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Growth, External Debt and Sovereign Risk in a Small Open Economy*

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Abstract

This paper constructs and analyzes an optimizing model of a highly-indebted small open economy. An important innovation in the model is the incorporation of sovereign risk through the specification of an upward-sloping foreign debt supply function. The model is used to examine the interaction between external debt and growth in response to various policies and exogenous disturbances. It is shown that structural policies intended to reduce the fiscal deficit or increase productivity can lead to tradeoffs in their effect on capital accumulation and the stock of debt.

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Summary

This paper specifies an analytical model of a small open economy that borrows abroad. The framework developed is of the (intertemporal optimizing) type that has been increasingly used to analyze a variety of issues in international macroeconomics. The paper examines the dynamic interaction between external debt and growth, as well as the impact effects of various economic disturbances and policies on certain key macroeconomic variables. An innovative and realistic feature of the model is the fact that it incorporates explicity the risk premia associated with lending to sovereign borrowers.

The paper considers the short-run, dynamic, and steady-state effects of shifts in the foreign interest rate, the risk premium, productivity and government expenditure on variables such as the stock of debt, the domestic capital stock, the domestic interest rate, the trade balance and consumption. In the long-run (and assuming that the government pursues a balanced-budget policy) an increase in international interest rates leads to a decline in the level of external debt. However, the effect upon domestic interest rates is a priori unclear, although under plausible conditions the rate is expected to increase less than proportionately. Under the same conditions, the domestic capital stock also falls. The effects upon the domestic trade balance and consumption are indeterminate. An increase in the risk premium reduces the level of external debt when domestic consumers are net debtors, and increases domestic interest rates while lowering the capital stock. A productivity disturbance has no effect upon the steady-state stock of external debt and the equilibrium trade balance. The effect upon the capital stock depends upon whether the disturbance is productivity-enhancing or not. If it is then the capital stock rises. Most of these results carry over to more general models.

The results of this paper also have implications for growth and adjustment in that they suggest that structural adjustment policies may involve tradeoffs insofar as their effects upon growth and the stock of debt. For example, a productivity-enhancing policy will raise the growth rate but will also increase the stock of external debt. Similar tradeoffs arise for changes in international interest rates, as well as for changes in government expenditure when domestic employment is endogenous in the model.

I. Introduction

The continuing debt crisis facing many developing countries in the current decade has underscored the need to understand the relationship between debt accumulation and development, as well as to design policies that foster external adjustment while increasing the growth of domestic output. There is little doubt that developing economies require external capital, since typically they cannot generate adequate resources domestically to achieve the growth rates that may lead to an improvement in living standards. Experience with external borrowing, however, has shown that repayments are not always made on time. Overborrowing, resulting from inadequate perceptions of domestic growth potential, has occurred on occasion. Long gestation lags in investment projects, or the inability to adequately price domestic infrastructural projects that have been financed by external debt, or a combination of both, has led to difficulties in meeting repayment commitments in some cases (Kharas (1983), Kharas and Shishido (1986)). International capital markets are likely to react to their perception of a country's ability to repay, and hence seek to reduce their exposure at a time when a country requires more capital (Sachs (1984)). This has led in some cases to a significant cutback in domestic development efforts, and hence a reduction in the rate of growth. The question of an appropriate debt strategy for sustained growth has therefore become a pressing one in the 1980s.

A related issue, that has also received considerable attention during this period of the debt crisis, is that of achieving a needed adjustment in the external accounts while sustaining or increasing the growth momentum in the economy. When external resources are not available to the extent necessary, adjustment is clearly required in domestic spending and resource mobilization. External financing from multilateral agencies such as the Fund and the Bank is available to allow policy packages to be implemented to facilitate such adjustments (Khan et.al. (1989)). The general consensus in academic and policymaking circles is that the desired policy mix for making such adjustments should be growth enhancing. However, there are as yet few fully-articulated analytical models that incorporate growth-debt-adjustment relationships.

The purpose of this paper is to develop a macroeconomic model for a small open developing economy, in order to study the dynamic interaction between debt and growth, as well as the effects of various policies and exogenous shocks on the rate of capital accumulation, the current account balance, and the stock of foreign debt. The framework we develop involves an intertemporal optimizing model of the type which has increasingly been used to analyze a variety of issues in international macroeconomics. 1/

1/ For recent examples of this approach see Brock (1987), Buiter (1984), Obstfeld (1982, 1988), Sen and Turnovsky (1989a, 1989b).

As mentioned above, despite the current preoccupation with the debt crisis, there are few existing macroeconomic models which allow the relevant issues to be addressed adequately. 1/ This is because most of the existing literature is based on uncovered interest parity with the supply of debt to the economy being perfectly elastic. On the other hand, research has shown that the risks associated with sovereign borrowing does result in lenders charging a risk premium as well as, on occasion, cutting off credit to borrowers. 2/ Thus, for a substantive analysis of debt-related issues the model should include an appropriate debt supply schedule that incorporates the risk premia associated with lending to sovereign borrowers. The model presented below incorporates such a realistic debt supply schedule. 3/ The analysis emphasizes the effects of shocks to the debt supply schedule on key macroeconomic variables. 4/ Among the shocks considered are changes in the level of foreign interest rates, as well as the perceptions of the risk premium associated with country debt. Furthermore, since fiscal policy is an important element in growth, debt accumulation and adjustment strategies, the model considers the impact of changes in government expenditure on these variables as well. Finally, the effects of a productivity shock are considered. Specifically the model enables us to analyze the impact of both demand management and structural policies on variables such as household consumption-saving decisions, investment decisions of the firm, and the current account, and hence debt accumulation. The model can also be used, as illustrated below, to study questions relating to the issue of the debt overhang, and for understanding the dynamics of a policy of debt relief.

The rest of the paper is organized as follows: Section II specifies the analytical framework. Section III discusses the long-run effects and dynamics of a balanced-budget policy, while debt-financed deficit policy is discussed in Section IV. Section V briefly discusses alternative specifications of the debt function. The concluding section summarizes

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1/ Reference should be made to a recent paper by Otani and Villanueva (1988) which analyzes the accumulation of capital and external debt in a neoclassical growth model. However, that paper adopts a very different approach and emphasizes different issues (i.e., the role of human capital formation) from those addressed in the present paper.

2/ See for example Eaton and Gersovitz (1981), Sachs (1984), Cooper and Sachs (1985). See Edwards (1984) for an empirical investigation of the risk premium.

3/ There are precedents for the type of debt supply function used in this paper. Eaton and Turnovsky (1983) incorporate such a function in their analysis of exchange rate dynamics under covered interest parity. Obstfeld (1982) also considers an upward sloping debt function in his intertemporal optimizing analysis of terms of trade shocks. However, most previous studies have not embedded such a debt supply function in an elaborate macroeconomic growth model such as the present one.

4/ The model can also be used, as illustrated below, to study questions relating to the issue of the debt overhang, or for understanding the dynamics of a policy of debt relief.

the main results of the paper and discusses some policy implications of the analysis. For convenience, all technical matters relating to the solutions are relegated to the Appendix.

II. The Analytical Framework

1. The structure of the economy

The economy we shall consider comprises three sectors: (i) consumers; (ii) firms; and (iii) the government. For analytical tractability all consumers and firms are assumed identical, enabling us to focus on the representative unit in each group. The economy produces a single traded good. The model is real, with the only financial asset held by domestic residents being a traded bond. 1/ International borrowing by the government is allowed, while consumers may borrow from or lend to the government, which is largely consistent with practice in borrowing countries. 2/

Unlike most macro-dynamic models of the type previously considered, our model incorporates sovereign risk via the introduction of an upward-sloping debt supply function. Eaton and Gersovitz (1981) have shown that because of the moral hazard associated with sovereign risk, lenders will charge a risk premium on lending to countries, and could, on occasion, also apply lending limits. 3/ This risk premium may vary directly with the stock of debt that the country holds since the probability of default is likely to increase with increased costs of servicing the debt; however, this could be offset if the costs associated with default also increase with debt accumulation. Sachs (1984), Sachs and Cohen (1982), and Cooper and Sachs (1985) have shown that a country, by adopting apparent growth-oriented policies, as well as policies that enhance its foreign exchange earning capacity, can shift the upward sloping debt supply curve outward, so that at each level of debt a lower risk premium is charged. Thus, for example, larger investment efforts or the adoption of efficiency-enhancing structural adjustment packages will result in a lower evaluation of country risk by lenders. The main conclusion of this literature is that the level of indebtedness of the country is likely to determine both the extent of additional credit that is likely to be made available, as well as the terms on which it will be made available. This proposition is best formalized by an upward sloping supply curve.

1/ Note that since the model is real, there are no prices or nominal variables that need be considered.

2/ Even though in some countries the private sector has borrowed abroad, implicit or explicit government guarantees have essentially underwritten this debt, making private debt indistinguishable from government debt, insofar as the foreign creditor is concerned.

