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"Currency Substitution and Financial Innovation"

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Abstract

This paper presents a cash-in-advance framework, with variable income velocity, where the domestic effects, as well as the international transmission, of financial innovation can be analyzed. In particular, the discussion emphasizes the role of currency substitution and of cross-border transfers of seigniorage in determining the general equilibrium effects of financial innovation.

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| <u>Table of Contents</u>   | <u>Page</u> |
|--|-------------|
| Summary  | iii         |
| I. Introduction  | 1           |
| II. The Basic Model  | 3           |
| III. Equilibrium in a Two-Country World                              | 8           |
| IV. Financial Innovation in the Presence of Currency<br>Substitution | 11          |
| 1. Global financial innovation                                       | 12          |
| 2. International transmission of financial<br>innovation             | 13          |
| V. Conclusions   | 16          |
| References   | 18          |

Summary

To analyze the macroeconomic effects of structural changes in the financial system this paper presents a cash-in-advance framework, in which the velocity of income varies. The financial innovation discussed in this paper encompasses changes in the technological environment affecting how individuals carry out financial transactions. The analysis deals with the transmission of financial innovation and the role played by currency substitution.

The analysis leads to several conclusions. First, cash-in-advance constraints influence the effective relative prices of consumption. Therefore, benefits from the transmission of financial innovation depend on how the innovation alters the relative cost of using different currencies and how it affects equilibrium terms of trade.

Second, financial innovation may lead the nominal and the real exchange rate to move in opposite directions. In particular, a country in which the relative cost of using foreign currency declines may face a nominal appreciation and a real depreciation of its currency.

Third, not only changes in relative prices, but also cross-border transfers of seigniorage, which occur because of currency substitution, assist the international transmission of the effects of financial innovation. Under suitable symmetry assumptions, these considerations in an analysis of country-specific financial innovation do not alter qualitatively the price effects that are obtained by an analysis of the effects of global changes.



## I. Introduction

The financial systems of industrial countries have undergone a process of substantial transformation during the last decade. Most of the changes have been the result of two events: technological progress in the area of financial services and substantial changes in the regulatory environment affecting financial markets. The importance of these structural changes, which has coincided with an increased interdependence across major industrialized economies, has not been adequately reflected in general equilibrium macroeconomic models. Few papers, notably Fisher (1983), provide set-ups where these issues could be properly analyzed.

The recent decision adopted within the European Monetary System to accelerate the process of economic integration by setting the objective of achieving a "single market" by 1992, has prompted renewed interest in the macroeconomic effects of major changes in financial regulations coupled with an almost complete liberalization of capital movements. In particular, in a European context, some attention is being focused on the role of currency substitution in the process of financial integration, particularly on the likelihood that it might lead to increased instability of monetary aggregates and have undesirable effects on the ability of governments to conduct economic policy.

This paper provides a framework in which the macroeconomic effects of structural changes in the financial system, and more importantly, their international transmission, as well as the role played by currency substitution, can be analyzed. In particular, the emphasis is placed on structural changes that affect the transactions demand for currencies. Therefore, the analysis is careful in motivating money only as an asset held for transactions purposes, since, as a store of value, money is return-dominated by interest-bearing assets. The model used in this paper, which is based on Feenstra (1986) and Guidotti (1987), attempts to minimize implicit theorizing by developing a cash-in-advance framework where income velocity is made variable by introducing inventory-type considerations a la Baumol (1952) and Tobin (1956). Unlike other cash-in-advance models with a variable income velocity (Svensson (1985a) and (1985b)), the model developed here maintains a variable velocity despite being a perfect-foresight model. Moreover, in this framework, currency substitution results endogenously. 1/

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1/ The term "currency substitution" may be understood in two ways: (1) that foreign money is used along with domestic money in transacting; and (2) that changes in the relative cost of holding one currency induces a change in the ratio of domestic to foreign money holdings demanded (Calvo and Rodriguez (1977)). While the model presented in this paper does not provide a theory of why foreign currency is used, it provides a theory of how the ratio of domestic to foreign demand holdings depends on factors that affect the relative cost of holding the two currencies.

The term "financial innovation" is used in this paper to encompass changes in the technological environment affecting the way individuals carry out their transactions. In particular, financial innovation, a consequence both of technical progress and of changes in financial regulations, affects the time-costs involved in transacting. <sup>1/</sup>

A number of insights emerge from the analysis. First, because of the nature of the cash-in-advance constraints faced by individuals, financial considerations are shown to affect the effective relative prices of consumption. Therefore, the international transmission of the effects of financial innovation, as well as the domestic effects, depends importantly on how financial innovation alters the relative cost of using different currencies and, hence, on how it affects equilibrium terms of trade.

Second, it is shown that financial innovation may lead to a negative co-movement between the nominal and the real exchange rate. In particular, a country in which there is a lowering in the relative cost of using the foreign currency may face a nominal appreciation and a real depreciation of its currency.

