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Optimal Fiscal Policy and Government Provision of Contingent Liabilities: The Example of Government Loan and Deposit Guarantees

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Abstract

The optimal provision of loan guarantees or deposit insurance is examined in the context of an overlapping generations model. It is demonstrated that even in the face of a market imperfection that precludes diversification of the private sector's loan portfolio to eliminate risk, full government guarantee of private sector loans (or deposits) is suboptimal. The results of the paper suggest that although some degree of guarantee is appropriate, the design of such policies should be tempered to avoid an inefficient level of capital accumulation.

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Contents

Summary

This paper develops a simple overlapping-generations model of a closed economy to examine the government's potential role in providing loan guarantees. The model assumes that firms facing random production shocks that, in some cases, reduce revenues below the level required to repay creditors, produce the consumption good. While equity portfolios can be perfectly diversified, market imperfections are assumed such that lenders cannot fully diversify loan portfolios. The government's activity is limited to either purchasing private sector output or providing loan guarantees. Financing takes place through lump-sum taxes or the issuance of government bonds.

The analysis begins by examining the ideal allocation between capital and consumption expenditure. It confirms that the ideal allocation distributes consumption over time so that utility, discounted by the rate of time preference, is equalized, while the level of savings and capital maximizes production of the consumption good. Curiously, while this consumption allocation can be achieved by providing full loan guarantees, such a policy would imply an over-capitalized economy and, therefore, less than maximal output. In fact, the optimal policy guarantees only a proportion of the private sector's loan portfolio.

The implication of the analysis is that the limited liabilities of equity owners may limit the government's ability to achieve the best risk-reduction policies through loan guarantees. The second-best policy offers only partial guarantees. Government policies that fully guarantee loans or deposits are suboptimal.

I. Introduction

Increasing attention has been paid to the impact of government non-cash policies that have future tax implications--namely, those policies that entail contingent liabilities. These liabilities may be defined as contracts to make specified payments at some point in the future, contingent on the realization of a particular (uncertain) event. The obvious and often cited examples include (unfunded) social security programs, loan guarantee programs, and health, deposit, and other government insurance programs. In each instance, the provision of the program (and the undertaking of the contingent liability) does not imply a current cash obligation but a future obligation that is contingent on a future event (such as old age or need), in turn implying the possibility of a contingent tax liability of the private sector. 1/ In each of these cases, even though no cash outlay may be undertaken initially, private sector activity may be affected by the provision of the liability. 2/

This point has been forcefully made in the context of the United States by a number of authors, including Boskin (1987), Boskin, Barham, Cone, and Ozler (1987), and Break (1982), who have argued that the increased provision of these contingent liabilities (particularly those associated with social security and loan guarantee programs) has caused the U.S. fiscal deficit, measured on a net wealth basis, to be seriously understated. 3/ For example, over fiscal year 1984, net contingent liabilities of the U.S. Government were estimated to have grown by US$548.4 billion to reach US$4,756.4 billion, versus an increase of US$257.5 billion in other liabilities, reaching US$1,979.0 billion at the end of the fiscal year. 4/

1/ More subtle examples include exchange rate guarantees, indexed interest rates or wages, foreign currency, and floating rate debt, in which the government's obligation is contingent on the behavior of financial markets. However, especially in the case of indexed debt, it could be argued that any nominal government obligation is contingent when defined in real terms.

2/ For example, it is often considered that social security programs have the effect of reducing current savings. In his exhaustive survey of the theoretical and empirical research related to the impact of social security schemes on economic activity, Atkinson (1987) notes that the empirical research to date is ambiguous--in some cases rejecting the pure Ricardian prediction that the private sector reacts to social welfare programs by simply reducing savings, and in other cases rejecting the alternate life-cycle hypothesis that savings would be only partially depressed, causing consumption to increase.

3/ Eisner (1984, 1986) is considerably more sanguine regarding the impact of contingent government liabilities on private sector activity, arguing that if such unfunded obligations are expected to be met through increased taxes, the net impact on private sector wealth will be zero.

4/ Eisner (1986).

Among the menu of contingent liabilities undertaken by governments, social security and social welfare-related programs have received the most attention, often disproportionately to their share of total government contingencies. For example, Eisner (1986) estimates that the value of U.S. Treasury contingent liabilities associated with annuity programs (such as social security, retirement, and hospital insurance) represented only 30 percent of total net contingency liabilities at the end of fiscal year 1984, whereas the remaining 70 percent was mostly associated with loan and credit guarantees and insurance (including deposit insurance) commitments reflecting, in part, recent legislation that reduced the government's social security liability. 1/

Loan guarantee programs vary in their characteristics; guarantees may cover 100 percent of the credit arrangement, or up to some fraction or fixed amount. 2/ Most credit and deposit guarantee schemes require some fee or premium, which may be a onetime or annual payment of an amount usually based on a percentage of the amount guaranteed. Often these programs are "funded," in the sense that some attempt is made to ensure that a reserve is set aside that matches the expected liability; nonetheless, the amount of insurance coverage may or may not be limited to the amount of the reserve. Participation in such programs may be voluntary, as is often the case with loan guarantees, or mandatory, as in the case of deposit insurance programs.

