning horizon. But since λ_1 in equation (19) may not equal λ_0 in equation (9), it remains to be seen whether the optimal tax rates of the two governments are the same. Substituting equation (19) into equation (17) reveals that they do:

$$\tau_1^{**} = \tau_2^{**} = \tau^{**} = g/(x - g). \tag{20}$$

In this sense, the optimal tax rate as determined by the government in period 0 is time consistent. 1/ A different way of looking at this result is to ask what amount of debt the government in period 1 would find it optimal to issue. Substituting equations (12), (13), and (20) into equation (15) yields

$${}_1b_2 = 0. \tag{21}$$

1/ It is straightforward to verify that the government in period 2 will again arrive at the same optimal tax rate.

Equation (21) states a basic implication of time-consistent taxation: if the tax rate set by the government in period t is time consistent, then governments in successive periods will not find it optimal to restructure the maturity of their inherited debt.

A still more telling implication of the above result can be obtained by investigating the consequence of a hypothetical situation in which the government is not able, for whatever reason, to issue debt with a sufficiently rich maturity structure. Suppose, for example, that the government in period 0 has only one-period bonds at its disposal so that its total debt issue, given by equation (12), consists entirely of bonds that will mature in period 1. In this case, it turns out that the optimal tax solution remains unchanged, but the government in period 1 will now find it optimal to issue ${}_1b_2 = {}_0b_2$; that is, it will duplicate the term structure that would have been issued by its predecessor had the government in period 0 been able to do so.

III. Extensions and Discussion

1. Collection costs

A striking result of the consumption tax discussed in the previous section is that, though the tax is distortionary, at the optimum, all distortions are eliminated because tax rates are equalized across periods. It might therefore be argued that it is for this reason that the time consistency problem does not arise. This conjecture, however, turns out to be not quite true. It can be shown that the consumption tax would still be time consistent, even if the optimal tax rates were to be different across periods.

To produce unequal optimal tax rates in a simple fashion, consider the incorporation of collection costs into the model. These costs are associated with the resources needed to administer the tax system, including all forms of deadweight losses. Hence, only part of the tax revenue collected in each period is assumed to be available to finance the given expenditure requirement. Suppose that, for every period, the collection cost (z_t) is proportional to the total revenue collected:

$$z_t = a_t \tau_t c_t, \qquad t = 0,1,2, \tag{22}$$

where $1 > a_t \geq 0$ is a period-specific cost coefficient. 1/ With these costs taken into account, the intertemporal budget constraint of the government in period 0 becomes

$$gR = \Sigma_{t=0}^{2}\rho_t\tau_t x_t - \Sigma_{t=0}^{2}\rho_t a_t\tau_t c_t = \Sigma_{t=0}^{2}(1-a_t)\rho_t\tau_t c_t. \tag{23}$$

The maximization of equation (8) subject to equation (23) gives the optimal tax rates for the three periods as

$$(1 + \tau_t) = (1 - a_t)/(A/B - g/x), \qquad t = 0,1,2, \tag{24}$$

where $A \equiv \Sigma_{t=0}^{2}\beta^t(1-a_t)$. Notice that the optimal tax structure expressed in equation (24) does give rise to distortions at the margin for consumption.

Is the consumption tax under the present circumstance time consistent? Applying equation (24) to the appropriate period-wise budget constraints of the government in period 0 (equations (2.1)-(2.3) modified for the presence of collection costs), one can solve for the total debt issuance in that period as

$$\Sigma_{t=1}^{2}\ {}_0b_t' = \Sigma_{t=1}^{2}\ {}_0b_t - [(1-a_0) - A/B]xR/B, \tag{25}$$

with a term structure of

$${}_0b_1' = {}_0b_1 + \{[(1 - a_1) - A/B]\beta/\rho_1 + [(1 - a_0) - A/B]r_1\}xR/B \tag{26}$$

and

$${}_0b_2' = {}_0b_2 - \{[(1 - a_1) - A/B]\beta + (1 - a_0) - A/B\}xR/(\rho_1 B), \tag{27}$$

where ${}_0b_1$ and ${}_0b_2$ are given in equations (13) and (14), respectively. Compared with the previous solution, the extra term in each of the three