Third, in addition to changes in relative prices, cross-border transfers of seigniorage, which occur because of the presence of currency substitution, play a major role in the international transmission of the effects of financial innovation. Under suitable symmetry assumptions, these considerations, when analyzing country-specific financial innovation, do not alter qualitatively the price effects that are obtained by analyzing the effects of global changes in a simplified pooling equilibrium.

Section II presents the basic model and characterizes the consumer's maximum problem under the assumption that individuals are required to use domestic money to purchase domestic goods and foreign money to purchase foreign goods. Unlike other cash-in-advance models (Helpman, 1981), currency substitution is generated in this model, despite the fact that the use of each currency is linked to a specific good, through the interaction between the two monies in affecting the total amount of time devoted to transaction activities. In fact, this framework is shown to generate similar results to other currency substitution optimizing models (Liviatan (1981); Calvo (1980 and 1985); Boyer and Kingston (1987); Végh (1988 and 1989)).

Section III characterizes the world economic equilibrium. Specific functional forms for the utility function and the transactions technology

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<sup>1/</sup> Calvo's (1980) analysis of the effects of "financial opening" in the context of a small open economy is similar to the analysis carried out in this paper. In Calvo's (1980) model, financial opening is assumed to generate changes in a "real liquidity" function which enters the utility function. Unlike this paper, in Calvo's (1980) currency substitution model, money serves both a store of value and as a medium of exchange.

generate a recursive structure which allows a simple analytical solution for the steady state equilibrium. Monetary considerations are shown to be important in the determination of the equilibrium terms of trade.

Section IV analyzes the effects of financial innovation. The effects of both global as well as country specific innovation are discussed. In particular, it is shown that financial innovation may produce a negative co-movement between the nominal and the real exchange rate. The importance of relative price effects, as well as of cross-border seigniorage transfers, for the international transmission of financial innovation is emphasized.

Section V contains some concluding remarks.

## II. The Basic Model

Consider a two-country world. Each economy is inhabited by identical, infinitely lived individuals. There are two goods, two monies and two bonds. The domestic representative individual's preferences are characterized by the following intertemporal utility function:

$$(1) \quad W = \sum_{t=0}^{\infty} \beta^t u(c_t, c_t^*, x_t)$$

where  $c$  and  $c^*$  denote consumption of the domestic and the foreign good, respectively, and  $x$  denotes leisure. The utility function  $u(\cdot)$  is assumed to be strictly concave, twice continuously differentiable, with positive and decreasing marginal utilities.

The representative individual holds four assets, a domestic and a foreign money, and a domestic and a foreign bond which pay a fixed interest rate of  $i$  and  $i^*$  per period, respectively. There is no uncertainty. For expositional convenience, assume that each period is divided into two subperiods. In the first subperiod, consumption takes place while in the second subperiod asset trading takes place. Following Guidotti (1987), we assume that the representative individual is subjected to a cash-in-advance constraint during the consumption subperiod. However, the individual is allowed to finance consumption by withdrawing money from bonds as in Baumol's (1952) or Feenstra's (1986) models. Specifically, we assume that the representative individual holds two accounts, one denominated in foreign and the other in domestic currency, respectively. While domestic currency is obtained from the stock of domestic bonds, foreign currency is obtained from the stock of foreign

bonds. 1/ In the second subperiod, individuals receive an endowment income, pay taxes, receive interest payments on their bond holdings, and engage in asset trade to choose the new composition of their asset portfolio.

Since the flow of consumption to be financed occurs continuously, cash withdrawals are evenly distributed within each period. Define by  $N$  and  $N^*$  the frequency of withdrawals in domestic and foreign currency, respectively. As in Guidotti (1987), instead of monetary transactions costs like in Baumol (1952) and Feenstra (1986), it is assumed that cash withdrawals involve a loss of leisure. Given an endowment  $T$  of time available, it may be used either as leisure or in withdrawing cash:

$$(2) \quad T = x_t + f(N_t, N^*_t); \quad f_N > 0, \quad f_{N^*} > 0$$

where  $f(\cdot)$  is a time-cost function with positive first derivatives. 2/ The fact that the time-costs involved in transacting depend on the frequencies of cash withdrawals,  $N$  and  $N^*$  (or, in the spirit of the Baumol-Tobin approach, the amount of times the consumer goes to the bank), implies that those costs are not proportional to the amount of cash withdrawn. 3/

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1/ This assumption could be relaxed without affecting qualitatively the ensuing analysis of the effects of financial innovation. The fact that, during the consumption subperiod, individuals can only withdraw currency from bonds in this predetermined way is in the spirit of cash-in-advance models; namely, while consumption takes place only a limited set of financial transactions (in this case only cash withdrawals) can be made.

2/ Equation (2) allows for fairly general forms of the transaction cost function  $f(\cdot)$ . Even if  $f(\cdot)$  is linear, however, the concavity of the utility function implies that the welfare cost of transacting are convex. The specific form of the transactions technology may depend on production decisions like choosing the optimal amount of ATM's per geographical area, the optimal type of services performed by ATM's in different locations, etc.

