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Valuation of Menu Items in Debt Restructuring

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Abstract

This paper outlines a procedure for calculating the cash value of "menu items" in debt restructuring proposals, including par and non-par exchanges, with enhancements consisting of either interest or principal guarantees. It is argued that under certain plausible assumptions interest and principal guarantees are directly equivalent to cash buy-backs. Using these assumptions, formulas to calculate the exchange ratios, resource requirements, interest rates, and net debt reduction for particular menu items are derived. It is shown that there is not a direct relationship between the exchange discount and the market price.

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Summary

This paper outlines a procedure for calculating the cash value of "menu items" in debt restructuring proposals, including par (when the face value of new debt issued equals the face value of old debt) and non-par exchanges, with enhancements consisting of either interest or principal guarantees.

The paper argues that under certain plausible assumptions, interest and principal guarantees are directly equivalent to cash buy-backs. Using these assumptions, formulas are derived to calculate the exchange ratios, resource requirements, interest rates, and net debt reduction for particular menu items.

The analysis of these formulas shows that exchange discounts consistent with fair arbitrage conditions are generally not equal to cash market discounts.

I. Introduction

This paper outlines a procedure for calculating the cash value of "menu items" in debt restructuring proposals, including par and non-par exchanges, with enhancements consisting of either interest or principal guarantees. It is argued that under certain plausible assumptions, interest and principal guarantees are directly equivalent to cash buy-backs. Using these assumptions, formulas to calculate the exchange ratios, resource requirements, interest rates, and net debt reduction for particular menu items are derived. It is shown that exchange discounts consistent with fair arbitrage conditions are generally not equal to cash market discounts.

II. Basic Assumptions

We start with two key assumptions about the resources and the behavior of the debtor country. We assume that there is a given (but possibly uncertain) flow of resources available to make debt-service payments, and that the debtor country is committed to meeting all its contractual obligations, subject only to this resource constraint. We can think of the available resources being determined by the trade surplus. A key implication of this assumption is that the present value of debt-service payments will be the same before and after any restructuring of the debt.

A final assumption used in this analysis is that the market value of debt is equal to the expected present value of all future interest and principal payments. This implies that creditors are risk neutral, and will value alternative debt instruments equally if their expected returns are equal, regardless of the time profile of the payments.

III. Valuation of Interest Guarantees

A central problem in valuing debt restructuring packages is determining the cash value to the creditors (or the cost to the debtors) of provisions which guarantee the payment of interest.

The problem may be put as follows: Suppose $100 million is put into an escrow account to guarantee one year's interest on a debt issue. If the guarantee is fully drawn in the first year, the value to the creditors is $100 million. But if the guarantee is not fully drawn until later, the present value to the creditors is less, because payments in the future are discounted at some rate. However, if the interest earned on the escrow account is added to the guarantee fund, the present value to the creditors remains the same (assuming that the creditors' discount rate is equal to the market rate of interest). This result only holds, however, if the full amount of the guarantee, including the accrued interest, is eventually paid out. Otherwise, the present value of the guarantee depends on the probability of full payment of interest owed by the debtor.

The technical problem of valuing interest guarantees when there is some probability that full payment of interest will be made by the debtor

without using the guarantee is both difficult and interesting. 1/ However, in our view this case is not particularly relevant, for the reasons discussed below. We assume instead that debtors and creditors will design arrangements in which creditors expect to receive the full cash value of an interest guarantee. Thus, a guarantee of $100 million in interest payments would be worth $100 million in present value to the creditors, and would require the debtors to put up $100 million in collateral.

There are two reasons for making this assumption. One is precisely because it greatly simplifies the analysis--it does not seem plausible that either creditors or debtors (much less third party analysts) could evaluate the cash value of a highly conditional guarantee with any degree of certainty, and they would therefore be unlikely to enter into such arrangements unless the outcome were reasonably certain. A second reason is that any guarantee in which the creditors do not expect to get the full cash value is inefficient, because the debtor must pay the opportunity cost of making resources available even if they are not drawn on. 2/ This inefficiency implies that it is in the interest of both parties to avoid an agreement in which the creditors do not receive the cash value of any enhancements provided. 3/

The net value of a guarantee to the creditors may be less than the nominal amount even if the guarantee funds are paid out in full, if there is some risk that the debtor may reduce the debt service payments it would otherwise have made. The reduction in the burden of the debt might lead to lower expected payments in states of the world in which the debtor could, but would no longer be obliged to, make payments required under the old debt arrangements. For example, the country might choose to skip the last interest payment, if the guarantee fund were not drawn on until that point, and instead use the funds for additional imports. 4/ Cases of

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1/ Several different approaches to this problem have been adopted. The actual interest payment can be modeled as an independent random variable [see Dooley (1988) and Folkerts-Landau (1989)], or as a stochastic process [Borensztein and Pennacchi (1989), Nocera (1989)]. Another approach is based on the discount rate implied by the market yield to maturity [Kahn (1989); Telljohann and Purcell (1989)].

