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External Adjustment and the Optimal Demand for International Reserves

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Abstract

This paper provides a theoretical underpinning for the major determinants of optimal reserve demand in the case where fundamental disequilibrium constitutes a key element governing reserve management. Emphasis is given to the role of reserves to smooth the process of economic adjustment by financing part of external disequilibrium, as well as to meet temporary random fluctuations in the excess demand for foreign exchange. The analysis incorporates this financing aspect of reserve holdings into a simple inventory model and discusses the optimal stock of reserves in the context of the optimal mix of adjustment and financing.

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Contents

Summary

Despite the shift to managed floating by the major industrial countries since the early 1970s, official reserve holdings have continued to increase in most countries and the maintenance of adequate reserves has remained an important macroeconomic issue, especially for developing countries. In recent years, this issue has attracted renewed attention as a result of a sharp increase in the number of countries suffering from persistent balance of payments disequilibrium and a consequent increase in economic programs oriented toward structural adjustment. The literature on the demand for reserves has focused, however, primarily on the case where the balance of payments is in fundamental equilibrium, and little theoretical underpinning has been provided for reserve demand in the case where a possible emergence of fundamental disequilibrium constitutes a key consideration in reserve management. The objective of this paper is to present a model to analyze the major determinants of the demand for reserves under such circumstances.

The model incorporates two theoretical implications that fundamental diseqilibrium and the associated structural adjustment to cope with this disequilibrium have for the role of reserves. First, reserves are held not only to meet temporary random fluctuations in the balance of payments--the principal role of official reserves assumed in the existing literature --but also to finance part of external disequilibrium so as to facilitate the adjustment process and alleviate the economic cost associated with external adjustment. Second, external adjustment designed to deal with fundamental disequilibrium is a dynamic process in the sense that it involves changes in key macroeconomic variables over time, and therefore the specification of the adjustment cost requires appropriate consideration of intertemporal changes in these aggregates. These implications are incorporated into a simple inventory model in which the optimal mix of adjustment and financing constitutes a key theoretical ingredient for deriving the optimal demand for reserves. The analysis shows that the demand for reserve holdings depends critically on the pace of external adjustment, which, in turn, is influenced by the cost of external adjustment measured in terms of an intertemporal decline in domestic expenditure. Thus, the paper shows that the optimal stock of demand depends not only on parameters, such as the expected scale of fundamental disequilibrium and the variability of the balance of payments, but also on dynamic factors affecting the cost of external adjustment.

I. Introduction

One of the issues that emerged during the course of discussions of reform of the international monetary system in the 1960s was related to the adequacy of official reserves to support the effective functioning of a fixed exchange rate regime. The theoretical discussion of this issue was led by Heller (1966) who pioneered the analysis of reserve demand in a stochastic framework and was further developed and elaborated in a number of papers (for example, Clark (1970), Kelly (1970), Hamada and Ueda (1977), and Frenkel and Jovanovic (1981)). Although the shift to a managed floating rate regime on the part of industrial countries since the early 1970s has tended to diminish interest in this topic, the determinants of reserve holdings remain an important issue and further analytical refinements are desirable for two reasons. First, a large number of countries, particularly developing countries, have continued to maintain fixed exchange rate regimes despite the adoption of floating rates by developed countries. Second, reserve holdings, on both global and individual country basis, have continued to increase during the period of the managed floating rate system. 1/ One area for further analytical refinements is concerned with relaxing the assumption of underlying payments equilibrium that has characterized most discussions of the demand for official reserves and extending the analysis so as to incorporate fundamental disequilibrium. The practical importance of analytical refinement in this direction has been heightened in recent years as a result of a sharp rise in the number of countries suffering from fundamental disequilibrium and a consequent increase in economic programs oriented toward structural adjustment. These developments give rise to a policy issue as to what is the appropriate level of reserve holdings to support the effective implementation of such external adjustment.

The major objective of this paper is to present a simple model in which fundamental disequilibrium constitutes a key element governing reserve management and to provide a theoretical underpinning for the major determinants of the demand for official reserves under such circumstances. The paper is organized as follows: Section II provides a brief review of the literature on the optimal holdings of official reserves as a background for the subsequent sections; Section III presents a model to consider the demand for official reserves in a small open economy and analyzes the major determinants of the optimal level of reserves; the last section presents concluding remarks, including some policy implications of the model.

1/ Available empirical studies suggest that the adoption of the floating rate regime has not reduced the global demand for official reserves appreciably, although it led to some shift in the demand in the early 1970s. For this point, see, for example, Williamson (1976) and Lizondo and Mathieson (1987).

II. Theory of Optimal Reserve Holdings: A Review

Official holdings of reserve assets 1/ provide an important means by which a national monetary authority can achieve the desired degree of exchange rate stability in the face of a variety of payments disturbances. The basic role of official reserves is to provide the monetary authority with the foreign assets needed to fill the gap between foreign receipts and payments at a desired exchange rate. In other words, official reserves function essentially as a buffer to accommodate the excess demand for foreign exchange at the exchange rate desired by the authorities. Given this role of official reserves, the question of the optimal stock of reserves has been addressed in terms of two different costs: the potential cost of external adjustment and the opportunity cost of reserve holdings. The cost of external adjustment is associated with insufficient holdings of official reserves and a consequent risk of reserve depletion which would force monetary authorities to undertake adjustments of the domestic economy in order to maintain the desired exchange rate. Such adjustment is costly in that it requires a compression of domestic expenditure relative to output in order to generate the change in the country's external position that would be needed in the absence of financing the payments gap by means of reserves. Alternatively, excessive holdings of official reserves imply a less productive use of a country's capital resources which are embodied in official reserves to the extent that the opportunity cost of reserve holdings exceeds the return on reserve assets.