3/ When thinking of time-costs, this is a natural assumption. In the context of monetary costs, as the original work of Baumol (1952) or as in Feenstra (1986), one could consider costs that are proportional to the amount of cash withdrawn.

Assuming that interest is paid on the initial stock of bond holdings, 1/ and using the price of domestic goods, P, as the numeraire, the consumer's intertemporal budget constraint is:

$$(3) \quad z_{-1} + \sum_{t=0}^{\infty} R^t (y_t + r_t) = \sum_{t=0}^{\infty} R^t \{ a^*_t q_t [(1+r^*_t) - (1+r_t)(q_{t-1}/q_t)] + i_t m_t + i^*_t q_t m^*_t + c_t + q_t c^*_t \}$$

where  $z_{-1}$  denotes the initial level of assets; R is the real domestic interest factor; y is the domestic endowment; r denotes real government transfers;  $a^*_t = A^*_t / P^*_{t-1}$  where  $A^*$  is the nominal value of foreign currency denominated assets and  $P^*$  is the price of the foreign good;  $q = eP^*/P$  denotes the terms of trade where e is the nominal exchange rate;  $m = M/P$  and  $m^* = M^*/P^*$  denote real domestic and foreign cash balances, respectively. By definition,  $(1+r_t) = (1+i_t)P_{t-1}/P_t$  and  $(1+r^*_t) = (1+i^*_t)P^*_{t-1}/P^*_t$ , where r and  $r^*$  denote the domestic and foreign real interest rates, respectively. 2/

In addition to the intertemporal budget constraint (3), the representative individual faces a cash-in-advance constraint during each period. With two currencies, the specification of the cash-in-advance constraint entails an assumption about the way transactions must be carried out. We assume that individuals are required to use foreign currency to purchase foreign goods while they are required to use domestic currency to purchase domestic goods, as in Stockman (1980), Lucas (1982), and Svensson (1985a). Thus, with two currencies, the individual faces two cash-in-advance constraints, which are given by: 3/

$$(4) \quad m_t N_t = c_t$$

$$(5) \quad m^*_t N^*_t = c^*_t$$

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1/ This assumption is made for simplicity and does not affect the results. If interest is paid on average bond holdings the budget constraint would be modified using the fact that average bond holdings (in nominal values),  $B^a$  and  $B^{*a}$ , are related to initial holdings, B and  $B^*$ , by the following equations:

$$B^a = B + M - (M/2)(N+1)$$

$$B^{*a} = B^* + M^* - (M^*/2)(N^*+1).$$

2/ The real interest factor R is defined as  $R^t = 1/((1+r_1)(1+r_2)\dots(1+r_t))$ .

3/ With no uncertainty and positive interest rates, the cash-in-advance constraints are always binding.

The L.H.S. of equations (4) and (5) is the total amount of (domestic and foreign, respectively) cash withdrawn per period; it equals the number of withdrawals multiplied by the amount of cash withdrawn each time.

In other cash-in-advance set-ups, like Helpman's (1981), assuming cash-in-advance constraints like equations (4) and (5) is equivalent to ruling out currency substitution. In this model, however, where income velocity is variable, currency substitution occurs because the use of both monies affects the amount of time that individuals spend transacting. In fact, as will be seen below, the type of currency substitution which results endogenously in this paper is similar to that of other optimizing currency substitution models (Liviatan (1981); Calvo (1980 and 1985); Boyer and Kingston (1987); Végh (1988 and 1989).

The representative consumer maximizes the value of utility function (1) subject to the time constraint (2), the lifetime budget constraint (3), and the cash in advance constraints (4) and (5). The first order conditions for an interior optimum imply:

$$(6) \quad [u_c^* - (u_x f_N^*/m^*)]/[u_c - (u_x f_N/m)] = q$$

$$(7) \quad (u_x f_N N/m)/[u_c - (u_x f_N/m)] = i$$

$$(8) \quad (u_x f_N^* N^*/m^*)/[u_c^* - (u_x f_N^*/m^*)] = i^*$$

$$(9) \quad [u_c - (u_x f_N/m)]_t = (R/\beta)[u_c - (u_x f_N/m)]_{t-1}$$

$$(10) \quad 1 + r_t = (1 + r_t^*)q_t/q_{t-1}$$

It can be shown that the solution to this consumer's problem is equivalent to a money-in-the-utility-function problem where the indirect utility function  $U(c, c^*, m, m^*) \equiv u[c, c^*, T - f(c/m, c^*/m^*)]$ , which implies that  $U_c = u_c - (u_x f_N/m)$ ,  $U_c^* = u_c^* - (u_x f_N^*/m^*)$ ,  $U_m = u_x f_N N/m$ , and  $U_m^* = u_x f_N^* N^*/m^*$ . 1/ In these expressions,  $N=c/m$  and  $N^*=c^*/m^*$ , from the cash-in-advance constraints (4) and (5).