In addition, governments (sometimes through their central banks) and government agencies offer exchange rate guarantees and/or credit guarantees to exporters, often as a means of promoting domestic export potential by reducing the share of risks associated with international trade borne by the private sector. 3/ In such cases, the financing often comes from private sector financial institutions, while the government agency provides a guarantee of interest and/or principal. The guarantee may be either to the exporter (suppliers' credit) or to the importer (buyers' credit). The guaranteed credit may be denominated either in the exporter's or the importer's currency, implying risk to the government of both default and exchange rate variation. In general, agencies supplying these guarantees require a premium, which may vary according to perceived risk.

Analytically, these guarantee programs are distinct from the annuity programs described above for a number of important reasons. First, these programs are generally explicitly intended to reduce risk rather than to provide income support. Second, whereas annuity programs usually entail the provision of funds in a nonspecific manner (that is, for normative reasons, there is often no means test for the provision of benefits) and participation is often compulsory, the amount of loan

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1/ Eisner correctly notes, however, that current accounting conventions prevent a strict comparison of social security programs with other contingencies.

2/ For a useful description of the characteristics of credit guarantee schemes, see Levitsky and Prasad (1987).

3/ See Brau and Puckahtikom (1985) for a description of such systems.

guarantees and other insurance schemes provided is tied to the voluntary consumption of a particular service. For example, deposit insurance payoffs are linked to the size of the deposit. Finally, loan guarantee and insurance programs have been most often targeted to businesses or directly to savings instruments, and would therefore have a more direct and different impact on investment decisions and capital accumulation than would the provision of social welfare programs.

For these reasons, the macroeconomic implications of these types of contingent liabilities will likely be significantly different from those associated with social security programs. Despite these differences, little analysis has been undertaken of the impact of government insurance programs on economic activity and welfare, and therefore, with regard to the optimal provision of these programs. Exceptions include papers by Fried (1983), who examines the interest rate impact of loan guarantees in a Tobinesque portfolio model, and Chaney and Thakor (1985), who derive the partial equilibrium incentive effects of government bailouts on firms' investment decisions. 1/

In what follows, the optimal provision of loan and deposit guarantees will be examined in an overlapping generations model of a closed economy originally introduced by Samuelson (1958), and later adapted by Diamond (1965) and Samuelson (1975), to examine optimal savings and social security issues. The model is similar to the stochastic variants developed by Blanchard (1985), in which individuals face uncertain lifetimes, and by Zeira (1988), in which labor income streams are uncertain. However, in the case described below, the source of the uncertainty is the return on loans to private sector firms rather than the length of life or the return to human capital. Production is assumed to be performed by firms (which may also be viewed as banks) that convert capital borrowed in the previous period into the current consumption good. However, since firms face a random production shock, some firms will make positive profits, returning the borrowed capital to lenders (or depositors) with interest and a profit to shareholders, whereas other firms will become bankrupt, returning only a proportion of their liabilities to lenders and nothing to shareholders.

Economic agents are assumed to choose among three savings vehicles: government bonds, equity shares of firms, and loans to firms. Government bonds are assumed riskless (in the absence of inflation), and, for analytic convenience, the risk associated with

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1/ A literature does exist with regard to the pricing of government loan guarantees and deposit insurance in the context of option-pricing models (see, e.g., Jones and Mason (1981) and Pennacchi (1987), respectively). A number of papers have also examined the impact of taxes and subsidies on private sector investment and risk taking. See, for example, Zeira (1988), Gordon (1985), and Mayshar (1984).

ownership of equity is assumed to be eliminated through diversification. However, owing to a market imperfection, investors are unable to fully diversify their portfolio of loans to firms and are thus assumed to face the above-mentioned default risk. In other words, equity markets are assumed to be relatively deep, permitting full diversification of risk, whereas investors in the loan market are assumed to face information or transactions costs that restrict diversification (e.g., geographic constraints). As will be demonstrated, it is this market imperfection that provides a role for the government's provision of the contingent liability.

The government chooses the level of lump-sum taxes, bond issuance, and interest rates, subject to an exogenous requirement to purchase a pure public good. Further, the government is assumed to be able to choose the level of the guarantee of private sector loans it will provide to firms. This approach makes it possible to extend the results to the case of deposit insurance, since, as stated above, firms may be viewed as banks, and their liabilities to the private sector as deposits. Section II introduces the model. Section III demonstrates the optimal provision of loan guarantees and examines the comparative static results of the issuance of this type of government contingent liability. Section IV summarizes the main results.

II. The Model

1. The firm and the stock market

It is assumed that at each point in time a continuum of identical firms exists whose number is normalized to unity. At time t, firms borrow funds in a perfectly competitive credit market at a rate $r_t^\ell - 1$, purchase capital, k_t, and produce the homogeneous consumption good for delivery in the following period according to a strictly concave stochastic production function, $\pi f(k_t)$, where π is a random variable distributed according to the marginal probability distribution, $p(\pi)$, over the interval $[0, 1]$.

