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**"The Linkage Between Speculative Attack and Target  
Zone Models of Exchange Rates"**

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Abstract

In this paper we generalize the target zone exchange rate as model formalized by Krugman (1988b). The main contributions of these pages consist of linking the recent developments in the theory of target zones to the mirror image theory of speculative attacks on asset price fixing regimes and in using aspects of that linkage to give an intuitive interpretation to the "smooth pasting" condition often invoked as a terminal condition. We aim to unify these two literatures by showing that the solution concepts in both are identical.

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Summary

Agents in the public or private sectors often follow one set of actions when their environment lies within some prescribed boundaries and switch to another set of actions when the boundaries are reached. Their recognition of the presence of the boundaries ties the two sets of actions together.

This paper is concerned with describing the actions of agents operating inside a target exchange rate zone. The authors' previous work studied economic behavior in similar situations but did not come to closed-form solutions. Recent research, however, has shown how to derive closed-form solutions for a variety of problems in controlled Brownian motion (that is, random movement with occasional interaction by a regulator). This research makes it easy to implement the idea that the anticipation of "bumping into" the boundaries generates important nonlinearities that are poorly modeled by linear approximations.

This paper generalizes the model of the target zone exchange rate. Its main contributions are in linking recent developments in the theory of target zones to the mirror-image theory of speculative attacks on asset price fixing regimes and in using aspects of that linkage to interpret this literature. The paper aims to unify these two theories by showing that the solution concepts in both are identical. It can be shown that in the target zone context, interventions similar to the speculative attacks must generally occur as a result of a policy to defend the zone.



## I. Introduction

Agents in the public or private sectors often follow one set of actions when their environment lies within some prescribed boundaries and switch to another set of actions when the boundaries are reached. Their recognition of the presence of the boundaries ties the two sets of actions together. In previous work, we have studied economic behavior in similar situations, but we found it difficult to produce closed form solutions. 1/

Recently, several authors have investigated this class of problems with a set of tools that readily generates closed form solutions in a stochastic setting. The new research, aimed at issues in exchange rate policy, has been due to Smith (1987), Krugman (1987, 1988), Smith and Smith (1988), Miller (1988), Klein (1988), and Froot and Obstfeld (1989). Additional work in a microeconomic setting has been carried out by Krugman (1987), Bertola (1987), and Dixit (1987, 1988).

All this recent work was partly stimulated by the publication of Harrison (1985), which shows how to derive closed form solutions for a variety of problems in controlled Brownian motion. Harrison's results make it easy to implement the idea that the anticipation of "bumping into" the boundaries generates important nonlinearities that are not well modeled by linear approximations.

In this paper, we will arbitrage yet another result from Harrison (1985) into the study of exchange rate target zones. We will generalize the target zone (TZ) exchange rate as model formalized by Krugman (1988b) and extended by Froot and Obstfeld (1989). 2/ The Krugman TZ bears some resemblance to the European Monetary System (EMS), where members agree to maintain their exchange rates within prespecified bands. While the EMS recognizes the possibility of occasional band shifts (devaluations), we will study a system in which the exchange rate bands are permanent. 3/

The main contribution of these pages consists of linking the recent developments in the theory of target zones to the mirror-image theory of

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1/ In Flood and Garber (1980, 1983b), we studied the impact of an endogenous triggering of monetary reform on the current price level in the German hyperinflation. In Flood and Garber (1983a), we considered the determination of the current floating exchange rate when the regime would switch to a fixed rate system at an endogenously determined, random time.

2/ The Krugman TZ is in spirit a simplified version of the TZ blueprint offered by Williamson and Miller (1987).

3/ The TZ models can readily be integrated with models of speculative attacks either of the buying or selling varieties, if a limit is placed on the amount of reserves or bonds that the authorities are willing to buy or sell to defend the zone.

speculative attacks on asset price fixing regimes. <sup>1/</sup> We also use aspects of this linkage to provide an intuitive interpretation of the "smooth pasting" condition, generally invoked as a terminal condition in this literature.

We aim to unify these two literatures by showing that the solution concepts in both are identical. Indeed, we can show that in the TZ context speculative attack-like interventions may occur as a result of a policy to defend the zone. <sup>2/</sup> Thus, Krugman's recent (1987, 1988) work is actually a step toward coming full circle on his (1979) contribution on balance-of-payments crises.

We derive our results in the two remaining sections. In Section II, we present the exchange rate TZ problem in a standard exchange rate model. The TZ literature developed thus far has considered only the case of infinitesimal interventions. A closed-form solution for the exchange rate for the case of discrete-sized interventions can be read directly from results in Harrison (1985), however. Therefore, we simply reproduce the relevant problem from Harrison and indicate those properties of Harrison's solution which convey economic intuition about boundary conditions. In Section III, we restudy the TZ problem in the manner of Krugman and of Froot and Obstfeld using the boundary conditions developed in Harrison for discrete interventions. We also extend the analysis to situations in which the policy authority may use discrete interventions of uncertain magnitude to keep the exchange rate within possibly stochastic boundaries. Finally, we also examine the interest rate implications of adopting a TZ in this model and conclude by placing the current analysis in a broader perspective.

