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Credibility and Nominal Debt: Exploring the Role of Maturity in Managing Inflation

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Abstract

This paper focuses on the role of debt maturity in managing the government's incentives to use opportunistic inflation to reduce the ex post real value of its nominal liabilities. The maturity structure of government debt is shown to be a powerful instrument to affect the time profile of the inflation tax base and, hence, to mitigate the distortions introduced by time inconsistency on taxation policies. The nature of the optimal policy is shown to be heavily dependent on the type of precommitment enjoyed by policymakers.

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I. Introduction

Despite the policy maker's best intuitions and the towering contribution by Tobin (1963) to the issue of the optimal debt maturity of government debt, it is perhaps fair to say that, until very recently, the subject has not attracted the attention of mainstream economics. One reason for this peculiar state of affairs is probably the fact that under (a) complete markets and (b) policy precommitment--which were important assumptions in mainstream economics until recent times--debt maturity does not change the set of equilibrium solutions. However, when either one of those key assumptions is relaxed, debt maturity has a role to play.

Lucas and Stokey (1983) have forcefully brought the debt maturity issue into center stage. They showed that by relaxing assumption (b) above, debt maturity could be utilized to affect the incentives of future policymakers. As a matter of fact, they showed a special example in which a careful choice of debt maturity neutralizes all the time inconsistency problems and, thus, allows the decentralization of the present government first-best plan. This is, undoubtedly, a big turnabout with respect to the role of debt maturity.

The discussion was further extended to a monetary economy by Persson, Persson and Svensson (1987). They showed that in such a context the maturities of both indexed (to the price level) and nonindexed government debt mattered, even in a perfect-foresight context. 1/ Assumption (a) above (complete markets) was relaxed by Calvo and Guidotti (1989) by assuming, instead, that bonds could be indexed to the price level, but not to any other characteristic of the state of nature like, for instance, government expenditure or terms of trade.

So far, the results obtained with these models are either proofs of very general propositions--with little insight on the characteristics of the associated debt maturity structure, e.g., Persson, Persson, and Svensson (1987), Calvo and Obstfeld (1988)--or they concern more specific maturity-structures, but the results are obtained in the context of special examples, like in Lucas and Stokey (1983) and Calvo and Guidotti (1989). Both types of approaches have yielded useful insights, although certainly more research is needed to close the gap between the two.

In the present paper we will pursue, in a more general context, some of the issues raised in Calvo and Guidotti (1989). However, we concentrate our attention on nominal debt--which is still the dominant form debt takes in practice--by leaving aside the question of indexation. For the sake of tractability, only the perfect-foresight case will be studied.

1/ Calvo and Obstfeld (1988) have shown, however, that their stronger claim that today-planner's optimum can be decentralized by choosing an appropriate maturity structure does not hold true in general.

The model aims at capturing the implications of nominal debt maturity in the simplest possible setup. We assume three periods. Government in period 0 has a given stock of debt which has to be rolled over (i.e., shifted to periods 1 and 2). Thus, assuming that nominal debt is the only form the debt can take, government-0 has the relatively simple task of only having to choose the maturity structure of its outstanding debt. As stated above, if there was full precommitment on the part of government 0--i.e., if future governments were bound by the announcements made by government 0--then maturity structure would be irrelevant. However, in the present paper we assume that government 0 has, at best, limited control on the behavior of future governments. Thus, government 0 has to take into account future governments' incentives to inflate away inherited debt.

We show that the nature of optimal policy is heavily dependent on the _type_ of precommitment. Thus, if government 0 can precommit the actions of government 1, but neither one can precommit those of government 2, then, once again, maturity structure is shown to be irrelevant. However, the lack of perfect precommitment leads governments to accelerate the rate of debt repayment beyond what would prevail under full precommitment. We call that the case of "debt aversion." This phenomenon, first noticed by Obstfeld(1989), arises because without full precommitment the cost of nominal debt is larger than the regular interest rate charges. With imperfect precommitment a higher nominal debt leads to higher inflation due to the future governments' futile (in equilibrium) attempt to inflate it away. This is the reason for the above-mentioned extra costs.

On the other hand, if government 1 can precommit the actions of government 2, but government 0 cannot place any constraint on any future government, then there exists a well-defined optimal maturity structure from the perspective of government 0. For example, if the demand for high-powered money is nil, we show that it is optimal to have only one-period debt. A positive demand for high-powered money induces a richer maturity spectrum, but it has to be sufficiently large to change the structure in favor of long-run debt. This is an interesting finding because it provides a rationale for the observed shortening of the maturity structure in several important countries (see, for instance, Alesina, Tabellini and Prati (1989)). This phenomenon is shown to be independent of debt aversion, because due to the assumption that government 1 can commit the actions of government 2, there is no extra social benefit associated with early repayment. Therefore, government 1 always finds it optimal to smooth completely over time conventional taxes independently of the choice of different maturities by government 0.

Finally, the paper examines the case where there is no precommitment. The analytical results are much less clear-cut here than in the previous situations because debt aversion interacts with optimal maturity. The absence of precommitment on the part of government 1 introduces debt aversion whenever government 1 has to issue new debt. This occurs because the nominal debt issued by government 1 is part of the inflation tax base of government 2. The choice of different maturities by government 0 interacts with debt aversion by affecting the amount of new debt to be issued by government 1 and, hence, it alters the time profile of conventional taxes. In addition, the choice of maturities affects the time profile of incentive-compatible inflation by altering the time profile of the inflation tax base. The problem is explored with the help of some numerical simulations. The main implication that springs up from our simulations is that lack of precommitment is associated with a relatively balanced maturity structure. This result connects with recent work by Giavazzi and Pagano (1989) and Alesina, Tabellini and Prati (1989) which also calls for a balanced maturity structure of the public debt, although their reasons are substantially different from those explored in this paper. The numerical simulations also suggest that optimal maturity lengthens in response to an increase in the stock of debt and to an increase in the demand for high-powered money, and shortens in response to an increase in government spending.

The paper is organized as follows. Section 2 presents the basic model and characterizes the equilibrium under full precommitment. Section 3 considers the first example of partial precommitment and shows how time inconsistency in government behavior distorts the intertemporal choice of conventional taxes leading to "debt aversion". Section 4 presents the second example of partial precommitment and shows how time inconsistency introduces a role for debt maturity. Section 4 analyzes the case of no precommitment where debt aversion interacts with optimal maturity. Section 5 contains the conclusions.