Equation (6) states that the marginal rate of substitution between consumption of the two goods equals the terms of trade. 2/ The marginal

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1/ See Guidotti (1987) for a more detailed discussion.

2/ In what follows we use the expressions "terms of trade" and "real exchange rate" interchangeably.

rate of substitution in consumption takes account of the fact that an increase in consumption of a particular good, for a given level of real cash balances, requires an increase in the amount of time spent transacting in the currency used to purchase that good. The increased time spent transacting is the result of an increase in the frequency of cash withdrawals. The presence of terms involving  $m$  and  $m^*$  in equation (6) implies that, unlike other cash-in-advance models, the terms of trade are affected by financial considerations. Moreover, perfect substitutability in consumption no longer implies that  $q$  equals unity at all times, as is the case in Helpman (1981).

Because, by the cash-in-advance constraints, the consumer must use domestic currency to purchase domestic goods and foreign currency to purchase foreign goods, equation (6) can be reinterpreted as showing that the "effective" prices of consumption goods are affected by financial considerations. Equation (6) can be written as:

$$(6') \quad u_c^*/u_c = q[N(N^*+i^*/N^*(N+1))] = \bar{q}$$

where  $\bar{q}$  denotes the "effective" terms of trade. 1/

In terms of the indirect utility function  $U(c, c^*, m, m^*)$ , equations (7) and (8) state that the marginal rate of substitution between real cash balances and consumption equals the opportunity cost of holding money. In this context the marginal utility of money represents the marginal utility of additional leisure obtained by decreasing the frequency of cash withdrawals through an increase in real cash balances. Given that domestic (foreign) currency is withdrawn from domestic (foreign) bonds, the opportunity cost of holding domestic (foreign) money is given by the domestic (foreign) nominal interest rate.

Equation (9) is a standard Euler equation. Equation (10) is the interest rate parity condition. Using the definition of the real interest rate and the terms of trade, equation (10) can be rewritten as:

$$(10') \quad 1 + i_t = (1 + i_t^*)e_t/e_{t-1}$$

Using the first order conditions that describe the behavior of the domestic consumer, and an analogous set of conditions describing the behavior of the foreign consumer, we proceed to characterize the world economy's equilibrium under a flexible exchange rate regime.

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1/ The interpretation of (6') is similar to that which obtains in models that introduce money in the production function or money as reducing transaction costs (Frenkel and Dornbusch (1973); McCallum (1983); Kimbrough (1986); and Végh (1989)).

### III. Equilibrium in a Two-Country World

This section characterizes the world economy's equilibrium under flexible exchange rates. Assuming for simplicity that there is no public borrowing or lending, the budget constraint faced each period by the domestic government is given by:

$$(11) \quad \tau_t + g = \mu_t m^s_t$$

where  $\mu_t = (M^s_{t+1} - M^s_t) / M^s_t$  denotes the rate of growth of the domestic money supply,  $M^s$ , and  $g$  denotes government expenditure which is assumed to be constant over time. An analogous constraint is faced by the foreign government. It is important to note that, since individuals in both countries hold both currencies, the domestic (foreign) government earns seigniorage from the holdings of domestic (foreign) money in the foreign (domestic) country. This is reflected in the R.H.S. of equation (11) where seigniorage is earned on the total quantity of money.

In a world of perfect capital mobility, assuming that the two countries have the same rate of time preference and that all exogenous variables are constant over time, it can be shown that there are no intrinsic dynamics in the model (Obstfeld and Stockman (1985)) and that each country is in a steady state equilibrium.

Goods and asset market equilibria imply the following conditions:

$$(12) \quad c + c^f = y - g \equiv z$$

$$(13) \quad c^* + c^{*f} = y^* - g^* \equiv z^*$$

$$(14) \quad M^s/P \equiv m^s = m + m^f$$

$$(15) \quad M^{*s}/P^* \equiv m^{*s} = m^* + m^{*f}$$

where a superscript "f" indicates that the variable corresponds to the foreign country, and time subscripts have been dropped for notational convenience since the economy is in a steady state equilibrium.

Equations (12) and (13) incorporate the assumption that each country is specialized in the production of one good and that each government consumes only the domestic good. This can be easily modified to consider other cases. One particular modification that is of interest is one that simplifies the world economy's equilibrium to a nonstochastic version of the "pooling equilibrium" discussed extensively in the literature (Lucas

(1982), Stockman (1980), Svensson (1985a), among many others). A "pooling equilibrium" is obtained when the two countries are identical in all respects (including wealth). Each country's endowment is composed of half of the endowment of each good (net of government expenditure), and each country rebates the seigniorage earned from foreign (domestic) money holdings to the foreign (domestic) individual (or, alternatively, the rates of money growth are equal to zero). In a pooling equilibrium, each country consumes half of the endowment of each good, and holds half of the stock of each currency.

