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Pricing an Interest Payment Guarantee--A Contribution
to Debt Reduction Techniques

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Abstract

This paper describes an approach for computing the market value of an interest guarantee on a bond where the principal is fully collateralized and which is exchanged for discounted sovereign debts. The cost of the insurance is determined on the basis of a simple option pricing model according to the theory of contingent claims. This method offers the advantage over previously proposed approaches by drawing a distinction between different classes of creditors that may wish to select different levels of insurance protection, recognizing thereby the leverage opportunities that arise from the existence of differing views on the credit risk of the sovereign borrower and different operational environments of the creditors.

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Summary

This paper describes an approach for computing the market value of an interest guarantee on a bond whose principal is fully collateralized and which is exchanged for discounted sovereign debt. The proposed methodology is based on the theory of contingent claims, according to which an investor who has purchased a bond has also implicitly purchased a contingent claim on the assets of the bond issuer. In the event of a default, the investor effectively has the right to liquidate the assets of the issuer of the bond and raise enough cash to recover his initial investment. This contingent claim can be valued as a put option on the assets of the bond issuer, with a predetermined strike price equal to the value of the bond. Thus, the value of the put represents the value of an insurance on the bond in that it provides to the investor the certainty of recovering his initial investment.

Whether or not the put option is actually available in the market and is purchased by the investor, it has a value exactly equal to the risk premium arising from the possibility of a default on the coupon. The premium on the put option is in fact determined directly from the portfolio equilibrium conditions and arbitrage constraints that underlie option pricing models. If the price of the option or the price of the insurance is determined on the basis of other considerations, so that the price is different from the one determined by the option pricing model, there would be riskless arbitrage opportunities that would restore equilibrium conditions.

This method offers the advantage over previously proposed approaches by drawing a distinction between different classes of creditors that might select various levels of protection, that is, different strike prices. This in turn allows debt reduction operations to take into account leverage opportunities that arise from the existence of different views in the market on the credit risk of the sovereign borrower, as well as the different operational environments of the creditors.

I. Introduction

In current discussions concerning debt reduction operations, it has often been argued that the exchange of old discounted debt for a new bond, where only the principal is collateralized by a riskless asset (e.g., zero-coupon U.S. Treasury bond), may not be acceptable to the creditor at the going discount in the secondary market, given the default risk that remains on the stream of interest payments. One solution that has been suggested is to guarantee to the creditor a minimum interest payment by setting up an insurance fund which could be tapped in the event of a partial or total interest payment default by the debtor. While the explicit enhancement of the new debt instrument offered by this type of guarantee would facilitate a profitable exchange for both parties involved, one remaining issue is the determination of the appropriate cost of the insurance scheme and, consequently, its value in the debt reduction operation.

The typical approach followed to determine the "equilibrium" value of the new debt instruments and thereby determine the exchange ratio between old debts and the new enhanced debts, has been to concentrate on the computation of the probability of default based on various assumptions about the statistical distribution of historical interest and principal repayments as reflected in the secondary market price of sovereign debts. Once this probability or expected average repayment has been determined, it is then incorporated in a standard bond-pricing technique to determine the present value of the old debt and thereby the cost of insuring the remaining interest payment as equal to the difference between the expected repayment and the contractual interest.

One potential problem with this approach is that it assumes that the average expected repayment computed as the average secondary market price accurately incorporates into one single price the views of all the creditors. It does not allow for different classes of creditors who may have different views of the probability of repayment or who may simply value the risk in a more comprehensive and multi-faceted framework that includes, for example, the regulatory, tax, and accounting environments, the specific relationship with the debtor country, the level of exposure, the amount of reserves, and the asset/liability position, etc., of the creditor. In addition, this approach does not take into account the impact of the debt reduction operation on the secondary market price. It is plausible to expect that, if the debt reduction operation is large enough and is successful, the debtor will be in a better position to service the remaining (lower) amount of the debt outstanding. This in turn should reduce the discount on the debt and thereby determine a different level of the secondary market price. All of these factors will determine an array of potential transaction prices (i.e., analogous to a demand

schedule) which will deviate, in some cases substantially, from the current market price and which would correspond to possible exchanges of old for new enhanced debts at different exchange ratios for different participants.

This paper suggests an alternative approach that could be applied in order to determine the "fair price" ^{1/} of the insurance for interest payments, with the ultimate goal of incorporating this cost into the price of the new debt and thereby obtaining an estimate of the exchange ratio at which old debts can be exchanged for new enhanced debts. The cost of the insurance is determined by pricing a put option on the bond, assuming that the creditor has been able to acquire the right to re-sell the bond in the event of a default at a predetermined (strike) price, for a predetermined period of time and according to current market conditions. This method explicitly allows the creditor to select the desired level of insurance by selecting an appropriate strike price of the put option, according to his own regulatory, tax and accounting environment and/or balance sheet position.

