IMF WORKING PAPER

WP/89/19

INTERNATIONAL MONETARY FUND

Research Department

Dual Exchange Markets Under Incomplete Separation: An Optimizing Model

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March 1, 1989

Abstract

This paper constructs and analyzes an optimizing model of dual exchange markets which are incompletely separated owing to the presence of fraudulent cross transactions. The model is used to examine the implications of certain shocks, including devaluation. Devaluation first leads to the emergence of a spread with the financial exchange rate being relatively appreciated vis-a-vis the commercial rate. Over time, the financial rate depreciates beyond the level of the commercial rate. In the final phase of adjustment, the spread declines continuously until a zero spread is restored.

JEL Classification Numbers

3116, 4312, 4314

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*This paper was initiated while Carlos Végh was in the Research Department. The authors acknowledge helpful comments by Guillermo Calvo, Pablo Guidotti, Mohsin Khan, and Jonathan Ostry on an earlier draft of this paper. The usual disclaimer applies with respect to any remaining errors.

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Summary

This paper constructs and analyzes a model of dual exchange markets which are incompletely separated owing to the presence of fraudulent transactions. Such transactions are known to be widespread in the countries that adopt such an exchange rate system. While the presence of illicit cross-transactions between exchange markets has been incorporated in previous theoretical analyses, the present treatment of the phenomenon is somewhat more realistic in its treatment of fraudulent transactions. At the same time, the costs of engaging in illicit activity are also more comprehensively treated in that the possibility of both indirect transactions costs (such as bribing of officials, for example) and compulsory forfeiture of illicitly-acquired foreign assets is recognized and taken into account in the analysis.

The framework is utilized to examine the effects of certain domestic disturbances,in particular, devaluation and real government expenditure disturbances. It is seen that in the stationary state the spread between the financial and commercial exchange rates is zero. In this sense, the viability of dual exchange rate regimes is clearly limited. Upon devaluation of the official rate, however, a spread does emerge with the (floating) financial rate being relatively appreciated vis-a-via the commercial rate. Thus, foreign currency is available at discount in the financial market. With the passage of time the discount diminishes progressively, turning into a premium in the financial market. The final phase of adjustment involves a declining premium until the steady state with a zero spread is ultimately restored. This oscillatory response is directly attributable to the presence of inter-market leakage and other optimizing models which abstract away from leakage are incapable of generating this type of response involving first, a financial discount and subsequently, a premium in the market. This type of adjustment is more than a theoretical curiosity and examination of several actual country experiences reveals a pattern that conforms with our theoretical predictions.

I. Introduction

There has been a great deal of renewed interest recently in the operation of multiple exchange rate regimes as represented, for example, by the number of recent contributions in this area (see Bhandari (1988) and Guidotti and Végh (1988) for detailed bibliographies). Typically, dual exchange markets encompass a fixed rate for trade transactions ("commercial rate") and a flexible rate for financial transactions ("financial rate"). An important feature of some of the new literature is the recognition that such multiple rate regimes often involve "leakage" between the two exchange markets. The presence of cross-transactions implies that despite the flexibility of the financial exchange rate, the capital account of the economy in question is non-zero and net accumulation/decumulation of foreign assets may thus occur. This process has been noted in the early literature (for example, Lanyi (1975)) and has been explicitly incorporated in non-optimizing models such as Bhandari and Decaluwe (1987), Lizondo (1987), and Guidotti (1988). The incorporation of the phenomenon of cross-transactions between the two markets has apparently not been extended to optimizing models. Thus, recent analyses of dual exchange markets incorporating optimizing behavior continue to assume that exchange markets can be, and in fact are, perfectly segmented (see for example, Guidotti and Végh (1988) and Obstfeld (1986)). 1/ In large part, this omission is due to the analytical difficulties encountered in attempting to deal with leakage in optimizing models, as these models become very complicated.

The present paper is an attempt to incorporate leakages into an optimizing framework. Specifically, the model we construct is similar in some respects to the complete separation model proposed by Guidotti and Végh (1988). At the same time, inter-market leakage is explicitly introduced into the framework. While such leakage is examined in non-optimizing models such as Bhandari and Decaluwe (1987), the present treatment of this phenomenon is somewhat more realistic in its treatment of overinvoicing/underinvoicing associated with fraudulent transactions. At the same time, the costs of engaging in illicit activity are also more comprehensively treated in that the possibility of both indirect transactional costs (such as bribing officials, for example) and compulsory forfeiture of illicitly acquired foreign assets is duly recognized.

The framework is utilized to examine the effects of certain domestic disturbances; in particular, devaluation and real government expenditure disturbances. Certain results of interest may be noted at this point. First, in the steady state the spread between the financial and commercial

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1/ The possibility of inter-market transactions is briefly noted in Flood and Marion (1988). Tornell (1988) also recognizes the possibility of leakage; however, the model he uses abstracts from consumption-savings decisions and illegal transactions are carried out by risk-neutral investors.

exchange rates is zero. 1/ This result is thus in conformity with Frenkel and Razin (1986) who also note the limited viability of dual rate regimes. In the absence of additional policies, upon devaluation of the commercial exchange rate, a spread does emerge with the financial rate being relatively appreciated vis-a-vis the commercial rate. Thus, foreign currency is available at a discount in the financial market. With the passage of time, the discount diminishes progressively, turning into a premium in the financial market. The final phase of adjustment involves a declining premium until the steady-state with a zero spread is ultimately restored. This non-monotonic response is directly attributable to the presence of inter-market leakage and other optimizing models which abstract away from leakage (for example, Obstfeld (1986) and Guidotti and Végh (1988)) are incapable of generating this type of response involving, first, a financial rate discount and, subsequently, a premium in the market.