In the general case, assuming that each country's net bond position is zero in the steady state equilibrium (i.e.,  $b + qb^* = 0$ ), the domestic economy's budget constraint reduces to the following expression:

$$(16) \quad c + qc^* + \mu^*qm^* = z + \mu m^f$$

where  $\mu^*$  is the rate of expansion of the foreign money supply. Equation (16) indicates that aggregate spending plus the payment of seigniorage to the foreign country must equal the sum of domestic output net of government expenditure plus seigniorage earned from foreign holdings of domestic currency.

In the steady state equilibrium, the domestic and foreign real interest rates equal the common rate of time preference (i.e.,  $\beta(1+r) = \beta(1+r^*) = 1$ ). Any divergence between the domestic and foreign nominal interest rates  $i$  and  $i^*$  is determined by differences between the domestic and foreign rates of monetary expansion since  $(1+i) = (1+\mu)/\beta$  and  $(1+i^*) = (1+\mu^*)/\beta$ . Moreover, a divergence between the domestic and foreign rates of monetary expansion generates a proportional change in the nominal exchange rate.

The configuration of the world economy's equilibrium is completed by adding the first order conditions that characterize consumer behavior to the system of equations (12)-(16).

In order to simplify the ensuing discussion and to sharpen intuition we adopt the following specific functional forms for the representative individual's utility function and transactions technology:

$$(17) \quad u(c, c^*, x) = \ln(c) + \ln(c^*) + \ln(x)$$

$$(18) \quad f(N, N^*) = N + \alpha N^*.$$

The parameter  $\alpha$  affects the relative marginal cost of using a particular currency and it may be greater or smaller than one; if  $\alpha > 1$ , then the cost of holding a given level of foreign relative money balances,  $m^*/c^*$ , is

larger than the cost of holding a given level of domestic relative money balances,  $m/c$ ; namely,  $\alpha u_x > u_x$ . (It is reasonable to think that  $\alpha \geq 1$  for the domestic economy, while  $\alpha \leq 1$  for the foreign economy.)

In addition to equations (12)-(16), the world economy's equilibrium is characterized by the set of first order conditions (4)-(10), taking into account equations (17) and (18), for each country. After some algebraic manipulation, equations (6)-(8) may be rewritten as follows:

$$(19) \quad c^* \phi q = \psi(\alpha, N, N^*) c$$

$$(7') \quad (N+i)N = ix$$

$$(8') \quad (N^* + \phi i)\alpha = \phi ix$$

where  $x = T - N - \alpha N^*$  by equation (2), and  $\psi(\alpha, N, N^*) = \alpha N^{*2}/N^2$ , where  $\psi_\alpha > 0$ ,  $\psi_N < 0$ , and  $\psi_{N^*} > 0$ . Equation (19) is obtained using the fact that equations (6)-(8) can be shown to imply that  $U_c^*/U_c = U_m^*/\phi U_m$ . A set of equations analogous to (19), (7'), (8') and the finance constraints (4) and (5) applies to the foreign economy.

Under the functional specification chosen in (17) and (18), the system is block recursive. In particular, equations (7') and (8') alone determine the equilibrium levels of  $N$  and  $N^*$  as functions of  $i$ ,  $i^*$ , and  $\alpha$ :

$$(20) \quad N = \hat{N}(i, i^*, \alpha) \quad , \quad \hat{N}_i > 0, \hat{N}_{i^*} < 0, \hat{N}_\alpha < 0$$

$$(21) \quad N^* = \hat{N}^*(i, i^*, \alpha) \quad , \quad \hat{N}^*_i < 0, \hat{N}^*_{i^*} > 0, \hat{N}^*_\alpha < 0$$

The interpretation of the sign of the partial derivatives of  $N$  and  $N^*$  is the following. An increase in domestic nominal interest rate (i.e., the opportunity cost of holding domestic currency) generates, ceteris paribus, an increase in the frequency of withdrawals of domestic currency, which for a given consumption level implies a fall in domestic real cash balances. The increase in  $N$  (which by the finance constraint (4) implies a fall in the ratio  $m/c$ ) requires spending more time transacting and, hence, increases the marginal utility of reducing the frequency of foreign currency withdrawals. Therefore, the partial derivative of  $N^*$  with respect to  $i$  has a negative sign. This interaction between the holdings of domestic and foreign currency through the amount of time used in financial activities explains the phenomenon of currency substitution in this model. An analogous interpretation applies to the signs of the derivatives of  $N$  and  $N^*$  with respect to the foreign nominal interest rate.

