WP/88/94

INTERNATIONAL MONETARY FUND

Research Department

Nominal Interest Rate Pegging Under Alternative Expectations Hypotheses

Prepared by Joseph E. Gagnon and Dale W. Henderson*

Authorized for Distribution by Michael P. Dooley

October 21, 1988

Abstract

Nominal interest rate pegging leads to instability in an IS-LM model with a vertical long-run Phillips curve and backward-looking inflation expectations. However, it does not lead to instability in several large multicountry econometric models, apparently primarily because these models have nonvertical long-run Phillips curves. Nominal interest rate pegging leads to price level and output indeterminacy in a model with staggered contracts and rational expectations. However, when a class of money supply rules with interest rate smoothing is introduced, and interest rate pegging is viewed as the limit of interest rate smoothing, the price level and output are determinate.

JEL Classification Numbers:
3110, 4300

*While the majority of the work on this paper was done outside, a substantial amount of the work was done and the final version of this paper written while Henderson was a Visiting Scholar in the Research Department of the International Monetary Fund. We have benefited greatly from discussions with Flint Brayton, Peter Isard, David Lindsey, Jaime Marquez, Bennett McCallum, John Taylor and Geoffrey Woglom. We received helpful comments from Guillermo Calvo, Elhanan Helpman, and Lars Svensson in the Research Department Seminar. All these individuals did their best to keep us on track but may not think that they succeeded. We also wish to thank Gwyn Adams for her able research assistance. This paper is scheduled for publication in Hooper et al. (forthcoming).

Contents

I. Introduction and Conclusions

For at least two centuries economists have been studying the policy of nominal interest rate pegging. 1/ In this paper we address two questions in the ongoing debate on the feasibility and desirability of this policy. First, does it lead to dynamic instability when inflation expectations are backward looking? Second, does it lead to price level indeterminacy when inflation expectations are rational? We derive several theoretical results and compare some of them with the econometric simulation results presented at the conference on Monetary Aggregates and Financial Sector Behavior in Interdependent Economies sponsored by the Board of Governors of the Federal Reserve System.

In some of the simulation experiments designed for the Federal Reserve conference, it is specified that nominal interest rates should be pegged. Results were reported at the conference for these experiments for all but one of the models with backward-looking expectations (the Federal Reserve Board's MPS model, the Federal Reserve Board's MCM model, the Project LINK World Model, the Japanese Economic Planning Agency's EPA model, and the OECD's INTERLINK model). Results were reported to us after the conference for a related experiment for the remaining model with backward-looking expectations (the United Kingdom National Institute's GEM model). These results provide empirical answers to the dynamic instability question.

No results were reported for the experiments with nominal interest rates pegged for the one model with rational expectations, the Taylor model. We can think of two possible explanations for why no results were reported. Taylor may have believed that nominal interest rate pegging would lead to price level indeterminacy in his model and that, therefore, there was no reason even to attempt the experiments. Alternatively, he may have believed that the experiments could be performed in principle but may have been unable to perform them for technical reasons.

Our paper is divided into six sections. In Section II we review the Wicksellian argument that with backward-looking inflation expectations an economy is dynamically unstable when the authorities peg the nominal interest rate. First, we show that this argument is correct in the most basic closed-economy IS-LM model with an expectations-augmented Phillips curve and adaptive expectations. The logic of the argument is clear. Consider an increase in the nominal interest rate peg. This increase raises the real interest rate because expected inflation does not change in the first period. The increase in the real interest rate lowers aggregate demand. The decrease in aggregate demand causes actual inflation to fall. The fall in actual inflation causes a decrease in next

---

1/ For references to both early and recent writings on the policy of nominal interest rate pegging, see McCallum (1986).

period's expected inflation. The decrease in next period's expected inflation causes larger decreases in next period's actual inflation and the following period's expected inflation. The process repeats itself, and both actual and expected inflation continue to fall. (In Appendix A we confirm that a two-country version of the model is also dynamically unstable.) Next, we consider an alternative version of the model with a Phillips curve more like those used in the econometric models. The alternative version is dynamically unstable under a reasonable assumption.

In Section III we ask whether the results for one of the simulation experiments with nominal interest rate pegging are consistent with the prediction of dynamic instability. The modelers were asked to report the effects of increasing the short-term nominal interest rate in one country by 100 basis points above the baseline throughout the simulation period while holding the short-term nominal interest rates in the other countries on the baseline. 1/ Many of the results seem inconsistent with the prediction of dynamic instability. Only the results for the United States in the MPS model are unambiguously consistent with this prediction. Results for a related experiment in the GEM model also seem to be consistent with this prediction.

Section IV is a consideration of two possible reasons for the apparent inconsistency of many of the simulation results with the prediction of dynamic instability. We were able to obtain equation listings for the MPS, MCM, EPA, and INTERLINK models. All of these models except the MPS model appear to have nonvertical long-run Phillips curves. We modify the theoretical model by making the long-run Phillips curve nonvertical and show that the resulting model may be dynamically stable. The MPS model and the EPA model allow for wealth effects on consumption. We add a wealth effect of the type in the EPA model to the theoretical model and show that the resulting model may be dynamically stable even in the presence of a vertical long-run Phillips curve.

1/ They were also asked to report the effects of simultaneously increasing the short-term nominal interest rates in the United States, Germany, and Japan by 100 basis points above the baseline throughout the the simulation period while holding the short-term nominal interest rates in the other countries on the baseline. We have not analyzed the results for this experiment closely. In the theoretical models we analyze, dynamic instability with nominal interest rate pegging under backward-looking expectations is a general characteristic; that is, the models are unstable no matter what the source of the shock. For this reason we decided to focus our attention on the results of only one of the simulation experiments with nominal interest rate pegging.

Next we turn to models with rational expectations. In their classic article, Sargent and Wallace (1975) show that with rational inflation expectations the price level is indeterminate if the authorities peg the nominal interest rate. 1/ Sargent and Wallace assume that the authorities peg the nominal interest rate by simply standing ready to exchange securities for money at the chosen rate. With nominal interest rate pegging in the Sargent-Wallace model there are multiple nonexplosive solutions for the price level, but the values of the real variables are uniquely determined.

In Section V we spell out the implications of nominal interest rate pegging in a Taylor (1980) model with staggered wage contracts and rational expectations. We obtain a result that is closely related to the Sargent-Wallace indeterminacy result. When the authorities peg the nominal interest rate in the Taylor model, there are multiple nonexplosive solutions for both nominal and real variables. The difference in results between Sargent-Wallace-type models and the Taylor model arises because of an important difference in assumptions. In Sargent-Wallace-type models the real variables depend on the nominal variables only through deviations between the actual and expected values of the nominal variables. However, in the Taylor model current values of the real variables depend on lagged values of the nominal variables.

Canzoneri, Henderson, and Rogoff (1983), Dotsey and King (1983), and McCallum (1986) all have concluded that a policy of pegging the nominal

---

1/ Sargent and Wallace (1975) claim to have established a much more general result regarding nominal interest rate policy rules. They assert that with rational expectations the price level is indeterminate for any rule in which the current nominal interest rate depends on lagged values of endogenous and exogenous variables. It is now clear that this assertion is incorrect.

McCallum (1981) puts forward a nominal interest rate policy rule for which the price level is determinate in a Sargent-Wallace model. In McCallum's rule the authorities give some weight to the objective of making the expected value of the money supply equal to a target value. As a result, the current nominal interest rate depends on the expected value of the current price level based on lagged information. According to McCallum, his result "does not formally contradict" the Sargent-Wallace assertion because the parameters in the Sargent-Wallace rule are "autonomous -- i.e. unrelated to behavioral parameters" while the parameters of his rule are related to behavioral parameters.

Sims (1988) puts forward a counterexample that does formally contradict the Sargent-Wallace assertion. In Sims' rule the current nominal interest rate depends on the lagged price level. For a range of values of the parameter relating the interest rate to the lagged price level, the price level is determinate in a Sargent-Wallace model.

interest rate by simply standing ready to exchange securities for money at the chosen rate is an incompletely specified policy. 1/ Canzoneri, Henderson, and Rogoff and McCallum introduce money supply rules with nominal interest rate smoothing into Sargent-Wallace-type models and show that when interest rate pegging is viewed as the limit of interest rate smoothing, the price level is determinate.

In Section VI we introduce money supply rules from a class with nominal interest rate smoothing into our Taylor model. We show that for each rule in this class the price level remains determinate as the smoothing parameter increases without limit. We also explain what happens when the target nominal interest rate is increased permanently or temporarily and the smoothing parameter is very large. A permanent increase in the target nominal interest rate must raise inflation by the same amount in the long run. The real interest rate is fixed in the long run, so if the nominal interest rate is to rise, the expected rate of inflation and, therefore, the actual rate of inflation must rise by the same amount. The policy may either raise or lower output in the short run. The policy may be implemented using different rules in the class we consider. While the different rules have the same implication for the nominal interest rate, they have different implications for output and the other variables.

II. The Basic Theoretical Model with Backward-Looking Expectations

In this section we review the Wicksellian argument that with backward-looking inflation expectations the economy is dynamically unstable when the authorities peg the nominal interest rate. We show that this argument is correct in the most basic closed-economy IS-LM model with an expectations-augmented Phillips curve and adaptive expectations. We also show that under a reasonable assumption this argument is robust to a replacement of the Phillips curve with one more like those used in the econometric models.

The basic model consists of the following three equations:

$$y_t = - \sigma(\bar{r} - \pi_t), \tag{1}$$

1/ In contrast, Benavie and Froyen (1988) have concluded that an interest rate pegging policy of the Sargent-Wallace type is a completely specified policy.

$$p_{t+1} - p_t = \alpha' y_t + \pi_t, \tag{2}$$

$$\pi_{t+1} - \pi_t = \rho(p_{t+1} - p_t - \pi_t). \tag{3}$$

y is (the logarithm of) output, r is the nominal interest rate, π is the expected rate of inflation, and p is (the logarithm of) the price level. It is assumed that (the logarithm of) the natural rate of output is zero. The bar over the nominal interest rate indicates that it is being pegged. Equation (1) is the goods market equilibrium condition. Output must equal aggregate demand which depends negatively on the expected real interest rate ($\sigma > 0$). Equation (2) is an expectations-augmented Phillips curve. The rate of inflation between today and tomorrow depends positively on today's gap between actual and natural output ($\alpha' > 0$) and on the expected rate of inflation between today and tomorrow. Today's price level is completely predetermined. We begin with this version of the expectations-augmented Phillips curve because it yields a very clear result. A more conventional version is analyzed below. Equation (3) is the expectations formation equation. Agents increase their expected rate of inflation by some fraction of the gap between actual inflation and expected inflation ($0 < \rho < 1$).

An increase in the nominal interest rate peg sets off a deflationary spiral. An increase in $\bar{r}$ in period t lowers y_t from equation (1). The drop in y_t lowers $p_{t+1}-p_t$ from equation (2). The fall in $p_{t+1}-p_t$ lowers π_{t+1} from equation (3). A decrease in π_{t+1} causes both $p_{t+2}-p_{t+1}$ and π_{t+2} to decrease by more as explained below. The process repeats itself, and both actual and expected inflation continue to fall.

Why does a decrease in π_{t+1} cause both $p_{t+2}-p_{t+1}$ and π_{t+2} to decrease by more? A decrease in π_{t+1} has a one-for-one direct effect on $p_{t+2}-p_{t+1}$ and also has a negative indirect effect on $p_{t+2}-p_{t+1}$ since it lowers y_{t+1}. A decrease in π_{t+1} has a one-for-one direct effect on π_{t+2} and also has a negative indirect effect by reducing $p_{t+2}-p_{t+1}-\pi_{t+1}$.

There is a more formal way of summarizing the behavior of the model. Substituting equation (1) into equation (2) yields

$$p_{t+1} - p_t = (1 + \alpha'\sigma)\pi_t - \alpha'\sigma\bar{r}. \tag{4}$$

Substituting equation (4) into equation (3) yields a first-order difference equation in π:

$$\pi_{t+1} = (1 + \rho\alpha'\sigma)\pi_t - \rho\alpha'\sigma\bar{r}. \tag{5}$$

The root of this equation is greater than one since ρ, α', and σ are positive. Therefore, the economy of equations (1), (2), and (3) is unstable. It follows from equation (5) that if $\bar{r}$ is increased π_t falls without limit. Equation (4) implies that if π_t falls without limit, $p_{t+1}-p_t$ also falls without limit.

In econometric models it is conventional to employ an expectations-augmented Phillips curve slightly different from equation (2). In the conventional Phillips curve, the rate of inflation between yesterday and today depends positively on today's gap between actual and natural output and on the expected rate of inflation between yesterday and today. Today's price level is not predetermined. An alternative version of the basic theoretical model with a conventional expectations-augmented Phillips curve is given by the following equations:

$$y_t = - \sigma(\bar{r} - \pi_t), \tag{6}$$

$$p_t - p_{t-1} = \alpha y_t + \pi_{t-1}, \tag{7}$$

$$\pi_t - \pi_{t-1} = \rho(p_t - p_{t-1} - \pi_{t-1}). \tag{8}$$

Under a reasonable assumption stated below, an increase in the nominal interest rate peg sets off a deflationary spiral in the alternative version just as in the first version. An increase in $\bar{r}$ in period t creates an excess supply of goods. y_t must fall to remove this excess supply. A fall in y_t is associated with decreases in p_t-p_{t-1} and π_t. A decrease in π_t causes both π_{t+1} and $p_{t+1}-p_t$ to decrease by more. The process repeats itself, and both expected and actual inflation continue to fall.

