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Policy Reform, Shadow Prices, and Market Prices

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Abstract

How should possible policy reforms and projects be assessed when prices give misleading signals? Revenues and costs at market prices then give distorted measures of social gains and losses and our appraisal should use social opportunity costs, or correctly defined, shadow prices. We show how shadow prices may be integrated into an analysis of tax and price reform, demonstrate the critical dependence of these prices on government policy, and analyze their relations with market prices. A conceptual framework for applied analysis is provided plus a detailed theoretical account of policy in a model with some fixed prices, rationing, and taxation.

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Contents

Summary

This paper develops a framework for applying the theory of public policy to an economy with distortions, particularly those resulting from price rigidities and quantity rationing. Where market prices do not reflect social opportunity costs, the assessment of policy must incorporate shadow prices that reflect the full repercussions of having less of a given good (this involves a model) and a criterion for evaluating these repercussions.

Defined in this way, shadow prices become a useful analytical tool not only for project evaluation but also for the theory of optimal policy and of policy reform, and for structuring thinking and data gathering on applied problems. Since shadow prices cannot be defined without specifying government behavior (tax and commercial policy, redistribution policies, quotas, rationing, and so on), policy choices should be appraised using the same criteria as for projects. Policy choice and shadow prices are then determined simultaneously.

The paper applies this conceptual framework to show how changes in public finance policy can be assessed to reflect full general equilibrium repercussions. It also provides appropriate methods for calculating such "macroeconomic" social opportunity costs as shadow exchange rates, shadow discount rates, shadow wage rates, premia on savings, and so on. Finally, the framework has been applied to the analysis of two important applied problems: privatization and the reform of rationing systems. The paper emphasizes the need for systematic economic analysis and shows how structured argument can help define social opportunity costs, provide rules for their calculation, integrate cost-benefit analysis and the theory of policy, and, finally, guide our thinking and judgment on immediate policy problems.

I. Introduction

Economists are often asked to give policy advice in situations where, it is claimed, prices give distorted or misleading signals. And many of them are fond of suggesting that governments should leave more to the market so that private agents can respond effectively to price incentives. While these positions are not necessarily contradictory their juxtaposition should lead us to ask some questions. What do we mean by misleading signals? Can we define satisfactorily an index of scarcity or value which is not misleading? How do we identify the social opportunity cost or shadow price of a commodity? How do these shadow prices compare with market prices and under what circumstances will they coincide? Should governments leave decisions to the market when prices give impaired signals? How can the government improve price, tax, or regulatory incentives? How can concern with income distribution be integrated systematically with measures to improve efficiency and the supply side? Generally, how can shadow prices and market prices be combined in the understanding and implementation of the reform of government policies?

The purpose of this paper is to provide a framework for the analysis of these questions and, where possible, some results which may suggest answers. These results will not generally be cast in terms of formulas or programs which can be speedily calibrated and implemented. On the other hand, they should provide the practitioner with a structured and productive way of thinking about the problems of how market distortions should influence proposals for reform and they should help identify the empirical questions that should be checked and the judgments that need to be made before deciding on a particular line of policy.

We shall begin the analysis (in Section II) by defining shadow prices and then set out a formal exposition of their properties in a model of an economy where markets do not work perfectly. Such models are generally more difficult to analyze than those with well-functioning competitive markets, but we shall try to make the discussion accessible and avoid unnecessary technicalities. We have to recognize, however, that the logic of our inquiry requires us to deal with models which do not have the simplicity of perfect competition because it is usually the departures from that framework that lead to the distortions under study. Further, the literature on shadow prices has been somewhat bewildering in its different definitions, methods, and models, and we shall attempt to provide a unified analysis within which many of the apparently disparate results and propositions can be understood. We shall then relate the theory and discussion to issues that are central to the Fund's concerns--government revenue and the macroeconomy (Section III) and the encouragement of efficient supply (Section IV). The effects of policy reform on income distribution, also a major issue for the Fund, will be central throughout the discussion in that the analysis systematically takes into account the effects of policy on households in different circumstances. A summary and concluding comments are provided in Section V. A more technical appendix covers price reform in a distorted economy.

It is fairly easy to see that in an economy with irremovable distortions, as, for example, those arising from the need to raise revenue, some market interventions by the government will be necessary and desirable. Thus no government should unconditionally surrender resource allocation to markets. The question then becomes "how should the government intervene?" There are a number of responses that should be ruled out straightaway. For example, it is not useful to suggest that each decision be subjected to a microscopic cost-benefit analysis. Governments simply do not have the resources or information, and the upshot would be cumbersome and constricting. But neither should one try to suggest that for a distorted economy no systematic economic guidelines for making reforms exist or that such a broad set of shadow prices can be justified that only crude judgments are left. We shall argue that there are general notions that are worth grasping, general rules that are worth applying, and a genuine intellectual framework that can structure inquiry into applied policy problems.

In the remainder of this section we shall review briefly the lessons of the standard theory of prices and policy for competitive economies and then identify some imperfections that may make a mess of, or distort, the role of prices as indicators of social opportunity costs or values. This will help to explain our choice of model for Section II and indicate which types of distortion are included and which are not.

As we have indicated, the purpose of our discussion makes imperfections central and these imperfections bring with them some complexity, at least relative to the perfectly competitive framework. Most of the difficulty comes from attempting to predict the consequences of policy changes and not from the normative evaluation of those consequences. Clearly, normative analysis cannot proceed in advance of the positive, but it can show how to assemble the results from positive analysis in a way that allows the key features for policy appraisal to be seen. Further, it allows investigation of the circumstances under which it may be acceptable to use simple rules that cut through some of the complexity.

It is convenient to approach the discussion of the social value of commodities in distorted economies by defining an undistorted system where shadow prices would coincide with market prices. It is a well-known result of classical welfare economies that under standard conditions (particularly the absence of externalities), a perfectly competitive equilibrium is Pareto efficient. In order for market prices to reflect the social value of commodities, the distribution of income should also be optimum according to the ethical judgments underlying the shadow price system. This additional requirement is often swept under the carpet, but this attitude is really hard to defend. Indeed, it is arguable that an essential role of government is to ensure that the standard of living of poorer groups receives some protection. Many would go further than this; redistribution is indeed a commonly articulated concern of governments. Incorporating this objective typically requires sustained attention to the distributional consequences of government policies. To achieve an optimum distribution of income without distorting the price system one needs to be able to redistribute resources and raise government revenue by lump-sum taxes and transfers (i.e., taxes and transfers that cannot be altered through the behavior of agents). Insofar as appropriate fiscal instruments of this kind are not available, shadow prices are liable to deviate substantially from market prices even in perfectly competitive economies.

Further distortions arise when the economy does not have the necessary features for Pareto efficiency described above and the following kinds of additional problems may occur:

1. Indirect and income taxes introduce a divergence between the prices paid by buyers and sellers. Thus, with indirect taxes; households equate their marginal rates of substitution to relative consumer prices and firms equate their marginal rates of transformation to relative producer prices; the resulting inequality between marginal rates of substitution in consumption and in production implies inefficiency. Similarly, taxes on labor income imply that the net and gross of tax wages are unequal, with the former relevant for the household and the latter relevant for the firm. Thus, the amount of goods required to compensate a household for an extra unit of work will be less than the marginal product of labor.

2. There may be externalities that are uncorrected by taxes, social customs, trading between those affected, or regulation.

3. Some goods may be directly allocated by government authorities and the quantities going to each agent may be such that their marginal social value in different uses is not equalized, i.e., the government may not allocate resources in a socially efficient manner. This example may include investment licensing and input or output quotas for enterprises.

4. Prices may be controlled so that they cannot adjust to excess demands or supplies. In this case some form of rationing, formal or informal, will occur, again causing marginal social values or opportunity costs to be unequal across uses (unless rations are optimally constructed).

5. There may be tariffs or trade controls which prevent marginal rates of transformation between goods produced domestically being equated to marginal rates of transformation through world trade, thus generating inefficiency.

6. Rapid inflation may prevent the adjustment of relative prices. If there is extensive indexing to one particular index, adjustments in relative prices may be difficult to achieve.

7. Some markets may not exist or be subject to significant transaction costs, such as forward and insurance markets, which may be substantially limited in scope. Trade is then prevented from playing its role in equalizing marginal rates of transformation and substitution.

8. Markets may be oligopolistic or monopolistic so that prices are not equal to marginal costs.

9. Problems of imperfect information and foresight may be involved. Thus, individuals may be able to trade goods but be unaware of their quality or their value. This generates different types of equilibrium (and creates problems for the existence of equilibrium) from the standard competitive variety and once again trade cannot be relied upon to equalize the relevant marginal rates.

The first seven of these sources of imperfection are either included in whole or in part in the models that follow or can be easily added to the models. The last two--involving monopoly, oligopoly, and imperfect information--are more problematic. The models to be analyzed here include price control, quotas, rationing, and taxes in a fairly general way and it is relatively straightforward to add externalities to them. They are essentially models where all agents are price takers and well informed but may be subject to a variety of quantity constraints. Our exposition will purposely avoid excessive detail and technicalities. The interested reader can find a more rigorous and complete treatment in Drèze and Stern (1987).

II. The Basic Theory

We want to derive a set of shadow prices reflecting the social value of commodities, in order to guide policy reform and the choice of public sector projects. To this end, the shadow price of a commodity is defined as its social opportunity cost, i.e., the net loss (gain) associated with having one unit less (more) of it. The losses and gains involved have to be assessed in terms of a well-defined criterion or objective, which is referred to as "social welfare." The evaluation of social welfare is naturally based (at least partly) on an assessment of the well-being of individual households, which has to be supplemented by interpersonal comparisons of well-being. The latter is embodied in what is called "welfare weights." This is not the place to debate which weights should be used--they should be discussed responsibly and intelligently but are ultimately value judgments depending, inter alia, on one's views of inequality and poverty.

It is difficult, however, to dispense with reliance on the notion of social welfare. Most practical examples of policy reform or public projects make some people better off and some worse off and we have to take a decision which trades off these gains and losses. Implicitly or explicitly we shall be using weights. In our judgment, attempts to produce cost-benefit tests for policy appraisal that avoid interpersonal comparisons have not got very far. For example, hypothetical transfers of the Hicks-Kaldor variety that could yield Pareto improvements are not relevant when such transfers do not take place; and in the event that such transfers do take place systematically, it is straightforward to incorporate them in the present framework. Also, assertions that "a dollar is a dollar is a dollar,"--that it is worth the same wherever it goes, so that money values across households can be simply added up--do not avoid value judgments but rather replace them with a specific one that says that all welfare weights (in terms of social marginal utilities of income) are equal. This is not only an ethically unappealing view of income distribution but risks serious logical inconsistencies (see, for example, Roberts (1980)).

In this paper we shall make extensive use of the Bergson-Samuelson social welfare function. This is a rather general approach and says simply that social welfare depends on household welfare. We shall not be constrained to any particular form of the social welfare function but shall present results in relation to a general function of the Bergson-Samuelson variety.

When the social opportunity cost or shadow price of a good is defined in terms of the marginal effect on social welfare of the availability of an extra unit, it leads directly to a "cost-benefit test," i.e., projects which make positive profits at shadow prices should be accepted because they increase welfare. Indeed, it should be clear (and see below) that no other definition of a shadow price can have this property. Thus, in this paper there will be one single definition of a shadow price--it is the increase in social welfare resulting from the availability of an extra unit of the specified commodity. Strictly speaking, one also has (generally) to state from which agent the extra unit comes or to which agent the extra unit accrues. For specificity, and because the interest here is primarily in public sector decisions, this agent shall be understood to be the public sector (see Drèze and Stern (1987) for further discussion).

In subsection II.1 we provide a formal definition of shadow prices and illustrate the basic ideas with simple examples. The model we shall use is set out and discussed in subsection II.2, and in subsection II.3 we describe the relationship between optimum policies, shadow prices, and policy reform. Guidelines for reforming policy and the rules satisfied by shadow prices are analyzed in subsection II.4.

1. Definitions and simple examples

Our definition of shadow prices requires us to calculate the effect of an extra unit of public supplies (the latter are represented by the vector z) on social welfare. The public supplies do not impinge directly on social welfare but only affect it through the variables that influence household welfare and demands: prices, wages, rations, and so on. 1/ We shall think of these variables as being of two types: control variables ($\underline{s}$) and parameters or predetermined variables ($\underline{\omega}$). The former are determined within the system, subject to the scarcity constraints that usages of goods cannot exceed availability and to any other constraints that may be relevant. The variables ω are fixed as parameters of the system. We shall, however, examine the consequences of shifts in these parameters.

We shall refer to the person or agency responsible for the evaluation of public decisions as the "planner." This does not imply that we think of the government as a well-tuned, harmonized whole, acting coherently and consistently in pursuit of a well-defined set of objectives, captured by a single social welfare function. The planner will usually be operating in a particular agency and may have to treat the responses of other government agencies (e.g., taxes set by the finance ministry or quotas specified by a trade ministry) as outside its control. This causes no problems for the analysis since such items can be included among the list of predetermined variables ($\underline{\omega}$). We shall assume, or rather recommend, that the planner chooses those variables which are in its control with respect to the same social welfare or objective function that is being used to evaluate changes in public supplies. This is simply a counsel of consistency for the planner. The planner's range of control may, however, be so limited that it essentially has no choice at all. Crudely, there may be exactly the same number of constraints as there are control variables. This does not affect the analysis and is retained as an important special case throughout. We shall refer to this case as one where the model is "fully determined."

