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Indexation and Maturity of Government Bonds: A Simple Model

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Abstract

The central issue of the paper is the optimality of different degrees of price indexation and maturity structures of government debt when markets are incomplete and policymakers face "credibility" problems.

The analysis shows that price indexation is useful because it affects the relevant inflation tax base and allows governments to strike the optimal balance between the gains from conventional tax smoothing and the inflation costs associated with time-consistent policies. On the other hand, debt maturity management matters because it allows governments to alter the time profile of the inflation tax base and to influence the intertemporal path of incentive-compatible inflation.

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Summary

This paper focuses on an important but somewhat neglected character in the inflation drama, namely, government bonds. It investigates the degrees of price indexation and debt maturity structures that are best when markets are incomplete and policymakers face "credibility" problems.

The paper uses a two-period model, in which the interest rate on government debt is not contigent on government expenditure, to analyze optimal indexation. It uses a three-period model to analyze optimal debt maturity. If indexation is optimal, the analysis shows it's best for the government to choose long-term maturities to smooth out inflation over time. If indexation is nonexistent or less than optimal, shorter maturities are preferable.

The analysis shows that indexation and maturity of government bonds may be powerful tools for policymaking. Price indexation is useful because it affects the relevant inflation-tax base--and removes the incentive to "inflate away" the real value of government bonds. On the other hand, debt-maturity mangement matters because it allow governments to alter the time profile of the inflation-tax base and to influence over time, inflation that is compatible with productive incentives.

I. Introduction

Very few economists would nowadays deny that the stock of money and the price level are closely related variables. In fact, cross-country empirical analyses (see, e.g., Vogel (1974), Lucas (1980), Lothian (1985), Calvo (1987)) and even a cursory look at the data eloquently shows that a nonbeliever in this basic "monetarist" proposition would have a hard time making his case (except, perhaps, for recent periods when the very definition of the relevant stock-of-money concept is somewhat controversial). However, if money growth is the main cause of inflation, and the relationship is well understood, why is it then that inflation has not been completely eradicated? A possible answer is that at times countries rely on the inflation tax as a source of fiscal revenue. In fact, Phelps (1973) has given rise to a literature which suggests that a sensible reliance on the inflation tax could even be socially optimal (see, e.g., Végh (1988), Guidotti and Végh (1988), and the references therein). Interestingly, however, there are many instances in which the inflation tax appears to be larger than any sensible social welfare function would dictate. The phenomenon becomes self-evident in hyperinflation episodes. 1/ An answer to this puzzle was given in Calvo (1978) where it is shown that money creation becomes an attractive fiscal revenue source when its present use has little effect on expectations about future monetary/fiscal policy (a characteristic of nonreputational rational-expectations equilibria). Thus, for each successive government it is optimal to engineer "high" inflation. But since individuals are assumed to be rational, they take this inflationary bias into account, leading to an almost total inability to collect revenue through the inflation tax. 2/

It is still too early to judge whether the rational-discretion approach is the key to explain the above-mentioned puzzle. Our present empirical knowledge, however, strongly suggests that precommitment-only models are unable to explain even the most basic stylized facts (see, e.g., Poterba and Rotemberg (1988), Obstfeld (1988)), and that the rational-discretion approach could provide some of the missing links. 3/ Thus, whatever is the final verdict, a sensible study of monetary policy issues appears to necessitate the use of rational-discretion methods. And this is the approach that this paper will take.

1/ Friedman (1971) was one of the first papers to squarely focus on this issue.

2/ Barro and Gordon (1983a,b) have expanded this line of research in several important directions.

3/ Obstfeld (1988) shows, for example, that a rational-discretion type model can explain the failure of the inflation-rate series to behave like a martingale, which is an implication of standard precommitment-only models with quadratic and time-separable utility functions (see Mankiw (1987)).

We will focus on a somewhat relegated character of the inflation drama, namely, government bonds. Its potential relationship with the inflationary process has been recently rediscovered--Keynes (1971) knew it well enough himself--due partly to the experience of some heavily indebted LDCs as well as that of some industrialized ones like Belgium and Italy. However, the theoretical work using post-Keynesian (New Classical?) tools has only started (see, for instance, Lucas and Stokey (1983), Persson and Svensson (1984), Persson, Persson and Svensson (1987), Bohn (1988), Calvo (1988), Obstfeld (1988)). Some of the results that emerge from this literature are quite striking. Take, for instance, the work of Lucas and Stokey (1983), and Persson, Persson and Svensson (1987). Their central message is that a careful managing of the maturity structure can be so effective that it could actually succeed in replicating the full-precommitment optimum in a rational-discretion world. This remarkable result is bought at the cost of making some unrealistic assumptions, 1/ but it strongly suggests that in a rational-discretion world, the structure of debt maturity could become a highly relevant policy instrument. 2/

The present paper relaxes the assumption of complete markets--to make the model more realistic--and introduces actual-inflation costs (i.e., anticipated and unanticipated inflation are both costly). The latter has been shown to be almost a necessary assumption in the context of the above-mentioned first-best-through-discretion literature (see Calvo and Obstfeld (1989) and Persson, Persson and Svensson (1989)). 3/ The model is, otherwise, very simple.

The central issue addressed in the paper is the impact and optimality of different price indexation coefficients and debt maturity structures. We first examine the indexation issue in terms of a two-period model where government in period 0 decides the proportion of total debt--a predetermined variable--that will be indexed to the price level. The only source of uncertainty is (exogenous) government expenditure. We study the cases in which the nominal interest rate can be indexed to the state of nature, and some of those in which it cannot. The optimal solution with a state-contingent interest rate is referred to as the first best, and is our benchmark for our consumer-surplus type welfare calculations.

1/ For example, existence of complete markets is assumed. It should also be noted that the problem may exhibit multiple equilibria, so decentralization of the good may be at risk (see Calvo (1988)).

2/ This contrasts sharply with the well-trained economist bias that "debt maturity is irrelevant, all that matters is present value."

3/ These costs play an important role in the Barro and Gordon (1983 a,b) papers. They are assumed away in Persson, Persson and Svensson (1987) but, as shown by Calvo and Obstfeld (1989), their absence prevents attaining the full-precommitment solutions under rational discretion, except in very special cases.

Most of the discussion in the paper deals with the realistic case in which the interest rate on government debt is not indexed to the state of nature (i.e., government expenditure). 1/ However, we allow debt to be indexed to the price level. The latter is a useful device whenever there are rational-discretion type elements, because price indexation lessens the incentives to use inflation to liquidate the real value of the outstanding debt--and thus lowers the deadweight-loss of conventional taxes associated with debt service. Like in the Gray-type papers (see, Gray (1973), Aizenman and Frenkel (1985)), however, full price indexation is shown not to be necessarily optimal, because the government would be completely prevented from applying the inflation tax on bonds to smooth out conventional taxes. There exists, therefore, a nontrivial optimization problem, and it is carefully discussed in Section 2 employing, as an intermediate step, the case in which the government in period 0 can precommit the actions of its successor (i.e., government 1).

In Section 3 we look at a three-period model in which government 0 can determine, in addition, the maturity structure of the outstanding debt, i.e., the amount and indexation characteristics of the debt obligations that government 0 passes along to governments 1 and 2. Interesting insights can already be shown in the context where government 1 can fully precommit the actions of government 2, but government 0 has to rely on "incentives" in order to influence next governments' (i.e., governments 1 and 2) policies. We find that optimal maturity depends very strongly on whether government 0 is able to index optimally. If the latter holds then it is optimal to issue only long-term maturity bonds. Otherwise, if no price indexation is possible, for example, there is a clear tendency for optimal debt maturity to have a strong short-term bias.

Section 4 shows several ways in which the above model's assumptions can be relaxed and, for the sake of realism, it examines the situation in which all debt is nominal (i.e., no price indexation is possible) and no government can precommit the actions of any of its successors. Interestingly, here we observe a reversion towards long-term debt.

In sum, then, with some precommitment and optimal indexation, long-term debt is optimal. With no indexation optimal debt maturity shortens considerably. But, with no precommitment the optimal term structure becomes longer once again. The intuition behind these different cases is carefully discussed, and numerical examples are shown to assess possible welfare implications.

Section 5 closes the main text of the paper with some conclusions and suggestions for further work. An Appendix collects the relatively more technical material.

1/ This is, incidentally, how incomplete markets will come into the picture.