In order to obtain these results, it must be assumed that the decrease in π_t associated with a fall in y_t in the goods market ($1/\sigma$) is greater than the decrease in π_t associated with a decrease in y_t through the Phillips curve and the expectations formation equation ($\rho\alpha$). That is, it must be assumed that $1-\rho\alpha\sigma > 0$. This assumption guarantees that a fall in y_t reduces excess supply. The negative direct effect of a unit decrease in y_t on excess supply is one. However, there is also a positive indirect effect. A unit decrease in y_t reduces p_t-p_{t-1} by α from equation (7), reduces π_t by $\rho\alpha$ from equation (8), and increases excess supply by $\rho\alpha\sigma$ from equation (6). Therefore, a fall in y_t reduces excess supply if and only if $1-\rho\alpha\sigma > 0$. This assumption also guarantees that a decrease in π_{t-1} leads to a larger decrease in π_t. A decrease in π_{t-1} would decrease π_t by the same amount if $p_t-p_{t-1}-\pi_{t-1}$ did not depend on π_t. A decrease in π_{t-1} raises the left-hand side of equation (8) and has no effect on $p_t - p_{t-1} - \pi_{t-1}$ because the decrease in π_{t-1} lowers $p_t - p_{t-1}$ by the same amount. However, $p_t - p_{t-1} - \pi_{t-1}$ does depend on π_t. A decrease in π_t lowers y_t by σ from equation (6), lowers $p_t - p_{t-1}$ by $\alpha\sigma$ from equation (7), and lowers the right-hand side of equation (8) by $\rho\alpha\sigma$. Therefore, a decrease in π_{t-1} leads to a larger decrease in π_t if and only if $1-\rho\alpha\sigma > 0$. Finally, this assumption guarantees that a decrease in π_{t-1} causes a larger decrease in $p_t - p_{t-1}$. A decrease in π_{t-1} has a one-for-one direct effect on $p_t - p_{t-1}$. It also has a negative indirect effect if and only if $1-\rho\alpha\sigma > 0$. It reduces π_t, and the reduction in π_t reduces y_t.

More formally, substituting equation (6) into equation (7) yields

$$p_t - p_{t-1} = \alpha\sigma\pi_t + \pi_{t-1} - \alpha\sigma\bar{r}. \tag{9}$$

Substituting equation (9) into equation (8) yields a first-order difference equation in π:

$$\pi_t = \frac{1}{1 - \rho\alpha\sigma}\pi_{t-1} - \frac{\rho\alpha\sigma}{1 - \rho\alpha\sigma}\bar{r}. \tag{10}$$

Under our assumption that $1-\rho\alpha\sigma > 0$, the root of this equation is greater than one. Therefore, the economy of equations (6), (7), and (8) is unstable. It follows from equation (10) that if $\bar{r}$ is increased π_t falls without limit. Equation (9) implies that if π_t falls without limit, $p_t - p_{t-1}$ also falls without limit.

Of course, the condition $1-\rho\alpha\sigma > 0$ is sufficient but not necessary for instability. The economy is unstable if and only if $1-\rho\alpha\sigma > -1$. However, if $1-\rho\alpha\sigma < 0$, expected and actual inflation oscillate between positive and negative values. Our assumption that $1-\rho\alpha\sigma > 0$ seems all the more reasonable because it rules out such implausible oscillatory behavior.

Chart 1 shows the paths of the price level and the inflation rate following an increase in the nominal interest rate peg in the alternative version. The parameter values used to generate Chart 1 were $\rho = 0.2$, $\alpha = 0.2$, and $\sigma = 1$.

III. Some Simulation Results

In this section we compare the results of one of the simulation experiments designed for the Federal Reserve conference with the predictions of the theoretical model. The modeling groups were asked to report the effects of permanently raising the short-term nominal interest rate by 100 basis points above the baseline in one country while holding short-term nominal interest rates fixed on the baseline in the remaining countries. Results for this experiment were reported for all the models with backward-looking expectations, with the exception of the GEM model. Results for a related experiment were reported for the GEM model after the conference.

The simulation experiments were performed in multicountry econometric models, but the theoretical model is a closed economy model. Therefore, it might seem inappropriate to compare the simulation results with the theoretical predictions. However, as we show in Appendix I, the theoretical finding that nominal interest rate pegging with backward-looking expectations leads to dynamic instability generalizes to the case of two countries with nominal interest rates pegged at different levels. For this reason we feel justified in proceeding with a comparison of the simulation results and the theoretical predictions.

Charts 2, 3, 4, 5, and 6 plot the deviations of price levels and inflation rates from the baseline for particular countries. The deviations for each country are the result of increasing the short-term nominal interest rate in that country by 100 basis points above the baseline while holding short-term nominal interest rates in other countries on the baseline.

Chart 2 shows results for the Federal Reserve's MPS model. The deviations of the U.S. price level and inflation rate are clearly consistent with the predictions of the theoretical model. Raising the short-term nominal interest rate generates an accelerating deflation.

Chart 3 shows results for the Federal Reserve's MCM model. It is not clear whether the deviations of the U.S., German, and Japanese price levels and inflation rates are consistent with the predictions of the theoretical model. Deflation may be accelerating or it may be approaching a steady state value. Clearly, even if deflation is accelerating, it is not accelerating nearly so rapidly as in the MPS model.

Chart 4 shows results for the Japanese Economic Planning Agency's EPA model. The deviations of the U.S., German, and Japanese price levels and inflation rates are clearly inconsistent with the results of the theoretical model. Price levels fall below baselines and then rise back toward them. The U.S. and Japanese inflation rates fall increasingly below their baselines for a while, then begin rising toward them, and eventually rise above them. The German inflation rate falls below its baseline, rises back to it, falls increasingly below it, rises back to it, and eventually rises above it. It is not clear whether the inflation rates are approaching new steady-state values, nor is it clear whether any new steady-state inflation rates would be above or below the baseline.

Chart 5 presents the simulation results for the OECD's INTERLINK model. The price levels and inflation rates in the United States, Germany, and Japan do not appear to behave as predicted by the theoretical

Chart 1

Adaptive Expectations Theoretical Model

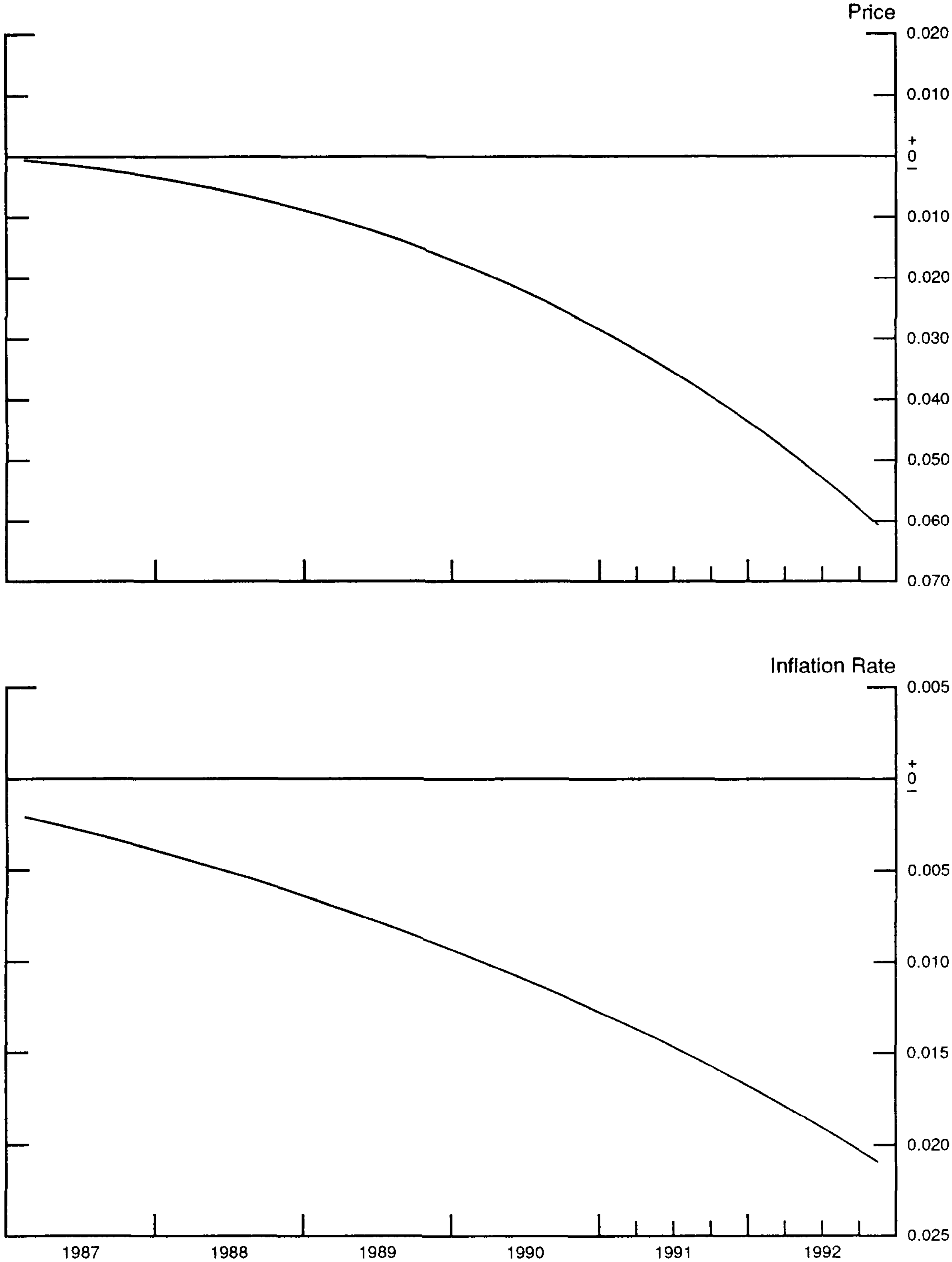
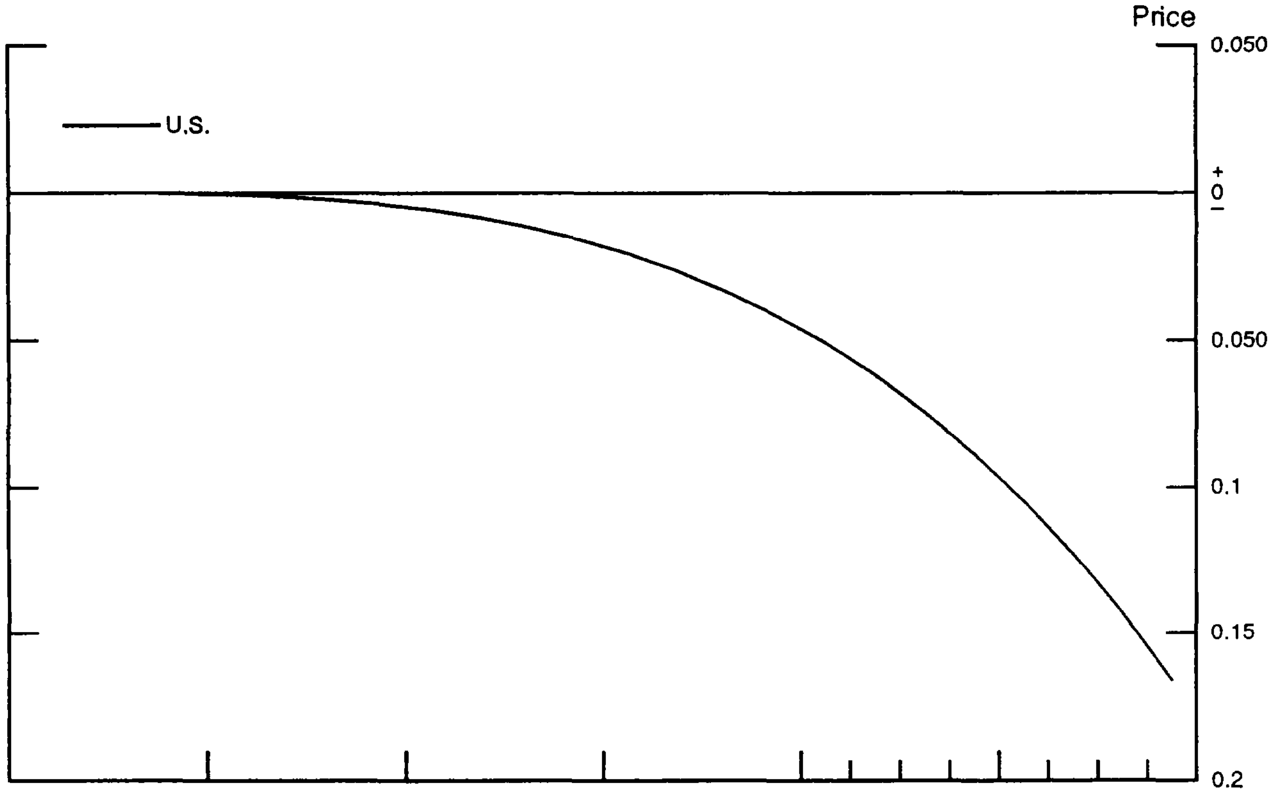


