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Real Exchange Rate and Output Variability: The Role of Sticky Prices

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Abstract

This paper investigates the relationships between the degree of price stickiness (inflexibility), the variability of output, and that of the real exchange rate in an open economy under flexible exchange rates and capital mobility. We show that there exists, in general, a critical degree of price inflexibility below which increased inflexibility of prices reduces the variability of output. We also show that, in general, as prices become more inflexible, the relationship between the variability of the real exchange rate and that of output will be nonmonotonic. That is, as the variability of the real exchange rate increases the variability of output will decline up to a point, and only then increase.

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Contents

Summary

This paper extends to an open economy with flexible exchange rates and perfect capital mobility the recent literature on the relationship between price stickiness and aggregate economic activity in a closed economy.

Despite the widespread incorporation of the assumption of sticky goods price adjustment in models of exchange rate determination, little effort has been devoted to examining the relationship between the degree of price stickiness, or the speed of goods price adjustment, and the variability of the real exchange rate and that of output. Such a link is important since a plausible inference often drawn is that the greater the degree of price stickiness, the more variable are the real exchange rate and output. Indeed, fluctuations in aggregate output are often attributed to the short-run rigidity of wages and prices, and various proposals and pleas to make wages and prices more flexible have often been advanced.

The paper shows that, in general, a critical degree of price inflexibility exists below which increased inflexibility of prices reduces the variability of output. It also shows that, as prices become more inflexible, the relationship between the variability of the real exchange rate and that of output will be "nonmonotonic," that is, as the variability of the real exchange rate increases, the variability of output will decline up to a point, and only then increase. A nonmonotonic relationship between the variability of output and the degree of price stickiness does not require the dependence of aggregate demand on the real interest rate, as has been argued in the context of a closed economy.

I. Introduction

The substantial variability of real exchange rates since the advent of floating rates has been extensively documented and analyzed. 1/ The large and persistent movements of real exchange rates have been characterized by substantially greater movements in nominal exchange rates than in national price levels. 2/ Such behavior is consistent with predictions made by a broad class of models--best exemplified by the Dornbusch (1976) model--that assume sluggishness in the adjustment of nominal price levels, compared with nominal exchange rates which are assumed to move flexibly to continuously equilibrate asset markets. 3/

Despite the widespread incorporation of the assumption of sticky goods price adjustment in models of exchange rate determination, little effort has been devoted to examining the relationship between the degree of price stickiness, or the speed of goods price adjustment, and the variability of the real exchange rate and that of output. 4/ Such a link is important since a plausible inference often drawn is that the greater the degree of price stickiness the more variable are the real exchange rate and output. Indeed, fluctuations in aggregate output are often attributed to the short-run rigidity of wages and prices, and various proposals and pleas to make wages and prices more flexible have often been advanced. However, it will be shown that the simple traditional model developed in the next section yields a 'U-shaped' relationship between the degree of price stickiness and the variability of output. up to a point, greater price stickiness actually reduces the variability of output. 5/6/

1/ For a wide-ranging review of the experience under floating exchange rates, see Goldstein (1984) and Obstfeld (1985).

2/ For a recent, extensive empirical study of the behavior of real exchange rates, see Mussa (1986).

3/ Obstfeld and Rogoff (1984), and Obstfeld and Stockman (1985) discuss various other contributions.

4/ An exception is Calvo (1982b) who, among other things, links real exchange rate overshooting to the average length of a price quotation in the economy.

5/ It is argued, implicitly or explicitly, by the literature on optimal wage and price adjustment and optimal contract length that certain policy changes may change the degree of price stickiness in the goods market. See, among others, Barro (1972), Gray (1978), Mussa (1981a, 1981b), Rotemberg (1983), and Sheshinski and Weiss (1983). This is a classic example of the Lucas (1976) critique.

6/ Various authors have presented empirical evidence arguing that there have been substantial changes in the degree of price stickiness in countries, and that there are differences across countries. In particular, it has been argued that the speed of adjustment of prices has substantially slowed, or that prices have become more inflexible. See Taylor (1980b, 1986), DeLong and Summers (1986a, 1986b), Cagan (1975), Gordon (1980, 1982) and Sachs (1980). Taylor (1986) argues that the increased stickiness of prices has tended to increase the variance of output around capacity.

The endogeneity of the real exchange rate has so far restricted consideration of the relationship between the variability of the real exchange rate and that of aggregate economic activity mainly to empirical studies. 1/ In contrast, this paper examines theoretical aspects of the relationship by exploring a particular channel, namely, the degree of price stickiness, through which changes in the variability of the real exchange rate and that of output may come about. The presence of the real exchange rate as an argument in standard aggregate demand functions would suggest that an increase in the variability of the real exchange rate would increase the variability of output. It is shown here, however, that the implied relationship between the variability of the real exchange rate and the variability of output around capacity is in general a nonmonotonic one. In particular, it is shown that increases in the variability of the real exchange rate will be accompanied by a decline in the variability of output up to a critical value, after which increases in the variability of the real exchange rate will be associated with increases in the variability of output.

The results of this paper parallel recent work by DeLong and Summers (1986a, 1986b) in a closed economy context. Their argument relies on the presence of the real interest rate in the aggregate demand function. Part of the contribution here is to show that a nonmonotonic relationship between the degree of price (in)flexibility and output may exist in an open economy with flexible exchange rates and perfect capital mobility even in the absence of the dependence of aggregate demand on the real interest rate.

1/ See the empirical studies of Branson and Love (1986, 1987) and Krieger (1987). There are, of course, several possible empirical and conceptual measures of the real exchange rate. It is often defined as the relative price of nontradable goods produced or consumed in a country in terms of tradable goods produced or consumed in that country. The use of this definition empirically, is hampered by the lack of good measures of the price of nontraded goods. Alternatively, the real exchange rate between two countries is often defined as the relative price of one country's consumption basket in terms of the consumption basket of the other country. See Lanyi and Suss (1982), for a discussion of the appropriateness of alternative definitions of the real exchange rate Here, the real exchange rate is defined as the relative price of the rest of the world's (composite) good in terms of the (composite) home good. Note that it coincides with the terms of trade.