2/ The inefficiency results if the opportunity cost of funds to the debtor country exceeds the risks free return. This will normally be the case unless the country has unlimited access to funds at concessional rates.

3/ However, in the course of negotiation, proposals might be presented which do not provide full cash value to the creditors, and evaluating these would obviously require this assumption to be relaxed.

4/ Formally, this is an example of an incentive incompatibility, in which the debtor has an incentive to renege on a contract once it has been negotiated. (The "contract" in this case includes a common expectations about the level of actual interest payments.) We assume that incentive incompatibilities are ruled out by the costs of "default," in this sense.

this type are ruled out by our earlier assumption that the debtor meets its contractual obligations subject only to its real resource constraint.

IV. Valuation of Principal Guarantees

Principal and interest guarantees are fully equivalent if we assume that the present value of debt service payments includes both interest due and amortization payments, whether paid directly to the creditors or into a sinking fund in the debtor country. Since the debt obligations we are considering are fixed-term securities, we must make some assumption about how they will be repaid. It is obviously unrealistic to assume that the resources needed to repay the principal will all be acquired in the year the debt matures. 1/ In this case, the probability that the principal guarantee is drawn on is the same as the probability that an interest guarantee of the same face value is drawn on. 2/ In our analysis of principal guarantees, as with interest guarantees, we assume that both creditors and debtors expect the guarantee to be fully paid out.

The question of incentive compatibility also arises in the case of a principal guarantee, since if the principal repayment were fully guaranteed the country might be tempted to withhold some payments it would otherwise make. The issues are precisely the same as before, and again we assume that the implicit contract can be enforced in one way or another.

V. Debt Exchanges

With these assumptions in place it is straightforward to evaluate a restructuring which involves the exchange of new debt for old, in which the new debt may be issued at an arbitrary interest rate, be exchanged for old debt at an arbitrary exchange ratio, and may be enhanced by a buy-back, an interest guarantee, or a principal collateralization. The starting point is the identity which equates the market value of debt before and after the exchange:

$$P \cdot D = P \cdot (r^*/r) \cdot D^* + E \qquad (1)$$

1/ The amortization of the debt might be "financed" by growth in the real domestic capital stock--this is formally equivalent to a sinking fund, even if no explicit payments are made.

2/ There is, however, a difference in the timing: a principal guarantee is only paid out when the bond matures, so its face value must be fully discounted to obtain its present value.

where the variables are defined as

D	face value of old debt offered in exchange
P	price of old debt <u>after</u> restructuring is announced (i.e., the price of "country risk", or 1 minus the market discount)
D^*	face value of new debt issued in exchange
r	contract interest rate on old debt
r^*	contract interest rate on new debt
E	enhancements to new debt (present cash value to creditors)

This equation sets the market value of old debt exchanged equal to the value of new debt issued, adjusting for any difference in the contract interest rates, and including the cash value of any enhancements, which can include cash buy-backs and interest or principal guarantees. For simplicity, we assume that the contract interest rates are fixed interest rates; on floating rate debt, r and r^* can be interpreted as spreads vis-á-vis a floating risk-free rate. 1/

The assumption that the same country risk P applies to both the new and the old debt implies that the probability of making debt-service payments on the new debt is the same as on the old. The market price of the new debt, however, will, in general, differ from P, because the new debt may have a different contractual interest rate, and because of the enhancements. 2/

1. <u>Par exchange</u>

In a par exchange, the face value of new debt issued is equal to the face value of old debt exchanged ($D^* = D$), and we can solve equation (1) for the interest rate the new debt would have to carry in order to be accepted by the market:

$$r^* = r \cdot (1 - E/(P \cdot D^*)) \qquad (2)$$

1/ The case in which floating rate debt is exchanged for fixed rate bonds requires an additional analysis of the reduction in interest rate risk.

2/ The market price of the new enhanced security is given by $P^* = P/e$, where e is the exchange ratio between the two securities, D^*/D, as defined in equation (9) below.