II. The Basic Model

Denote by b_{ij} the output value, at time i, of nominal public debt issued in period i with maturity in period j. The government is assumed to have a three periods horizon. In the last period (period 2), the government inherits a given stock of maturing nominal debt issued in the previous two periods. In addition to repaying the maturing debt, the government finances a constant (exogenous) flow of expenditure, g, by levying a distorting tax, x, and by using the revenue from inflation. Hence, the flow budget constraint in period 2, expressed in output values, is given by:

$$(1)\quad x_2 + S(\Pi_2) = g + b_{12}I_{12}/\Pi_2 + b_{02}I_{02}/\Pi_1\Pi_2$$

where I_{ij} denotes the nominal interest factor (i.e., one plus the corresponding interest rate) between periods i and j, and Π_i denotes the inflation factor in period i (i.e., $P_i = P_0\Pi_1\Pi_2....\Pi_i$, where P is the price level), and $S(\Pi)$ is the inflation tax on money balances. We assume throughout the paper that $S(\Pi) \equiv k(\Pi-1)/\Pi$, where "k" denotes a constant (interest-inelastic) demand for money. 1/

In period 1, the government's flow budget constraint is given by:

$$(2) \quad x_1 + S(\Pi_1) + b_{12} = g + b_{01}I_{01}/\Pi_1$$

Equation (2) states that the government may finance expenditure plus the amortization of debt issued in period 0 maturing in period 1 by resorting to conventional taxation, the inflation tax, or by issuing new nominal debt.

In order to sharply focus on the debt-maturity issue, we assume that, at period 0, the only decision faced by the government is choosing the maturity structure of a given initial stock of public debt, whose output value is denoted by b: 2/

$$(3) \quad b = b_{01} + b_{02}$$

where b_{0j} (j=1,2) may, in principle, take negative as well as positive values. Positive values show a debt position.

The optimal choice of instruments by the government responds to the objective of minimizing the value of the following intertemporal cost function:

$$(4) \quad L = V(x_1) + H(\Pi_1) + R^{-1}[V(x_2) + H(\Pi_2)]$$

where the time preference discount factor is assumed to be equal to the (assumed constant) real interest rate factor, R, and the functions $V(\cdot)$ and $H(\cdot)$ are assumed to be strictly convex. 3/ 4/

1/ As will become clear below, the presence of a "genuine" revenue effect of inflation is important. The assumption that the demand for money is interest-inelastic introduces this effect in the simplest way. Assuming that the demand for money is interest elastic would make the analytical presentation more complex without providing additional insights.

2/ It is assumed that the price level at period zero is a predetermined variable. This makes the output value of the initial stock of debt an exogenous variable.

3/ In addition, $V'(0) = 0$ and $H'(1) = 0$.

4/ Notice that we are making the cost of inflation a function of actual inflation, not just expected inflation. In this we follow Barro and Gordon (1983). For some microfoundations, see Calvo (1988).

To characterize government behavior it is necessary to assume something about its ability to precommit future actions at any given point in time. The first-best situation is one in which the government at period zero has the ability to make all sorts of precommitments about future policies. In such a case, policies chosen at period 0 are, by definition, time-consistent (Kydland and Prescott (1977); Calvo (1978)).

In many instances, however, the government is unable to make credible precommitments; in such a case, policies which are optimal in a first-best situation at time zero would not be optimal to follow in the future, and, hence, would be infeasible. In the present framework, a time-inconsistency problem arises because the existence of nominal debt induces future governments to attempt to partially repudiate those obligations through inflationary means. Such partial repudiation, is, however, just an illusion. In a world of rational agents any incentive the government has to "inflate away" its nominal obligations is reflected in higher nominal interest rates at the time those obligations are contracted. Ex-post, the nominal value of those obligations is predetermined and provides the basis for those incentives. But if individuals know the government's incentives, the equilibrium interest rate would exactly cover them against the resulting, opportunistic, inflation. 1/

Since no real gains result from excess inflation, it is in the interest of the government to take into account, at time zero, those future incentives to liquidate nominal debt through inflation. Accounting for those incentives provides the basis for the formulation of time-consistent, or second-best, policies in a world where precommitment is not possible.

Before analyzing the government's behavior under time inconsistency and characterizing the optimal role of debt maturity in formulating time-consistent policies, we discuss briefly the first-best case where complete precommitment on the part of the government is possible and where, therefore, policies chosen at period 0 are credible.

The intertemporal government budget constraint in period zero is given by:

$$(5) \quad bR + g[(1+R)/R] = x_1 + S(\Pi_1) + R^{-1}[x_2 + S(\Pi_2)]$$

1/ The assumption of perfect certainty helps showing in a dramatic way the revenue-ineffectiveness of expected inflation when applied to the stock of nominal bonds. Under uncertainty, however, the government would be able to collect inflation tax on nominal bonds as long as inflation is unanticipated. On average, of course, unexpected inflation would anyway collect nothing from nominal bonds. For a discussion, see Calvo and Guidotti (1989).

Equation (5) is obtained by combining equations (1)-(3), taking into account that, under perfect foresight, $I_{01}=R\Pi_1$, $I_{02}=R^2\Pi_1\Pi_2$, and $I_{12}=R\Pi_2$ (recall that R is the real interest factor which has been assumed to be constant over time). It simply says that the government is constrained to make the present value of expenditure (including debt obligations) equal to the present value of taxes (including the inflation tax).

With full precommitment, the government minimizes social loss in equation (4) subject to budget constraint (5). It is obvious that, for this problem, the maturity structure of the public debt is irrelevant. Moreover, the first-best choice of tax and inflation rates implies:

(6) $x_1 = x_2 = x$

(7) $\Pi_1 = \Pi_2 = \Pi$

(8) $H'(\Pi) = V'(x)S'(\Pi)$

Equations (6) and (7) imply that it is optimal to achieve perfect smoothing of tax and inflation rates over time. Equation (8) implies that the marginal cost of inflation equals, at the optimum, the cost reduction from the (conventional) tax cut induced by the associated larger inflation tax.

In the following sections we discuss how the above problem is modified in the presence of time inconsistency of government behavior. As mentioned earlier, the potential for time inconsistency exists because the presence of nominal debt provides the government an incentive to resort, in the future, to inflation in order to reduce the real value of nominal debt obligations. Time inconsistency has two major implications. First, it alters the optimal intertemporal allocations of taxes. Second, it introduces a role for the maturity structure of nominal public debt.

III. Partial Precommitment I: A Case of "Debt Aversion"

Consider a simple case of partial precommitment where the government in period 0 is able to make commitments about period-1 variables but cannot precommit period-2 variables. This implies that, in fact, there are only two interesting decision points: one at period 2 where x_2 and Π_2 are decided, and one at period 0 where the period-1 policy variables (i.e., x1, Π_1, and b_{12}), as well as the maturity structure of initial debt, are chosen. One way to think about partial precommitment is to consider a situation where government administrations are able to make precommitments during the periods in which they are in power, but cannot make precommitments about future administrations' policies. This case may, thus, reflect a situation where there is a change of government at the end of period 1 and the incoming administration stays in power in period 2.