II. The Model

Consider a country producing a basket of goods that is, in principle, entirely tradable in world markets. The domestically-produced (composite) good is assumed to be differentiated from, and an imperfect substitute in consumption for, the rest of the world's (composite) good. Domestic producers are hence assumed to be able to actively set the price of their good. The country is, however, assumed to be small, in the sense of being a price taker in the market for the imported good. Further, domestic producers are assumed to price their good solely in terms of domestic currency. 1/

The level of output produced is assumed to equal the demand for output. The demand for domestic output is posited to be a positive function of the real exchange rate, representing the ratio of competitors' output price in the world market to the price of domestic output, and a negative function of the real interest rate:

$$y_t = y_t^d = d[e_t+P_t^*-P_t] - \sigma[i_t-(E_tP_{t+1}-P_t)], \tag{1}$$

where $d>0$ and $\sigma>0$, y_t denotes (the log of) output at time t; e_t is the (log of the) spot exchange rate, i.e., the number of units of the domestic currency for one unit of the foreign currency; P_t^* is the (log of the) foreign currency price of the rest of the world's good; P_t is the (log of the) price of the domestically-produced good; i_t is the domestic nominal interest rate; and E_t represents the mathematical expectations operator, conditional on information available at time t.

The equilibrium condition in the money market, which we assume always to hold is

$$M_t - P_t = -\alpha i_t + \Phi y_t, \tag{2}$$

where α and Φ are > 0 and M_t is the (log of the) money supply at time t. With perfect capital mobility and perfect substitutability between domestic and foreign currency assets, the interest rate at home is equal to the foreign interest rate, adjusted for the expected rate of depreciation of the home currency:

$$i_t = i_t^* + [E_t(e_{t+1})-e_t]. \tag{3}$$

1/ These are the traditional, rather strong assumptions in most sticky goods price, flexible exchange rate models. They imply an asymmetry in the domestic pricing of domestic and foreign goods. An exception is Flood and Hodrick (1984). They, however, assume that all price setters are perfectly synchronized and prices are revised at the end of each period, i.e., all prices are perfectly flexible with an exactly one period lag.

The money supply process is assumed to be given by

$$M_t = M_{t-1} + \epsilon_t, \tag{4}$$

where ϵ_t is a white noise innovation in the money supply process and $E_t(\epsilon_{t+s}) = 0$ for all $s = 1,2,\ldots$; $V(\epsilon) = \sigma^2_{\epsilon}$.

Note that the only shocks to this economy are monetary. It is straightforward to introduce a real shock to the goods market in equation (1). However, little is added to the analysis, since the monetary shock is a demand shock and the results below are not affected in any significant manner.

Sticky goods price adjustment

The model is closed by specifying the dynamic adjustment process for the aggregate price level at home. The price of the domestically-produced good is assumed to be sticky in that it is completely predetermined at a point in time and adjusts only slowly over time to equilibrate the goods market. The particular price adjustment rule employed is that posited by Mussa (1981, 1982): 1/

$$DP_t = E_t[D\overline{P}_t] + (1-\tau)[\overline{P}_t - P_t], \text{ where } 0 < \tau < 1. \tag{5}$$

D represents the forward difference operator, i.e., $DX_s = X_{s+1} - X_s$, and $\overline{P}_t$ represents the equilibrium price level. The rate of inflation, DP_t, is therefore posited to equal the sum of the expected rate of change of the equilibrium price level and a fraction of disequilibrium in the goods market. Here the extent of disequilibrium in the goods market is measured as the difference between the equilibrium price level and the actual price level. In general, the extent of disequilibrium in the goods market could be measured in price or quantity terms. The formulation in equation (5) has been chosen primarily for analytical tractability. 2/

Now, in equation (5), $(1-\tau)$ represents the proportion of disequilibrium in the goods market that is dissipated by changes in the price level in one time period. It, therefore, represents a measure of the speed of adjustment of prices in the economy, or a measure of the degree of price flexibility. As is brought out more clearly below, τ then represents a measure of the degree of price stickiness or inflexibility.

1/ See Obstfeld and Rogoff (1984) for a discussion of appropriate, forward-looking, sticky goods price adjustment rules.

2/ We note, as is brought out more clearly below, that the reduced-form solution for the aggregate price level generated as a consequence of adopting (5) is exactly that in Chadha (1987b), where a much more complicated and appealing two-part price setting mechanism is employed, and corresponds to the reduced form solution in Rotemberg (1982) where the aggregate price adjustment rule is derived from microfoundations.

The equilibrium price level, $\bar{P}_t$, is defined as the price level that solves

$$y_t = y_t^d = d(e_t + P_t^* - P_t) - \sigma[i_t - (E_t P_{t+1} - P_t)] = \bar{y}, \tag{6}$$

where $\bar{y}$ denotes fixed capacity output. The unique, convergent solution for $\bar{P}_t$ is:

$$\bar{P}_t = \frac{[d+\sigma/\alpha]}{[d+\sigma+\sigma/\alpha]} \sum_{i=0}^{\infty} \left[\frac{d+\sigma/\alpha}{d+\sigma+\sigma/\alpha}\right]^i E_t M_{t+i}, \tag{7}$$

where fixed capacity output, $\bar{y}$, the foreign nominal interest rate, i^*, and the foreign price level, P^*, have all been 'normalized' to equal zero. Under the assumption that the process generating the money supply is that given by equation (4) it follows, as would be expected, that the equilibrium price level is simply equal to the money supply, i.e.,

$$\bar{P}_t = M_t. \tag{7'}$$

III. Solution

The Appendix establishes the existence of a unique, convergent solution. It is then straightforward to solve the model by the method of undetermined coefficients. The solution is

$$P_t = \tau P_{t-1} + (1-\tau)M_{t-1}, \tag{8}$$

$$e_t = (1-\Theta)P_t + \Theta M_{t-1} + \Theta \epsilon_t, \tag{9}$$

where,

$$\Theta = \frac{1 + \alpha(1-\tau)}{\alpha(1-\tau) + \Phi\sigma(1-\tau) + \Phi d} > 0. \tag{10}$$

The aggregate price level in equation (8) is, therefore, a weighted average of last period's price level and last period's money stock. An increase in τ increases the weight given to last period's price level and reduces the weight given to last period's money stock. Maintaining the assumption that prices are predetermined, consider the maximum degree of price flexibility permitted, i.e., when $(1-\tau) = 1$ or $\tau = 0$. In this case, prices are flexible with a one-period lag, i.e., prices are set prior to the opening of goods markets in each period. The solution for the current price level is then simply the lagged value of the money stock, which is the time t-1 expectation of the steady state price level. In this sense, an increase in τ implies slower adjustment of the price level to monetary innovations and hence provides a dynamic measure of the degree of price stickiness in the economy.