1/ The introduction of collection costs in this manner is, admittedly, somewhat ad hoc in spirit, though not without precedence in the literature (see, for example, Barro (1979)). The period-specificity of the cost coefficient is for illustrative purposes only.

equations (25)-(27) indicates that the tax-smoothing function of debt has now been modified somewhat in order to take tax collection costs into account.

The government in period 1, upon inheriting the above debt structure, is assumed to maximize as before equation (18), but is now subject to the following budget constraint:

$$\Sigma_{t=1}^{2}\ {}_{0}b_t' + g(R-1) = \Sigma_{t=1}^{2}(1 - a_t)\rho_t\tau_t c_t. \tag{28}$$

After some tedious algebra, the solutions for the optimal tax rates in this maximization problem turn out to be exactly the same as those expressed in equation (24), proving that the consumption tax, even when it is optimal to have different rates across periods, remains a form of taxation that is time consistent.

2. Labor-leisure choice

The existing literature on the time consistency of optimal policy generally incorporates labor-leisure choice into the analysis, since the allowable form of taxation in this literature has invariably been a tax on wages. To extend the analytical framework in Section II to the case of a variable labor supply, reinterpret x as the time endowment of the representative consumer. Assuming, for simplicity, the transformation between leisure and output is unity, the resource constraint of the economy becomes

$$x = c_t + y_t + g, \qquad t = 0,1,2, \tag{29}$$

where y_t is the amount of leisure demanded in period t. The consumer now maximizes

$$U = \Sigma_{t=0}^{2}\beta^t[\gamma\ln c_t + (1-\gamma)\ln y_t], \qquad 1 > \gamma > 0, \tag{30}$$

subject to

$$xR = \Sigma_{t=0}^{2}\rho_t[(1+\tau_t)c_t + y_t]. \tag{31}$$

Because of the log-linearly additive specification of the utility function, the demand for leisure is independent of the tax on consumption. Specifically,

$$y_t = \beta^t(1-\gamma)xR/(\rho_t B), \qquad t = 0,1,2. \tag{32}$$

It immediately follows that optimal consumption taxation remains to be time consistent under the present circumstance, with the optimal tax rate set at

$$\tau^* = g/(\gamma x - g), \tag{33}$$

which is again constant over time. Equation (33) indicates that, for the same given level of government expenditure as before, the optimal tax rate is now higher than that given by equation (11), reflecting the consequence of a smaller tax base (as leisure is not taxed).

3. Discussion

The foregoing extensions to the basic model of Section II illustrate two important insights concerning the time consistency problem. First, in the collection-costs example, it is shown that time consistency need not require a constant tax rate over time. In that example, the optimal tax rate in any given period may differ from the rest, but solely as a result of an attribute specific to that period, namely, the cost coefficient a_t. Hence, as long as the time profile of this coefficient does not change, governments in successive periods would always arrive at the same optimal rate for each period. Secondly, in the labor-leisure choice extension, the consumption tax continues to be time consistent, even though a choice variable in the consumer's utility function escapes taxation, because the demand for the untaxed commodity is independent of the tax rate on the taxed commodity. Under a more general specification of the utility function, however, Zee (1988b) has shown that time consistency would require the use of the consumption tax in conjunction with a wage tax. 1/

It is possible to deduce a general conclusion from the above discussion. For optimal intertemporal taxation to be time consistent, it is necessary and sufficient to have a tax structure that is backward independent, i.e., the optimal tax rate in any period t does not in any way depend on attributes related to periods prior to period t. When this condition is operative, the optimal policy as viewed by governments in different dates is bound to be the same even as time elapses, as long as the intertemporal economic environment remains unchanged.