3/ Stiglitz and Weiss (1981) have shown that even in cases of individual borrowing risk premia or credit constraints or both may exist because of informational asymmetries.

To incorporate this feature of international borrowing, we assume that the interest rate, i(z), charged on foreign debt, z, is:

$$i(z) = i_o + i_1\omega(z) \qquad \omega'(z) > 0,\ \omega''(z) > 0 \qquad (1)$$

where i_o is the interest rate prevailing internationally and $i_1\omega(z)$ is the country specific risk premium which varies directly with the stock of debt, z, held by the country. The possibility of a cutoff in debt is captured by the assumption that the function $\omega(z)$ is convex ($\omega''(z) > 0$). In this case, the fact that the interest rate rises at an increasing rate with the level of indebtedness means that at some point it becomes prohibitive, so that an effective lending limit is reached. 1/ With this specification, an exogenous increase in foreign interest rates is captured by an increase in i_o, while an exogenous shift in country-specific risk can be represented by a shift in the parameter i_1. 2/

Equation (1) results from the combination of an interest parity relationship, $i = i_o + \dot{e}$ and an equation relating to the determination of the relevant foreign interest rate, i.e., $i_o = i_w + \rho$, when i_w is the world interest rate and ρ is the risk premium attached to the home country. Recalling that $\dot{e} = 0$ (since this is a fixed exchange rate model) and assuming a specific form for the risk premium, i.e., $\rho = i_1\omega(z)$, yields equation (1). 3/ Interest parity (given fixed exchange rates) requires that the domestic interest rate be equalized with the foreign interest rate. Hence expression (1) above will also represent the interest rate prevailing at home, at which domestic consumers can borrow or lend to the government.

a. Consumers

The representative consumer chooses his consumption and bond holdings to maximize the intertemporal utility function:

1/ This enables us to incorporate the Eaton and Gersovitz (1981a,b) argument.

2/ This formulation which postulates the cost of debt to depend upon the absolute level of foreign debt is similar to that adopted by Eaton and Turnovsky (1983) and Obstfeld (1982). Alternative specifications which scale the absolute level of national debt by variables such as output or the capital stock will be considered later. Such specifications are intended to endogenize creditworthiness of the debt or country; see for example, Sachs (1984). However, as will be seen below, our essential qualitative results are not altered by these modifications.

3/ It should also be noted at this point that the risk premium attached to the home country could alternatively be modelled as a function of total government debt. However, as Dooley (1987) points out, this reformulation does not lead to any substantive change in the analysis.

$$\int_0^{\infty} U(x,\bar{\ell})\ e^{-\beta t}dt \qquad U_x > 0\ ,\ U_{xx} < 0 \tag{2}$$

subject to the budget constraint:

$$x+\dot{b} = w\bar{\ell} + \pi + i(z)b - T \tag{3}$$

and the initial condition:

$$b(0) \equiv b_o$$

where x is consumption, $\bar{\ell}$ is labor supply assumed to be fixed, β is the consumer rate of time discount, w is the wage rate, π is profit distributed by firms to households, b is the stock of debt (or bonds) held by domestic residents, and T denotes lump sum taxes. 1/ The utility function is assumed to be concave. 2/ The present-value Hamiltonian for the consumer problem may be written as:

$$H = e^{-\beta t}\{U(x,\bar{\ell}) + \lambda\ [w\bar{\ell} + \pi + i(a-b)b - T - x - \dot{b}]\} \tag{4}$$

where a denotes the total stock of government debt ($a \equiv z + b$), and λ is the Lagrange multiplier associated with the budget constraint. The required optimality condition for the individual's consumption decision is: 3/

$$U_x = \lambda \tag{5}$$

The optimal dynamic path is determined by the budget constraint, along with

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1/ For simplicity, labor is assumed to be fixed. Since in a developing country context the endogeneity of labor is not likely to be a critical issue, this assumption is not viewed as being particularly restrictive.

2/ If $b > 0$ the consumers are creditors while if $b < 0$ then they are debtors. Examination of the budget constraint shows that if consumers are creditors, then acquisition of increasingly costly debt by the government adds to disposable income and vice versa.

3/ Subscripts and primes (') denote derivatives.

$$\beta - \frac{\dot{\lambda}}{\lambda} = i - i'b \tag{6}$$

as well as the transversality condition:

$$\lim_{b\to\infty} \lambda b e^{-\beta t} = 0 \tag{7}$$

Equation (6) determines optimal bond accumulation by equating the marginal rate of return to consumers on consumption (the left hand side) to the marginal cost of an additional unit of debt facing consumers. 1/

b. *Firms*

Firms produce output using capital and the fixed supply of labor through a production function $y = f(k,\bar{\ell})$ which is assumed to possess standard neoclassical properties. Net profits of the firm at each point in time are, therefore, given by:

$$\pi(t) = f(k,\bar{\ell}) - w\bar{\ell} - C(I) \tag{8}$$

where I is the rate of investment.

The function C(I) represents installation costs associated with the purchase of I units of new capital. It is assumed to be an increasing convex function of I: $C'>0$, $C''>0$. In addition, we assume $C(0) = 0$, $C'(0) = 1$ so that the total cost of zero investment is zero, and the marginal cost of the initial installation is unity. This formulation of the installation function follows the original specification of adjustment costs introduced by Lucas (1967) and Gould (1968). 2/ More recent work by Hayashi (1982) and others postulates an installation function which depends upon k as well as I. This modification makes little difference to our analysis and for simplicity we retain the simpler specification.

1/ Note that the cost of debt depends on whether the consumer is a net creditor or a net debtor. In the former case the marginal cost exceeds the interest rate; in the latter case, the opposite is true.

2/ Note that this specification implies that in the case where disinvestment may occur that $C(I) < 0$ for low rates of disinvestment. This may be interpreted as reflecting the revenue obtained as capital is sold off. The possibility that all changes in capital are costly can be incorporated by introducing sufficiently large fixed costs, so that $C(0) > 0$. This does not alter our analysis in any substantive way.

The firm's optimization problem is to

$$\text{Maximize} \int_0^{\infty} \pi(t)e^{-\int_0^t i(s)ds}dt = \int_0^{\infty} [f(k,\bar{\ell})-w\bar{\ell}-C(I)]e^{-\int_0^t i(s)ds}dt \quad (9)$$

subject to

$$\dot{k} = I \quad (10)$$

and the initial condition

$$k(0) = k_o$$

Two further points about this specification of the firm's optimization problem should be noted. First, (10) abstracts from physical depreciation. Second, the firm is assumed to finance investment purely from retained earnings and therefore does not need to borrow.

Writing the Hamiltonian function for firms as

$$H = e^{-\int_0^t i(s)ds}[f(k,\bar{\ell}) - w\bar{\ell} - C(I)] + qe^{-\int_0^t i(s)ds}(I-\dot{k}) \quad (11)$$

where q is the Lagrange multiplier associated with the accumulation equation (10), the optimality conditions are:

$$C'(I) = q \quad (12)$$

$$f_k = -\dot{q} + i(z)q \quad (13)$$

$$w = f_{\bar{\ell}}(k,\bar{\ell}) \quad (14)$$

together with the transversality condition

$$\lim_{t\to\infty} qke^{-\int_0^t i(s)ds} = 0 \quad (15)$$

Equation (12) determines the level of investment in each period by equating the marginal cost of investment to the shadow price of capital. Equation (13) can be rewritten as:

$$\frac{f_k(k,\bar{\ell})}{q} + \frac{\dot{q}}{q} = i(z) \tag{16}$$

Here the left hand side, which equals the sum of the marginal physical product of capital deflated by q, and the percentage rate of change in the shadow price of capital, i.e., $\dot{q}/q$, is the rate of return on investing in a unit of capital. Optimality requires that this return should be equated, at each point in time to the interest rate. 1/ In view of the assumption about labor supply, equation (14) requires that the wage rate be set equal to the marginal physical product of the fixed supply of labor.

c. Government

The remaining agent in the domestic economy is the government, which operates in accordance with its budget constraint

$$\dot{a} = g + i(z)a - T \tag{17}$$

Thus, government spending, g, plus the interest obligations on outstanding government debt i(z)a, must be financed either by additional taxes, which we assume are of the lump sum type, or by issuing additional debt. Of course, the government retains the choice of borrowing at home or abroad. i.e., $\dot{a} = \dot{z} + \dot{b}$.

2. Macroeconomic equilibrium

Summing the consumers budget constraint (3), the firm's profit relation (8), and the government budget constraint (17), and noting that $\dot{z} = \dot{a} - \dot{b}$, the rate of decumulation of foreign debt, $-\dot{z}$, is equal to the current account balance. 2/

$$-\dot{z} = f(k) - x - C(I) - g - i(z)z \tag{18}$$

1/ This is ensured by an appropriate adjustment in q at each point in time.

2/ Hereafter the labor variable will be suppressed for convenience.

The current account balance is simply the sum of the trade balance, $f(k)-(x + C(I) + g)$, plus the service account, which in this case is only debt service, $-i(z)z$.