The solution for the exchange rate in equation (9), noting (10), shows clearly that the response of the exchange rate to innovations in the money supply is in the same direction as the innovations. The exchange rate can, however, overshoot or undershoot its new long-run value depending on whether θ is greater or less than 1; it overshoots if $\theta>1$ and undershoots if $\theta<1$. This in turn implies that overshooting or undershooting depends on whether

$$1 \gtrless \Phi d + \Phi\sigma(1-\tau). \qquad (11)$$

An increase in the degree of price stickiness reduces the value of the terms in the right-hand side of equation (11); therefore, the stickier are prices, the greater the likelihood that the exchange rate will overshoot its new long-run value in response to an innovation in the money supply.

IV. Real Exchange Rate and Output Variability

This section examines the variability of the real exchange rate and output as functions of the degree of price stickiness, and the association between the variability of the real exchange rate and that of output as the degree of price stickiness changes. It is shown that the relationship between the degree of price stickiness and the variability of output is nonmonotonic. There are, however, two distinct channels through which changes in the degree of price stickiness bring about a nonmonotonic effect on the variability of output. The first channel requires only the presence of the real exchange rate as an argument in the aggregate demand function. The second relies on the presence of the real interest rate as an argument in the aggregate demand function and operates through the Mundell-Tobin effect, explained more fully below. The latter was put forward by DeLong and Summers (1986a, 1986b), who presented simulations of Taylor's (1979, 1980a) model for the U.S. economy, and was proved in Chadha (1987a), in a closed economy context. It is, therefore, useful to consider simplified versions of the aggregate demand function in equation (1), so as to highlight the alternative channels.

We first consider the case where aggregate demand depends only on the level of the real exchange rate. We then examine the case where aggregate demand depends only on the real interest rate. It is shown in this context that there is a nonmonotonic association between the variability of the real exchange rate and that of output. We finally consider the case where aggregate demand depends on both the real exchange rate and the real interest rate.

1. Aggregate demand dependent only on the real exchange rate ($\sigma = 0,\ d > 0$)

Aggregate demand is here posited to be a function simply of the level of the real exchange rate, that is

$$y_t = y_t^d = d[e_t + P_t^* - P_t]. \qquad (12)$$

The solutions for the exchange rate and the aggregate price level are as in equations (8) and (9) with Θ_1 replacing Θ, where

$$\Theta_1 = \frac{1 + \alpha(1-\tau)}{\alpha(1-\tau) + \Phi d} > 0, \tag{13}$$

is the coefficient on the innovation in the money supply in the solution for the exchange rate. The exchange rate, therefore, overshoots or undershoots depending on whether

$$1 - \Phi d \gtrless 0. \tag{14}$$

Employing the solution for the exchange rate and the price level in equations (8) and (9), the time paths of the real exchange rate and output may be written as:

$$(e_t - P_t) = \tau(e_{t-1} - P_{t-1}) + \Theta_1 \epsilon_t, \tag{15}$$

$$y_t = \tau y_{t-1} + \Theta_1 d \, \epsilon_t. \tag{16}$$

The real exchange rate and output move on impact in the direction of the innovation in the money supply. The effect of an increase in the degree of price stickiness, i.e., an increase in τ, is clearly to increase persistence or inertia in consecutive levels of both the real exchange rate and output. The effect of an increase in the degree of price stickiness on the impact effects depends, however, on whether the exchange rate overshoots or not since

$$\frac{\delta \Theta_1}{\delta \tau} = \frac{\alpha(1-\Phi d)}{[\alpha(1-\tau)+\Phi d]^2}, \tag{17}$$

$$\text{and sign } \frac{\delta \Theta_1}{\delta \tau} = \text{sign } (1-\Phi d), \tag{18}$$

which is positive if the exchange rate overshoots, (see equation (14)), and negative if the exchange rate undershoots. Figure 1 plots the impulse response of output to a positive monetary shock at time T for alternative degrees of price stickiness.

The variability of the real exchange rate and that of output may, from equations (15) and (16), be written as:

$$V(e-P) = \Theta_1^2 \frac{1}{(1-\tau^2)} \sigma_\epsilon^2, \tag{19}$$

$$V(y) = \Theta_1^2 \frac{d^2}{(1-\tau^2)} \sigma_\epsilon^2 = V(e-P)d^2. \tag{20}$$

It follows from equation (20) that, in this case, the variability of output and that of the real exchange rate change in the same direction as the degree of price stickiness changes. Now

$$\frac{\delta V(y)}{\delta \tau} = \frac{2\Theta_1^2 d^2 \sigma_\epsilon^2}{(1-\tau^2)} \{f(\tau)\}, \tag{21}$$

$$\text{where } f(\tau) = \frac{\tau}{(1-\tau^2)} + \frac{\alpha(1-\Phi d)}{[1+\alpha(1-\tau)][\alpha(1-\tau)+\Phi d]} \tag{22}$$

If the exchange rate overshoots, i.e., if $(1-\Phi d) > 0$, then $f(\tau) > 0$ for all values of τ, so that the variability of output is a strictly positive function of the degree of price stickiness. If, on the other hand, the exchange rate undershoots, it can be shown that

$$\text{sign } f(\tau) = \text{sign } g(\tau), \text{ where} \tag{23}$$

$$g(\tau) = \alpha^2\tau^3 - [2\alpha(1+\alpha)]\tau^2 + [(1+\alpha)(\alpha+\Phi d)]\tau + \alpha(1-\Phi d).$$

<u>Proposition 1</u>: If the exchange rate undershoots, then there exists a degree of price stickiness $\tau = \tau_1$, such that $g(\tau) < 0$ (i.e., $\frac{\delta V(y)}{\delta \tau} < 0$) for all $\tau < \tau_1$; similarly, there exists a $\tau = \tau_2$ such that $g(\tau) > 0$ (i.e., $\frac{\delta V(y)}{\delta \tau} > 0$) for all $\tau > \tau_2$, and where $\tau_2 \geq \tau_1$.