Chart 2
MPS Model



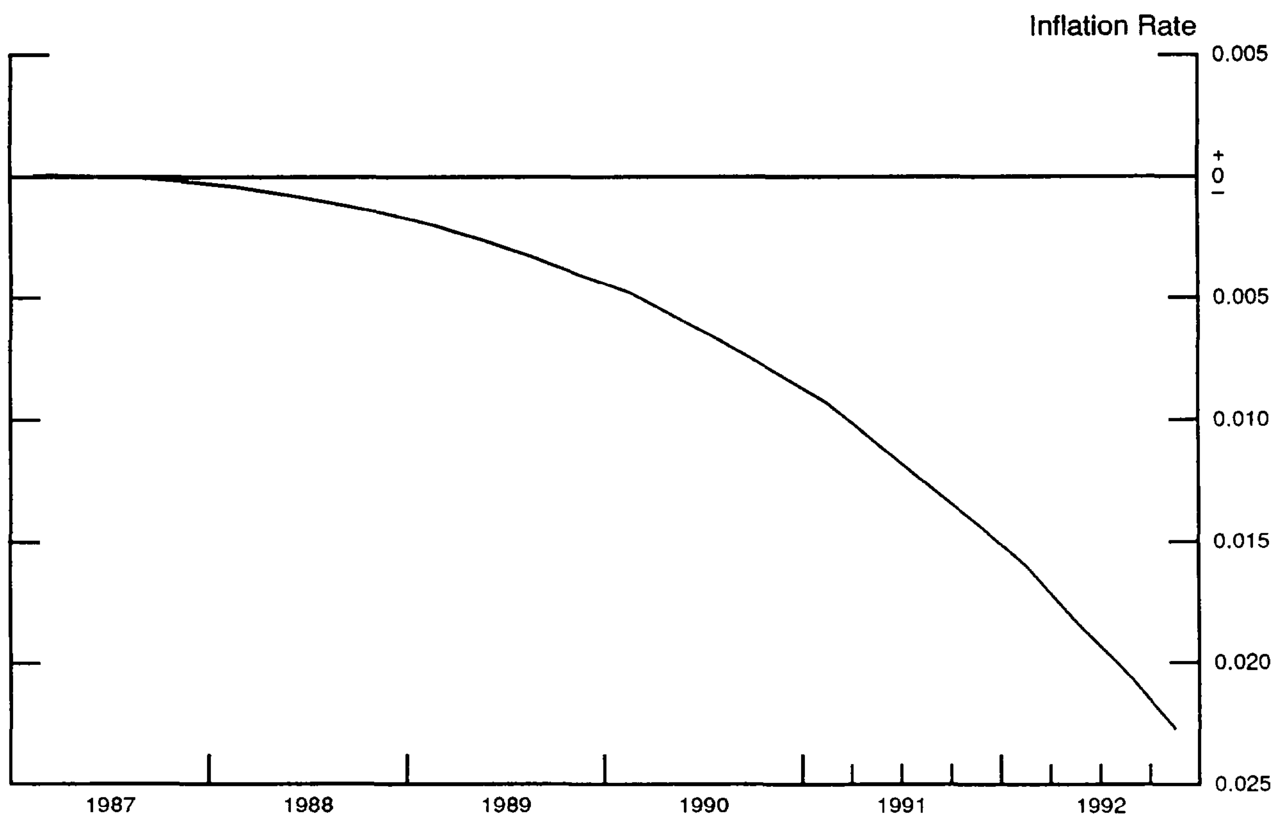


Chart 3
MCM Model

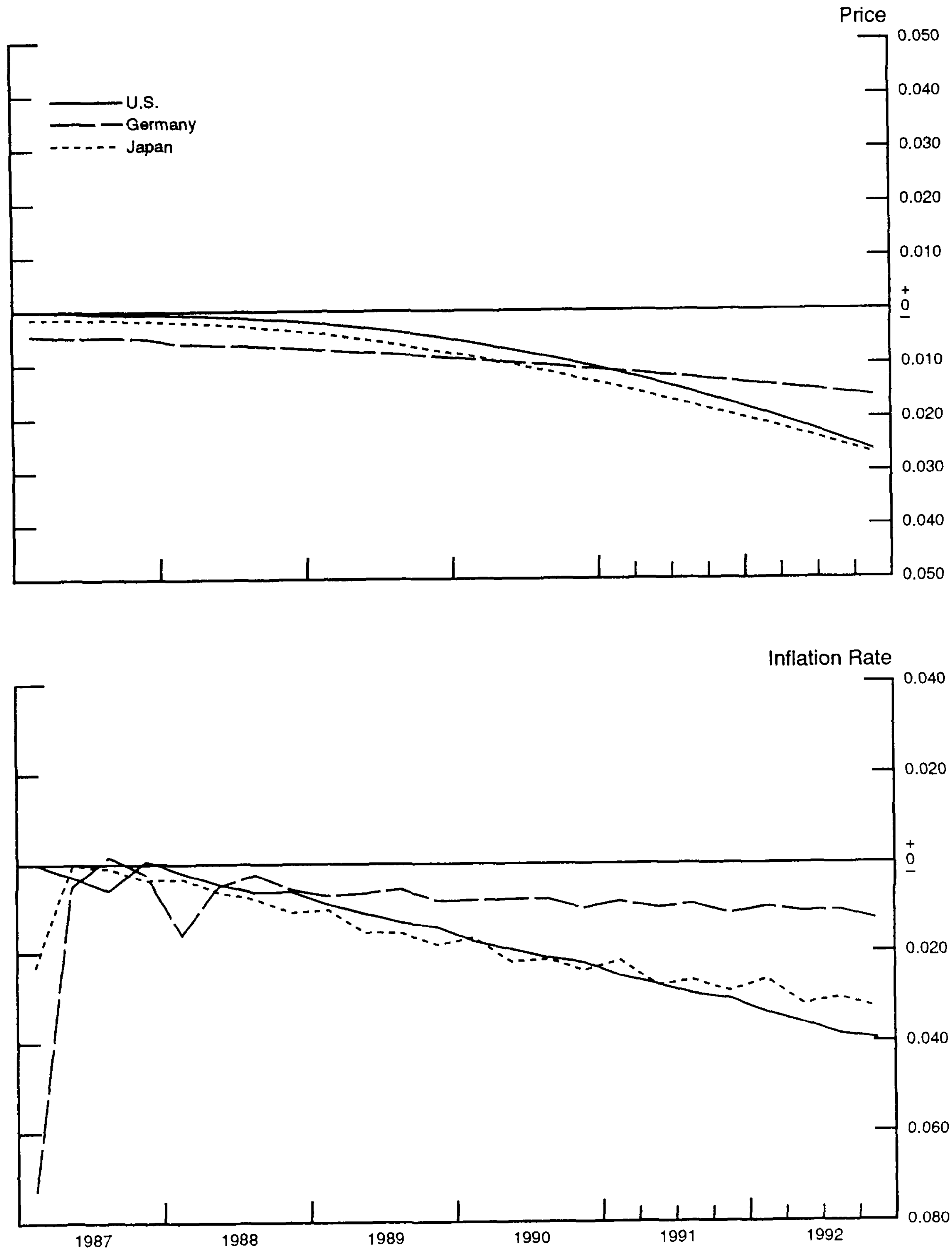
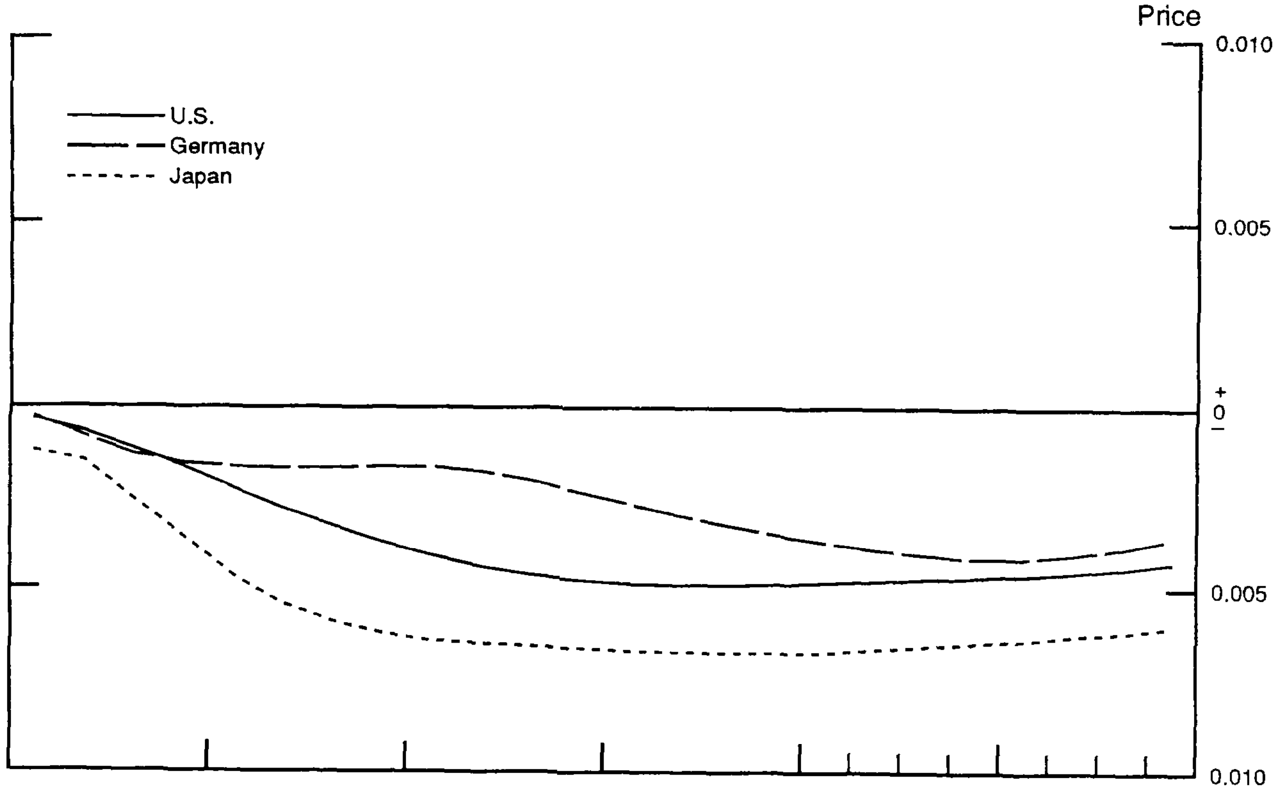


Chart 4
EPA Model



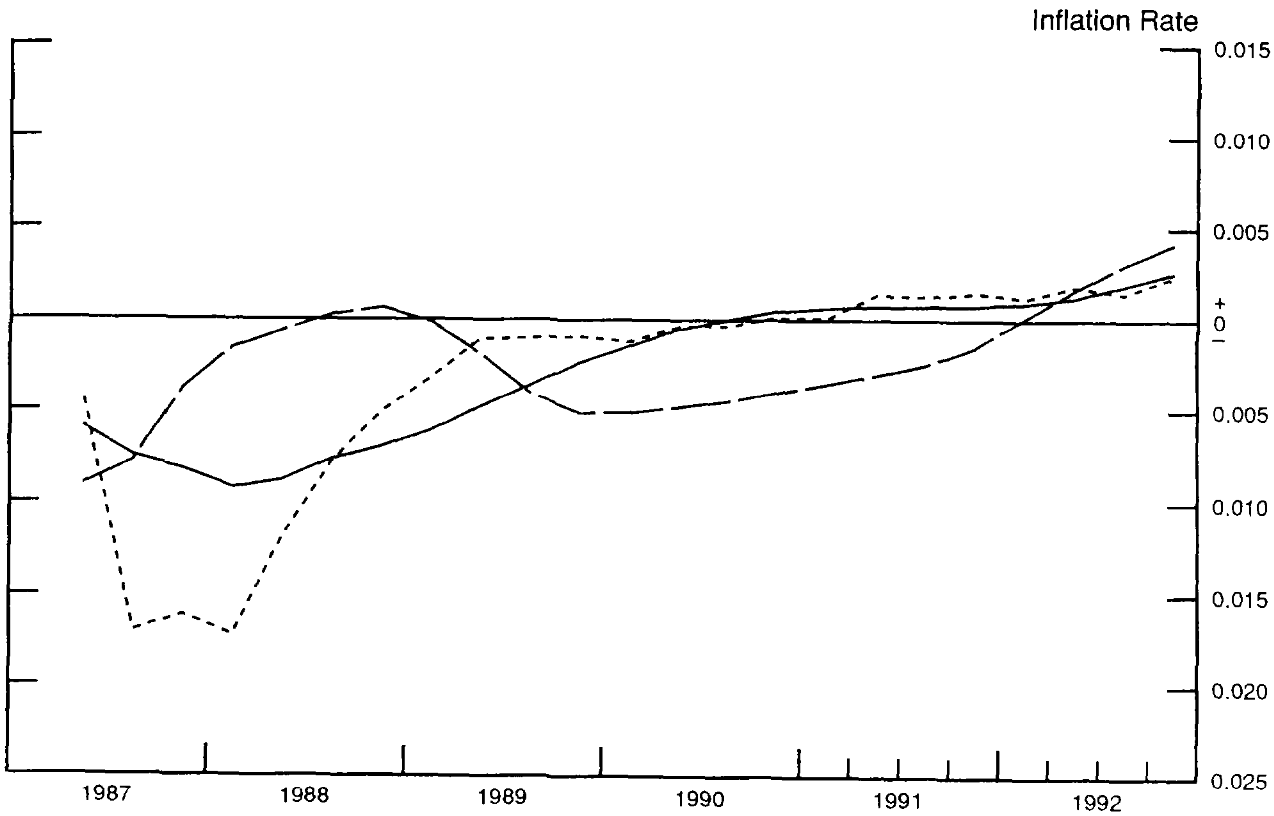
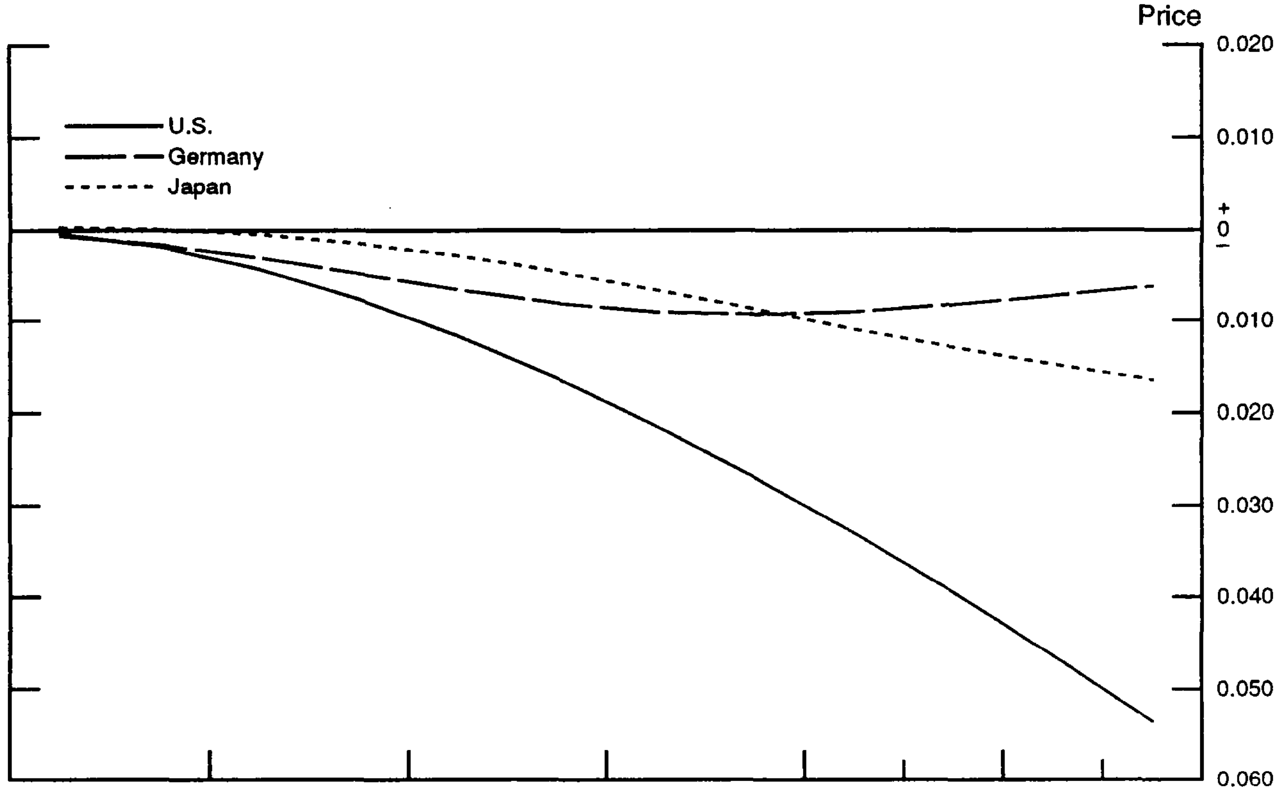