II. A Two-Period Model

Consider a two-period economy. Government in period 0 has a debt b (>0 unless otherwise explicitly stated) which passes on to government in period 1. To simplify, we assume throughout that this decision has already been taken. We will first study the case in which the government in period 0 can completely control the actions of the government in period 1 (full precommitment).

In period 1 the relevant government budget constraint is:

$$(1) \qquad x = g + (1-\theta)b(1+i^*) + \theta b(1+i)/(1+\pi) - k\pi/(1+\pi)$$

where i^* is the one-period international rate of interest, g is (exogenous) government spending, x are conventional taxes, i and π are the one-period nominal interest rate and rate of inflation (i.e., $\pi = P_1/P_0 - 1$, where P_t is the price level in period t = 0, 1) and, finally, $(1-\theta)$ is the share of b which is indexed to the price level and $k\pi/(1+\pi)$ is the inflation tax on cash balances. 1/ Implicit in the above formulation is the assumption of perfect capital mobility. This is reflected in the second term of the R.H.S. of equation (1), where the interest rate on indexed debt is equated to the international one. 2/ Variables x, g, and b are measured in terms of (homogenous) output.

From the perspective of period 1, variables θ, b, i^* and i are predetermined. On the other hand, variables π, the rate of inflation, and x, taxes, are in principle under the control of government in period 1, subject to budget constraint (1). However, in the case of full precommitment we assume that also those last two variables are chosen by the government in period 0.

We will allow for stochastic shocks on g. In period 0, therefore, the value of g is, in principle, unknown. However, we assume that government in period 0 knows its probability distribution. Social welfare, as seen by government in period 0, is a negative function of taxes and inflation or deflation. More concretely, we assume the loss function, ℓ, takes the following form:

$$(2) \qquad \ell = E\,[(Ax^2 + \pi^2)]/2$$

1/ We assume the demand for real monetary balances k (≥ 0) is completely inelastic with respect to its opportunity cost. Relaxation of this assumption is possible but results are not significantly affected.

2/ We assume the existence of only one homogenous good, and the strict validity of P.P.P. Furthermore, we assume that international prices are constant, implying that i^* is also the <u>real</u> international rate of interest.

where A is a positive parameter and E is the expectations operator based on information available in period 0 which, by assumption, is the full structure of the model except for the realization of g.

Suppose first that the nominal interest rate, i, could be made contingent on the state of nature, g. The formal problem faced by government in period 0 is, thus, to minimize (2) by choosing θ and schedules i(g) and π(g), subject to budget constraint (1), and

$$(3) \qquad E\,[1+i(g)]/[1+\pi(g)] = 1+i^{*}.$$

The last condition is equivalent to saying that investors are risk neutral in terms of output.

A quick look at this problem immediately reveals that the optimal solution exhibits constant x and π. In particular, if the base of the (conventional) inflation tax, k, is zero, then optimal inflation is zero (i.e., equal to Friedman's optimal inflation in the present context), and i(g) is chosen so as to keep x constant subject to (1), which in the present context boils down to

$$(1') \qquad x = g + (1-\theta)b(1+i^{*}) + \theta b[1+i(g)]$$

Notice that i(g) is effective as a tool to keep x constant only if debt indexation to the price level is less than perfect, i.e., if $\theta \neq 0$. The constant optimal value of x is, of course, unique because, by (3) and (1'),

$$(4) \qquad x = \bar{g} + b(1+i^{*})$$

where $\bar{g}$ is expected government expenditure. This shows, incidentally, that the degree of price indexation is irrelevant as long as it is not perfect. 1/

State-contingent contracts are not easy to write or to enforce, particularly when moral hazard considerations are present. In the above optimal solution, for example, the rate of interest must fall as g rises. Hence, if the government had any control on g, it might be to its advantage to increase g and pretend that it was due to unavoidable circumstances. This shows why it is relevant to examine the case in which the nominal interest rate is independent of the state of nature.

Letting i denote the fixed nominal interest rate on the non-indexed portion of the domestic debt, equation (3) becomes:

$$(3') \qquad E\,(1+i)/[(1+\pi(g)] = 1+i^{*}$$

1/ In fact, full indexation, i.e., $\theta=0$, could also achieve the same optimum if we allowed for the interest rate on indexed debt to be a function of the state of nature g.

and the government loses an important tax-smoothing device, since now it is only the rate of inflation that could be employed for that purpose. In using it, however, the government gives up inflation smoothing, which is costly.

Let us assume that the share θ satisfies $0 \leq \theta \leq 1$. Then it can be readily seen that an optimal solution to the above minimization problem (subject now to a fixed i and (3')) calls for setting $\theta = 1$, i.e., no debt indexation. This remarkable result is due to the following facts: (1) fluctuations of π are costly; and (2) the base of the inflation tax--particularly the third term of the R.H.S. of (1)--increases with θ. Therefore, with the same inflation cost, the larger is θ, the larger will be the capability to smooth out taxes with inflation (more on this below).

Setting $\theta = 1$, the minimization problem faced by government in period 0 becomes:

(5a) $\quad$ Min ℓ with respect to $\pi(.)$ and i

subject to

(5b) $\quad x(g) = g + b(1+i)/[(1+\pi(g)] - k\pi(g)/[1+\pi(g)]$

and to (3'). One can easily check that at an optimum solution inflation, π, is an increasing function of government expenditure, g. This is true even when k, i.e., the base of the conventional inflation tax, is equal to zero. This is a point worth emphasizing, because when k=0 the <u>expected</u> revenue from inflation is zero. To prove this, note that the latter is given by

(6) $\quad E\, b(1+i)/[(1+\pi(g)] = b(1+i^{*})$

where the equality follows directly from (3'). Hence, changes in the rate of inflation have, on average, no effect on revenue. Despite this, however, π varies with g because it is a way to smooth out x (although, by previous considerations, $\bar{x}$ is invariant with respect to the π schedule).

The above discussion assumed that θ lies between zero and one. However, there is in principle no natural bound for θ. If $\theta > 1$, for example, the amount of indexed debt, $1-\theta$, would be a negative number, meaning simply that the government is a net <u>creditor</u> in indexed bonds. Interestingly, as noted above, the larger is θ, the larger will be the base of the inflation tax. We already noticed that a bigger θ enlarges the base but not the expected revenue from inflation. However, a larger base implies that the same smoothing of x could now be obtained by smaller changes in π. Thus, a larger θ always lowers minimum loss (this is the same argument behind the proof that $\theta = 1$ is optimal when $0 \leq \theta \leq 1$). In the limit as θ becomes infinitely large (or small, for that matter), the variance of π necessary to smooth out x goes to zero, and <u>perfect</u>

smoothing of x and π is possible. As a matter of fact, one can show that in the limit the optimal value of the loss function equals that obtained if the nominal rate of interest, i, could be made contingent on the state of nature, g.

A more inflexible nominal rate of interest, i, is equivalent to decreasing the number of available contracts. Therefore, it is to be expected that when i is independent of the state of nature the remaining instruments will tend to be used more intensively in order to compensate for such a loss. In the example the only loss of making i inflexible is its power to smooth out x. There is no expected revenue loss because equation (3) holds. That is why an unbounded enlargement of θ is enough to recover the smaller degrees of freedom of an inflexible i. It should be noted, however, that this device of increasing the base of the inflation tax to smooth out x gives rise to the temptation to use "unanticipated" inflation or deflation in order to increase government revenues. This is ruled out by assumption in the present case, but will play a key role in the ensuing discussion.

To show the above results in a simpler manner and to increase our understanding of more complicated scenarios, we will linearize the budget constraint (1). Using the first-order terms of the Taylor series corresponding to the R.H.S. of equation (1), and expanding at the point $i=\pi=i^*=0$, we get 1/

$$(7) \qquad x = g + (1-\theta)b + \theta b(1+i-\pi) - k\pi$$

We can now solve problem (5) substituting (7) for (5b). Given θ, we prove in the Appendix that optimal π and x satisfy: 2/

$$(8) \qquad \pi(g) = \frac{A(\theta b+k)}{1+A(\theta b+k)^2}(g-\bar{g}) + \frac{Ak}{1+Ak^2}(\bar{g}+b)$$

$$(9) \qquad x(g) = \frac{1}{1+A(\theta b+k)^2}(g-\bar{g}) + \frac{1}{1+Ak^2}(\bar{g}+b)$$

The above equations confirm the intuitions discussed in the previous paragraphs. Thus, by (8), inflation is positively (negatively) correlated with the level of government expenditure if the base of the inflation tax (i.e., $\theta b+k$) is positive (negative) and likewise, by (9), taxes increase with g. Furthermore, as θ becomes infinitely large (in absolute value),

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1/ Henceforth we will assume, without loss of generality, that the international interest rate $i^*=0$.