A firm's profits at t+1 are equal to $\pi f(k_t) - r_t^\ell k_t$. If the production shock is "good," the value of production is sufficient to pay off the firm's creditors and provide a profit to shareholders. In this case, the production shock is $\pi \geq \pi^* \equiv r_t^\ell k_t/f(k_t)$. If the production shock is "bad," that is, if $\pi < \pi^*$, then profits distributed to shareholders are zero and the firm's output is devoted to partially paying off its creditors.

It is assumed that firms are owned by a perfectly diversified stock market mutual fund. Therefore, since risk is eliminated as far as the firms' owners are concerned, the firms will be directed to conduct business in a risk-neutral manner, choosing a level of debt and capital at time t, which maximizes the firm's expected profit at t+1. Given the above assumptions, the expected profit-maximizing level of debt will satisfy

$$\int_{\pi^*}^{1} [\pi f'(k_t)-r_t^{\ell}]p(\pi)d\pi = 0, \tag{1}$$

which, assuming that the second-order conditions are satisfied, implies a demand for debt/capital that is decreasing in the loan rate. Note that owing to the asymmetric risk (no negative profits), the loan rate will exceed the expected marginal product of capital.

Since all firms are identical ex ante, facing identical constraints at t, their demand for capital/loans will be identical, so that $k_t^i = k_t$. Aggregating over all firms yields the aggregate loan demand:

$$\int_0^1 k_t di = k_t.$$

Similarly, whereas individual firms suffer individual production shocks at t+1, they are assumed to be independently distributed. This--assuming that many firms are evenly distributed over the unit interval--implies that aggregate output at t+1 will be

$$\int_0^1 \pi f(k_t)p(\pi)d\pi = f(k_t)\int_0^1 \pi p(\pi)d\pi. \tag{2}$$

Firms' "equity" is assumed to be traded in a stock market dominated by a single mutual fund, which owns the productive capacity of the economy. 1/ A share of the mutual fund (s_t) may be purchased during period t at price h_t. Total dividends paid out by the mutual fund (δ_{t+1}) are paid at the beginning of the following period and will be equal to the sum of the profits of all firms able to meet their interest obligations; that is, the profits of all firms whose production shocks π are greater than π^*. These dividends at time t are

$$\delta_t = \int_{\pi^*}^{1} [\pi f(k_{t-1})-r_{t-1}^{\ell}k_{t-1}]p(\pi)d\pi. \tag{3}$$

Owing to the mutual fund's diversified portfolio, dividends will be a nonrandom function of the current capital stock. The second-period proceeds from the individual's current stock purchase will be $s_t(h_{t+1} + \delta_{t+1})$ in period t+1.

1/ This is an analytical convenience. An equivalent assumption would allow investors to diversify their portfolios without restriction.

2. The consumer's choice set

It is assumed that the lifetime of economic agents may be subdivided into two periods. At each point in discrete time, therefore, the population comprises two segments, the current young and old, each consisting of a continuum of individuals whose population size is normalized to unity.

In the first period of life, agents are assumed to receive an exogenous labor income (y), which is used to finance lump-sum taxation (τ_t), purchases of a nonstorable homogeneous consumption good (c_t), and the acquisition of savings vehicles. 1/ These include either government bonds (b_t) or loans to the private sector (k_t), both of which are purchased at par, or the stock of the market portfolio (s_t) purchased at price h_t. Thus, the ith individual's first-period budget constraint is

$$c_t^1 = y - \tau_t - b_t - k_t - h_t s_t. \tag{4}$$

In the second period of life, the agent's sole source of income is the return on savings. Government bonds yield r_t with certainty at t+1. However, as discussed above, loans to firms are risky; if the firm to which an agent has loaned funds suffers a sufficiently bad production shock, it will default on at least part of its liability. In principle, given the assumption of independent shocks and a continuum of firms, a riskless portfolio of loans could be derived through diversification. In this case, it will be assumed that, unlike the case of the stock market, market imperfections exist to preclude agents from diversifying their private sector loan portfolio. Examples of similar real-world constraints include the inability--owing to geographic, information, and transactions cost considerations--of individuals to perfectly diversify bank deposit portfolios, or, similarly, the inability of banks to achieve a perfect diversification of loan portfolios. At time t, the ith individual of the current young invests k_t as a loan to firm i at the market-determined interest rate, r_t^ℓ. Since both creditors (the current young) and borrowers (firms) are homogeneous ex ante, the assumption of perfect competition in credit markets assures a single loan rate.