## II. A Model of Target Zones

The TZ is a nonlinear compromise between fixed exchange rates and freely flexible exchange rates. The basic idea of the exchange rate TZ is that a country or group of countries sets explicit margins within which exchange rates will be allowed to fluctuate. While the exchange rate is within those boundaries, policy can be directed toward other goals. When a boundary is reached, the policy maker focuses resources on maintaining it. The TZ does not preclude foreign exchange interventions inside the boundaries. Indeed, the TZ studied by Krugman simply does not specify government behavior inside the TZ boundaries. Of course, this is

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<sup>1/</sup> For some literature on speculative attacks, see Salant and Henderson (1978), Krugman (1979), Salant (1983), Flood and Garber (1984a, b), and Obstfeld (1984, 1986).

<sup>2/</sup> Because of the upper and lower bounds in a TZ, these may be either buying or selling attacks as in Grilli (1985).

the point of the TZ. While the exchange rate is inside the band, policy can be directed as desired toward goals other than fixing the exchange rate. 1/

Krugman (1988) was able to characterize the behavior of the exchange rate within a target zone when exchange rates are driven by regulated Brownian motion. Regulated Brownian motion is Brownian motion with occasional intervention by a regulator, which we identify as a policy authority. In Krugman's case, the unregulated fundamentals follow nondrifting Brownian motion. The intervention which regulates the process is triggered by the edges of the TZ, which are symmetric about zero. Froot and Obstfeld (1989) extend Krugman's results to the case of fundamentals following Brownian motion with constant drift and to nonsymmetric zones. 2/

We extend this analysis to allow both a discrete intervention and the possibility that intervention may be triggered randomly. While both extensions are required before models of this type can be applied to data, the ability to obtain solutions for discrete interventions is the more interesting result. This extension provides the intuitive link between the earlier literature on discrete attacks on foreign exchange reserves and clarifies the latitude available to a policy authority while maintaining the TZ.

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1/ There is no claim that such a nonlinear setting of policy would be welfare improving compared to linear nonsterilized intervention in response to a set of indicator variables. In fact, for fairly general quadratic welfare functions, the optimal policy is linear. Therefore, the case for a TZ must rely on, for example, welfare's being poorly modeled by a quadratic function, the optimal settings of the coefficients in a linear policy rule being difficult to achieve (with the TZ providing a better welfare outcome than inappropriate coefficients in a linear policy), or possibly some multilateral negotiating aspects of a TZ providing a better chance of reaching a near Pareto optimal policy than would be true for a linear policy.

2/ Froot and Obstfeld (1989) have also extended these results to the case of a different forcing process, the Ornstein-Uhlenbeck process, and to the case of absorbing barriers. The absorbing barriers case was studied by Flood and Garber (1983a) with closed form solutions first derived by Smith (1987).

To study the behavior of the exchange rate inside the band, Krugman and Froot and Obstfeld use a standard law of motion for a flexible price exchange rate model: 1/

$$(1) \quad x(t) = k(t) + \alpha E[dx(t)]/dt, \quad \alpha > 0.$$

$x(t)$  is the logarithm of the exchange rate,  $\alpha$  can be interpreted as the Cagan interest rate semi-elasticity, and the expectation operator is conditioned on current information. Only information about the forcing variable  $k(t)$  is relevant to the exchange market. In a standard monetary approach model,  $k(t)$  is a linear combination of the logarithms of foreign and domestic money supplies, real incomes, money demand disturbances and real exchange rate movements.

$k(t)$  can be controlled by intervention of the monetary authorities. Specifically, the authorities can control  $k(t)$  to assure that

$$(2) \quad x^u > x > x^l$$

where  $x^u$  and  $x^l$  are the upper and lower bounds of the exchange rate TZ, respectively. Krugman and Froot and Obstfeld assume that the authorities interfere with the motion of  $k$  only when  $x$  reaches the boundaries of the TZ. 2/ When the exchange rate is inside the TZ boundaries,  $k$  follows a random walk with drift that is independent of the exchange rate: 3/

$$(3) \quad dk = \eta dt + \sigma dz$$

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1/ The law of motion would be generated in the exchange rate models of Bilson (1978), Frenkel (1978), and Mussa (1978). The same model was adapted by Flood and Garber (1983a) to a continuous time stochastic environment with future exchange rate fixing. The underpinning of the law of motion would be a Cagan money demand function in two countries, purchasing power parity, and open interest parity. Other models, of course, would produce laws of motion for the exchange rate of a similar form with different interpretations of the forcing variables.

2/ Both  $x$  and  $k$  are functions of time, however, we will usually suppress the time dependence notation in the following presentation.