2/ The following results are based on a linearization of equation (3'), as given by equation (16) below.

functions $\pi(g)$ and $x(g)$ tend to become completely independent of g, i.e., perfect smoothing of π and x can be arbitrarily approximated by setting θ sufficiently large (in absolute value).

A payoff of linearizing the budget constraint is that the second term of equation (8) can be interpreted as the certainty-equivalent optimal full-precommitment inflation tax on cash balances when g equals its expected value, $\bar{g}$. For, under perfect certainty and full precommitment, the budget constraint (1) becomes (recalling that $i^{*}=0$):

$$(10) \qquad x = g + b - k\pi.$$

Thus, minimizing ℓ, given by equation (2), with respect to π subject to equation (10) calls for setting π equal to the second term of the R.H.S. of equation (8). Consequently, optimal inflation associated with government expenditure g--as given by equation (8)--is the sum of certainty-equivalent optimal inflation and a linear term in the level of government expenditure.

Employing equations (8) and (9) in expression (2) yields that, given θ, at its minimum the loss function satisfies:

$$(11) \qquad \ell(\sigma,p) = \left[\frac{\sigma^2}{1+A(\theta b+k)^2} + \frac{1}{1+Ak^2}(\bar{g}+b)^2\right]\frac{A}{2}$$

where σ^2 is the variance of government expenditure, g, and p stands for "precommitment." Clearly, at optimum (given θ), social loss is an increasing function of the variance and expected value of government expenditure. Moreover, the relationship with respect to θ is ambiguous but, confirming our earlier discussion, the loss ℓ can be set arbitrarily close to its minimum (or, rather its infimum) with respect to θ by setting θ sufficiently large. Finally, one can easily show that at the limit (i.e., as θ converges to plus or minus infinity), the value of the loss function converges to its optimum without uncertainty--as asserted for the nonlinear case.

We now turn to examine the case of no precommitment with respect to π. We assume that government in period 1 (or government 1, for short) has the same utility function as that of government 0, but knows the value of g. On the other hand, government 1 takes the nominal interest rate and the degree of indexation as given. Therefore, government 1 faces a problem without uncertainty; the only variables under its control are the rate of inflation π and taxes x. The solution to this problem is of relevance for government 0 given that its choices of i and θ will affect π and x. 1/

1/ For the sake of brevity, we will concentrate our discussion on the case in which the nominal interest rate i is not state-contingent.

Formally, the problem faced by government 1 is to minimize

(12) $\ell_1 = [Ax^2 + \pi^2]/2$

with respect to π and x, subject to equation (7). The first-order condition with respect to π (taking into account constraint (7)) is

(13) $Ax(\theta b+k) = \pi$

where $\theta b+k$ is the total base of the inflation tax in period 1 (i.e., cash balances k plus non-indexed debt θb), and Ax is, by (12), the marginal cost of conventional taxes, x. Similarly, π is the marginal cost of inflation. Thus, equation (13) simply tells us that at optimum government 1 will choose the rate of inflation so that a marginal increase in π will reduce the cost of taxation by as much as it increases the cost of inflation. By (7) and (13), we get

(14) $$\pi = \frac{A(\theta b+k)}{1+A(\theta b+k)^2}\left[g + b(1+\theta i)\right]$$

and

(15) $x = \pi/[A(\theta b+k)]$

As in the full precommitment case here also π and x are increasing (decreasing) functions of the level of government expenditure, g, when the base of the inflation tax is positive (negative).

We now return to period 0 in order to examine the problem faced by government 0. To simplify the analysis, we assume that equilibrium condition (3') can be approximated by:

(16) $E\ (i-\pi) = i^*$

Thus, by equations (14)-(16), we get the following equilibrium relationships:

(17) $$\pi(g) = \frac{A(\theta b+k)}{1+A(\theta b+k)^2}(g-\bar{g}) + \frac{A(\theta b+k)}{1+Ak(\theta b+k)}(\bar{g}+b)$$

(18) $$x(g) = \frac{1}{1+A(\theta b+k)^2}(g-\bar{g}) + \frac{1}{1+Ak(\theta b+k)}(\bar{g}+b)$$

Interestingly, the first terms of the R.H.S. of equations (17) and (18) are the same as in equations (8) and (9), corresponding to full precommitment. The second terms are equal only if θ=0, full indexation. In general, however, it can be shown that the second term of the R.H.S. of the inflation equation (17) is the optimal rate of inflation under certainty <u>without precommitment</u> and when g equals the expected value of government expenditure, $\bar{g}$. As in previous cases, there exists a positive (negative) association between the rate of inflation and government expenditure if the base of the inflation tax is positive (negative), and between the latter and taxes. It is worth noting that, unlike the full precommitment case, perfect-certainty inflation--given by the second term on the R.H.S. of equation (17)--under incomplete indexation (i.e., $\theta \neq 0$) would be different from zero, even when there is no demand for real monetary balances (i.e., $k=0$). This is so because when $k=0$ government in period 1 can still collect revenue from inflation on account of the share of total debt which is not indexed to the price level--although, as shown below, on average government 1 will not be induced to employ this source of revenue. To see this, notice that, by (18), when $k=0$ we have:

$$(19) \qquad E\,x = \bar{g} + b.$$

which implies, recalling that the international rate of interest was assumed to be zero, that on average conventional taxes, not the inflation tax, will bear the brunt of financing government expenditure and debt amortization.

By (2), (17), and (18), the expected loss of government 0 is given by:

$$(20) \quad \ell(\sigma) = \left[\frac{\sigma^2}{1+A(\theta b+k)^2} + \frac{1+A(\theta b+k)^2}{[1+Ak(\theta b+k)]^2}(\bar{g}+b)^2\right]\frac{A}{2}$$

Comparing (20) with (11), we notice that

$$(21) \qquad \ell(\sigma) - \ell(0) = \ell(\sigma,p) - \ell(0,p) \equiv \mu(\sigma)$$

which means that the cost differential between total and partial precommitment is entirely due to the differential that would occur under perfect certainty (i.e., $\sigma=0$). Clearly, the minimum of $\ell(0)$ is attained at $\theta=0$, and, furthermore, $\ell(0,p)=\ell(0)$ at $\theta=0$, i.e., with full indexation. Therefore, recalling (21), the extra social loss associated with the smaller degree of precommitment (i.e., $\ell(\sigma)-\ell(\sigma,p)$) is captured by the increased value of $\ell(0)$ when $\theta \neq 0$, and it is, thus, no longer true in general that ℓ is minimized by setting θ unboundedly large in absolute value.

Obviously, by equation (21),

$$(22) \qquad \ell(\sigma) = \mu(\sigma) + \ell(0)$$

To develop the intuition behind the optimal choice of θ, consider first the effect of θ on $\ell(0)$--i.e., the loss if $\sigma=0$. Without uncertainty, social loss is minimized with full indexation, which ensures that inflation is set to its optimal full-precommitment value. When $\theta \neq 0$, the presence of nominal debt (credit) affects the temptation of government 1 to inflate (deflate). Since, with no uncertainty, inflation collects revenue on cash balances only, the presence of non-indexed debt (combined with partial precommitment) serves no useful purpose and, hence, results in a larger $\ell(0)$. It is interesting to note that if the demand for real cash balances is zero, i.e., k=0, then $\ell(0)$ goes to infinity as θ grows without bound (in absolute value), but converges to a finite value if k>0. The reason for this is that when k=0 then, obviously, inflation does not collect any revenue. Thus, for instance, if θ goes to plus infinity the temptation of government 1 to inflate away the swelling nominal debt is not offset by any other indirect cost. Hence, recalling that inflation costs are quadratic, optimal inflation from the perspective of period 1 goes to plus infinity, which explains the unbounded growth of $\ell(0)$. On the other hand, if k>0, larger inflation implies larger revenues. As π grows without bound, those revenues will eventually be so large that will have to be disposed of through negative taxes, i.e., through subsidies, which, by (2), are socially costly. In the limit as π goes to infinity the associated subsidies would also go to infinity, which, recalling the loss function (2), would provoke an unbounded growth in social costs. However, since government 1 internalizes the relationship between unbounded π and infinite social costs, this puts some additional restraint on its temptation to inflate away nominal debt as θ goes to infinity. This helps to explain why the social loss as perceived by government 0, $\ell(0)$, does not go to infinity as θ grows without bound (in absolute value). 1/

By (21), the uncertainty-related part of social loss, μ, is the same as in the full-precommitment case, which shows that basically the same arguments that were discussed in connection with that case apply here. It is worth recalling that this term vanishes as θ becomes infinitely large in absolute value. In view of the above discussion about $\ell(0)$, it clearly follows that optimal θ could now turn out to be finite. For example, we show in the Appendix that if k=0, then in the realistic case in which $\sigma<\bar{g}+b$ the optimal solution calls for perfect indexation, i.e., $\theta=0$. This is a dramatic illustration of the importance of precommitment. In this example, lack of precommitment implies complete renunciation from the use of the inflation tax for (conventional) tax-smoothing purposes.