Given that the ith firm is subject to random production shocks, the return on loans to that firm bears a default risk. For all production shocks, $\pi < \pi^*$ [$\equiv r_t^\ell k_t / f(k_t)$], output is insufficient to fully repay outstanding credit. Thus, in the absence of government loan guarantees, the payoff to a loan of k_t is $r_t^\ell k_t$ for $\pi \geq \pi^*$, and $\pi f(k_t)$ for $\pi < \pi^*$. This is illustrated in Figure 1. In the absence of a loan guarantee, the payoff to creditors is the line 0bcd, with the amount varying

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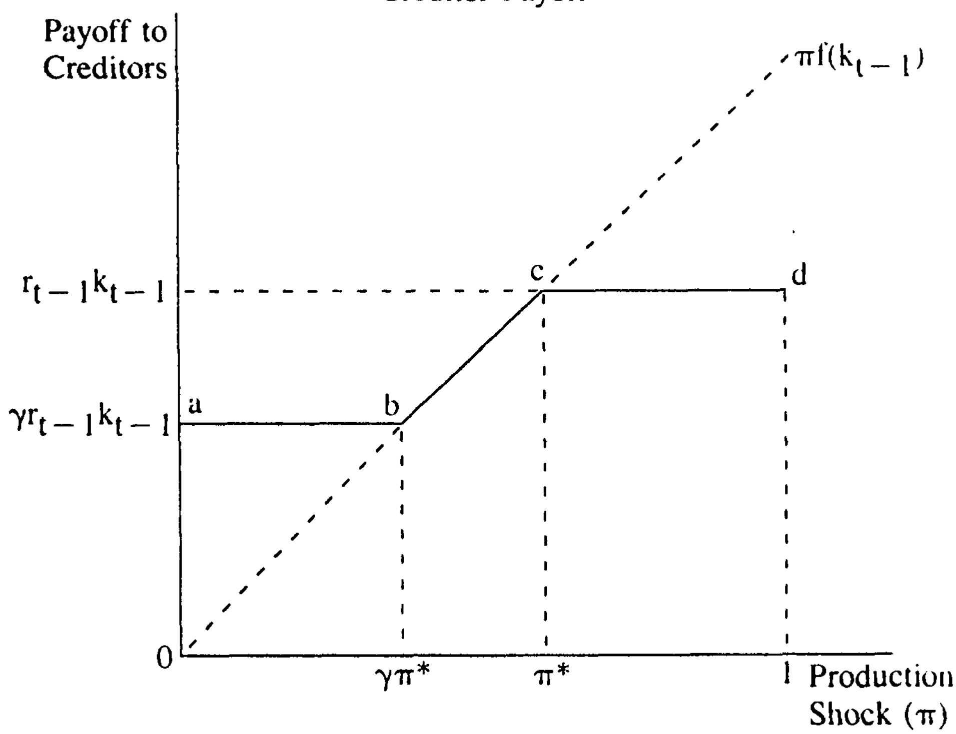
1/ Note that to simplify the analysis, consideration of labor in the profit function of firms is ignored. Thus, first-period income may be considered either as "home" production or as subsumed in firms' production functions.

Figure 1.
Creditor Payoff



depending on the production shock and the amount invested. Now suppose that the government guarantees a proportion γ $(0 \leq \gamma \leq 1)$ of the private sector's loans. If the firm's production shock is sufficiently good $(\pi \geq \pi^*)$, then, as before, no default occurs. If the guarantee scheme is not complete $(\gamma < 1)$, then, for modestly bad shocks (i.e., for π only slightly less than π^*) the value of production will still exceed the government's guarantee, and the payoff to creditors will be $\pi f(k_t)$. However, if the value of production is less than the government's minimum guarantee (i.e., $\pi f(k_t) \leq \gamma r_t^{\ell} k_t$, or if $\pi < \gamma\pi^* \equiv \gamma r_t^{\ell} k_t / f(k_t)$) then the creditor's payoff is $\gamma r_t^{\ell} k_t$. Figure 1 illustrates the effect of the guarantee where the new locus of credit payoffs is abcd. The guarantee scheme reduces the creditor's uncertainty with regard to low payoffs; as the guarantee index γ approaches unity, uncertainty is eliminated, whereas at the opposite extreme, as γ approaches zero, the level of uncertainty is maximized. 1/

At the beginning of their second period of life, agents discover the return to their loan portfolio and inelastically purchase the consumption good. The second-period budget constraint is then

$$c_{t+1}^2 = \begin{cases} r_t b_t + r_t^{\ell} k_t + (h_{t+1}+\delta_{t+1}) s_t & \pi \geq \pi^* \\ r_t b_t + \pi f(k_t) + (h_{t+1}+\delta_{t+1}) s_t \quad \text{for} & \gamma\pi^* < \pi < \pi^* \\ r_t b_t + \gamma r_t^{\ell} k_t + (h_{t+1}+\delta_{t+1}) s_t & \gamma\pi^* \leq \pi, \end{cases} \qquad (5)$$

where second-period consumption will be reduced to the extent that firms default on first-period loans, and increased to the extent that these loans are guaranteed by the government.

3. The market equilibrium

The current young are assumed to choose a planned consumption path that maximizes the expected value of a separable utility function $U(c_t^1) + \beta U(c_{t+1}^2)$, where β is the rate of time preference, subject to the budget constraints described above. 2/ Using the budget constraints to substitute for consumption in the utility function and maximizing with respect to b_t, k_t, and s_t yield the following first-order conditions:

1/ This is easily confirmed by differentiating the expression for the variance of the creditor's payoff with respect to γ.