3/ Krugman assumes a zero drift parameter. Froot and Obstfeld argued that the policy specification in Krugman is incomplete since both discrete jumps in  $k$  and infinitesimal interventions are compatible with a given TZ. They base their discussion on a similar point made in Obstfeld and Stockman's (1985) discussion of an indeterminacy in Flood and Garber (1983a). Nevertheless, they consider only the case of infinitesimal intervention. It turns out that this case implies intervention only at the boundaries of the TZ.



where  $\eta$  and  $\sigma$  are constants and  $z$  is a standard Wiener process, implying that the variance of  $k$  over any period  $t$  is  $\sigma^2 t$  and that  $(dk)^2 = \sigma^2 dt$  (see Harrison (1985) page 65).

# 1. An exchange rate solution with discrete intervention

We now transcribe from Harrison (1985) the closed form exchange rate solution when intervention is discrete. Consider a situation in which the Brownian motion with drift process  $k$  is controlled as in Figure 1. When it is between the bounds  $k^u$  and  $k^l$ ,  $k$  follows the Brownian motion process of equation (3). When  $k$  hits the upper bound  $k^u$ , an intervention throws  $k$  discontinuously back to the interior point  $Q = k^u - I^u$ , where  $I^u$  measures the magnitude of intervention. At  $Q$ , the process resumes the random motion given in equation (3). If  $k$  hits the lower bound  $k^l$ , it is thrown back to the point  $q = k^l + I^l$ . Of course, when  $I^u = I^l = 0$ , we have the case of Froot and Obstfeld: asymmetric reflecting barriers with infinitesimal interventions. When, in addition,  $\eta = 0$  and  $k^u = -k^l$ , we have the symmetric case of Krugman.

These discontinuous shifts in  $k$  can be interpreted as interventions that occur to maintain the exchange rate  $k$  within its prescribed bounds. Krugman implicitly and Froot and Obstfeld explicitly assume that such interventions are infinitesimal; they associate the attainment of  $x^u$  with the simultaneous attainment of  $k^u$ . We doubt, however, that agents operating in a TZ would be sure that any intervention would be infinitesimal and would take place only at the TZ boundary. In the generalization to discrete interventions, these two events will not coincide. 1/

How will the exchange rate,  $x$ , behave under such circumstances? Note that the exchange rate solution for the differential equation (1) can be written as:

$$(4) \quad x = g(k(0)) = \frac{1}{\alpha} E \left[ \int_{\tau=0}^{\infty} k(\tau) \exp(-\frac{1}{\alpha} \tau) d\tau \mid k(0) \right]$$

The analytical solution of this integral, as well as the layout in Figure 1, is presented in Harrison (1985), Chapter 3, page 52, problem 13, for the case where units are chosen so that  $x^l = 0$ . From special cases of this solution, it is easy to arbitrage most of the results of the literature applying controlled Brownian motion to economics. For current purposes, however, we note that the general functional form of the solution is exactly that of Krugman and Froot and Obstfeld:

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1/ Apparently actual interventions by the Bundesbank in defense of the EMS margins neither take place only at the declared exchange rate margins nor is the size of the intervention always related to the proximity of the margins.

$$(5) \quad x = g(k) = k + \alpha\eta + A \exp [\lambda_1 k] + B \exp [\lambda_2 k]$$

where

$$\lambda_1 = - \left( \frac{1}{\sigma^2} \right) \left[ \left( \eta^2 + \frac{(2\sigma^2)}{\alpha} \right)^{\frac{1}{2}} + \eta \right]$$

and

$$\lambda_2 = \left( \frac{1}{\sigma^2} \right) \left[ \left( \eta^2 + \frac{(2\sigma^2)}{\alpha} \right)^{\frac{1}{2}} - \eta \right]$$

For a given TZ the constant terms A and B are identical to those of Krugman and Froot and Obstfeld, though they are apparently determined by different boundary conditions.

We graph Harrison's solution for the function  $g(k)$  for the case of discrete intervention in Figure 2. The functional form in equation (5) is invariant to the size of the intervention; and  $I^u$  and  $I^l$  will affect neither A nor B for a given TZ. Also, it can be shown with some manipulation that Harrison's solution for  $x$  as a function of  $k$  is continuous at  $k = k^u$  and  $k = k^l$ , so that  $g(k^u) = g(k^u - I^u)$  and  $g(k^l) = g(k^l + I^l)$ . We will present an intuitive argument for this result and use it as a boundary condition below.

## 2. A derivation of the functional form of the exchange rate solution

Our presentation of Harrison's solution for  $g(k)$  without any development is a bit mysterious. Therefore, we now follow Krugman and Froot and Obstfeld to develop the functional form of the solution presented by Harrison. Suppose that the exchange rate solution is  $x = g(k)$ . Our problem is to develop the  $g(k)$  function explicitly.