More generally, it is important to note here that our list of "control variables" includes what one may wish to call the "endogenous variables" of the system, i.e., the variables whose value is determined by the constraints of the problem. To put it crudely again, when there are $\underline{I}$ constraints, a vector of $\underline{K}$ control variables can often be interpreted as consisting of $\underline{I}$ "endogenous variables," and ($\underline{K}$-$\underline{I}$) variables under the effective and direct control of the planner. It is, however, neither necessary nor useful to give special attention to endogenous

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1/ The assumption that public supplies do not directly affect household welfare may sound restrictive (especially in the case of public goods), but in fact it is not. Household welfare depends on consumption and not on supplies as such, and the link between consumption and supplies will be provided by the scarcity constraints of the model.

variables separately from other control variables. Indeed, the identification of endogenous variables is often arbitrary from a purely formal point of view. For instance, in a model where the only constraints are the scarcity constraints and the control variables consist of prices and indirect taxes, it does not matter whether we choose to describe prices as being determined by the market-clearing process and taxes by the planner, or the other way round. From the formal point of view, it is enough to note that both prices and taxes should be treated as "control variables," given that the scarcity constraints enter the model explicitly. This point is important for a correct interpretation of the models analyzed in this paper (for further elaboration, see Drèze and Stern (1987)).

We are now in a position to define shadow prices. We write social welfare $\underline{V}$ ($\underline{s}$; $\underline{\omega}$) as a function of $\underline{s}$ and $\underline{\omega}$, and think of the "planner's problem" as that of choosing $\underline{s}$ to solve

$$\text{Maximize } \underline{V}\ (\underline{s};\ \underline{\omega}) \text{ subject to } \underline{E}\ (\underline{s};\ \underline{\omega}) - \underline{z} = 0, \tag{2.1}$$

where $\underline{E}$ ($\underline{s}$; $\underline{\omega}$) is the vector of net demands arising from the private sector. The constraints expressed by (2.1) are the scarcity constraints, which say that available supplies must match demands. There may in practice be constraints in addition to the scarcity constraints. These constraints may, in part, be captured by considering some variables as predetermined, and to this extent they are included in our formulation; but where they cannot be modeled in this way they should be included as constraints additional to the scarcity constraints. To keep things simple, we shall avoid including them in the analysis that follows (see Drèze and Stern (1987) for further discussion).

Given $\underline{z}$ and $\underline{\omega}$, the solution to (2.1) gives a level of social welfare which we write as $\underline{V}^*$ ($\underline{z}$; $\underline{\omega}$)--the maximum level of social welfare associated with the production plan $\underline{z}$ (given $\underline{\omega}$). The shadow price $\underline{\nu}_i$ of the i^{th} good is defined by

$$\nu_i \equiv \frac{\partial V^*}{\partial z_i}. \tag{2.2}$$

Thus, ν_i is precisely the increase in social welfare associated with a unit marginal increase in z_i; or the social opportunity cost in terms of social welfare of a marginal unit reduction in z_i. Alternatively, relative shadow prices represent marginal rates of substitution in the social utility function V^* (.) defined on the space of commodities.

A project is a small change in public supplies dz (private projects are discussed in Section IV). We can see from equation (2.2) that the value of a project dz at shadow prices νdz (i.e., $\sum_i \nu_i\, dz_i$) is equal to dV*, so a project increases social welfare if, and only if, it makes a profit at shadow prices. Thus, the cost-benefit test "accept the project if it is profitable at shadow prices" correctly identifies all those projects that are desirable in the sense of increasing social welfare. 1/ Our definition of shadow prices is, of course, designed with precisely this property in view, and it can be seen that any set of relative prices that fails to coincide with the relative shadow prices defined by equation (2.2) cannot possess the same property. Notice, on the other hand, that it is only relative shadow prices that matter, and we can always scale the vector ν up or down by a positive multiple without changing anything of substance. Finally, it must be stressed that the shadow prices discussed in this paper are appropriate only for the evaluation of small projects: for large changes, differential techniques based exclusively on first-order terms are no longer adequate. 2/

The change in welfare from an extra unit of public supplies comes about through the resultant changes in the variables which affect household welfare. These changes will, of course, be determined by the structure of the economy, including, in particular, the policy instruments available to the government. The link between shadow prices and the tools at the government's disposal may be illustrated using two simple examples. Consider an economy with a single consumer that supplies labor to produce corn. The only firm is owned entirely by the government. In the first example the government can control the economy fully in the sense that it can allocate labor and corn in both production and consumption subject only to the availability of each good. The control variables are the quantities of corn and labor in both production and consumption. In the second example the government has to work through the price system and can only obtain labor by hiring at each wage the amount the worker-consumer wishes to supply at that wage. Further, it cannot levy lump-sum taxes or make lump-sum transfers. It is clear that in the second case the powers of the government are more limited than in the first, and the overall levels of welfare it can achieve will be lower. It is also true that the marginal rates of substitution in terms of social welfare, or relative shadow prices, will be different in the two cases.

The position is illustrated in Figures 1 and 2. In the first case (Figure 1) the government has complete control over the economy and chooses the first-best X. The relative shadow prices are given by the loss of labor (i.e., extra labor required of the worker) that would hold social welfare constant if an extra unit of corn became available. In other words we can ask, "what shift of the origin would leave social

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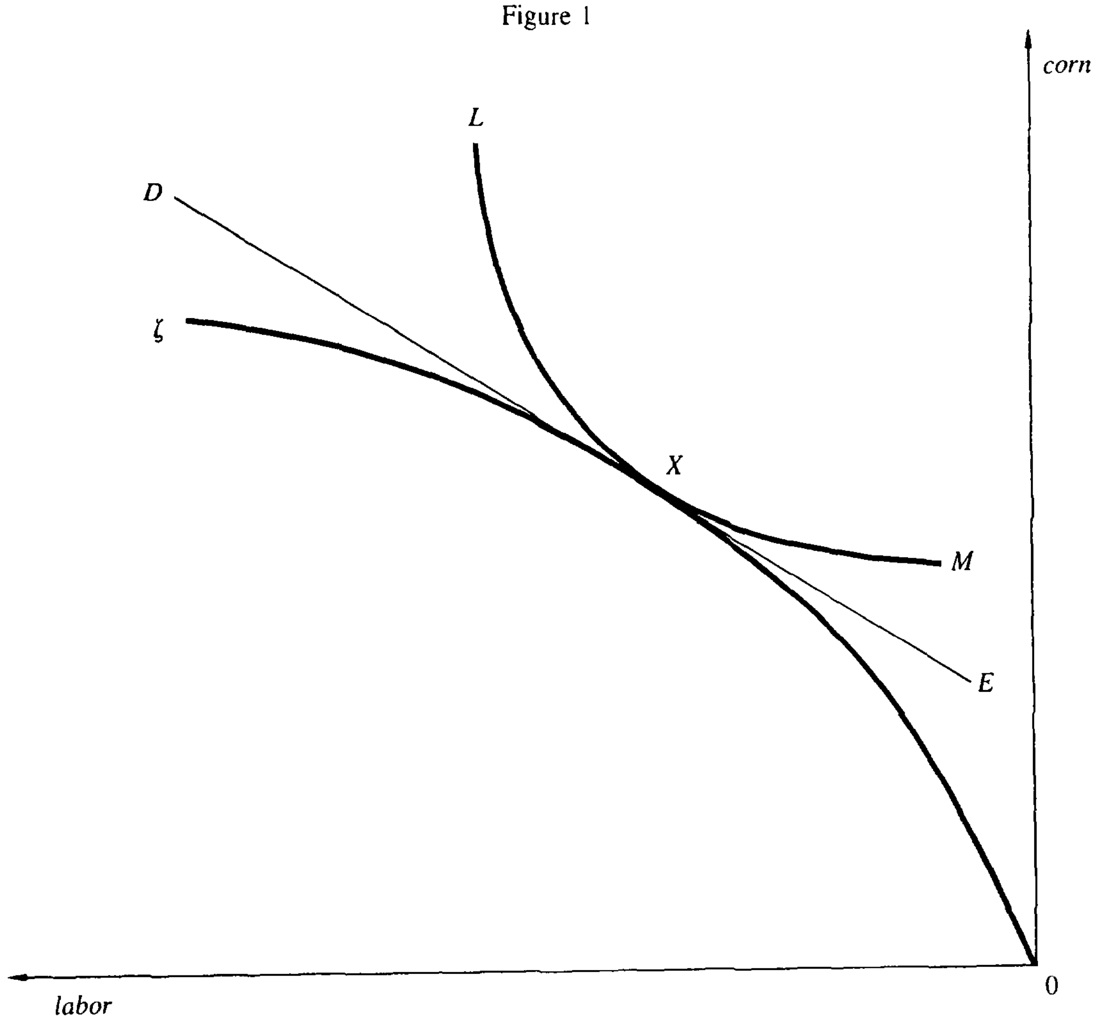
1/ Strictly speaking, if a project makes exactly zero profits at shadow prices, one should check second-order terms to decide whether it is socially desirable or not.

2/ See, for example, Hammond (1983) for a discussion of the problems raised by the analysis of large projects.

Shadow Prices Equal To Consumer Prices

Figure 1



Shadow Prices Equal To Producer Prices

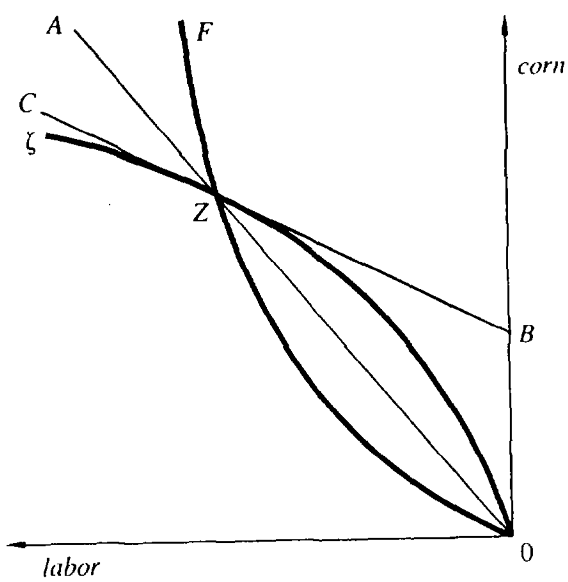


Figure 2(a)

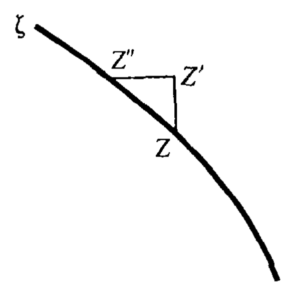


Figure 2(b)

welfare unchanged?" The changes envisaged in response to the increased availability of a unit of corn must be achievable making the best possible use of the tools at the government's disposal and observing the scarcity constraints. If an extra unit of corn becomes available, then utility could be held constant by a marginal increase in labor from X along the line DE toward D. The gradient of the line DE gives the marginal rate of substitution in social welfare and hence relative shadow prices.

In the second case the government can choose only points along the offer curve OF in Figure 2(a). Since utility increases as we move up the offer curve (higher real wages), the optimum will be at Z where the offer curve cuts the production frontier OG. Relative consumer prices (the real wage to the consumer) are represented by the gradient of the line OZA and the marginal rate of transformation in production (the marginal product of labor here) is given by the gradient of BC. Thus, marginal rates of substitution for the consumer and marginal rates of transformation for the producer are not equal. The former is equal to the consumer price ratio, and we may think of the latter as the producer price ratio. The difference is a tax, or if it is negative, a subsidy. Here it is a subsidy that is financed by the profits of the firm (at the given producer price ratio) of OB in terms of corn. If we now have an extra unit of corn, what will be the increase in labor required to maintain utility constant given the tools at the government's disposal, i.e., when we are restricted to movements along the offer curve? It is clear that constant utility must involve a return to Z since any other point on the offer curve involves higher or lower utility. Thus the consumer price ratio must remain unchanged after the adjustment, and the accommodation to the changes must be on the production side. This means that the change in labor in response to an extra unit of corn must be one that takes us back to the production frontier so that production can adjust to take us back to Z. The increase in consumptior by one unit is represented in Figure 2(b) by the move to Z' and the increase in labor to regain the frontier by the move Z' to Z". Production can then be adjusted to bring us back to Z with no utility change. The marginal rate of substitution in social welfare is now seen to be given by the gradient of BC or relative <u>producer</u> prices.

It is straightforward to generalize these results to higher dimensions and several consumers, and some related results are discussed below. These simple examples, however, illustrate the important point that shadow prices depend on the tools at the government's disposal: we have seen that there will be cases where shadow price are equal to consumer prices and others where they are equal to producer prices. We are also going to find examples where they are averages of the two, or obey more complicated formulas.

2. The model

We shall now set out the basic model in which the properties of shadow prices are investigated. This model is, in effect, a mixture of fix-price and flex-price models (in the language of Hicks) where prices in some markets are free to adjust but in others are not. Households and firms are price takers but may be subject to quantity rationing in certain markets. This formulation will allow us to capture at once the important phenomena of price responses as well as quantity rationing. In particular, the familiar themes of the literature on temporary equilibrium with quantity rationing, such as Keynesian unemployment, are firmly within our framework.