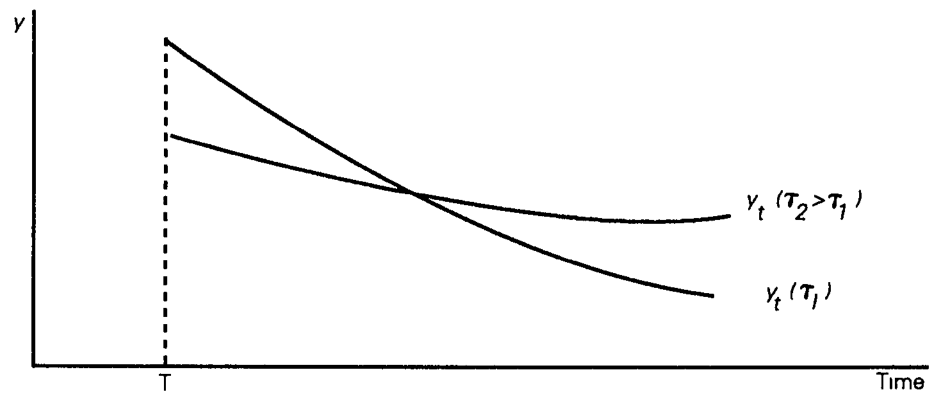
<u>Proof</u>: $g(0) = \alpha(1-\Phi d) < 0$; $g(1) = \Phi d > 0$. Given that $g(\tau)$ is a continuous, single-valued function, proposition 1 follows.

A corollary of proposition 2 is that for 'small' values of τ, $g(\tau) < 0$ and for 'large' values of τ, $g(\tau) > 0$. Thus, we have shown that when the exchange rate undershoots, increases in the degree of price stickiness reduce the variability of output and the real exchange rate up to a critical degree of price stickiness, τ_1. It is only after the degree of price stickiness exceeds a critical value, $\tau_2 (\geq \tau_1)$, that increases in

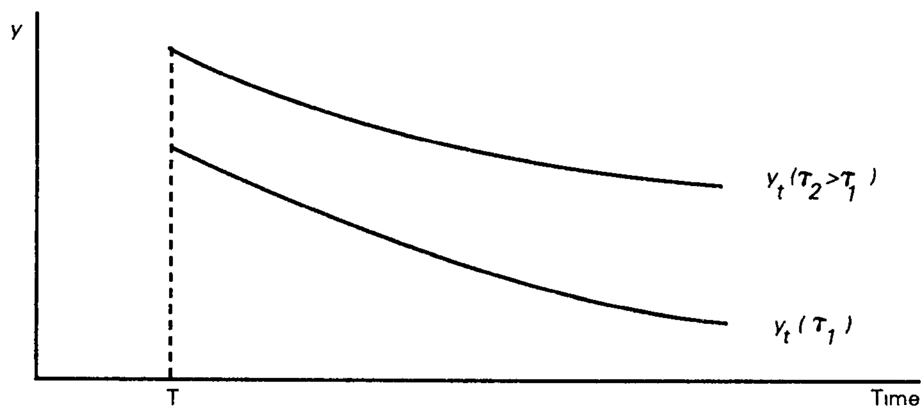
Figure 1

Impulse response of output with alternative degrees of price stickiness

a. Case of exchange rate undershooting



b. Case of exchange rate overshooting



the degree of price stickiness increase the variability of the real exchange rate and output. 1/

It is possible to show that this result can be expected to hold, in general, and does not depend on the specific form of the pricing rule employed in equation (5). Substituting the expression for output in equation (12) into the money market equilibrium condition in equation (2) and rearranging, the exchange rate may be written as

$$e_t = \frac{1}{(\alpha + \Phi d)} \sum_{i=0}^{\infty} \left[\frac{\alpha}{\alpha + \Phi d}\right]^i E_t[M_{t+1} - (1-\Phi d)P_{t+1}] \quad (24)$$

Recalling equations (2) and (3), if one is to think of the exchange rate as moving to continuously equilibrate the money market, note that this equilibrium is conditional on the expected future path of the exchange rate. Since the future exchange rate depends on future money market equilibrium, the level of the current exchange rate is, as equation (24) clearly brings out, a function of the expectation of all future variables affecting money market equilibrium. To bring out the effect of an increase in the degree of price stickiness on the exchange rate (and consequently output) it is useful to consider the impact of a positive monetary shock at time t, where the economy is assumed to be in equilibrium at time t-1. The expectation of all future money supplies in equation (24) is, recalling equation (4), simply the current level of the money supply. If prices are stickier, in the sense that the general price level tends to adjust more slowly to the current level of the money stock (the new long run expected steady state value of the price level), then this would imply that each expected future price level term in (24) would be smaller. Now, if $(1-\Phi d) > 0$, this would imply a larger impact effect on the exchange rate and if $(1-\Phi d) < 0$ a smaller impact effect on the exchange rate. Since the price level is predetermined and output depends on the real exchange rate, the magnitude of the impact effect on the nominal exchange rate translates directly to the magnitude of the impact effect on output.

2. Aggregate demand dependent only on the real interest rate ($\sigma > 0$, $d = 0$)

To bring out most clearly the effect of the real interest rate on the relationship between the degree of price (in)flexibility and the variability of the real exchange rate and that of output, it is useful to

1/ Whether the exchange rate overshoots or undershoots is an empirical matter. Papell (1985) finds empirical cases of exchange rate overshooting and undershooting.

consider the case where aggregate demand depends only on the real interest rate

$$y_t = y_t^d = -\sigma[i_t - (E_t P_{t+1} - P_t)] \ . \qquad (25)$$

In this case the solution to the system may be written as

$$P_t = \tau P_{t-1} + (1-\tau)M_{t-1} \ , \qquad (26)$$

$$e_t = (1-\Theta_2)P_t + \Theta_2 M_{t-1} + \Theta_2 \varepsilon_t, \qquad (27)$$

where

$$\Theta_2 = \frac{1 + \alpha(1-\tau)}{\alpha(1-\tau) + \Phi\sigma(1-\tau)} > 0. \qquad (28)$$

The condition for overshooting of the exchange rate is

$$1 - \Phi\sigma(1-\tau) > 0, \qquad (29)$$

and clearly the stickier are prices, the greater the likelihood that this relationship will hold. Moreover, the stickier are prices the greater the magnitude of overshooting. In this case

$$(e_t - P_t) = \tau(e_{t-1} - P_{t-1}) + \Theta_2 \varepsilon_t, \qquad (30)$$

$$y_t = \tau y_{t-1} + \Theta_2 \sigma(1-\tau)\ \varepsilon_t \ . \qquad (31)$$

$$\frac{\partial \Theta_2}{\partial \tau} = \frac{1}{(1-\tau)^2(\alpha+\Phi\sigma)} > 0 \ , \qquad (32)$$

An increase in the degree of price stickiness, therefore, increases the persistence and impact effect on the real exchange rate. For output, on the other hand, whereas an increase in the degree of price stickiness increases persistence or inertia in consecutive levels of output, it <u>reduces</u> the impact effect of a monetary shock, since

$$\frac{\delta[\Theta_2 \sigma(1-\tau)]}{\delta\tau} = \frac{-\alpha\sigma}{(\alpha+\Phi\sigma)} < 0. \qquad (33)$$

The reason is the Mundell-Tobin effect. The equations describing the point-in-time equilibrium of the system, (2) and (25), can be graphed in the real interest rate and output plane as the traditional IS and LM curves, respectively, for a given rate of inflationary expectations. Consider a positive innovation in the money supply at time t. Holding expectations of inflation constant, the increase in the nominal money supply translates to a real increase in the money supply, shifting the LM curve down. Now, given that in this model expectations of inflation are formed rationally, the increase in the money stock brings about expectations of inflation that cause a "second" shift of the LM curve downward. As we have proved by equation (33), and as would be intuitively expected, the stickier are prices the smaller the expected inflation differential and the smaller this "second" shift; hence, the higher the real interest rate and the lower the level of output on impact.