If  $g(k)$  is the solution then  $Edx/dt$  can be derived from the appropriate differential of the  $g(k)$  function. The  $g(k)$  function and thus the exchange rate is driven by  $k$ , which follows a Weiner process.

Figure 1  
Controlled Brownian Motion with Discrete Intervention

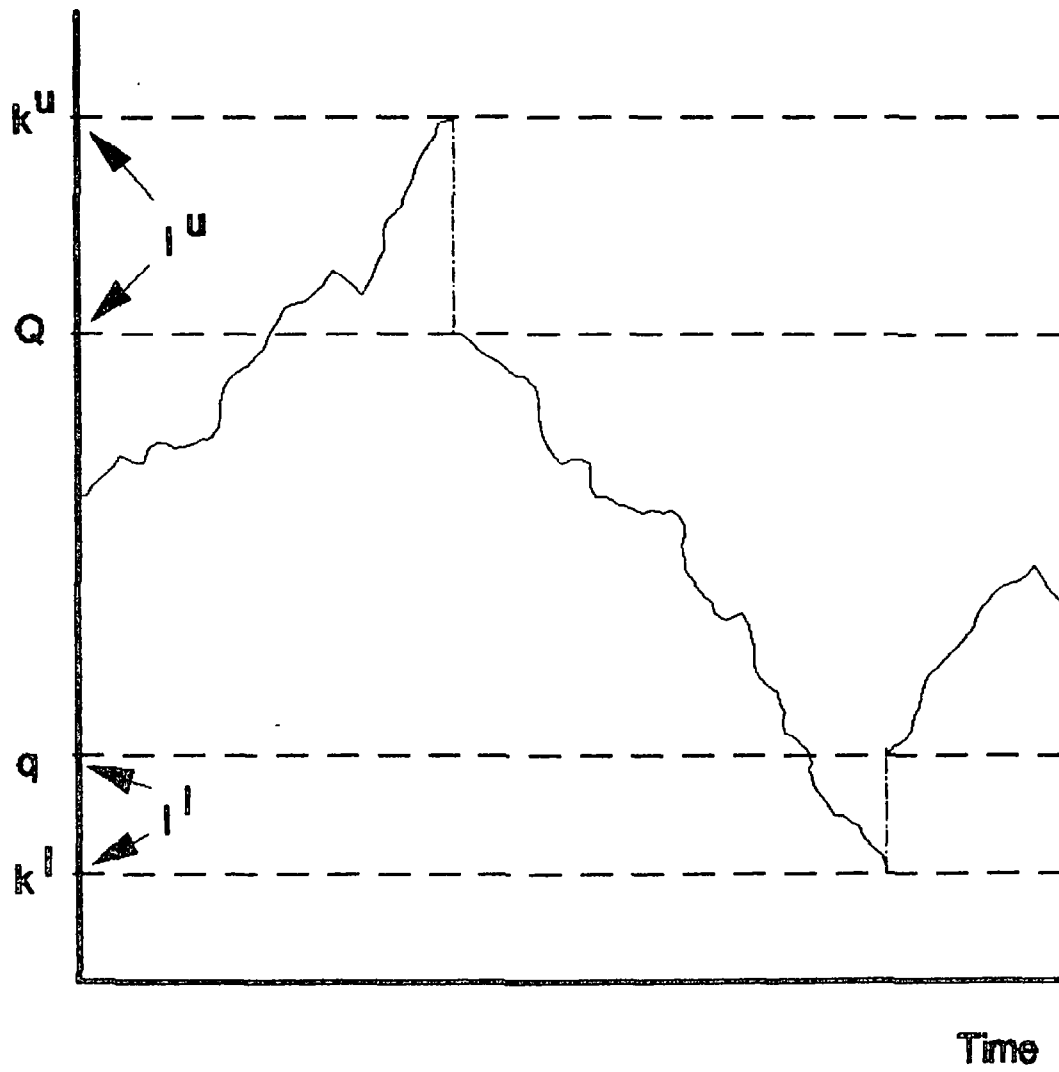
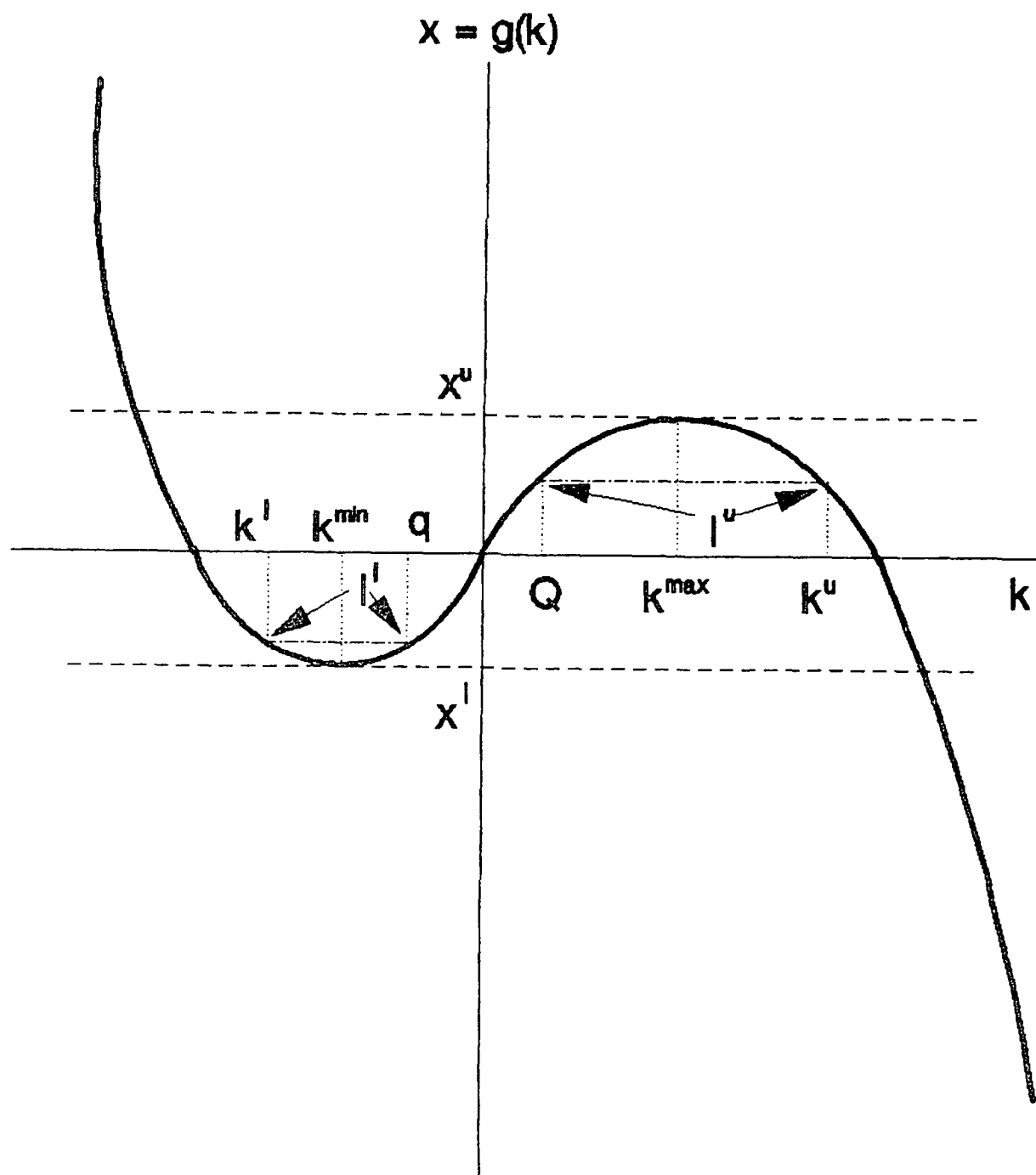