Against the background of these two opposing cost considerations, the question of the optimal stock of reserves has been viewed as a problem of minimizing the economic cost associated with reserve holdings. Consequently, the analysis has been focused on how to specify these two costs, especially the cost of external adjustment which is an important determinant of the optimal stock of reserves. While there have been various specifications of the cost of adjustment in the models of reserve holding behavior, 2/ there are a number of features that are common to these models. First, in the absence of policy adjustment, the change in official reserves is viewed as a random variable which is characterized by a zero mean and a constant variance; in other words, the balance of payments is assumed to be in

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1/ In this paper, reserves are defined as foreign assets held by a country's monetary authorities and do not include those held by financial institutions. The latter, however, can be an important supplement to official reserves to the extent that they can finance other items in the balance of payments in response to certain policy actions, e.g., an increase in domestic interest rates.

2/ For useful reviews of the theoretical framework of demand for official reserves, see Clower and Lipsey (1968) and Williamson (1973).

fundamental equilibrium although subject to random fluctuations. Second, the probability that reserves will fall below a target level constitutes a critical factor in determining the potential cost of external adjustment. A key theoretical issue therefore is to determine this probability, which is related to reserve stocks through the application of various probability theories. 1/ Third, it is generally assumed that external adjustment policies are composed of expenditure-reducing policies, 2/ and consequently the marginal cost associated with external adjustment is measured in terms of a potential decline in domestic income (or expenditure) or the variability of income; the former is generally approximated by the inverse of the marginal propensity to import.

While the existing literature provides some insights into the theoretical underpinnings for the demand for reserves, there are a number of issues that need to be addressed for further study. One such issue relates to relaxing the assumption of fundamental equilibrium and examining how the optimal conditions for reserve holdings will be altered if allowance is made for fundamental disequilibrium; the latter is defined broadly as a situation where a country cannot expect to keep its overall external position in balance over the medium term without modifying macroeconomic policies so as to reduce expenditure relative to income. 3/ The explicit introduction of fundamental disequilibrium and the possibility of structural adjustment designed to correct fundamental disequilibrium have two important theoretical implications.

First, the implementation of structural adjustment highlights the financing aspect of reserve holdings which can facilitate the adoption of such adjustment measures and help alleviate the cost of external adjustment. More specifically, a principal function of official reserve holdings is to smooth the adjustment process by financing part of the external disequilibrium as adjustment measures are put into place and take effect, as well as to cushion temporary random disturbances, which do not require the implementation of adjustment

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1/ For example, the contributions by Heller (1966) and Hamada and Ueda (1977) are based on a random walk theory, while Clark (1970) applies the Chebychev inequality.

2/ Expenditure-switching policies are generally excluded from discussion because of the difficulty in quantifying the adjustment costs stemming from the use of such policies. The use of direct control measures on international trade is also excluded because of their adverse effect on resource allocation.

3/ The causes of fundamental disequilibrium can be numerous, reflecting both domestic and external disturbances, e.g., exogenous shifts in relative prices, foreign demand, and domestic production and demand.

measures. 1/ This aspect of official reserve holdings has not been fully incorporated into those models that assume fundamental equilibrium. The role of official reserves as a means of financing external imbalance while adjustment takes place arises from the need to "buy time" to design and implement adjustment policies, to take account of the time lags before such policies generate the intended effects, and, perhaps most important, to avoid an abrupt and substantial external adjustment in a short period of time. These considerations suggest that the question of how to mix adjustment and financing constitutes a key issue that needs to be addressed explicitly in the analysis of the demand for reserves. 2/

Second, external adjustment designed to deal with fundamental disequilibrium is, by its nature, a dynamic process in the sense that it involves changes in key macroeconomic variables over time. This implies that the economic cost associated with external adjustment would be better measured in terms of a cumulative decline in economic welfare during an adjustment period rather than in terms of a static approximation, such as the marginal propensity to import. 3/ In particular, given the importance of income and expenditure as determinants of the current account, the specification of the adjustment cost would require appropriate considerations to inter-temporal changes in these aggregates. 4/

1/ Part of external disequilibrium could also be financed through recourse to external borrowing, and the extent to which monetary authorities of a country hold reserves to meet fundamental disequilibrium will depend on the availability and terms of external borrowing. The importance of external borrowing to alleviate the cost of external adjustment is analyzed by Martin and Selowsky (1988).

2/ This point was also stressed by Clark (1970) and E. Claassen (1975), in which the speed of adjustment is treated as an endogenous variable that is determined along with the optimal reserve stock. However, their optimal conditions for the speed of adjustment differ from those derived in this paper because of the different specification of the economic cost of external adjustment. This point is explained in more detail below.

3/ It might be noted that the appropriateness of the marginal propensity to import as a proxy for the marginal cost of adjustment has not necessarily been supported by empirical studies, as the coefficient of this variable generally gives the wrong sign in regression equations.

4/ This point, namely, that incorporation of the possibility of fundamental disequilibrium into the analysis would require an inter-temporal model, is also stressed by Williamson (1973) in his comments on Niehans (1970) who explored implicitly an intertemporal optimizing approach in connection with the discussion of the utility of reserves. An interesting attempt is also made by Martin and Selowsky (1988) to specify the cost of adjustment in a dynamic framework.

Against the background of these two implications, a simple model for reserve demand which incorporates fundamental disequilibrium as a key variable is presented below.

III. The Model

1. Major assumptions

The model is based on a number of simplifying assumptions to facilitate the presentation, although some of these assumptions can be easily relaxed without affecting the substance of the analysis. It is assumed that the country is a small open economy that operates under a fixed exchange rate regime and produces traded goods (including services) that are not perfectly substitutable for foreign goods. The prices of both domestic and foreign goods are set exogenously at a constant level under the assumption of a small country. The economy is in a stationary state and output is constant in the absence of exogenous changes in parameters affecting production. The country's balance of payments is assumed to adjust in part automatically to exogenous disturbances that result in fundamental disequilibrium, as the private sector modifies production and expenditure decisions in response to changes in exogenous conditions, but full adjustment in a short time frame is constrained by a number of factors, some of which have been mentioned above. Part of balance of payments disequilibrium could be financed through external borrowing, but there is the budget constraint in the sense that the country cannot rely on borrowing indefinitely and therefore has to initiate external adjustment eventually to cope with fundamental disequilibrium.