As a background to facilitate an understanding of this approach, Section II describes a standard method in the finance literature for determining the risk premium on a private bond, i.e., an unguaranteed debt instrument issued by a private borrower. Section III then applies this approach to the debt instruments of sovereign borrowers. The relevance of some of the assumptions of the model utilized in the paper are discussed in Section IV. Section V presents some preliminary empirical results and Section VI provides some concluding remarks.

II. Valuation of Credit Risk in the Capital Market

In normal capital market conditions, the market valuation of the credit risk of a debt instrument, for example a bond, is embodied in the contractual interest of the instrument as a spread over the current market rate (e.g., Libor) or over what is perceived in the market as being a riskless interest rate (e.g., the interest rate on U.S. Treasury debt instruments). Bonds that are exposed to the risk of default will sell at a discount relative to other bonds that are regarded as being free from this risk, with the discount being directly proportional and positively correlated with the probability of default on the debt--the higher this probability, the higher the discount from the nominal face value of the bond. Thus, the discount, or

^{1/} Throughout the paper, the term "fair price" denotes a zero net present value transaction and thus, a price at which there are no arbitrage opportunities.

equivalently, the higher yield, will explicitly represent the additional reward required by the holder of the bond in order to compensate for the probability of default relative to a riskless type of investment.

Determining the market value of the compensation required to bear the credit risk on a bond traded in the secondary market is a relatively straightforward process. If the price of the default-free bond and the price of the risky bond with identical contractual conditions are known, the value of the credit risk can be estimated directly as the difference between these two prices using the following identity:

$$(1) \quad \text{Value of Credit Risk} = \text{Price of Default-free Bond} - \text{Price of Risky Bond } \underline{1/}$$

However, there may be circumstances arising from the structure of the specific market examined or other considerations in which the view of the individual investor on the credit risk of the bond may differ from that of the market. In this case, the individual investor considering the purchase of the bond will have to determine the price at which he is ready to transact and which reflects his own evaluation of the risk premium.

Using the modern theory of contingent claims, a number of authors, including Augros (1985), Brennan and Schwartz (1983), Merton (1974, 1977), have applied directly the classical portfolio arbitrage conditions that form the basis of option pricing models in order to estimate the risk premium charged on a bond as a compensation for the probability of default. The intuitive observation at the base of this approach is that, according to the legal framework existing in most financial markets, an investor who has purchased a private unguaranteed bond has also implicitly purchased a contingent claim on the assets of the bond issuer. 2/ In case of a default, the investor effectively has the right to liquidate the assets of the issuer of the bond and raise enough cash to recover his initial investment. Of course, the

1/ In practice, this difference may also reflect interest rate risk. However, to simplify the analysis and clearly isolate the value of the credit risk, the interest rate risk is omitted, i.e., it is assumed that the (spot) yield curve used to discount the bond is flat and that the short-term interest rate is constant with respect to time. Alternatively, it could be assumed that the McCaulay duration is identical for both bonds.

2/ See R. Brealy and S. Myers, Principles of Corporate Finance, MacGraw-Hill, 1981, pp. 425-431.

market value of these assets may be lower than the value (principal plus interest accrued) of the bond, in which case the investor will incur a loss.

However, if the investor were able to purchase a contract in the market which, in the event of a default, would give him the certainty to be able to sell the asset of the bond issuer at a price equal to the value of the bond (or, the price paid by the investor), then he would have completely hedged the credit risk on the bond. The value of this contract is equivalent to the value of a put option on the assets of the bond issuer, whereby the investor purchases the right (which would be exercised in case of a default) but not the obligation to sell these assets in the market at a predetermined (strike) price equal to the nominal value of the bond. Thus, the value of the put represents the value of insurance on the bond in that it provides the investor the certainty of recouping his initial investment.

Accordingly, rewriting (1) as an equation where the value of the credit risk is determined by the cost of the put, the fair price of the bond will be equivalent to:

$$(2) \quad \begin{array}{lcl} \text{Price of} & \text{Price of Bond} & \text{Value of} \\ \text{Risky Bond} & = \text{Assuming no Default} & - \text{the Put} \end{array}$$

More formally, using the standard bond pricing formula, this relationship can be represented as:

$$(3) \quad Pr = \left[\sum_{t=1}^m \frac{C_t}{(1 + R_t)^t} + \frac{N}{(1 + R_m)^m} \right] - P(E, X, T)$$

where:

- Pr = Price of the bond
- Ct = Coupon on the default-free bond at time t
- Rt, m = Discount rate for period t and m
- N = Face value of the bond
- P(.) = Value of the put
- X = Spot price of the risky bond
- E = Strike price of the risky bond
- T = Maturity of the put option
- m = Maturity of the bond

The first term in the brackets represents the sum of the nominal coupons each discounted back to give the present value; the second term represent the face value of the bond discounted back to give the

present value according to the discount factor for the maturity of the instrument. From equation (3), it follows that pricing the risky bond is reduced to a two-step process: first, the value of the bond is determined assuming zero probability of default; second, the value of a put on the bond issuer's assets with a strike price equal to the value of the bond is computed.