Figure 2  
Exchange Rates with Discrete Interventions





Therefore we must apply I to differentiation to  $g(k)$ . According to Harrison (1985) page 65 we obtain: 1/

$$(6) \quad dx = g'(k)dk + (1/2)g''(k)dk^2$$

Now take the expectation of each side of equation (6), conditional on current information. Inside the TZ,  $E[dk] = \eta dt$  since  $Edz = 0$ . Since current  $k$  is known to agents,  $Eg'(k)dk = g'(k)Edk = g'(k)\eta dt$ . Further  $dk^2 = \sigma^2 dt$  (see Harrison, page 65) so that equation (6) may be rearranged to become:

$$(7) \quad E[dx]/dt = g'(k)\eta + (1/2)g''(k)\sigma^2$$

Now substitute from equation (7) into equation (1) to find:

$$(8) \quad x = g(k) = k + \alpha[g'(k)\eta + (1/2)g''(k)\sigma^2]$$

Equation (8) is a second order linear differential equation, which we solve by: (1) finding a particular solution; (2) combining the particular solution with the homogenous solution; and (3) applying appropriate boundary conditions.

A particular solution is obtained by ignoring the TZ. Without the TZ equations (1) and (3) require  $x = k + \alpha\eta$ , and this is a particular solution to equation (8). The homogenous part of the solution is  $x = A\exp[\lambda_1 k] + B\exp[\lambda_2 k]$ . The sum of the two parts of the solution is the general solution given in equation (5) with  $\lambda_1$  and  $\lambda_2$  as above. Obtaining a complete solution requires application of the appropriate boundary conditions.