The excess demand for foreign exchange (or the overall balance of payments deficit) during a given time period (defined as a basic time period, e.g., a quarter or a year) is assumed to be a random variable; 1/ the monetary authority is uncertain about the size of the demand that will actually occur, but is assumed to know the probability distribution of this demand, for example, through past experience. 2/ The probability distribution of the excess demand for foreign exchange

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1/ In what follows, the excess demand for foreign exchange refers to the excess demand for foreign exchange by the private sector (or the overall balance of payments deficit) unless otherwise indicated.

2/ Strictly speaking, the demand for official reserves must be analyzed in relation to expected future disturbances, not in relation to those of the past. But the size and duration of future disturbances could perhaps be gauged, as a rough approximation, by examining past disturbances that are manifested in changes in official reserves. On this point, see, for example, Triffin (1961).

in a given time period is characterized by a normal distribution, 1/ with a constant mean (μ) and a constant variance (σ^2), and changes in the excess demand are assumed to be serially uncorrelated. 2/ However, to incorporate the case of fundamental disequilibrium, the mean is not necessarily assumed to be zero; if it is positive, there is a strong possibility over time that official reserves would be eventually depleted in the absence of external adjustment to correct the disequilibrium. Assuming that the capital account is in balance in net terms 3/ and that domestic absorption and output, both of which are treated as random variables, are stochastically uncorrelated, the expected value of the excess demand for foreign exchange in a given time period in the absence of external adjustment can be expressed as a function of the expected values of income and expenditure: 4/ 5/

$$(1) \qquad \mu = A - Y, \quad \text{or} \quad A = Y + \mu$$

where, Y: the expected value of domestic income in a given time period in the absence of adjustment;
A: the expected value of domestic absorption in a given time period in the absence of adjustment.

1/ Olivera (1969) also assumes a normal distribution as an approximation of the density function of the excess demand for foreign exchange in his discussion of the optimal demand for official reserves.

2/ The empirical evidence concerning the stochastic nature of changes in official reserves has been mixed and further investigation is needed. For example, Kenen and Yudin (1965) observed that monthly changes in official reserves in selected 14 countries during 1958-62 displayed a significant serial correlation and stressed that the balance of payments in the short run should also be described as a Markov process rather than a simple random walk variable. The existence of a serial correlation was also stressed by Archibald and Richmond (1971), although they argued that the serial correlation was rather characterized by heteroskedasticity. On the other hand, Streeter (1970) found that, using quarterly and annual data for the period of 1958-67, there was virtually no support for the autoregressive hypothesis and strong evidence for the normality hypothesis. On this latter point, see Williamson (1973).

3/ It is assumed that there are no speculative capital outflows in response to the emergence of fundamental disequilibrium and consequent possible declines in official reserves.

4/ It should be noted that the nominal values of these aggregates are equal to the real values, because domestic prices are assumed to be exogenously set at a constant level under the assumption of a small country.

5/ In what follows, macroeconomic variables, such as income (or output) and expenditure, refer to their expected values unless otherwise indicated.

Policy actions to achieve external adjustment are assumed to be aimed at reducing the mean of the excess demand for foreign exchange to zero through an unspecified set of measures 1/ which are designed to curtail the expected value of domestic absorption while raising the expected value of output.

It is assumed that a key task of reserve management is to determine the desired amount of reserves to be held by the monetary authority at the beginning of a given time period. The adequacy of reserve holdings is reviewed and assessed each period by the monetary authority, taking into account the need for external adjustment in response to likely exogenous disturbances. The length of time required for external adjustment, namely, the adjustment period, is a policy variable that is determined by the monetary authority as a part of the adjustment strategy to correct the external imbalance. As a general case, it is assumed that the monetary authority plans to complete the external adjustment in T periods. 2/ In other words, external disequilibrium will be eliminated, on average, by (μ/T) per unit time period, and external adjustment will be undertaken at an average speed of $\gamma = (1/T)$. 3/ Under this adjustment strategy, the expected value of the excess demand for foreign exchange (x) in period t after the initiation of external adjustment is specified as:

$$(2) \qquad \begin{aligned} E[x_t] &= (1 - \gamma t)\mu \qquad \text{for } t = 0, 1, \ldots, T-1 \\ &= 0 \qquad\qquad\quad \text{for } t = T, T+1, T+2, \ldots \end{aligned}$$

where, x_t: the excess demand for foreign exchange, a random variable, in period t;
T : the length of the adjustment period;
$\gamma = 1/T$ $(0 < \gamma < 1)$: the speed of external adjustment.

Noting the definition of the balance of payments given by (1), this external adjustment can be expressed alternatively in terms of the adjustment of domestic expenditure relative to output:

1/ In general, these adjustment measures will include exchange rate policy, although the extent of the recourse to this policy will depend on the nature and causes of such disequilibrium. It should be noted that the adequacy of official reserves will be substantially influenced by the degree of exchange rate flexibility pursued by the monetary authority, but this issue is not discussed in this paper.

2/ Namely, the length of the adjustment period is equal to T periods.

3/ These assumptions imply that the monetary authority envisions a certain adjustment strategy over a definite time period which includes its assessment of optimal policies and their likely economic impact on the balance of payments.

$$(3) \qquad A_t = Y_t + (1 - \gamma t)\mu$$

where, A_t: the expected value of domestic absorption in period t;
Y_t: the expected value of real income (or output) in period t.