The government in period 2 (government 2, for short), faces budget constraint (1), where the only non pre-determined variables are Π_2 and x_2. Furthermore, since planning is done at period 2, the planner is not constrained by the perfect foresight conditions that $I_{02}=R^2\Pi_1\Pi_2$ and $I_{12}=R\Pi_2$. 1/ Given the objective of minimizing the value of the cost function $H(\Pi_2) + V(x_2)$ subject to equation (1), the optimal choice of period 2 inflation is given by:

$$(9) \quad \Pi_2 H'(\Pi_2) = V'(x_2)[b_{02}I_{02}/\Pi_1\Pi_2 + b_{12}I_{12}/\Pi_2 + \Pi_2 S'(\Pi_2)]$$

Equation (9) differs from equation (8) because the absence of precommitment in previous periods about period-2 variables leaves government 2 free to resort to inflation in order to reduce the output value of its obligations (recall that, at that point in time, the nominal value of outstanding debt, as well as the nominal interest rate factors I_{12} and I_{02}, are pre-determined variables). In addition to the "genuine" revenue gain associated with a higher inflation tax on money balances, which also shows up in equation (8), the R.H.S. of equation (9) takes into account the (conventional) tax cut associated with the fall in the real value of debt obligations maturing in period 2. The R.H.S of equation (9) shows that, from the perspective of period 2, the base of the inflation tax is not just high-powered money but also the stock of nominal debt obligations maturing in period 2.

The gains resulting from reducing the real value of government obligations are, however, an illusion. The market perceives the future incentive to inflate on the part of the government and, at the time the nominal debt is being issued, nominal interest rates reflect, point for point, future inflation. This implies that for government bonds issued at time zero we have, in equilibrium:

$$(10a) \quad I_{01}=R\Pi_1$$

$$(10b) \quad I_{02}=R^2\Pi_1\Pi_2$$

Furthermore, the equilibrium interest rate for debt issued at time 1 satisfies:

1/ These conditions will hold, nevertheless, in equilibrium. See equations (10) and (11) below.

(11) $I_{12}=R\Pi_2$

Since market interest rates reflect actual equilibrium inflation, we have, combining equation (9) with equations (10), (11), and (1):

$$(9') \quad \Pi_2 H'(\Pi_2) = V'(x_2)[b_{02}R^2 + b_{12}R + \Pi_2 S'(\Pi_2)] =$$

$$= V'(x_2)(x_2 - g + k)$$

Government 0 can precommit x_1 and Π_1 but cannot precommit period-2 variables. The formulation of its time-consistent policy takes into account that in equilibrium government 2 chooses Π_2 according to equation (9'), which shows that Π_2 is a function of the stock of nominal debt maturing in period 2. The problem at time zero is to minimize social loss in equation (4) subject to budget constraint (5) and the incentive-compatibility constraint (9'). This problem involves only the choice of x_1, x_2, Π_1, and Π_2, and is entirely independent of the maturity structure of initial debt; i.e., debt maturity is irrelevant. The first-order conditions for optimization imply:

$$(12) \quad V'(x_1) = V'(x_2) + \mu\{V''(x_2)[x_2-g+k] + V'(x_2)\}R$$

$$(13) \quad H'(\Pi_1) = V'(x_1)S'(\Pi_1)$$

$$(14) \quad H'(\Pi_2) = V'(x_1)S'(\Pi_2) + \mu[\Pi_2 H''(\Pi_2)+H'(\Pi_2)]R$$

where μ is the Lagrange multiplier associated with the incentive compatibility constraint (9'). Equations (12)-(13) imply that, compared to the first-best optimum, it is no longer optimal to completely smooth out taxes over time. In particular, equation (12) shows that altering the intertemporal distribution of taxes--which requires changing the amount of nominal obligations maturing in period 2--has an effect on period-2 inflation through the incentive-compatibility constraint. The L.H.S. of equation (12) correspond to the cost reduction from the cut in x_1 associated with a marginal increase in b_{12}. The first term of the R.H.S of equation (12) is the cost provoked by the higher x_2 called for by the increase in b_{12}. The second term of the R.H.S. of equation (12) is the effect on the incentive-compatibility constraint (9'). The multiplier μ, which we will show must be positive, represents the marginal social loss derived from increasing the gains from inflation perceived by government 2.

Since the presence of nominal debt introduces a distortion by providing an incentive to attempt (in vain) to reduce its real value through inflation, it is reasonable to expect that, in a time-consistent equilibrium, the government at time zero will find it optimal to reduce the amount of nominal obligations left in period 2 by raising tax revenues in period 1 relative to what was optimal under full precommitment. This phenomenon is what we call "debt aversion."

To verify that the presence of nominal debt induces "debt aversion," consider the simple case where $k=0$, (i.e., $S=0$); the proof for the case where $k > 0$ is presented in the appendix. If $k=0$, we claim that $x_1 > x_2$, which implies debt aversion since in the first-best optimum $x_1 = x_2 = x$. To prove the claim consider equations (9'), (12)-(14), and (5), where the terms involving the inflation tax on cash balances are set to zero. We will show by contradiction that x_1 must be different from x_2, and that x_1 cannot be smaller than x_2. The same argument also shows that $\mu>0$.

We will first show why, at the optimum, $x_1 \neq x_2$. Suppose that $x_1 = x_2$. Then, from equation (12), this would imply that $\mu = 0$, which also implies, from equation (14), that $\Pi_1 = \Pi_2$. However, the only case in which the equality between tax and inflation rates is consistent with equations (9') and (13) is when $x_2 = g$. This, however, is inconsistent with budget constraint (5) as long as $b > 0$ (as assumed), since tax revenues would suffice only to finance government spending.

Next, suppose that $x_2 > x_1$. Since $V''(x) > 0$, the latter implies $V'(x_2)>V'(x_1)$. Thus, by equation (12), $\mu\{V''(x_2)(x_2-g)+V'(x_2)\} < 0$. First, if $x_2>x_1$, it must be true that $V'(x_2)>0$ (i.e., $x_2 > 0$ to be able to raise revenue). Second, $x_2>g$, because otherwise tax revenues would not be enough to finance expenditures. Hence, $\mu < 0$. From equation (14) this implies that $H'(\Pi_2)<0$, which in turn implies, from equation (9') that $x_2 < g$. Again, this is inconsistent with the budget constraint since we started with $x_1<x_2$.

Therefore, the time-consistent equilibrium exhibits higher taxes in period 1 relative to period 2. Moreover, since when $k=0$ the second order condition for the choice of Π_2 requires $b_{02}R+b_{12}>0$, then, by equation (9'), at equilibrium $H'(\Pi_2)>0$. Since Π_1 is at its first-best level (i.e., where $H'(\Pi_1) = 0$), $\Pi_2 > \Pi_1 = 1$. 1/

The fact that $x_1 > x_2$ implies that it is optimal to lower the amount of debt to be left for the government in period 2 relative to the case of full precommitment. The intuition is clear; by equation (9') period-2 taxes are more costly than with full precommitment because they are linked to period-2 inflation. Therefore, if with full precommiment it was optimal to smooth completely taxes over time, now it is optimal to use less period-2 taxes relative to period-1 taxes. The lower debt maturing

1/ Recall that $\Pi=1$ is the first best level of inflation when $k=0$.

in period 2--because x_2 is lower than its first-best level 1/--has the effect of reducing (but not eliminating) the future incentive to increase inflation above the first-best optimum. Notice that the distortion of the intertemporal distribution of taxes occurs even though the discount rate in the government's objective function is equal to the real interest rate. 2/ Moreover, in this example, while the time profile of conventional taxes is downward-sloping, the opposite holds true for the time profile of inflation.