### III. Boundary Conditions and the Explicit Exchange Rate Solution

Though the explicit solution is available with some manipulation of Harrison's result, we prefer for expositional purposes to develop the solution for the problem of discrete interventions along the intuitive lines provided by Krugman and Froot and Obstfeld. To solve for A and B in equation (5), Froot and Obstfeld (following Krugman) impose the "smooth

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1/ The stochastic differential of  $x$  is slightly different from the differential we normally encounter because though the path of the variable  $k$  is continuous, it is not differentiable in the normal sense. Consequently the normal rules of differentiation are not applicable and we must use a slightly different derivative concept, the I to derivative as in equation (6).

pasting conditions," a requirement that  $g'(k^u) = 0$  and  $g'(k^l) = 0$ . These two conditions provide sufficient boundary information to determine A and B.

To see that these conditions are valid in the cases studied by Krugman and Froot and Obstfeld, let us consider Figure 2, the diagram of Harrison's solution for discrete interventions. Define  $k^{\max}$  and  $k^{\min}$  as the solutions of  $g'(k) = 0$ . Note that  $k^u > k^{\max} > Q$  and  $k^l < k^{\min} < q$ . Assuring that the endpoints of the segment defined by  $I^u$  always lie on the solution curve at the same vertical height, let  $I^u$  converge to zero. Similarly, let  $I^l$  converge to zero.  $Q$  and  $k^u$  will then converge to  $k^{\max}$  and  $q$  and  $k^l$  will converge to  $k^{\min}$ . Therefore, for infinitesimal interventions,  $g'(k^u) = g'(k^l)$ , the smooth pasting condition. The smooth pasting condition is applicable for infinitesimal interventions, but it is inapplicable for finite interventions. Similarly, the assumption that the TZ boundary points are attained simultaneously with the  $k$  boundary points is true only in the case of infinitesimal intervention.

# 1. Continuity at the boundaries

What is the intuition behind the continuity of the exchange rate in the presence of discrete foreseen interventions? This is a natural requirement familiar from the speculative attack literature. If there were a discontinuity in response to an anticipated intervention i.e., if  $g(k^u)$  did not equal  $g(k^u - I^u)$ , there would be a foreseeable profit at an infinite rate. Speculators would act to remove this opportunity.

Recall that in the speculative attack literature of Krugman (1979) or Flood and Garber (1984), domestic credit is usually rigged with upward drift. A foreseen attack on a fixed exchange rate system is timed to prevent an expected discontinuity in exchange rates. In the context of equation (1) exchange rate continuity in the face of an expected attack on the fixed rate regime requires the exchange rate to be the same in the instant just before and the instant just after the attack. Yet, the expected rate of change of the exchange rate would jump discontinuously from zero to a positive number, driving down money demand and accommodating the sudden loss of reserves at the moment of the attack.

The logic for a discrete foreseen intervention in a TZ is identical. When the distance from  $k$  to  $k^u$  is infinitesimally small, there will be a shift from  $k^u$  to  $Q$  with probability one. Since the expected value of the integral in equation (4) takes account of the shift, its value will not change when the shift occurs. Now evaluate equation (1) at  $k^u$  and  $Q$ , respectively, and subtract the two results. Since the exchange rate is the same at the two  $k$  values, the result is:

$$(9) \quad k^u - Q = I^u = -\alpha \left[ \left. \frac{Edx}{dt} \right|_{k=k^u} - \left. \frac{Edx}{dt} \right|_{k=Q} \right]$$



The discontinuous shift in  $k$  (e.g., the reduction in foreign reserves or bond holdings of the policy authority) exactly offsets the discontinuous shift in expectations.

In this TZ problem, the shift in the expected depreciation adjusts to the arbitrary magnitude of the intervention. In the speculative attack literature, the magnitude of the intervention adjusts to the specified shift in expected depreciation rates rigged into the problem. Otherwise, the problems are identical.

Indeed, there is even a run on reserves in the TZ model. When  $k$  (domestic credit) reaches a high enough level, speculators will approach the policy authority to convert domestic currency for reserves, thereby forcing an intervention of the prescribed magnitude. This monetary alignment crisis (MAC Attack) serves to preserve the TZ, however.

The relevant  $(A, B)$  pair for a specific intervention strategy that supports a particular TZ can be found by noting that  $x^u = g(k^{\max})$  and  $x^l = g(k^{\min})$ , where  $x^u$  and  $x^l$  are the announced TZ bounds. For arbitrary values of  $A$  and  $B$ , we can find  $k^{\max}$  and  $k^{\min}$  the critical values of  $g(k)$ , as functions of  $A$  and  $B$ . Substituting these functions into  $x^u = g(k^{\max})$  and  $x^l = g(k^{\min})$  yields two equations in the unknowns  $(A, B)$ . In the Appendix we sketch more completely the determination of  $A$  and  $B$  for a particular intervention strategy.

The continuity condition requires that  $g(k^u) = g(k^u - I^u)$  and  $g(k^l) = g(k^l + I^l)$ . These two conditions are also sufficient to determine  $A$  and  $B$  in equation (5) as functions of the policy quadruple  $(k^u, Q, q, k^l)$ . The pair  $(A, B)$  associated with a specific TZ can therefore be generated by an infinite number of combinations of values for the inner and outer bounds on  $k$ .