The variability of the real exchange rate and output are

$$V(e-P) = \Theta_2^2 \frac{1}{(1-\tau^2)} \sigma_\varepsilon^2 , \tag{34}$$

$$V(y) = \Theta_2^2 \frac{\sigma^2(1-\tau)^2}{(1-\tau^2)} \sigma_\varepsilon^2 = V(e-P)[\sigma^2(1-\tau)^2] . \tag{35}$$

It is straightforward to show that, in this case

$$\frac{\delta V(e-p)}{\delta\tau} = \sigma_\varepsilon^2 \left[\frac{2\tau}{(1-\tau^2)^2} + \frac{2}{(1-\tau^2)} \cdot \frac{\delta\Theta_2}{\delta\tau}2\right] > 0 , \tag{36}$$

so that the variability of the real exchange rate unambiguously increases as the degree of price stickiness rises. The effect of an increase in the degree of price stickiness on the variability of output is, however, ambiguous since

$$\frac{\delta V(y)}{\delta\tau} = \frac{2\sigma_\varepsilon^2 \cdot \sigma^2[1+\alpha(1-\tau)]}{(\alpha+\Phi\sigma)^2 (1-\tau^2)} \{\tau + \alpha\tau - \alpha\} \gtreqless 0 . \tag{37}$$

Therefore

$$\frac{\delta V(y)}{\delta\tau} \gtreqless 0 \text{ as } \tau \gtreqless \frac{\alpha}{1+\alpha} . \tag{38}$$

The variance of output, therefore, declines with increases in the degree of price stickiness until a critical degree of price stickiness (τ^C), given by $\tau^C = \alpha/(1+\alpha)$, after which it increases.

The above has shown that as prices get stickier, the variance of the real exchange rate unambiguously increases but the variance of output declines until a critical degree of price stickiness, after which it increases. There is therefore an implied nonmonotonic association between the variability of output and the variability of the real exchange rate as the degree of price stickiness changes. The variability of output declines as the variability of the real exchange rate increases up to a point, after which it increases.

3. The General Case

In the general case, the variability of the real exchange rate and output may be written

$$V(e-P) = \frac{\Theta^2}{(1-\tau^2)} \sigma_\varepsilon^2 , \tag{39}$$

$$V(y) = V(e-P) \; [d + \sigma(1-\tau)]^2 , \tag{40}$$

where Θ is defined in equation (12).

It can be shown that the derivatives of both the real exchange rate and output are ambiguous with respect to τ. Little further is gained by examining the conditions that determine the signs so we do not present them here. Instead we present simulations of the variability of the real exchange rate and that of output, and the implied association between the variability of the real exchange rate and output for reasonable parameter values. Figures 2 and 3 plot the variance of output and the real exchange rate as functions of the degree of price stickiness, respectively. The variance of innovations in the money supply has been normalized to unity. The values of the other parameters are noted in each figure. Initially, the variance of output declines substantially in percentage terms as the degree of price stickiness increases. It starts to rise only after a certain degree of price stickiness. For the parameter values here it turns out that the variability of the real exchange rate increases monotonically with increases in the degree of price stickiness. Figure 4 plots the variability of output against the variability of the real exchange rate for the values in Figures 2 and 3. In Figure 4 the variance of output declines as the variance of the real exchange rate increases, up to a critical value; only beyond this point does it start to rise.