This type of oscillatory adjustment of the spread is much more than a mere theoretical curiosity. Examination of several actual country experiences reveals precisely such behavior of the spread. For example, Chart 1 which is reproduced from Kamin (1988) depicts the pattern of the spread computed as an average for selected countries which engaged in devaluation. 2/ Inspection of this Chart reveals the presence of an oscillatory pattern of adjustment in the spread after devaluation at t=1, which is in broad conformity with our theoretical predictions. In addition to predicting an oscillatory pattern of adjustment in the spread, the model also suggests a similar pattern for the capital account. Specifically, devaluation of the commercial exchange rate is followed by successive phases of capital inflows and outflows. Such a pattern is somewhat harder to corroborate because of the difficulties in obtaining data on illegal capital movements. There are other detailed results of interest, in particular those relating to a change in real expenditure, that are discussed below. These non-monotonic movements in the spread and the capital account may have important implications for policy-makers. For instance, a policy of devaluation (undertaken perhaps, to achieve unification of exchange rates) will clearly appear counter-productive during the phase of increasing spreads, which as indicated above, occurs in the second phase of adjustment. In addition, it is also apparent that devaluation will generate (sequentially) both capital inflows and outflows and proper account of this possibility needs to be taken in any liberalization program that envisions devaluation as one of its ingredients.

In what follows, Section II presents a simple optimizing model of a dual exchange rate economy where no leakage is assumed to occur. This model is intended to serve as a benchmark case in order to facilitate comparison with the more substantive model incorporating leakage that is developed in Section III. The last section offers some concluding

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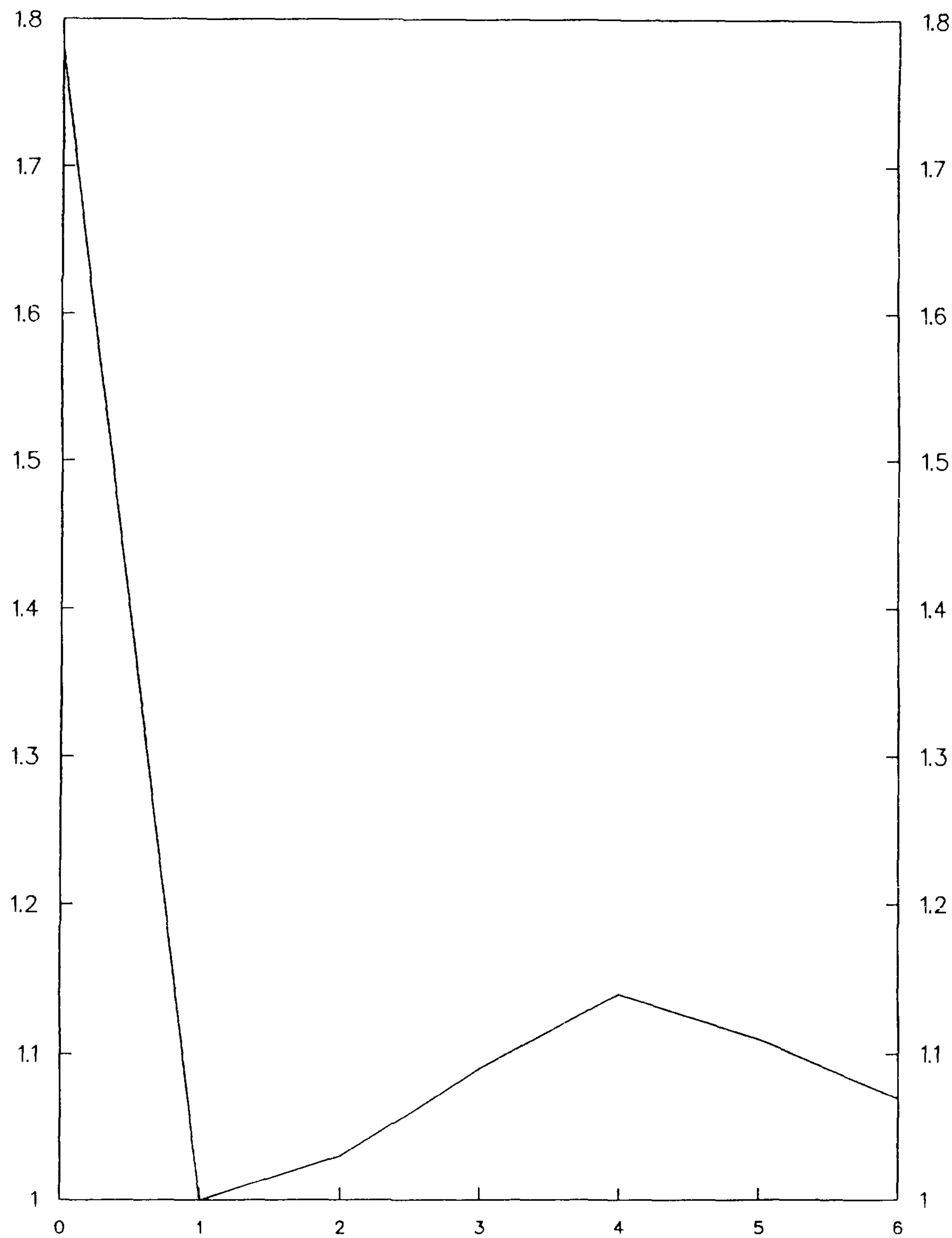
1/ In non-optimizing models such as Lizondo (1987), for example, a steady-state spread may be indefinitely maintained.