1/ A similar argument can be made for the case in which θ goes to minus infinity, because the temptation to generate an infinitely large <u>de</u>flation can be shown to be pared down by the unboundedly large costs of the associated subsidies.

We show in the Appendix that when $k>0$ then optimal $\theta>0$. 1/ The comparative statics of this case are relatively straightforward. Thus, we show in the Appendix that optimal debt indexation to the price level, i.e., $1-\theta$, decreases with the variance of g, σ^2, and increases with the expected value of g, $\bar{g}$, and the level of debt, b. Notice, incidentally, that the negative association between the variance of real shocks and indexation is in line with the findings of Gray (1976) and related literature (e.g., Aizenman and Frenkel (1985)). In contrast with that literature, however, in the present model the real shock is intertwined with monetary decisions, so that the reduced form contains elements that could also be identified with monetary shocks. 2/

The intuition behind the above-mentioned comparative statics results is as follows. An increase in σ affects only $\mu(\sigma)$ in equation (22) and, ceteris paribus, increases the variability of government revenues. The latter enhances the attractiveness of widening the base of the inflation tax through less debt indexation (in order to smooth out the increased variability). A higher expected g, on the other hand, only affects the term $\ell(0)$ in equation (22) and, ceteris paribus, brings about higher expected taxes and inflation. The latter raises the marginal costs of nominal debt and partial precommitment--which explains why it is optimal to increase the indexation parameter $1-\theta$. Finally, by equation (20), a higher debt level increases expected inflation, taxes, and the (total) base of the inflation tax, i.e., $\theta b+k$. The first two effects lead to higher indexation for the same reasons discussed in connection with a larger expected government expenditure. The implications of a bigger inflation tax base, on the other hand, are harder to explain, but, obviously, under those conditions the marginal cost of reducing θ--which lowers the inflation tax base--ought to be smaller, reinforcing the previous effects.

III. A Three-Period Model

We turn now to examine an extension of the above example to a three-period world in order to be able to examine the optimal maturity of government debt. As before, the government of period 0 passes on to future governments b units of debt (in real terms). However, it now has

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1/ With perfect certainty, optimal $\theta = 0$ because we have assumed that the demand for money is interest inelastic. It can be shown, however, that if the demand for money is interest elastic, then optimal $\theta<0$. An optimal $\theta<0$ is consistent with Persson, Persson and Sevensson's (1987) notion that governments should leave net nominal claims on the private sector to their successors.

2/ It would be interesting to extend this example in order to account for shocks on k. However, the exercise is not trivial because the problem appears to be inherently nonlinear.

to decide the quantities that will be allocated to each future government, i.e., governments 1 and 2. We denote those quantities by b_{01} and b_{02}, respectively, and, of course, constraint them to satisfy:

$$(23) \qquad b = b_{01} + b_{02}$$

For the sake of brevity, we assume that the interest rate on the non-indexed portion of these debt instruments is not state-contingent, and we denote them by i_{01} and i_{02}, respectively.

Let us begin by examining the case of full precommitment, that is to say, the case in which government 0 can predetermine all the policies of governments 1 and 2. To simplify, we further assume that the international interest rate, i^*, is constant over time, and that both i^* and the demand for real cash balances, k, are equal to zero. 1/ Hence, linearizing (like in equation (7)) the budget constraint faced by government 0, we get:

$$(24) \qquad x_1 + x_2 = g + b + \theta b_{01}(i_{01} - \pi_1) + \theta b_{02}(i_{02} - \pi_1 - \pi_2)$$

where g is the sum of government expenditure in periods 1 and 2, and the subscripts on x and π indicate their timing.

Objective function (2) is now written as follows:

$$(25) \qquad \ell = E\ [Ax_1{}^2 + \pi_1{}^2 + Ax_2{}^2 + \pi_2{}^2]/2$$

We assume that the state of nature is fully specified by g, and that the latter is known in period 1. We will allow government 0, however, to design its inflation policy as a function of the state of nature (this is similar to the second exercise developed in Section 2).

The first optimization problem that will be examined is the minimization of ℓ in equation (25) with respect to π and x for each period and state of nature, and with respect to i_{0j} for j=1, 2, given θ and b_{0j} for j=1, 2, and subject to budget constraint (24) and the no-arbitrage conditions:

$$(26a) \qquad E\ (i_{01} - \pi_1) = 0$$

$$(26b) \qquad E\ (i_{02} - \pi_1 - \pi_2) = 0$$

Conditions (26) are linearizations like in equation (16), and take into account the assumption of a zero international interest rate, i.e., $i^*=0$.

We show in the Appendix that at optimum:

1/ The appendix contains the formulas for k>0. In the text, however, the reader will be alerted whenever substantive results change if k>0.

(27) $$x_1 = x_2 = x(g) = \frac{g - \bar{g}}{2 + A(\theta b)^2 + A(\theta b_{02})^2} + \frac{\bar{g} + b}{2}$$

(28) $$\pi_1(g) = A\theta b(x - E\,x)$$

(29) $$\pi_2(g) = A\theta b_{02}(x - E\,x)$$

Equation (27) states that it is optimal to smooth out taxes completely over time, a standard result under full precommitment when the real rate of interest is equal to the rate of discount, as in the present case. Furthermore, by equations (27)-(29), there exists a positive association between government spending and taxes, and a relationship between government spending and inflation whose sign depends, as in the previous section, on the signs of the respective inflation tax bases (i.e., recalling equation (24), θb for period-1 inflation and θb_{02} for period-2 inflation). 1/

By equations (25), (27)-(29), expected loss of government 0 satisfies:

(30) $$\ell(\sigma,p) = \{\sigma^2/[2+A(\theta b)^2+A(\theta b_{02})^2] + (\bar{g} + b)^2/2\}A/2$$

where the definition for precommitment loss $\ell(\sigma,p)$ in equation (30) substitutes for that in equation (11). As in the precommitment case of Section 2, the optimal indexation policy would be to set θ unboundedly large in absolute value, and the reasons are the same. The new character in this play, however, is the level of debt issued in period 0 and that matures in period 2, i.e., b_{02}; or, recalling (23), the maturity structure of the debt issued by government 0. First we note that if there is no uncertainty, i.e., σ=0, then the maturity structure has no effect. This "irrelevancy" result is quite familiar in the presence of complete markets and full precommitment. With uncertainty, however, markets become incomplete because we do not allow governments to make state-contingent interest contracts. Therefore, it is to be expected that debt maturity begins to matter, and that is precisely what is implied by equation (30). Remarkably, the role of b_{02} is very similar to that of θ. In fact, the optimal policy consists of setting non-indexed b_{02} unboundedly large (in absolute value) which, in view of (23), implies that it is optimal to swap an infinite amount of non-indexed short against non-indexed long-term maturity debt, or vice versa. The principles behind this result are essentially the same that we discussed in connection with optimal indexation under precommitment. Examination of budget constraint (24) shows that, for example, an increase in b_{02} at the expense of b_{01} increases the inflation tax base in period 2, which permits to obtain the same path of x with smaller fluctuations in π_2 or, alternatively, to

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1/ Expected inflation in both periods is zero. This follows from the assumption that the conventional base of the inflation tax, k, is assumed to be zero, but it is also a consequence of our linearizations.

reduce the fluctuations of x with the same path of π_2. Given the discussion in Section 2, it should by now be obvious that an unboundedly large b_{02} can attain perfect smoothing of x with (in the limit) no inflation cost, thus mimicking the complete-markets solution.