As indicated by (3), developments in domestic absorption during the adjustment period depend in part on how domestic output will respond to adjustment policies. External adjustment aimed at eliminating fundamental disequilibrium typically includes supply side measures designed to improve resource allocation and increase productivity and efficiency, and it will be appropriate to incorporate the effect of these measures. Denoting the supply effect of external adjustment policy in given period by θ, then the expected value of output during the adjustment period is specified as:

$$(4) \qquad \begin{aligned} Y_t &= Y + \theta\gamma\mu t \qquad \text{for } t = 0, 1, \ldots, T\text{-}1 \\ &= Y + \theta\mu \qquad \text{for } t = T, T\text{+}1, T\text{+}2, \ldots \end{aligned}$$

where, Y: the expected value of output in the absence of adjustment, namely,
$Y = A - \mu$
θ: a constant coefficient, where $0 < \theta < 1$, representing the output effect of external adjustment. 1/

The specification of (4) shows that the increase in output during the adjustment period will be greater, the larger is the coefficient $\theta\gamma\mu$, which represents a crude indicator of the overall output effect of external adjustment. In the long run when the balance of payments deficit has been eliminated, the level of output will be larger than the initial level by $\theta\mu$ and is independent of the speed of adjustment. Substituting (4) into (3), the expenditure function can be rewritten as:

$$(5) \qquad \begin{aligned} A_t &= Y + \mu - (1 - \theta)\gamma\mu t \qquad \text{for } t = 0, 1, \ldots, T\text{-}1 \\ &= Y + \theta\mu \qquad \text{for } t = T, T\text{+}1, T\text{+}2, \ldots \end{aligned}$$

It is seen from (5) that under the assumed adjustment strategy, expenditure is reduced by $(1 - \theta)\gamma\mu$ in a given period. The scale of

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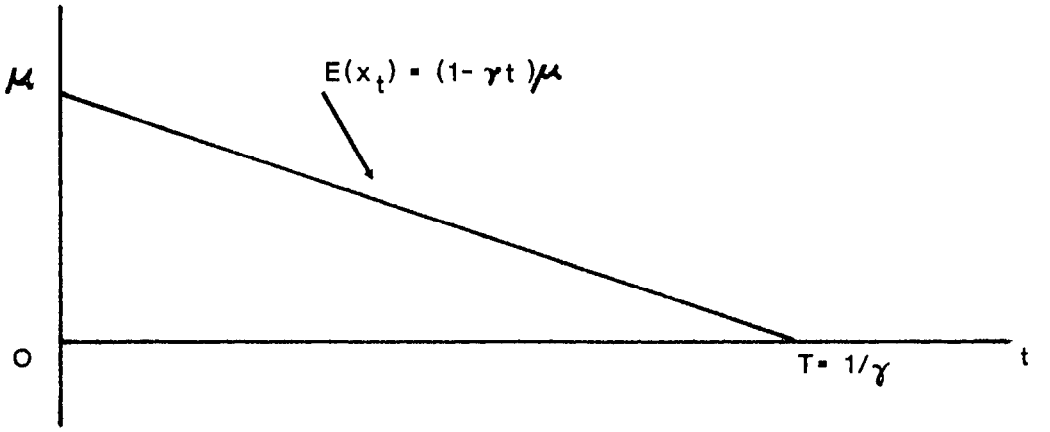
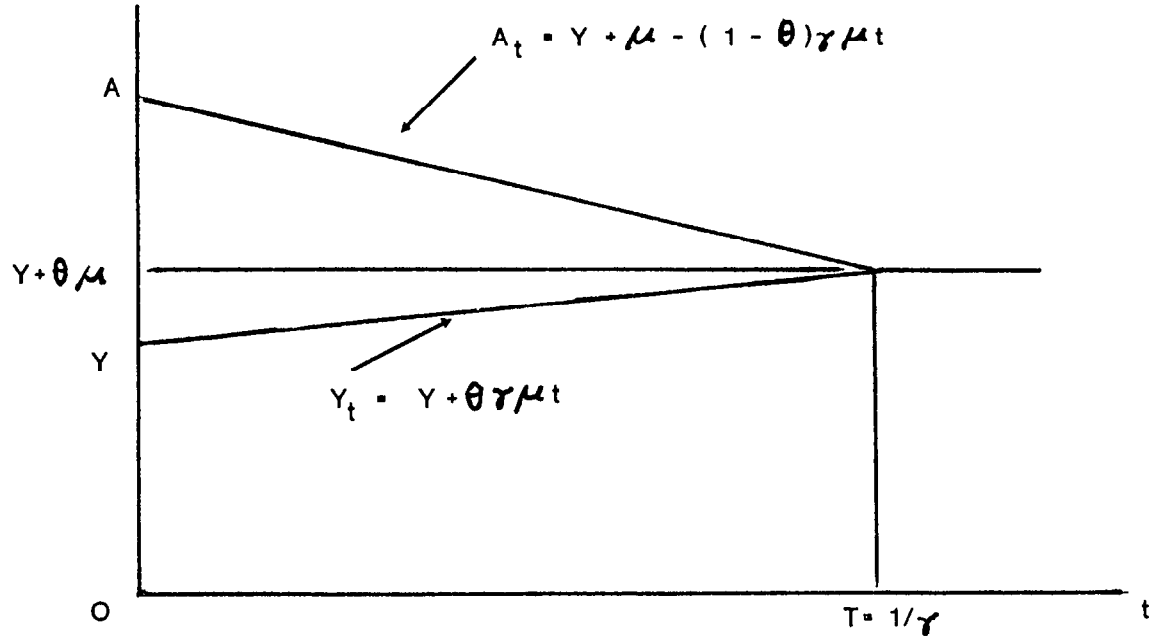
1/ The size of θ will depend partly on the type and extent of external adjustment policies implemented by the monetary authority and the economy's supply elasticity in response to these measures. The assumption that θ is less than unity is made to keep the model dynamically stable.

Chart 1

The Expected Excess Demand for Foreign Exchange and the Speed of Adjustment. 1/



Domestic Absorption and Output



1/ The speed of adjustment (γ) is defined as the inverse of the length of the adjustment period (T).

expenditure reduction will be smaller, the larger is the supply response to structural adjustment measures (θ). Chart 1 illustrates the dynamic movements in the balance of payments, domestic absorption, and output that are given by (2), (4), and (5).

On the basis of these basic assumptions with regard to the excess demand for foreign exchange and the external adjustment strategy, the next subsection presents an analytical framework to determine the demand for reserves and shows that the demand for reserve holdings depends critically on the pace of external adjustment undertaken by the monetary authority. This is followed by a subsection that discusses major factors affecting the speed of external adjustment and derives the conditions for optimal reserve holdings. In the discussion, it is shown that the cost of external adjustment is a key variable influencing the authority's decision concerning the speed of adjustment.