2/ See Kamin (1988) for details.

CHART 1

BLACK MARKET PREMIUM



Source: Table 2 – *Devaluation, Exchange Controls, and Black Markets for Foreign Exchange in Developing Countries* by Steven B. Kamin.

observations and also notes possible caveats and extensions to the present analysis.

II. The Dual Exchange Rate Economy without Cross-Transactions

This Section discusses the case of a dual exchange rate economy wherein the two exchange markets are perfectly segmented. The model discussed here is a particular case of the more general framework of the next section. Nevertheless, it is presented to enable one to appreciate the novel features introduced by the presence of leakage in the more general setting.

The framework is a simplified small country version of the two-country (complete separation) cash-in-advance model discussed by Guidotti and Végh (1988). The exchange rate regime in place involves a pegged commercial rate ($\bar{E}$) and a freely flexible financial rate (Q). Because the financial rate is flexible and because of complete market separation, the real quantity of external assets (i.e., bonds) in the hands of the private sector is fixed at $\bar{b}$. 1/ By contrast, as seen in the next Section, the possibility of leakage permits domestic residents to purchase or sell foreign bonds. The ratio of the commercial rate to the financial rate is denoted by q; namely $q \equiv (Q/\bar{E})$, which can be interpreted, as will be seen below, as the real (domestic) price of bonds. The world interest rate is a constant equal to r. For purposes of this Section, it will be assumed that there is only one (non-storable) good in the world whose domestic price is given via a purchasing power parity relationship, i.e., $P = \bar{E}P^*$, where P and P^* denote the domestic and foreign price level respectively. In what follows, the foreign price level P^* is set equal to unity for simplicity.

The representative domestic consumer is endowed with a constant stream of the good (y). The consumer must use money to acquire the good; namely $M = \alpha Pc$, where M stands for nominal money balances, c for consumption, and $\alpha > 0$. 2/

The consumer faces the following optimization problem:

$$\underset{\{c_t, m_t, b_t\}_{t=0}^{\infty}}{\mathrm{MAX}} \int_0^{\infty} U(c_t)\exp(-\delta t)dt$$

1/ The foreign price level is assumed to be constant so that both the nominal and real quantities of foreign bonds in the hands of domestic residents is fixed.

2/ Feenstra (1985) derives cash-in-advance constraints in continuous-time models. For applications, see Calvo (1986, 1987a).

subject to:

$$\dot{a} = y + \frac{(r + \dot{q}_t)}{q_t} q_t b_t - c_t + \tau_t \tag{1}$$

$$m_t = \alpha c_t \tag{2}$$

$$a_t \equiv m_t + q_t b_t \tag{3}$$

where a dot over a variable denotes its time derivative; the utility function, U(.), is assumed to be increasing, twice-continuously differentiable, and strictly concave; δ denotes the constant rate of time preference; $m \equiv M/P$ denotes real money balances; and τ stands for real transfers from the government. Equation (1) is the asset accumulation relationship; note that the real return on bonds equals $(r+\dot{q})/q$ (hereafter denoted by ρ) which is the domestic real rate of interest. As is well known, a dual exchange rate regime drives a wedge between the world and the domestic real interest rates. Equation (2) is the cash-in-advance constraint. Equation (3) defines real wealth.

The other actor in this economy is the government. It faces the following budget constraint:

$$\tau_t = rf_t - \bar{g} \tag{4a}$$

where f denotes holdings of interest-bearing reserves and $\bar{g}$ indicates the (constant) stream of real government spending. 1/

Money supply is given by:

$$m = f + \frac{D_0}{\bar{E}} \tag{4b}$$

where D_0 denotes the initial stock of domestic credit.

Solving the consumer's problem, imposing the equilibrium condition $b_t = \bar{b}$, and taking into account (4), the following differential equation system is obtained:

1/ As pointed out by Obstfeld (1986), the fact that reserves earn interest implies that, under fixed exchange rates, devaluation is neutral.

$$\dot{M} = \bar{E}y - (\frac{1}{\alpha} - r)M + r\bar{E}\,\bar{b} - rD_o - P\bar{g} \tag{5}$$

$$\dot{\rho} = \frac{U_{cc}}{\lambda\alpha^2 P} M - (\frac{1+\alpha\rho}{\alpha})(\delta-\rho) \tag{6}$$

$$\dot{q} = \rho q - r \tag{7}$$

where time subscripts have been dropped for convenience; and a subscript, unless otherwise indicated, indicates a partial derivative.

As can be observed, the system is block recursive. In fact, equation (5), by itself, determines the behavior of M. Assuming that $[(1/\alpha)-r]>0$, equation (5) provides the only negative root of the system. Since there is only one state variable, M, the system exhibits saddle-path stability. As shown in Guidotti and Végh (1988), M and ρ, as well as ρ and q, move in opposite directions.