By combining the optimality conditions derived for the individual sectors above, together with the accumulation equations, macroeconomic equilibrium can be described by the following equations:

$$U_x(x) = \lambda \tag{19a}$$

$$C'(I) = q \tag{19b}$$

$$\dot{\lambda} = \lambda[\beta - i(z) + i'(z)(a-z)] \tag{20a}$$

$$\dot{q} = i(z)q - f_k(k) \tag{20b}$$

$$\dot{k} = I(q) \tag{20c}$$

$$\dot{z} = x + C(I) + g - f(k) + i(z)z \tag{20d}$$

$$\dot{a} = g + i(z)a - T \tag{20e}$$

The pair of static equations (19a) and (19b) determine consumption as a function of the marginal utility of consumption, λ,

$$x = x(\lambda) \qquad x' < 0 \tag{21a}$$

and investment ($I = \dot{k}$) as a function of its shadow price q

$$I = I(q) \qquad I' > 0 \tag{21b}$$

The latter equation will be recognized as being the Tobin q theory of investment. Substituting for x and I into (20a) - (20e) describes the dynamics of the economy.

When capital is perfectly mobile internationally, so that the interest rate facing the small economy is fixed at the given world rate, the dynamic structure simplifies drastically. In order for a steady-state equilibrium to exist in such an economy, the discount rate, β, must equal the world interest rate, and this implies that the marginal utility of consumption must remain constant, at say, $\bar{\lambda}$. The dynamics of the accumulation of capital k and its shadow price q, which constitute the core dynamics of the economy, are jointly determined by (20c) and (20b), respectively. The long-run equilibrium in such an economy has the property

that it depends upon the initial stocks of real and financial assets. This in turn means that temporary shocks have permanent effects. 1/

The fact that the economy faces an upward sloping supply function for debt changes the dynamics in a fundamental way. With the domestic interest rate depending upon the stock of national debt, the discount rate β is no longer tied to the world interest rate, and the marginal utility of consumption, λ, is not constant. Instead, it depends upon the accumulation of both foreign debt z and government debt a, and thereby the dynamics of the entire economy become highly interdependent.

In order to complete the specification of the dynamics, government budgetary policy needs to be specified. As a useful benchmark, which is analytically tractable, we shall focus first on the balanced-budget policy

$$\dot{a} = 0,$$

or $$T = g + i(z)\bar{a}$$

where lump sum taxes T are continually adjusted to finance expenditures. 2/ Using this model we analyze the long-run and short-run effects of a number of disturbances, focusing on the tradeoffs involved between the rate of capital accumulation, on the one hand, and the accumulation of foreign debt, on the other.

III. Balanced Budget Policy

1. The long run

The dynamics of the system involve forward-looking behavior. The short-run transition is therefore determined in part by the long-run steady state. Hence, it is convenient to begin with a consideration of the latter. Since the government is assumed to maintain a balanced budget, its stock of debt remains fixed at say, $\bar{a}$.

The steady state of the economy is reached when all variables cease to change, i.e., $\dot{\lambda} = \dot{q} = \dot{k} = \dot{z} = 0$. With no depreciation of capital, steady-state investment is zero, which in turn implies that the shadow price of investment, (q) is equal to 1. Denoting the steady-state values by tildes, the long run can be described by:

1/ See Sen and Turnovsky (1989a) for an example of such a model.

2/ In Section IV below, we shall also discuss a form of debt financing. In order to be sustainable in the long run, this needs to be accompanied by a once-and-for-all change in lump sum taxes.

$$U'(\tilde{x}) = \tilde{\lambda} \tag{22a}$$

$$i(\tilde{z}) - i'(\tilde{z})(\bar{a}-\tilde{z}) = \beta \tag{22b}$$

$$f_k(\tilde{k}) = i(\tilde{z}) = i_o + i_1\omega(\tilde{z}) \tag{22c}$$

$$f(\tilde{k}) - \tilde{x} - g = i(\tilde{z})\tilde{z} \tag{22d}$$

$$\tilde{T} = g + i(\tilde{z})\bar{a} \tag{22e}$$

These equations jointly determine the steady-state values of the marginal utility $\tilde{\lambda}$, consumption $\tilde{x}$, capital stock $\tilde{k}$, national debt $\tilde{z}$, and lump sum taxes $\tilde{T}$.

The equilibrium defined by (22a) - (22e) has a particularly simple recursive structure. The equilibrium level of the foreign debt $\tilde{z}$ is determined by (22b) which requires that consumers equate the costs (or returns) of buying or selling the marginal bond to their rate of time preference. Since the debt schedule facing the economy and hence the consumer, is upward sloping, the cost (or return) of holding a marginal bond equals the interest rate plus the impact of the additional bond on the interest rate. Having determined $\tilde{z}$, the domestic rate of interest is determined by the debt supply schedule. Equation (22c) then determines the long-run capital stock by equating the marginal physical product of capital to the domestic interest rate. At the same time, equation (22e) determines the required lump sum taxes necessary to finance government expenditures and interest on outstanding government debt. Equation (22d) is the steady-state balance of payments equilibrium condition. In the long run, the country must run a balance of trade surplus of sufficient magnitude to finance the interest on the outstanding foreign debt. Given the capital stock, and hence output, and the level of interest payments, this determines the level of consumption $\tilde{x}$. Finally, given $\tilde{x}$, the equilibrium marginal utility of consumption $\tilde{\lambda}$ is determined by (22a).

Table 1 summarizes the long-run effects of shifts in the exogenous variables namely: the foreign interest rate (i_o), the risk premium (i_1), a productivity shock (θ), and an increase in government expenditure (g), on the stock of debt, the capital stock, the domestic interest rate, the trade balance, consumption, and taxes. 1/

An increase in the level of international interest rates, represented by an upward shift in i_o raises the marginal return or cost to consumers from holding an additional bond. Since in the long-run this marginal return must equal the fixed rate of time discount β, this requires a reduction in the former, to offset the effect of the higher i_o, and maintain equality with the latter. This is brought about by a reduction in

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1/ The condition $2i'-i''\tilde{b} > 0$ given in Table 1 is essentially a stability condition; see Appendix.

Table 1. Long-Run Effects

		Increase in i_o	Increase in i_1	Increase in θ	Increase in g
Foreign debt	$\tilde{z}$	$-\frac{1}{D}$	$\frac{\omega'\tilde{b}-\omega}{D}$	0	0
Capital stock	$\tilde{k}$	$-\frac{(i'-i''\tilde{b})}{f_{kk}D}$	$\frac{i'(\omega'\tilde{b}+\omega)-i''\omega\tilde{b}}{f_{kk}D}$	$\frac{f_{k\theta}}{f_{kk}}$	0
Interest rate	$\tilde{i}$	$\frac{i'-i''\tilde{b}}{D}$	$\frac{i'(\omega'\tilde{b}+\omega)-i''\omega\tilde{b}}{D}$	0	0
Trade balance		$-\frac{[i-\tilde{z}(i'-i''\tilde{b})]}{D}$	$i\frac{d\tilde{z}}{di_1}+z\frac{d\tilde{i}}{di_1}$	0	0
Consumption	$\tilde{x}$	$\frac{f_k(i'-i''\tilde{b})}{f_{kk}D}$	$\left(\frac{i}{f_{kk}}-\tilde{z}\right)\frac{d\tilde{i}}{di_1}$	$f_\theta-\frac{f_k f_{k\theta}}{f_{kk}}$	-1
		$+\frac{i-\tilde{z}(i'-i''\tilde{b})}{D}$	$-i\frac{d\tilde{z}}{di_1}$		
Lump sum taxes	$\tilde{T}$	$\frac{\bar{a}(i'-i''\tilde{b})}{D}$	$\bar{a}\frac{d\tilde{i}}{di_1}$	0	1

$D = 2i' - i''\tilde{b} > 0$

the level of external debt, $\tilde{z}$. However, the effect upon the domestic interest rate is subject to countervailing considerations and is a priori unclear. On the one hand, the upward shift in the debt supply schedule leads to a substitution away from foreign debt to domestic debt, putting upward pressure on the domestic interest rate. On the other hand, the reduction in the long-run stock of external debt lowers the risk premium, thereby tending to reduce the domestic rate. The overall effect depends upon which of these factors dominates, and this in turn depends critically upon the nature of the risk premium function $\omega(z)$. Intuition would suggest that the domestic interest rate will, on balance, rise, and this in fact is likely to be so under plausible conditions. For example, if the debt supply schedule is linear ($i'' = 0$) or if domestic private residents hold no bonds ($\tilde{b} = 0$), then a rise in the more costly foreign debt does indeed result in a rise in the domestic interest rate, though not by the full amount. 1/ If $i'' > 0$ and domestic private residents are net debtors ($\tilde{b} < 0$), the upward pressure on the domestic interest rate is increased. On the other hand, if $i'' > 0$ and domestic private residents are large creditors ($\tilde{b} > 0$), the response of the domestic interest rate is reduced, and indeed it may now fall. 2/

The response of the domestic capital stock depends upon that of the domestic interest rate. Taking the more plausible case where the latter rises, the equilibrium capital stock falls, while the financing requirement of the government rises. As noted, in the long run, the trade surplus must finance the interest costs $i(\tilde{z})\tilde{z}$ of the foreign debt. While one effect of the higher foreign interest rate is to reduce the stock of foreign debt, this is likely to be offset by a higher domestic interest rate. The net effect on total interest payments is unclear and depends upon the function $\omega(z)$. In the case where this is linear, the decline in foreign debt dominates the higher domestic interest rate and overall interest payments decline. Assuming this case prevails, the long-run trade balance which was, and remains, in surplus to meet the economy's interest obligations, is reduced. What happens to domestic consumption is uncertain, even in the simplest case. For example, while the linear debt schedule leads to a reduction in the capital stock and hence output, less output is now devoted to interest payments on international debt. 3/ The

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1/ If the debt supply function is horizontal, i.e., $i_1 = 0$ and $i(z) = i_o$, then any increase in the external interest rate would translate into an equiproportionate rise in domestic interest rates.