2. Determinants of official reserve holdings

The basic analytical framework used in this paper is an inventory approach in that the minimization of the "inventory cost" associated with reserve holdings is a key condition to derive the demand function for reserves. As already noted, official reserves are held to meet the expected excess demand for foreign exchange (or the overall balance of payments deficit) at the desired exchange rate over an adjustment period. Under the adjustment policy specified by (2), the excess demand for foreign exchange over the entire adjustment period (x) is given by a sum of random variables x_t:

$$(6) \qquad x = x_0 + \ldots + x_{T-1}$$

$$\text{and} \quad E[x] = \lambda = \mu(1 + 1/\gamma)/2$$
$$V[x] = \epsilon^2 = \sigma^2/\gamma \quad \underline{1}/$$

1/ The key assumption for the derivation of E[x] and V[x] is that the excess demand for foreign exchange is serially uncorrelated. Under this assumption, E[x] and V[x] are given by:

$$E[x] = E[x_0] + \ldots + E[x_{T-1}]$$
$$V[x] = V[x_0] + \ldots + V[x_{T-1}]$$

Substituting $E[x_t] = (1 - \gamma t)\mu$ and $V[x_t] = \sigma^2$ into the above equations gives:

$$E[x] = \mu(T - \gamma T(T - 1)/2)$$
$$V[x] = T\sigma^2$$

Noting $T = 1/\gamma$, these give the formulae presented in the text.

The stochastic excess demand for foreign exchange defined by (6) has two implications worth noting. First, the expected value and variance of the excess demand for foreign exchange are related positively to the length of the adjustment period (T) which is decided by the monetary authority to correct fundamental disequilibrium, because the duration of the adjustment period varies inversely with the speed of adjustment (γ). Consequently, the expected value and the variance of the excess demand for foreign exchange would be larger (smaller), the longer (shorter) is the length of the adjustment period. Second, financing requirements associated with the correction of fundamental disequilibrium are reflected in the expected value E(x). This variable represents the expected value of the cumulative balance of payments deficit over the adjustment period that would be financed through drawdown on official reserves to alleviate the costs of external adjustment. As shown in the specification of E(x), such financing requirements would be larger, the slower is the adjustment speed or the longer is the length of the adjustment period.

Given the stochastic excess demand for foreign exchange, the desired level of official reserves to be held at the beginning of a given time period is derived by minimizing the expected cost associated with reserve holdings, which is comprised of two elements. The first element is the opportunity cost of reserve holdings, which depends on the yield differential between reserves and alternative investment assets, as well as the amount of reserves to be held; that is,

$$(7) \qquad C_1 = rR$$

where, C_1: the opportunity cost of reserve holdings;
R: the amount of reserves to be held at the beginning of a given time period;
r: the marginal opportunity cost of reserve holdings which is given by the yield differential between reserves and alternative investment assets.

The second element is concerned with the cost incurred by the monetary authority when the demand for foreign exchange actually exceeds reserve holdings and official reserves are depleted. Since fundamental disequilibrium is dealt with in this model by external adjustment so as to prevent the emergence of a depletion of reserves, such a depletion is related primarily to random fluctuations in factors affecting the balance of payments. Given the temporary and random nature of reserve depletion, an appropriate policy option for the monetary authority to meet the excess demand for foreign exchange in the event of reserve depletion will be to borrow either on inter-

national capital markets or through official sources. 1/ External borrowing also seems to be a viable option in view of the recent rapid development of international capital markets and the ready availability of foreign exchange compared with the generation of foreign reserves through external adjustment, although the scope for market borrowing by a number of highly indebted countries is limited at present. Possible recourse to external borrowing suggests that the potential cost of reserve depletion will vary positively with the amount of external borrowing and the rate of interest charged on such borrowing. Thus, the potential cost is specified as:

$$(8) \qquad C_2 = 0, \quad \text{if } x < R$$
$$= i(x - R), \quad \text{if } x > R$$

where, C_2: the potential cost of reserve depletion;
i: the marginal cost of external borrowing, which is given by the rate of interest charged on external borrowing; it is assumed that i is larger than r; 2/
$(x-R)$: the amount of external borrowing.

Since x is a random variable that is characterized by a normal distribution with the mean of λ and the variance of ϵ^2, 3/ the expected

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1/ This assumption differs from the standard stochastic approach in which reserve depletion is assumed to induce the adoption of external adjustment policies. The importance of the role of external borrowing in meeting possible reserve depletion, however, has recently been recognized in Dooley, Lizondo, and Mathieson (1989). As noted in the text, the scope for market borrowing by highly indebted countries are now limited, and they will have to rely more on official sources, including Fund credit. It should also be noted that external borrowing in this context refers to "compensatory" borrowing by monetary authorities, which will be registered below the line and will not affect the capital account. Needless to say, such external borrowing would be undertaken when signs of possible reserve depletion emerge rather than after reserves have actually been depleted.

2/ More realistically, the marginal cost of external borrowing (i) could be assumed to be a positive function of the amount of external borrowing (x-R).

3/ This feature reflects that the random variable x is a compound random variable which is composed of T independent random variables x_t. For this point, see the definition of x given in (6).

value of the potential cost of reserve depletion, which varies negatively with the amount of reserves, is given by: 1/

$$(9) \quad E[C_2] = \int_R^{\infty} i(x - R)f(x)dx$$

$$= i\lambda - iR(1 - F(R)) - i\int_{-\infty}^{R} xf(x)dx$$

where,

$$F(x) = \int_{-\infty}^{x} f(x)dx,$$

$$\text{and } f(x;\ \lambda,\ \epsilon^2) = \frac{1}{\epsilon\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\frac{x - \lambda}{\epsilon}\right]^2}$$

$F(x)$: the cumulative probability distribution of x;
$f(x)$: the probability density function of x.

Given (7) and (9), the expected value of the total cost associated with reserve holdings is specified as:

$$(10) \quad E[C] = E[C_1 + C_2]$$

$$= rR + i\lambda - iR(1 - F(R)) - i\int_{-\infty}^{R} xf(x)dx$$

where, C: total cost associated with a given level of reserve holdings.