1. Devaluation

Consider an unexpected and permanent devaluation; i.e., an increase in $\bar{E}$. 1/ The path of the different variables is illustrated in Figure I. The increase in $\bar{E}$, through the law of one price, implies a rise in the domestic price level that reduces real money balances. Owing to the assumption of a freely fluctuating financial rate coupled with complete market separation the public is prevented from exchanging bonds for money (which would have been possible under fixed exchange rates). Thus, the public cannot instantaneously restore its real money balances. Therefore, it follows from the cash-in-advance constraint that consumption also falls on impact, thus generating a current account surplus. Given that the economy will be running current account surpluses during the adjustment path, real money balances and hence consumption will also be increasing over time. 2/ The public foresees this increasing consumption path and bids down the real price of bonds, q, by attempting to sell bonds in order to smooth out consumption over time. Given that, in the aggregate, this is not feasible, the real domestic interest rate rises on impact and remains above its steady-state level throughout the adjustment path in order to induce such a consumption path before returning to its initial level.

The fact that there is only one state variable, M, ensures that the adjustment of all variables is monotonic. In particular, q rises towards

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1/ It is assumed, as it is usually the case, that the Central Bank does not monetize the nominal capital gains on reserves but instead creates a fictitious non-monetary liability.

2/ Note that, even if consumption rises over time, it is initially at a lower level than before the disturbance.

its initial value, given by (r/δ). 1/ Both nominal money balances and consumption are higher in the new steady state since the economy runs current account surpluses during the adjustment path. 2/

2. Decrease in government spending

Consider an unanticipated and permanent decrease in government spending. The path of the different variables is the same as that illustrated in Figure I except for the behavior of consumption. Since real money balances are given on impact, the decrease in government spending is translated one-for-one into a current account surplus. Consumption remains unchanged on impact but increases over time because money flows into the country during the adjustment period. Individuals foresee this increasing consumption path and try to smooth out consumption over time by selling bonds, thus bidding down their real price, q. Subsequently, q increases over time to its initial value. Naturally, in the aggregate, this increasing consumption path cannot be smoothed out so that the domestic real interest rate rises on impact and remains above its steady-state level to induce such a path. Consumption is higher in the new steady state due to the current account surpluses run by the economy. Again, the adjustment of all variables, in particular that of q, is monotonic.

III. The Dual Exchange Rate Economy with Leakage

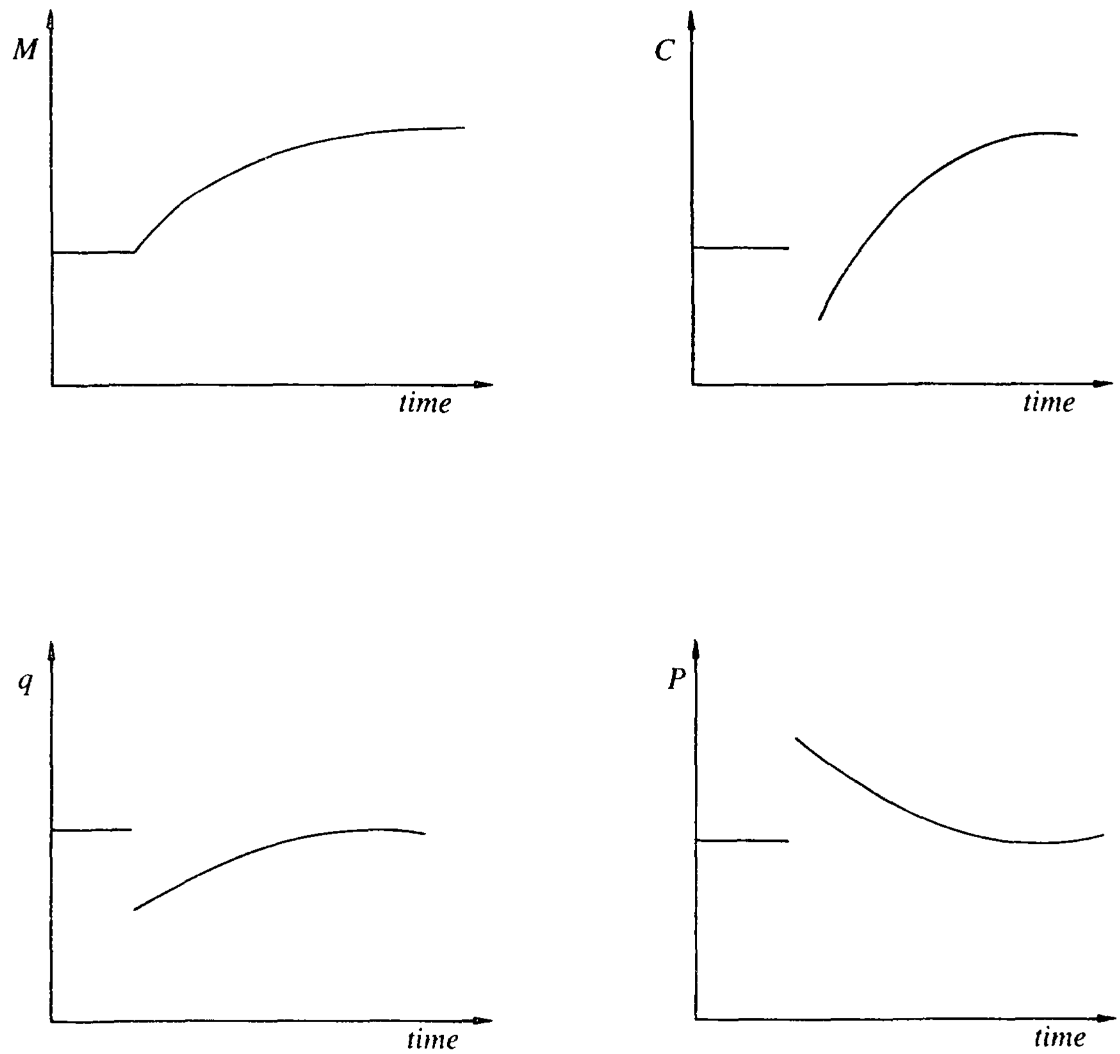
This Section modifies the benchmark model of the previous section in order to allow for the possibility of cross-transactions (leakage) between the two exchange markets. The economy is endowed with an exportable good (y) which it does not consume. Instead, the economy consumes an importable good (C). It is convenient to begin the description with a specification of the budget constraint of the representative individual in nominal terms. Specifically, this is given by (only new variables are specified below; otherwise, the definitions of the previous section apply):