The notional price of the bond estimated according to equation (3) may not be equal to the actual price quoted in the secondary market; as the formula shows, P_r will depend on the value of the put which, in turn, will depend on the value selected of E , X , and T . While in normal circumstances X will be determined according to existing market conditions, E and T will be selected according to the views of investors on the credit risk. If an investor anticipates a high probability of default on the bond, he will choose a higher level of protection (insurance) by selecting a strike price E closer to the nominal value of the bond and for a longer maturity T . This, in turn, will raise the value of the put option (or, the value of the insurance), thereby lowering the price that the investor is willing to pay. By contrast, if the investor anticipates a low level of probability of default, he will select a lower strike price E and maturity T , thereby reducing the cost of the put, or the cost of the insurance. It follows that, if the value of the bond estimated by the investor is different from the price actually quoted in the market, the investor will then decide whether or not he wants to transact and to which extent.

In practice, the investor may not have to value the put himself; given the highly developed level of capital markets, he may be able to actually purchase the put option on the bond either over-the-counter or in an organized exchange. ^{1/} In this case, the investor could implement a simpler hedging transaction whereby he is covering the risk on the bond by purchasing a put option. The investor would then subtract from the nominal value of the bond (price of the bond assuming no default) the cost of the hedging tool and thereby determined the "fair" price of the bond and compare it with the current secondary market price at which the bond is actually traded. Alternatively, if there are no prices in the market for such an option, the investor could buy a put on an interest bearing instrument which is highly correlated with the market value of the bond, thereby purchasing a synthetic put option on the bond which would broadly have similar profit/loss characteristics to the true put option.

^{1/} In other words, the investor may be able to find a financial agent ready to write the put option at the desired conditions (i.e., strike price, maturity, amount, etc.)

III. Application to Sovereign Debt

The methodology described in the previous section can be directly applied to determine the risk premium on bonds issued by sovereign borrowers. More specifically, a similar valuation methodology can be implemented for determining the risk premium and, consequently, the price of a bond where the principal is fully collateralized and which has been issued by a heavily indebted country in exchange for old discounted debts. In this case, the interest payment would be the only component exposed to the risk of default, as the investor would be certain of recouping at least the nominal face value of the bond equal to the value of the collateral pledged. By analogy to what was described in Section II, the value of the put would represent the risk premium necessary for compensating for the possibility of default on the interest payment or, equivalently, it would represent the cost of insuring the coupon on the bond.