IV. Partial Precommitment II: Optimal Debt Maturity

Assume that the government in period 1 has the ability to precommit period-2 policy variables, 3/ but let us also assume--in order to differentiate this case from the one in the previous section--that at period zero the government is unable to make any commitments about future policies. This case may, thus, reflect a situation where there is a change of government at the end of the first period (i.e., period zero) and the incoming administration stays in power for the following two periods.

Government 1 minimizes the value of cost function (4) subject to the following intertemporal budget constraint:

$$(15)\quad b_{02}I_{02}/R\Pi_1\Pi_2 + b_{01}I_{01}/\Pi_1 + g(1+R)/R) = x_1 + S(\Pi_1) + R^{-1}[x_2+S(\Pi_2)]$$

Equation (15) is similar to equation (5), except that since planning is done at time 1, the perfect-foresight conditions $I_{02}=R^2\Pi_1\Pi_2$ and $I_{01}=R\Pi_1$ are not imposed.

The first-order conditions for an interior optimum imply:

$$(16)\quad x_1 = x_2 = x$$

$$(17)\quad H'(\Pi_1) = V'(x)\{S'(\Pi_1) + [b_{02}I_{02}/R\Pi_1\Pi_2 + b_{01}I_{01}/\Pi_1](1/\Pi_1)\}$$

$$(18)\quad H'(\Pi_2) = V'(x)[S'(\Pi_2) + (b_{02}I_{02}/\Pi_1\Pi_2)(1/\Pi_2)]$$

1/ The fact that x_2 is lower than its first-best level follows from the fact that, since we have shown that $x_1 > x_2$, the opposite would imply excess government revenues.

2/ See Obstfeld (1988) where debt aversion is shown to arise even though bonds are fully indexed to the price level.

3/ All that is needed is the ability to make precommitments about inflation in period 2.

As in the full-precommitment case (Section 2), equation (16) indicates that complete tax smoothing over time is optimal. This result is the consequence of a "separation" property that exists between the optimal choice of conventional taxes and inflation rates. This separation property derives from two facts. First, the cost function is separable in x and Π. Second, the government is able to borrow (or lend) between periods 1 and 2, without being subject to a time-inconsistency problem. As a result, changes in the intertemporal distribution of taxes can be achieved through the use of b_{12}, without affecting at all the choice of inflation rates.

It is important to notice, however, that while the above-mentioned separation property makes it optimal to completely smooth out taxes over time, as in the first-best, the level of x differs from the first-best because as will be argued, in general, the choice of Π_1 and Π_2 differs from that of the first-best optimum. However, if k=0 (i.e., there is no inflation tax on cash balances), then x would be the same as in the full precommitment case of Section 2.

Equation (17) and (18), as was the case with equation (9) in Section 3, reflect the fact that, since no precommitment is possible at time 0, government 1 is left free to resort to inflation in order to reduce the output value of its obligations (recall that, at that point in time, the nominal value of outstanding debt, b_{01} and b_{02}, as well as the nominal interest rate factors I_{01} and I_{02}, are pre-determined variables). Equations (17) and (18) state that, at the optimum, the marginal costs of increasing inflation in period 1 and 2 are equated to decline in the tax costs due to the revenue gains of the associated inflation tax. The base of the inflation tax, however, includes debt obligations maturing in periods 1 and 2. In particular, while period-1 inflation reduces the real value of total government debt issued in period 0, period-2 inflation may only be used to reduce the real value of nominal debt maturing in period 2. This implies that, from the point of view of government 1, it is always optimal to set Π_1 greater than Π_2 whenever b_{01} is positive.

Since in equilibrium there are no revenue gains from excess inflation--i.e., conditions (10) and (11) hold--the government's budget constraint (15) reduces to equation (5); thus, at equilibrium, revenue from inflation is given by the inflation tax on cash balances only.

Interestingly, unlike the examples of Sections 2 and 3, the maturity structure of debt at time zero is no longer irrelevant. The maturity structure of nominal debt has a role to play because it influences the optimal choice of inflation rates in periods 1 and 2 by government 1 (who, by assumption, is able to precommit future policies).

Consider, therefore, the decision faced by government 0. Using equation (10), equations (17) and (18) boil down to:

$$(17') \quad \Pi_1 H'(\Pi_1) = V'(x)[\Pi_1 S'(\Pi_1) + bR]$$

(18′) $\Pi_2 H'(\Pi_2) = V'(x)[\Pi_2 S'(\Pi_2) + b_{02}R^2]$

Dividing (18′) by (17′), and assuming away division by zero, the following expression obtains:

(19) $b_{02}/b = \Pi_2[H'(\Pi_2) - V'(x)S'(\Pi_2)]/\Pi_1[H'(\Pi_1) - V'(x)S'(\Pi_1)]R$

Equation (19) implies that, if $b_{02}R <(=) b$ then $\Pi_1 >(=) \Pi_2$. This results from the fact that $f(\Pi) \equiv \Pi[H'(\Pi) - V'(x)S'(\Pi)]$ is an increasing function given that $H''(\Pi) > 0$ and $S''(\Pi) < 0$.

Equations (17′), (5), and (19) characterize the choice of taxes and inflation rates in period 1 as functions of the maturity structure of initial debt, b_{02}/b: 1/

(20a) $x = \hat{x}(b_{02}/b,\ b,\ g)$; $\partial x/\partial(b_{02}/b) < 0$, $\partial x/\partial b > 0$, $\partial x/\partial g > 0$

(20b) $\Pi_1 = \hat{\Pi}_1(b_{02}/b,\ b,\ g)$; $\partial\Pi_1/\partial(b_{02}/b) < 0$, $\partial\Pi_1/\partial b > 0$, $\partial\Pi_1/\partial g > 0$

(20c) $\Pi_2 = \hat{\Pi}_2(b_{02}/b,\ b,\ g)$; $\partial\Pi_2/\partial(b_{02}/b) > 0$, $\partial\Pi_2/\partial b > 0$, $\partial\Pi_2/\partial g > 0$

Hence, a change in the maturity structure which increases b_{02}/b generates a fall in x and Π_1, and an increase in Π_2. This implies that the increase in b_{02}/b has two effects. First, it induces a substitution between Π_1 and Π_2; more specifically, there is a stronger incentive to use Π_2 relative to Π_1. Second, the substitution of Π_2 for Π_1 generates an increase in the present value of seigniorage, $S(\Pi_1)+R^{-1}S(\Pi_2)$, generating a fall in x. 2/

1/ We assume that the conditions of the Implicit Function Theorem obtain. Moreover, the following results assume existence of a regular minimum (i.e., a minimum where the second-order sufficient conditions are satisfied).

2/ The intuition behind the effects of a change in b_{02} can be obtained through contradiction. *Using the fact that $H''>0$ and $S''<0$, we can observe* that equation (17′) implies that Π_1 and x move always in the same direction. Consider now the effects of an increase in b_{02}, ceteris paribus. If an increase in b_{02} increases x, it must also increase Π_1. However, from equation (18′), it can be seen that the increase in both x and Π_1 implies an increase in Π_2, which is inconsistent with budget constraint (5). Therefore, an increase in b_{02} must reduce x and Π_1. It is clear that, if x and Π_1 fall, only an increase in Π_2 is consistent with both equations (18′) and (5).