Chart 5
INTERLINK Model



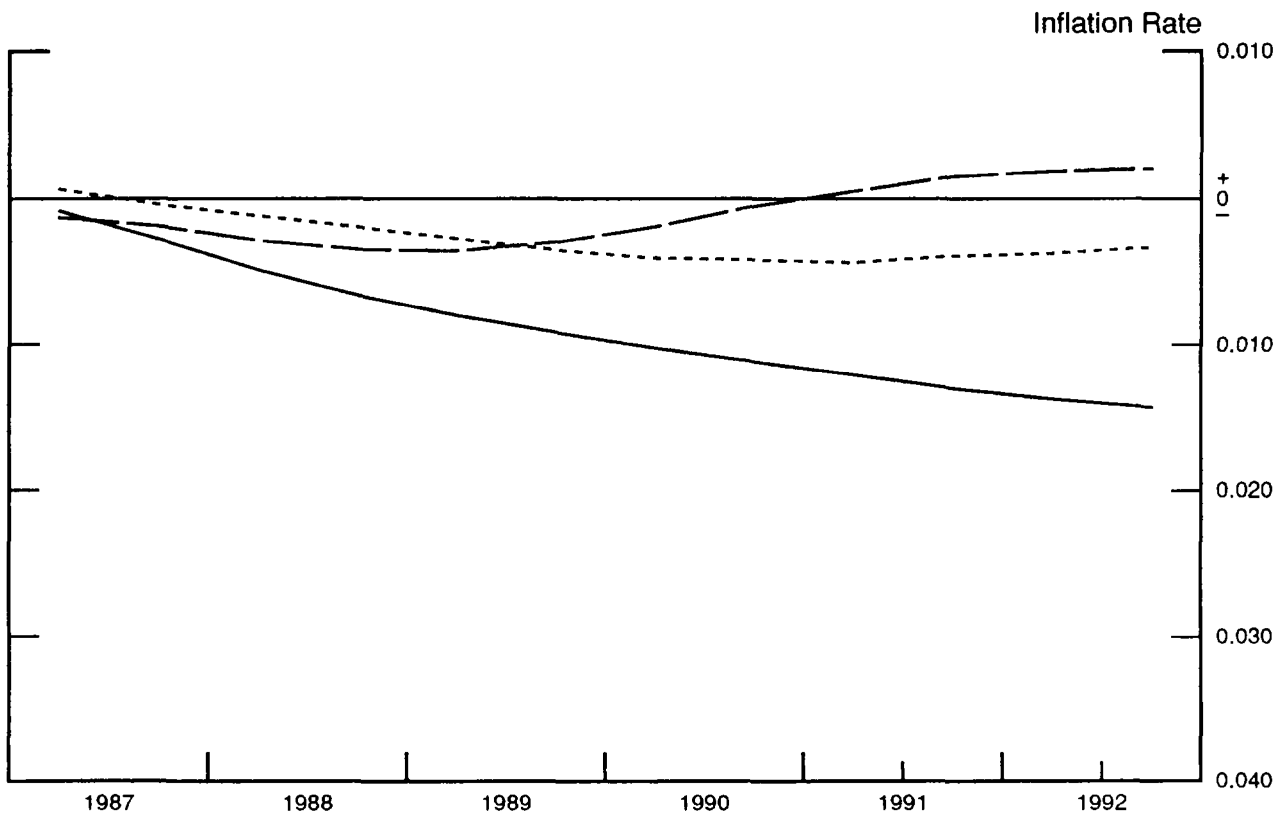
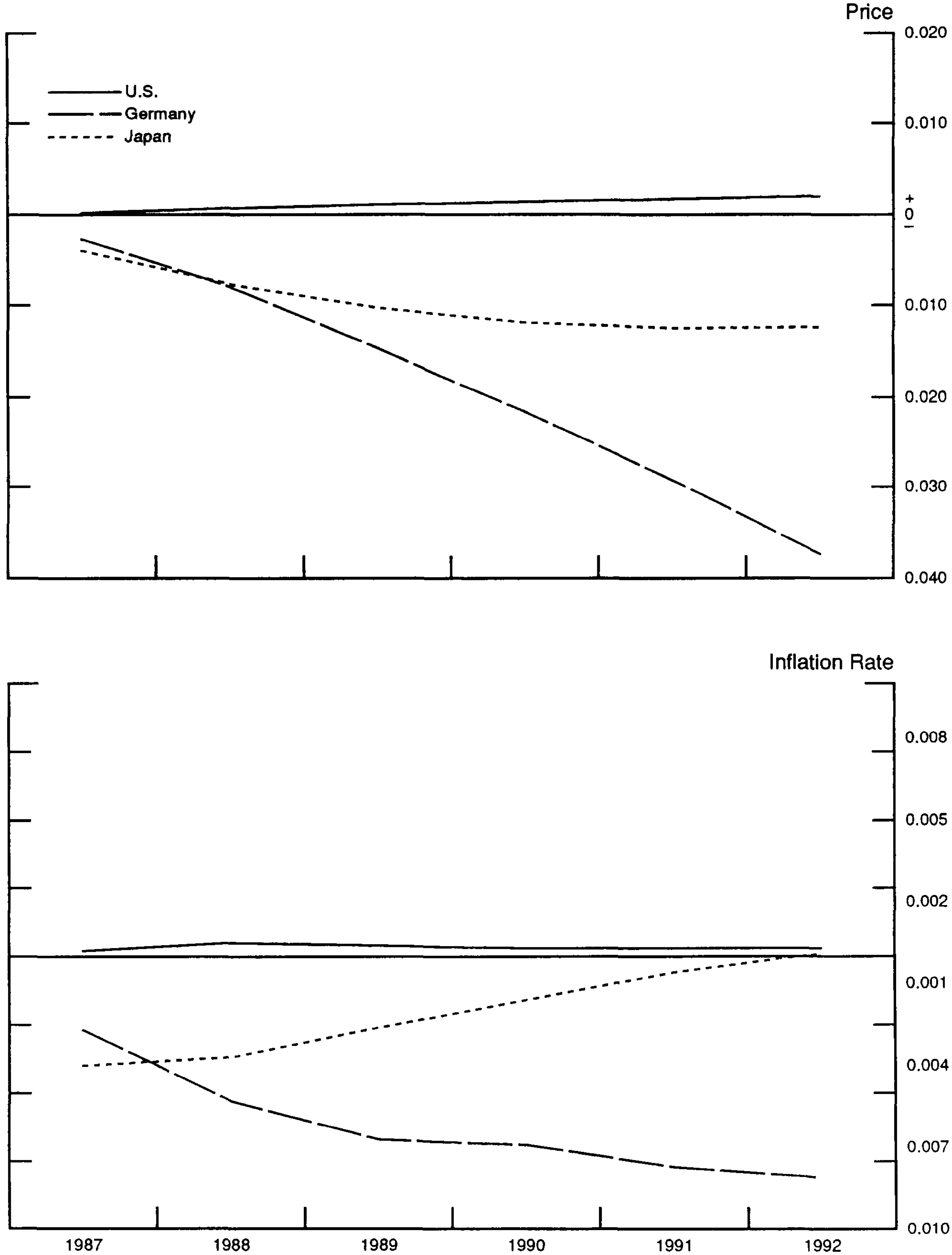


Chart 6
Project LINK Model



model. All three inflation rates appear to be stabilizing. It looks as though the U.S. and Japanese inflation rates are stabilizing at values below the baseline. In the INTERLINK simulation as in the EPA simulation, the German inflation rate is the most erratic of the three. It is not clear whether the German inflation rate is stabilizing at a value above or below the baseline.

Finally, Chart 6 shows the results for Project LINK's world econometric model. None of the price levels and inflation rates appear to behave as predicted by the theoretical model. There appears to be a permanent decrease in the German inflation rate and a very slight permanent increase in the U.S. inflation rate. The Japanese inflation rate first drops and then gradually drifts back to its baseline value.

Results were reported to us for a related experiment for the GEM model after the conference. In this experiment, the short-term nominal interest rates in all the G-7 countries were raised by 100 basis points above the baseline throughout the simulation period. The results for this experiment were consistent with the theoretical model. Inflation rates became increasingly negative over time. 1/

Since so many of the results were inconsistent with the theoretical model, we decided to take a closer look at the empirical models. Of the five models in Charts 2 through 6, we were able to obtain equation listings for four: MPS, MCM, EPA, and INTERLINK. We conclude that the most important difference between the theoretical model and the empirical models is that all of the empirical models except the MPS have nonvertical long-run Phillips curves. 2/ In every case that we could check, stable inflation-rate trajectories were associated with models that allow a permanent trade-off of output for inflation. A second important difference between our theoretical model and the empirical models is that the MPS and EPA models allow for wealth effects on consumption. In the next section we demonstrate that the dynamic paths shown in Charts 2 through 6 are not inconsistent with the theoretical model when it is modified to include either a nonvertical Phillips curve or a wealth effect.

1/ Simon Wren-Lewis of the United Kingdom National Institute reported these results to us by letter.

2/ We would like to thank Flint Brayton of the MPS modeling group for suggesting that the differences between the results for the MPS model and those for the other models represented at the Conference might be due to differences in the slopes of their long-run Phillips curves.

IV. Modifications to the Basic Theoretical Model

First, we modify the basic theoretical model to allow for a nonvertical Phillips curve. The modified model is

$$y_t = -\sigma(\bar{r} - \pi_t), \tag{11}$$

$$p_t - p_{t-1} = \alpha y_t + \omega\pi_{t-1}, \tag{12}$$

$$\pi_t - \pi_{t-1} = \rho(p_t - p_{t-1} - \pi_{t-1}). \tag{13}$$

We assume that $0 < \omega \leq 1$. If $\omega = 1$, the model of equations (11) through (13) is identical to the alternative version of Section II, and if $0 < \omega < 1$, the Phillips curve is nonvertical.

Analysis of the modified model reveals that the impact effect of a higher interest rate is the same as in the alternative basic model. We continue to assume that $1-\rho\alpha\sigma > 0$, so an increase in $\bar{r}$ in period t causes y_t to fall. A fall in y_t is associated with decreases in p_t-p_{t-1} and π_t. However, with $0 < \omega < 1$ a decrease in π_t may cause both π_{t+1} and $p_{t+1}-p_t$ to decrease by less as explained below. If it does, the system is stable and inflation levels off at a rate below its original value.

With $0 < \omega < 1$ a decrease in π_{t-1} may cause both π_t and p_t-p_{t-1} to decrease by less. In these circumstances a unit decrease in π_{t-1} not only raises the left-hand side of equation (13) by one unit but also raises the right-hand side by $\rho(1-\omega) < 1$ units through its direct and indirect effects on $p_t-p_{t-1}-\pi_{t-1}$. A unit decrease in π_t not only lowers the left-hand side of (13) by one unit but also lowers the right-hand side by $\rho\alpha\sigma < 1$ units through its indirect effect on $p_t-p_{t-1}-\pi_{t-1}$. Therefore, a decrease in π_{t-1} leads to a smaller decrease in π_t if and only if $1-\omega > \alpha\sigma$. It can be shown that a decrease in π_{t-1} causes a smaller decrease in p_t-p_{t-1} if and only if $1-\omega > \alpha\sigma$.

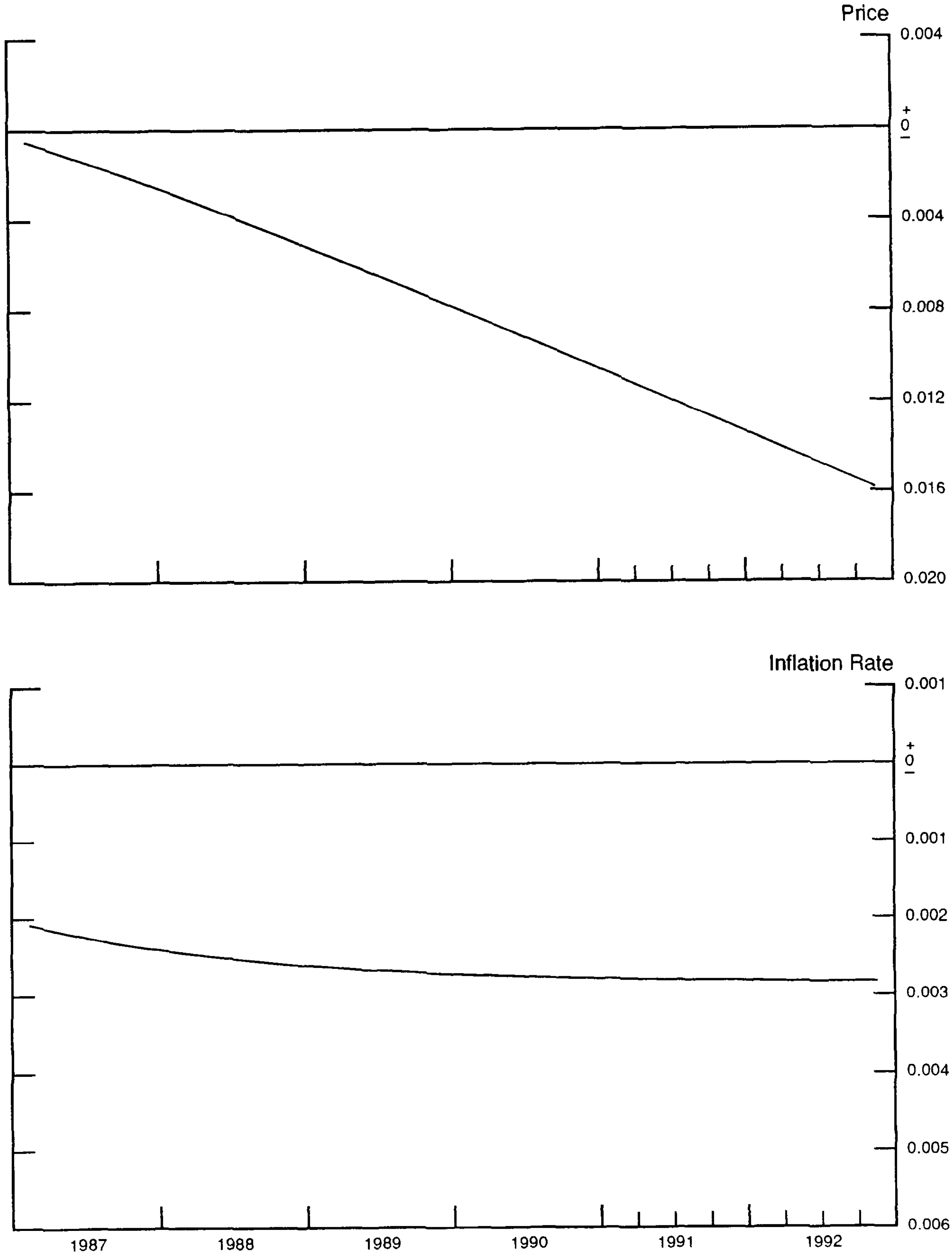
More formally, we reduce the model to a difference equation in π as we did in Section II:

$$\pi_t = \frac{1 - \rho(1-\omega)}{1 - \rho\alpha\sigma}\pi_{t-1} - \frac{\rho\alpha\sigma}{1 - \rho\alpha\sigma}\bar{r}. \tag{14}$$

If ω is small enough that $1-\omega > \alpha\sigma$, the root of equation (14) will be less than one, and the system will be stable. Chart 7 plots the response of the model to the same interest rate shock used in Chart 1. The parameter values are the same as those in Chart 1, with the addition of $\omega = 0.1$.