Equation (10) shows that the larger is the amount of reserves to be held, the smaller will be the expected value of the potential cost of reserve depletion but the greater will be the opportunity cost of reserve holdings. This suggests that there is a level of official reserves which minimizes the total cost associated with reserve holdings. This level of official reserves can be derived by partially

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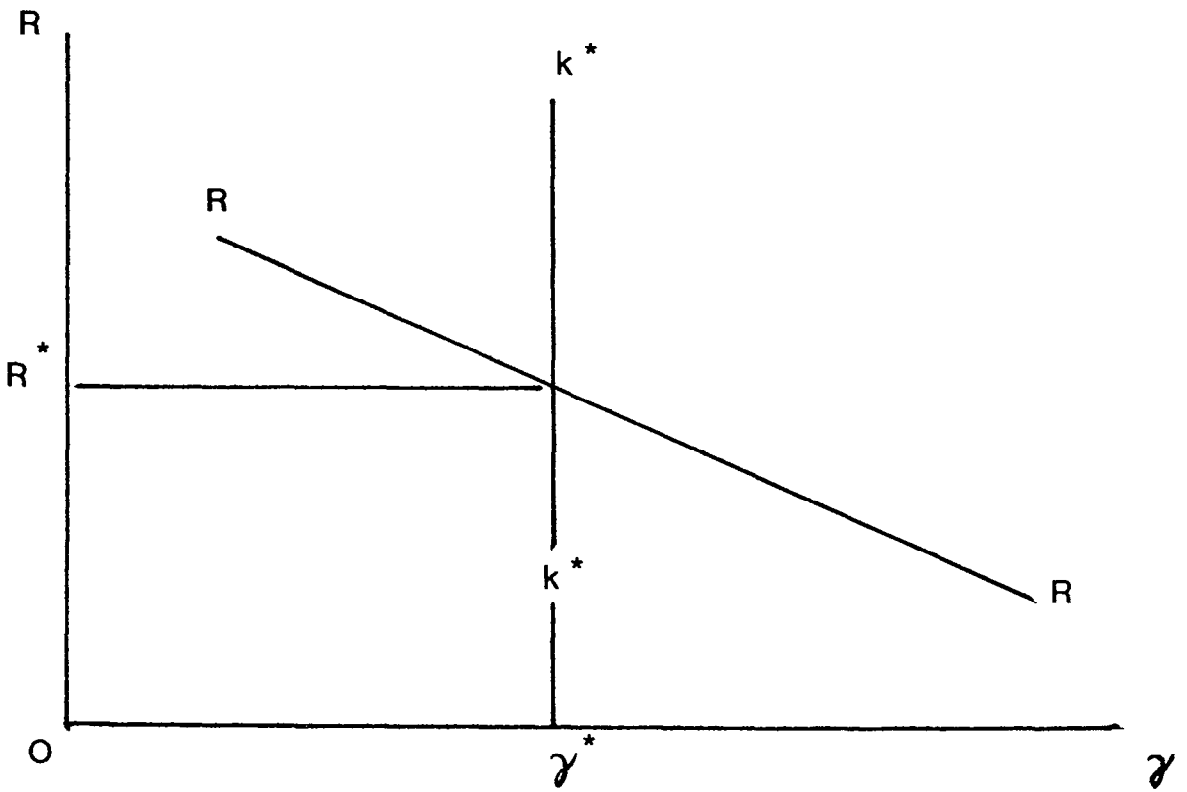
1/ This specification of the potential cost of reserve depletion is based on the specification of "depletion penalty" developed by Arrow, Harris, and Marschak (1951) in the discussion of optimal inventory policy.

Chart 2

Optional Reserve Holdings and Speed of Adjustment



differentiating (10) with respect to R. 1/ Since the cumulative probability function of x depends partly on the speed of adjustment, the desired level of official reserves (R) that minimizes E[C] can be expressed as a function of γ and other parameters:

$$(11) \qquad R = R(\gamma, \mu, \sigma^2, i, r),$$

$$\frac{\partial R}{\partial \gamma} < 0, \quad \frac{\partial R}{\partial \mu} > 0, \quad \frac{\partial R}{\partial \sigma^2} > 0, \quad \frac{\partial R}{\partial i} > 0, \quad \frac{\partial R}{\partial r} < 0. \text{ 2/}$$

The relationship between R and γ given in (11) is illustrated in Chart 2 (the RR curve), which depicts the combination of these two variables which satisfies the minimum cost condition. Chart 2 shows that desired reserve holdings are negatively related to the speed of adjustment, given the expected scale of fundamental disequilibrium, the variability of the excess demand for foreign exchange, the cost of external borrowing, and the marginal opportunity cost of reserve holdings. It can be easily shown that an increase in the expected scale of fundamental disequilibrium, the variability of the excess demand for foreign exchange, and external borrowing cost will shift the RR curve upward at a given speed of adjustment and will increase the demand for official reserves; thus, the absence of fundamental disequilibrium will shift the RR curve downward and reduce the need for reserves compared with the case where the balance of payments experiences fundamental disequilibrium. On the other hand, a higher opportunity cost of reserve holdings will shift the RR curve downward at a given speed of adjustment and will reduce the demand for official reserves.

3. The optimal speed of adjustment and official reserves

The discussion in the preceding section indicates that there is an infinite number of possible combinations of R and γ which meet the minimum cost condition, and the determination of optimal reserve

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1/ The partial differentiation of (12) gives the following first order condition to minimize the total cost associated with reserve holdings.

$$\frac{\partial E(C)}{\partial R} = r - i(1 - F(R)) = 0$$

Noting that F(R) depends on γ, μ, and σ^2, the first order condition is rewritten as (11) in the text. The second order condition is satisfied because $F'(R) = f(R)$ is always positive. A detailed mathematical appendix is available on request.