An increase in both initial debt, b, and government expenditure induce government 1 to use the three instruments, i.e., x, Π_1, and Π_2, to meet the higher revenue needs. In addition, both the increase in b and the increase in x raise the incentive for government 1 to use inflation.

The time-consistent choice of the optimal maturity structure of the initial stock of public debt by government 0 responds to the objective of minimizing social loss in equation (4) subject to the incentive-compatibility constraints (20).

A few things can be said at an intuitive level before looking at the general solution of this problem. Suppose that k=0. In this case, it can be seen that equations (5) and (17') determine x and Π_1 independently from b_{02}/b; namely, issuing long term debt affects only the choice of inflation in period 2. Since the tax base for period-1 inflation is the total stock of debt, Π_1 is independent from the debt maturity. Moreover, since in equilibrium there are no revenue gains associated with inflation, the optimal choice of maturity is clear: government 0 should issue only short term debt (i.e., $b_{02}=0$). Issuing long-term debt would only raise the inflation tax base in period 2--hence resulting in higher period-2 inflation--with no effect on the choice of x and Π_1.

One important aspect of this solution should be noticed. In this model, debt has two functions. First it is used to smooth out conventional taxes and inflation over time. Second, it affects the future incentive to resort to inflation. From the point of view of taxation smoothing, given that short term debt can be issued in every period, the possibility of issuing long-term debt adds nothing to the opportunity set. From the point of view of the incentives to inflate, since the government in period 1 can issue b_{12} without being subject to the problem of time-inconsistency, long term debt is always dominated by the alternative of issuing successively the equivalent amount of short term debt. Hence, the solution of setting $b_{02}=0$ reflects the principle that it is optimal to use the comparative advantage that the government has in period 1 to make pre-commitments about Π_2.

Let us tackle the more general case with k>0 and consider intuitively why it is no longer optimal at time 0 to issue only short-term debt (i.e., it is no longer optimal to set $b_{02}=0$). Assume for the sake of the argument that R=1. If all debt is issued with short-term maturity, it can be seen from equations (5), (17') and (18') that, compared to the first-best optimum, Π_1 is "too high," while x and Π_2 are "too low." This reflects the fact that the incentive to inflate boosts up period-1 (conventional) inflation tax relative to its first-best level and, hence, lower levels of x and Π_2 are needed to balance the budget. In addition, from the intra-temporal first-order conditions (17') and (18'), it can be seen that, when $b_{02}=0$, while the cost of Π_1 exceeds the true associated gains (i.e., $H'(\Pi_1) > V'(x)S'(\Pi_1)$), this is not the case for Π_2 (where the above relation holds with equality). Therefore, given the convexity of the cost function, an increase of Π_2 coupled with a fall in Π_1 could improve welfare. This is precisely what an increase in b_{02} (from $b_{02}=0$) does; for it induces the government to increase Π_2 and to reduce Π_1 and x. The fall in x and Π_1 occur because, when k>0, the increase in Π_2 has a positive effect on government revenues which was not present in the case where k=0. This shows that the presence of a positive demand for money provides the link between changes in Π_2, induced by different debt maturity structures, and changes in x and Π_1. When k=0, we have shown that debt maturity could not be used to affect x and Π_1.

Two important points follow from the above discussion. First, it is optimal to issue both short and long term debt. Second, since when k=0 it was optimal to set $b_{02}=0$, the presence of a positive (conventional) inflation tax base lengthens the optimal maturity of government debt.

The first order condition that, along with the set of incentive compatibility constraints (20), characterizes the optimal time-consistent policy equilibrium is given by:

$$(21)\quad [(1+R)/R][\Pi_1 H''(\Pi_1)+H'(\Pi_1)+V'(x)S'(\Pi_1)][H'(\Pi_2)-V'(x)S'(\Pi_2)]$$

$$+ [bR+\Pi_1 S'(\Pi_1)]V''(x)[H'(\Pi_2)S'(\Pi_1)-H'(\Pi_1)S'(\Pi_2)] = 0$$

Equation (21) shows that, at the time-consistent equilibrium, $H'(\Pi_2)>V'(x)S'(\Pi_2)$ from equation (18'), and $H'(\Pi_1)/H'(\Pi_2)>S'(\Pi_1)/S'(\Pi_2)$ from equations (17') and (18'). These inequalities imply that $0<(b_{02}/b)^o<1$, where $(b_{02}/b)^o$ denotes the optimal debt maturity structure. 1/

V. Optimal Debt Maturity under No Precommitment

Unlike Sections 3 and 4, we now assume that *no* government has the ability to precommit future policies. Therefore, the formulation of a time-consistent policy at time 0 requires to take into account all future reaction functions with respect to present policies. An interesting feature of this section's example is that it combines the role of debt maturity with the issue of debt aversion discussed in Section 3.

In period 2, the government's behavior is analogous to that described in Section 3. Government 2 minimizes the value of cost function $H(\Pi_2) + V(x_2)$ subject to equation (1), where the only non pre-determined variables are x_2 and Π_2. The optimal choice of period-2 inflation is given by equation (9), which reflects the government's incentive in period 2 to use inflation to reduce the real value of nominal debt obligations maturing in period 2. Since market interest rates fully reflect equilibrium inflation, the choice of Π_2 is given by equation (9'), which we re-write below for convenience:

1/ Notice that if $b_{02}=b$, then $\Pi_1=\Pi_2$ and $H'(\Pi_1)/H'(\Pi_2)=S'(\Pi_1)/S'(\Pi_2)$.

$$(9') \quad \Pi_2 H'(\Pi_2) = V'(x_2)[b_{02}R^2 + b_{12}R + \Pi_2 S'(\Pi_2)] =$$

$$= V'(x_2)(x_2 - g + k)$$

Government 1 can choose Π_1, x_1, and b_{12}, but cannot make commitments about period-2 variables. The formulation of its time-consistent policy takes into account the fact that government 2 decides Π_2 according to equation (9'). *It is important to notice that the reaction function of* government 2, given by equation (9'), is affected by the nominal debt issued by government 1, b_{12}.

The time-consistent policy for government 1 is the one that minimizes the value of cost function (4), subject to the intertemporal budget *constraint (15) and the incentive compatibility constraint (9') for* government 2. By substituting equations (1) and (2), where condition (11) holds, for x_1 and x_2 in the cost function and in the incentive compatibility constraint (9'), the problem reduces to choosing b_{12}, Π_1 and Π_2 to minimize social loss in equation (4) subject to equation (9').