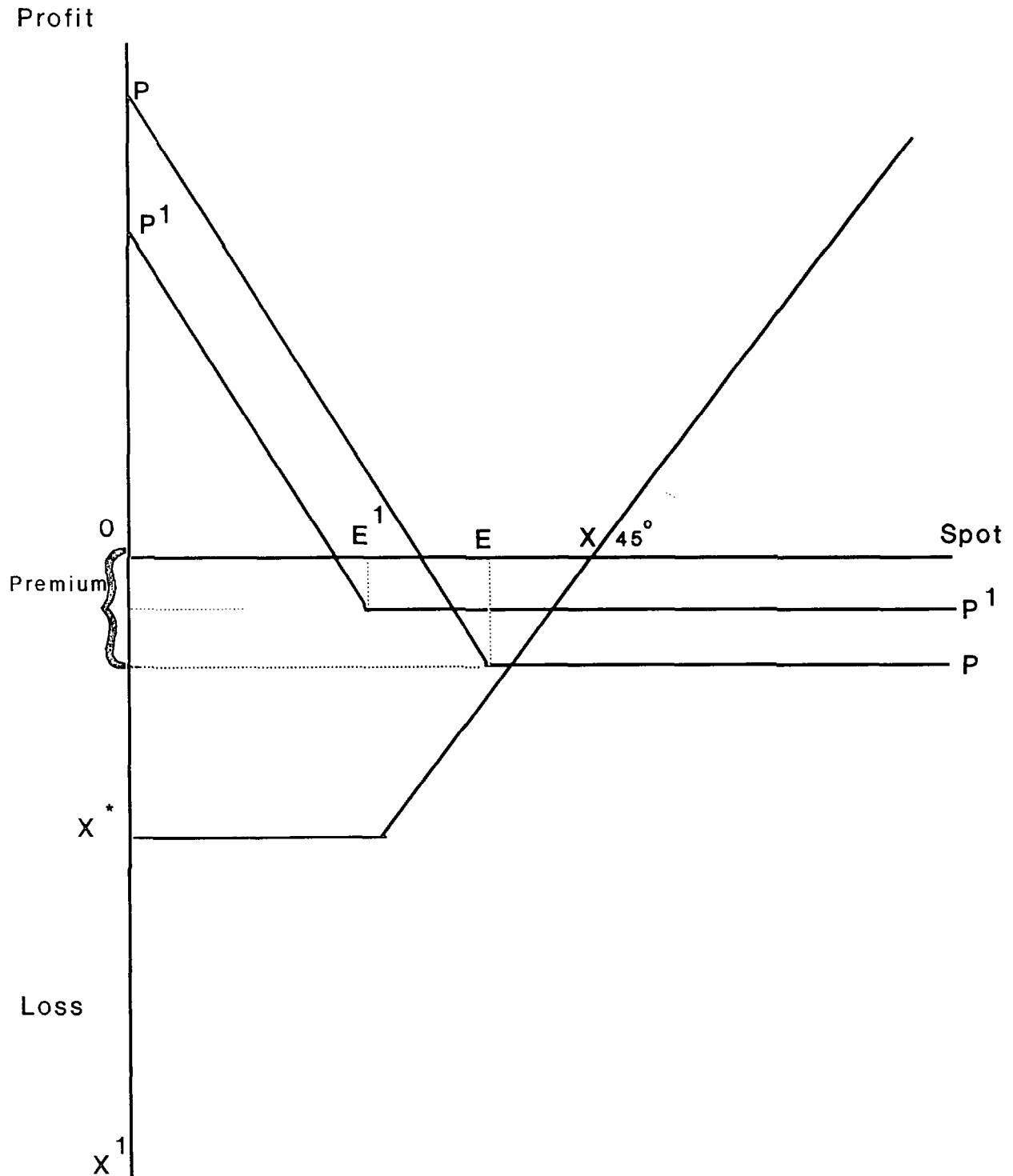
If the put option were actually available in the market and if the bond investor were to follow this hedging strategy by purchasing it, at any point in time during the life of the bond the value of the option would be:

$$\text{Value of the Put} = \text{Max } [0; E - \text{Pr}^*]$$

where E is the strike price of the option and Pr^* is the current price of the bond in the secondary market. In other words, if the bond trades in the secondary market at a price lower than E, the investors could exercise the put option and secure a gain equal to the difference between the nominal value of the bond (which is also assumed to be the strike price of the option) and the price of the bond in the secondary market, i.e., $E - \text{Pr}^*$. This profit would be, however, completely offset by the loss on the bond, given that at the same time the value of the bond held by the investor has declined by the same amount, i.e., $\text{Pr} - \text{Pr}^*$, where Pr is the nominal value of the bond equal by assumption to the strike price E. If, however, the bond trades at or above the strike price E, the investor would not exercise the option as it would have a zero value.

The investor's profit and loss potential throughout the life of the put option are represented in Graph 1. The vertical axis represents the profit/loss position of the investor for various movements in the price of the bond in the secondary market, which is plotted on the horizontal axis. If the debtor is current in his

Graph 1
PROFIT / LOSS POSITION



interest payment, the value of the bond will be at or above par 1/ (100 percent of face value, point X on the horizontal axis) and the option will have a zero value. If, however, the debtor defaults partially or totally on the interest payment, the value of the bond will decline in the secondary market below par (line XX', lower panel), with the decline directly proportional to the amount of default on the interest. In this particular case, the maximum loss that the creditor could incur is equal to X* where:

$$X^* = X - \frac{N}{(1 + R_m)^m}$$

or the price paid for the bond minus the present value of the principal, equal in turn to the present value of the collateral.

However, by purchasing a put option on the bond (line PP), the creditor is able to hedge (insure) his position from a possible decline in the value of the bond. If the debtor does not fulfill his interest payment obligation, the price of the bond declines in the secondary market, but the loss for the creditor is fully compensated by the increased value of the put option (minus the cost of the put).

Line p^1 p^1 in the graph represents the possible profit and loss outcome for a lower level of insurance. In this case, the value of the put is determined on the basis of a strike price E^1 lower than E, thus reducing the value of the option and at the same time the cost of the insurance, i.e., the premium is less for the put with the lower strike price.