2/ The positive sign of the partial derivative of R with respect to σ^2 assumes $(R-\lambda) > \epsilon$.

holdings requires an additional condition to determine the most desired speed of adjustment. While the monetary authority's decision on the choice of the pace of external adjustment is likely to be influenced by a variety of economic factors, it is assumed that the following two factors are particularly important: (i) the extent of the compression of domestic expenditure that is required for external adjustment and (ii) the expected change in the variability of the excess demand for foreign exchange or the balance of payments deficit (namely, V(x) in (6)). The importance of the former factor is obvious, as it represents a typical proxy for the cost of external adjustment. On the other hand, the importance of the latter factor reflects the underlying relationship that an increase in the variability of the excess demand for foreign exchange, which varies negatively with the speed of adjustment as shown in (6), will require larger official reserves to pursue prudent reserve management policy and therefore will increase the cost of reserve holdings indirectly.

Since the relationship between the variability of the excess demand for foreign exchange and the speed of adjustment is already given by (6), the remaining task is to specify how the cost of external adjustment in terms of the decline in domestic expenditure is related to the speed of adjustment. In view of the dynamic nature of external adjustment to correct fundamental disequilibrium, the extent of the compression of domestic expenditure will be best measured in terms of the present value of its intertemporal decline:

$$(12) \qquad s = \sum_{t=0}^{\infty} (A - A_t)/(1+\rho)^t$$

where, s: the present value of an intertemporal decline in expenditure;
ρ: a social discount rate.

Substituting (5) into (12), the present value of the cumulative decline in domestic expenditure is given by

$$(13) \qquad s = \sum_{t=0}^{\infty} (A - A_t)/(1+\rho)^t$$

$$= ((1+\rho)/\rho^2)(1-(1+\rho)^{-T})(1-\theta)\mu/T$$

$$= s(\gamma, \mu, \theta, \rho) \qquad \text{(since } T = 1/\gamma\text{)},$$

$$\frac{\partial s}{\partial \gamma} > 0, \quad \frac{\partial s}{\partial \mu} > 0, \quad \frac{\partial s}{\partial \theta} < 0, \quad \frac{\partial s}{\partial \rho} < 0. \quad 1/$$

It is seen from (13) that the cost of external adjustment, which is measured in terms of the present value of the cumulative decline in expenditure, varies positively with the speed of adjustment and the scale of external disequilibrium. On the other hand, it is related negatively to the supply response to structural adjustment policies and the size of the social discount rate; this latter relationship implies that the larger the social discount rate, the smaller will be the cost of external adjustment because the disutility associated with a future decline in expenditure diminishes.

Given (13), the monetary authority's key objective is to choose the speed of adjustment that will minimize the present value of the intertemporal decline in domestic expenditure while limiting the excessive volatility of the balance of payments which arises from external adjustment. In this connection, it is important to note that there is a trade-off relationship between these two factors in that a relatively fast (slow) external adjustment will reduce (increase) the variability of the demand for foreign exchange during the adjustment period, but will aggravate (moderate) the intertemporal decline in domestic expenditure. This trade-off relationship implies that there exists a speed of external adjustment that minimizes the total economic cost arising from these two factors and is therefore optimal for the monetary authority. Assuming that certain social disutility is associated with each of these two factors, this optimal speed of adjustment can be derived by minimizing the following social disutility function:

$$(14) \qquad W = j_1 s(\gamma, \mu, \theta, \rho) + j_2(\sigma^2/\gamma)$$

where, j_1: the marginal social disutility associated with the intertemporal decline in expenditure;
j_2: the marginal social disutility associated with the variability of the balance of payments.

1/ The negative sign of the partial derivative of s with respect to ρ assumes $T < 1/\rho$.

Partially differentiating (14) with respect to γ, the optimal speed of adjustment is given by: 1/

$$(15) \qquad \gamma^* = \gamma^*(\mu, \sigma^2, \theta, \rho),$$

$$\frac{\partial\gamma^*}{\partial\mu} < 0, \ \frac{\partial\gamma^*}{\partial\sigma^2} > 0, \ \frac{\partial\gamma^*}{\partial\theta} > 0, \ \frac{\partial\gamma^*}{\partial\rho} > 0.$$

Equation (15) is presented as the k^*k^* curve in Chart 2, and it can be shown that the k^*k^* curve will shift to the left and the pace of external adjustment pursued by the monetary authority will be slower, the larger is the expected magnitude of fundamental disequilibrium. In contrast, the k^*k^* curve will shift to the right and the monetary authority will opt for a faster speed of external adjustment, when the variability of the demand for foreign exchange and the social discount rate are high and the output response to adjustment policies is greater. Once the optimal speed of adjustment is determined, the corresponding optimal level of reserves is obtained by substituting (15) into (11):

$$(16) \qquad R^* = R^*(\mu, \sigma^2, \theta, i, r, \rho),$$

$$\frac{\partial R^*}{\partial\mu} > 0, \ \frac{\partial R^*}{\partial\sigma^2} > 0, \ \frac{\partial R^*}{\partial\theta} < 0, \ \frac{\partial R^*}{\partial i} > 0, \ \frac{\partial R^*}{\partial r} < 0, \ \frac{\partial R^*}{\partial\rho} < 0.$$

The condition (16), which corresponds to R^* in Chart 2, indicates that optimal reserve holdings depend on a number of parameters, including the expected size of fundamental disequilibrium, the variability of the excess demand for foreign exchange, the supply effect of external adjustment, the social discount rate, the cost of external borrowing, and the opportunity cost of reserve holdings. It can be shown from comparative static analysis that an increase in the expected magnitude of fundamental disequilibrium and the cost of external borrowing will lead to an increase in the optimal demand for

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1/ The first order condition for the minimization of (14) is given by:

$$(1+\rho)(1-\theta)\mu((1+\rho)^T-1-T\ln(1+\rho))/(\rho^2T^2(1+\rho)^T) = (j_2/j_1)\sigma^2$$

where $T = 1/\gamma$.

The second order condition and the impact of changes in the major explanatory variables on the speed of adjustment is given by:

$$2(1+\rho)^T - 1 - (T\ln(1+\rho)+1)^2 > 0.$$

official reserves. On the other hand, a rise in the opportunity cost of reserve holdings and the social discount rate and a larger supply response to adjustment measures will have the opposite effect on the optimal demand for official reserves. The effect of an increase in the variability of the excess demand for foreign exchange is not necessarily deterministic because of its divergent impacts on the RR and the k^*k^* curves. However, an increase in the variability will induce larger reserve holdings by the monetary authority to the extent that the marginal social disutility associated with such a variability is relatively small.