An increase in  $\alpha$  raises the marginal cost of using foreign currency; namely, the amount of time associated with a given frequency  $N^*$  is higher. As a result,  $N^*$  falls implying by equation (5) that the ratio  $m^*/c^*$  increases. Since the increase in  $\alpha$  raises the total amount of time devoted to transactions activities (hence, reducing leisure), it is also optimal to reduce  $N$  because the marginal utility of holding domestic currency increases as leisure falls. 1/

Given  $N$  and  $N^*$  determined from equations (7') and (8'), and  $N^f$  and  $N^{*f}$  determined by an analogous set of equations corresponding to the foreign country, equation (19), along with the following conditions, determines the equilibrium values of  $c$ ,  $c^*$ , and  $q$ .

$$(22) \quad \psi^f(\alpha^f, N^f, N^{*f})(z-c) = \phi q(z^*-c^*)$$

$$(16') \quad c + q[1 + (\mu^*/N^*)]c^* = z + \mu(z-c)/N^f$$

Equation (22) is the foreign country's counterpart of equation (19), combined with the goods market equilibrium conditions (12) and (13). Similarly, equation (16'), which is the foreign country's counterpart of equation (16), is obtained by combining equations (16), (12), (13), and the cash-in-advance constraints (4) and (5).

Given  $c$ ,  $c^*$ , and  $q$  determined by equations (19), (22), and (16'), equations (12) and (13) determine  $c^f$  and  $c^{*f}$ , and the finance constraints determine the level of real cash balances in the two countries. Finally, the money market equilibrium conditions determine the price levels and the nominal exchange rate, as a function of the domestic and foreign money supplies.

#### IV. Financial Innovation in the Presence of Currency Substitution

In this section we analyze the effects of changes in the parameter  $\alpha$  which affects the form of the transaction technology given in equation (18). As indicated earlier, changes in this parameter may reflect technical progress in the financial system as well as changes in the regulatory environment.

As such, these changes, which we encompass under the expression "financial innovation," may occur simultaneously in all countries or be country-specific. In what follows, we use the "pooling equilibrium" to discuss the effects of global financial innovation, while we use the more

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1/ One can show that  $\partial(\alpha N^*)/\partial\alpha = -\partial x/\partial\alpha > 0$ . This implies that, ceteris paribus, an increase in  $\alpha$  increases the amount of time devoted to cash withdrawals and, hence, reduces leisure.

general set-up to discuss the domestic effects of country-specific financial innovation as well as their international transmission.

1. Global financial innovation

A "pooling equilibrium" is characterized by the fact that each individual consumes half of the world's endowment of each good, and holds half of each money supply. The equilibrium is described by the following equations: 1/

$$(23) \quad c = c^f = y ; c^* = c^{*f} = y^* ; m = m^f = m^s/2 ; m^* = m^{*f} = m^{*s}/2$$

$$(24) \quad m = \Omega(i, i^*, \alpha)y ; \Omega_i < 0, \Omega_{i^*} > 0, \Omega_\alpha > 0$$

$$(25) \quad m^* = \Omega^*(i, i^*, \alpha)y^* ; \Omega^*_i > 0, \Omega^*_{i^*} < 0, \Omega^*_\alpha > 0$$

$$(26) \quad q = \psi[\alpha, \theta, \hat{N}(i, i^*, \alpha), \hat{N}^*(i, i^*, \alpha)]y/y^*\phi$$

$$(27) \quad e = (M^s/M^{*s}\phi)\alpha\hat{N}^*(i, i^*, \alpha)/\hat{N}(i, i^*, \alpha)$$

where  $\Omega \equiv \hat{N}^{-1}$  and  $\Omega^* \equiv \hat{N}^{*-1}$ . Equations (24) and (25) are the money demand functions, which exhibit unitary income elasticities. As a result of the cash-advance-constraints, the demand for domestic currency is a function of domestic but not of foreign output, while the opposite is true of the demand for foreign currency. Equation (26) shows the equilibrium terms of trade being affected by foreign and domestic output and, as previously discussed, by financial variables. Equation (27) is the equation determining the nominal exchange rate. Interestingly, under the functional forms given in equations (17) and (18) the nominal exchange rate is independent of domestic and foreign output.

Consider the effects of a global increase in the relative cost of transacting in the foreign currency, represented by an increase in  $\alpha$ . From the money demand equations, it easily seen that an increase in  $\alpha$  raises the demand for both currencies. This reflects the fact that the frequency of withdrawals of both currencies falls. The frequency  $N^*$  falls because, as withdrawing foreign currency uses more time, it is optimal to reduce the consumption velocity of foreign money (i.e.,  $N^*$ , since by equation (5)  $N^* = c^*/m^*$ ), hence increasing average foreign currency balances. Since the increase in  $\alpha$  makes transacting, in general, more costly because there is a reduction in leisure, it is optimal to reduce

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1/ In a "pooling equilibrium" the values of the parameter  $\alpha$  is the same in the two countries. Assume, for simplicity, that  $g = g^* = 0$ .

also the frequency of withdrawals of domestic currency, and, hence, to increase the average holdings of domestic money. It can be shown that, as one would expect, the ratio of foreign to domestic cash balances falls.