It is important to note that irrespective of whether the put option is actually traded on financial markets and can be purchased by an investor, its value is equal to the risk premium arising from the possibility of a default on the coupon. The cost of the put option is in fact determined directly from portfolio equilibrium conditions and arbitrage constraints which underlie option pricing models and which reflect existing global market conditions of the bond. 2/ In other words, if the price of the option or, equivalently, the price of the insurance for the same strike price were determined on the basis of other considerations and if, consequently, the price were different than the one determined by the option pricing model, then there would

1/ Assuming that C_t is equal to R_t and that the (spot) yield curve is flat.

2/ See F. Black and M. Scholes "The Pricing of Option and Corporate Liabilities," Journal of Political Economy, May-June 1973,.

be riskless arbitrage opportunities which would restore equilibrium conditions. Accordingly, the price of the bond determined on the basis of equation (3) will incorporate precisely the risk premium necessary for compensating for the risk of default on the coupon or, equivalently, it will incorporate the exact cost of the insurance.

IV. Description of the Option Pricing Model

The option pricing model utilized for a preliminary and purely experimental determination of the cost of the put is the standard Black-Scholes model, which is described in Appendix I. Other models, such as that developed by Merton (1974), could have been used for the determination of the put and thereby the value of the risk premium on the bond. ^{1/} However, the purpose of this analysis is to demonstrate the validity of using the put option framework to calculate the cost of the insurance; extension or variation of the option pricing model would obviously include greater precision in the computation of the cost of insuring the interest payment.

As is always the case with option pricing models, one of the critical steps is to evaluate the realism of the underlying assumptions of the model selected. The Black-Scholes model is based on a simplifying assumption when applied to debt instruments as, in its basic version, it does not take into account the interest rate component of the underlying debt instrument on which the put option is valued and it assumes a constant short-term interest rate. The value of the underlying debt instrument (price of the bond) in fact will vary during the maturity of the option reflecting two major factors: first, the accrued interest (the payment of the coupon) and second, changes in the market interest rate structure. ^{2/}

One straightforward solution to the first problem is to select a strike price for the put option which reflects the interest accrued on the bond, or, in other words, utilizes the forward price of the bond. If the put option is a European-style of option, i.e., it can be exercised only at maturity, then the forward price (or flat price of the bond plus accrued interest) for the maturity date of the option can be determined unambiguously. Thus, the empirical estimation of the

^{1/} Augros (1985) has demonstrated that in some specific circumstances the solution to the Merton model is actually identical to the Black-Scholes. See Augros (1985), p.211.

^{2/} In terms of equation (3), this simply takes into account possible variations in the discount factor R_t during the life of the bond.

costs of the insurance presented in this paper were determined assuming that the put is a European-style option.

As regards the second problem, the additional generality desired from incorporating changes of the interest rate structure and therefore assuming a variable interest rate environment appears somewhat limited for the specific case analyzed in this paper. Movements of the price in the secondary market will be caused primarily by credit risk considerations rather than interest rate risk. Indeed, the deep discount on the debt instruments of heavily indebted countries reflects primarily the perception among the creditors of an increasingly high credit risk more than a substantial increase in the market interest rate relative to the original contractual interest rate. While there should exist a negative correlation between the movement in the market interest rate and the secondary market price of the debts (the higher the Libor rate, the lower the secondary market price), the magnitude of this correlation appears to be quite low. Therefore the inclusion of this additional source of volatility in the determination of the cost of the put option does not add any significant information and would appear to have only a small impact on the final results. 1/

A source of complication arises from the estimation of the variance of the price of the bond on the basis of which the cost of the put option is determined. According to the theoretical approach described in Section II, this cost reflects the cumulative probability distribution of all possible intrinsic values of the put option, i.e., the difference between the strike price and the market value of the bond issuer's assets at expiration. Since the strike price is fixed in advance, the only unknown element is the probability distribution of the value of these assets. In the specific case analyzed in this paper, it would indeed be arduous to determine precisely the probability distribution of the market value of the assets of a sovereign borrower. It can be argued, however, that the secondary market price of debts of heavily indebted countries will be highly correlated with the country's capacity to fulfill its financial obligations and, consequently, with its "value" as estimated directly

1/ While it is relatively difficult in this case to perfectly isolate the interest rate impact on the secondary market prices from the credit risk impact given the complexity of this market, a preliminary statistical analysis shows a relatively modest level of correlation between the one year Libor rate and the secondary market price of debt instruments of most of the heavily indebted countries during the period from March 1986 to June 1989.

by market participants. 1/ Thus, the variance of this price may provide a reasonable assessment of the range of uncertainty existing in the market over the probability of default on the bond reflecting explicitly the views of the different creditors. In other words, it may provide relevant information regarding the magnitude of the range of prices at which different classes of creditors are ready to engage in a secondary market transaction. While the average secondary market price may change radically and therefore may not be representative of the true "value" of the country, the dispersion around the average price would appear to represent the range of uncertainties reflecting the specific circumstance of each class of creditors.