Chart 7

Theoretical Model with Nonvertical Phillips Curve



Now we modify the basic model to allow for a wealth effect in the goods market equation. In order to focus on the implications of the wealth effect separately from the nonvertical Phillips curve, we return to the vertical Phillips curve of the alternative version of the basic model. The modified model is

$$y_t = -\sigma(\bar{r} - \pi_t) + \theta(w_t - p_t), \tag{15}$$

$$p_t - p_{t-1} = \alpha y_t + \pi_{t-1}, \tag{16}$$

$$\pi_t - \pi_{t-1} = \rho(p_t - p_{t-1} - \pi_{t-1}), \tag{17}$$

$$m_t - p_t = \eta y_t - \lambda \bar{r}, \tag{18}$$

$$W_t = (1 + \bar{r})W_{t-1} - \bar{r}M_{t-1} - \tau P_t Y_t. \tag{19}$$

There are some new variables that must be defined. W_t and w_t are the level and the logarithm of nominal wealth. M_t and m_t are the level and logarithm of nominal money balances. It is assumed that nominal wealth is the sum of nominal money balances and nominal government bonds which are not assigned a symbol. P_t is the price level. The demand for output has been modified by adding a wealth effect. There are also some new equations. Equation (18) is the money demand function. The demand for real balances varies directly with output and inversely with the nominal interest rate. When the nominal interest rate is pegged, the nominal money supply is endogenous. Equation (19) is the government budget constraint written as a wealth evolution equation. It states that this period's nominal wealth (nominal money balances plus nominal government bonds) is equal to one plus the nominal interest rate times last period's nominal wealth minus the nominal interest rate times nominal money balances (on which no interest is paid) minus tax revenues. Government spending is assumed to be zero for simplicity. The evolution of wealth in our model is similar to the evolution of wealth in the EPA model. In the EPA model, nominal wealth is obtained by cumulating nominal household savings using the short-term nominal interest rate.

An increase in a nominal interest rate peg has additional effects in the wealth-effect model. An increase in the peg in period t raises period t real wealth in two ways through its negative effects on period t output and inflation. First, it lowers real taxes. Second, it raises the real value of beginning of period wealth by lowering prices. An increase in the peg in period t also raises period t+1 real wealth in two ways. First, it raises period t+1 real interest payments directly. Second, by lowering period t output, it raises the period t demand for real bonds at the expense of the demand for real money balances, thereby raising period t+1 interest payments indirectly. The increases in period t and period t+1 real wealth tend to offset the effect of the rise in the peg on output and inflation.

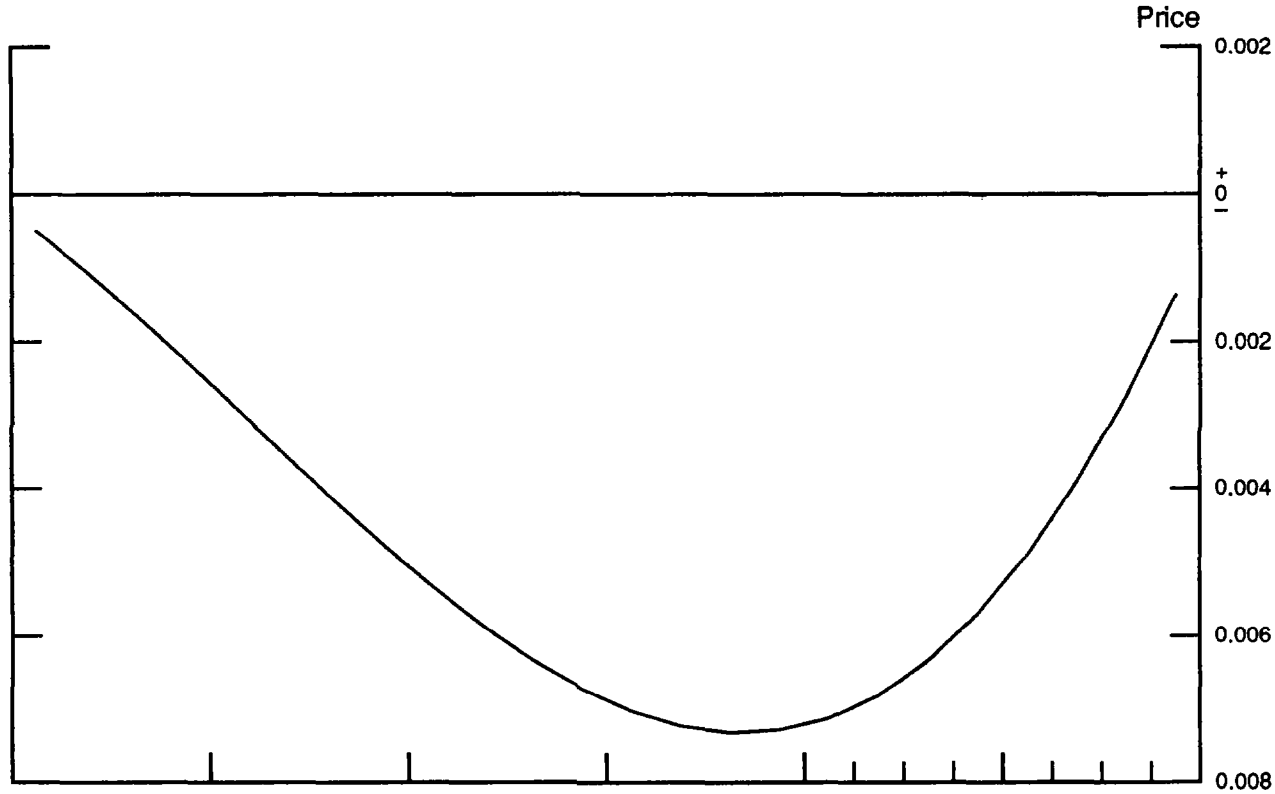
Because of the combination of levels and logarithms in equations (15) through (19), it is not possible to obtain an analytic solution. Rather than studying the model's local stability properties using a log-linearization of the government budget constraint, we trace out the implications of some specific parameter values using numerical simulations. When θ is quite small, the qualitative behavior of the wealth-effect model is the same as that of the basic model. That is, increasing an interest rate peg leads to a deflationary spiral. However, when θ is somewhat larger, the qualitative behavior of the wealth-effect model is quite different.

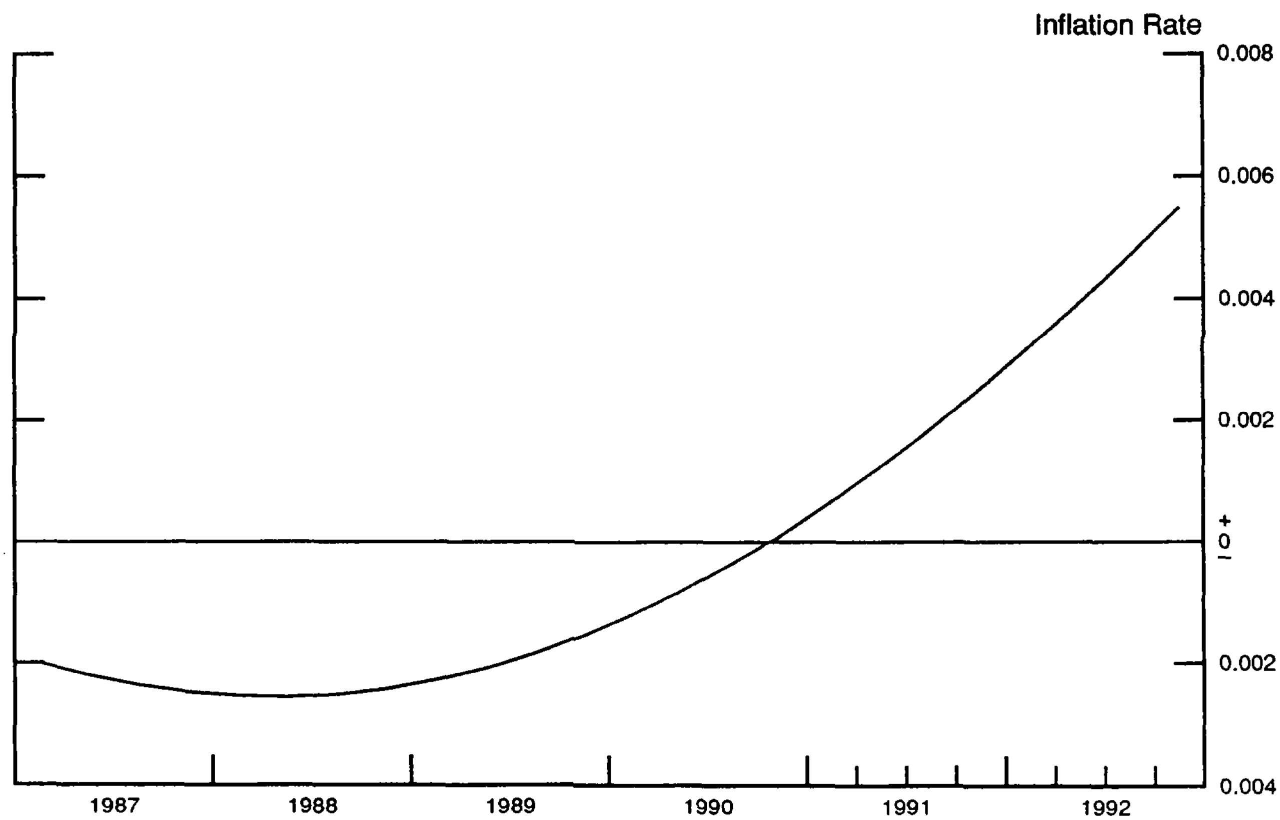
Chart 8 shows the paths of the price level and the inflation rate following an increase in a nominal interest rate peg in the wealth-effect model. The parameter values used to generate Chart 8 are the same as those used to generate Chart 1, with the addition of $\theta = 0.1$, $\eta = 1$, and $\lambda = 4$. As is apparent, with this set of parameters the theoretical model generates the prediction that the price level will drop, fall farther, and then rise continuously and that the inflation rate will drop and then rise. What is not apparent from Chart 8 is that the inflation rate asymptotically approaches a new steady state value that is higher than the old by an amount not quite equal to the increase in the nominal interest rate peg. Indeed we found that the wealth-effect model is stable for a wide range of parameters.

A comparison of Chart 8 with Chart 4 reveals that the EPA results are consistent with the wealth-effect model. As Chart 2 demonstrates, the MPS model is not stable under interest rate pegging despite the inclusion of a wealth effect. As noted above, it is possible for a model with a small enough wealth effect to be unstable. However, the MPS model is unstable for a different reason. 1/ Its measure of wealth includes equities and land. There is a direct effect of changes in the interest rate on the prices of these assets. A higher interest rate causes the nominal value of equities and land to fall by enough that real wealth falls even though the price level falls.

1/ This conclusion is based on an analysis of the behavior of real wealth in the MPS model by Flint Brayton.

Chart 8
Theoretical Model with Wealth Effect





V. Nominal Interest Rate Pegging with Rational Expectations

Next, we consider the implications of nominal interest rate pegging in models with rational expectations. We use a model adapted from Taylor (1980):

$$y_t = -\sigma[\bar{r}_t - ({}_tp_{t+1} - p_t)], \tag{20}$$

$$x_t = \frac{1}{2}(x_{t-1} + {}_tx_{t+1}) + \gamma(y_t + {}_ty_{t+1}), \tag{21}$$

$$p_t = \frac{1}{2}(x_{t-1} + x_t). \tag{22}$$

A subscript t before a variable denotes the expectation of that variable conditional on period t information. The pegged interest rate, $\bar{r}_t$, is dated because both past and expected future interest rates enter into the solution. Equation (20) is the goods market equilibrium condition. It is identical to the goods market equilibrium condition used above except that it embodies the assumption that inflation expectations are formed rationally, not adaptively. Equation (21) tells how firms and workers making contracts in period t set the contract nominal wage x_t. Firms and workers enter into nominal wage contracts that last for two periods. They agree on a contract nominal wage that is the same in both periods. It is assumed that exactly half of all workers enter into new two-period contracts in each period, so wage contracts overlap. It is also assumed that the excess demand for labor in period t is proportional to the deviation of output in period t from its natural rate of zero. If the sum of the excess demand for labor in period t and expected excess demand in period t+1 is zero ($y_t + {}_ty_{t+1} = 0$), firms and workers making contracts at time t try to maintain the relative wage of the workers over the life of the contract. They set x_t equal to the average of the wage being received by the workers who entered into contracts in period t-1 (x_{t-1}) and the wage that they expect will be received by these same workers when they enter into new contracts in period t+1 (${}_tx_{t+1}$). If the sum of current and expected future excess demand is positive or negative, firms and workers making contracts in period t agree to raise or lower the relative wage of the workers. Equation (22) describes the determination of the price level in period t. It is assumed that each unit of labor produces one unit of output and that firms set the price of output equal to the wage they pay so that the markup is zero. Therefore, the price level in period t is equal to the average of the contract wages negotiated in periods t-1 and t.