The increase in the demand for both currencies prompts a fall in both  $P$  and  $P^*$ . The terms of trade also fall, as the effect of the reduction in  $N^*$ , in equation (26), outweighs the direct effect of  $\alpha$  and the effect via the reduction of  $N$ . The rationale behind the fall in the terms of trade is that the increase in the marginal cost of using foreign currency affects the relative cost of consuming foreign goods. Because of the cash-in-advance constraints which imply that the consumer must use domestic currency to purchase domestic goods and foreign currency to purchase foreign goods, the effective price of consumption is affected by monetary considerations. In particular, an increase in the cost of using foreign currency indirectly raises the relative effective price of consuming foreign goods. As a result, an excess supply of foreign goods would be generated at the initial relative market price, inducing a fall in equilibrium terms of trade. This can be seen easily from equation (6').

For an initial level of  $q$ , the monetary term of the effective relative price  $q$  increases following the increase in  $\alpha$ . This leads to a fall in the equilibrium level of  $q$ .

Interestingly, it can be shown that the nominal exchange rate,  $e$ , increases following an increase in  $\alpha$ ; namely, there is a nominal devaluation of the domestic currency. This reflects the fact that the relative demand for domestic money falls as withdrawing domestic currency has become relatively less costly compared to the foreign currency. Since

$$(28) \quad e = (M^S/M^{*S})(qm^*/m),$$

it is clear that the increase in the ratio  $m^*/m$  outweighs the fall in the terms of trade,  $q$ .

The negative co-movement of the nominal exchange rate and the terms of trade implies that the increase in the nominal exchange rate is outweighed by the drop in the price ratio  $P^*/P$ . Such negative co-movement shows clearly the distinction between monetary and real effects. While the nominal devaluation occurs because of a change in the relative demand for monies, the fall in the terms of trade reflects a change in the relative effective prices of consumption goods.

## 2. International transmission of financial innovation

In this section we discuss the effects of an increase in  $\alpha$  that occurs only in the domestic country. Comparing the present exercise with that carried out in the previous subsection, it is obvious that there

exist some important similarities. Two additional considerations, however, are important in interpreting this section's results. First, unlike the pooling equilibrium, there are "transfer problem" criteria. Second, unlike the pooling equilibrium, there are additional wealth effects resulting from changes in the net transfers of seigniorage across countries present because of currency substitution.

As was discussed above, an increase in  $\alpha$  results in a higher consumption velocity of circulation of both currencies; namely,  $N$  and  $N^*$  both increase. The effects on the levels of consumption and the terms of trade are found by calculating the comparative statics of the system composed by equations (19), (22) and (16'):

$$(29) \quad dc/d\alpha = - (1/\Delta') q\phi z \{c(1+\mu^*N^{*-1})(d\psi/d\alpha) - \mu^*N^{*-1}\phi qc^*\hat{N}^*_\alpha\}$$

$$(30) \quad dc^*/d\alpha = (1/\Delta') (\mu^*/N^{*2}) \{\phi qc^*\hat{N}^*_\alpha [\psi(z^*-c^*) + \psi^f c^*]\} < 0$$

$$(31) \quad dq/d\alpha = (q/\Delta') \{c(d\psi/d\alpha) [\phi(1+\mu N^{f-1}) + \psi^f(1+\mu^*N^{*-1})] \\ - (\psi^f - \psi) (\mu^*/N^{*2}) \phi qc^*\hat{N}^*_\alpha\}$$

where  $\Delta' > 0$  and  $d\psi/d\alpha = \psi_\alpha + \psi_N \hat{N}_\alpha + \psi_{N^*} \hat{N}^*_\alpha < 0$ .

First, consider the effects of an increase in  $\alpha$  abstracting from the changes in net seigniorage transfers across countries. This is obtained by setting  $\mu$  and  $\mu^*$  equal to zero. 1/ When the revenues from seigniorage are zero, an increase in  $\alpha$  increases domestic consumption of the domestic good, has no effect on domestic consumption of the foreign good, and, as in the pooling equilibrium, reduces the terms of trade. As in the case of global financial innovation, the fall in the terms of trade reflects an increase in the effective relative price of foreign goods, which results in an increase in the ratio of domestic to foreign goods consumed.

The demand for domestic money in the domestic economy increases because of two reasons. First, the shift in consumption towards domestic goods requires the use of more domestic money. Second, there is a decrease in  $N$ , as the increase in  $\alpha$  makes transactions more costly in terms of time and leisure becomes more valuable. Similarly, given that  $N^*$  falls while  $c^*$  remains unchanged, there is an increase in the domestic demand for foreign currency.

The effects of financial innovation in the domestic economy are transmitted to the foreign economy. Through the goods market equilibrium

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1/ This implies that  $\phi = 1$ .

condition, it is easy to see that the level of consumption of domestic goods in the foreign country falls, while foreign consumption of foreign goods remains unchanged. The fall in  $c^f$  reflects the fall in the terms of trade. Since  $N^f$  and  $N^{*f}$  are unaffected by the financial innovation in the home country, it is clear that the foreign demand for domestic currency falls due to the lower consumption of domestic goods, while the foreign demand for foreign currency remains unaffected.