In addition, given the relative thinness of the current secondary market, the variance of the price may very well provide an explicit upper limit for the level of volatility in this market. According to some analysts, in fact, current prices quoted by major market makers are only indicative and most likely represent the price of the last transaction more than an equilibrium average price at which demand and supply converge. Given the extremely low level of liquidity relative to the total amount of debt outstanding, single transactions of a relatively small amount may move the price radically, thus resulting in wide fluctuations, i.e., a higher level of variance. 2/ Consequently, it would appear reasonable to expect that if more financial agents were to engage in secondary market transactions, the increased liquidity would result in smaller price fluctuations, although measured around an entirely different level of average price. This in turn implies that the value of the put determined on the basis of the variance of

1/ Most of the option pricing models applied to the determination of the risk premium on private bonds generally assume that the Modigliani-Miller theorem holds true, i.e., they assume that the value of a corporation is independent from its financial structure (See J.C. Augros (1985), p.209 and R.C. Merton (1974-1977)). This implicitly presupposes that, in the case of a default, the assets of the bond issuer can be readily converted into cash to repay the outstanding debts. By contrast, the assets of a sovereign borrower may not be readily liquidated in the case of a default, thus requiring a generalizing assumption for the specific case analyzed.

2/ For a more detailed description of the factors affecting the secondary market price, see M. Blackwell, and S.E. Nocera, "Debt/Equity Swaps," in Analytical Issues in Debt, ed. by Jacob Frenkel (International Monetary Fund, forthcoming 1989).

secondary market will identify an upper limit for the cost of the insurance for predetermined levels of strike and spot prices. 1/

V. Some Preliminary Empirical Estimates

Table 1 lists some preliminary estimates of the value of the put option and the consequent cost of the insurance for a selected group of debtor countries. The premium of the option was determined assuming an underlying 7-year bond with a face value of \$100, a coupon of 10 percent paid annually, and a discount factor of 10 percent which is assumed to remain constant throughout the life of the bond. Accordingly, the spot price of the bond assuming no probability of default and on the basis of which the value of the option is determined is equal to \$100. 2/

The strike price of the option was selected according to two illustrative scenarios, each reflecting a specific class of creditors. In the first case, it was assumed that creditor A, given his own specific circumstances, wishes to insure the full value of the bond or, equivalently, wishes to insure a full repayment of the coupon. In this case, the strike price E that he chooses will be \$100 and the option will be priced at the money (first column of Table 1) i.e., with a strike price equal to the spot price. In the second case, it was assumed that creditor B, again given his own specific circumstances, is willing or able to accept a greater default risk and, therefore, chooses a lower level of insurance. As an example, it was assumed that creditor B wishes to insure a premium repayment equal to 75 percent of the coupon, thus choosing a strike price of \$87.33 (second column). Given the assumption of a spot price equal to \$100, the option is

1/ Given the construction of the option pricing model, a higher level of volatility measured by the standard deviation will result in a higher cost of the option per unchanged strike and spot prices (see Table 1).

2/ The put option cannot be valued on the basis of the current secondary market price. While the new bond can be considered as a "derivative" product of this market, it explicitly has an enhanced value for the creditor as, irrespective of the credit risk existing on the interest payment, the principal is fully collateralized and, consequently, the present value of the new bond is, in most cases, higher than the current secondary market price. In addition, if the debt reduction operation is substantial enough and successful, it is conceivable to speculate that the new bond may trade at par in the secondary market.

Table 1. Cost of a Put for Selected Countries

(In cents per dollar)

Country	Annualized Standard Deviation of the Price (In Percentage)	At-the-money Put <u>1/</u>	Out-of-the-money Put <u>2/</u>
Brazil	20.40	4.08	1.35
Mexico	13.44	2.10	0.20
Argentina	20.26	4.07	1.16
Philippines	12.82	1.80	0.12

1/ Spot price and strike price equal to \$100

2/ Spot price equal to \$100 and strike price equal to \$87.33 reflecting 75 percent of the coupon.

Table 2. Cost of the Insurance for Various Combinations
of Strike Prices and Expected Spot Prices

(In cents per dollar)

Expected Spot Price	Strike Price			
	100	90	80	70
100	1.40	-	-	-
90	5.43	1.89	-	-
80	12.27	5.65	1.44	-
70	21.47	12.76	5.47	-
60	-	22.27	13.35	5.59

out-of-the-money and therefore the cost of the insurance for creditor B is lower than the one for creditor A.