The first order conditions for b_{12}, Π_1, and Π_2 are given by:

$$(22) \quad - V'(x_1) + V'(x_2) + \mu\{V''(x_2)(x_2-g+k) + V'(x_2)\}R = 0$$

$$(23) \quad - V'(x_1)[(b_{01}I_{01}/\Pi_1^2)+S'(\Pi_1)] + H'(\Pi_1) - R^{-1}V'(x_2)(b_{02}I_{02}/\Pi_1^2\Pi_2)$$

$$- \mu\{V''(x_2)(x_2-g+k) + V'(\Pi_2)\}(b_{02}I_{02}/\Pi_1^2\Pi_2) = 0$$

$$(24) \quad R^{-1}\{H'(\Pi_2) - V'(x_2)[(b_{02}I_{02}/\Pi_1\Pi_2^2)+S'(\Pi_2)]\}$$

$$- \mu\{V''(x_2)(x_2-g+k) + V'(x_2)\}[(b_{02}I_{02}/\Pi_1\Pi_2^2)+S'(\Pi_2)]$$

$$- \mu[\Pi_2 H''(\Pi_2)+H'(\Pi_2)] = 0$$

First order conditions (22)-(24) boil down to the first order conditions under the case of partial precommmitment of Section 4 when μ equals zero. The analysis of Section 3, however, is useful to interpret the additional terms appearing in equations (22)-(24), which represent the effects of the decision variables on the incentive compatibility constraint (9'). Equation (22), which corresponds to the choice of b_{12}, is the same as equation (12). The first term reflects the cost reduction from the tax cut in period 1 made possible by the increase of b_{12}. Similarly, the second term in (22) reflects the cost from the higher period-2 conventional taxes necessary to finance the repayment of b_{12}. The third term, where μ is the Lagrange multiplier associated with equation (9'), is the effect of borrowing on the government's incentive to inflate in period 2; namely, higher taxes associated with higher b_{12}, as well as the larger stock of nominal obligations in period 2, increase the government's incentive to raise future inflation.

The first and third terms in equation (23) represent the cost reduction from the tax cut allowed by the fall in the real value of nominal obligations maturing in period 1 associated with an increase in Π_1. The second term is the direct cost of increasing Π_1. Similarly to what was discussed above, the third term in equation (23) reflects the effect of Π_1, via a change in x_2, on the incentive-compatibility constraint.

The first term in equation (24) is the direct cost of increasing Π_2, while the second term reflects the tax cut associated with the reduction in the real value public debt maturing in period 1. The remaining terms represent the effects of changes in Π_2 on the incentive-compatibility constraint.

Since the government's incentive to resort to inflation in period 1 is recognized by the market, the nominal interest rates applying to debt issued in period 0 satisfy condition (10). Using (10), the first order conditions (22)-(24) can be written as:

$$(25) \quad \Pi_1 H'(\Pi_1) = V'(x_1)[\Pi_1 S'(\Pi_1) + bR]$$

$$(26) \quad \frac{V'(x_1) - V'(x_2)}{\Pi_2 H'(\Pi_2) - V'(x_1)[(b_{02}/b)bR^2 + \Pi_2 S'(\Pi_2)]} = \frac{\{V''(x_2)[x_2 - g + k] + V'(x_2)\}}{[\Pi_2 H''(\Pi_2) + H'(\Pi_2)]\Pi_2}$$

Equation (26) indicates that, as in Section 3 and unlike the case of partial precommitment studied in Section 4, it is usually not optimal to smooth completely taxes over time. Recall the discussion made in Section 4. When government 1 was able to make precommitments the result of complete tax smoothing was dependent on the fact the government was able to issue b_{12} without a time-inconsistency problem. Therefore, changes in the intertemporal distribution of taxes could be supported by borrowing (or lending) without affecting the choice of inflation rates. In the present case, changes in b_{12} affect the choice of Π_2, as can be seen from the incentive compatibility constraint (9') breaking down the separation result encountered in Section 4.

Further analysis of equation (26) shows that the intertemporal distribution of taxes under no-precommitment is directly related to whether the government borrows or lends in period 1. Using equation (9'), equation (26) can be written as:

$$(26') \quad \frac{V'(x_1) - V'(x_2)}{b_{12}R} = \frac{V'(x_1)[V''(x_2)(x_2 - g + k) + V'(x_2)]}{\{SOC\}}$$

where $\{SOC\}>0$ is the second order condition for the choice of Π_2 in period 2. *Since the right hand side of (26') is positive, the following* relationship obtains: 1/

$$(27) \quad x_1 \geq x_2 \quad \text{iff} \quad b_{12} \geq 0$$

The above inequality indicates that the only time-inconsistency problem which matters to government 1 for altering the intertemporal distribution of taxes is the one concerning b_{12}. In particular, if $b_{12} > 0$, government 2 is provided with an additional (to b_{02}) incentive to increase Π_2. As a result, if without that time-inconsistency problem, it was optimal for government 1 to choose $x_1 = x_2$, now government 1 internalizes part of the cost of period-2 inflation and finds it optimal to increase taxes in period 1 and decrease taxes in period 2 to reduce its borrowing in period 1. The opposite reasoning applies if $b_{12} < 0$, since a negative b_{12} provides an incentive to deflate.

The possibility expressed in equation (27) that, in equilibrium, period-1 conventional taxes could be lower than period-2 conventional taxes may appear quite surprising if one expects the "debt aversion" result of Section 3 to carry over to the present case. In particular, if $k=0$, then $x_1<x_2$ implies that it is optimal to postpone, instead of anticipating, tax revenue collection. The underlying intuition, however, is clear. Consider what are the differences between the problem faced by government 1 in this section and that faced in Section 3. The only difference lies in the government's budget constraint, which in Section 3 was given by equation (5) while in this section is given by equation (15). To sharpen intuition let us focus on the case where $k=0$. In the problem studied in Section 3, debt aversion always occurs because x_2, being positively linked to Π_2 through equation (9'), is more costly than x_1 relative to the first-best case. Moreover, if $k=0$, then budget constraint (5) is independent of Π_1 and Π_2. In the problem studied in this section,

1/ Notice that, by equations (22) and (27), $\mu>(=)(<)0$ if $b_{12}>(=)(<)0$.

Π_2--in addition to being linked to x_2 by equation (9')--affects budget constraint (15) by altering the real value of nominal debt maturing in period 2. From government 1's perspective this effect, which exists only if $b_{02}>0$, goes in the direction of making Π_2, and therefore x_2, less costly. Hence, depending on the amount of long-term debt inherited by government 1 it is perfectly plausible to encounter a situation in which the debt aversion result is overturned because government 1 has a sufficiently large incentive to raise x_2 (and Π_2) relative to x_1. Interestingly, by equation (27), we know that government 1 has debt aversion only when it wants to issue new debt.