We have H consumers (indexed by $h=1, \ldots, H$), one private firm, and one public firm. It is straightforward to extend the model to allow for an arbitrary number of private and joint public-private firms (see Drèze and Stern (1987)), but we are trying to keep the model as simple as possible while retaining its essential features. In a similar spirit we shall ignore externalities. There are I goods (indexed by $i=1, \ldots, I$), distinguished, if necessary, by time and contingency. The private firm trades at prices given by the vector p, and its net supply vector is y. We follow the usual convention that a negative supply by a firm represents a demand for an input and a negative demand by a household represents a supply by that household. The firm may be subject to constraints on its choice of inputs or outputs. These are represented by an I vector of lower bounds $\bar{y}_-$ and a similar vector of upper bounds $\bar{y}_+$; the 2I vector $(\bar{y}_-, \bar{y}_+)$ is called $\bar{y}$. The relevant binding constraint for good i, if any, is referred to as $\bar{y}_i$ (the upper and lower bounds cannot both be binding unless they are equal). The firm's profit-maximizing supply $y(p, \bar{y})$ is then a function of the prices and quotas which it faces. Its pretax profits $\pi(p, \bar{y})$ are also a function of p and $\bar{y}$ and are distributed to households according to the shares θ^h. We denote $1 - \sum_h \theta^h$ by ζ, which represents the government's share in the private firm including any profits tax.

Households face prices $q \equiv p+t$ where t is the vector of indirect taxes (linear factor taxes are included within the components of t but there are no taxes on intermediate transactions). The h^{th} household receives lump-sum income $m^h \equiv r^h + \theta^h\pi$, where r^h is a lump-sum transfer from the government (which may, of course, be zero or negative). Each household chooses a utility-maximizing consumption vector x^h subject to its budget constraint and quantity constraints that are represented by vectors of quotas $\bar{x}^h_-$, $\bar{x}^h_+$ as with the firm. We denote the 2I vector $(\bar{x}^h_-, \bar{x}^h_+)$ by $\bar{x}^h$, and the relevant binding constraints by $\bar{x}^h_i$. The demands x^h of household h are then functions of q, $\bar{x}^h$, and m^h. They are written as $x^h(q, \bar{x}^h, m^h)$, but the identities $q \equiv p+t$ and $m^h \equiv r^h + \theta^h\pi$ should be kept in mind throughout.

The vector of net government supplies is z as before, and the vector of net imports is denoted by n. There is a given endowment F of foreign exchange, and world prices are fixed at p^w. The scarcity constraints are then

$$\sum_h x^h - y - n - z = 0 \tag{2.3}$$

$$p^w n - F = 0. \tag{2.4}$$

We have chosen to write equation (2.4) separately from equation (2.3) to bring out the role of foreign trade more sharply. A more unified and elegant treatment, dispensing with equation (2.4), can be obtained by treating (somewhat artificially) foreign exchange as a separate commodity and net imports as the net supplies of a separate firm (see Drèze and Stern (1987) for details). However, for greater transparency we shall treat foreign exchange and net imports explicitly.

The vector s of control variables consists of K variables drawn from the following list:

$$(p_i),\ (t_i),\ (r^h),\ (\bar{x}_i^h),\ (\bar{y}_i),\ (\theta^h),\ (n_i). \tag{2.5}$$

The variables included in s are under the planner's control, but they must be chosen subject to the constraints (2.3) and (2.4). The more variables s contains, the larger the degree of freedom for the planner. Crudely speaking, if there are just (I + 1) control variables then the planner has no real choice since equations (2.3) and (2.4) represent (I + 1) constraints. This is the "fully determined" case to which we have already referred. If there are more than (I + 1) variables in the list s, then the planner has a genuine choice, which it takes so as to maximize social welfare. The variables in equation (2.5) which are not in s are predetermined, or are parameters, and are denoted as before by ω.

Finally, the social welfare function takes the Bergson-Samuelson form $W(u^1, u^2, \ldots, u^H)$ where u^h is the level of utility of the h^{th} household. This can also be written as:

$$V(s;\ \omega) \equiv W(\ldots.,\ v^h(q,\ \bar{x}^h,\ \bar{m}^h),\ \ldots), \tag{2.6}$$

where v^h is the "indirect utility function" relating individual welfare to prices, rations, and income.

In the remainder of this paper (except Section V), we shall assume some familiarity with the elementary properties of the functions $x^h(q, \bar{x}^h, m^h)$, $y(p, \bar{y})$, $v^h(y, \bar{x}^h, m^h)$, and $\pi(p, \bar{y})$, i.e., with the basic theory of competitive demand and supply in the presence of quantity constraints or rationing. We also assume knowledge of simple optimization techniques using Lagrange multipliers.

3. Optimum policies, shadow prices, and policy reform

Nearly all our results are derived from the first-order conditions for the maximization of social welfare subject to the scarcity constraints (2.3) and (2.4). We shall examine, in particular, the Lagrange multipliers associated with the scarcity constraints. It should be clear that this is a natural way to proceed since these Lagrange multipliers will tell us precisely how much social welfare goes up if we have a little extra public supply, and this corresponds exactly to our definition of shadow prices. To see this, let us write the Lagrangean L of the social welfare maximization problem as

$$L(s;\omega, z, F, \lambda, \mu) \equiv V(s;\omega) - \lambda[E(s;\omega) - z] - \mu[p^w n - F], \qquad (2.7a)$$

where μ is a Lagrange multiplier on the foreign exchange constraint, λ a vector of Lagrange multipliers on the scarcity constraints, and

$$E(s;\omega) \equiv \sum_h x^h - y - n. \qquad (2.7b)$$

Now let $V^*(z,F;\omega)$ denote, as before, the maximum social welfare associated with given values of z, F, and ω. A standard result of optimization theory (known as the "envelope theorem") states that the gradient of the "maximum value function" V* is identical to the gradient of the Lagrangean L. Therefore we have

$$\lambda \equiv \frac{\partial L}{\partial z} \equiv \frac{\partial V^*}{\partial z} \equiv \nu, \qquad (2.8a)$$

which confirms our earlier assertion. 1/ Similarly we can at once derive

$$\frac{\partial V^*}{\partial F} \equiv \frac{\partial L}{\partial F} \equiv \mu \tag{2.8b}$$

so that μ can be interpreted as the marginal social value of foreign exchange: it measures the increase in social welfare resulting from the availability of an extra unit of foreign exchange. Notice that the shift in V^* following a change in z works through the effect of a change in s on household utilities. However, the shift in L is at constant s (see the arguments of L in equation (2.7a)), and we can ignore any effects operating on L through s precisely because s has been chosen optimally--this is the "envelope property."

Our writing of the constraints (2.3) and (2.4) separately to replace the constraint in (2.1) means that the evaluation of a project (dz, dF) is now given by whether or not

$$\sum_{i=1}^{I} \nu_i dz_i + \mu\, dF > 0, \tag{2.9}$$

where we accept the project if equation (2.9) holds. Note that the foreign exchange component dF will be zero unless the project is tied to a direct gift of foreign exchange: the (direct and indirect) effects of a project on foreign exchange will be captured by the first term in the left-hand side of equation (2.9) and should not be counted again through the component dF.

As we have already pointed out, it is only the relative shadow prices in the vector $(\nu_1, \nu_2, \ldots, \nu_I, \mu)$ that matter, since we can always rescale this vector by rescaling the social welfare function V(.). For example, doubling V would double all shadow prices but would not change anything of substance in the analysis. In other words, we

1/ Notice that there are, generally speaking, many equivalent ways of expressing a set of constraints in an optimization problem. In this context, rewriting the constraints in an equivalent manner would change the Lagrange multipliers but not the shadow prices. Thus, it is important to recognize that our definition of shadow prices comes first, and we have shown that they happen to be equal to Lagrange multipliers if the constraints on the problem are written in an appropriate way.

are free to choose a unit of account for our shadow prices. One common practice in the literature has been to do the accounting in terms of foreign exchange so that μ is set equal to one (see Little and Mirrlees (1974)).

The control variables are chosen to maximize social welfare subject to the scarcity constraints. The first-order conditions for maximization are:

$$\frac{\partial L}{\partial s_k} \equiv \frac{\partial V}{\partial s_k} - \nu \frac{\partial E}{\partial s_k} = 0 \tag{2.10}$$

if the k^{th} control variable is any element in the list (2.5) excluding (n_i). The first-order conditions for (n_i) are discussed below. The condition (2.10) has a very natural interpretation: it tells us that at an optimum the direct effect on social welfare of a marginal adjustment in any control variable should be equal to the marginal social cost of the extra net demands generated. Thus, the shadow prices are picking up the welfare effects of the full general equilibrium adjustments in the system. In this sense they are sufficient statistics for the general equilibrium responses: they summarize what we need to know about those consequences for the purpose of policy evaluation.

The first-order conditions (2.10), together with those for net imports (n_i) where relevant, give us a set of K first-order conditions, where K is the number of control variables. These K conditions, together with the (I + 1) scarcity constraints (2.3) and (2.4), determine the values of the K control variables s_k at the optimum and the (I + 1) shadow prices. Thus, we can see that to speak of "rules determining shadow prices" and "rules determining optimum policies" is to describe the same set, or a subset of the same set, of conditions but from a different viewpoint. Moreover, we retain the "fully determined" case, with K = (I + 1), as a special case. Here, the scarcity constraints uniquely determine the controls, and the condition (2.10) represents a set of equations that we must solve for the shadow prices given these values of the controls.

As we have already emphasized, when K is greater than (I + 1), one could think of a particular subset of control variables K - (I + 1) in number as the variables directly under the control of the planner, and the remaining (I + 1) as determined endogenously from the general equilibrium system. This may occasionally be helpful in interpretation and to fix ideas but it is somewhat artificial from the analytical point of view because one could, in principle, think of any particular (K - (I + 1)) subset of the controls (s_k) as being directly controlled by the planner. Unless otherwise stated, therefore, we shall simply speak of the (s_k) as being K controls chosen subject to (I + 1) constraints.

We can now discuss the value of a shift in one of the parameters ω_k, i.e., a "reform." Suppose, for example, that we can contemplate a marginal shift in some tax or grant ω_k that was previously outside the control of the planner. To evaluate such a possibility, we want to know its effect on social welfare, i.e., $\frac{\partial V^*}{\partial \omega_k}$. Applying the "envelope theorem" once again we find that

$$\frac{\partial V^*}{\partial \omega_k} \equiv \frac{\partial L}{\partial \omega_k} \tag{2.11}$$

or, using equation (2.7)

$$\frac{\partial V^*}{\partial \omega_k} \equiv \frac{\partial V}{\partial \omega_k} - \nu \frac{\partial E}{\partial \omega_k} \tag{2.12}$$

when ω_k is one of the variables in equation (2.5) (excluding (n_i)). This tells us that we evaluate whether or not the change is welfare improving by first taking the direct effect $(\frac{\partial V}{\partial \omega_k})$ and then comparing it with the cost at shadow prices of the extra demands $(\frac{\partial E}{\partial \omega_k})$ generated by the change. Again, the partial derivatives involved are calculated for constant s and the shadow prices capture for us the relevant general equilibrium consequences of the change.

An example may help clarification. Suppose we are interested in a marginal reform of the old-age pension. The direct benefit of an increased pension is the social value of an extra unit of income to the pensioners. In deciding whether the reform is worthwhile, this direct benefit is compared with the cost at shadow prices of meeting the extra demands for goods by the old-age pensioners arising from the extra income. Of course, if ω_k became a control variable we would go on adjusting it to the point where its marginal contribution to welfare was zero as embodied in the first-order conditions (2.10).

The result (2.12) is of great generality, and it will be used repeatedly in the remainder of this paper. It is convenient, as well as natural, to refer to the gradient of L (or, equivalently, of V*) with respect to any parameter ω_k as the "marginal social value" (MSV) of that parameter. By analogy, and for presentational purposes, we shall also speak of the gradient of L with respect to a control variable s_k as its MSV. The first-order conditions (2.10) thus simply require that the MSV of a control variable should be zero.