The volatility is derived on the basis of monthly data on secondary market prices quoted by Salomon Brothers from March 1986 to June 1989, computed as the standard deviation of the natural logarithm of the price ratio, 1/ or:

$$(4) \quad \sigma = \left[\frac{1}{n-1} \sum_{i=1}^t (x_i - \mu)^2 \right]^{1/2} \quad [12]^{1/2}$$

where:

$$(5) \quad \mu = \frac{1}{n} \sum_{i=1}^t x_i$$

$$(6) \quad x_i = \ln \frac{\text{Price } t}{\text{Price } t-1}$$

The riskless interest rate utilized in the option pricing model is the one year U.S. dollar Libor equal to 9.0 percent as of July 28, 1989. For simplicity, the maturity T was set to one year. 2/

Having determined the value of the put and therefore the cost of the insurance, each creditor will transfer it to the debtor subtracting it according to (3) from the value of the new bond assuming no default and, thereby, determining the price that he is willing to

1/ The standard deviation can be computed under different assumptions on the statistical distribution. For simplicity, it was assumed that the secondary market price follows a continuous random walk.

2/ Obviously the maturity T can be raised to cover at the limit the entire seven years, raising the value of the put and the cost of the insurance accordingly. While this would provide a more precise computation of the cost of the insurance and of the price of the bond, it may not reflect the most commonly proposed terms of the insurance guarantee which is envisaged to be implemented for two to three years.

bid for the new debt. For example, in the case of Brazil, creditor A will bid a price of

$$\$95.92 = \$100 - \$4.08$$

while creditor B will bid a price of

$$\$98.65 = \$100 - \$1.35$$

These results are only illustrative as they will depend entirely on the strike price selected and, therefore, on the level of insurance coverage selected by each creditor.

Of course, it is possible to envisage a variety of scenarios according to various combinations of strike prices selected and various scenarios of spot prices. ^{1/} The various combinations of strike and spot prices will represent the various levels at which each creditor will stand ready to transact, and can perhaps be viewed as analogous to a demand schedule. Table 2 illustrates the cost of some possible combinations of strike and spot prices for Mexico.

For example, if a creditor is not convinced that a debt reduction operation or an eventual structural adjustment program will allow the debtor to become current in servicing his debts (i.e., if he expects that the new bond will trade below par after it is issued); if, in addition, his total credit exposure is relatively modest compared with other assets on his balance sheet (i.e., if the creditor can comfortably absorb the potential loss) and if, in the end, he is ready to sell at or slightly above the ongoing price in the secondary market, he may choose a relatively lower level of protection, say a strike price of 80, thus reducing the cost of the insurance.

Having determined the cost of insurance in light of the perception of the debtor default risk and the exposure to the country, the creditor will transfer it directly to the debtor, subtracting it according to (3) from the value of the new bond assuming no default, thereby determining the price at which it is willing to buy the new debt and, subsequently, the exchange ratio between old and new debt:

^{1/} These will not be clearing prices in the market, which in equilibrium will have only one price. These should instead be considered as benchmarks against which each investor will compare the nominal price of the bond and determine the price that he is willing to bid.

$$(7) \quad Pr = \text{Value of the Bond} - \text{Cost of the Put}$$

$$(8) \quad \text{Exchange ratio} = Pold/Pr$$

Following the above example of a strike price of 80 and a spot price of 70, and assuming that the new bond evaluated under no possibility of default will sell at par, the fair price for the investor would be equal to:

$$94.53 = 100 - 5.47$$

and the exchange ratio equal to:

$$0.74 = 70/95.53$$

given that 70 is the theoretical level of secondary market price estimated by the investor.

The cost of the puts listed in Table 2, although depending on the combination of strike and spot prices, will be relevant for the debtor in the debt reduction negotiation process, as they provide a method of determining the exchange ratio of new for old debts that correspond to the specific circumstances of the creditor or class of creditors. In particular, if the debtor is negotiating in a non-concerted environment (i.e., negotiating bilaterally with each creditor), then he would presumably attempt to transact first at the margin with the creditor offering the higher price.

VI. Conclusions

It should be emphasized that the approach described in this paper does not require actually carrying out the transaction in the market, i.e., it does not require an option seller and an option buyer. Whether the option is actually written or not, its "fair" market value and, consequently, the cost of the insurance, can be computed directly and reflects portfolio equilibrium conditions and arbitrage constraints in the secondary market for debt instruments. Accordingly, the price of the put or, equivalently, the cost of the insurance can be determined ex-ante, regardless of the future outcome and, consequently, it can be incorporated directly into the value of the bond without relying on a statistical determination of an average expected repayment computed on the basis of the current secondary market prices. As was noted in the Introduction, these prices reflect a variety of factors some of which are related to the specific circumstances of the creditors. Consequently, they may yield a distorted estimation of the

average expected repayment or, conversely, the average probability of default.