Equations (25), (26), (9'), and (5) characterize the time-consistent policy for government 1, as a function of the maturity structure of initial debt and the exogenous variables of the model: 1/

(28a) $$x_1 = \hat{x}_1(b_{02}/b,\ b,\ g)$$

(28b) $$x_2 = \hat{x}_2(b_{02}/b,\ b,\ g)$$

(28c) $$\Pi_1 = \hat{\Pi}_1(b_{02}/b,\ b,\ g)$$

(28d) $$\Pi_2 = \hat{\Pi}_2(b_{02}/b,\ b,\ g)$$

To study the optimal choice of the maturity structure of initial debt by government 0 let us examine how conventional taxes and inflation rates respond to changes in debt maturity in the reaction functions summarized by equations (28). Notice that equation (9') implies that Π_2 and x_2 always move together. Similarly, equation (25) implies that Π_1 and x_1 also move together. Moreover, the government's budget constraint (5), along with the above two statements, implies that Π_2 and Π_1 must move in opposite direction. The intuition behind these relationships should be clear by now. Therefore, the only relationship that needs to be understood is that between b_{02} and Π_2. Interestingly, the effect of b_{02} on Π_2 entails a relationship between governments 0 and 2, through government 1. An increase in b_{02} may not necessarily imply a higher incentive to inflate for government 2 if government 1 responds by reducing b_{12} enough to generate a fall in the inflation tax base of government 2. If an increase in b_{02} is not offset by a reduction in b_{12}, and results in a higher incentive to inflate for government 2, then it generates an increase in Π_2, and, by previous considerations, a fall in Π_1, an increase

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1/ Again, we assume that the conditions of the Implicit Function Theorem obtain, and we assume existence of a regular minimum.

in x_2, and a fall in x_1. 1/ In the appendix we show that if the incentive compatibility constraint (9') is linearized, i.e., only first-order effects are taken into account, it is always the case that a higher b_{02} induces an increase in Π_2.

The optimal maturity structure for the initial stock of government debt is the one that minimizes social loss in equation (4) subject to the incentive-compatibility constraints (28). The first-order condition, which along with (28), determines the optimal maturity structure, $(b_{02}/b)^*$, is given by:

$$(29)\quad H'(\Pi_1)[\partial\hat{\Pi}_1/\partial(b_{02}/b)] + V'(x_1)[\partial\hat{x}_1/\partial(b_{02}/b)] +$$

$$R^{-1}(H'(\Pi_2)[\partial\hat{\Pi}_2/\partial(b_{02}/b)] + V'(x_2)[\partial\hat{x}_2/\partial(b_{02}/b)]) = 0$$

To investigate how optimal debt maturity depends on exogenous variables like b, k, and g, we simulate numerically the model. For the numerical example we use the following quadratic cost function:

$$(30)\quad L = x_1^2 + (\Pi_1-1)^2 + R^{-1}[x_2^2 + (\Pi_2-1)^2]$$

The results are summarized in Table 1. The benchmark case is characterized by the following parameter values: the initial stock of debt, b, is assumed to be equal to 20 percent of GNP, the demand for high-powered money, k, equals 10 percent of GNP, and government expenditure, g, equals 40 percent of GNP. The real interest rate (as well as the discount rate in cost function (30)) is assumed to be equal to zero, i.e., R=1.

The simulations show that it is optimal to issue both short and long-term debt. In the cases considered, optimal debt maturity calls for a share of long term to total debt between 30 and 45 percent. The resulting inflation rates show that always $\pi_1>\pi_2$, a result that is consistent with Obstfeld's (1989) dynamic analysis of seigniorage. In this model, however, the downward-sloping time profile of inflation follows from the presence of non-indexed debt and it is not necessarily linked to inflation tax revenue considerations or to the presence of debt aversion. 2/ In fact, even if k=0 (i.e., the inflation tax collects no

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1/ In the numerical simulations presented in the next section an increase in b_{02} has always a positive impact on Π_2.

2/ Note that if in this model debt were indexed, as in Obstfeld (1988), there would be no time-inconsistency problem because we are assuming that the demand for money is interest-inelastic.

Table 1. Optimal Debt Maturity Under no Precommitment

	Benchmark Case	b		k		g	
		10	30	5	15	30	50
$b0_2/b$	38.4	31.2	42.9	34.3	42.0	45.6	33.0
π_1	12.6	7.9	17.7	11.0	14.1	10.3	14.8
π_2	8.5	5.9	11.3	6.6	10.2	7.0	9.8

Note: Numbers are expressed in percentage points.

revenue) and $x_1 \leq x_2$ (there is no debt aversion) it can be shown that $\pi_1 > \pi_2$ because the base for Π_1 (i.e., bR) exceeds at the optimum the base for Π_2 (i.e., $b_{02}R^2 + b_{12}R$).

The simulations presented in Table 1 illustrate the effects of changes in b, k, and g. They indicate that optimal debt maturity lengthens with increases in b and k, while it shortens with increases in g. The intertemporal distribution of inflation appears also to be affected in a systematic way. An increase in b raises the ratio π_1/π_2, while the increase in k reduces it. An increase in g appears to cause only a slight upward movement in the ratio π_1/π_2.

VI. Conclusions

This paper has provided relatively simple examples of how the maturity structure of public debt matters when the government has incentives to use inflation ex-post to reduce the real value of its nominal liabilities. The nature of the optimal policy has been found to be quite sensitive to the type of precommitment enjoyed by the government and, hence, to the type of incentive-compatibility constraints at stake.

The analysis suggests that when governments are partially able to precommit policies the optimal maturity structure of nominal debt tends to be short. On the other hand, when no precommitment exists, our simulations suggest that it is optimal to have a relatively balanced maturity structure. The simulations also suggest that optimal maturity is longer the higher is the level of public debt, and shorter the higher is government spending.

The examples provided in this paper should be viewed as first steps toward a more general characterization of the role of maturity in managing incentive-compatible inflation over time. These first steps, however, show that the problem of characterizing the time-consistent policy becomes analytically complex even when the basic structure of the economy is kept at a minimum.

Appendix

I. Debt Aversion

In this section we prove that debt aversion occurs for the general case of Section 3 in which $k>0$. If $k>0$, debt aversion implies that period-1 revenues--including the inflation tax--in the second-best equilibrium are higher than in the first-best optimum, implying that less debt is left in period 2 compared to the first-best case. Define the first-best allocation by (x^*,Π^*). Since there is complete smoothing over time of conventional taxes and inflation, period-1 (and 2) revenues are equal to:

$$\text{(I.1)}\quad x^* + S(\Pi^*) = g + R^2b/(1+R) \equiv c$$

Hence, "debt aversion" is characterized by:

$$\text{(I.2)}\quad x_1 + S(\Pi_1) > c$$

where (x_1, x_2, Π_1, Π_2) is the second-best (time-consistent) optimum.

For convenience, we re-write (and re-number) below the equations that characterize the time-consistent equilibrium:

$$\text{(I.3)}\quad \Pi_2H'(\Pi_2) = V'(x_2)[x_2 - g + S(\Pi_2) + \Pi_2S'(\Pi_2)] = V'(x_2)(x_2 - g + k)$$

$$\text{(I.4)}\quad V'(x_1) = V'(x_2) + \mu\{V''(x_2)[x_2-g+k] + V'(x_2)\}R$$

$$\text{(I.5)}\quad H'(\Pi_1) = V'(x_1)S'(\Pi_1)$$

$$\text{(I.6)}\quad H'(\Pi_2) = V'(x_1)S'(\Pi_2) + \mu[\Pi_2H''(\Pi_2)+H'(\Pi_2)]R$$

$$\text{(I.7)}\quad bR + g[(1+R)/R] = x_1 + S(\Pi_1) + R^{-1}[x_2 + S(\Pi_2)]$$

Proposition 1: $x_1 > x_2$.