2/ Assuming a convex debt function of the form $i = i_o + i_1 z^{\alpha}$, $\alpha > 1$, this will be so if the ratio $\tilde{b}/\bar{a} > 1/\alpha$.

3/ These results may be usefully compared with the long-run effects of an increase in the foreign interest rate under the limiting assumption of uncovered interest parity. In such a case, the domestic interest rate rises by the same amount as does the foreign interest rate, leading to a larger fall in the domestic capital stock than in the present case. The stock of external debt can be shown to decline by an amount which is proportional to the reduction in the capital stock, with the resulting effect on the long-run trade balance being ambiguous, depending upon the stock of external debt.

net effect on consumption depends upon which of these influences dominates. Moreover, since utility is a function of consumption, an upward shift in the cost of debt may, or may not, lead to a corresponding reduction in steady-state welfare.

Turning to the impact of an increase in the risk premium i_1, we see that this will raise or lower the marginal return or cost to consumers from holding additional debt, according to whether $\omega - \omega'\tilde{b} \gtrless 0$. In the case where domestic consumers are net debtors ($\tilde{b} < 0$), this quantity is certainly positive, raising the marginal cost to them of incurring additional debt. As a result, they reduce their level of debt (i.e., increase $\tilde{b}$) so as to maintain the equality between the marginal cost of debt and the fixed rate of time preference β. However, if consumers are net creditors, the marginal return to holding more debt may either rise or fall with i_1, even for the simplest debt function, depending upon the level of $\tilde{b}$. In the case where $\tilde{b}$ is sufficiently small so that $\omega - \omega'\tilde{b} > 0$, external debt will still fall. However, if $\tilde{b}$ is sufficiently large to reverse this inequality, a higher risk premium will lead to a higher equilibrium stock of external debt. 1/

The response of the domestic interest rate depends upon whether the upward shift in the marginal cost of debt brought about by i_1 more than offsets the decline resulting from the likely reduction in the stock of external debt. Again this depends upon the form of the function $\omega(z)$. For plausible debt supply functions, such as the linear, or constant elasticity type, the interest rate will certainly rise. However, a decline cannot be ruled out, although it is even less likely to occur than in response to an upward shift in i_o. The stock of capital moves counter to the interest rate and therefore, almost certainly will fall. The equilibrium trade balance will rise or fall to cover external payments commitments. Focusing on the linear debt supply function, if domestic residents are net debtors, external debt as well as interest payments decrease and therefore, the trade surplus declines; otherwise it could move in either direction. Finally, the effects on long-run consumption and utility are unclear, although both will decline if net interest payments increase.

Column 3 of Table 1 summarizes the effect of a productivity shock, θ, which is introduced as a shift operator in the production function. 2/ Such a disturbance has no effect on the steady-state stock of external debt and the equilibrium trade balance too, remains unaffected. The effect on the capital stock depends on the sign of $f_{k\theta}$, the impact of the productivity shock, θ, on the marginal physical product of capital. If

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1/ For example, for the constant elasticity convex debt function $i = i_o + i_1 z^{\alpha}$, $\alpha \geq 1$, the quantity $\omega - \omega'\tilde{b} = \tilde{z}^{\alpha-1}[\tilde{z} - \alpha\tilde{b}]$. The criterion determining whether external debt falls or rises with i_1 depends upon whether the ratio of the country's external debt to the net credit of its private sector is greater or less than α, i.e., whether $\tilde{z}/\tilde{b} \gtrless \alpha$.

2/ For this case, the production function is changed to $f(k,\theta)$.

the shock is productivity-enhancing, capital stock will increase. The effect on consumption depends on whether or not the shock increases output i.e., $f_\theta \gtrless 0$, and whether or not capital stock increases.

The last column of Table 1 summarizes the impact of a change in government expenditures. Since, in this model all government expenditures are merely for consumption purposes, and the government budget is balanced, a change in government expenditure affects only consumption by the private sector. Any increase (decrease) in government expenditures has to be matched fully by an increase (decrease) in taxation and hence a commensurate decrease (increase) in private consumption.

2. Transitional dynamics

The transitional dynamic behavior of the economy is determined by the system of differential equations, (14a) - (14g). The solution of this fourth-order dynamic system is described in the Appendix. In the short run the stock variables, k and z may be regarded as predetermined. Any response to shocks will, therefore, impact on the two shadow prices, q(0) and $\lambda(0)$. As shown in the Appendix, these prices are determined by the expected long-run responses of both the capital stock and the foreign debt. In this respect, the dynamics of the system are forward-looking.

In this section we will examine the transitional effects of exogenous shocks, namely, shifts in the debt schedule, productivity shocks, and changes in the government expenditure levels. The shocks that we will consider are all unanticipated permanent shocks. Table 2 summarizes the qualitative short run effects of the various shocks.

a. Increase in the foreign interest rate

The previous section showed that an increase in i_o leads to a long-run decline in external debt and almost certainly in the stock of capital as well. Assuming this to be so, both of these long run effects contribute to an immediate fall in the shadow price of investment q(0), and therefore in the level of investment. 1/ The rate of capital accumulation, therefore slows down as a result of this shock. With the stock of foreign debt fixed instantaneously, a one percentage point increase in the foreign interest rate leads to a corresponding immediate one percentage point increase in the domestic interest rate, although over time the domestic rate will decline to the smaller steady-state response. The higher short-term interest with the fixed stock of debt means that the cost of servicing the debt also immediately increases, although it too

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1/ For obvious reasons we restrict our discussion to the plausible case where the increase in the foreign interest rate leads to an increase in the long-run domestic interest rate and a corresponding decline in the long-run capital stock. The perverse case, where the long-run interest rate declines can be analyzed similarly but is of little practical interest

Table 2. Qualitative Short-Run Effects

		Increase in			
		i_o	i_1	θ	g
Debt accumulation	$\dot{z}(0)$	−	< 0 if $\frac{d\tilde{z}}{di_1} < 0$? if $\frac{d\tilde{z}}{di_1} > 0$	+	0
Investment	$\dot{k}(0)$	−	−	+	0
Interest rate	$i(0)$	+(=1)	$+\omega(\tilde{z})$	0	0
	$\frac{di(0)}{dt}$	−	?	+	0
Trade balance		+	< 0 if $\frac{d\tilde{z}}{di_1} < 0$? if $\frac{d\tilde{z}}{di_1} > 0$	−	0
Consumption	$x(0)$	?	?	?	−(= −1)
Lump sum taxes	$T(0)$	+(=$\bar{a}$)	+	0	−(= −1)

declines over time, as both the interest rate and the stock of debt decline to their steady-state values.

The effect of an increase in i_o^* on initial consumption is unclear. Equation (5) shows that the consumption varies inversely with the marginal utility of consumption, λ. However, we are unable to establish unambiguously the initial response of $\lambda(0)$, and hence that of consumption. Various factors are at work. First, the short-run marginal utility depends upon the long-run marginal utility, $\tilde{\lambda}$, which may either increase or decrease, as previously discussed. Secondly, since we know that the long-run capital stock declines, capital will decumulate over the transition path. This would tend to increase consumption over the transitional path. An offsetting impact on consumption, however, results from the fact that long-run external debt is known to decline, and hence the debt stock must also be reduced in the transition. Since the long-run debt of the government is known to be fixed at $\bar{a}$, the private supply of debt has to increase and this may occur at the expense of consumption.

The impact effect of the increase in external interest rates on the trade balance tends to increase the surplus. Although the debt-servicing cost increases on impact and the effect on consumption is uncertain, the decline in investment alone is sufficient to increase (reduce) the surplus (deficit).