It is useful to indicate how these results differ from the existing literature. It should be first noted that the existing literature provides little theoretical underpinning on the determinants of reserve demand when fundamental disequilibrium is explicitly taken into account as an important factor in reserve management, because it is primarily concerned with the demand for official reserves in the case where the balance of payments is in fundamental equilibrium. The only exception in this regard is Hamada and Ueda (1977), which analyzes the effect of fundamental disequilibrium in the context of a random walk theory. However, their approach differs from the model presented in this paper in that it does not include external adjustment as an integral part of the analytical framework and its impact on the demand for reserves is not incorporated into the analysis in a coherent way.

The importance of the pace of external adjustment as a factor affecting the demand for reserves is recognized by the existing literature, especially Clark (1970) and Claassen (1975). However, the theoretical implication of fundamental disequilibrium for official reserve holdings is not explored by these papers, as external adjustment is viewed as a means to correct a temporary deviation in official reserves from the desired level (Clark (1970)) or as a disturbance factor which increases the variability of the balance of payments (Claassen (1975)), rather than as a way to eliminate fundamental disequilibrium.

Another feature of the model presented in this paper is that external adjustment designed to alleviate external imbalances is viewed as a dynamic process and that the cost of adjustment is measured in terms of an intertemporal change in economic welfare. Therefore, the explicit specification of the intertemporal paths of income and expenditure during the adjustment period is treated as an important theoretical ingredient to derive the demand function for reserves. Consequently, the optimal conditions for reserve holdings are viewed as depending not only on static parameters, such as the variability of the balance of payments and the opportunity cost of reserve holdings, but also on dynamic factors, especially the pace of external adjustment and the social time preference rate.

IV. Concluding Remarks

The demand for official international reserves is a complex topic which cannot be described fully in a simple analytical model. Such complexity reflects partly the fact that a country's need for reserves is influenced by its macroeconomic objectives, the pace of economic adjustment in the face of external disturbances, and uncertainty over the scale of such disturbances--variables which are not necessarily easily quantifiable. The complexity is especially pronounced when fundamental disequilibrium becomes a major consideration in reserve management because the discussion of the adequacy of reserve holdings requires an explicit assessment of an adjustment strategy that would be taken by monetary authorities and its impact on key macroeconomic variables, including the balance of payments. In view of these complexities, it should be emphasized that the model presented in this paper is a tentative attempt to incorporate the effect of fundamental disequilibrium on reserve management and to provide a theoretical underpinning for major determinants of the demand for official reserves.

In this paper, the demand function of official reserves was derived through the application of an inventory model in which the question of optimal reserve holdings is viewed essentially as the problem of minimizing the cost of reserve holdings. It was shown that the consideration of fundamental disequilibrium would increase the demand for official reserves compared with the case where the balance of payments is in fundamental equilibrium and that such an increase stems from the greater need for financing to alleviate the cost of adjustment. 1/ This financing need is associated with the high costs of achieving a full and immediate adjustment of domestic expenditure and income in response to structurally related exogenous disturbances. A basic motive of reserve holdings is therefore to smooth the process of economic adjustment by allowing part of external disequilibrium to be financed by drawdown of official reserves. The demand for reserves based on this motive essentially differs from the reserve demand which is associated with the need to meet random fluctuations in the excess demand for foreign exchange so as to make external adjustment unnecessary.

Given this role of reserves, it was stressed that the demand for official reserves to meet fundamental disequilibrium would depend, among others, on the expected scale of such disequilibrium and the pace of external adjustment undertaken to eliminate the disequilibrium. The

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1/ In this connection, it is worth noting that official reserves held by oil-importing countries increased substantially following the rises in oil prices in the 1970s, which created a major fundamental disequilibrium in the balance of payments in these countries.

latter in turn depends importantly on the economic costs of the adjustment, which play a key role in determining the appropriate mix of financing and adjustment. The discussion in this paper suggested that given the dynamic nature of external adjustment, the measurement of adjustment cost would require the consideration of intertemporal changes in domestic expenditure and the associated social disutility. In general, however, it is difficult to know to what extent the policymakers of a country can effectively assess such intertemporal social disutility. It is this difficulty that tends to make the analysis of the reserve demand associated with fundamental disequilibrium more judgmental and the concept of reserve demand somewhat elusive. Nonetheless, it is important to recognize that the reserve demand to meet fundamental disequilibrium cannot be analyzed appropriately without the explicit consideration of dynamic adjustment cost, however broad and general the concept of such cost might be.

The discussion of this paper was primarily focused on the major determinants of the demand for official reserves. However, given the effect of fundamental disequilibrium on the demand for reserves--fundamental disequilibrium will increase the desired level of official reserves--an important issue arises as to how best a country can meet such an increase in reserve requirements. Although the full discussion of this issue is beyond the scope of this paper, the issue has become more important since the last decade, as an increasing number of countries have experienced prolonged payments problems, in part as a result of structurally related external disturbances. For some countries, the need for larger reserves may be met through generating current account surpluses or through increased recourse to borrowing on international capital markets. However, for a large number of countries, especially those with high external indebtedness, the scope for meeting increased reserve requirements through these channels appears to be limited in view of the heavy social cost associated with external adjustment and the already high credit risk perceived by commercial banks. In these circumstances, it seems important and appropriate that increased consideration should be given to other less costly avenues to meet reserve demand, including a more active use of the SDR system.

Finally, it should be noted that the model presented in this paper was constructed on the basis of a number of simplifying assumptions. In particular, no explicit consideration was given to the effect on the demand for reserves of the flexible adjustment of exchange rates, the absence of budget constraint on external borrowing, and the existence of the serial correlation in the excess demand for foreign exchange. The relaxation of some of these assumptions remains an important theoretical issue for further research to extend the analysis of the demand for reserves.

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