We could also solve the equation for the level of resources required to support a new debt issue (of size D^*) at a given interest rate:

$$E = (1 - (r^*/r)) \cdot P \cdot D^* \quad (3)$$

or for the size of the new debt issue, given the interest rate and the level of enhancements:

$$D^* = (E/P)(r/(r-r^*)) \quad (4)$$

Finally, equation (1) can be solved for the market price (of old debt) at which the new restructured debt would have the same value as the old debt:

$$P = (E/D^*) \cdot (r/(r-r^*)) \quad (5)$$

Comparing the price defined in equation (5) with the actual market price provides a measure of the attractiveness of a particular restructuring plan to both creditors and debtors. Another measure of the gains from a particular plan is the net debt reduction (NDR), defined as the change in the contractual value of the debt outstanding, less the value of the enhancements:

$$NDR = D - (r^*/r) \cdot D^* - E \quad (6)$$

Here the value of old debt exchanged is equal to the new debt issue, which is scaled by the ratio of the new to old interest rates to obtain its equivalent value in terms of old debt.

2. Non-par exchange

In a non-par exchange, the exchange ratio is defined as the number of units of new debt received in exchange for one unit of old debt:

$$e = D^*/D \quad (7)$$

The discount on new debt is defined as one minus the exchange ratio:

$$d = (1 - e) \quad (8)$$

Equation (1) can be solved for the rate at which debt would be exchanged, given the size of the new issue and its contract interest rate:

$$e = D^*/D \quad (9)$$

$$= 1/((r^*/r) + E/(P \cdot D^*))$$

One can also solve for the interest rate, given the exchange ratio:

$$r^* = r \cdot ((1/e) - (E/P \cdot D^*)) \quad (10)$$

or for the resources needed to support a new issue:

$$E = P \cdot D^* \cdot ((1/e) - (r^*/r)) \tag{11}$$

The implied price associated with a given non-par exchange is:

$$P = (E/D^*) / ((1/e) - (r^*/r)) \tag{12}$$

and the net debt reduction is given by:

$$NDR = D^*/e - (r^*/r) \cdot D^* - E \tag{13}$$

The exchange ratio, e, or exchange discount, d, refers to the face value of old debt exchanged for face value of new debt. It does not measure an exchange of old debt for cash. The market cash discount, in contrast, is the difference between the market price of debt and its par value, (1-P). The exchange discount does not directly reflect the cash value of a particular restructuring, and therefore is not a meaningful measure to consider in isolation. It can be seen from (12) that the cash discount, (1-P), depends both on the exchange discount and the level of enhancements.

VI. Change in the Market Price with a Buy-Back

The formulas presented above assume that the market price of old debt, P, fully reflects the probability of payment of debt service on the new debt. For this to be the case, the price of old debt must be the market price prevailing after the restructuring plan is announced. In general, debt restructuring plans are accompanied by some amount of concessional financing, which improves the ability of the country to service its outstanding debt whether or not that debt is restructured. The market price of debt will rise once the amount of this concessional financing is known to the market.

The market price following the announcement of a debt restructuring plan will, under the assumptions given above, be the same as the price which would result from an equivalent buy-back, that is, a buy-back of E dollars of old debt, where E is the cash value of the enhancements. This price is given by the expected present discounted value of debt-service payments following the buy-back, divided by the stock of debt outstanding after the buy-back.

The market price of debt will typically either rise or remain unchanged following a buy-back, depending on how the buy-back is financed. If the funds needed for the buy-back are a pure grant to the debtor country, the price will rise by the ratio of the buy-back to the original debt stock. On the other hand, if the debtor country must borrow the funds for the buy-back at the market rate, and if the buy-back loan is senior to existing debt (so that it is serviced in full) there will be no

change in the market price, because the funds available to make debt-service payments will be reduced in the same proportion as the stock of debt. In general, the increase in the price will be somewhere between these extremes. 1/

The increase in the price of debt, if any, will occur as soon as a plan is announced (or as soon as the market expects it to be announced), which is usually well before the debt restructuring actually takes place. In evaluating alternative menu items in debt restructuring packages, it is frequently appropriate to assume that the total grant value to the debtor of the package is already known to the market, so that the current market price can be used in calculating alternative restructuring options as shown above.

VII. Illustrative Examples

The following table presents a hypothetical menu of debt restructuring options based on the preceding analysis. We start by assuming that the market price of debt for the debtor country is $0.40 (40 percent of par), and that the debt bears a contractual interest rate of 10 percent. The country is assumed to be considering a debt restructuring which involves enhancements of between $100 and $400 million, and the price of $0.40 is assumed to reflect the market's valuation of this plan.