An interesting theme that emerges from the above arguments, and that appears to be relatively novel in the public policy literature, is that when markets are incomplete, a tax that collects nothing on average could be socially desirable, and, perhaps more surprising, that "artificially" increasing the base of the tax through, for instance, offsetting subsidies could help achieving the complete-markets optimal fiscal policy solution. In our problem, the artificiality of the optimal inflation tax base is glaringly apparent when b=0. Initially there is no debt of any kind and so, in principle, there is no inflation tax base either (recall that k=0); however, the government in period 0 will find it optimal to increase non-indexed long-term debt (i.e., increase the period-2 inflation tax base), say, by making short-term loans to the private sector--and the more, the better!

As in the two-period example, however, the above type of optimal solution raises serious questions about policy credibility. A key element of the solution is the blowing up of some component of the inflation tax base. The objective is, of course, to be able to smooth out taxes with a minimum of inflation cost; however, government 1 may be tempted to inflate more (less) than it was optimal from the perspective of government 0 if the inflation tax base is positive (negative). It is, therefore, important to study solutions that take into account the temptation of governments 1 and/or 2 to depart from the original plan. Our next exercise moves in that direction by relaxing the assumption that the policies of government 1 can be precommitted by government 0, but still assuming that government 1 can fully precommit the policies of government 2.

The optimal problem of government 1 under the above precommitment assumptions is essentially equivalent to the one we discussed in Section 2 when government 0 can precommit the policies of government 1. The details, however, are somewhat different because government 1 can choose both (π_1, x_1) and (π_2, x_2) taking as predetermined the interest rates and indexation coefficient on the government-0 debt. When government 0 takes into account the optimal response of government 1 then, as shown in the Appendix, we have that $x_1 = x_2 = x$ for all g, and

$$(31) \qquad x(g) = \frac{g - \bar{g}}{2 + A(\theta b)^2 + A(\theta b_{02})^2} + \frac{\bar{g} + b}{2}$$

$$(32) \qquad \pi_1(g) = A\theta b x$$

$$(33) \qquad \pi_2(g) = A\theta b_{02} x$$

Comparing (27) with (31) we note that the optimal tax formula is the same as with full precommitment. 1/ Moreover, comparing (32)-(33) with (28)-(29), we note that with full precommitment, expected inflation is zero in both periods, while in the present case expected inflation depends on expected taxes and the inflation tax base for each of the two periods. The intuition for this should be clear from the above discussion, since the temptation to inflate or deflate is obviously related to the corresponding inflation-tax base.

Consequently, expected loss at time 0, taking equations (25) and (31)-(33) into account is:

$$(34)\quad \ell(\sigma) = \left[\frac{\sigma^2}{2+A(\theta b)^2 + A(\theta b_{02})^2} + \frac{2+A(\theta b)^2 + A(\theta b_{02})^2}{4} (\bar{g}+b)^2 \right] \frac{A}{2}$$

where the definition for $\ell(\sigma)$ replaces that of equation (20).

Clearly, if there is no uncertainty, then it is optimal to fully index debt to the price level, i.e., set θ=0, like in the two-period example. Failing that, a second best is to set b_{02}=0, meaning that all debt should exhibit short-term maturity, i.e., $b=b_{01}$. 2/ This is an illustration of the possible optimality of short-term maturity debt when governments are tempted to generate "excessive" inflation--a theme that will recur in the more general cases examined in the ensuing simulations. 3/

When there is uncertainty (i.e., $\sigma>0$), then optimal θ could be larger than zero and, thus, maturity begins to matter. Notice, by (32), that if $b_{02}=b$ then inflation will be constant over time, which is a valuable feature for our planners. Surprisingly, the optimal solution actually requires setting $\pi_1=\pi_2$, even when k>0 (see the Appendix). The intuition for this result is that long-term debt is part of the inflation-tax base for π_1 and π_2, while the real value of short-term debt can only be affected by π_1. Thus, when all debt is long-term, government 1 will make equal use of both instruments, which is an efficient way to collect the inflation tax. This may still not be optimal, however, because the base could be too large or too small. But the size of the inflation tax can be directly controlled by the indexation parameter, $1-\theta$. This is the reason why a first-best type property (like $\pi_1=\pi_2$) holds in a second-best world.

1/ As shown in the appendix, this does not hold with k>0.

2/ As shown in the appendix, if k>0 then optimal $b_{02}>0$.

3/ Spaventa(1987), for example, suggests that the temptation to inflate away the debt could be behind the decision by some governments, like Italy, to load up the short-term end of the maturity spectrum.

The picture changes drastically if governments are constrained to have a degree of debt indexation different from the optimal. As the ensuing simulations will illustrate, when no indexation is possible (i.e., θ=1), then optimal debt maturity shortens by a considerable amount. This can also be explained in an intuitive manner. When no indexation is possible and the unconstrained optimal solution calls for some debt indexation, then the inflation tax bases are large relative to their unconstrained optima. Under the present conditions, the monetary authority cannot lower the inflation tax base for period 1. However, by shortening the debt maturity structure, it can still lower the inflation-tax base for period 2--which is, quite naturally, what the second-best solution requires.

Table 1 shows some simulations to develop some feeling about the possible empirical relevance of these issues. In our experiments we assume that the average (conventional) tax revenue is 40 percent of GNP, 1/ and that the standard deviation of shocks to government expenditure (i.e., σ) is 3 percent of GNP. Finally, we chose A=0.5 in order to make the inflation rates associated with the case of (1) no debt indexation; (2) 100 percent debt-to-GNP ratio; and (3) k=15 percent of GNP quantitatively similar to the annual inflation rates for Italy in the 1980s. Columns denoted by ℓ is the extra cost in relation to the first best, and it is given in terms of the extra government expenditure that would have to be incurred under first-best conditions, as a percentage of GNP, to generate the same rise in social cost. The inflation columns correspond to expected inflation, and are expressed as percentages per period. The numbers for the b and b_{02} columns are percentages of GNP and b, respectively. Numbers in parenthesis correspond to k=8 percent of GNP, while the others are generated under the assumption that k=15 percent of GNP.

The salient features of Table 1 are that there is a substantial cost associated with government 0 losing its ability to precommit the rates of inflation for periods 1 and 2; the cost is about 1.1 percent of GNP and does not vary with the initial debt level. 2/ 3/ Furthermore, optimal indexation is larger than 95 percent in all cases, despite having chosen a relatively high standard deviation for g (3 percent of GNP).

1/ This is equivalent to setting g+b=0.8. Revenues constancy is assumed in order to isolate the total equilibrium revenue requirements from changes in the level of debt, b, which is the focal point of our analysis.

2/ As can be seen in the appendix, if $\bar{g}$+b is held constant then only the product θb matters at optimum (recall, also, that b_{02}=b at optimum). Thus, in Table 1, changes in b generate changes in θ that leave θb unchanged. This explains the invariance of cost with respect to b.

3/ By way of comparison, notice that the above-mentioned loss would be zero if there were no uncertainty (i.e., σ=0). Hence, these costs are intimately related to the randomness of g.

Table 1. Simulations Under Full and Partial Precommitment

	$b_{02} = b$				θ = 100 percent				θ=100 percent $b_{02} = b$	
b	E π_{fb}	ℓ	θ	E π	ℓ	b_{02}	E π_1	E π_2	ℓ	E π
25	3 (1.6)	1.1 (1.1)	3.6 (1.9)	3.1 (1.7)	1.37 (1.38)	5 (2.5)	7.8 (6.5)	3.2 (1.7)	1.6 (1.65)	7.8 (6.5)
50	3 (1.6)	1.1 (1.1)	1.8 (0.96)	3.1 (1.7)	2.1 (2.2)	4 (2)	12.6 (11.4)	3.3 (1.8)	3.1 (3.2)	12.4 (11.3)
100	3 (1.6)	1.1 (1.1)	0.9 (0.48)	3.1 (1.7)	5.1 (5.3)	4.8 (2.5)	21.9 (21.1)	3.8 (2.0)	8.3 (8.9)	21.2 (20.7)

Note: π_{fb} is expected inflation under full precommitment.

Social loss increases if no debt indexation is possible, and even more so if, in addition, all debt is of long-term maturity. The importance of this effect grows significantly with the debt level, and relatively big losses are found when the debt is 50 percent of GNP or larger. As asserted before, Table 1 shows that optimal debt maturity shortens significantly if no indexation is possible. In the examples, optimal short-term debt is never less than 95 percent of initial debt. Finally, when indexation is not possible, inflation rates are not equalized and could be substantially different, depending on the initial debt.