Figure 2

Variance of output as a function of the degree of price stickiness

d=0.5; Sigma=3; Alpha=3; Phi=1

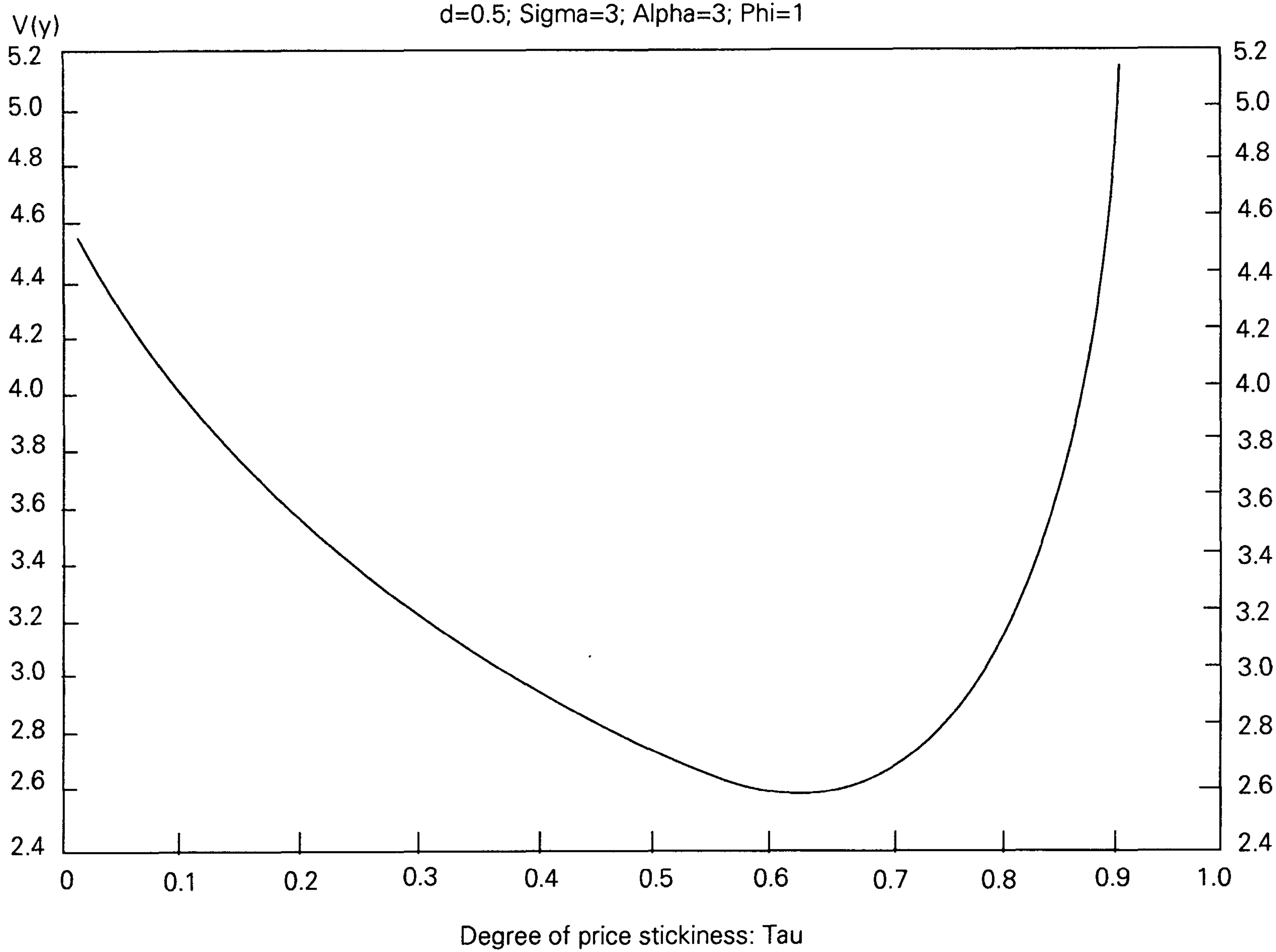
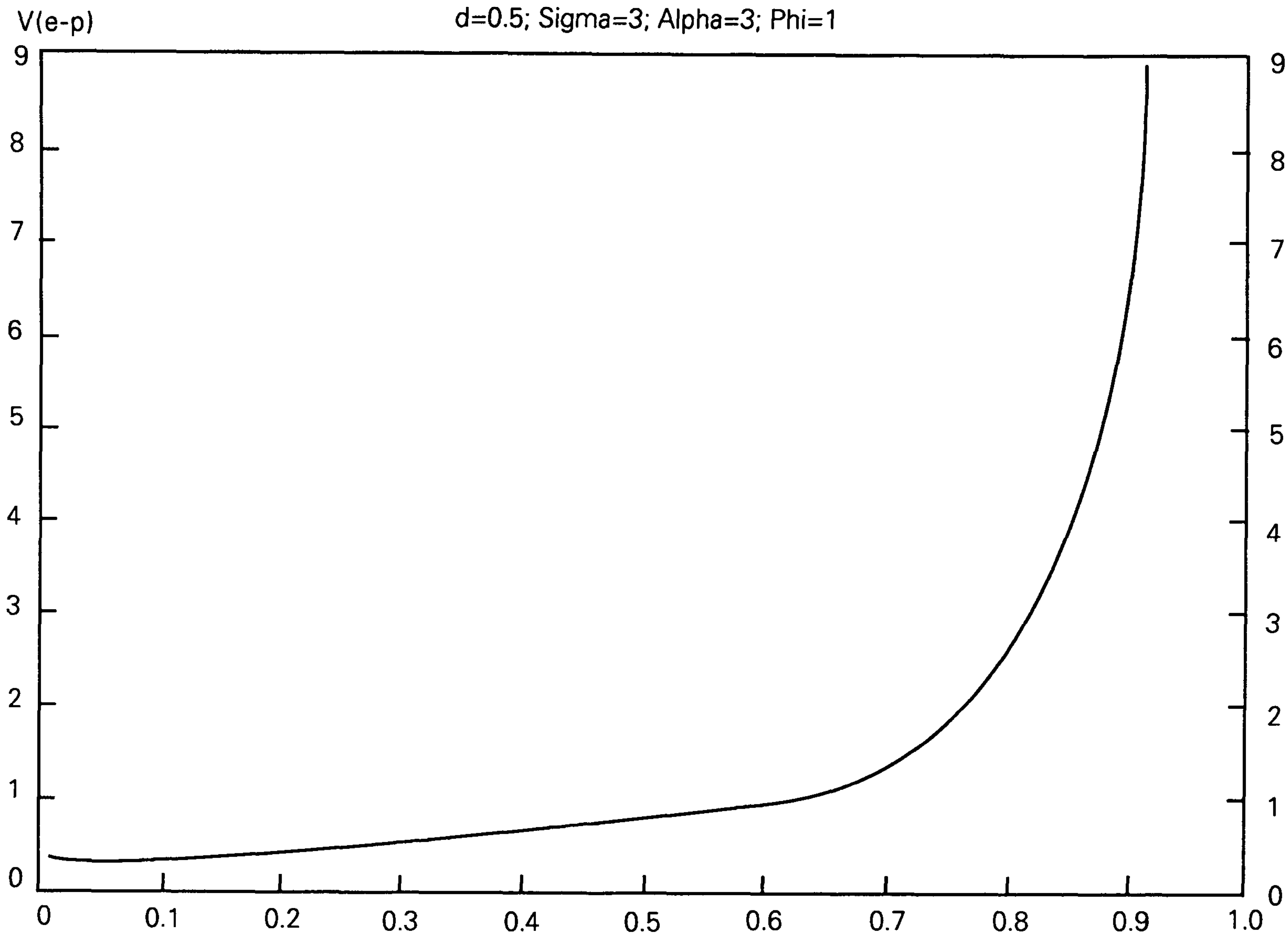


Figure 3

Variance of the real exchange rate as a function of the degree of price stickiness