Our Taylor model can be expressed as a system of three first-order difference equations:

$$\begin{bmatrix} x_t \\ {}_tp_{t+1} \\ {}_ty_{t+1} \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 1/\sigma \\ -2/\gamma & 2/\gamma & -(1+\sigma\gamma)/\sigma\gamma \end{bmatrix} \begin{bmatrix} x_{t-1} \\ p_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1/\gamma \end{bmatrix} \bar{r}_t. \tag{23}$$

The first equation can be obtained by solving equation (22) for x_t. The second can be obtained by solving equation (20) for ${}_tp_{t+1}$. The third can be obtained by making use of the first two and equation (21). Shift the first equation forward one period, take expectations based on period t information, and substitute the resulting expression for ${}_tx_{t+1}$ into equation (21). Now substitute the expression for ${}_tp_{t+1}$ given by the second equation into equation (21) and solve the resulting equation for ${}_ty_{t+1}$.

In equations (23) the lagged contract wage, x_{t-1}, is a "sticky" variable. It is predetermined and, therefore, is not free to move at time t. Price and output, p_t and y_t, are "jump" variables. They are free to move at time t.

The solutions for x_{t-1}, p_t, and y_t take the form

$$x_{t-1} = C_1v_{11}u_1^t + C_2v_{12}u_2^t + C_3v_{13}u_3^t + Q_x(t), \tag{24}$$

$$p_t = C_1v_{21}u_1^t + C_2v_{22}u_2^t + C_3v_{23}u_3^t + Q_p(t), \tag{25}$$

$$y_t = C_1v_{31}u_1^t + C_2v_{32}u_2^t + C_3v_{33}u_3^t + Q_y(t). \tag{26}$$

The C_j *are arbitrary constants to be determined by boundary conditions.* The u_j are the roots of the characteristic equation of system (23). The v_{ij} are the elements of the characteristic vector corresponding to the characteristic root u_j. $Q_x(t)$, $Q_p(t)$, and $Q_y(t)$ are the particular solutions for x_{t-1}, p_t, and y_t respectively.

The characteristic equation of the system (23) is

$$u^3 + (1 + B)u^2 - (1 + 2B)u - (1 - B) = 0, \tag{27}$$

$$B = \frac{1}{\gamma\sigma}.$$

The u_j are

$$u_1 = 1, \quad u_2 = \frac{-(2 + B) + \left[8B + B^2\right]^{1/2}}{2}, \quad u_3 = \frac{-(2 + B) - \left[8B + B^2\right]^{1/2}}{2}. \tag{28}$$

u_2 always lies between -1 and +1. To see this, subtract 8B from the term inside the square brackets to obtain an expression exactly equal to -1. Since 8B is positive, u_2 must be larger than -1. Now, instead of subtracting 8B, add 16 to the term inside the square brackets. The resulting expression is greater than u_2, and it is exactly equal to 1. Since B is always positive, it is clear that u_3 is always less than -1.

The elements of the characteristic vectors corresponding to the u_j are

$$v_{1j} = 1, \qquad v_{2j} = \frac{1 + u_j}{2}, \qquad v_{3j} = \frac{\sigma(u_j - 1)(1 + u_j)}{2}, \tag{29}$$

where the vectors have been normalized by setting the v_{1j} equal to unity. Note that all the v_{ij} except v_{31} are nonzero.

Sargent and Wallace (1975) show that with nominal interest rate pegging the price level is indeterminate in their model even when explosive solutions for the endogenous variables are ruled out. 1/ We obtain a closely related result in our Taylor model.

First we rule out explosive solutions for x_{t-1}, p_t, and y_t. We assume that if $\bar{r}_t$ is growing, it is growing at a less than exponential rate. If a variable is growing at a less than exponential rate, we say that it is growing at a nonexplosive rate. Since the exogenous variable in our system is growing at nonexplosive rate, it seems reasonable to rule out explosive solutions for the endogenous variables. It can be shown

---

1/ Explosive solutions are often referred to as speculative bubbles. Ruling out explosive solutions is the same thing as making a no-speculative-bubbles assumption.

that if $\bar{r}_t$ is growing at a nonexplosive rate, it is possible to construct nonexplosive particular solutions for x_{t-1}, p_t, and y_t. 1/ Even if the particular solutions are nonexplosive, the solutions for x_{t-1}, p_t, and y_t will be explosive unless $C_3 = 0$ because $|u_3| > 1$. We assume that $C_3 = 0$.

Next we show that there are a multiplicity of nonexplosive solutions for x_{t-1}, p_t, and y_t. Given that $v_{31} = C_3 = 0$, the solutions for the endogenous variables at time zero take the form

$$x_{-1} = C_1 v_{11} + C_2 v_{12} + Q_x(0), \tag{30}$$

$$p_0 = C_1 v_{21} + C_2 v_{22} + Q_p(0), \tag{31}$$

$$y_0 = C_2 v_{32} + Q_y(0). \tag{32}$$

The v_{ij} and $Q_x(0)$, $Q_p(0)$, and $Q_y(0)$ are known functions of the parameters and the exogenous variable. x_{-1} is known at time zero. However, p_0, y_0, C_1, and C_2 are unknown. Since there are three equations but four unknowns, there is a multiplicity of solutions for p_0, y_0, C_1, and C_2. Since there is a multiplicity of solutions for C_1 and C_2, there is a multiplicity of nonexplosive solutions for x_{t-1}, p_t, and y_t.

The existence of a multiplicity of nonexplosive solutions for the endogenous variables with nominal interest rate pegging does not imply that all paths are feasible. Each value of C_2 implies unique values for p_0, y_0, and C_1 and, therefore, unique paths for x_{t-1}, p_t, and y_t.

In his discussion of multiple solutions McCallum (1986) distinguishes between "indeterminacy" and "nonuniqueness." For McCallum a model exhibits "indeterminacy" if it has the type of multiple solution in which real variables are uniquely determined and nominal variables are underdetermined, and a model exhibits "nonuniqueness" if it has any other type of multiple solution. In the models of Sargent and Wallace (1975), Canzoneri, Henderson, and Rogoff (1983), and McCallum (1986) with nominal interest rate pegging the behavior of the real variables is the same along every nonexplosive path for the economy. While the path of the price level is indeterminate, each given price path is consistent with only one path of the money supply. Sargent-Wallace-type models with nominal interest rate pegging exhibit what McCallum calls indeterminacy because

---

1/ For a description of how to obtain particular solutions see Blanchard and Kahn (1980).

real variables depend on nominal variables only through deviations between the actual and expected values of these variables. Our Taylor model with nominal interest rate pegging does not exhibit what McCallum calls indeterminacy despite the fact that it embodies the assumption of rational expectations. Different feasible price paths are always associated with different paths of output. This result arises because in the Taylor model the current values of real variables depend on lagged values of nominal variables.

Although McCallum's distinction may be useful in some contexts, to us the more natural distinction is between multiple solutions that are explosive and those that are not explosive. Sargent and Wallace (1975), Taylor (1977), and Blanchard and Kahn (1980) all have used the nonexplosiveness condition to rule out some rational expectations solutions. We believe that it is useful to say that a model exhibits indeterminacy if it has a multiplicity of nonexplosive solutions whether or not real variables are uniquely determined. Of course, our Taylor model does exhibit what we call indeterminacy.

VI. Nominal Interest Rate Pegging as the Limit of Nominal Interest Rate Smoothing Under Rational Expectations

In previous sections we have assumed that the authorities peg the nominal interest rate by simply standing ready to exchange securities for money at the chosen rate. According to Canzoneri, Henderson, and Rogoff (1983), Dotsey and King (1983), and McCallum (1986), when the authorities peg the nominal interest rate in this way, they are pursuing an incompletely specified policy. In this final section we introduce money supply rules from a class with interest rate smoothing into our Taylor model and trace out the implications of viewing nominal interest rate pegging as the limit of nominal interest rate smoothing.

The modified version of our Taylor model is given by

$$y_t = -\sigma[r_t - ({}_tp_{t+1} - p_t)], \tag{33}$$

$$x_t = \frac{1}{2}(x_{t-1} + {}_tx_{t+1}) + \gamma(y_t + {}_ty_{t+1}), \tag{34}$$

$$p_t = \frac{1}{2}(x_{t-1} + x_t), \tag{35}$$

$$m_t - p_t = \eta y_t - \lambda r_t, \tag{36}$$

$$m_t = m_0 + \mu t + \beta(r_t - \bar{r}_t). \tag{37}$$

Equations (33) through (35) can be obtained by rewriting equations (20) through (22), replacing $\bar{r}_t$ with r_t. Equation (36) is the same standard formulation of the money demand function that appeared as equation (18) in our wealth-effects model in Section IV. Equation (37) is a representative of a class of money supply rules with nominal interest rate smoothing. If the market nominal interest rate, r_t, equals the target nominal interest rate, $\bar{r}_t$, then (the logarithm of) the money supply follows a linear trend. 1/ When r_t exceeds $\bar{r}_t$, the money supply increases by a fixed proportion, β, of the gap. We call the parameter m_0 the initial money supply, the parameter μ the trend growth rate of the money supply, and the parameter β the interest rate smoothing parameter. 2/

As before, the model can be expressed as a series of three first-order difference equations:

$$\begin{bmatrix} x_t \\ {}_tp_{t+1} \\ {}_ty_{t+1} \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & [1 + 1/(\beta+\lambda)] & [1/\sigma + \eta/(\beta+\lambda)] \\ -2/\gamma & [2 - 1/(\beta+\lambda)]/\gamma & [-\gamma - 1/\sigma - \eta/(\beta+\lambda)]/\gamma \end{bmatrix} \begin{bmatrix} x_{t-1} \\ p_t \\ y_t \end{bmatrix} \tag{38}$$

1/ Of course, since the logarithm of the money supply grows arithmetically, the money supply itself grows exponentially.

2/ By inspection of equations (36) and (37) we can see that in a steady state with constant y and r, both m and p must grow at rate μ. Furthermore, if the long-run values of y and r are 0 and $\bar{r}$, respectively, then equation (33) requires that $\mu = \bar{r}$. This relationship was imposed in the subsequent simulations, but it is not necessary for a solution to equations (38). If $\mu \neq \bar{r}$, r will gradually approach μ.

$$+ \begin{bmatrix} 0 & 0 & 0 \\ -1/(\beta+\lambda) & -\mu/(\beta+\lambda) & \beta/(\beta+\lambda) \\ 1/\gamma(\beta+\lambda) & \mu/\gamma(\beta+\lambda) & -\beta/\gamma(\beta+\lambda) \end{bmatrix} \begin{bmatrix} m_0 \\ t \\ \bar{r}_t \end{bmatrix} .$$

In order to obtain these equations, first substitute the right-hand side of equation (37) for m_t in equation (36). Next solve the modified version of equation (36) for r_t and substitute the resulting expression for r_t into equation (33). Then follow the steps described in the last section using the modified version of equation (33) and equations (34) and (35). As before, x_{t-1} is a sticky variable, and p_t and y_t are jump variables.

The form of the solutions for x_{t-1}, p_t, and y_t for the model of system (38) is given by equations (24), (25), and (26). Of course, the characteristic roots, characteristic vectors, and particular solutions for the model of equations (38) will be different from those for the model of equations (23).

The characteristic equation of the system (38) is a cubic, and we have been unable to obtain analytical expressions for the roots. However, in Appendix B we show that this equation has three distinct real roots, one inside the unit circle and two outside the unit circle. Let u_1 represent the root inside the unit circle, and let u_2 and u_3 represent the roots outside the unit circle.

As before, we begin by ruling out explosive solutions for x_{t-1}, p_t, and y_t. Consider the exogenous variables of the system of equations (38). m_0 is a constant. t is growing at a nonexplosive rate. We assume that if $\bar{r}_t$ is growing, it is growing at a nonexplosive rate. Since the exogenous variables in our system are growing at nonexplosive rates, it seems reasonable to rule out explosive solutions for the endogenous variables. It can be shown that if the exogenous variables are growing at nonexplosive rates, it is possible to construct nonexplosive particular solutions for x_{t-1}, p_t, and y_t. Even if the particular solutions are nonexplosive, the solutions for x_{t-1}, p_t, and y_t will be explosive unless $C_2 = C_3 = 0$ because $|u_2|$, $|u_3| > 1$. We assume that $C_2 = C_3 = 0$.

Next we show that there is a unique nonexplosive solution for x_{t-1}, p_t, and y_t. Given that $C_2 = C_3 = 0$, the solutions for the endogenous variables at time zero take the form

$$x_{-1} = C_1 v_{11} + Q_x(0), \tag{39}$$

$$p_0 = C_1 v_{21} + Q_p(0), \tag{40}$$

$$y_0 = C_1 v_{31} + Q_y(0). \tag{41}$$

The v_{ij} and $Q_x(0)$, $Q_p(0)$, and $Q_y(0)$ are known functions of the parameters and the exogenous variables. x_{-1} is known at time zero. However, p_0, y_0, and C_1 are unknown. Since there are three equations and three unknowns, there is a unique solution for p_0, y_0, and C_1. Since there is a unique solution for C_1, there is a unique nonexplosive solution for x_{t-1}, p_t, and y_t. 1/

We wanted to learn more about the properties of system (38). For example, we wanted to know whether increasing the value of β reduces the gap between the market interest rate and the interest rate target at every point in time after an increase in the interest rate target. Since we have been unable to obtain analytical expressions for the roots of the characteristic equation, we performed some numerical simulations of the system using the Fair-Taylor extended path algorithm. 2/ The results of these simulations are shown in Charts 9 through 12. The parameter values used in the simulations are $\gamma = 0.2$, $\sigma = 1$, $\eta = 1$, and $\lambda = 4$. These values are in the range of long-run elasticities in Taylor's multicountry model.