IV. Interpretations and Extensions

The previous analysis has given us yet another example of the importance of precommitment for macroeconomic policy. We showed that with full precommitment, debt indexation and maturity structure are close substitutes, because a large nominal debt (in absolute value)--achieved either by swapping short against long-term maturity bonds, or by indexation, e.g., by swapping nominal against real debt--is useful to smooth out fluctuations in conventional taxes. However, with partial precommitment, a careful choice of indexation and maturity are both necessary for an optimum. Interestingly, the optimal policy is quite sensitive to the instruments that can be controlled. Thus, if one can freely choose indexation and maturity, then it is optimal to concentrate all debt on long-term maturity bonds; but, on the other hand, if it is not possible to index, then our simulations suggest that optimal maturity will shorten by a considerable amount.

The last model that was examined in Section 3 assumes that government 0 cannot precommit the actions of future governments. Government 1, instead, is assumed to be able to precommit the policies of government 2. This "partial precommitment" assumption can be easily relaxed if government 1 is free to choose the indexation level on its own debt. By assumption, uncertainty is fully resolved in period 1, and, hence, government 1 operates under perfect certainty. Thus, it is easy to see that if the nominal debt inherited by government 2 equals b_{02}, and its total debt (including the fully indexed part) is equal to the optimal level corresponding to the case in which government 1 can precommit the actions of government 2 (the partial-precommitment case), then government 2 will actually replicate the policy that government 1 would like it to follow. In other words, the model's implications apply equally to the case in which no government can tie the hands of any future administration. 1/

If optimal indexation is not possible, then the inability to precommit has a substantial effect on the nature of optimal policy. Consider, for example, the case in which debt indexation to the price level is not possible (i.e., θ=1). By previous discussion, if government 1 can precommit the actions of government 2, then most of the debt (in fact, 100 percent of the debt if k=0) should optimally be placed in the form of short-term bonds. However, when no precommitment is possible, the latter-type of policy tends to generate "too much" tax revenue in period 1. This is so, because, in contrast to the partial-precommitment case, when government 1 considers transferring part of its inherited debt to government 2, it realizes that it simultaneously increases government-2 debt-liquidation incentives. Thus, if following the prescriptions of the partial-precommitment model of Section 3, government 0 puts most of its debt in the form of short-term maturity instruments, government 1 will tend to pay a share of total debt which is higher than that of the partial-precommitment optimum (i.e., the optimal solution when government 1 can precommit the policies of government 2). Consequently, government 0 will find it optimal to issue more long-run debt than in the partial-precommitment case.

The analytics behind the above intuitions are presented in the Appendix. Table 2 shows some simulations corresponding to the case in which government 1 cannot precommit the actions of government 2, there is

1/ This equivalency, however, would break down if uncertainty is not fully resolved in period 1.

Table 2. Simulations Under Partial and No Precommitment

	Partial precommitment				No precommitment.				Partial precommitment $b_{02} = b$		No precommitment $b_{02} = b$		
b	ℓ	b_{02}	π_1	π_2	ℓ	b_{02}	π_1	π_2	ℓ	π	ℓ	π_1	π_2
25	0.3 (0.3)	1.5 (0.6)	7.8 (6.5)	3.0 (1.6)	0.4 (0.4)	43.6 (39.2)	7 8 (6.5)	5.3 (4.0)	0.6 (0.6)	7.8 (6.5)	0.4 (0.4)	7.8 (6.5)	5.6 (4.3)
50	1.15 (1.2)	2.4 (1.2)	12.6 (11.4)	3.1 (1.7)	1.41 (1.5)	50.6 (47.2)	12.5 (11.4)	7.6 (6.4)	2.2 (2.3)	12.4 (11.4)	1.5 (1.6)	12.4 (11.4)	8.3 (7.1)
100	4 3 (4.5)	4.1 (2.1)	21.9 (21.1)	3.6 (2.0)	5.1 (5.5)	60.4 (57.3)	21.3 (20.7)	12.5 (11.4)	7.7 (8.3)	21.2 (20.7)	5.4 (5.6)	20.7 (20 4)	14.1 (13.1)
												$b_{02} = 0$	
											5.8 (6.1)	23.5 (22.8)	10.3 (9 5)

no debt indexation (i.e., θ=1), and perfect certainty prevails. 1/ For the sake of comparison, all other parameters are the same as in Table 1.

Confirming the above discussion, the table shows that under no precommitment the maturity structure tends to become more uniform--in fact, numbers range from 40 to 60 percent--and long-term debt increases with the size of the initial debt. Finally, maturity is important (in terms of social cost) in the case of partial precommitment (i.e., when government 1 can precommit the policies of government 2), but its relevance diminishes considerably when there is no precommitment.

These results suggest that there may be some wisdom behind the observed trend towards short-term debt that accompanied the recent accumulation of domestic government debt. Although we do not provide a rationale for the existing high proportion of nominal debt (except for the somewhat unrealistic case in which the monetary authorities can precommit the future rates of inflation), our simulations suggest that with low levels of debt indexation it may be optimal and important to shorten the maturity structure if some precommitment is feasible. On the other hand, this policy would not be optimal if governments could not precommit future inflation. In that case, however, maturity-structure mistakes do not appear to be very costly. Hence, as a rule of thumb, we reach the tentative conclusion that when government debt is not indexed to the price level short maturities look like a reasonably fair bet: they could make a sizable contribution to social welfare and, at worst, their costs appear to be low.

V. Final Remarks

1. In a world of complete markets where policy makers can make credible announcements, price indexation can easily be substituted by other policies, and the maturity structure of government debt is totally irrelevant. Departures from that Alice-in-Wonderland case, however, could turn these instruments into extremely useful tools for policy making. Our analysis suggests that if indexation is credible--in the sense that it is not expected to be nullified by interest taxation or sheer debt repudiation--then there exists a variety of realistic cases in which it is optimal to index a high proportion of the debt to the price level. Moreover, the effect of optimal indexation is further enhanced by choosing long debt maturities. Our simulations show that the contribution of long maturities to welfare could be quite important when the debt-to-GNP ratio exceeds 50 percent. But, on the other hand, if indexation was not possible and there was a relatively low degree of credibility of policy

1/ Uncertainty does not seem to be essential for the following discussion. It should be noted, however, that even under our previous linearizations, the optimum problem of government 1 becomes highly nonlinear, which substantially complicates the analysis of the stochastic case.

announcements then, interestingly, it was hard for us to find realistic examples in which the marginal cost of selecting a "wrong" maturity structure exceeded 0.3 percent of GNP.

2. Our discussion revealed that the relevant base for the inflation tax includes all the nominal government liabilities, not just the noninterest-bearing part. The reason is rather obvious: the price level affects the real value of the whole set of nominal assets. Some economists, however, would prefer to subtract the interest-bearing part because in equilibrium the interest rate on those assets may tend to include the expected rate of inflation, point for point, and, consequently, in equilibrium no inflation revenue will be collected on that account. Although we do not disagree, in principle, with this methodology for calculating the actual proceeds from the inflation tax, our analysis shows that it could be very misleading to abstract from the interest-bearing part of government nominal liabilities if one is trying to come up with a "positive" theory of inflation: they are as much a temptation to inflate as high-powered money. In fact, we showed examples where inflation is positive even though at equilibrium it collects no revenue whatsoever--i.e., there is no demand for high-powered money--and it is first-best optimal to generate zero inflation. This inflationary potential of government bonds is precisely one of the fundamental reasons why debt maturity may matter. By changing the maturity structure of nominal government debt, the policy maker changes the time profile of the inflation base, and is thus able to affect the incentive-compatible inflation path.

3. The paper made enough assumptions to ensure uniqueness of equilibrium solutions. However, uniqueness is not a very robust property of these types of models. In fact, multiplicity of equilibria is relatively easy to generate when there exists a positive stock of non-indexed bonds (see Calvo (1988a,b)). The intuition is that the nominal interest rate reflects inflationary expectations. Thus, if the public expects "high" inflation then, ex post, the government will be tempted to validate those expectations; for, otherwise, the real interest rate will be "high," calling for "high" distorting conventional taxes. But, on the other hand, if inflationary expectations are "low" then, for similar but opposite reasons, the government could be led to generate "low" inflation. Consequently, the monetary authority may end up being the passive reflector of average opinion.

A way to try to avoid the non-uniqueness problem is to index the entire stock of bonds to the price level, but this could be costly in an uncertain environment. Hence, maturities are once again likely to be of some help. We conjecture that, when non-uniqueness is a problem, the optimal maturity structure will tend to be shorter than otherwise. This appears to be a promising area for future research. 1/

1/ Some progress on this front has been made by Giavazzi and Pagano (1988).