$$\dot{A} = \bar{E}y + I(Q-\bar{E}) - \bar{E}h(I) - \gamma(qb)Qb + \bar{E}br + \dot{Q}b - \bar{E}c + \bar{E}\tau \qquad (8)$$

1/ The behavior of q will be dramatically altered when leakages between the commercial and financial markets are incorporated, as will be shown in the next Section.

2/ As noted by Obstfeld (1986), the fact that steady-state consumption is higher does not mean that a devaluation is welfare improving. To the contrary, the losses in utility attributable to diminished consumption in the short run outweigh the gains from a higher steady-state consumption. This follows from the fact that, for given resources, a constant path of consumption is preferable to a non-constant path.

Figure I: Effects of a devaluation under complete market separation



where A represents nominal wealth and I measures the real amount of leakage which is characterized by fraudulent overinvoicing/underinvoicing of exports. To the extent that the financial rate is relatively depreciated vis-a-vis the commercial rate, the term $I(Q-\bar{E})$ measures the addition to the consumer's resources resulting from successfully underinvoicing exports. Similarly, if the financial rate is relatively appreciated with respect to the commercial rate, $I(Q-\bar{E})$ measures the profits made from overinvoicing exports. It should be noted that, at an aggregate level, the stock of legally acquired bonds is fixed. Therefore, in the aggregate, the actual stock of bonds changes solely as a result of overinvoicing/underinvoicing of exports. The fraudulent activity is not without costs, however, and (8) recognizes two types of costs imposed upon the trader. First, h(I) is intended to capture recurring transactional costs as would be incurred, for example, in the corrupt procurement of official approval or acquiescence in a de facto illegal transaction. In what follows, we assume a specific functional form for h, i.e., $h(I) = \frac{I^2}{2(1-\phi)}$ where $0 \leq \phi < 1$. 1/ We interpret ϕ as the indirect transactional cost associated with the maintenance of illicit activity and a rise in ϕ indicates an increase in such costs. The second variety of cost imposed upon the trader is a forfeiture penalty given by the term $\gamma(qb)Qb$, where $0 \leq \gamma < 1$ is the fraction of bonds forfeited and is assumed to be an increasing function of qb; that is, the stock of bonds valued at the financial rate. 2/ Specifically, due to the administrative burdens associated with attempting to distinguish lawfully from illicitly acquired securities, it is assumed that the authorities seize and confiscate a portion of all bonds in the possession of the trader. 3/ The remaining terms in (8) are similar to those in equation (1) in the simplified model. Thus, $\bar{E}br$ is the interest proceeds on holding foreign assets, $\dot{Q}b$ is the capital gain term while $\bar{E}\tau$ and $\bar{E}c$ refer to nominal transfers and consumption respectively.

1/ This functional form permits considerable simplicity in that the resulting expressions are linear.

2/ Note that the present model is not stochastic and as such, there is no uncertainty regarding the eventual detection by the authorities of the fraudulent activities. It will also be assumed that γ, the fraction of bonds confiscated takes the linear form $\gamma(qb) = \beta qb$, where β is a constant which is small enough to ensure that γ remains below unity. Thus, the larger the amount of bonds in the possession of the trader, the higher the portion of bonds that is forfeited.

3/ A more natural assumption would be that the confiscation carried out by the authorities applies only to those bonds illegally acquired. This assumption, however, would render the model analytically intractable. While the assumptions just spelled out are undoubtedly special ones, they are needed for analytical tractability. If these assumptions were not made, it would be necessary to resort to numerical methods which are necessarily limited in generality.

For purposes of analysis it is preferable to recast (8) into real terms analogously to (1), taking into account the functional forms for $h(I)$ and $\gamma(bq)$. Thus, after some rearrangement, the budget constraint may be expressed as

$$\dot{a} = y + (q-1)I - \frac{I^2}{2(1-\phi)} + (r+\dot{q}-\beta bq^2)b - c + \tau \tag{9}$$

where $\dot{a} \equiv \dot{A}/\bar{E}$ and $q \equiv Q/\bar{E}$.