2/ Note that since it is assumed that government expenditure is a pure public good not affecting the marginal utility of consumption, it can be ignored in the analysis of the consumer's problem.

$$U'(c_t^1) = r_t\beta\,\{\int_{\pi^*}^{1} U'(c_{t+1}^2)p(\pi)d\pi + \int_{\gamma\pi^*}^{\pi^*} U'(c_{t+1}^2)p(\pi)d\pi$$

$$+ \int_{0}^{\gamma\pi^*} U'(c_{t+1}^2)p(\pi)d\pi\} \tag{6}$$

$$U'(c_t^1) = r_t^{\ell}\beta \int_{\pi^*}^{1} U'(c_{t+1}^2)p(\pi)d\pi + f'(k_t)\beta \int_{\gamma\pi^*}^{\pi^*} \pi U'(c_{t+1}^2)p(\pi)d\pi$$

$$+ \gamma r_t^{\ell}\beta \int_{0}^{\gamma\pi^*} U'(c_{t+1}^2)p(\pi)d\pi \tag{7}$$

$$r_t = \frac{h_{t+1} + \delta_{t+1}}{h_t}, \tag{8}$$

where c_t^1 and c_{t+1}^2 are as defined by equations (4) and (5). Equations (6) and (7) require that the marginal reduction in the first-period utility from the purchase of government bonds and private sector loans, respectively, be compensated by an increase in expected second-period utility. Equation (8) requires that the rate of return on the two riskless assets be equal. Equations (6)-(8) will determine the ith individual's demand for government bonds, private sector loans, and stocks, given the first-period endowment, current interest rates and taxes, and the expected level of stock prices at t+1. 1/

Whereas the rate of return on the riskless government bond must be identical to the rate of return on the similarly riskless stock, given the assumed riskiness of private sector loans, the rates of return on capital and bonds will diverge. Equations (6) and (7) may be manipulated to derive the risk premium on private sector loans:

$$\frac{r_t - r_t^{\ell}}{r_t^{\ell}} = \frac{\{\int_{\gamma\pi}^{\pi^*} (\frac{f'(k_t)\pi}{r_t^{\ell}} - 1)U'(c_{t+1}^2)p(\pi)d\pi + (\gamma-1)\int_{0}^{\gamma\pi^*} U'(c_{t+1}^2)p(\pi)d\pi\}}{\int_{0}^{1} U'(c_{t+1}^2)p(\pi)d\pi},$$

1/ Given the perfect substitutability between bonds and capital, the second-order conditions for a maximum are not strictly satisfied (the quadratic form is zero or negative semidefinite) when derived from the three first-order conditions. However, viewing these two assets in the aggregate, and assuming constant absolute risk aversion, assures an interior maximum.

which, given the firm's demand for capital (from equation (1)), will be nonnegative for $0 \leq \gamma < 1$ and zero for $\gamma = 1$, so that the risk premium over the rate on the riskless government bond is minimized when the guarantee is maximized.

To close the model, the goods and stock markets equilibrium must be specified. The stock market will clear when the demand for stocks equals the supply--that is, $s_t = 1$. Equilibrium in the goods market requires that the supply of goods will equal the exogenous income of the current young plus the sum of firms' production. The demand for goods equals the sum of individuals' consumption, current investment in capital, plus any government expenditure, g_t. Aggregating across firms and individuals yields the following (since individuals and firms differ only with respect to productivity shocks, the index of integration is π):

$$\int_0^1 \pi f(k_{t-1})p(\pi)d\pi + \int_0^1 y d\pi = \int_0^1 (c_t^1+c_t^2)d\pi + \int_0^1 k_t d\pi + g_t.$$

Using the consumers' budget constraints to substitute for c_t^1 and c_t^2, and performing the indicated integration and rearranging yields

$$\tau_t - g_t - \int_0^{\gamma\pi^*} [\gamma r_{t-1}^{\ell} k_{t-1} - \pi f(k_{t-1})]p(\pi)d\pi = r_{t-1}b_{t-1} - b_t, \qquad (9)$$

which is the government's budget constraint. In this case, tax revenue less government purchases of the consumption good less the subsidy on loan defaults equals the net revenue from bond sales.

Equations (1), (3), (6)-(9), and the definition of π^* represent seven deterministic difference equations in ten variables (k_t, b_t, r_t, r_t^{ℓ}, δ_t, h_t, π^*, g_t, τ_t, and γ). It will be assumed that a perfect-foresight equilibrium exists, defined as a time path of k_t, b_t, r_t, r_t^{ℓ}, h_t, δ_t, and π_t^*, given a preannounced time path of g_t, τ_t, and γ, and given the values of the state variables b_{t-1}, r_{t-1}, and k_{t-1}, which satisfy these equations. Furthermore, it is assumed that for constant time paths of the exogenous variables, there exists a unique perfect-foresight equilibrium, which converges to the steady state described below.