4. We need to develop a better intuition about inflation costs, in particular, and debt-repudiation costs, in general. They are central to the analysis and essential for policy implications. In our model, for example, if there is no uncertainty then the optimal solution is to index the government debt fully to the price level. This is the best one can do because full indexation removes the incentive to "inflate away" the real value of government bonds--a completely wasteful activity in equilibrium. However, in more realistic economies, inflation is not the only policy that can be employed to lower the real value of government debt. Wealth or interest taxes, for example, could yield the same effects. Consequently, in a more general framework complete neutralization of the inflation tax on bonds may give rise to other forms of debt repudiation, which may turn out to be more costly. Until the microeconomics of these costs is better understood, therefore, we should be very cautious about the relevance of our policy conclusions. 1/

1/ Rogers (1986) is a forerunner in this line of research.

Technical Notes

I. Two-Period Model, Full Precommitment

Government 0 chooses functions $\pi(g)$, $x(g)$, and a constant i to minimize ℓ, given by equation (2), subject to budget constraint (7), and equilibrium condition (16), for given θ. The first order conditions of this minimization problem imply that:

$$\text{(I.1)} \quad - Ax(\theta b+k) + \lambda + \pi = 0$$

$$\text{(I.2)} \quad A\theta bEx - \lambda = 0$$

where λ is the Lagrange multiplier associated with equilibrium condition (16). The fact that costs are quadratic and the constraints are linear ensures that ℓ attains a unique (global) minimum. Using equations (7), (16), and (I.2), equation (I.1) can be written as:

$$\text{(I.3)} \quad - A(\theta b+k)[g+b(1+i^*)+\theta b(E\pi-\pi)-k\pi]$$

$$+ \pi + A\theta b[\bar{g}+b(1+i^*)-kE\pi] = 0$$

We conjecture that optimal $\pi(g)$ is a linear function:

$$\text{(I.4)} \quad \pi(g) \equiv B + Cg$$

where B and C are unknown positive constants. To verify the conjecture we find unique values of coefficients B and C for which equation (I.3), combined with (I.4), holds as an identity. This yields the following values of B and C:

$$\text{(I.5a)} \quad B = \frac{Akb(1+i^*)}{1+Ak^2} - \frac{A\theta b[1-kA(\theta b+k)]}{(1+Ak^2)[1+A(\theta b+k)^2]} \bar{g}$$

$$\text{(I.5b)} \quad C = A(\theta b+k)/[1+A(\theta b+k)^2]$$

Using (I.5a) and (i.5b), and assuming that $i^*=0$, optimal $\pi(g)$ is given by equation (8). Similarly, using (I.1), (I.2) and (7), it follows that also optimal $x(g)$ is given by equation (9). Moreover, taking into account the fact that $E X^2 = E [X - EX]^2 + (EX)^2$ for $X=x,\pi$, and equations (8) and (9), it is easy to check that expected loss at time 0 is given by equation (11).

The optimal value of the non-indexation parameter θ is that which minimizes ℓ in equation (11). As discussed in the text, ℓ can be set arbitrarily close to its infimum with respect to θ by setting θ sufficiently large in absolute value.

II. Two-Period Model, No Precommitment

Government 0 chooses optimal θ to minimize ℓ in equation (20). As discussed in the text, equation (20) is obtained by taking into account the optimal response of government 1. Consider first the case in which k=0. The minimization problem faced by government 0 is equivalent to choosing z to minimize the value of the following function h(z):

$$\text{(II.1)}\quad h(z) = \alpha/z + \beta z$$

where α and β are positive constants and $z \epsilon [1,\infty)$. Let us study the same problem with the exception that $z \epsilon (0,\infty)$. From equation (II.1) we have

$$\text{(II.2)}\quad h'(z) = -\alpha/z^2 + \beta$$

$$\text{(II.3)}\quad h''(z) = 2\alpha/z^3 > 0$$

for z>0. Hence, h(·) is strictly convex and its global minimum is attained where

$$\text{(II.4)}\quad h'(z) = 0, \quad \text{i.e., at } z = (\alpha/\beta)^{\frac{1}{2}}$$

Hence, at minimum,

$$\text{(II.5)}\quad h(z) = 2(\alpha/\beta)^{\frac{1}{2}}.$$

If z is constrained such that $z \geq 1$ then, due to the convexity of h and equation (II.4), we have

$$\text{(II.6)}\quad \underset{z\geq 1}{\operatorname{argmin}}\, h(z) = 1 \quad \underline{\text{iff}} \quad (\alpha/\beta)^{\frac{1}{2}} \leq 1$$

In terms of loss function (20), $\alpha \equiv \sigma^2$, $\beta \equiv (\bar{g}+b)^2$, and $z \equiv 1+A(\theta b+k) = 1+A\theta b$ since k=0. Therefore, equation (II.6) implies that, in the case in which k=0, optimal θ=0 if and only if $\sigma \leq \bar{g}+b$, as asserted in the text.

Consider the case in which k>0. Minimizing the value of loss function (20) with respect to θ is equivalent to choosing z to minimize the value of f(z), defined by:

$$\text{(II.7)}\quad f(z) = -\frac{\alpha}{\beta} + \beta\frac{z}{[1+k(z-1)^{\frac{1}{2}}A]^2}$$

where $z \geq 1$ (note that $f(z) \equiv h(z)$ when k=0.)

It can be checked that

$$\text{(II.8)}\quad f(1) < f(\infty) \quad \text{iff} \quad \alpha + \beta(1 - 1/k^2A) < 0$$

(II.9) $f'(z) = -\alpha/z^2 + \beta(w-kA)/w(1+kw)^2$

where $w = [A(z-1)]^{1/2}$. Hence,

(II.10) $\lim_{z \downarrow 1} f'(z) = -\infty.$

Thus, if equation (II.8) holds, minimum exists and optimal $z > 1$.

When minimum occurs at $z<\infty$ we have $f'(z)=0$ which, by equation (II.9), requires

(II.11) $w > kA.$

Since, in terms of loss function (20), $z \equiv 1 + A(\theta b+k)$ and $w = A(\theta b+k)$, equation (II.11) implies that optimal θ satisfies

(II.12) $\theta b > 0.$

Thus, if $b>0$ then optimal $\theta>0$.

The comparative statics with respect to σ and $\bar{g}$ are easily computed from the first order condition $f'(z)=0$. Using the definitions of α and β, condition (II.11), and the fact that at a minimum $f''(z)>0$, it can be checked that optimal θ increases with σ and decreases with $\bar{g}$.

III. Three-Period Model, Full Precommitment

Government 0 chooses functions $\pi_1(g)$, $\pi_2(g)$, $x_1(g)$, $x_2(g)$, and constants i_{01} and i_{02} to minimize ℓ in equation (25), subject to the budget constraint

(III.1) $x_1 + x_2 = g + b + \theta b_{01}(i_{01}-\pi_1)$

$\qquad + \theta b_{02}(i_{02}-\pi_1-\pi_2) - k(\pi_1+\pi_2)$

and equilibrium conditions (26a) and (26b), for given θ. The first order conditions for this minimization problem include:

(III.2) $Ax_1 - \mu = 0$

(III.3) $Ax_2 - \mu = 0$

(III.4) $\pi_1 - \mu(\theta b+k) - \lambda_1 - \lambda_2 = 0$

(III.5) $\pi_2 - \mu(\theta b_{02}+k) - \lambda_2 = 0$

(III.6) $E\,\mu\theta(b-b_{02}) + \lambda_1 = 0$

(III.7) $E\ \mu\theta b_{02} + \lambda_2 = 0$

where $\mu(g)$ is the multiplier associated with constraint (III.1) and λ_1 and λ_2 are the multipliers associated with equilibrium conditions (26a) and (26b). Existence of a global minimum is ensured by the fact that costs are quadratic and the constraints are linear. The above first order conditions imply

(III.8) $x_1 = x_2 = x$

(III.9) $\pi_1 = A\theta b(x - Ex) + Akx$

(III.10) $\pi_2 = A\theta b_{02}(x - Ex) + Akx$

From equations (III.1) and (III.8)-(III.10) we obtain:

(III.11) $E\ x = (\bar{g}+b)/2(1+Ak^2)$

(III.12) $E\ \pi_1 = E\ \pi_2 = Ak(E\ x)$.