The representative consumer's maximization problem may now be stated as:

$$\underset{\{c_t, m_t, b_t, I_t\}_{t=0}^{\infty}}{\text{MAX}} \int_0^\infty U(c_t)\exp(-\delta t)dt$$

subject to (9) and

$$m = \alpha c \tag{10}$$

$$a \equiv m+qb \tag{11}$$

The optimality conditions for this control problem are given by:

$$U_c = \lambda(1+\alpha\rho) \tag{12}$$

$$\dot{\lambda} = \lambda(\delta-\rho) \tag{13}$$

$$I = (1-\phi)(q-1) \tag{14}$$

$$\dot{q} = \rho q-r+2\beta q^2 b \tag{15}$$

where:

$$\rho \equiv \frac{r}{q} + \frac{\dot{q}}{q} - 2\beta bq \tag{16}$$

Equation (12) is the familiar condition equating the marginal utility of consumption to the marginal utility of wealth, λ, times the cost of consumption. The cost of consumption is given by its price (unity), plus the opportunity cost of holding the α units of money needed to purchase a unit of the good, $\alpha\rho$. The expression for the domestic real interest rate, given by (16), is worth noting since it is affected by the presence of the forfeiture penalty. If the forfeiture rate were zero, i.e., $\beta=0$, the domestic real interest rate would be the same as in the previous case. When β is positive, however, the forfeiture rate affects the real return on bonds because the consumer is deprived of a fraction βbq of his bond holdings. Thus, the real return on bonds now depends upon the level of bond holdings. Equation (13) describes the path of the co-state variable. Equation (14) indicates the instantaneous level of leakage that results from falsifying export invoices. As one would expect, a positive (negative) spread induces underinvoicing (overinvoicing), which results in asset accumulation. Equation (15) indicates the path of q which is influenced by the forfeiture rate.

It will be assumed that the bonds that are seized are returned to the consumer in the form of lump-sum transfers. As before, government spending is exogenously given. The government budget constraint is thus:

$$\tau = rf + \beta q^2 b^2 - \bar{g} \tag{17}$$

In equilibrium, the stock of bonds held by the private sector can only change as a result of fraudulent transactions; namely,

$$\dot{b} = I \tag{18}$$

Combining the consumer's optimality conditions (9) - (15) with (17), (18), and the definition of money supply (given by (4b)) the following four-equation dynamic system is obtained:

$$\dot{M} = \bar{E}y - (\frac{1}{\alpha} - r)M - \frac{1}{2}(1-\phi)(q^2-1)\bar{E} + r\bar{E}b - rD_o - \bar{E}\,\bar{g} \tag{19}$$

$$\dot{\rho} = \frac{U_{cc}}{\lambda\alpha^2}\frac{\dot{M}}{\bar{E}} - \left\{\frac{1+\alpha\rho}{\alpha}\right\}(\delta-\rho) \tag{20}$$

$$\dot{q} = \rho q - r + 2\beta b q^2 \tag{21}$$

$$\dot{b} = (1-\phi)(q-1) \tag{22}$$

Equation (19) is the balance of payments equation. The presence of leakage implies that, in contrast to the simpler model (recall equation (5)), the level of q affects the change in money balances. This in turn implies that the balance of payments equation no longer determines by itself the behavior of real money balances. Equation (20) describes the evolution of the real domestic interest rate. Equations (21) and (22) are familiar by now (note that (22) results from substituting (14) into (18)). The presence of leakage introduces some additional technical difficulties for the analytical solution to the dynamic system (19) - (22) since there are now two state variables in the economy, M and b. 1/

Linearizing the system (19) - (22) around the steady state, we obtain:

$$\begin{bmatrix} \dot{M} \\ \dot{\rho} \\ \dot{q} \\ \dot{b} \end{bmatrix} = \Gamma \begin{bmatrix} M-\bar{M} \\ \rho-\bar{\rho} \\ q-\bar{q} \\ b-\bar{b} \end{bmatrix} \tag{23}$$

where

$$\Gamma \equiv \begin{bmatrix} -(\frac{1}{\alpha}-r) & 0 & -(1-\phi)\bar{E} & r\bar{E} \\ -\frac{U_{cc}}{\lambda\alpha^2\bar{E}}(\frac{1}{\alpha}-r) & \frac{1+\alpha\delta}{\alpha} & -\frac{U_{cc}}{\lambda\alpha^2}(1-\phi) & \frac{U_{cc}r}{\lambda\alpha^2} \\ 0 & 1 & 2r-\delta & 2\beta \\ 0 & 0 & (1-\phi) & 0 \end{bmatrix} \tag{24}$$

$$\bar{M} = \frac{\bar{E}\left[y + r\left(\frac{r-\delta}{2\beta}\right) - r\left(\frac{D_o}{\bar{E}}\right) - \bar{g}\right]}{(\frac{1}{\alpha}-r)} \tag{25a}$$

1/ The solution procedure used is based on Calvo (1987b).

$$\bar{\rho} = \delta \tag{25b}$$

$$\bar{q} = 1 \tag{25c}$$

$$\bar{b} = \frac{r-\delta}{2\beta} \tag{25d}$$

Equations (25) describe the steady-state values of the system. The steady-state value of q is unity, unlike the no-leakage case where it could differ from unity depending on the ratio (r/δ). A value of q different from unity would be inconsistent with a steady state because there would be net variations in the private sector's stock of bonds. The steady-state stock of bonds is constant (in the sense that it does not depend on either $\bar{g}$ or $\bar{E}$) because their real return depends on the amount that is held and thus, in the steady state, there is a unique level of bond holdings that enables the consumer to equate the real return of bonds to the marginal rate of time preference. The steady-state value of nominal money balances does not differ, conceptually, from the no-leakage case in that it depends positively on $\bar{E}$ and negatively on $\bar{g}$. Furthermore, if $r=\delta$, nominal (and real) money balances would be the same with or without leakage.