III. The Steady State

The economy's steady-state equilibrium can be described by the definition of π^* and the following equations:

$$\int_{\pi^*}^{1}[\pi f'(k)-r^{\ell}]p(\pi)d\pi = 0 \tag{1'}$$

$$\delta = \int_{\pi^*}^{1} [\pi f(k)-r^{\ell}k]p(\pi)d\pi \tag{3'}$$

$$U'(y-b-k-h-\tau) = r\beta \; \{\int_{\pi^*}^{1} U'(br+r^{\ell}k+h+\delta)p(\pi)d\pi$$

$$+ \int_{\gamma\pi^*}^{\pi^*} U'[br+\pi f(k)+h+\delta]p(\pi)d\pi + \int_{0}^{\gamma\pi^*} U'(br+\gamma r^{\ell}k+h+\delta)p(\pi)d\pi\} \tag{6'}$$

$$U'(y-b-k-h-\tau) = r^{\ell}\beta \int_{\pi^*}^{1} U'(br+r^{\ell}k+h+\delta)p(\pi)d\pi$$

$$+ f'(k)\beta \int_{\gamma\pi^*}^{\pi^*} \pi U'[br+\pi f(k)+h+\delta]p(\pi)d\pi$$

$$+ \gamma r^{\ell}\beta \int_{0}^{\gamma\pi^*} U'(br+\gamma r^{\ell}k+h+\delta)p(\pi)d\pi \tag{7'}$$

$$h(r-1) = f(k)\int_{\pi^*}^{1} (\pi-\pi^*)p(\pi)d\pi \tag{8'}$$

$$\tau - g - \int_{0}^{\gamma\pi^*} [\gamma r^{\ell}k-\pi f(k)]p(\pi)d\pi = b(r-1) \tag{9'}$$

Equation (1') represents firms' profit-maximizing condition (i.e., their implicit demand for capital investment) and equation (3') is the steady-state definition of investors' dividends. Equations (6')-(8') are the steady-state equivalents to the consumers' first-order conditions, and equation (9') corresponds to the government's budget constraint.

1. Optimal policy

To avoid the complications associated with the (admittedly important) dynamic implications of changes in policy, it will be assumed that the government seeks to maximize the expected level of steady-state welfare of the representative agent, subject to the market-clearing constraints and the menu of instruments available to the government. This would require the maximization of the expected value of $U(c^1) + \beta U(c^2)$ subject to equations (1'), (3'), and (6')-(9'). In general, given the assumed market imperfection, the resultant optimal policy will be the second-best policy, relative to the command optimum, defined as the maximum level of welfare achievable, given the technical (rather than the market) constraints of the economy. Nonetheless, it will be instructive to derive this latter allocation of consumption and capital to use as a benchmark in evaluating the optimal policy subject to the constraints of the market economy.

The technical constraint on the economy will be that the sum of consumption plus capital replenishment, $(c^1 + c^2 + g + k)$, equals the production of the consumption good $(y + \int_0^1 \pi f(k)p(\pi)d\pi)$. Clearly, social welfare (as defined above) is maximized when the standard conditions,

$$U'(c^1) = \beta U'(c^2) \tag{10}$$

and

$$\int_0^1 \pi f'(k)p(\pi)d\pi = \underline{\pi} f'(k) = 1, \text{ where } \underline{\pi} \equiv \int_0^1 \pi p(\pi)d\pi \tag{11}$$

are satisfied. The first condition requires that the marginal utility of consumption--discounted by the rate of time preference--in both periods be equal; the second condition states that the "expected," or average, marginal product of capital should equal one--the marginal consumption cost of investing an additional unit of capital. This latter condition requires that output net of capital inputs be maximized and is simply the standard golden rule for optimal capital accumulation (applied to a nongrowing economy). Thus, the command optimum allocation is to maximize output net of investment (equation (11)) and allocate that output so as to maximize utility (equation (10)).

Surprisingly, the following can be shown:

Proposition: Setting the real rate of interest to zero ($r=1$) and fully guaranteeing loans ($\gamma=1$) does not yield the command optimum allocation.

This proposition is easily proven if one notes that $r = 1$ implies that equilibrium $\pi^* = 1$ (from equation (8')), in turn requiring that $r^\ell k/f(k) = 1$; whereas $\gamma = 1$ requires that $r^\ell = 1$ (from equations (6') and (7')). Therefore, the market equilibrium level of capital, given $r = \gamma = 1$, will be k^m, which satisfies $k^m = f(k^m)$, which, in general, will not be consistent with that level of capital, k^g, which satisfies equation (11). The following result is also evident:

Corollary: The market equilibrium characterized by full loan guarantees and a zero real rate ($\gamma = r^\ell = r = 1$) is overcapitalized relative to the command optimum allocation.