Using (III.1) and (III.8)-(III.12) one can verify that optimal $x(g)$, $\pi_1(g)$, and $\pi_2(g)$ are linear functions that satisfy

(III.13) $$x(g) = \frac{g - \bar{g}}{2+A(\theta b+k)^2 + A(\theta b_{02}+k)^2} + E\ x$$

(III.14) $$\pi_1(g) = \frac{A(\theta b+k)(g - \bar{g})}{2+A(\theta b+k)^2 + A(\theta b_{02}+k)^2} + E\ \pi_1$$

(III.15) $$\pi_2(g) = \frac{A(\theta b_{02}+k)(g - \bar{g})}{2+A(\theta b+k)^2 + A(\theta b_{02}+k)^2} + E\ \pi_2$$

Using equations (III.13)-(III.15), one can check that expected loss at time 0 satisfies:

(III.16) $$\ell(\sigma,p) = \left[\frac{\sigma^2}{2+A(\theta b+k)^2+ A(\theta b_{02}+k)^2} + \frac{\bar{g}+b}{2(1+Ak^2)} \right]\frac{A}{2}$$

Hence, as discussed in the text, social loss can be set arbitrarily close to its infimum with respect to θ and b_{02} by making θ or b_{02} arbitrarily large in absolute value.

IV. Three-Period Model, Partial Precommitment

After the realization of g, government 1 chooses x_1, π_1, x_2, and π_2 to minimize ℓ in equation (25) subject to budget constraint (III.1), taking as predetermined θ, b_{01}, b_{02}, i_{01}, and i_{02}. After taking into account equilibrium conditions (26a) and (26b), the formulas describing the reaction function of government 1, faced by government 0, when k>0 are:

$$\text{(IV.1)}\quad x(g) = \frac{g - \bar{g}}{2+A(\theta b+k)^2+A(\theta b_{02}+k)^2} + \frac{\bar{g} + b}{2+Ak[\theta(b+b_{02})+2k]}$$

$$\text{(IV.2)}\quad \pi_1(g) = A(\theta b+k)x$$

$$\text{(IV.3)}\quad \pi_2(g) = A(\theta b_{02}+k)x$$

where the fact that, at optimum, $x_1=x_2=x$ has been used. From equations (25) and (IV.1)-(IV.3), expected loss at time 0 satisfies:

$$\text{(IV.4)}\quad \ell(\sigma) = \left[\frac{\sigma^2}{2+A(\theta b+k)^2+ A(\theta b_{02}+k)^2} + \frac{2+A(\theta b+k)^2+ A(\theta b_{02}+k)^2}{\{2+Ak[\theta(b+b_{02})+2k]\}^2}(\bar{g}+b)^2\right]\frac{A}{2}$$

Government 0 chooses θ and b_{02} to minimize ℓ in equation (IV.4). This problem is equivalent to minimizing F(y,n), defined below, with respect to y and n:

$$\text{(IV.5)}\quad F(y,n) = \frac{\alpha}{2 + A(n^2+ y^2)} + \frac{2 + A(n^2+ y^2)}{[2 + Ak(n + y)]^2}\beta$$

where $n,y \in (-\infty,\infty)$ except where $2+A(n+y)=0$. If (y^*,n^*) minimizes F(y,n) then (y^*,n^*) must also solve the following problem:

$$\text{(a)}\qquad \underset{n,y}{\mathrm{Min}}\ F(y,n)$$

(IV.6) subject to

$$\text{(b)}\qquad n + y = n^* + y^*$$

Under equation (IV.6b) we have,

$$z' \equiv 2 + A(n^2+y^2) = 2 + A[n^2 + (n^*+y^*-n)^2]$$

which attains its minimum where

(IV.7) $n = (n^* + y^*)/2$, i.e., at $z' = 1 + (A/2)(n^*+y^*)^2$.

Consequently, we can express problem (IV.6) as follows:

$$\text{(IV.8)} \quad \begin{aligned} &\text{(a)} \quad \min_{z'} \ \alpha/z' + \beta' z' \\ &\text{subject to} \\ &\text{(b)} \quad z' \geq 1 + (A/2)(n^* + y^*)^2 \end{aligned}$$

where

(IV.9) $\beta' \equiv \beta/[2+Ak(n^*+y^*)]^2$.

By equation (II.5), if problem (IV.8) has an interior solution, we have that the minimum value of the function in (IV.8a) is

(IV.10) $2(\alpha/\beta)^{\frac{1}{2}}/[2+Ak(n^*+y^*)]$.

Hence, solutions to the original problem do not lead to interior solutions of problem (IV.8), because otherwise the value of F could be lowered by changing (n^*+y^*). Thus, by (IV.8), at optimum

(IV.11) $z' = 1 + (A/2)(n^* + y^*)^2$

which, by definition of z' and equation (IV.7), requires setting

(IV.12) $n = y$.

Interestingly,

$$\text{(IV.13)} \quad F(n,n) = \frac{1}{2}\left[\frac{\alpha}{1 + An^2} + \frac{1 + An^2}{(1 + Akn)^2}\beta\right].$$

Minimizing the value of F(n,n) with respect to n is equivalent to minimizing the value of f(z) in equation (II.7) with respect to z. In terms of loss function (IV.4), $y = \theta b+k$ and $n = \theta b_{02}+k$. Thus, the above discussion implies, by equation (IV.12), that if $\theta \neq 0$ then $b=b_{02}$, otherwise if $\theta=0$ then the maturity structure does not matter. Moreover, since the choice of n to minimize the value of F(n,n) in equation (IV.13) is equivalent to choosing optimal θ in the two-period model, then optimal θ in the three-period model is the same as in the two-period model. By equation (II.12), when $k>0$ and $b>0$, optimal $\theta>0$. Finally, by equation (IV.2) and (IV.3), $\pi_1=\pi_2$ at optimum.

V. Three-Period Model, No Precommitment, σ=0, θ=1

Government 2 chooses x_2 and π_2 to minimize ℓ_2 given by

(V.1) $\ell_2 = (Ax_2^2 + \pi_2^2)/2$

subject to the budget constraint

(V.2) $x_2 = g/2 + b_{02} + b_{12} + b_{02}(i_{02}-\pi_1-\pi_2) + b_{12}(i_{12}-\pi_2) - k\pi_2$

where b_{12}, and i_{12} denote (nominal) debt issued in period 1 with maturity in period 2, and the interest rate applying to b_{12}, respectively. It can be verified that at optimum

(V.3) $\pi_2 = A(b_{02} + b_{12} + k)x_2$.

With perfect certainty, equilibrium requires (recalling that i^*=0)

(V.4) $i_{12} = \pi_2$.

Government 1 chooses x_1, π_1, and b_{12}, taking into account the optimal response of government 2, which chooses x_2 and π_2 as described above. The minimization problem of government 1 is equivalent to choosing x_1, x_2, π_1, π_2, and b_{12} to minimize social loss

(V.5) $\ell = (Ax_1^2 + \pi_1^2 + Ax_2^2 + \pi_2^2)/2$

subject to equations (V.2), (V.3), (V.4), and the period-1 budget constraint

(V.6) $x_1 = g/2 + b_{01} + b_{01}(i_{01}-\pi_1) - k\pi_1 - b_{12}$.

The solution to this minimization problem, along with equilibrium conditions

(V.7a) $i_{01} = \pi_1$

(V.7b) $i_{02} = \pi_1 + \pi_2$,

characterizes the reaction function of government 1 faced by government 0.

Equation (V.3), along with the following set of equations, characterizes the reaction function of government 1:

(V.8) $x_1 + x_2 = g + b - k(\pi_1+\pi_2)$

(V.9) $$\frac{x_1 - x_2}{\pi_2 - Ax_2(b_{02}+k)} = \frac{2x_2 - g + k(1 + \pi_2)}{1 + A(b_{02}+k)[x_2-g+k(1+\pi_2)]}$$

$$(\text{V}.10) \qquad \pi_1 = Ax_1(b+k) - \frac{Ab_{02}x_2(x_1-x_2)}{2x_2-g+k(1+\pi_2)}$$

Government 0 chooses b_{02} to minimize ℓ in equation (V.5) taking into account the optimal responses of governments 1 and 2. Formally, this is equivalent to choosing x_1, x_2, π_1, π_2, and b_{02} to minimize ℓ in equation (V.5) subject to incentive compatibility constraints (V.3) and (V.8)-(V.10). Simulations presented in Table 2 are computed by solving numerically the minimization problem of government 0.

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