The first case is a par exchange with interest collateralization. As discussed earlier, the present value of the interest guarantee is assumed to be equal to the amount of the resources committed to the guarantee fund. Panel 1 of the table shows the interest rate on the new bonds which would yield the same expected return as on the old bonds, which is calculated using equation (2) above. The larger the size of the new bond issue, the higher is the interest rate, since the share of the resources used for the interest guarantee going to each bond is smaller. Similarly, the larger the amount of resources used for the guarantee, the lower is the interest rate on the new bonds. The panel also shows the net reduction in debt, calculated using equation (6).

Panel 2 shows the exchange discount on new debt for a non-par exchange of debt with the same contractual interest rate. The discount is calculated using equations (8) and (9), setting $r^* = r$. The net debt reduction, calculated using equation (13), is the same as for the par exchange for a given amount of resources.

The last case shown, in panel 3, is a principal collateralization. Here the face value of the new debt issue is not arbitrary, but instead depends on the level of resources used, since we assume that the full amount of the principal is secured by zero-coupon bonds. The interest

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1/ See Dooley, Symansky, and Tryon (1989). Only with very special assumptions will the market price fall when a buy-back takes places.

Table 1. Sample Table of Menu Items for Debt Restructuring

Market price = $0.40 Interest rate = 10 percent

1. Par Exchange with interest collateralization

Interest rate on new debt (percent):

Face value of new debt	Resources for collateral $100m	$200m	$300m	$400m
$2 billion	8.75	7.50	6.25	5.00
$3 billion	9.17	8.33	7.50	6.67
$4 billion	9.38	8.75	8.13	7.50
$5 billion	9.50	9.00	8.50	8.00
Net debt reduction	$150m	$300m	$450m	$600m

2. Non-par exchange with interest collateralization

Exchange discount on new debt:

Face value of new debt	Resources for collateral $100m	$200m	$300m	$400m
$2 billion	0.11	0.20	0.27	0.33
$3 billion	0.08	0.14	0.20	0.25
$4 billion	0.06	0.11	0.16	0.20
$5 billion	0.05	0.09	0.13	0.17
Net debt reduction	$150m	$300m	$450m	$600m

3. Non-par exchange with principal collateralization

	Resources for collateral $100m	$200m	$300m	$400m
20 Year coupons				
New debt issue	$670m	$1350m	$2020m	$2690m
Discount factor	0.27	0.27	0.27	0.27
30 Year coupons				
New debt issue	$1740m	$3490m	$5230m	$6980m
Discount factor	0.13	0.13	0.13	0.13
Net debt reduction	$150m	$300m	$450m	$600m

rate used to calculate the cost of the principal guarantee is assumed to be the same as the contract rate on the old debt (10 percent), so that, for example, $200 million in resources can secure a principal amount of $1350 million at a maturity of 20 years, and $3490 million at a maturity of 30 years. The exchange discounts on the new bonds are then calculated using equations (8) and (9), as above.

These examples illustrate the equivalence of different forms of debt enhancement in terms of the reduction of net debt achieved with a given amount of resources used for the enhancement. Debt restructuring packages with a wide range of options, in which each option is fairly priced, can be designed using this framework by adjusting the level of resources assigned to each option (or some other variable) so as to yield the same net debt reduction. Restructuring options combining principal and interest guarantees can also be derived.

In practice, restructuring plans are sometimes designed with the enhancements specified in terms of the number of years of interest or the percentage of the principal which is guaranteed under the restructuring. In this case the value of the enhancements is given by:

$$E = x \cdot D^* + n \cdot r^* \cdot D^* \tag{14}$$

where x is the price of a zero coupon bond paying $1 at the appropriate maturity and n is the number of years of interest to be guaranteed. Substituting into equation (1), and letting $D = D^*/e$, we have:

$$P/e = P \cdot (r^*/r) + x + n \cdot r^* \tag{15}$$

Note that the face value of the new debt issue, D^*, drops out; the equation holds for any amount of debt exchanged. Equation (15) can be solved for P, e, or any other variable of interest, as before.

As an example, consider a non-par exchange with a full principal guarantee in 20 years and six months' interest guaranteed. Using the numerical example of Table 1, the price of the zero coupon bond (x) at 10 percent annual interest is 0.1486, and n would be 0.5. Rearranging equation (15) and setting $r^* = r$ we have:

$$e = P / (P + x + n \cdot r^*) \tag{16}$$

$$= 0.4 / (0.4 + 0.1486 + 0.5(0.1))$$

$$= 0.668$$

which implies an exchange discount of 0.332. Using equation (14), the total resources required for a non-par exchange of $2 billion of new debt exchanged for $3 billion of old debt would be approximately $400 million. This can be seen in the second panel of Table 1 which shows that a non-par exchange with $400 million in resources implies an exchange discount of 0.33.

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