1/ It is worth pointing out that, even with the Cobb-Douglas utility function as specified in equation (30), the use of the wage tax alone would not produce time-consistent optimal tax rates. This is because the wage tax, unlike the consumption tax, affects the present value of the consumer's income endowment, and, through it, affects the consumption demand in every period. This effect is apart from the price channel that has already been made inoperative by the assumed utility function.

IV. Concluding Remarks

Although Persson and Svensson (1984) has provided an insightful "government cash flow" interpretation of the time consistency problem as originally analyzed by Lucas and Stokey (1983), a simple and intuitive explanation of why optimal policy can be time inconsistent in a barter economy without capital still seems to elude researchers. The analysis in the present paper sheds some light on this issue.

Unlike taxing wages, which is the commonly assumed form of taxation in the time consistency literature, a tax on consumption does not alter the present value of the consumer's income endowment, and therefore makes it more likely that the derived optimal tax structure would be backward independent--a necessary and sufficient condition for time consistency. Indeed, this paper has shown that under a variety of circumstances, the consumption tax would be time consistent, implying the neutrality of the term structure of debt. For small, open economies that must face given intertemporal prices, taxing consumption rather than wage income 1/ provides a way out of the time consistency quandary that invariably results from a regime of wage income taxation alone.

The above consideration is particularly relevant in the context of the current international economic environment, in which tax reform has become an increasingly important aspect of a country's overall structural adjustment strategy to promote growth. To the extent that time consistency is regarded as a desirable property for formulating optimal policy (if only for the purpose of ensuring policy creditability), the present analysis suggests that the prevailing trend of adopting some form of consumption-based taxation (such as the value-added tax) in many recent tax reforms is a movement in the right direction.

1/ Or in conjunction with taxing wage income if the supply of labor is a function of the consumption tax. For details, see Zee (1988b).

A General Proof of Time Consistency of Optimal Consumption Taxation

This Appendix presents an analysis of optimal intertemporal taxation on consumption with many of the restrictive assumptions made in Section II relaxed. 1/ It is shown here that the basic results of that section continue to hold under a more general model of consumption taxation. 2/ In particular, optimal consumption taxes are shown to be time consistent, so that the term structure of government is neutral.

Let α_t be the government expenditure/income ratio in period t, that is,

$$g_t = \alpha_t x_t, \qquad t = 0,1,\ldots,T, \tag{A1}$$

and h_t the rate of growth of income in period t, that is,

$$x_t = (1 + h_t)x_{t-1}, \qquad t = 1,2,\ldots,T, \tag{A2}$$

with T being the (arbitrarily) given number of periods in the model. With the behavior of income and government expenditure governed by equations (A1) and (A2), the consumer's intertemporal budget constraint becomes

$$x_0\Omega = \Sigma_{t=0}^{T} \rho_t(1 + \tau_t)c_t, \tag{A3}$$

whereas the government's intertemporal budget constraint can be written as

$$x_0G = \Sigma_{t=0}^{T} \rho_t\tau_t c_t, \tag{A4}$$

where $\Omega \equiv \Sigma_{t=0}^{T} \rho_t\delta_t$,

$G \equiv \Sigma_{t=0}^{T} \alpha_t\rho_t\delta_t$,

$\delta_0 \equiv 1$, and

1/ Notations used here are consistent with those employed in Section II. Variables introduced in that section are not redefined in this Appendix.

2/ For a general analysis that incorporates labor-leisure choice, see Zee (1988b).

$$\delta_t \equiv \Pi_{i=1}^{t}(1 + h_i), \qquad t = 1,2,\ldots,T.$$

The representative consumer maximizes the utility function

$$U = U(c_1,c_2,\ldots,c_T), \tag{A5}$$

subject to equation (A3). The first-order conditions are

$$\partial U/\partial c_t = \mu p_t, \qquad t = 0,1,\ldots,T, \tag{A6}$$

where $p_t \equiv (1 + \tau_t)\rho_t$ and μ is the relevant Lagrangean multiplier. Equations (A3) and (A6) then allow one to solve for the system of consumption functions as

$$c_t = c_t(p_0,p_1,\ldots,p_T,x_0\Omega), \qquad t = 0,1,\ldots,T. \tag{A7}$$

Substituting equation (A7) into equation (A5) results in the indirect utility function

$$V = V(p_0,p_1,\ldots,p_T,x_0\Omega), \tag{A8}$$

which is the maximand of the government, subject to equation (A4).