Since the demand for foreign currency increases at home and remains unchanged abroad, the world demand for foreign currency increases. This reduces the equilibrium price level of the foreign good,  $P^*$ . The effect on the equilibrium price of the domestic good is ambiguous because the demand for domestic currency increases at home but decreases abroad. The expression for the effect of  $\alpha$  on  $(m+m^f)$  is the following:

$$(32) \quad d(m+m^f)/d\alpha = (N^{-1} - N^{f-1})(dc/d\alpha) - (c/N^2)\hat{N}_\alpha$$

The sign of (32) depends on a transfer problem criteria; namely, the difference between the consumption velocities of circulation of the domestic currency in the two countries. If, at the initial equilibrium, those velocities are equal, i.e.,  $N=N^f$ , an increase in  $\alpha$  results in an increase in the world demand for the domestic currency. In this case, the equilibrium price of the domestic good decreases.

Also the effect of  $\alpha$  on the nominal exchange rate depends on transfer problem criteria, as can be seen from the following expression for the equilibrium exchange rate: 1/

$$(33) \quad e = (M^S/M^{*S})[\theta(qm^*/m) + (1-\theta)(qm^{*f}/m^f)] \\ = (M^S/M^{*S}\phi)[\theta(\alpha N^*/N) + (1-\theta)(\alpha N^{*f}/N^f)]$$

where  $\theta = m/(m+m^f)$ . Two effects may be distinguished. First there is a fall in the ratio  $qm^*/m$ . Second, there is an increase in  $\theta$ . If, at the initial equilibrium, the two countries have the same ratio of domestic to foreign monies the second effect disappears and the increase in  $\alpha$  results in an increase in the nominal exchange rate, as in the previous section. In this case, there is a negative co-movement between the nominal exchange rate and the terms of trade; namely, an increase in  $\alpha$  generates a nominal devaluation of the domestic currency but a real appreciation in the sense that there is a fall in the equilibrium level of  $q$ .

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1/ Equation (32) is also valid when  $\mu$  and  $\mu^*$  are different from zero.

Consider next the additional effects that appear because of the transfer of seigniorage across countries. <sup>1/</sup> These are the terms in equations (29)-(31), which involve the rates of monetary expansion. Since a higher  $\alpha$  induces an increase in the domestic demand for foreign currency, there is more seigniorage paid by the domestic to the foreign country. In addition, since there is a fall in foreign holdings of domestic money, the domestic country receives less seigniorage from abroad. These two effects imply an increase in net transfers from the domestic to the foreign economy that has the expected associated wealth effects. For the domestic economy, these higher net seigniorage outflows imply a net fall in consumption of foreign goods, and a negative effect on consumption of domestic goods. The net effect on  $c$ , however, is ambiguous and depends on the magnitude of this income effect. In the foreign country, the effects are reversed, leading to a net increase in consumption of foreign goods and an ambiguous effect of consumption of domestic goods.

With these additional effects of seigniorage, the effect of  $\alpha$  on the terms of trade is ambiguous and depends on a transfer problem criteria. If, at the initial equilibrium,  $\psi^f = \psi$  the increase in  $\alpha$  reduces  $q$  as was the case in the previous subsection.

Having analyzed the implications of country-specific financial innovation, it is worth mentioning that the framework can be used to analyze the effects of unifying financial regulation. The unification of financial regulations, in this set-up, would imply studying the combined effects of two country-specific changes, moving from an initial situation in which  $\alpha > 1$  and  $\alpha^f < 1$  to a situation in which  $\alpha = \alpha^f = 1$ .

## V. Conclusions

This paper has presented a framework in which the domestic effects, as well as the international transmission, of financial innovation and the role of currency substitution can be analyzed.

The analysis provides a number of insights on both aspects. First, because of the nature of the cash-in-advance constraints faced by individuals, financial considerations are shown to affect the relative effective prices of consumption and, hence, equilibrium terms of trade.

Second, analyzing the effects of global innovation that alters the relative cost of transacting in one currency, it has been found that such innovation may lead to negative co-movements between the nominal and the real exchange rate. This negative co-movement is explained by analyzing, on the one hand, the effects of global financial innovation on the

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<sup>1/</sup> The importance of currency substitution for the cross-border payment of seigniorage has been analyzed by Fisher (1982), where some estimates of the size of these transfer for several groups of countries are calculated.

relative demand for currencies, and, on the other hand, the effects on the effective relative price of consumption goods. These price effects may be altered when financial innovation is country-specific, because of transfer-problem criteria. However, if, initially, the two countries are symmetric enough, the price effects of country-specific financial innovation are qualitatively the same as when innovation is global.

Third, the international transmission, as well as the domestic effects, of financial innovation, depends importantly on how it affects the cross-border transfers of seigniorage, which occur because of the presence of currency substitution, in addition to how it affects the equilibrium terms of trade. In particular, it is shown that a change in the technological (or, regulatory) environment which reduces the cost of transacting in the foreign currency induces, provided the two countries are initially similar, a nominal appreciation and real depreciation of the domestic currency, and results in a net seigniorage inflow from the rest of the world.

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