It follows from (23) that

$$\text{Trace of } \Gamma = 3r > 0 \tag{26a}$$

$$\text{Determinant of } \Gamma = (1-\phi)\left(\frac{1}{\alpha} - r\right) 2\beta \left(\frac{1+\alpha\delta}{\alpha}\right) > 0 \tag{26b}$$

The sign of (26b) follows from the assumption that $[(1/\alpha)-r]>0$. 1/ From (26a) it follows that there is at least one positive characteristic root. Given (26a), (26b) indicates that there are either four positive roots or two positive and two negative roots. The former case can be ruled out since it would imply instability of the system. The remaining case, i.e., two positive and two negative roots, ensures that, given initial conditions for M and b, there exists a unique convergent continuous path that satisfies (19) - (22)--provided, of course, that in the general solution the arbitrary scalars that multiply the two positive roots are set to zero. Since it has been found that there are two negative roots (or, more, precisely, two roots with negative real parts), the question of whether these roots can be either real or complex or both

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1/ Recall that this condition is necessary in the no-leakage model, which is a particular case of the present one, to ensure existence of the only negative characteristic root.

has to be addressed. The characteristic polynomial associated with matrix Γ is given by:

$$P(\mu) = \mu^4 - 3r\mu^3 + \left[-2\beta(1-\phi) + (\delta+r)(2r-\delta)-(\frac{1}{\alpha} - r)(\frac{1+\alpha\delta}{\alpha}) - D(1-\phi)\right]\mu^2$$

$$+ \left\{ (1-\phi)[2\beta(\sigma+r)+Dr] + (2r-\delta)(\frac{1}{\alpha} - r)(\frac{1+\alpha\delta}{\alpha}) \right\}\mu \tag{27}$$

$$+ (1-\phi)2\beta(\frac{1}{\alpha} - r)(\frac{1+\alpha\delta}{\alpha})$$

$$\text{where } D \equiv - \frac{U_{cc}}{\lambda\alpha^2}$$

Figure II depicts $P(\mu)$. Two pieces of information yield the conclusion that the two negative roots are real. First, it can be shown from (27) that, for any parameter configuration, $P(0)>0$ and second, $P\{-T \equiv -[(1/\alpha)-r]\}<0$. This implies, as Figure II makes clear, that the two negative roots are real roots, one being larger (in absolute value) than $-[(1/\alpha)-r]$ and the other being smaller. Denote these roots by μ_1 and μ_2, respectively. Letting $h_i = (h_{i1},h_{i2},h_{i3},h_{i4})$ be the eigenvector associated with root μ_i, i=1,2, and taking into account that $\Gamma h_i=h_i\mu_i$, where Γ is given in (24), we obtain:

$$\left[-(\frac{1}{\alpha} - r)-\mu_i\right]h_{i1} - \bar{E}(1-\phi)h_{i3} + \bar{E}rh_{i4} = 0 \tag{28a}$$

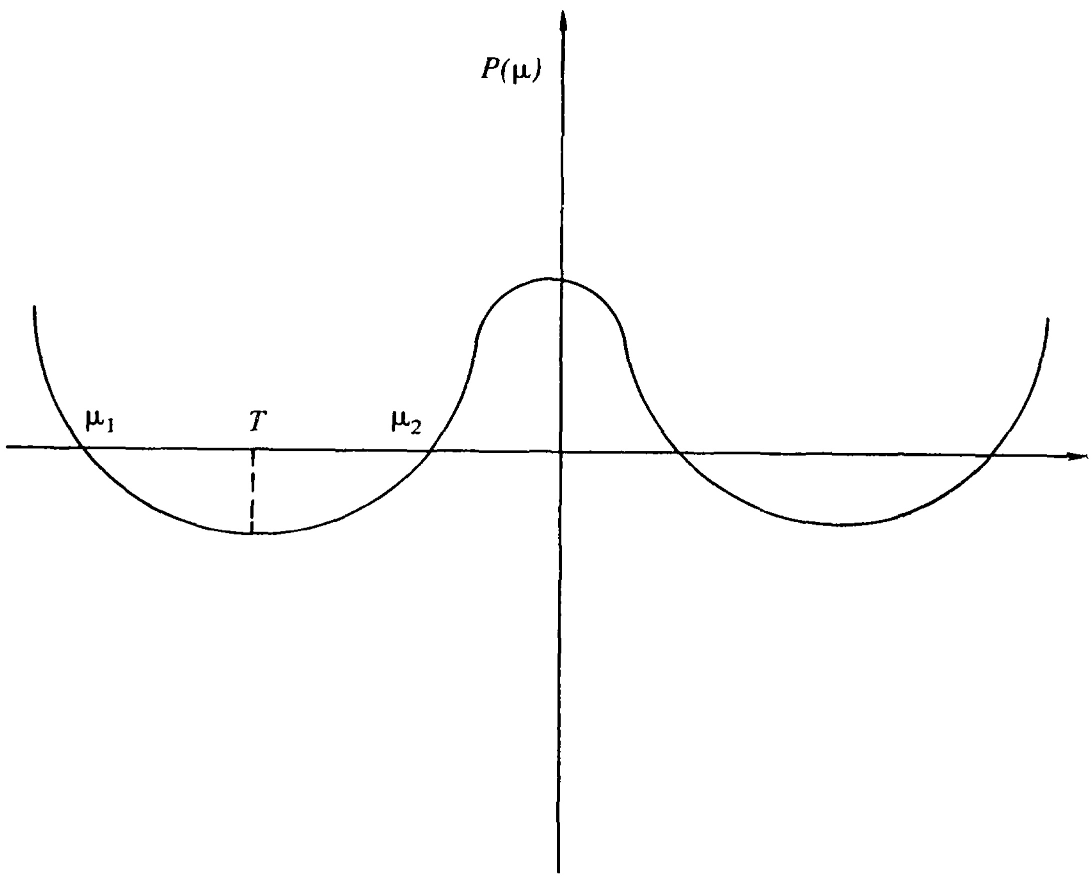
$$h_{i2} + (2r-\delta-\mu_i)h_{i3} + 2\beta h_{i4} = 0 \tag{28b}$$