This corollary is easily demonstrated diagramatically if the production function has the usual characteristics of convexity and $f(k=0) = 0$ and $f'(k=0) = \infty$ (Figure 2). The production function crosses the 45° line at $f(k^m) = k^m$, the market equilibrium, given $r = \gamma = 1$. However, since the assumptions regarding the production technology require that the slope of $f(k)$ decline from infinity to less than unity before crossing the 45° line, then k^g defined by $f'(k^g) = 1/\underline{\pi}$ (where $\underline{\pi}$ is defined by equation (11)) must be less than k^m. 1/

Thus, to summarize, the ideal--or command optimum--allocation is to distribute consumption between generations so as to equalize the level of marginal utility (discounted by the rate of time preference) over time while setting the level of savings and capital at the golden rule level, which maximizes net production of the consumption good. It was demonstrated, however, that this allocation could not be duplicated in the market economy, even with full loan guarantees and a zero real rate. Although a zero real rate did equalize discounted marginal utility across time and yielded the appropriate distribution of consumption, the provision of full loan guarantees resulted in an overcapitalized, and therefore suboptimal, steady-state equilibrium.

These results followed from the limited liability that firms were assumed to enjoy, which implied that they did not "value" losses incurred appropriately. Since firms' profits do not symmetrically account for bad production shocks, their demand for loans/capital is too high, resulting in excessive capital accumulation and a lower level of net output. Note that this result does not necessarily hinge on the assumed association between investors' and managers' objective functions. As long as any bankruptcy and/or limited liability is allowed, a similar result will follow.

However, despite the foregoing discussion, the following result can be demonstrated:

<u>Proposition</u>: The command optimum capital stock can be supported as a market equilibrium and is characterized by a zero real rate of interest on government debt ($r=1$), less than full loan guarantees ($\gamma < 1$), and a positive real rate on private debt, such that $r^{\ell} = f(k^g)/k^g$, where k^g is the golden rule level of capital that satisfies equation (11).

The proposition that the steady-state equilibrium supports the command optimum capital stock can be demonstrated by imposing the condition that $r = 1$ and equation (11) on equations (1') and

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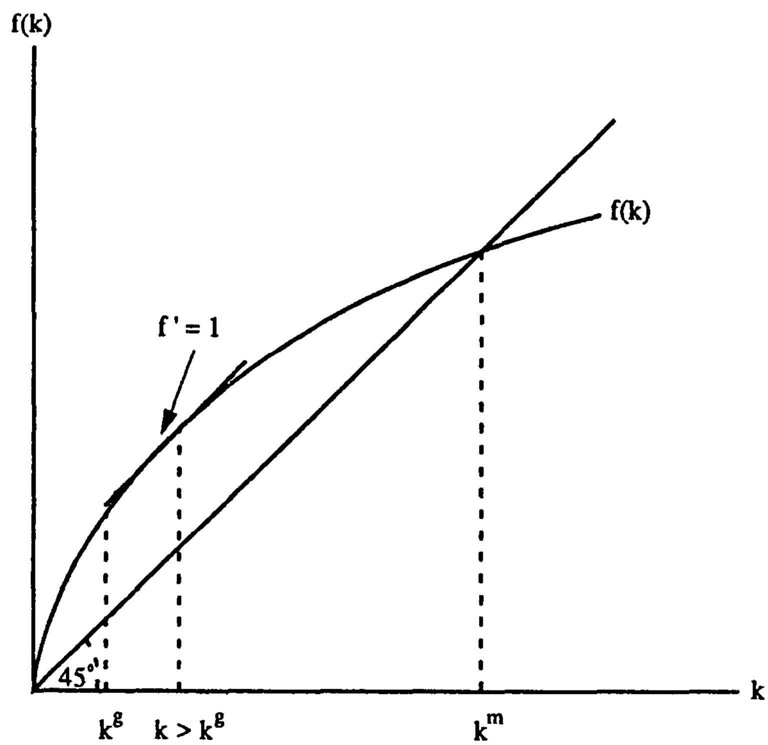
1/ Note that at $k = k^m$ aggregate output is more than exhausted and government and private consumption is financed solely through labor income.

Figure 2

Equilibrium Capital Stocks



(6')-(9'). The resultant system will determine the levels of k, b, r^{ℓ}, τ, γ, and h, which also satisfy equation (10). The equilibrium real rate of interest on loans/capital follows from equation (1').

Thus, since the welfare achieved at the command optimum cannot be exceeded, the government's optimal policy is simply to encourage the golden rule level of output and the welfare-maximizing distribution of consumption by setting the real rate on riskless government debt to the rate of real growth (i.e., zero) and offer less than full loan guarantees. As discussed above, although a full loan guarantee will engender the appropriate supply of capital/loans from consumers, given the firms' limited liability, the demand for loans will be too high, provoking excessive capital accumulation. Similarly, the optimal level of loan guarantee will, in general, be greater than zero, so as to encourage the appropriate level of loan supply. The following can be demonstrated:

Corollary: The optimal provision of loan guarantees will be nonzero.

This is easily confirmed by imposing the optimality conditions that r = 1 on equations (6') and (7'). If optimality also requires that γ = 0, then the limits of integration collapse, such that equations (6') and (7') imply

$$\int_0^1 [1-\pi f'(k)]U'(c^2)p(\pi)d\pi = 0, \tag{14}$$

which cannot hold, given the usual assumption of a concave utility function.