In this sense, simply announcing a specific TZ is not a well-specified policy. A particular TZ can be supported by an infinity of intervention strategies. This incompleteness is not an indictment of the TZ policy, which was designed as a part of a broader policy. Indeed, the incompleteness of the TZ allows policy to be directed toward other goals with only occasional attention to the maintenance of the TZ.

## 2. A discrete intervention with $Q < q$

An interesting case occurs when  $Q < q$  as depicted in Figure 3. In this case, the exchange rate can float downward toward the floor of the TZ even though  $k$  approaches its upper bound,  $k^u$ . When  $k$  reaches  $k^u$  a large intervention to reduce  $k$  occurs although the exchange rate is near or even at its minimum level. Thus, in a situation in which an intervention to purchase reserves appears necessary to prevent breaching the floor of the zone, the intervention instead consists of a large reserve sale aimed at preventing the breaching of the zone's ceiling.

### 3. Intervention uncertainty

In our analysis so far we have assumed a preannounced intervention strategy defending a sharp TZ, i.e., a preannounced pair  $(I^u, I^l)$ . In practice, however, it is unlikely that the intervention strategy would be precisely preannounced and we find interesting the possibility of "soft zones," which we formalize as a probability distribution concerning the boundaries of the TZ. <sup>1/</sup> In this situation we need to examine some additional possibilities to find a formula for  $x$  inside the TZ. In particular, recall that a "solution" for the exchange rate must be of the form given by equation (3). To use equation (3) in the current setting let:

$$(10) \quad E[dx]/dt = \int \int E([dx|n,y]/dt) f(n,y) dndy,$$

where  $n$  is an intervention strategy  $y$  is a TZ boundary pair and  $f(n,y)$  is the distribution function giving the appropriate probability density for any  $n,y$ . For any  $n$  and for any  $y$  we can solve the model conditionally as above to get the appropriate values of  $A$  and  $B$ . We then calculate the unconditional expected rate of change of  $x$  from equation (10) and substitute the result into equation (1) to find the value of the exchange rate with stochastic intervention policy.

Since the actual intervention can surprise private agents, intervention can result in seemingly perverse exchange rate movements. To see this point examine Figure 2. The figure was predicated on the exchange authority's intervening in amount  $I^u$  when  $k$  reaches its upper bound. If instead the authority were to intervene by less than  $I^u$  then the currency would actually depreciate.

### 4. The volatility of exchange and interest rates

Krugman (1988) showed that the TZ will stabilize the exchange rate. This result occurs because the function  $g(k)$  is almost everywhere less responsive to  $k$  than is the functional relation between  $x$  and  $k$  in a pure floating regime.

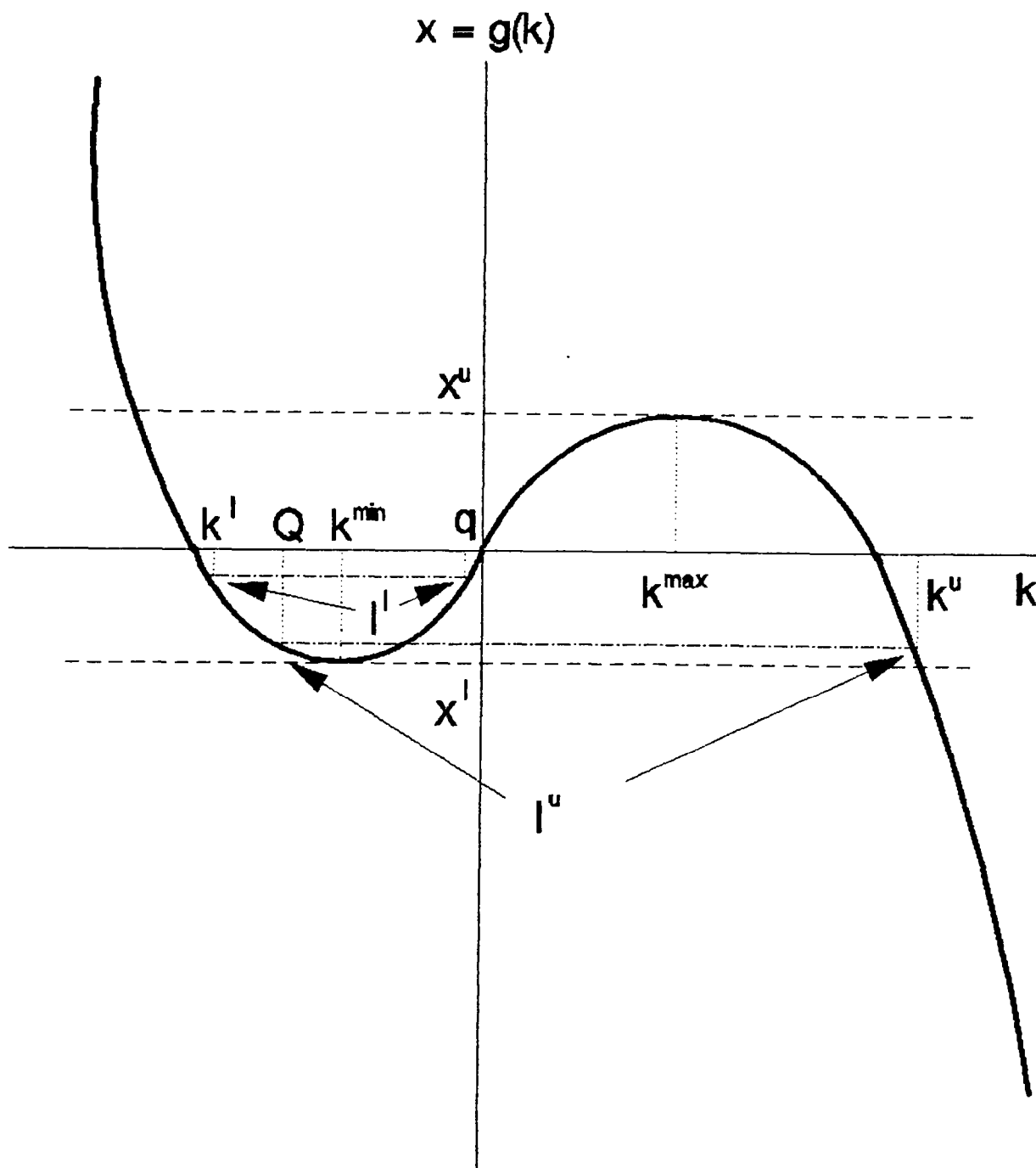
Yet, the possibility of achieving some exchange rate stability without actually having to intervene has the disturbing appearance of a free lunch. Where does the volatility go?