4. Rules for policies and shadow prices

We shall now examine the particular form of the rules for optimum policies and shadow prices which arise from the model with objective function as in equation (2.6) and constraints as in equations (2.3) and (2.4). For this purpose it may be helpful to rewrite the Lagrangean (2.7a) explicitly as

$$L(.) \equiv W(\ldots, v^h(p+t, \bar{x}^h, r^h + \theta^h \pi), \ldots) - \nu \left[\sum_h x^h(p+t, \bar{x}^h, r^h + \theta^h \pi) - y(p, \bar{y}) - n - z\right] - \mu[p^w n - F]. \tag{2.13}$$

We can now consider, for variables in the list (2.5), the first-order conditions (2.10) as well as the MSVs (2.12).

a. Transfers to households, r^h

$$\frac{\partial L}{\partial r^h} = \frac{\partial W}{\partial v^h}\frac{\partial v^h}{\partial r^h} - \nu \frac{\partial x^h}{\partial r^h}. \tag{2.13a}$$

This tells us that the MSV of a transfer to household h is the direct effect (the private marginal utility of income of household h, $\frac{\partial v^h}{\partial r^h}$, multiplied by the rate of change of social welfare with respect to the utility of household h, $\frac{\partial W^h}{\partial v^h}$) less the cost at shadow prices of meeting the extra demands by the household ($\frac{\partial x^h}{\partial r^h}$) arising from the extra income. We shall refer to $\frac{\partial W^h}{\partial v^h} \cdot \frac{\partial v^h}{\partial r^h}$ as the social marginal utility of income or <u>welfare weight</u> of household h and denote it by β^h; we also write

$$b^h \equiv \beta^h - \nu \frac{\partial x^h}{\partial r^h} \tag{2.13b}$$

so that b^h stands for the MSV of a transfer to household h. If b^h is greater than 0 we would want to increase the transfer if an upward adjustment were possible, and if b^h is less than 0, we would want to decrease it. If the transfer to household h is a control variable, then the first-order conditions (2.10) require that b^h should be equal to zero.

b. Net imports, n^i

$$\frac{\partial L}{\partial n_i} = \nu_i - \mu p_i^w. \tag{2.14}$$

Recalling our earlier discussion of control variables and "endogenous" variables, it can be seen that the variable n_i will be a control variable if it is either set by the planner as an optimum quota on net trade or the level of net imports for that commodity is determined endogenously within the system (as when trade in the commodity is unrestricted--although possibly subject to tariff--and net imports meet the gap between domestic demands and supplies). From the formal point of view there is no difference between these two situations. Where n_i is a control variable, the first-order conditions (2.10) imply

$$\nu_i = \mu p_i^w. \tag{2.15}$$

This says that the shadow price of good i is equal to its world price multiplied by the MSV of foreign exchange. Thus, relative shadow prices within the relevant set of goods are equal to relative world prices.

We have, therefore, derived a standard rule for relative shadow prices of traded goods in cost-benefit analysis--they are equal to relative world prices. The model we have constructed allows us to see clearly the conditions under which this result holds. The crucial feature is that public production of any of the commodities under consideration can be seen as displacing an equivalent amount of net imports, and moreover, the general equilibrium repercussions involved are mediated exclusively by the balance of payments constraint. Formally, this can be seen from the fact that in equation (2.13) the vector n enters the Lagrangean in exactly the same capacity as the vector z, except for its presence in the balance of payments constraint.

That this is the crucial condition should be intuitively clear from the economics of the problem. An extra demand for an imported good will generate a foreign exchange cost and thus create a cost given by its world price multiplied by the MSV of foreign exchange. If there is <u>no other</u> direct effect in the system then all the relevant general equilibrium repercussions are captured in the MSV of foreign exchange and we have found the shadow price of the good. If, however, there is a direct effect of increasing net imports somewhere else in the system then the world price (multiplied by μ) will no longer be equal to the shadow price.

One example of the latter complication would arise with an exported good for which world demand was less than perfectly elastic. One might think it sufficient in this case simply to replace the world price by the marginal revenue in the shadow price formula (2.15). This is correct if the domestic price of the good can be separately controlled. If it cannot (because, say, the indirect tax on the good is predetermined and cannot be influenced by the planner), then the calculation of the shadow price should take into account the effects on private sector supplies and profits of the fall in the world price as a result of extra public production.

A second example would arise if there were separate foreign exchange budgets for different sectors which were set outside the control of the planner. If a sector is identified by its group of commodities, then within a sector relative shadow prices of traded goods would be equal to relative world prices, but across sectors this would not apply, since the MSV of foreign exchange across sectors would no longer be equal. This is a further example of the importance for shadow prices of being clear as to just what the planner controls. If the planner also controlled the allocation of foreign exchange across sectors, then the correct rule would be to allocate across sectors so that the MSV of foreign exchange across sectors would be equal, and then relative shadow prices for traded goods would be equal to relative world prices for all goods.

c. <u>Producer rations, $\bar{y}_i$</u>

$$\frac{\partial L}{\partial \bar{y}_i} = \nu \frac{\partial y}{\partial \bar{y}_i} + b \frac{\partial \pi}{\partial \bar{y}_i} \tag{2.16}$$

where

$$b \equiv \sum_h \theta^h b^h \tag{2.17}$$

and b^h is as in equation (2.13b). In equation (2.17), b may be intuitively understood as the MSV of (a transfer to) private profits. The expression (2.16) is most easily interpreted if we think of good i as the output of the firm and of $\bar{y}_i$ as the amount it is required to produce. Then an increase in the required output will require extra inputs, and the overall change in the production vector will be $\frac{\partial y}{\partial \bar{y}_i}$ with value at shadow prices given by the first term on the right-hand side of (2.16). The increase in required output will also affect the profits of the firm and this is captured in the second term on the right-hand side of (2.16)--b is the weighted sum of the MSVs of transfers to households with weights given by the shares of the households in the profits of the firm. Using the properties of the functions y and π, and noting in particular that $\partial y_i/\partial \bar{y}_i \equiv 1$, we can rewrite (2.16) as

$$\frac{\partial L}{\partial \bar{y}_i} = \nu_i - MSC_i + b(p_i - MC_i) \tag{2.18}$$

where

$$MC_i \equiv - \sum_{j \neq i} p_j \frac{\partial y_j}{\partial \bar{y}_i} \tag{2.18a}$$

is the marginal (private) cost of good i (i.e., the value at market prices of the inputs required to produce an extra unit), and

$$MSC_i \equiv - \sum_{j \neq i} \nu_j \frac{\partial y_j}{\partial \bar{y}_i} \tag{2.18b}$$

is its "marginal social cost" (i.e., the value at shadow prices of the same inputs).

If $\bar{y}_i$ is a control variable then the expression in (2.18) must be zero and we have

$$\nu_i = MSC_i - b\ (p_i - MC_i). \tag{2.19}$$

This states that the shadow price of good i is its marginal social cost, corrected by the MSV of the relevant profit effects.

As in the earlier discussion of trade, we can interpret the fact that $\bar{y}_i$ is a control variable in two different ways. First, the private firm may be subjected to a quota which is directly (and optimally) set by the planner. Second, the output of the private firm may be determined endogenously, with the private firm adjusting to demand (this is often called a "Keynesian equilibrium" for the firm). In both cases, the intuition behind the shadow price rule (2.19) is that a change in public production of the relevant good results in a corresponding displacement of private production.

d. Producer prices, p_i

If a small change in the public production of a commodity is accommodated by a change in its market price p_i, complicated adjustments in both consumption and private production will follow. The corresponding shadow price expression will then be quite complicated as well. Roughly speaking, the main result here is that the shadow price of a good whose market clears by price adjustment can be seen as a weighted average of the shadow prices of its complement and substitute commodities, the weights being given by the relevant price elasticities (in addition, the social value of income effects has to be taken into account). Under simplifying assumptions, shortcuts can be obtained, such as more elementary "weighted average rules" involving marginal social costs on the production and consumption sides. The correct expressions can be derived from the first-order conditions for maximization with respect to the producer price p_i. They involve a little more technicality than would be appropriate for this paper and are therefore not pursued here (for details, see Appendix and Drèze and Stern (1987)).

e. Consumer rations, $\bar{x}_i$

The social value of adjusting a consumer ration can be analyzed in an analogous manner to producer rations. The details are not presented here. One can show that if preferences are such that the i^{th} good is (weakly) "separable" from the others (i.e., a change in the consumption of the i^{th} good does not affect marginal rates of substitution among the others), then the first-order condition for an optimum ration of that good reduces to

$$\nu_i = \beta^h \rho_i^h - b^h q_i \tag{2.20}$$

where ρ_i^h is the "marginal willingness to pay" for good i by household h (the marginal utility of good i divided by the marginal utility of income).

The expression (2.20) for the shadow price of a rationed good clearly brings out the two components of the gain in social welfare associated with extra public production of that good. First, the consumer who benefits from the extra ration available enjoys an increase in utility measured (in money terms) by his marginal willingness to pay (ρ_i^h), and the social valuation of this change in private utility is obtained by using the usual "welfare weights" (β^h). Second, a transfer of income occurs, as the same individual is required to pay the consumer price (q_i) for the extra ration; the social evaluation of this second effect relies on the marginal social value of a transfer to the concerned consumer (b^h).

This formulation can be adapted to give us shadow prices for important special cases, e.g., public goods and the hiring of labor in a rationed labor market (see Section III). Public goods, for instance, can be modeled here as a rationed commodity--everyone has to consume the same amount, i.e., that which is made available--with a zero price. Analogously to expression (2.20) we then obtain the shadow price of a public good as $\Sigma \beta^h \rho_i^h$, the aggregate, welfare-weighted, marginal willingness-to-pay.

If the separability assumption is violated, expression (2.20) will involve an extra term that captures the MSV of the resulting substitution effects. Alternatively, this extra term can be seen as reflecting the effect on the "shadow revenue" of the government--a notion explored in detail in Section III.

f. <u>Indirect taxes, t_i</u>

$$\frac{\partial L}{\partial t_i} = - \sum_h \beta^h x_i^h - \nu \frac{\partial x}{\partial q_i} . \tag{2.21}$$

This can be reformulated after decomposing $\frac{\partial x}{\partial q_i}$ into an income and a substitution effect (and using the symmetry of the Slutsky terms) to give

$$\frac{\partial L}{\partial t_i} = - \sum_h b^h x^h + (q - \nu) \frac{\partial \hat{x}_i}{\partial q} \tag{2.22}$$

where $\hat{x}_i^h(q, \bar{x}^h, u^h)$ is the <u>compensated</u> demand of consumer h for good i, and $\hat{x}_i \equiv \Sigma \hat{x}_i^h$. Thus, the j^{th} component of the vector $\frac{\partial \hat{x}_i}{\partial q}$ is the compensated response in aggregate consumption of good i to the price of good j. If t_i is a control, then we have

$$\tau^c \frac{\partial \hat{x}_i}{\partial q} = \sum_h b^h x_i^h \tag{2.23}$$

where $\tau^c \equiv q - \nu$ is a vector of "shadow consumer taxes" equal to the difference between consumer prices and shadow prices. If shadow prices (ν) are equal to producer prices (p), then τ^c reduces to the indirect tax vector t and expression (2.23) precisely amounts to the familiar "many-person Ramsey rule" (see, for example, Diamond (1975) and Stern (1984)).

We see, therefore, that expression (2.23) gives us a remarkably simple generalization of the standard optimum tax rules. The kind of economies where the standard rules are derived happen, in fact, to be economies for which shadow prices are proportional to producer prices (as in Diamond and Mirrlees (1971)). What we have now seen is that the same rules apply in much more general economies if we simply replace producer prices by shadow prices.

Much of the discussion of the structure of indirect taxes and the balance between direct and indirect taxation also carries through on replacing actual taxes by shadow taxes. One can show, for example, that if there is an optimum poll tax, households have identical preferences with linear Engel curves, and labor is separable from goods, then shadow taxes should apply at a uniform proportionate rate (see Drèze and Stern (1987) and Stern (1987)). If shadow prices are not proportional to producer prices, this means that actual indirect taxes should not be uniform.

III. Shadow Prices and the Macroeconomy

We shall now investigate the relation between the theory developed so far and more familiar consideration of public finance and macroeconomic policy. These two themes are treated in subsections III.1 and III.2, respectively.

1. Public finance and the shadow revenue

If the public sector trades at prices p we can calculate its net revenue R_p as the sum of profits in the public sector (pz), the value of the foreign exchange sold (ψF, where ψ is the exchange rate), revenue from indirect taxes (tx), tariff revenue ($(p - \psi p^w)n$), the government's share of private profits ($\zeta\pi$) which includes profits taxes, and lump-sum taxes ($-\sum_h r^h$):

$$R_p \equiv pz + \psi F + tx + (p - \psi p^w)n + \zeta\pi - \sum_h r^h. \tag{3.1}$$

It is important to recognize that this expression for R_p gives us the net revenue for the government in a way that is different from the standard accounting procedures in a number of important respects. First, it takes the revenue and expenditure accounts together. If we were to separate the elements of R_p in expression (3.1) we might think of ($- pz + \sum_h r^h$) as government expenditure and of the remaining terms as revenue, with the net revenue being the difference between the two. However, there is nothing compelling in this particular way of making the distinction since, in fact, many different components of R_p could turn out to be either positive or negative. Second, R_p represents net government revenue expressed in present value terms--expenditures and revenues occurring in the future are taken together with those occurring now and the use of present value prices converts them to a common unit of account. Thus, the government may actually run a deficit this year and this will be covered by surpluses at some stage in the (possibly distant) future. Third, there is no difference here between current and capital accounts for the government. This is an aspect of the inter-temporal nature of the net revenue R_p--purchases and receipts are noted as and when they occur, irrespective of whether they might be labeled "current" or "capital."

Now using the identity

$$py \equiv (1 - \zeta)\,\pi + \zeta\pi \tag{3.2}$$

and the aggregate budget constraint for consumers

$$qx \equiv \sum_h r^h + (1 - \zeta)\pi, \tag{3.3}$$

we easily obtain

$$R_p \equiv p(z + y + n - x) + \psi(F - p^w n). \tag{3.4}$$

This gives us Walras' law for this economy, i.e., the government budget constraint is balanced ($R_p = 0$) if, and only if, the value of the net excess demands at producer prices (including the excess demand for foreign exchange) is zero. This value of net excess demands at producer prices can be zero whether or not markets clear, but if they do clear, then it must be zero. Hence, in this economy the government's budget is balanced as soon as the scarcity constraints are met and we do not have to impose budget balance as a separate constraint.