However, it is clear that the optimal provision of loan guarantees will imply a second-best consumption allocation from the perspective of the representative agent. This is to say that while the equilibrium level of capital maximizes output, the residual uncertainty from a less than full government guarantee of loans will reduce expected utility below that which would be achieved if the command optimum were attainable as a market equilibrium. 1/ Thus, while loan guarantees offer the government the avenue for provoking the golden-rule level of capital, thereby maximizing output, this instrument does not permit a full offset to the distortion caused by the limited liability that firms are assumed to enjoy.

1/ This can easily be confirmed by examining the command optimum to determine whether making second-period consumption random, subject to the golden rule level of output, increases expected utility.

2. Comparative statics

The complexity of the conditions defining the steady state unfortunately limits the ability to analyze the impact of policy shocks. However, Tables 1 and 2 illustrate the impact of changes in loan guarantees on the level of capital, private sector interest rates, and stock prices under the assumption that the real rate on government bonds is held constant at its golden rule level (i.e., r = 1). The sign of the comparative statics is evaluated at three levels of the index of loan guarantee: zero, its golden rule level, and unity. Table 1 contains the comparative static results, given that the level of government expenditures is also held constant. As indicated, the effect of an increase in the degree of loan guarantee would generally be as expected. The capital stock would tend to increase as the loans become relatively less risky, and the real rate of interest on such loans would tend to fall. The effect on stock prices and taxes is, for the most part, uncertain, depending on the degree of risk aversion. However, it would appear that at $\gamma = 1$ a decrease in the rate of loan guarantee would tend to reduce taxes. In other words, as would be expected, the provision of the guarantee is a net subsidy and requires tax revenue to balance the primary budget.

The results of the alternative experiment, changing the degree of loan guarantee while holding taxes constant, are presented in Table 2. The comparative static results are qualitatively unchanged, except that the effect on stock prices of the increase in loan guarantees (evaluated at g = 1) is determined to be negative. That is to say, stock prices tend to fall in the face of increased loan guarantees. 1/

IV. Conclusion

A simple overlapping generations model of a closed economy was developed to examine the government's potential role in providing loan guarantees. The consumption good was assumed to be produced by firms facing random production shocks that, in some cases, reduced revenues below the level required to repay creditors. Although equity portfolios could be perfectly diversified, market imperfections were assumed such that lenders could not fully diversify loan portfolios. The government's activity was limited to either purchasing private sector output or providing loan guarantees. Financing was through lump-sum taxes or the issuance of government bonds.

First, the command optimum allocation was examined. This was confirmed to be equivalent to the usual golden rule level of capital and an allocation of consumption that equilibrated the discounted marginal utility of consumption over time. It was also determined that although

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1/ Of course, this is simply due to a wealth effect, since profits, and therefore dividends, are unaffected by the loan guarantee at r = 1.

Table 1. Comparative Static Results ($r = 1$, $g = \bar{g}$)

	$\gamma = 0$	$\gamma = \gamma^g$	$\gamma = 1$
$\frac{\partial k}{\partial \gamma}$	$= 0$	> 0 1/	> 0
$\frac{\partial r^\ell}{\partial \gamma}$	$= 0$	< 0	< 0
$\frac{\partial h}{\partial \gamma}$	$= 0$	$\gtrless 0$	> 0
$\frac{\partial \tau}{\partial \gamma}$	$= 0$	$\gtrless 0$	> 0

1/ Sufficient conditions that:

$k > \gamma(f - k)/k\rho$, $k > \gamma\beta[\rho f(\gamma-1) + \frac{f}{k}\rho(f-k)]$

and

$(\pi - f'')\pi < 1$ for $\gamma^g \leq \pi \leq 1$,

where ρ is the measure of absolute risk aversion.

Table 2: Comparative Static Results ($r = 1$, $\tau = \bar{\tau}$)

	$\gamma = 0$	$\gamma = \gamma^g$	$\gamma = 1$
$\frac{\partial k}{\partial \gamma}$	$= 0$	> 0 1/	> 0
$\frac{\partial r^\ell}{\partial \gamma}$	$= 0$	< 0	< 0
$\frac{\partial h}{\partial \gamma}$	$= 0$	$\gtrless 0$	$\gtrless 0$ 2/
$\frac{\partial g}{\partial \gamma}$	$= 0$	$\gtrless 0$	< 0

1/ Sufficient conditions that:

$$k > \gamma(f-k)/k\rho,\ k > \gamma\beta[\rho f(\gamma-1) + \frac{f}{k} + \rho(f-k)]$$

and

$$(\pi - f'')\pi < 1 \text{ for } \gamma^g \le \pi \le 1,$$

where ρ is the measure of absolute risk aversion.

2/ Sufficient condition is that $1 - \rho f > 0$.

this consumption allocation could be achieved as a market equilibrium through the provision of full loan guarantees, such a policy would imply an overcapitalized economy and, therefore, less than maximal output. In fact, the optimal policy was to guarantee only a proportion of the private sector's loan portfolio. The implication of the analysis is that the limited liabilities of equity owners may limit the government's ability to achieve first-best risk-reduction policies through loan guarantees. The second-best policy will be to offer only partial guarantees; government policies that fully guarantee loans or deposits are suboptimal.

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