The contrast between the short-run and long-run effects of an upward shift in the cost of debt can be usefully summarized at this point. An increase in i_o leads to long-run declines in both the capital stock and the level of external debt. Despite a somewhat higher interest rate, the long-run cost of debt servicing declines and this requires a smaller trade surplus. In the short run, prior to any adjustment in the stock of debt, the domestic interest rate responds by the full amount of the increase in the foreign rate. Consequently, debt servicing requirements in the short run are increased. At the same time, since higher interest rates in the long run lead to a reduced capital stock, investment must decline. The cumulative effect of these changes is that in the short run the trade balance improves to meet the increased interest payments, as well as to allow the stock of foreign debt to decline.

Further characterization of the transitional dynamic paths followed by k and z can be obtained by using the dominant eigenvalue method suggested by Calvo (1987). Writing the solutions (A5) and (A6) in the Appendix, for k and z in the form

$$k = \tilde{k} + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}$$

$$z = \tilde{z} + \phi(\mu_1)A_1 e^{\mu_1 t} + \phi(\mu_2)A_2 e^{\mu_2 t}$$

we have

$$\frac{z-\tilde{z}}{k-\tilde{k}} = \frac{\phi(\mu_1)A_1 e^{(\mu_1-\mu_2)t} + \phi(\mu_2)A_2}{A_1 e^{(\mu_1-\mu_2)t} + \phi(\mu_2)}$$

where $\phi(\mu_1) > 0$, $\phi(\mu_2) < 0$ are defined in the Appendix. Since μ_2 is the dominant stable root (i.e., $0 > \mu_2 > \mu_1$), it follows that as $t \to \infty$ $(z - \tilde{z})/(k - \tilde{k}) \to \phi(\mu_2) < 0$. That is, z and k asymptotically approach their respective steady-state values along a ray having a negative slope = $\phi(\mu_2)$. The initial phase of the path can be determined by evaluating dz/dk, and d^2z/dk^2 at the initial instant t = 0. In the present case of the upward shift in the cost of debt, i_o, we have already shown that $dz(0)/dk(0) > 0$, and imposing additional weak conditions we can further establish that $d^2z(0)/dk(0)^2 < 0$.

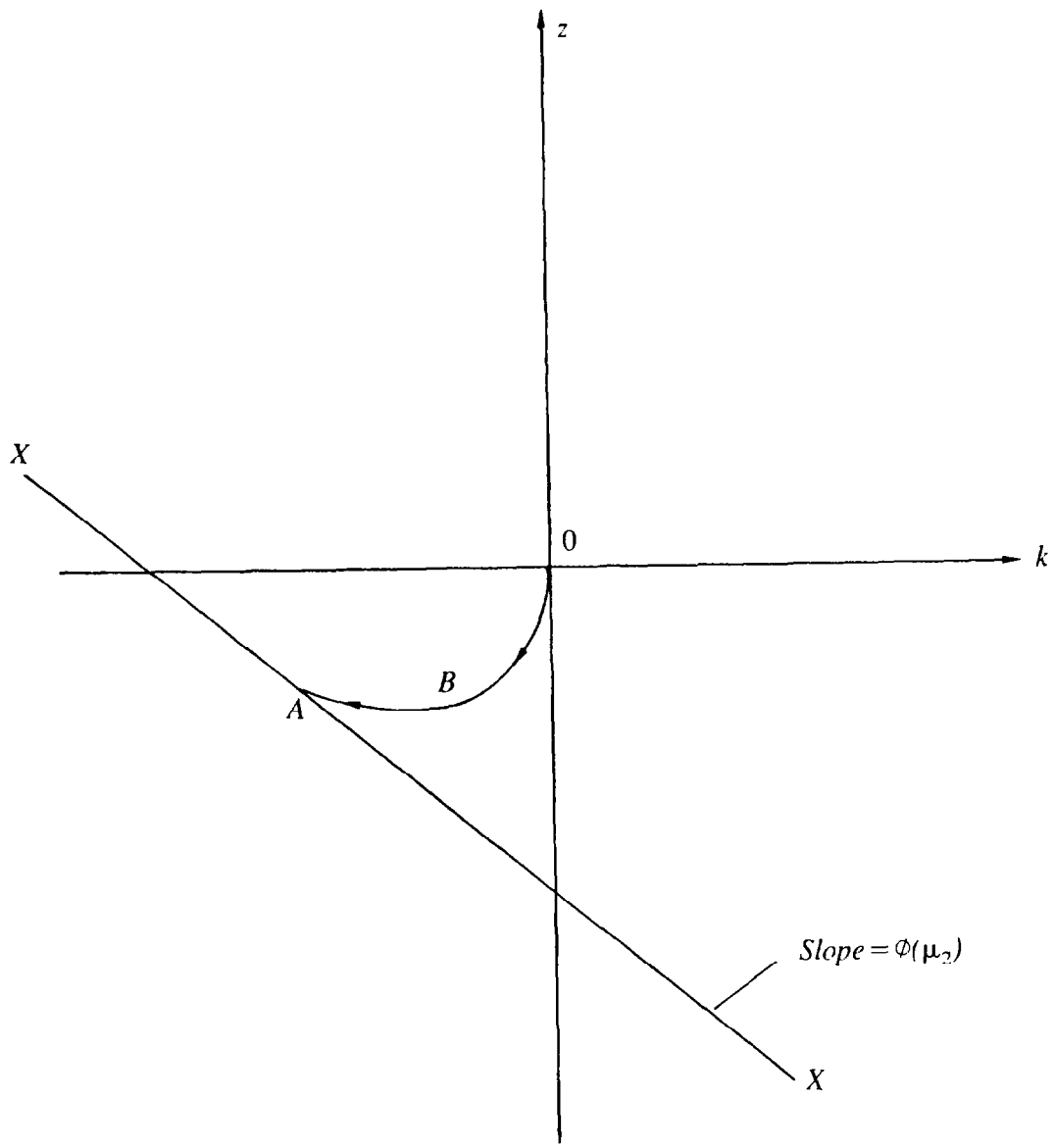
The transitional path followed by k and z is illustrated in Figure 1. The starting point is at the origin, with the new steady-state equilibrium being at A, having lower stocks of both physical capital and foreign debt. In the limit, this is approached along the locus XX having the negative slope $\phi(\mu_2)$. Initially, the declining capital stock and external debt causes the system to move in a southwesterly direction from 0. The convexity of this locus, and the subsequent convergence along the negative ray implies that the stock of foreign debt overadjusts during the transition. After declining to the point B, z then begins to increase to its new equilibrium level. Intuitively, the convexity of the transitional locus OA implies an increasing decline in k relative to z over time. This in turn leads to an eventual decline in output, which exceeds the decline in other components of the current account. The initial current account surplus eventually becomes a deficit and at that time additional external debt is incurred. 1/

b. Increase in risk premium

The qualitative short-run responses to an increase in the marginal cost of debt, i_1, are given in Column 2 of Table 2. As before, these responses also depend on the long-run response of the level of external debt, which as shown in Section 3.2a depends upon $(\omega'\tilde{b}-\omega)$. If $\omega'\tilde{b} < \omega$, so that $\tilde{z}$ falls, the results are qualitatively the same as those for an increase in i_o and for essentially the same reasons. Once again the short-run interest rate exceeds the long-run. If, on the other hand, $\omega'\tilde{b} > \omega$, so that long-run debt increases, the short-run responses are less clear. Even though one can establish that there will be an initial decline in the

1/ This contrasts with the dynamics under the limiting case of uncovered interest parity when the paths followed by z and k can both be shown to be monotonic.

Figure 1: Transitional Path in Response to
Upward Shift in Cost of Debt



shadow cost of capital q(0), causing an immediate decumulation of capital, it is now possible for this to be accompanied by a current account deficit leading to an increase in foreign debt. In any event, even if the external debt does not decline on impact, it will have to rise at some point during the transitional path.

c. Productivity shock

The productivity shock is simpler to analyze. The reason is that since it has no long-run effect on national debt, the only long-run response driving short-run behavior is the change in the capital shock. In order to be concrete, assume $f_{k\theta} > 0$, i.e., the productivity shock impacts positively on the marginal physical product of capital. In this case, the increase in the long-run level of the capital stock will lead to an instantaneous increase in the shadow price q(0) causing investment demand to increase and an increased accumulation of capital to take place. Moreover, since consumption increases in the long run, the steady-state value of the marginal utility of consumption, $\tilde{\lambda}$, is now lower. Instantaneously $\lambda(0)$ will likely fall, though by a smaller amount than in the long-run. Consequently, though consumption increases immediately it will not increase by the full amount of the increase expected in the steady state. This is so because the expansion in the economy stimulates consumption as capital is accumulated over the transition.