The above formulation of the optimal intertemporal consumption tax problem is formally identical to an optimal commodity tax problem over T + 1 goods in a static framework. Thus, one would expect that the optimal consumption tax structure is a proportional one. The first-order conditions for the government's maximization problem in period 0 are

$$\partial V/\partial \tau_i = \lambda_0\rho_i[c_i + \Sigma_{t=0}^{T}\, \rho_t\tau_t(\partial c_t/\partial p_i)], \; i = 0,1,\ldots,T. \tag{A9}$$

However, by Roy's identity,

$$\partial V/\partial \tau_i = -\mu\rho_i c_i, \qquad i = 0,1,\ldots,T. \tag{A10}$$

Therefore,

$$\Sigma_{t=0}^{T} \rho_t \tau_t (\partial c_t / \partial p_i) = -c_i (\mu + \lambda_0)/\lambda_0, \quad i = 0,1,\ldots,T. \tag{A11}$$

From equation (A3), it can be shown that consumption in any period i is expressable as

$$c_i = - \Sigma_{t=0}^{T} \rho_t (1 + \tau_t)\,(\partial c_t / \partial p_i), \qquad i = 0,1,\ldots,T. \tag{A12}$$

Substituting equation (A12) into equation (A11) yields

$$\Sigma_{t=0}^{T} \rho_t \tau_t (\partial c_t / \partial p_i) = [(\mu + \lambda_0)/\lambda_0] \Sigma_{t=0}^{T} \rho_t (1 + \tau_t)(\partial c_t / \partial p_i),$$

$$i = 0,1,\ldots,T. \tag{A13}$$

It follows immediately from equation (A13) that

$$\tau_t^{*}/(1+\tau_t^{*}) = (\mu + \lambda_0)/\lambda_0 \equiv \theta_0, \qquad t = 0,1,\ldots,T. \tag{A14}$$

The optimal tax structure given by equation (A14) displays an extreme form of proportionality: the tax rate is constant over time. Utilizing the budget constraints of the consumer and the government, one can obtain explicit solutions for the tax rates as

$$\tau_0^{*} = \tau_1^{*} = \ldots = \tau_T^{*} = \tau^{*} = G/(\Omega - G). \tag{A15}$$

Equation (A15) is the analogue of equation (10) in Section II.

The government in period 1 faces an optimization calculus similar to that faced by its predecessor, and, therefore, its optimal tax structure likewise takes a form similar to that of equation (A14):

$$\tau_t^{**}/(1 + \tau_t^{**}) = (\mu + \lambda_1)/\lambda_1 \equiv \theta_1, \qquad t = 1,2,\ldots,T, \tag{A16}$$

which implies that a constant tax rate ($\tau_1^{**} = \tau_2^{**} = \ldots = \tau_T^{**} = \tau^{**}$) continues to be optimal. However, the two optimal rates, as determined by the governments in the two successive periods, will be equal to each other, that is, $\tau^{*} = \tau^{**}$, if and only if $\theta_0 = \theta_1$. To show that this latter condition in fact holds, note first that the intertemporal budget constraint facing the government in period 1 is

$$\Sigma_{t=1}^{T}\ {}_{0}b_{t} + x_{0}(G - \alpha_{0}\rho_{0}\delta_{0}) = \Sigma_{t=1}^{T}\rho_{t}\tau_{t}c_{t}, \tag{A17}$$

where the first term on the left-hand side of equation (A17) denotes the total amount of debt issued by the government in period 0. But definitionally,

$$\Sigma_{t=1}^{T}\ {}_{0}b_{t} = x_{0}\alpha_{0}\rho_{0}\delta_{0} - \rho_{0}\tau^{*}c_{0}. \tag{A18}$$