On the other hand, we do have to ask about the process by which the government budget is balanced because we must specify the control variables, i.e., we must say which of the taxes, transfers, quotas, and so on are under the control of the planner. As we have seen, we get different rules for shadow prices depending on which variables are controlled and which are exogenous. Notice also that we are treating all government activities--projects, transfers, and the like--as being financed out of a common "pot" or general revenue. Where there are separate budget constraints applying to specific government activities, they have to be incorporated explicitly into the analysis (see Section IV).

Note also, however, that there are many alternative ways of defining "government revenue." Indeed, when a transaction takes place between two agents, the prices which they respectively face can be arbitrarily split between the "transaction price" and agent-specific taxes. In the earlier definition of government revenue, producer prices were implicitly taken as "transaction prices," but any other vector of prices could be considered. In particular, if we use shadow prices as transaction prices, we obtain an alternative and--it turns out--extremely useful definition of government revenue which we shall call <u>shadow revenue</u> (denoted by R_ν):

$$R_\nu \equiv \nu z + \mu F + (q-\nu)x + (\nu-\mu p^w)n + \zeta\pi - \sum_h r^h + (\nu-p)y. \tag{3.5}$$

This definition is analogous to expression (3.1), with ν and μ respectively replacing p and ψ, and the natural addition of the extra term $(\nu-p)y$. As in Section II, the components of the vector $q-\nu$ (also denoted by τ^c) are referred to as "shadow consumer taxes," and similarly the components of the vector $\nu-q$ (also denoted by τ^p) are referred to as "shadow producer taxes." Clearly, if $\nu=p$ then $\tau^p = 0$ and $R_\nu = R_p$. Using expressions (3.2) and (3.3) again, we have

$$R_\nu \equiv \nu(z + y + n - x) + \mu(F - p^w n) \tag{3.6}$$

much as in expression (3.4). It follows, in particular, that $R_\nu = 0$ when the scarcity constraints are satisfied. Moreover, from expressions (3.6) and (2.7) or (2.13) we have

$$L \equiv V + R_\nu \tag{3.7}$$

and, therefore, for shifts in parameters ω_k

$$\frac{\partial V^*}{\partial \omega_k} \equiv \frac{\partial L}{\partial \omega_k} \equiv \frac{\partial V}{\partial \omega_k} + \frac{\partial R_\nu}{\partial \omega_k}. \tag{3.8}$$

This last expression provides us with an interesting and important way of evaluating policy reforms. Take again the example of a small increase in the old-age pension which we used to illustrate expression (2.12). We can evaluate this reform by first assessing the direct effect on pensioner households, as evaluated through the social welfare function, and then adding the increase in <u>shadow</u> revenue (or equivalently subtracting the loss in shadow revenue) resulting from the reform.

This simple result shows us that a very common method of costing a policy change, i.e., asking about its effect on the government deficit, is mistaken in a distorted economy. In the United Kingdom, for example, in the last few years measures to reduce unemployment--tax cuts, public expenditure programs, employment subsidies, and the like--have been extensively discussed, and it has been common practice to evaluate them in terms of the implied savings or losses of public revenue. This is, indeed, the spirit of many simplistic applications of the fashionable "cost-effectiveness" criterion. Equation (3.8) shows us that this approach is incorrect. If we are to cost our policies in order to take full account of their general equilibrium impact we should be using shadow revenue and not actual revenue. We should emphasize again that this does not neglect issues of financing since Walras' law tells us that satisfying the scarcity constraints will imply budget balance by whatever methods have been incorporated into the specification of the model. What equation (3.8) provides us with is a method of evaluating policy reforms which, once shadow prices have been calculated, no longer requires full computation of general equilibrium responses--since the latter are fully captured by the immediate impact on (or partial derivative of) shadow revenue.

2. Some macroeconomic considerations

We now turn to some important macroeconomic aspects of shadow price systems: savings, foreign exchange, shadow wages, and shadow discount rates. Again, the reader interested in details is referred to Drèze and Stern (1987).

It is often argued that developing countries should place a special premium on savings, i.e., that for some reason savings are too low and that it is important to provide measures to increase them. This position is not always clearly argued--to increase savings is to increase the welfare of the future generations relative to current ones. The statement that savings are too low implies a judgment on the intertemporal allocation of consumption which may not be easy to make. Moreover, savings rates in developing countries are now commonly around 20 percent, which does not immediately suggest that savings are too low.

It is possible, within our framework, to give precision to the notion of a premium on savings. We can write the expression for b^h (2.13b) as

$$b^h = \beta^h - \sum_{i,\tau} \frac{\nu_{i\tau}}{q_{i\tau}} MPC^h_{i\tau} \tag{3.9}$$

where, $MPC^h_{i\tau}$ is the marginal propensity of household h to spend on the i^{th} good in period τ,

$$MPC^h_{i\tau} \equiv q_{i\tau} \frac{\partial x^h_{i\tau}}{\partial m^h} . \qquad (3.10)$$

We can then see that the social value (b^h) of a transfer to household h is higher the higher is its propensity to spend on goods in periods when the shadow price is low relative to the market price (low $\nu_{i\tau}/q_{i\tau}$). If current period commodities are relatively more valuable than future period commodities, then, other things being equal, there will be a higher value attached to transfers to those with a higher propensity to save.

The model we have described so far contains a shadow price on foreign exchange, μ, and it is tempting to compare it with the exchange rate ψ, i.e., the price of a unit of foreign exchange. The simple comparison, however, contains no information since we can choose any absolute level of μ by rescaling the shadow price vector (ν,μ) without changing anything real. The marginal social value of foreign exchange will, however, depend on the way in which the balance of payments is secured; an example will both illustrate this and provide a possible definition of a premium on foreign exchange which would be of substance.

Suppose that an import quota applies to the first good and that this quota can be set by the planner, or (equivalently for our purposes) it adjusts endogenously to ensure the balance of payments. Then n is a control variable and from expression (2.14) we have

$$\nu_1 - \mu p^w_1 = 0. \qquad (3.11)$$

Suppose further that the producer price for the good is simply the import price (ψp^w_1 in domestic prices) plus a tariff t^f_1 so that

$$p_1 = \psi p^w_1 + t^f_1. \qquad (3.12)$$

Then

$$\frac{\mu}{\psi} = \left(\frac{\psi p^w_1 + t^f_1}{\psi p^w_1}\right) \frac{\nu_1}{p_1} \qquad (3.13)$$

using expressions (3.11) and (3.12). Therefore, if the tariff is positive we have

$$\frac{\mu}{\psi} \text{ is greater than } \frac{\nu_1}{p_1} \qquad (3.14)$$

or the shadow price of foreign exchange as a proportion of the market price is greater than the corresponding ratio for good 1. We may then say that there is a premium on foreign exchange in the sense of expression (3.14) and that this premium can be measured by the term in brackets in (3.13), i.e., the extent to which the producer price of good 1 is above the world price. The analysis may be generalized in a straightforward way if the balance of payments is achieved not solely through the first good but by adjusting the quotas for a fixed bundle of goods--the premium is then given by the ratio between the value at producer prices and the value at world prices of that bundle. This idea is commonly embodied in manuals on cost-benefit analysis (see, for example, Dasgupta, Marglin, and Sen (1972), who use a shadow exchange rate along these lines, or Little and Mirrlees (1974), who work in terms of a standard conversion factor for converting the value of broad groups of commodities from market prices to shadow prices).

The appropriate cost of labor is a further topic that has received great attention in discussions of policies and shadow prices in developing countries. It is often argued, for example, that if market wages are kept above the marginal product of labor elsewhere, then there will be a bias against employment, and techniques of production will be more capital-intensive than they "should" be. The model that we are using embodies fixed prices and rationing and can, therefore, be used to derive an expression for shadow wages in the context just described.

The shadow wage will depend on just how the market for labor (indexed hereafter by ℓ) functions. For a simple but important example we shall think of a model where total labor is fixed and residual labor not employed in the formal sector is absorbed in self-employment. We can think of a peasant farm or family firm owned by a single household. We shall index this firm by g, and it is to be distinguished from the single private firm of the model as previously defined which we shall think of as being a formal sector firm employing labor at a wage p_ℓ. We consider the suppliers of labor to be rationed in the amount of work they can sell to the formal sector (i.e., they would like to work more). Under these assumptions, we can regard employment in firm g, the peasant farm (owned by a single household h) as being determined by an endogenous quota $\bar{y}^g_\ell$. The corresponding first-order condition (just as in expression (2.16)) is then

$$0 = \nu \frac{\partial y^g}{\partial \bar{y}^g_\ell} + b^h \frac{\partial \pi^g}{\partial \bar{y}^g_\ell} \qquad (3.15)$$

where π^g is the profit in the g^{th} firm (which goes entirely to the h^{th} household). We can rewrite the right-hand side of expression (3.15) (as in (2.18)) to give

$$\nu_\ell = MSP^g_\ell - b^h(p_\ell - MP^g_\ell) \tag{3.16}$$

where MP^g_ℓ is the marginal product of labor in firm g $\left(-\sum_{j\neq\ell} p_j \frac{\partial y^g_j}{\partial y^g_\ell}\right)$

and MSP^g_ℓ is its marginal social product $\left(-\sum_{j\neq\ell} \nu_j \frac{\partial y^g_i}{\partial y^g_\ell}\right)$. This is

precisely the shadow wage of Little-Mirrlees (1974, pp. 270-71).

The interpretation of expression (3.16) should be intuitively clear. The social cost of employing labor is the social value of what it would otherwise have produced less the marginal social value of the gains in income for household h. One can also consider different types of alternative activity where, for example, those who are not employed in the formal sector do not work but receive unemployment benefit. Then MSP^g_ℓ is replaced by the value of leisure, or reservation wage, adjusted by the welfare weight, and the income change is now the difference between the market wage and the unemployment benefit. The model can be adapted to deal with several interesting examples of unemployment. Migration equilibria can also be captured through extra constraints of the form, say, $v^R(.) = v^U(.)$, where R and U are indices for social and urban households, respectively.

The last example of a broad macroeconomic issue that we shall discuss here is the shadow discount rate. In order to define this we must make the intertemporal features of the problem more explicit. Assuming the project does not come along with a gift of foreign exchange ($dF = 0$) we can think of its social value, S, as νdz. Indexing now explicitly on time we have

$$S \equiv \nu dz \equiv \sum_\tau \nu_\tau dz_\tau = \sum_i \sum_\tau \nu_{i\tau} dz_{i\tau} \tag{3.17}$$

where $dz_{i\tau}$ is the change in public supplies of good i in period τ, dz_τ is the vector $(dz_{i\tau})$, and similarly for $\nu_{i\tau}$ and ν_τ. When we discuss discounting we focus on the shadow price (or marginal social value) of the numeraire commodity in year τ relative to its shadow price in other years. This, of course, requires the specification of a numeraire relative to which social profitability is measured in each year. Formally we may write

$$\nu_{i\tau} \equiv \bar{\nu}_{i\tau}\, \alpha_\tau \tag{3.18}$$

where $\bar{\nu}_{i\tau}$ is the vector of shadow prices for year τ normalized relative to the numeraire and α_τ is the shadow discount factor. The shadow discount rate, ρ_τ, is then defined as

$$\rho_\tau \equiv \frac{\alpha_\tau - \alpha_{\tau+1}}{\alpha_{\tau+1}} . \tag{3.19}$$

The process of going through expressions (3.17)-(3.19) is essentially unavoidable in defining the social discount rate, since the notion precisely concerns the rate at which the marginal social value of the numeraire is falling over time. If we take commodity i as the numeraire, then the shadow discount rate simply becomes

$$\rho_\tau = \frac{\nu_{i\tau} - \nu_{i,\tau+1}}{\nu_{i,\tau+1}} . \tag{3.20}$$

It is clear from equation (3.20) that the choice of commodity to be numeraire will affect the shadow discount rate unless the relative shadow prices of alternative numeraire commodities are constant over time, i.e., if ρ_τ is the shadow discount rate using i as numeraire, and ρ'_τ is the shadow discount rate when j is used as numeraire, then

$$\rho_\tau \equiv \rho'_\tau \text{ if,and only if, } \frac{\nu_{i\tau}}{\nu_{j\tau}} = \frac{\nu_{i,\tau+1}}{\nu_{j,\tau+1}} . \tag{3.21}$$

We cannot, therefore, answer the question "what should be the shadow discount rate" without being told, or without our choosing, what the numeraire is to be. And the apparent difference between the shadow discount rates proposed in alternative methods of cost-benefit analysis should not mislead us into thinking that the differences are necessarily real--alternative methods may simply involve different units of account.

One particularly easy and transparent choice for the numeraire in each year is foreign exchange. Trading in foreign exchange from one year to the next (i.e., borrowing and lending on world capital markets) can be seen as a form of production activity which we can think of as being undertaken by a public sector firm. If this firm is maximizing its profits at shadow prices, as it should (see Section IV), then its marginal rate of transformation of foreign exchange in the future into foreign exchange now will be given by the relevant interest rates ruling in the world capital markets. The rate of fall of the social value of a unit of foreign exchange is then equal to the interest rate on world capital markets. Therefore, the shadow discount rate will be equal to the rate of interest on world capital markets when foreign exchange in each year is the numeraire. If any other numeraire is chosen, then the shadow discount rate will not be equal to the world interest rate unless the relative shadow prices of that good and foreign exchange stay constant over time.