In addition, this method offers the advantage of taking into account leverage opportunities arising from tangible differences existing between the various creditors. As was shown above, the technique utilized to compute the cost of the insurance enables each creditor to select the appropriate level of insurance protection by choosing a specific strike price on the put option which reflects his own evaluation of the credit risk undertaken and, more importantly, his distinctive financial and operational circumstances. These circumstances may differ across the classes of creditors in light of difference in the regulatory, tax and accounting environment, the level and opportunity cost of loan loss reserves, the asset/liability position, and the overall financial strategy. The implicit advantage is that the debtor may be able to exploit the differences existing between the various creditors, particularly if the debt reduction negotiation process is carried out on a non-concerted basis given the voluntary nature of debt reduction operations.

While actual option transactions are not needed for this approach to be operational, it nonetheless appears that the potential for the development of such an option market does exist. There are in fact economic agents interested in a decline of the value of the debt in the secondary market. Most direct investors and portfolio investors in heavily indebted countries are interested in a possible decline of the price in order to carry out debt/equity swaps. Their incentive to enter into a debt conversion operation arises directly from the deep discount at which the debt trades in the secondary market and the fact that the nominal value of this debt can then be exchanged in the debtor country for the equivalent in domestic currency. These investors would therefore be natural put option writers as, if the price declined and the option were exercised (i.e., if the bond is sold by the creditor at the predetermined price to the option writer), they could utilize the acquired bond for a swap against equity in the defaulting country. Debt conversions during the period from 1983-87 totalled approximately \$10 billion; as a purely speculative estimate of the size of this potential market, if option writers covered the risk on the option by

delta-hedging, 1/ there would be a potential contract value of approximately \$20 billion of put options.

In addition, according to market sources call options on sovereign debts already exist in the market. Reportedly, potential direct investors contemplating the possibility of a debt/equity swap as well as brokers of sovereign debts have been net purchasers of these type of options in order to fix the price at which they would acquire the debt instrument in the secondary market before the actual transaction can be completed. Thus, as outlined in Appendix I, a synthetic put option position (i.e., a combination of instruments which would have the same profit/loss characteristics of a put option) could be constructed by simply applying the following call-put parity condition:

$$P(E,X,T) = C(E,X,T) + Ee^{-rt} - Pr$$

In order to acquire the put option, the investor would have to purchase a call with the same strike price, valued on the same spot and with the same maturity of the put; borrow the discount amount of the strike price and invest in the bond. The profit/loss characteristics of this portfolio would be exactly the same as the one yielded by the put.

As a final remark, it should be noted that the determination of the correct cost of the insurance through this type of approach is open to further refinements. In particular, the actual cost of the insurance and the fair price of the bond will depend on the specific strike price selected which, in turn, can be more precisely determined only by the debtor and the creditor in the debt reduction negotiation process. In addition, the price of the put or, equivalently, the cost of the insurance, could be computed under alternative option pricing models and under various scenarios of riskless interest rates and maturity of the put option (duration of the insurance). Also, and more importantly, the volatility or the standard deviations of the secondary market price should be based on a robust assessment of the structure of the secondary market particularly with regard to its liquidity, market players, settlement conventions, etc. of which little is known so far.

1/ The delta of an option is the first partial derivative of the premium with respect to the spot. It is normally utilized by option writers to determine the amount of position (long or short) that they should have on the underlying cash instrument in order to hedge the option position. When the option is at-the-money, i.e., when the strike price is the same as the spot, the delta is equal to 0.50.

Black-Scholes option pricing model

The value of a European style call option 1/ on an underlying financial instrument with a spot price equal to S, a strike price equal to E and a maturity equal to T is determined as:

$$C(S, E, T) = SN(d_1) - Ee^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln \frac{S}{E} + \left[r + \frac{(\sigma)^2}{2} \right] T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

and where:

σ = standard deviation of the natural logarithm of the ratio of spot movements.

r = riskless interest rate

$N(d_1)$ = Normal cumulative distribution of d_1

$N(d_2)$ = Normal cumulative distribution of d_2

Once the call value has been determined, the value of the put can be derived from the call-put parity condition based on the same spot S, strike price E and maturity T:

$$P(S, E, T) = C(S, E, T) + Ee^{-rT} - S$$

1/ Cannot be exercised before maturity.

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