Hence, upon substitution of equation (A18) into equation (A17), the latter can be rewritten as

$$x_{0}G = \Sigma_{t=1}^{T}\rho_{t}\tau_{t}c_{t} + \rho_{0}\tau^{*}c_{0}. \tag{A19}$$

Applying the result of equation (A16) to equation (A19), one obtains

$$x_{0}G = \theta_{1}\Sigma_{t=1}^{T}\rho_{t}(1 + \tau_{t}^{**})c_{t} + \rho_{0}\tau^{*}c_{0}. \tag{A20}$$

The consumer's intertemporal budget constraint (equation (A3)), when viewed from the perspective of period 2 and with tax rates set according to equation (A16), can be written as

$$\Sigma_{t=1}^{T}\rho_{t}(1 + \tau_{t}^{**})c_{t} = x_{0}\Omega - \rho_{0}(1 + \tau^{*})c_{0}. \tag{A21}$$

Consequently, equations (A20) and (A21), together, imply

$$x_{0}G = \theta_{1}x_{0}\Omega + \rho_{0}\tau^{*}c_{0}(1 - \theta_{1}/\theta_{0}), \tag{A22}$$

where use is made of equation (A14). Dividing equation (A22) through by x_0G, utilizing the result of equation (A15), and rearranging terms, one can show that

$$[1 - \rho_{0}\tau^{*}c_{0}/(x_{0}G)] = [1 - \rho_{0}\tau^{*}c_{0}/(x_{0}G)]\theta_{1}/\theta_{0}, \tag{A23}$$

which can hold if and only if $\theta_0 = \theta_1$, as the left-hand side expression of equation (A23) does not equal zero (see equation (A4)). This completes the proof that the optimal tax rate chosen by the government in period 1 is the same as that chosen by its predecessor. Since the methodology of the proof is applicable to any pair of successive periods, optimal consumption taxation is therefore time consistent.

References

Barro, Robert J., "On the Determination of the Public Debt", Journal of Political Economy, Vol. 87 (October 1979), pp. 940-71.

Calvo, Guillermo A., "On the Time Consistency of Optimal Policy in a Monetary Economy," Econometrica, Vol. 46 (November 1978), pp. 1411-28.

_____, and Maurice Obstfeld, "Time Consistency of Fiscal and Monetary Policy: A Comment," (mimeographed, 1988).

Fischer, Stanley, "Dynamic Inconsistency, Cooperation, and the Benevolent Dissembling Government," Journal of Economic Dynamics & Control, Vol. 2 (February 1980), pp. 93-107.

Hall, Robert E., "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," Journal of Political Economy, Vol. 86 (December 1978), pp. 971-87.

Kydland, Finn E., and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy, Vol. 85 (June 1977), pp. 473-91.

Lucas, Jr., Robert E., and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy Without Capital," Journal of Monetary Economics, Vol. 12 (July 1983), pp. 55-93.

Persson, Mats, Torsten Persson, and Lars E.O. Svensson, "Time Consistency of Fiscal and Monetary Policy," Econometrica, Vol. 55 (November 1987), pp. 1419-31.

Persson, Torsten, and Lars E.O. Svensson, "Time-Consistent Fiscal Policy and Government Cash Flow," Journal of Monetary Economics, Vol. 14 (November 1984), pp. 365-74.

_____, "International Borrowing and Time-Consistent Fiscal Policy," Scandinavian Journal of Economics, Vol. 88 (1986), pp. 273-95.

Zee, Howell H., "The Sustainability and Optimality of Government Debt," Staff Papers, International Monetary Fund, Vol. 35 (December 1988a).

_____, "Time-Consistent Optimal Intertemporal Taxation," (mimeographed, 1988b).