Chart 9 shows the paths of the price level, inflation rate, and interest rate in response to simultaneous, one-time increases of 0.01 in the target interest rate, $\bar{r}$, and the trend rate of money growth, μ. 3/ The interest rate converges quickly to its new target value even in the

---

1/ Reinhart (1988) studies an alternative money supply rule with nominal interest rate smoothing using a continuous-time model with staggered contracts of the type suggested by Calvo (1983). We have analyzed a continuous-time version of our proposed money supply rule using Reinhart's model and find that the price level remains determinate as the smoothing parameter approaches infinity.

2/ We used the Fair-Taylor extended path algorithm because it was convenient to do so. Since our Taylor model is linear, we could have used the solution formulae in Blanchard and Kahn (1980).

3/ The impact period was normalized at t=0, so that the shocked money supply equals the baseline money supply in the impact period in the absence of interest rate smoothing (β=0).

Chart 9
Rational Expectations, Staggered Contract Model

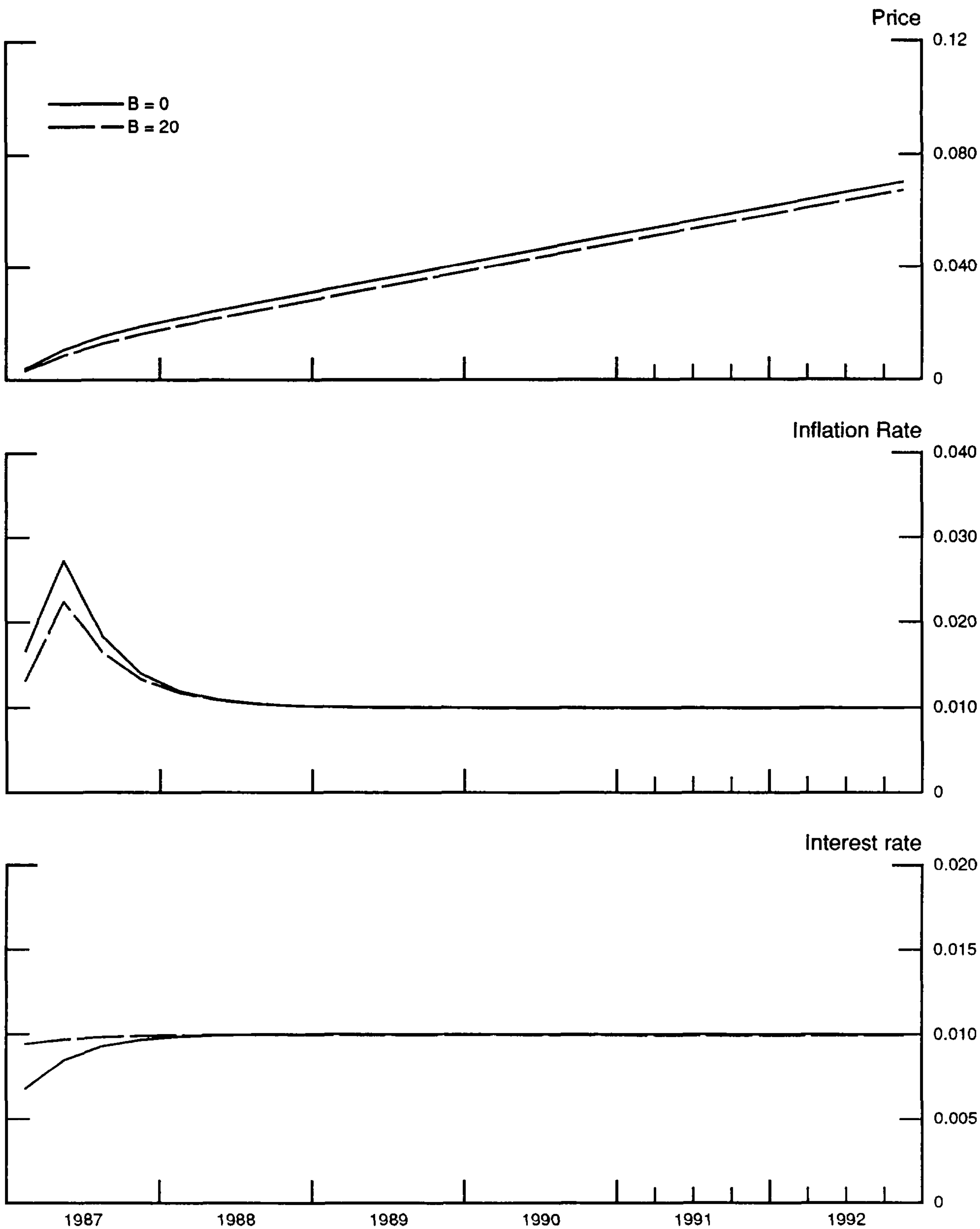


Chart 10
R.E. Model with Delay in Money Growth

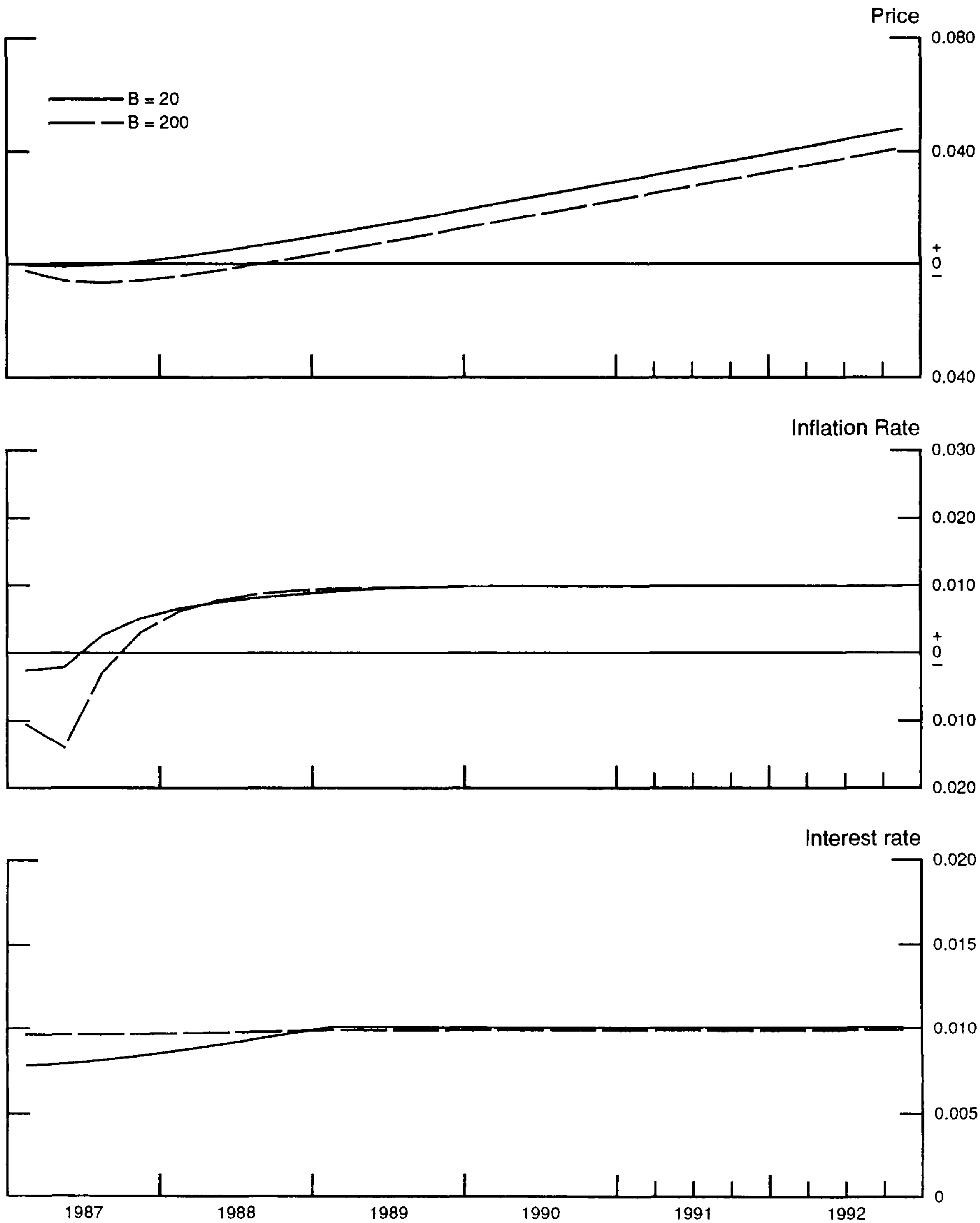


Chart 11

R.E. Model with Initial Money Supply Contraction

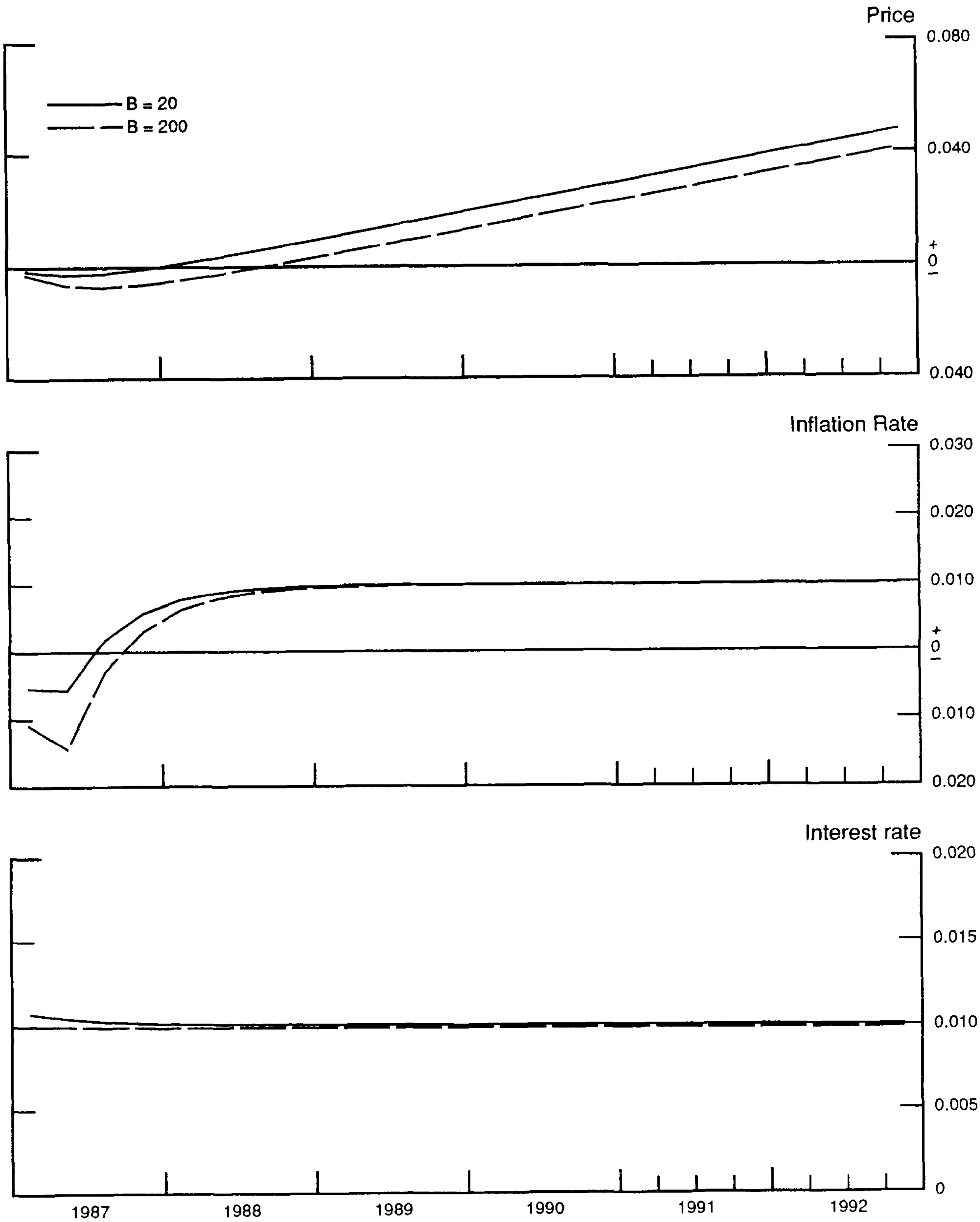
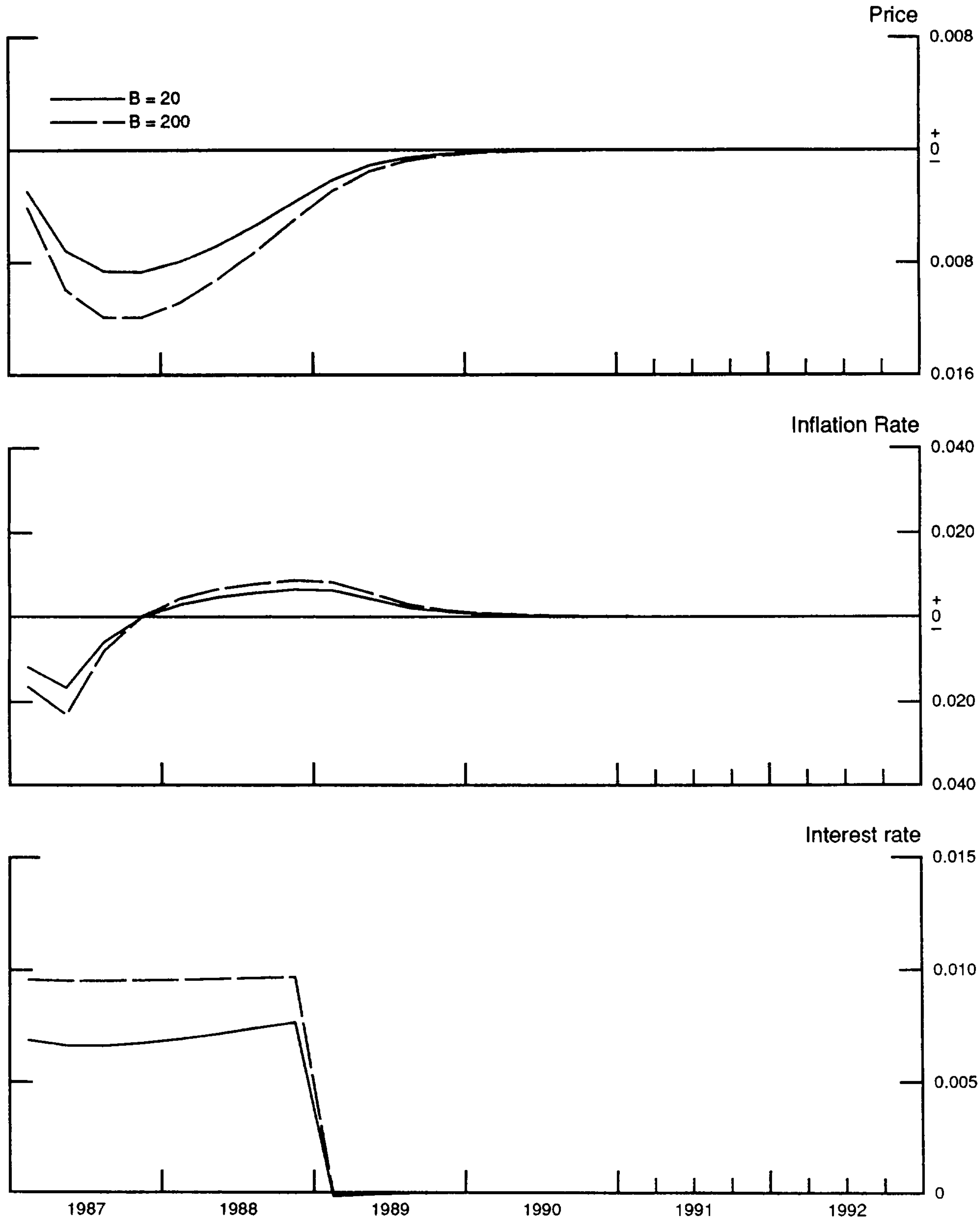