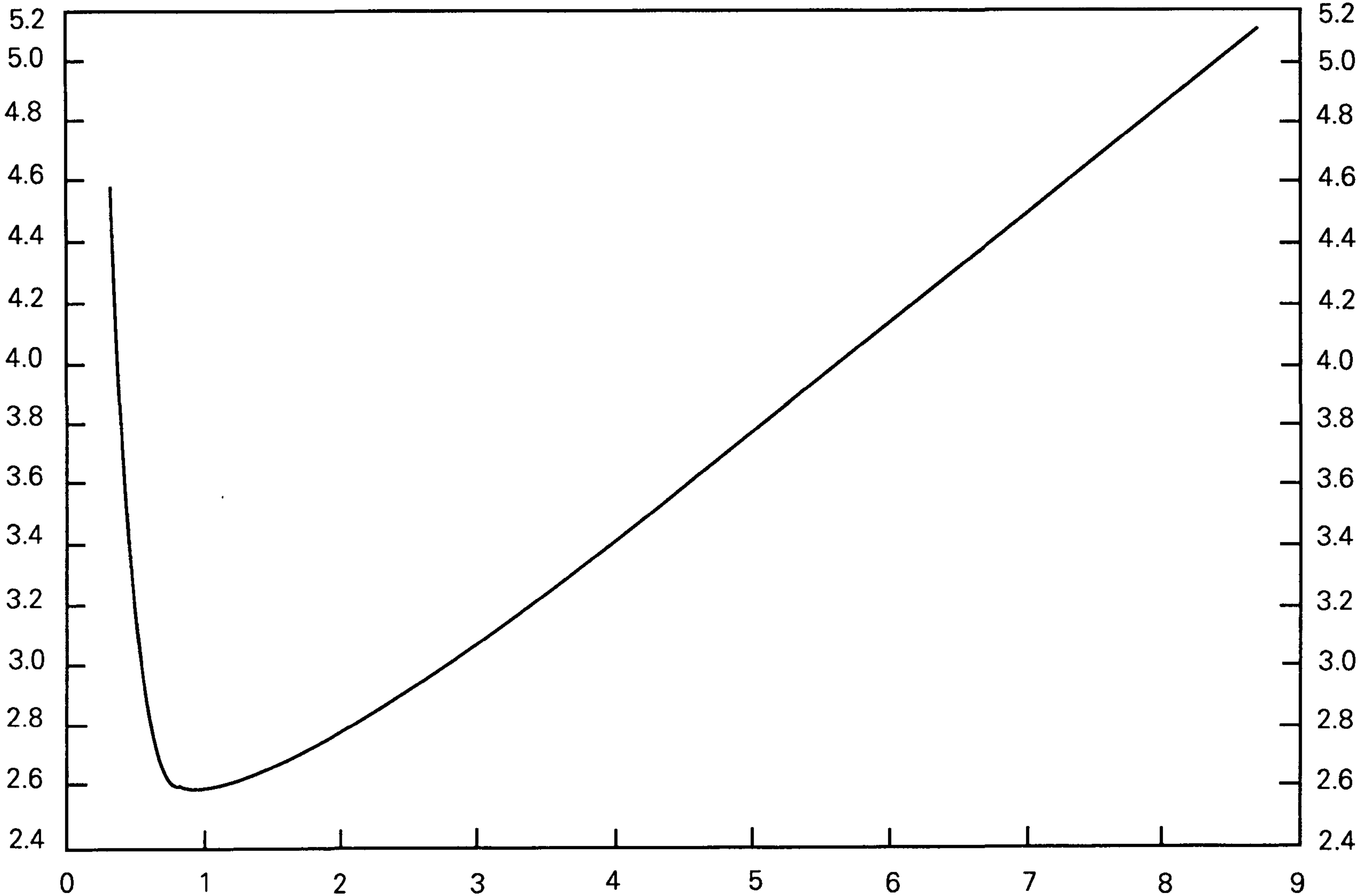


Degree of price stickiness: Tau

Figure 4

Variance of output and the real exchange rate

d=0.5; Sigma=3; Alpha=3; Phi=1



Variance of the real exchange rate

V. Conclusion

This paper has investigated the links between the degree of price stickiness or (in)flexibility and the variability of output and that of the real exchange rate, in an open economy under flexible exchange rates and perfect capital mobility. We have shown that there exists, in general, a critical degree of price inflexibility below which increased inflexibility of prices reduces the variability of output, and that there is a nonmonotonic association between the variability of output and the variability of the real exchange rate as the degree of price stickiness changes. In addition we have shown that the existence of a nonmonotonic relationship between the variability of output and the degree of price stickiness does not require the dependence of aggregate demand on the real interest rate, as has been argued in a closed economy context. A nonmonotonic relationship is shown to exist here when aggregate demand depends only on the real exchange rate, and if the economy is characterized by nominal exchange rate undershooting in response to a monetary shock.

Our findings have important policy implications. It has frequently been argued that fluctuations in aggregate output are caused by the short run rigidity of wages and prices, and hence that wages and prices should be made more flexible in order to reduce the variability of output. Given the analysis here, and as has been previously pointed out in a closed economy context, clearly such proposals may not be appropriate at all times. In order to determine the appropriateness of such proposals it is important to determine the degree of price stickiness in the economy. If the degree of price stickiness is less than a critical value (τ^c), increased flexibility of prices would increase the variability of output. Only if this degree of price stickiness is greater than τ^c would proposals for increased flexibility be appropriate for the purpose of reducing the variability of output.

APPENDIX

The purpose of this appendix is to provide a proof for the existence of a unique, convergent solution of the model developed in the text. The dynamics of the system may be described by a pair of stochastic difference equations in the price level and the exchange rate:

$$\begin{bmatrix} P_{t+1} \\ e_{t+1} \end{bmatrix} = \begin{bmatrix} \tau , & 0 \\ \frac{[1-\Phi(d+\sigma)+\Phi\sigma\tau]}{\alpha + \Phi\sigma} , & \frac{[\alpha+\Phi(d+\sigma)]}{\alpha + \Phi\sigma} \end{bmatrix} \begin{bmatrix} P_t \\ e_t \end{bmatrix} + \begin{bmatrix} (1-\tau)M_t \\ \frac{-[1-\Phi\sigma(1-\tau)]}{\alpha + \Phi\sigma} M_t \end{bmatrix} , \qquad \text{(A1)}$$

or simply

$$S_{t+1} = AS_t + F_t. \qquad \text{(A2)}$$

In solving the model, as is traditional in rational expectations models, we impose that the solution be convergent in the expected value sense, constrained on information when the forecast is made. The existence of a unique, convergent solution to (A2) requires that the matrix A possess exactly one characteristic root inside the unit circle given that there exists one predetermined variable, the domestic price level, in the system.

Proposition A1: The matrix A possesses exactly one characteristic root inside the unit circle.

Proof: The characteristic equation of the matrix A may be written as

$$X^2 - [\tau + 1 + \frac{\Phi d}{\alpha + \Phi\sigma}]X + \tau[1 + \frac{\Phi d}{\alpha + \Phi\sigma}] = 0.$$

Denoting the roots by X_i it is straightforward to note that

$$X_1 = \tau < 1 \ , \ \text{and } X_2 = 1 + \frac{\Phi d}{\alpha + \Phi\alpha} > 1.$$

References

Barro, R.J. (1972), "A Theory of Monopolistic Price Adjustment", Review of Economic Studies, (1972), pp. 17-26.