The values of the broad variables we have been examining here may be seen as major determinants of the appropriate level of investment, its allocation across industries, and its intensity in different factors of production. But it would not be correct to see these variables as exogenous so that the chain of causation flows from them to the appropriate level of investment. One should think of the shadow prices and the quantities as being determined simultaneously within the same model. And neither would it be correct to think of just one of these variables, say, the shadow discount rate, as determining or being determined by the size of the investment budget. Whether or not a project should be accepted depends on the whole vector of shadow prices and not on one single aspect of them, so that the overall level of investment as determined by this method of project selection depends on all the shadow prices.

IV. Shadow Prices, Market Prices, and the Private Sector

Our concern in the previous section was with public revenue and macroeconomic considerations; we now examine the implications of our theory for policy toward the private sector. We shall be particularly concerned with the relationship between public and private production and with defining the circumstances and sense in which certain market prices may be reliable guides to policymaking even in distorted economies. We begin the discussion by examining (in subsection IV.1) the relation between projects and plans; we then look (in subsection IV.2) at efficiency in the public sector and between public and private sectors; project appraisal for private firms and for public firms with separate budget constraints is examined in subsection IV.3. In subsection IV.4 we discuss the relation between market prices and shadow prices. Finally, in subsection IV.5 we ask about price reform and, in particular, examine the question of whether the price of a good in excess demand should be increased.

1. Projects and plans

So far, our examination of public production decisions has been confined to small projects defined as charges dz undertaken from an arbitrary initial public production plan. In particular, while we have assumed that (subject to scarcity constraints) the policy tools available to the planner were chosen optimally, we have not assumed that the public production plan itself had been optimized. Thus, our theory of shadow prices and policy appraisal does not assume either the optimization of public production or even the knowledge of public production possibilities.

It is clear, therefore, that since the theory we have presented applies with an arbitrary public production plan, it applies in particular to the situation where the initial public production also happens to be a socially optimum one. Of course, the values taken by shadow prices are generally different if evaluated at a different public production plan; but the rules determining them will not change as long as the controls available to the planner are the same. This point is worth emphasizing, because it has caused no small confusion in the literature (see Drèze and Stern (1987) and Dinwiddy and Teal (1987) for further elaboration).

Moreover, while shadow prices have been defined for an arbitrary public production plan, it is important to realize that they provide crucial signals for the improvement and optimization of public production decisions. This is so not only because, as we have seen, shadow prices allow a straightforward identification of socially desirable projects; in addition, it can be shown that under fairly general conditions a socially optimum public production plan is one that maximizes profits at shadow prices. To see this, let Z represent the set of feasible public production plans. Let also z^* be a socially optimum production plan (formally, a production plan which maximizes $V^*(z;\omega)$ within Z), and ν^* the corresponding vector of shadow prices. If, at z^*, some feasible project dz existed with ν^*dz greater than zero, then z^* would not be optimum, since ν^*dz greater than zero indicates that dz increases welfare; hence, it must be true that at z^* no feasible project dz shows a profit at shadow prices. If Z is convex, the latter in turn implies that z^* maximizes shadow profits (in Z) at the shadow prices ν^*, since otherwise a small move in the direction of any production plan with greater shadow profits would represent a feasible and socially profitable project. Thus, when public production possibilities are convex, a socially optimum public production plan is one that maximizes shadow profits.

2. Public efficiency and private efficiency

Given some initial public production plan, we have a set of shadow prices ν. These shadow prices should be used by all public sector firms except (i) public sector firms facing an independent revenue constraint (discussed in the next subsection), and (ii) public sector firms that generate externalities. All public sector firms to which these two qualifications do not apply should, moreover, choose a production plan that maximizes shadow profits (if they know their production set and the latter is convex). For those firms taken together, therefore, production should be efficient. This may seem an unremarkable result but it can have quite strong implications--see, for example, subsection IV.4.

There is no general reason to suppose that public sector and private sector firms taken together should be efficient, although under certain circumstances this may be desirable. For example, we can show that when we make some quite strong assumptions shadow prices ν will be proportional to producer prices p and government and private firms taken together should be efficient (see subsection IV.4).

3. Private firms and budget-constrained public firms

The projects (dz) we have considered so far have been explicitly in the public sector. The government may also wish to appraise private sector projects--for example, for the purpose of granting licenses. The essential difference in the model between a public and a private project representing the same change in net supplies lies in who receives the profits; this difference is reflected in the nature of the prices appropriate for appraising public and private projects.

Formally, we can introduce into the model a private firm (indexed by zero) whose production plan y^o is regarded as a vector of predetermined variables. A private project dy^o then induces a change in welfare dV, where

$$dV = \frac{\partial L}{\partial y^o} dy^o = \nu dy^o + b^o(p dy^o) \tag{4.1}$$

where b^o is the average of the marginal social values of transfers (b^h) for the shareholders of the firm, weighted by their shares in the firm ($b^o \equiv \sum_h \theta^{oh} b^h$ where θ^{oh} is the share of the h^{th} household). Thus,

$$dV = (\nu + b^o p)\, dy^o. \tag{4.2}$$

The appropriate price vector for the evaluation of private projects is therefore a straightforward weighted sum of shadow prices for public projects and of market prices, with weights reflecting the marginal social value of private profits. It should be noted that it is possible, or even likely, for some firms that b^o will be negative; indeed shareholders are rarely regarded as priority targets for income transfers from the government.

A similar result arises if the firm belongs to the public sector but is subject to a budget constraint of the form

$$\rho y^o = \bar{\pi}^o \tag{4.3}$$

for some price vector ρ. The analysis proceeds much as in the previous problem. The Lagrangean now includes a term $b^o(\rho y^o - \bar{\pi}^o)$, where b^o is the Lagrange multiplier on constraint (4.3), and for a project dy^o we have

$$dV = (\nu + b^o \rho)\, dy^o. \tag{4.4}$$

The appropriate price vector for evaluating projects in this public firm is then a weighted sum of the shadow price vector and the vector of prices defining the firm's budget constraint--the latter may, of course, be the vector p of market prices. Note, however, that for a feasible project dy^o preserving expression (4.3), i.e., a project satisfying $\rho dy^o = 0$, the expression (4.4) reduces to $dV = \nu dy^o$. In other words, an alternative and simpler way of formulating socially desirable production decision rules for budget-constrained public firms is to state that such firms should seek to improve profits at ordinary shadow prices within the possibilities compatible with their budget constraint. Budget-constrained public firms are quite common in practice where public firms have performance criteria related to profit, or are separately organized, and this case is therefore of some importance.

We can use the analysis of this section to examine the issue of "privatization." Suppose it is suggested that some production be transferred to the private sector in the sense that the public sector production plan is modified by dz and the private sector is relied upon to fill the gap. If the private sector does things differently from the public sector then the private firm's production change dy^o may not be exactly the converse of dz. The social value of the change is then, using expression (4.2),

$$dV = (\nu + b^o p)\, dy^o - \nu dz, \tag{4.5}$$

which may be written

$$dV = \nu(dy^o - dz) + b^o p\, dy^o. \tag{4.6}$$

The social value of the change then consists of an efficiency effect associated with the difference in the production changes dy^o and dz evaluated at shadow prices and a distributional effect associated with the transfer of profits. If government income is valued highly relative to that of the profit receivers then the second term in expression (4.6) will be negative. This will have to be adjusted if there is a payment from the purchasers of the privatized activity since this represents a transfer to the government. If the payment is less than the (discounted) value of the profits stream then there is a net outflow of public funds which would probably count negatively.

One then has to ask whether this loss of funds is outweighed by any efficiency gain. Leaving aside the empirical evidence on private versus public efficiency (which appears ambiguous), it must be emphasized that the difference in production vectors should be evaluated at shadow prices. There are, moreover, no incentives for private firms to economize on inputs at shadow prices whereas government firms can, in principle, be directed to do so. The issues included in this analysis do not therefore provide a strong presumption that privatization will yield net social benefits. There are obviously wider and important issues omitted in this analysis, including information and organization problems in public relative to private firms, but they should be set carefully against those that we have succeeded in capturing here.

4. Market prices and shadow prices

We derived in Section II a number of rules that should be satisfied by shadow prices and saw that these rules varied according to which variables could be controlled by the planner. We now focus on the relationship between shadow prices and market prices. Throughout this section it must be borne in mind that we are concerned with relative prices--thus, when we speak, say, of shadow prices and market prices being "equal," we really mean "proportional."

It is fair to say that the conditions that ensure that (relative) shadow prices are equal to (relative) producer prices are, generally, rather restrictive. Perhaps the most important example of a set of conditions that ensures this equality is given by the well-known model of Diamond and Mirrlees (1971), where (i) all goods can be taxed and indirect taxation is fully under the control of the planner; (ii) private production is competitive and production sets are convex; (iii) private profits, if any, are fully taxed; and (iv) no quantity rationing applies to private producers (the only exception being that a quantity signal may determine the production plan of industries with constant returns within their supply correspondence). Roughly speaking, when these assumptions are satisfied the private sector is essentially under full government control: by setting the appropriate set of producer prices the government can induce the private firm to produce at any relevant point that it wishes, and this has no direct repercussions on the consumer sector since profits are fully taxed and consumer prices can be manipulated separately. The optimum will therefore be the same as if the private firm was part of the public sector, and the marginal rates of transformation in the private firm will therefore be equal to shadow prices. They are also equal to market prices (since private production is competitive), and therefore shadow prices are equal to market prices. For a formal derivation of the result using the techniques of the model of Section II, see Drèze and Stern (1987).

An alternative set of conditions ensuring the equality of shadow prices and producer prices, which does not involve the restrictive assumption of optimum indirect taxation, consists of the conditions underlying the so-called nonsubstitution theorem: constant returns to scale, a single scarce factor, no joint production, and competition (without rationing) among private producers. This is an application of Diamond and Mirrlees (1976)--see Drèze and Stern (1987) for further discussion.

Apart from restrictive models of the kind involved in these two examples, one would not usually expect to find shadow prices equal to producer prices. But we can still ask whether shadow prices, or a subset of them, will coincide with other kinds of market prices. The principal case of this coincidence is where shadow prices for traded goods are equal to world prices. We saw in Section II that the conditions for this to be true are fairly general, and basically involve the relevant goods being traded without quota, or the quotas being optimally selected.

Where shadow prices are not equal to world prices, we can in many cases see them as a weighted average of marginal social costs of goods drawn from production and from consumption (see subsection II.4), with weights reflecting quantities drawn from each side. This is not the same, however, as a weighted average of consumer and producer prices. One can derive rules along these lines (see Drèze and Stern (1987)), but the weighting procedure is much more complicated and involves averaging (using matrices of demand derivatives) across all markets taken together.

A case where market prices and shadow prices have a strong link, although they do not coincide, is related to the existence of constant-returns-to-scale-firms. If there are no quantity constraints on such firms (other than the "scale factor," determining a firm's production plan within its supply correspondence) and they make zero profits, then one can show that they should also make zero profits at shadow prices. Intuitively, one can understand the result as follows. A small public project using the same input and output proportions as a private firm operating under constant returns could be accommodated in the general equilibrium if the production plan of that firm was correspondingly displaced. This public project would then have no effect on social welfare since no household welfare level has changed. It should therefore "break even" at shadow prices. But this project was simply a scale-down version of the activity of the private firm and therefore that firm would also break even at shadow prices. From a formal point of view this result can be derived by examining the first-order conditions for the scale factor of the firm, which may be regarded as a control variable.

The result does not allow us to say that producer prices are equal to shadow prices since the condition on one firm provides only one linear constraint on the I vector of shadow prices. If it holds for several constant-returns-to-scale firms then it can narrow down considerably the difference between shadow and producer prices. It must be remembered, however, that the condition is only relevant if the constant-returns firms are operating at a positive scale at the optimum level of the controls. We would not expect to find producer prices equal to shadow prices unless the conditions of one of the two examples above are met.

When there are quotas on outputs of firms the question naturally arises of whether the good should be devalued in the shadow price system relative to the producer price system. From expression (2.19) one can show (abstracting from distributional considerations) that the accounting ratio (ν_i/p_i) of a good i in excess supply is lower than a weighted average of the accounting ratios of its inputs by a fraction measuring the discrepancy between price and marginal cost. Hence, the intuitive presumption that a good in excess supply is overvalued in the producer price system does have some content. A discussion of how prices should be reformed in this context is provided in the next subsection.

We have seen, therefore, that while it is not usually true that shadow prices coincide with market prices in a distorted economy, quite a lot can be said about how they may diverge.

5. Price reform in a distorted economy

Should we lower the price of a good in excess supply and increase the price of a good in excess demand? The average economist's first reaction would be to say "yes," but it would, or must be, acknowledged that where distortions exist elsewhere in the system the answer should not be taken for granted. However, the basic principles of policy analysis in distorted economies can point us to what might go wrong and direct any empirical inquiry that is necessary to check whether the economist's initial response is reliable. We provide in this subsection a verbal account of the analysis. The technical details are provided in the Appendix.