There is no immediate effect on the interest rate or interest payments. The higher level of investment and the likely increase in consumption, leads to a deterioration in the trade balance and, therefore, in the current account. Foreign debt begins to accumulate, resulting in a rising interest rate. However, this is only temporary. Over time, as capital is accumulated, output increases. Eventually, output exceeds the levels of domestic demand by an amount in excess of that of additional interest payments leading to a surplus on the current account. At that time foreign debt is reduced and eventually returns to its original level. The adjustment path followed by capital and external debt is illustrated in Figure 2. The dynamics of debt are mirrored in the behavior of interest rates. Increasing current account deficits and accumulating debt lead to rising domestic interest rates on account of the increasing risk premium. When the current account switches from a deficit to a surplus and the process of debt reduction begins, interest rates start to decline, also returning to their pre-shock levels.

d. Increase in government expenditure

Since the long-run stocks of $\tilde{k}$ and $\tilde{z}$ remain unaffected by changes in government expenditures, and since the long-run behavior of these state variables determine the dynamics, changes in government expenditures do not affect any variable other than consumption and revenues. Again, because of the balanced-budget assumption, as government expenditures increase (decrease) a corresponding increase (decrease) in revenues occurs, leading to a corresponding decrease (increase) in consumption.

IV. Debt-Financed Deficit Policy

Up to this point we have assumed that the government's budget is continuously balanced. This assumption is a simplifying one, made in part to maintain analytical tractability. 1/ The present section briefly considers a modification wherein the domestic government finances its deficit with the issuance of debt instruments. 2/

At the outset it may be noted that such a debt-finance policy requires an accommodating one-for-all adjustment in the initial level of lump sum taxes, in order to be sustainable in the long-run. Formally, this arises from the fact with the stock of government debt constrained to adjust continuously, the number of unstable roots to the dynamic system (3) exceeds the number of "jump" variables (2). In order to obtain a viable solution therefore, an additional "jump" variable is required. This may be accomplished by appropriately choosing the initial level of lump-sum taxes. The steady-state of the model is now defined by: 3/

$$U_x(\tilde{x}) - \tilde{\lambda} = 0 \tag{23a}$$

$$i_o + i_1\tilde{z} - i_1(\tilde{a}-\tilde{z}) = i_o + i_1\,(\tilde{a}-2\tilde{b}) = \beta \tag{23b}$$

$$f_k(\tilde{k}) = i_o + i_1\tilde{z} \tag{23c}$$

$$f(\tilde{k}) - \tilde{x} - g = (i_o + i_1\tilde{z})\tilde{z} \tag{23d}$$

$$T = g + (i_o + i_1\tilde{z})\tilde{z} \tag{23e}$$

$$\psi_1(\tilde{z}-z_o) + \psi_2(\tilde{k}-k_o) + \psi_3(\tilde{a}-a_o) = 0 \tag{23f}$$

where equations (23a) - (23e) are identical to equations describing the steady-state for the balanced-budget case while (23f) is an additional relationship linking changes in government debt, total national debt and capital stock that (as shown in the Appendix) must additionally be satisfied. The coefficients ψ_i's are defined as:

$$\psi_1 \equiv \tilde{a}i'[\mu_1\mu_2 - I'f'' - i(\mu_1+\mu_2-i)] > 0$$

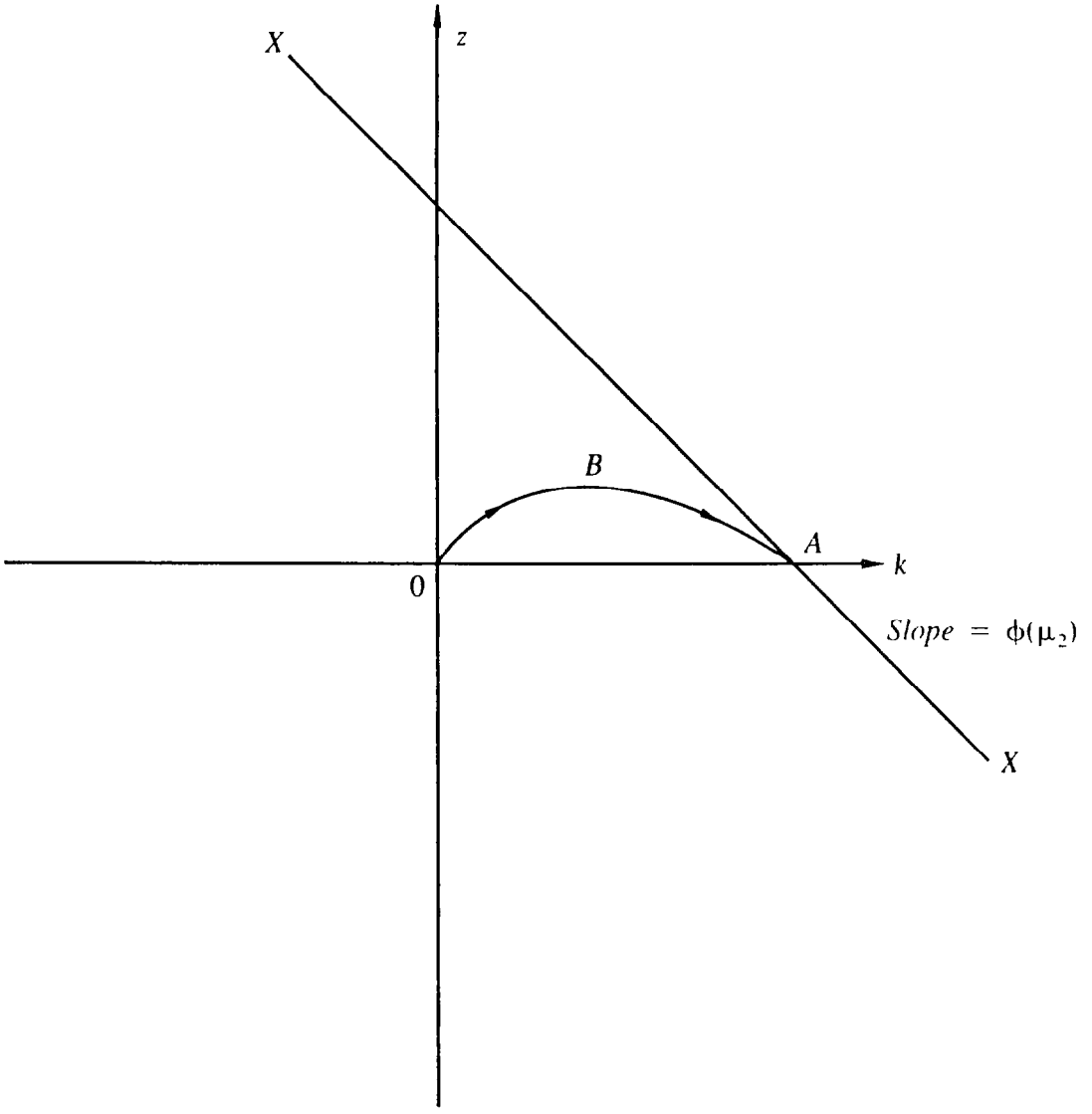
$$\psi_2 \equiv \tilde{a}i'^2 I'\phi(\mu_1)\phi(\mu_2) < 0$$

1/ As discussed in the Appendix, the dynamic structure of the balanced-budget variant involves four differential equations. With debt finance, the resulting dynamics is of the fifth order; the additional source of dynamics is the evolution of the stock of government debt $\dot{a}$.

2/ The technical details are discussed in the Appendix.

3/ For expositional convenience we restrict our discussion to the case where the debt schedule is linear.

Figure 2: Transitional Path in Response to Productivity Shock



$$\psi_3 \equiv -(\mu_1-i)(\mu_2-i)(\mu_1+\mu_2-i) > 0$$

where in turn μ_1, μ_2 are the stable (negative) roots of the dynamic system (see Appendix for details) while $\phi(\mu_i)$ are defined as:

$$\phi(\mu_i) \equiv \frac{\mu_i(\mu_i-i)+f''I'}{I'i'}$$

The steady-state stock levels of $\tilde{k}$, $\tilde{z}$ and $\tilde{a}$ are jointly determined by equation (23b), (23c) and (23d) and depend (via (23f)) upon the initial conditions k_o, a_o and z_o. Because of this, even temporary changes can give rise to permanent effects. 1/ Once $\tilde{k}$ and $\tilde{z}$ are determined as above, the balance of payments condition (23d) determines steady-state consumption $\tilde{x}$, while $\tilde{\lambda}$ may now be obtained from (23a). The required accommodation in lump-sum taxes is determined from (23e). Once T is set in this manner, it is not thereafter adjusted and the subsequent deficit is bond financed while the stock of government bonds follows an adjustment path. The short-run and long-run effects of the various disturbances can be analyzed in a manner similar to that previously and may be briefly summarized.

An upward shift in the cost of debt, i_o, will lower both the steady-state stock of foreign debt and physical capital, as previously. The long-run interest rate will also increase, though by less than the increase in i_o. Long-run government debt may either rise or fall. This is because the higher interest rate, accompanied by the lower foreign debt may or may not raise the marginal return (or cost) of purchasing an additional bond to consumers. If it does, $\tilde{a}$ will have to rise, in order to maintain the equality of the marginal return with the fixed rate of consumer discount β. But if this marginal return falls, then $\tilde{a}$ will have to fall for the same reason.