Proof:

1. $x_1 \neq x_2$. Suppose $x_1=x_2$. Then, equation (I.4) implies that $\mu\{V''(x_2)(x_2-g+k)+V'(x_2)\}=0$, which can only be true if $\mu=0$ since the term in brackets depends on $V''(x_2)$ which may be chosen arbitrarily. If $\mu=0$,

then equations (I.5) and (I.6) imply that $\Pi_1=\Pi_2$. Moreover, equations (I.4) and (I.6) imply that $x-g+S(\Pi)=0$, which is inconsistent with budget constraint (I.7) if $b>0$. Hence, $x1 \neq x_2$.

2. If $k=0$, then $x_1 > x_2$. This has already been proven in the text.

3. Using a), b), the property that the second-best equilibrium is continuous in k, we conclude by the Mean Value theorem that if $k>0$, then $x_1 > x_2$.

Q.E.D.

<u>Proposition 2 (Debt Aversion)</u>: $x_1 + S(\Pi_1) > x^* + S(\Pi^*) = c$.

<u>Proof</u>:

1. $x_1 + S(\Pi_1) \neq c$, $x_2 + S(\Pi_2) \neq c$. Suppose not, i.e.,

(I.8a) $x_1 + S(\Pi_1) = c$

(I.8b) $x_2 + S(\Pi_2) = c$.

Let

(I.9) $H'(\Pi_2) - V'(x_2)S'(\Pi_2) = \alpha$.

Notice that equations (I.9) and (I.3) are identical if $\alpha \equiv V'(x_2)(c-g)/\Pi_2$, where $c > g$ by definition if $b>0$.

Consider the system of equations (I.9), (I.4)-(I.7). If $\alpha=0$, then this system yields the first-best allocation (x^*, Π^*). In particular, we know that the first-best allocation satisfies (I.8a) and (I.8b). Since, we are supposing that (I.8b) holds, then since $x2 = c - S(\Pi_2)$ the L.H.S of equation (I.9) is an increasing function of Π_2.

Let now $\alpha>0$ at the value at which the above system of equations yields the second-best equilibrium. Since the L.H.S. of equation (I.9) is an increasing function of Π_2, then $\Pi_2 > \Pi^*$, which implies by equation (I.8b) that $x_2<x^*$.

Consider what happens with x_1 and Π_1. By equation (I.5), we know that x_1 and Π_1 move together. Hence, if $x_1 \neq x^*$, then equation (I.8a) does not hold, providing a contradiction since it would be inconsistent with (I.8b) and budget constraint (I.7).

To establish that in fact $x_1 \neq x^*$, suppose that $x_1 = x^*$ and prove by contradiction. Since equation (I.3) implies that x_2 and Π_2 move together, the only way in which the budget constraint could be satisfied is if also $x_2 = x^*$ and $\Pi_2 = \Pi^*$. But we have already established that $\Pi_2 > \Pi^*$.

2. If $k=0$, then $x_1 > x_2$ by Proposition 1. Also, since $k=0$, the inequality $x_1 + S(\Pi_1) > c > x_2 + S(\Pi_2)$ is met.

3. Using 1), 2), the property that the equilibrium is continuous in k, we conclude by the Mean Value theorem that if $k>0$, then $x_1 + S(\Pi_1) > c$.

Q.E.D.

II. Optimal Maturity under No Precommitment

In this section we show that, by linearizing the incentive compatibility constraint (9'), an increase in b_{02} in system (28) generates, *ceteris paribus*, *an increase in* Π_2 *and* x_2 *and a fall in* Π_1 *and* x_1. After linearizing, equation (9') can be written as:

$$\text{(II.1)} \qquad x_2 = \theta \Pi_2 + Z(k, g)$$

where θ is a positive constant and Z is a linear function of k and g. Using equation (II.1) instead of (9'), the reaction function (28) would be characterized by equation (II.1) and the following set of equations:

$$\text{(II.2)} \qquad - V'(x_1) + V'(x_2) + \mu R = 0$$

$$\text{(II.3)} \qquad \Pi_1 H'(\Pi_1) = V'(x_1)[bR + \Pi_1 S'(\Pi_1)]$$

$$\text{(II.4)} \qquad \Pi_2 H'(\Pi_2) - V'(x_1)[b_{02}R^2 + \Pi_2 S'(\Pi_2)] - \theta[V'(x_1) - V'(x_2)] = 0$$

$$\text{(II.5)} \qquad bR + g[(1+R)/R] = x_1 + S(\Pi_1) + R^{-1}[x_2 + S(\Pi_2)]$$

It is straightforward to verify that, using equations (II.1)-(II.5), $\partial\Pi_2/\partial b_{02} > 0$, $\partial x_2/\partial b_{02} > 0$, $\partial\Pi_1/\partial b_{02} < 0$, and $\partial x_1/\partial b_{02} < 0$ in system (28).

References

Alesina, Alberto, Prati, Alessandro and Guido Tabellini, "Debt Panics and Debt Management in Italy," unpublished (June 1989).

Barro, Robert J., and David B. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy, Vol. 91 (August 1983), pp. 589-610.

Calvo, Guillermo A., "Servicing the Public Debt: The Role of Expectations," The American Economic Review, Vol. 78, No. 4 (September 1988).

_____, "Controlling Inflation: The Problem of Non-Indexed Debt" (Washington: International Monetary Fund), Working Paper No. WP/88/29 (March 1988).

_____, "On the Time Consistency of Optimal Policy in a Monetary Economy," Econometrica, Vol. 46 (November 1978), pp. 1411-28.

Calvo, Guillermo A., and Maurice Obstfeld, "Time Consistency of Fiscal and Monetary Policy: A Comment," Institute for International Economic Studies, Seminar Paper No. 427 (Stockholm, Sweden, January 1989).

Calvo, Guillermo A., and Pablo E. Guidotti, "Indexation and Maturity of Government Bonds: A Simple Model" (Washington: International Monetary Fund), Working Paper No. WP/89/46 (May 1989).

Giavazzi, Francesco, and Marco Pagano, "Confidence Crises and Public Debt Management," unpublished (Universita' di Bologna, CEPR, NBER, and Universita' de Napoli, August 1988).

Kydland, Finn E., and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy, Vol. 85 (1977).

Lucas, Robert E., Jr., and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy Without Capital," Journal of Monetary Economics, Vol. 12 (July 1983), pp. 55-94.

Obstfeld, Maurice, "Dynamic Seigniorage Theory: An Exploration," NBER Working Paper 2869 (February 1989).

Persson, Mats, Torsten Persson, and Lars E.O. Svensson, "Time Consistency of Fiscal and Monetary Policy," Econometrica, Vol. 55 (November 1987), pp. 1419-32.

Tobin, James, "An Essay on the Principles of Debt Management," in Fiscal and Debt Management Policies, reprinted in J. Tobin, Essays in Economics, Vol. 1, Prentice-Hall 1963 (North-Holland, Amsterdam, 1971).