In a distorted economy one has to check the income distribution and allocative consequences of a price reform--one cannot assume that policy elsewhere will sort out any untoward effects. Consider the case of a good--for simplicity, a consumption good--in excess demand, with producers supplying as much as they like at the current (and exogenously given) price but with consumers being rationed. Suppose we now raise the price. We would expect producers to produce more and consumers to consume the extra that is produced. 1/ For a small change consumers would still be rationed and would still be prepared to pay more for the good. Crudely speaking, the cost of the extra production is less than the value the consumers attach to the extra amount produced. That is the simple answer, but it takes no account of any distortions and we must ask what might go wrong.

First, the change has income distribution consequences. An increase in the price of a rationed good results in a transfer of income from the consumers to the producers of that good. The government has to judge whether the transfers involved are beneficial. If, for example,

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1/ Note, however, that there are important exceptions to this rule. For instance, in a situation of Keynesian unemployment a decline in wages may fail to increase employment.

the consumers come from better-off groups and the producing firms are owned by the government, which could dispose of the extra profits as it wished, then the income distribution consequences may be beneficial. But it would also be easy to construct examples leading to the opposite conclusion, and indeed, in many cases, consumption goods are rationed in the first place because price increases are prevented on distributional grounds.

The allocative consequences are likely to be positive to the extent that more of the rationed good is produced: there is, as we have seen, a valid general presumption that the shadow price system puts rationed goods at a premium (relative to the producer price system). However, it is possible that greater production will divert resources away from goods with a still higher relative social value. For example, if both air conditioners and fans are rationed to consumers, then raising the price of air conditioners may divert resources away from fans, which are more socially valuable. More generally, greater production of a rationed good may entail adverse allocative consequences if inputs that have a high social value are involved. A similar reasoning can be applied to commodities for which firms rather than households are rationed (e.g., certain types of skilled labor) and to intermediate goods.

These examples show (i) that one must be careful; (ii) that one can see what it is that has to be checked; (iii) that it should not be too difficult in practice to come to a judgment; and (iv) that the basic presumptions on price adjustment are generally fairly reliable. On this last point it is worth trying to invent examples in which things can go wrong. Some are possible, but many seem fairly contrived. One also finds that where the argument does fail, other methods of sorting out the problems underlying the failure are naturally suggested. For example, if both fans and air conditioners are rationed, the price of both should perhaps be raised. And when a price increase hits a poor group, that group might be supported more directly in another way. This last point perhaps raises the most serious doubts because income-support mechanisms in poor countries are not well developed. But even then, in some cases, the producer's price could be raised while the consumer price could be held constant.

It is easy to see that a similar kind of analysis can be used to examine whether the price of a good in excess supply should be lowered. In the case of a final consumption good, excess supply means that consumers are not rationed but producers cannot sell as much as they may wish at the current price. As before, the income distribution effects have to be examined (if producers are poor and consumers are rich we may not want to lower the price), as does the issue of whether consumers will demand more goods with a high social value if the price of the rationed good is lowered. For example, if tonic is very scarce we might not want to lower the price of gin even though it is in excess supply.

The type of analysis provided in this paper shows how the crucial conditions can be systematically checked (see also the Appendix). The analysis can also be used to investigate different combinations of policy reforms. For example, in the case of a good in excess supply we may wish to reduce the producer price without lowering the consumer price so that production and consumption are constant but tax revenue is increased and the profits of the producing firms are reduced. This would be a simple means of transferring revenue from these firms to the government. Similarly, where a good is in excess demand, we may want to raise prices for consumers but not for firms. And one can consider various reorganizations of rations. All these combinations can be discussed straightforwardly within the theory.

V. Summary and Concluding Comments

Our purpose has been to examine the theory of public policy in an economy with distortions, particularly those resulting from price rigidities and quantity rationing. We have seen that the use of shadow prices, defined as social opportunity costs, can provide both a unifying theme for that theory and a simplification of results, in the sense that they summarize rather complicated general equilibrium effects. The social opportunity cost of a good is the net loss (gain) in social welfare associated with increasing its use (reducing its production) in the public sector. Applying this notion obviously requires a definition of social welfare (we have used one based on the welfare of the constituent households) together with a model of how the economy adjusts to changes in the use and production of commodities in the public sector. We have shown that, defined in this way, shadow prices provide an instructive analytical tool not only for project evaluation but also for the theory of optimum policy and of policy reform, and for structuring our thinking and data gathering on crucial applied problems. Our paper has in parts been technical and we therefore provide in this concluding section a verbal summary of the main results and arguments.

The rules satisfied by shadow prices can and should be derived from the maximization of social welfare with respect to the policy tools under the planner's control and subject to all the constraints restricting the choices involved, including, in particular, the scarcity constraints. The choice involved in this optimization may or may not be broad, depending on how many control variables there are relative to the number of constraints; at one extreme, there may be no real choice at all, if the constraints are so restrictive that only one feasible option is really available to the planner. Whatever the degree of freedom involved, shadow prices and optimum policies are determined together by the scarcity constraints and by the conditions for optimum policies. Thus, shadow prices and the theory of policy are part and parcel of the same problem and theory.

We saw in Section II that shadow price rules can be very different depending on which controls are at the planner's disposal; to put it another way, shadow prices can be very sensitive to the way in which the economy responds to a change in public production. This is hardly surprising, but it is, nevertheless, a crucial point to bear in mind.

On the other hand, the rules for shadow prices do not depend on whether or not public production is itself optimized. The values taken by shadow prices do, of course, depend on the public production levels at which they are evaluated; but the rules (or formulas) involved in the computation remain the same.

We found that we could use the same theoretical framework as that used for optimum policy to analyze the theory of marginal reform, i.e., the appraisal of small policy revisions from a given status quo. The effect on social welfare of a small change in a policy tool that had previously been regarded as fixed can be calculated as the change in the value of the Lagrangean associated with the earlier maximization problems. When only the scarcity constraints are relevant, we calculate the direct effect on households of the policy change and subtract the cost at shadow prices of meeting the extra demands generated. For example, suppose we are considering whether we should increase a transfer to a group of elderly individuals. The social benefit is the increased welfare of the group receiving the transfer, which must be set against the cost at shadow prices of meeting the extra demands generated by the transfer. Moreover, and this is very important, we have seen (in subsection III.1) that for any policy change the social value of excess demands generated is precisely equal to the implied loss of "shadow revenue" to the government. Roughly speaking, the shadow revenue is to the nominal revenue what shadow prices are to market prices. This important concept brings the notion of government revenue firmly within the general theory and provides a firm theoretical underpinning to fashionable but often poorly understood analytical tools such as that of "cost-effectiveness."

Shadow prices provide sufficient statistics for policy analysis of a kind that gives us valuable flexibility in discussing underlying assumptions. For example, if we have a fully articulated model of the economy it may be easy to change an elasticity of substitution, but difficult to change an assumption about how the labor market works. On the other hand, we could consider directly the possibility that the shadow wage is higher or lower than we had previously been assuming. In general, many formal models may generate a given set of shadow prices, and we can speculate informally on the type of assumptions that might lead to the revision we are considering without the burden of working through the detail of the completely specified model.

Among the rules derived in Section II, those concerning the use of world prices for traded goods and the marginal social costs of production for nontraded goods were of particular importance. The first of these is very robust and general and would apply in any model where an increase in the public production (use) of a commodity results in an equivalent reduction of net imports (exports) and where the general equilibrium effects operate entirely through the balance of payments. This is generally true if there are no quotas and world prices are fixed. If world prices depend on the quantities imported or exported (but there are still no quotas), world prices have to be replaced by marginal costs or revenues on the world market (and we assume domestic prices can be controlled independently of world prices). In every case, world prices (or marginal costs/revenues) have to be multiplied by the "marginal social value of foreign exchange," unless foreign exchange happens to be the numeraire in the shadow price system.

The second rule is much less general, and will usually depend on the assumption that an extra unit of the relevant commodity comes exclusively from extra production. If this is not true then we have to look at the social opportunity costs applying to different sources, and only exceptionally (with extensive optimization of taxes and production) will the shadow values associated with the different sources be the same.

The calculation of a set of shadow prices will require considerable knowledge of the economy in order to make sensible judgments on how it will react to an extra unit of public supply, particularly in regard to whether goods are properly deemed to be traded and to the degree to which extra demand for a nontraded good implies extra output. But just as careful monitoring of the macroeconomic system can be built on a study of national accounts information so too can the same information provide a basis, with experienced knowledge of the economic system, for a broad set of shadow prices. This is not something to be constructed in a brief mission but to be created, and maintained, at the desk of an international organization or in a planning ministry. If used critically it could provide valuable checks on, or inputs to, the recommendations of missions in terms of price and tax reform, their implications for the value of expanding output in different sectors, and so on. While we think such calculations of shadow prices are likely to be helpful, we would not press them as being necessarily the main contribution of theory. We would rather stress its importance as providing a set of guiding principles and questions for thinking about real policy problems.

The generalizations of the rules for optimum taxation were also of importance to an understanding of appropriate tax policy. The standard rules for optimum taxes in a competitive economy applied directly to a distorted economy provided actual taxes were replaced by the difference between market prices and shadow prices. This suggests that we use the tax system to compensate for differences between market prices and shadow prices generated by distortions elsewhere in the system. Thus, ceteris paribus, we should have a lower tax on nontraded labor-intensive goods if the shadow price of the relevant type of labor was judged to be particularly low relative to its market price. 1/

The ideas and theory developed in Section II and the first part of Section III were applied in the second half of the paper to show, first, how important shadow prices should be calculated, and second, to provide a framework for structuring the analysis of crucial problems in price and tax reform and privatization.

The analysis of subsection III.2 focused first on the provision of a coherent notion of a premium on savings: if, compared with the market price system, the shadow price system puts a premium on future goods relative to current goods, then those households with a higher propensity to save (i.e., to spend on future goods) will, ceteris paribus, be seen as having higher priority for income transfers. We were also able to provide a clear meaning for the idea of a premium on foreign exchange in terms of the extent to which the shadow price of foreign exchange relative to the exchange rate is above the ratio of shadow prices to market prices for a basket of goods. This in turn could be measured by the ratio of domestic prices to world prices for a basket of goods.

We saw also how to calculate shadow wages and discount rates. The standard theories of shadow wages, broadly speaking, survive transplantation to a more general framework. The central point emphasized in the discussion of the shadow discount rate was care with its definition. The shadow discount rate is the rate of fall in the social (present) value of the numeraire and will depend crucially on which numeraire is chosen. If, for example, foreign exchange in the hands of the government is the numeraire, then the borrowing and lending rates available to the government on world capital markets will be useful indications of the shadow discount rate. But it is misleading to try to discuss the shadow discount rate without a specific statement of the numeraire.

In Section IV of the paper we showed how the approach could be used to structure the analysis of important applied problems. Two critical issues in the appraisal of privatization fit neatly into the framework. Privatization involves a transfer of profits from the public to the private sector and also typically involves changes in production techniques and levels. The first aspect is evaluated as any other income transfer, taking into account the welfare weights for recipient groups and the marginal propensity to spend on goods with high or low shadow prices. The second is evaluated by calculating the value at shadow prices of the production change. The first contribution is likely to be negative, since the recipients of profits usually belong to higher-income groups, and whatever they do with their profits (including saving them) can, if nothing else, be replicated by the government. The second

1/ It does not mean that the correct tax is simply the difference between market and shadow price.

term is more problematic. If public policy and public production are carefully and efficiently integrated, then it is always true that privatization leads to a reduction in the social value of the production plan of the corresponding firm, since optimum public production precisely requires the maximization of shadow profits. The usual counterargument is that adequate information and incentives may not exist to make this integration a reality, so that the production decisions of public firms may turn out to be no less socially suboptimal than those of private firms. There is a real issue here, which goes much beyond the scope of the present paper. But the important point from our analysis is that the comparative production performance of public and private firms can only be correctly assessed with reference to the social values of commodities rather than with reference to their market prices. Assertions that private firms are more "efficient" than public firms do not carry much weight where shadow prices diverge substantially from market prices and public firms can be directed, at least to some extent, to take the discrepancies into account.

The theory, moreover, strongly suggests that substantial discrepancies of this kind are a common feature of distorted economies. Nor would it be reasonable to expect that some of the distortions involved (e.g., involuntary unemployment) could be easily removed.

Our final application was to the question of whether the price of a good in excess demand should be raised (or that of a good in excess supply lowered). We saw that the answer depended on the distributional pattern of consumption and production and on whether net demand was switched to or from goods with high shadow prices. If producers of a good are poor and consumers rich, then the distributional consequences of increasing the price of a good in excess demand would be favorable. There is some validity in the general presumption that the shadow price of a good in excess demand is "high" (relative to its consumer price) so that the effect of raising its price to producers is to switch production toward a good with a high shadow price. There is some reason, therefore, to suppose that the policy of raising the price would be justified. Analysis has, however, warned us to check the distributional effects and whether the inputs involved in increasing production might be at an even higher premium in the shadow price system than the rationed good itself.

Outside "perfect" economies it is possible to provide counterexamples to most propositions. This does not, however, mean that nothing can be said, that anything goes, that there are no rules, and that we can cast aside systematic economic analysis. We have tried to show in this paper how structured argument can define social values, provide rules for their calculation, integrate cost-benefit analysis and the theory of policy, and, finally, guide our thinking and judgment on immediate policy problems.