The short-run responses to an upward shift in the debt schedule can be analyzed as before. Again we can establish that the long-run declines in $\tilde{k}$ and $\tilde{z}$ generate short-run decumulations in these quantities. The interest rate rises more in the short run and the higher short-run interest costs to the government causes a short-run increase in the rate of accumulation of government debt.

1/ The impact of changes in the z_o, i.e., changes in the stock of debt held by the country, can be used to study the dynamics of the effects of debt forgiveness schemes. In this model, interest relief schemes are equivalent to negative foreign interest rate shocks.

While the changes in the marginal cost of debt i_1 and government expenditure give rise to virtually the same responses as before, more substantive differences arise with respect to a productivity shock. Not only does it lead to a long-run increase in capital stock as before, but now this is accompanied by higher long-run levels of both national and government debt. The higher marginal productivity of capital resulting from the increase in productivity raises the domestic interest rate, thereby encouraging more borrowing from abroad. The higher level of external debt raises the marginal rate of return of additional bonds to domestic consumers, and government debt will have to rise in order to maintain equality with β. The initial responses are also in the direction of the long-run effects.

V. Alternative Specifications of the Debt Function

As noted earlier, it has been argued that country creditworthiness may not be a function of only the stock of debt alone. Factors that serve to increase productivity or growth, or foreign exchange earnings, may all serve to indicate an increased capacity to service debt, and hence shift the debt supply schedule outwards. To incorporate this idea, the debt function can be altered in at least two interesting ways. Note that the above argument basically requires debt to be scaled in some way in its effect on the rate of interest. The central point appears to be that creditors are not as concerned with the absolute amount of debt held by the country as with its ability to service the debt, which is better represented by debt relative to some measure of servicing capacity. Thus, alternative specifications might be to use either debt-output (z/y) or debt-capital ratio (z/k) in (1) above. 1/

With employment fixed and output dependent on capital alone, these two modifications are almost identical. Moreover, neither of them alters our results in any fundamental way. For example, if one were to use the relationship

$$i(z/k) = i_o + i_1\omega(z/k)$$

1/ This would transform (1) to

$$i(z/y) = i_o + i_1\omega(z/y) \qquad (1')$$

or,

$$i(z/k) = i_o + i_1\omega(z/k) \qquad (1")$$

respectively.

and assume a balanced government budget, one finds that the two critical steady-state equilibrium relationship (22b), (22c) are modified to

$$i(\tilde{z}/\tilde{k}) - i'(\tilde{z}/\tilde{k})(\bar{a}-\tilde{z})/\tilde{k} = \beta \qquad (22\acute{b})$$

$$f_k(\tilde{k}) = i_o + i_1\omega(\tilde{z}/\tilde{k}) \qquad (22\acute{c})$$

The interdependence between z and k in the determination of the interest rate now destroys the recursivity in the long-run equilibrium noted earlier. These two equations now jointly determine the steady-state stocks of capital and external debt. The changes in these equilibrium levels then drive the short-run dynamics as before. Generally, the qualitative behavior of the system is changed little. The only point worth noting is that a productivity disturbance will now affect the steady state stocks of both debt and capital.

VI. Conclusion

Although it has long been recognized that small developing economies can borrow only at a premium, and therefore face an upward sloping supply of debt, most existing macro-dynamic models of such economies treat the supply of debt as being infinitely elastic. This paper has departed from this assumption and analyzed the dynamic consequences of various kinds of disturbances in an economy facing an upward sloping supply of debt. The presence of such a constraint changes the dynamics in fundamental ways from those which occur under the more usual, but less realistic, assumption of perfect capital mobility.

In order to highlight the issues involved, we have based most of our analysis on the simplest such model and then modified it in alternative ways. The basic model is one in which the government maintains a balanced budget, the cost of debt rises with its absolute level, and employment remains fixed. We have emphasized the tradeoffs which exist between the rate of capital accumulation, on the one hand, and the rate of accumulation of foreign debt, on the other, and analyzed how these respond to the various exogenous disturbances.

In this basic model, the only sources of disturbances which involve dynamic tradeoffs are disturbances either to the supply of debt function, or the production function. To some extent, the effects on the economy are sensitive to the form of the debt supply function, in particular its degree of convexity, as well as the net asset position of domestic private residents. The main findings, summarized as follows, are based on what we view as being the most plausible assumptions:

(1) An upward shift in the supply of debt, which may be taken to represent an exogenous increase in the world interest rate, leads to a long-run decline in external debt, a partially higher domestic

interest rate, and a lower capital stock. The combination of the lower external debt and the only partial response of the domestic interest rate results in a decline in the long-run cost of servicing the external debt, implying a reduction in the long-run trade surplus. In the short run, the domestic interest rate responds fully to the higher foreign interest rate, causing total interest costs to rise. The rate of investment falls, generating a sufficient increase in the trade surplus for the stock of external debt to begin to decline, along with the capital stock. The transitional path for foreign debt is non-monotonic. At some stage, the trade surplus becomes a deficit, and further debt is accumulated as the new steady-state equilibrium is approached.

(2) An increase in the marginal cost of debt (risk premium), while almost certainly raising the long-run domestic interest rate and lowering the capital stock, may or may not lower long-run external debt as well. Whether the stock of debt is reduced or not depends upon the ratio of the country's external debt to the net credit of its private sector. If this ratio is greater than some critical value (which depends the form of the debt supply function), then foreign debt is reduced in the long run and vice versa. The trade balance adjusts in the direction of a surplus if the long-run impact of the shock on debt accumulation is positive; i.e., if the country is not heavily indebted. Otherwise, the effect is uncertain.

(3) A positive productivity disturbance, resulting from say, structural efficiency-enhancing measures, will raise the long-run stock of capital, while leaving the long-run level of external debt and interest rate unchanged. In the short run capital is now more productive and begins to accumulate. A current account deficit is therefore generated and foreign debt is accumulated, leading to a higher interest rate. The accumulation of debt, however, is only temporary. Eventually as output increases, the current account deficit turns to a surplus and debt, along with the interest rate, returns to its original level.

(4) A fiscal expansion, taking the form of an increase in government expenditure has virtually no effect, either in the short run or in the long run. All that occurs is that consumers immediately offset the additional government expenditure with an equal reduction in private consumption.

Most of these conclusions carry over to the more general models, though with modifications in some instances. For example, in the case of a bond-financed deficit, the only substantive change that occurs is that a productivity shock now has a long-run effect on the level of external debt as well. Increased capital accumulation as a result of a positive productivity disturbance will now be accompanied by both higher stocks of external and government debt in the long run.

Modifying the debt supply function and specifying it in terms of debt relative to some measure of the country's economic performance such as output or capital stock also leaves our results mostly unchanged. Again the main difference is that the long-run stock of foreign debt will no longer be independent of the productivity shock. As capital or output increases, the marginal cost of debt declines, allowing a higher long-run stock of debt.

In the current version of the model, the role for fiscal policy is limited. Both the long-run stock of debt and the long-run stock of capital are determined independently of government expenditure. Since it is the changes in these long-run equilibrium stocks which drive the short-run dynamics, the latter are also independent of government expenditure. Endogenizing employment and/or including a role for government expenditure in either investment or consumption could be interesting avenues to pursue for a better understanding of the real effects of fiscal policy.

In conclusion, it is worthwhile to summarize some implications of this analysis for growth and adjustment. The model discussed in this paper, suggests that structural adjustment policies may involve tradeoffs insofar as their effects upon growth and the level of debt. For example, a productivity-enhancing policy will increase the growth rate but will also increase the stock of external debt initially. An exogenous increase in the world interest rate leads to a long-run decline in external debt; however, this is at the cost of a lower capital stock. On the other hand, an increase in the risk premium also reduces the capital stock but may or may not lower the stock of external debt. Similar tradeoffs are observed for government expenditure policy when domestic employment is permitted to be endogenous. Finally, as shown, the model lends itself readily to analyzing the dynamics of the effects of alternative debt relief strategies.

APPENDIX

1. Dynamic properties with balanced budget

Under the assumption of a balanced budget the dynamic structure of the system (20) is a fourth order system, which may be expressed in linearized form about the steady-state equilibrium

$$\begin{bmatrix} \dot{\lambda} \\ \dot{q} \\ \dot{k} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -\lambda(2i'-i''b) \\ 0 & i & -f_{kk} & i' \\ 0 & I' & 0 & 0 \\ x' & I' & -f_k & i+i'z \end{bmatrix} \begin{bmatrix} \lambda-\tilde{\lambda} \\ q-\tilde{q} \\ k-\tilde{k} \\ z-\tilde{z} \end{bmatrix} \tag{A1}$$

where the elements appearing in the matrix are evaluated at steady state. The dynamic properties of the economy depend upon the eigenvalues of the characteristic equation of (A1), namely

$$\frac{\mu(\mu-i)}{\mu(\mu-i)-[i'z\mu+x'a_{14}]} = \frac{\mu(\mu-i)+f_{kk}I'}{I'i'} \equiv \phi(\mu) \tag{A2}$$

where the element $a_{14} \equiv -\lambda(2i'-i''b)$.