Branson, W.H. and J. Love, "Dollar Appreciation and Manufacturing Employment and Output", NBER Working Paper No. 1972, (July 1986).

———— "U.S. Manufacturing and the Real Exchange Rate", Mimeo., Princeton University, 1987.

Buiter, W.H. and M. Miller, "Real Exchange Rate Overshooting and the Output Cost of Bringing Down Inflation", European Economic Review, 1982, pp. 18, 85-123.

Cagan, P. "Changes in the Recession Behavior of Wholesale Prices", Explortations in Economic Research, (1975), 2: 54-104.

Calvo, G.A., "Staggered Contracts in a Utility-Maximizing Framework", Discussion Paper Series No. 130, Columbia University, (1982a).

————, "Real Exchange Rate Dynamics with Fixed Nominal Parities : on the Economics of Overshooting and Interest Rate Management with Rational Price Setting", Discussion Paper Series No. 162, Columbia University, (1982b).

————, "Staggered Contracts and Exchange Rate Policy", in J. Frenkel, ed., Exchange Rates and International Macroeconomics, (Chicago), pp. 235-51, (1983).

Chadha, B., "Is Increased Price Inflexibility Stabilizing?" mimeo, Columbia University, New York, (1987a), forthcoming, Journal of Money, Credit and Banking.

————, "Contract Length, Monetary Policy and Exchange Rate Variability", Journal of International Money and Finance, Volume 6, Number 4, (1987b), pp. 491-504.

DeLong, B. and L.H. Summers, "The Changing Cyclical Behavior of Economic Activity in the United States", in R.J. Gordon, ed., The American Business Cycle, Continuity and Change, National Bureau of Economic Research, (1986a), pp. 679-734.

————, "Is Increased Price Flexibility Stabilizing?", American Economic Review, Volume 76, (1986b), 5:1031-44.

Dornbusch, R., "Expectations and Exchange Rate Dynamics", Journal of Political Economy, (1976), 84: 1161-1176.

Driskill, R.A. and S. Sheffrin, "Is Price Flexibility Destabilizing?", American Economic Review, (1986), Volume 76, 4: 802-807.

Flood, R.P. and R.J. Hodrick, "Exchange Rate and Price Dynamics with Asymmetric Information", International Economic Review, Vol. 25, (1984), p. 3.

Frenkel, J., "Flexible Exchange Rates, Prices and the Role of News", Journal of Political Economy, (1981), 89: 665-705.

Goldstein, M., "The Exchange Rate System: Lessons of the Past and Options for the Future", IMF Occasional Paper, No. 30, (1984).

Gordon, R.J., "A Consistent Characterization of a Near Century of Price Behavior", American Economic Review, 70: 243-49, (1980).

————, "Price Inertia and Policy Ineffectiveness in the United States, 1890-1980", Economic Journal, (1982b), 92:13-44.

Gray, J.A., "On Indexation and Contract Length", Journal of Political Economy, Feb, 1978), pp. 1-18.

Krieger, Reva, "Real Exchange Rates, Sectoral shifts, and Aggregate Unemployment", mimeo., Columbia University, (1987).

Lanyi, A. and E.C. Suss, "Exchange Rate Variability: Alternative Measures and Interpretation", Staff Papers, International Monetary Fund, Volume 29, No. 4, (1982), pp. 527-560.

Lucas, Robert E., Jr., "Econometric Policy Evaluation: A Critique", in The Phillips Curve and Labor Markets, edited by Karl Brunner and Allan Meltzer. Amsterdam: North Holland, (1976).

Meese, R.A., "Is the Sticky Price Assumption Reasonable for Exchange Rate Models?", Journal of International Money and Finance, (1984), pp. 3: 131-39.

Mundell, R.A., "Inflation and the Real Interest Rate", Journal of Political Economy, (1963), pp. 71, 280-283.

Mussa, M., "Sticky Individual Prices and the Dynamics of the General Price Level", Carnegie Rochester Conference Series on Public Policy, (1981a), pp. 15, 261-296.

————, "Sticky Prices and Disequilibrium Adjustment in a Rational Model of the Inflationary Process", American Economic Review, December (1981b), pp. 71, 1020-27.

————, "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications", Carnegie Rochester Conference series on Public Policy, (1986), 25, 117-214.

Obstfeld, M., "Relative Prices, Employment, and the Exchange Rate in an Economy with Foresight", Econometrica, (1982), pp. 50, 1219-1242.

————, "Floating Exchange Rates: Performance and Prospects", Brookings Papers on Economic Activity, (1985), p 2.

Obstfeld, M., and K. Rogoff, "Exchange Rate Dynamics with Sluggish Prices under Alternative Price Adjustment Rules", International Economic Review, Vol 25, (1984), p. 1.

Papell, D., "Activist Monetary Policy, Imperfect Capital Mobility and the Overshooting Hypothesis", Journal of International Economics, (1985), 18: 219-240.

Rotemberg, J., "Sticky Prices in the United States", Journal of Political Economy, Vol, 90, (1982), pp. 1187-1211.

————, "Aggregate Consequences of Fixed Costs of Price Adjustment", American Economic Review, Vol 73, No. 3, (1983), pp. 433-36.

Sachs, J., "The Changing Cyclical Behavior of Wages and Prices, 1890-1976", American Economic Review, 1980, 70:78-90.

Sheshinski, E. and Y. Weiss, "Inflation and Costs of Price Adjustment", Review of Economic Studies, 1977, pp. 54, 287-303.

————, "Optimum Pricing Policy Under Stochastic Inflation", Review of Economic Studies, 1983.

Taylor, J.B., "Staggered Wage Setting in a Macro Model", Papers and Proceedings of the American Economic Association, May 1979, pp. 108-113.

————, "Aggregate Dynamics and Staggered Contracts", Journal of Political Economy, Feb (1980a).

————, "Output and Price Stability: an International Comparison", Journal of Economic Dynamics and Control, (1980b), pp. 2, 109-132.

————, "Improvements in Macroeconomic Stability: The Role of Wages and Prices", in R.J. Gordon, ed., The American Business Cycle, Continuity and Change, National Bureau of Economic Research, 1986, pp. 639-678.

Tobin, James, "Keynesian Models of Depression and Recession", Papers and Proceedings of the American Economic Association, May 1975, pp. 195-202.