To address this question, let us assume that the domestic interest rate  $i$  equals the constant foreign interest rate plus the expected rate of

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<sup>1/</sup> We assume an information structure such that there is no learning about the nature of intervention or the boundaries of the zone. In our version both the nature of intervention and the soft boundaries will be random exogenous variables drawn from a time-invariant distribution.

Figure 3  
Large Discrete Intervention with  $Q < q$





depreciation. Since the foreign interest rate is constant, the only volatility in the domestic interest rate stems from movements in  $E[ds]$ . Then

$$(11) \quad V(i(k), t) = \alpha^2 V(g(k) - k, t)$$

where  $V(y, t)$  is the variance of the variable  $y$  over an interval of length  $t$ .

For both fixed and floating exchange rates  $V(i(k), t) = 0$  in the constant drift cases that we have considered. Since  $g(k) - k$  is not constant for the TZ, however,  $V(i(k), t) > 0$  in the target zone. Thus the exchange rate becomes less volatile at the expense of raising the interest rate volatility.

#### IV. Conclusion

Public and private sector agents often adopt one set of actions when some indicator is within certain bounds and switch to another set of actions when the bounds are reached. Examples of such action patterns are widespread in policy making circles and seem to describe a possibly important part of the policy making process not captured linear rules.

The purpose of the present paper was to extend the literature to the possibility of finite-sized and stochastic interventions in defense of the TZ. We found the terminal conditions required both for the finite-size interventions and for the infinitesimal interventions to be most easily understood by relating them to the mirror image problem of describing an anticipated speculative attack.

Explicit Formulas for the TZ

In this Appendix we sketch the development of the explicit formulas required to find A and B when the TZ is defended by finite interventions. Recall equation (9) in the text. This equation shows the shift in the expected rates of change of the exchange rate required to accommodate an intervention of size  $I^u$ . An analogous formula holds, but is not displayed, for a finite intervention from below in size I. Note that the two expected rates of exchange rate change in equation (9) may be developed in terms of A and B by using equations (5) and (7), which imply:

$$\begin{aligned} \left. \frac{Edx}{dt} \right|_{k=k^u} &= \eta[1 + \lambda_1 A \exp \lambda_1 k^u + \lambda_2 B \exp \lambda_2 k^u] \\ &+ (\sigma^2/2)[\lambda_1^2 A \exp \lambda_1 k^u + \lambda_2^2 B \exp \lambda_2 k^u], \end{aligned}$$

$$\begin{aligned} \left. \frac{Edx}{dt} \right|_{k=Q} &= \eta[1 + \lambda_1 A \exp \lambda_1 Q + \lambda_2 B \exp \lambda_2 Q] \\ &+ (\sigma^2/2)[\lambda_1^2 A \exp \lambda_1 Q + \lambda_2^2 B \exp \lambda_2 Q] \end{aligned}$$

These two equations may be substituted into equation (9) to give one equation in A and B. A second equation in A and B may be developed by considering intervention at the lower bound, which may be finite as above or infinitesimal as in Krugman and Froot and Obstfeld.

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