Chart 12

R.E. Model with Temporary Interest Rate Increase



absence of any interest smoothing (β=0). However, a higher value of β does indeed keep the interest rate closer to its new target value immediately after the shock. 1/ Note that the inflation rate jumps above its long-run value on impact. It makes sense that inflation should exceed the money growth rate for a while because real balances must be lower in the new steady state with a higher nominal interest rate. Output also rises on impact because of the drop in the expected real interest rate.

We were intrigued by the expansionary effect of a simultaneous increase in the nominal interest rate target and the trend money growth rate. Therefore, we conducted three experiments to determine whether an increase in the nominal interest rate target would always have an expansionary effect in the Taylor model with our class of money rules. In each experiment the increase in the interest rate target was implemented using a different money supply rule in our class. In the first experiment, we raised the interest rate target but held the trend money growth rate at its previous value for 8 quarters before raising it to the level of the new interest rate target. In the second, we raised the trend money growth rate simultaneously with the interest rate target but dropped the initial money supply by an amount equal to 8 quarters' growth at the new rate. In the third, we raised the interest rate target for 8 quarters and lowered it back to its initial value thereafter. In this experiment we did not change the trend money growth rate.

The results of these experiments are plotted in Charts 10 through 12. In every case, the higher interest rate target is deflationary in the initial periods. As the charts show, a much higher value of the interest smoothing parameter was necessary to keep the market interest rate within 5 percent of its target.

Our analysis of a Taylor model with money supply rules from a class with nominal interest rate smoothing yields two main conclusions. First, for each member of the class, nominal and real variables remain determinate as the interest rate smoothing parameter increases without limit. That is, nominal interest rate pegging viewed as the limit of nominal interest rate smoothing is consistent with price level determinacy. Second, different members of the class are consistent with

---

1/ The larger the value of β, the longer the forecast horizon and, therefore, the more computation time needed to achieve convergence of the extended path algorithm. This fact is not important if the model being analyzed is linear because the solution formulae in Blanchard and Kahn (1980) can be used instead of the extended path algorithm. However, it might be quite important if the model being analyzed is nonlinear because the extended path algorithm must be employed.

hitting the same nominal interest rate target but imply different paths for other nominal and real variables. That is, the values of the contract wage, the price level, and output depend not only on the nominal interest rate target but also on the other parameters of the money supply rule. The second conclusion confirms the main result of the previous section: the same nominal interest rate is consistent with a multiplicity of nonexplosive paths for the contract wage, the price level, and output. The second conclusion also has an important implication for the design of simulation experiments that call for nominal interest rate pegging in rational expectations models. The designers need to specify all the parameters of the money supply rule, not just the path of the target nominal interest rate.

This appendix demonstrates that a two-country version of the basic model is dynamically unstable. The model is given by the following equations:

$$y_t = -\sigma(\bar{r} - \pi_t) + \delta(e_t + \overset{*}{p}_t - p_t), \tag{42}$$

$$\overset{*}{y}_t = -\sigma(\overset{*}{\bar{r}} - \overset{*}{\pi}_t) - \delta(e_t + \overset{*}{p}_t - p_t), \tag{43}$$

$$p_{t+1} - p_t = \alpha' y_t + \pi_t, \tag{44}$$

$$\overset{*}{p}_{t+1} - \overset{*}{p}_t = \alpha' \overset{*}{y}_t + \overset{*}{\pi}_t, \tag{45}$$

$$\pi_{t+1} - \pi_t = \rho(p_{t+1} - p_t - \pi_t), \tag{46}$$

$$\overset{*}{\pi}_{t+1} - \overset{*}{\pi}_t = \rho(\overset{*}{p}_{t+1} - \overset{*}{p}_t - \overset{*}{\pi}_t), \tag{47}$$

$$\varepsilon_t + \overset{*}{\pi}_t - \pi_t = -\phi(e_t + \overset{*}{p}_t - p_t), \tag{48}$$

$$\varepsilon_t = \bar{r} - \overset{*}{\bar{r}} . \tag{49}$$

Variables with asterisks are foreign-country variables. y and $\overset{*}{y}$ are (the logarithms of) outputs, $\bar{r}$ and $\overset{*}{\bar{r}}$ are nominal interest rates, π and $\overset{*}{\pi}$ are expected rates of inflation, p and $\overset{*}{p}$ are (the logarithms of) the home currency price of home output and the foreign currency price of foreign output, e is (the logarithm of) the exchange rate defined as the home currency price of foreign currency, and ε is the expected rate of depreciation of the home currency. It is assumed that (the logarithms of) the natural rates of output are zero. The bars over the nominal interest rates indicate that they are being pegged. For simplicity it is assumed that the parameters for the two countries are the same.

According to equations (42) and (43), outputs in the two countries are equal to demands for those outputs. Demands depend negatively on expected real interest rates. Demand for the home good depends positively on the relative price of the foreign good or real exchange rate, and demand for the foreign good depends negatively on this variable.

According to equations (44) and (45), in each country the inflation rate depends positively on the gap between actual and natural output and on the expected rate of inflation. According to equations (46) and (47), expectations about inflation are formed adaptively; in each country the expected rate of inflation is increased by a fraction of the gap between actual inflation and expected inflation. According to equation (48), expectations about the rate of depreciation of the home currency in real terms are formed regressively; if the real exchange rate is above its long-run equilibrium value of zero, the expected rate of depreciation is negative. The combination of the assumption that expectations about inflation are formed adaptively and the assumption that expectations about the rate of real depreciation are formed regressively may seem somewhat anomalous. However, these assumptions are combined in at least one of the econometric models. According to equation (49), assets denominated in home and foreign currency are perfect substitutes; the expected rate of depreciation of the home currency must equal the interest differential in favor of the home currency.

Substituting (49) into (48), using the resulting equation to eliminate the real exchange rate from (42) and (43), and substituting the modified versions of (42) and (43) into (44) and (45) yields a pair of first-order difference equations in π and $\overset{*}{\pi}$:

$$\begin{bmatrix} \pi_{t+1} \\ \overset{*}{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + D + E & -E \\ -E & 1 + D + E \end{bmatrix} \begin{bmatrix} \pi_t \\ \overset{*}{\pi}_t \end{bmatrix} + \begin{bmatrix} -(D + E) & E \\ E & -(D + E) \end{bmatrix} \begin{bmatrix} \bar{r} \\ \bar{\overset{*}{r}} \end{bmatrix} \tag{50}$$

$$D = \rho\alpha'\sigma, \qquad E = \frac{\rho\alpha'\delta}{\phi}.$$

From inspection of equations (50) it is clear that when $\delta=0$, the equations are independent and identical to equation (46) in the text. The characteristic roots of the system (50) are 1+D and 1+D+E. Both roots lie outside the unit circle for all positive values of the parameters. Therefore, the system is unstable.

In this appendix we use Sturm's method to prove that the characteristic polynomial of the Taylor model with a money supply rule that incorporates nominal interest rate smoothing has three distinct real roots, one inside the unit circle and two outside the unit circle. The characteristic polynomial is

$$u^3 + (1 + F - G)u^2 - (1 + 2F + 2G)u - (1 - F + G), \tag{51}$$

$$F = \frac{1}{\gamma\sigma} + \left[\frac{1}{\lambda + \beta}\right]\frac{\eta}{\gamma}, \qquad G = \frac{1}{\lambda + \beta}.$$

Sturm's method is a way of determining the number of distinct real roots of a polynomial that lie between any two real numbers. 1/ In order to apply Sturm's method it is necessary to construct the Sturm functions $f_0(u)$ through $f_n(u)$, where n is the order of the polynomial. $f_0(u)$ is simply the polynomial itself. $f_1(u)$ is the first derivative of $f_0(u)$. To obtain $f_i(u)$, $i \geq 2$, divide $f_{i-1}(u)$ into $f_{i-2}(u)$ using polynomial long division; $f_i(u)$ is defined to be the remainder multiplied by -1.

The Sturm functions for the polynomial (51) are

$$f_0(u) = u^3 + (1 + F - G)u^2 - (1 + 2F + 2G)u - (1 - F + G), \tag{52}$$

$$f_1(u) = 3u^2 + 2(1 + F - G)u - (1 + 2F + 2G), \tag{53}$$

$$f_2(u) = \frac{1}{9}\{[8 + 16F + 8G + 2(F - G)^2]u + (8 - 2F^2 + 2G^2 - 12F + 8G)\}, \tag{54}$$

$$f_3(u) = \frac{144}{H}[F^2 + 8F + 4FG + 10F^2G + 6FG^2 + FG(F - G)^2], \tag{55}$$

$$H = [2(1 + F - G)^2 + 6(1 + 2F + 2G)]^2.$$

1/ For a discussion of Sturm's method see Baumol (1959).

Table 1

	$u = -\infty$	$u = -1$	$u = +1$	$u = +\infty$
$f_0(u)$	-	+	-	+
$f_1(u)$	+	-	?	+
$f_2(u)$	-	?	?	+
$f_3(u)$	+	+	+	+
$V(u)$	3	2	1	0

The number of distinct real roots between u_j and u_k, where $u_j < u_k$ can be determined in three steps. First, evaluate $f_i(u)$, $i = 0,\ldots,n$, for u_j and u_k. Second, determine $V(u)$, the number of times $f_i(u)$ changes sign as i runs from 0 to n, for u_j and u_k. Third, subtract $V(u_k)$ from $V(u_j)$.

The signs of the $f_i(u)$ and the values of $V(u)$ for the polynomial (1) for u equal to $-\infty$, -1, $+1$, and $+\infty$ are shown in Table 1. $V(-1) = 2$ independently of the sign of $f_2(-1)$. $f_1(1)$ and $f_2(1)$ are

$$f_1(1) = 4(1 - G), \tag{56}$$

$$f_2(1) = 16 + 4G^2 + 8G + 4F(1 - G). \tag{57}$$

If 1-G is positive, then both $f_1(1)$ and $f_2(1)$ are positive, so $V(1) = 1$. If 1-G is negative, then $f_1(1)$ is negative and $f_2(1)$ may be either positive or negative, but in either case $V(1) = 1$.

The polynomial (1) has three distinct real roots since $V(-\infty) - V(\infty) = 3$. One of these roots lies inside the unit circle since $V(-1) - V(1) = 1$. Therefore, the other two roots must lie outside the unit circle.

References

Baumol, William J., Economic Dynamics, (New York: The Macmillan Company, 1959).

Benavie, Arthur, and Richard T. Froyen, "Price Level Determinacy and Nominal Interest Rate Pegging " (University of North Carolina at Chapel Hill, unpublished paper, February 1988.).

Blanchard, Olivier J., and Charles M. Kahn, "The Solution of Linear Difference Models Under Rational Expectations," Econometrica, 48 (July 1980), 1305-11.

Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," Journal of Monetary Economics, 12 (September 1983), 383-98.

Canzoneri, Matthew B., Dale W. Henderson, and Kenneth S. Rogoff, "The Information Content of the Interest Rate and Optimal Monetary Policy," Quarterly Journal of Economics, 98 (November 1983), 545-66.

Dotsey, Michael, and Robert King, "Monetary Instruments and Policy Rules in a Rational Expectations Environment," Journal of Monetary Economics, 12 (September 1983), 357-82.

Hooper, Peter, Karen H. Johnson, Donald L. Kohn, David E. Lindsey, Richard D. Porter, and Ralph W. Tryon, eds., Monetary Aggregates and Financial Sector Behavior in Interdependent Economies, Board of Governors of the Federal Reserve System (forthcoming).

McCallum, Bennett T., "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations," Journal of Monetary Economics, 8 (March 1981), 319-29.

_______, "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine, "Journal of Monetary Economics, 17 (January 1986), 135-60.

Reinhart, Vincent, "Interest Rate Smoothing and Staggered Contracting," Board of Governors of the Federal Reserve System, (unpublished manuscript, June 1988.).

Sargent, Thomas J., and Neil Wallace, "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," Journal of Political Economy, 83 (April 1975), 241-254.

Sims, Christopher A., letter to Thomas J. Sargent, (June 1988)

Taylor, John B., "Conditions for Unique Solutions in Stochastic Macroeconomic Models with Rational Expectations," Econometrica, 45 (September 1977), 1377-1385.

________, "Aggregate Dynamics and Staggered Contracts," Journal of Political Economy, 88 (February 1980), 1-24.