Assuming $2i'-i''b > 0$, the following properties can be established:

(1) The product of the four roots is positive, implying that there are either 0, 2 or 4 positive roots.

(2) The sum of the roots is positive, ruling out the case of 0 positive roots.

(3) The coefficient of μ in (A2) is negative, ruling out the possibility of all roots being positive.

We are therefore left with the case of 2 positive and 2 negative roots, which may be ordered as follows

$$\mu_1 < \mu_2 < 0 < \mu_3 < \mu_4 \tag{A3}$$

The dynamics is therefore a saddle point, with the stock variables k and z evolving gradually over time, and the shadow prices λ, q being allowed to undergo instantaneous jumps in response to new information.

APPENDIX

The quantities $\phi(\mu_1)$, $\phi(\mu_2)$, as defined in (A2) are both critical parts of the solution. By direct evaluation of the characteristic equation, one can establish that

$$\phi(\mu_1) > 0 > \phi(\mu_2) \tag{A4}$$

where μ_1, μ_2, are ordered as above.

We shall focus our analysis on stable adjustment paths beginning from given initial capital stock k_o and stock of national debt z_o. The solutions for k, z, λ, and q along such paths are

$$k = \tilde{k} + \frac{[\phi(\mu_2)d\tilde{k} - d\tilde{z}]}{\phi(\mu_1)-\phi(\mu_2)} e^{\mu_1 t} + \frac{[d\tilde{z} - \phi(\mu_1)d\tilde{k}]}{\phi(\mu_1)-\phi(\mu_1)} e^{\mu_2 t} \tag{A5}$$

$$z = \tilde{z} + \frac{\phi(\mu_1)[\phi(\mu_2)d\tilde{k} - d\tilde{z}]}{\phi(\mu_1)-\phi(\mu_2)} e^{\mu_1 t} + \frac{\phi(\mu_2)[d\tilde{z} - \phi(\mu_1)d\tilde{k}]}{\phi(\mu_1)-\phi(\mu_1)} e^{\mu_2 t} \tag{A6}$$

$$\lambda = \tilde{\lambda} + \frac{a_{14}\phi(\mu_1)}{\mu_1}\left[\frac{\phi(\mu_2)d\tilde{k} - d\tilde{z}}{\phi(\mu_1)-\phi(\mu_2)}\right] e^{\mu_1 t} + \frac{a_{14}\phi(\mu_2)}{\mu_2}\left[\frac{d\tilde{z} - \phi(\mu_1)d\tilde{k}}{\phi(\mu_1)-\phi(\mu_2)}\right] e^{\mu_2 t} \tag{A7}$$

$$q = \tilde{q} + \frac{\mu_1}{I'}\left[\frac{\phi(\mu_2)d\tilde{k} - d\tilde{z}}{\phi(\mu_1)-\phi(\mu_2)}\right] e^{\mu_1 t} + \frac{\mu_2}{I'}\left[\frac{d\tilde{z} - \phi(\mu_1)d\tilde{k}}{\phi(\mu_1)-\phi(\mu_2)}\right] e^{\mu_2 t} \tag{A8}$$

where $d\tilde{k} = \tilde{k} - k_o$, $d\tilde{z} = \tilde{z} - z_o$ denote the long-run changes in k and z from their respective initial starting points.

One can eliminate $e^{\mu_1 t}$, $e^{\mu_2 t}$, to define the two locuses

$$\lambda - \tilde{\lambda} = \frac{-a_{14}}{\phi(\mu_1)-\phi(\mu_2)}\left[\frac{\phi(\mu_2)}{\lambda_2} - \frac{\phi(\mu_1)}{\mu_1}\right](z - \tilde{z}) - \frac{a_{14}\phi(\mu_1)\phi(\mu_2)}{\phi(\mu_1)-\phi(\mu_2)}\left[\frac{1}{\mu_1} - \frac{1}{\mu_2}\right](k - \tilde{k}) \tag{A9}$$

$$q - \tilde{q} = \frac{\mu_1 - \mu_2}{I'[\phi(\mu_1)-\phi(\mu_2)]}(z - \tilde{z}) + \frac{\mu_2\phi_1(\mu_1) - \mu_1\phi_2(\mu_2)}{I'[\phi(\mu_1)-\phi(\mu_2)]}(k - \tilde{k}) \tag{A10}$$

These are both three dimensional planes relating the respective instantaneous shadow prices to the slowly evolving dynamic variables k and z. They are the analogues to the usual two dimensional stable adjustment paths associated with saddle points.

The solutions reported in equations (A5) - (A8) form the basis for the analysis of the short-run dynamics in response to the various disturbances presented in Section IV. The different shocks identified differ simply in terms of how they impact on the long-run equilibrium stock of capital and national debt.

2. Dynamic properties with debt-financed deficit

When the government's budget is permitted to be out of balance, via the use of debt financing, the dynamics of the economy are governed by a fifth-order system which may be stated in linearized form as follows:

$$\begin{pmatrix} \dot{\lambda} \\ \dot{q} \\ \dot{k} \\ \dot{z} \\ \dot{a} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -\lambda(2i'-i''b) & \lambda i' \\ 0 & i & f_{kk} & i' & 0 \\ x' & I' & 0 & 0 & 0 \\ x' & I' & f_k & (i+i'z) & 0 \\ 0 & 0 & 0 & \tilde{a}i' & i \end{pmatrix} \begin{pmatrix} \lambda-\tilde{\lambda} \\ q-\tilde{q} \\ k-\tilde{k} \\ z-\tilde{z} \\ a-\tilde{a} \end{pmatrix} \tag{A11}$$

This system may be shown to have two stable roots (μ_1, $\mu_2 < 0$) and three unstable roots (μ_3, μ_4, $\mu_5 > 0$). Analogous to (A5) - (A8), the stable solution, starting from initial stocks k_o, z_o is

$$k = \tilde{k} + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t} \tag{A13}$$

$$z = \tilde{z} + \phi(\mu_1) A_1 e^{\mu_1 t} + \phi(\mu_2) A_2 e^{\mu_2 t} \tag{A14}$$

$$\lambda = \tilde{\lambda} + \frac{i'}{\mu_1}\left[\frac{a_{14}}{i'} + \frac{\lambda i'\tilde{a}}{\mu_1 - i}\right] \phi(\mu_1) A_1 e^{\mu_1 t} + \frac{i'}{\mu_2}\left[\frac{a_{14}}{i'} + \frac{\lambda i' a}{\mu_2 - i}\right] \phi(\mu_2)\, A_2 e^{\mu_2 t} \tag{A15}$$

$$q = \tilde{q} + \frac{\mu_1}{I'} A_1 e^{\mu_1 t} + \frac{\mu_2}{I'} A_2 e^{\mu_2 t} \tag{A16}$$

$$a = \tilde{a} + \frac{\tilde{a}i'}{\mu_1 - i} \phi(\mu_1)\, A_1 e^{\mu_1 t} + \frac{\tilde{a}i'}{\mu_2 - i} \phi(\mu_2)\, A_1 e^{\mu_2 t} \tag{A17}$$

where $\phi(\mu_i)$ and a_{14} are defined as previously and for notational convenience A_1 and A_2 are given by

$$A_1 = \frac{\phi(\mu_2)d\tilde{k}-d\tilde{z}}{\phi(\mu_1)-\phi(\mu_2)} \; ; \quad A_2 = \frac{d\tilde{z}-\phi(\mu_1)d\tilde{k}}{\phi(\mu_1)-\phi(\mu_2)} \tag{A18}$$

The assumption that the stock of government debt evolves continuously from the initial level a_o, can be shown to imply the relationships

$$\psi_1(z(t)-z_o)+\psi_2(k(t)-k_o)+\psi_3(a(t)-a_o)=0 \tag{A19}$$

$$\psi_1(\tilde{z}-z_o)+\psi_2(\tilde{k}-k_o)+\psi_3(\tilde{a}-a_o)=0 \tag{A19'}$$

where

$$\psi_1 \equiv \tilde{a}i'[\mu_1\mu_2 - I'f_{kk} - i(\mu_1 + \mu_2 - i)] > 0 \tag{A20}$$

$$\psi_2 \equiv \tilde{a}i'^2 I'\phi(\mu_1)\phi(\mu_2) < 0 \tag{A21}$$

$$\psi_3 \equiv -(\mu_1 - i)(\mu_2 - i)(\mu_1 + \mu_2 - i) > 0 \tag{A22}$$

The significance of (A19) is that the dynamics of government debt a becomes tied to that of k and z. There are in fact only two linearly independent unstable roots. As a further consequence, (A19') imposes an additional constraint on the steady state equilibrium, one which involves the initial points k_o, z_o and a_o.

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