Price Reform in a Distorted Economy

The first reaction of many economists when they see excess demand for a good is to say that the price should be raised, or if it is in excess supply, that the price should be lowered. We know that one must take care with such proposals in a distorted economy, and the purpose of this appendix is to characterize the circumstances under which the initial or "obvious" response is correct. We shall point out what the analyst must check in practice to see whether the simple rule is valid. This provides some technical detail for the verbal account given in Section IV.5. We begin with the case of excess supply and rationed producers and then look at excess demand and rationed consumers.

We consider the producer price of good i, p_i to be an exogenous parameter, which is currently fixed at a level which is "too high" in the sense that the consumers (who are not rationed for this good) demand less at the price they face (q_i) than the firm would like to supply at the price p_i. This is captured by the existence of a binding constraint $\bar{y}_i$ applying to the firm's production of good i. The variable $\bar{y}_i$ acts as the equilibrating variable on the i^{th} market.

Since $\bar{y}_i$ is an endogenous variable we have equation (2.19). To reiterate the latter

$$\nu_i = MSC_i - b(p_i - MC_i) \tag{2.19}$$

where MSC_i is the value at shadow prices of the marginal inputs required to produce an extra unit of the i^{th} good (see (2.18b)). If b is small enough, i.e., profit taxes are not too far from optimal, then given $p_i > MC_i$ we may write

$$\frac{\nu_i}{p_i} \approx \sum_{j \neq i} [\alpha_j \frac{\nu_j}{p_j}] \left(\frac{MC_i}{p_i}\right) < \sum_{j \neq i} \alpha_j \frac{\nu_j}{p_j} \tag{A.1}$$

where α_j is the share of the j^{th} input in the marginal cost (the α_j sum to one for $j \neq i$). Equation (A.1) says that, under the given assumption, the accounting ratio (ν_i/p_i) for the i^{th} good is less than a weighted average of the accounting ratios of the inputs, the weights being given by the respective shares in marginal cost. In this sense the good in excess supply is devalued in the shadow price system relative to the market price system.

We want now to look at the marginal social value of lowering the market (producer) price, p_i, of a good which is in excess supply. The 'standard reaction' is justified if $\partial V^*/\partial p_i < 0$. From (2.11) we have

$$\frac{\partial V^*}{\partial p_i} = \frac{\partial L}{\partial p_i} \tag{2.11}$$

which gives us, using (2.7) and (2.8),

$$\frac{\partial V^*}{\partial p_i} = -\sum_h \beta^h x_i^h - \nu \left(\frac{\partial x}{\partial q_i} - \frac{\partial y}{\partial p_i}\right) + by_i. \tag{A.2}$$

If we decompose $\frac{\partial x_j}{\partial q_i}$ into income and substitution effects we have

$$\frac{\partial V^*}{\partial p_i} = \nu\left(\frac{\partial y}{\partial p_i} - \frac{\partial \hat{x}}{\partial q_i}\right) - d_i \tag{A.3}$$

where $\frac{\partial \hat{x}}{\partial q_i}$ is the vector of aggregate substitution effects (s_{ij}), and d_i is a "net distributional characteristic," defined by $d_i \equiv \sum_h b^h x_i^h - by_i$. The latter captures the pure income effects associated with an increase in the i^{th} price. If, as we are assuming, the i^{th} output is rationed then $\frac{\partial y}{\partial p_i}$ is zero (using the symmetry of supply responses) and $\frac{\partial V^*}{\partial p_i}$ is given by

$$\frac{\partial V^*}{\partial p_i} = -\left(\nu \frac{\partial \hat{x}}{\partial q_i} + d_i\right). \tag{A.3a}$$

The sign of d_i depends on how far the consumers and shareholders are seen as transfer deserving. More precisely, d_i will be more likely to be positive the poorer are the consumers and the more they spend their income on goods with low shadow prices (and conversely with shareholders). If we use the index $h = 0$ for the government, with $b^0 \equiv 0$ (since a transfer from the government to the government naturally has zero marginal social value) we can also write d_i as

$$d_i \equiv H \operatorname{cov}(b^h, e^h)$$

where, using the earlier notation $\zeta \equiv 1 - \sum_h \theta^h$,

$$e_i^h \equiv x_i^h - \theta^h y_i \qquad (h \neq 0)$$

$$e_i^o \equiv - \zeta y_i - z_i.$$

We can think of e^h as that part of net excess demand 'arising' from the h^{th} household and then d_i is given by the covariance across households between net excess demand and the marginal social value of income. In particular, if no strong correlation between these two is expected, the distributional characteristic can be ignored.

It remains to examine the sign of $\nu \frac{\partial \hat{x}}{\partial q_i}$. We have from the homo geneity of the compensated derivatives

$$- \nu \frac{\partial \hat{x}}{\partial q_i} = (q - \nu) \frac{\partial \hat{x}}{\partial q_i} \tag{A.4}$$

$$= (s_{ii}\, q_i) \left(\left(\frac{q_i - \nu_i}{q_i}\right) - \sum_{j \neq i} \gamma j \left(\frac{q_j - \nu_j}{q_j}\right)\right) \tag{A.5}$$

where $s_{ii} < 0$ and

$$\gamma j = - \frac{s_{ij}\, q_j}{s_{ii}\, q_i} \tag{A.6}$$

(from the homogeneity of the compensated derivatives $\sum_{j \neq i} \gamma j = 1$). Thus when d_i is "small," we can say that the price p_i should be lowered $(\partial V^* / p_i < 0)$, i.e., the standard reaction is correct, if

$$\left(\frac{q_i - \nu_i}{q_i}\right) > \sum_{j \neq i} \gamma j \left(\frac{q_j - \nu_j}{q_j}\right). \tag{A.7}$$

But from (A.1) we have

$$\left(\frac{p_i - \nu_i}{p_i}\right) > \sum_{j \neq i} \alpha_j \left(\frac{p_j - \nu_j}{p_j}\right). \tag{A.8}$$

The basic economic issues embodied in our question "should we raise the price of a good in excess supply?" are now embodied in (A.3)-(A.8). We have already indicated the determinants of the sign of the distributional term in (A.3a), $-d_i$. We may call $(q-\nu)\frac{\partial \hat{x}}{\partial q_i}$ the allocative term in $\frac{\partial V^*}{\partial p_i}$ (but see below). The own price effect $\frac{\partial \hat{x}}{\partial q_i}$ will be negative and we think, intuitively, of the shadow price ν_i of the good being low since it is in excess supply. This leads us to suppose that $(q-\nu)\frac{\partial \hat{x}}{\partial q_i}$ will indeed be negative provided the own-price term is not swamped by cross-price effects. Condition (A.8) tells us that the shadow producer tax rate on good i is greater than a weighted average of the shadow producer tax rates applying to the inputs. In order for the allocative term to be negative we need the shadow consumer tax rate on good i to be greater than a weighted average of the consumer tax rates applying to the other goods (see (A.7)). The former condition being true militates in favor of the latter being true as well. For example, if there are only two goods so that α_j and γ_j are both 1, and taxes are proportional (q_j is proportional to p_j) then (A.8) implies (A.7).

The formal analysis of this section lends some support to the simple idea that we should lower the price of a good in excess supply but also gives us an understanding of the conditions under which this conclusion might be overturned. It is tempting to regard the distributional considerations as summarized in the term $-d_i$ and the allocative in the term $-\nu\frac{\partial \hat{x}}{\partial q_i}$. If, for example, the producers of good i are regarded as deserving, then this would weaken (through $-d_i$) the case for lowering the price. The distributional aspect, however, is not captured solely in $(-d_i)$ since from (2.19) we see that the valuation (b) of marginal transfers to the firm enters ν_i, and thus $\partial V^*/\partial p_i$. On substituting from (2.19) into (A.2) and examining the coefficient on b one can immediately see (using $\partial y/\partial p_i = 0$) that the coefficient on b in the resulting expression for $\partial V^*/\partial p_i$ is $(p_i - MC_i)\partial q_i + y_i$. If we assume all extra supplies came from domestic production then this is simply the effect of the price increase on the firm's profits (note that the demand response comes in here precisely because the producer cannot choose the output level). Hence if we were to regard producers as more deserving (higher b) then this would militate in favor of a price increase (i.e., would increase $\partial V^*/\partial p_i$) provided that the elasticity of demand is not so high that profits are thereby reduced. If the good is predominantly consumed by households with low welfare weights β^h, e.g., the rich, then we can see from (A.2) or (A.3) that this works to increase $\partial V^*/\partial p_i$ and is a factor against the standard reaction (which involves $\partial V^*/\partial p_i < 0$). From the allocative viewpoint, we may wish to raise the price if this would lead consumers to substitute toward goods with low shadow prices.

Faced with a practical question, the formal analysis tells us where to look to check whether the presumption that the price should be lowered is sound, i.e., at the incomes of producers and consumers and at the substitution behaviour of the consumers. For example, if cotton weavers are poor, consumers of cotton cloth are rich, and the shadow price of polyester (cotton substitute) is low then there might be an argument for raising the price of cotton cloth even though it is in excess supply, provided demand is not thereby so reduced that their net incomes fall. The example, however, points in two important directions. First, it is not very easy to find plausible counter-examples to the standard presumption, and second, the problems generating the counter-examples can sometimes be solved in other ways. Thus, one could try to support the incomes of cotton weavers by retraining them to other jobs. And one could shift consumption toward goods with low shadow prices by changing the prices of those goods themselves rather than of substitutes. Of course, if these steps are actually taken, this will be reflected in the value of $\partial V^*/\partial p_i$ itself.

A similar kind of analysis can be used to examine the question of whether the price of a good in excess demand should be raised. In this case we can think of suppliers being unrationed and producing and selling as much as they wish at a controlled price p_i but consumers, buying at q_i, are rationed. We have (A.2) and (A.3) as before but now $\frac{\partial \hat{x}}{\partial q_i}$ is zero. The standard answer that the price should be raised is supported if $\frac{\partial V^*}{\partial p_i}$ is positive. The relevant contributions are d_i and $\nu \frac{\partial y}{\partial p_i}$. We have indicated previously the determinants of d_i: this is more likely to be positive the less "deserving" the producers and the more "deserving" the consumers.

The sign of $\nu \frac{\partial y}{\partial p_i}$, which can also be written as $(\nu-p)\frac{\partial y}{\partial p_i}$, is more likely to be positive the more producers switch toward goods with a high shadow price. The effect through the i^{th} good would point in this direction since $\frac{\partial y_i}{\partial p_i}$ is positive (the basic "law" of supply) and one would expect the shadow price of the good in excess demand to be high. In an analogous manner to the previous analysis we can discuss whether

$\nu \frac{\partial y}{\partial p_i}$ will, in fact, be positive by examining the condition for the optimum (i.e., endogenous) consumer rations. The rations are adjusted to clear the market given the supply. If the ration which clears the market operates on a single consumer, (i.e., $\bar{x}_i^h$ adjusts) then the first-order condition is

$$0 \quad \beta^h(\rho_i^h - q_i) - \nu_i - \sum_{j \neq i} \nu_j \left(\frac{\partial x_j^h}{\partial \bar{x}_i^h}\right) \tag{A.9}$$

where ρ_i^h is the marginal willingness to pay for good i:

$$\rho_i^h \equiv \frac{\partial u^h}{\partial x_i^h} \Big/ \frac{\partial v^h}{\partial m^h} .$$

We are supposing that the good is rationed to the h^{th} household so that $\rho_i^h > q_i$ and we therefore have (using (A.9))

$$\frac{\nu_i}{q_i} > \sum_{j \neq i} \frac{\nu_j}{q_j} \hat{\alpha} j \tag{A.10}$$

where $\hat{\alpha} j$ is $\frac{q_j}{q_i} \frac{\partial x_j}{\partial \bar{x}_i}$;

(so that $\sum_{j \neq i} \hat{\alpha} j = 1$) which is analogous to (A.1). Analogously to (A.4) - (A.7) we have

$$\nu \frac{\partial y}{\partial p_i} = p_i \frac{\partial y_i}{\partial p_i} \left(\frac{\nu_i}{p_i} - \hat{\gamma} j \frac{\nu_j}{p_j}\right) \tag{A.11}$$

where

$$\hat{\gamma} j = \frac{-p_j \partial y_j / \partial p_i}{p_i \partial y_i / \partial p_i} \text{ and } \sum_{j \neq i} \hat{\gamma} j = 1 \tag{A.12}$$

[37e

from profit mazimization.

The analysis of (A.11) proceeds (using (A.10)) in a similar manner to that of (A.5). A valid general presumption exists to the effect that raising the price of a good in excess demand is beneficial. One can establish this for the one-consumer economy where taxes are proportional. But there are exceptions as well. Exceptions may arise where the good is primarily consumed by those who are particularly "deserving" or produced by the "less deserving" and the price increase leads to a greater use of inputs which have high shadow prices. Again these are issues on which one can form empirical judgements, may be fairly rare in practice and arise in circumstances where the problems generating them might be tackled in other ways. Nonetheless, the possibility that the standard rule is a mistake is a real one and should be checked.

We have analyzed here the validity of the simple reform of adjusting the producer price with a constant tax rate so that the consumer price is also raised. It is also natural to think of other possible reforms such as adjusting the producer price but holding consumer price constant. In the case of lowering the price of a good in excess supply this amounts to increasing the tax by the same amount that the producer price is lowered. Hence the change in welfare is $\frac{-\partial V^*}{\partial p_i} + \frac{\partial V^*}{\partial t_i}$. One can consider a variety of possibilities of producer price and tax changes as well as changes in the rationing rules.

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