$$(1-\phi)h_{i3} - \mu_i h_{i4} = 0 \tag{28c}$$

Since, by definition, an eigenvector cannot be a zero vector, equations (28) imply that all components of h_i, i=1,2, are different from zero. 1/ Having established this fact, we can now characterize the behavior (outside the steady state) of the ratio of the paths of q and b.

1/ For instance, if $h_{i1} = 0$ then (28a) implies that $(1-\phi)\ h_{i3} = rh_{i4}$. Substituting the latter into (28c) yields $(r-\mu_i)h_{i4} = 0$ whence it follows that $h_{i4} = 0$. This in turn implies, from (28a) that $h_{i3} = 0$ which (from (28b)) implies that $h_{i2} = 0$. The other cases are straightforward.

Figure II: Characteristic polynomial



Recall that the general solution for q and b is given by (after setting to zero the constants (ω) corresponding to the positive roots):

$$q_t-\bar{q} = \omega_1 h_{13}\exp(\mu_1 t) + \omega_2 h_{23}\exp(\mu_2 t)$$
$$b_t-\bar{b} = \omega_1 h_{14}\exp(\mu_1 t) + \omega_2 h_{24}\exp(\mu_2 t) \tag{29}$$

Along equilibrium paths which result from setting ω_i, i=1,2, equal to zero, we obtain (recall that we have established that all h_i's are different from zero):

$$\frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{h_{13}}{h_{14}} \tag{30a}$$

$$\frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{h_{23}}{h_{24}} \tag{30b}$$

It follows from (28a) and (28c) that:

$$\frac{h_{13}}{h_{14}} = \frac{\mu_1}{1-\phi} < 0 \tag{31a}$$

$$\frac{h_{23}}{h_{24}} = \frac{\mu_2}{1-\phi} < 0$$

These special paths are very useful in characterizing the dynamic behavior of the system. More specifically, the path $(q_t-\bar{q}/b_t-\bar{b})$ when μ_i is the greater (in absolute value) of the two negative roots will be of particular interest. Such a path is called "the dominant eigenvector ray" (Calvo (1987)). Recalling that μ_1 is the maximum negative root (in absolute value), the Appendix demonstrates the following proposition for the cases examined in this paper:

For any initial values of q and b that satisfy (25e) and (25d),

$$\lim_{t\to\infty} \frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{h_{23}}{h_{24}} < 0$$

The implications of this proposition are illustrated in Figure III. From (22), we can determine the direction of the movement of b in the two zones divided by the $\dot{b} = 0$ line. The above proposition indicates that the slope of the path $(q_t-\bar{q})/(b_t-b)$ eventually converges to the slope of the dominant eigenvector ray. This means that, depending on the value of q after the jump following an unanticipated disturbance, the adjustment path will follow either the arrowed curve going from A to 0 or that going from B to 0. It can be seen that in either case, the adjustment of both variables is non-monotonic. From this, the behavior of M can be derived by considering the (M,b) plane. Proceeding as before, it can be shown that

$$\lim_{t\to\infty} \frac{M_t-\bar{M}}{b_t-\bar{b}} = \frac{h_{21}}{h_{24}} > 0 \text{ , where } \frac{h_{21}}{h_{24}} = \frac{\bar{E}(\mu_2-r)}{[-(\frac{1}{\alpha}-r)-\mu_2]} > 0$$

The implication of the last statement for the adjustment path will become clear when we consider next the effects of a devaluation, of a decrease in government spending, and of a change in administrative regulations (parametrized by the parameter β).

1. Devaluation

Consider an unexpected and permanent devaluation; i.e., an increase in $\bar{E}$. The upper panel of Figure IV illustrates the behavior of M and b. The initial steady state is $(\bar{b},\bar{M}_0)$ (point A). As follows from (25), $\bar{b}$ remains constant across steady states while M increases so that the new steady state is given by $(\bar{b},\bar{M}_1)$ (point B). The arrowed curve describes the dynamic path of both variables from A to B. The arrowed curve cannot cross the dominant eigenvector ray (which slopes positively, as shown earlier) because if it did so, it would stay on it indefinitely since the dominant eigenvector ray is a solution to the differential system. It is clear that M increases monotonically towards its new steady state. The

Figure III: Adjustment paths of b and q

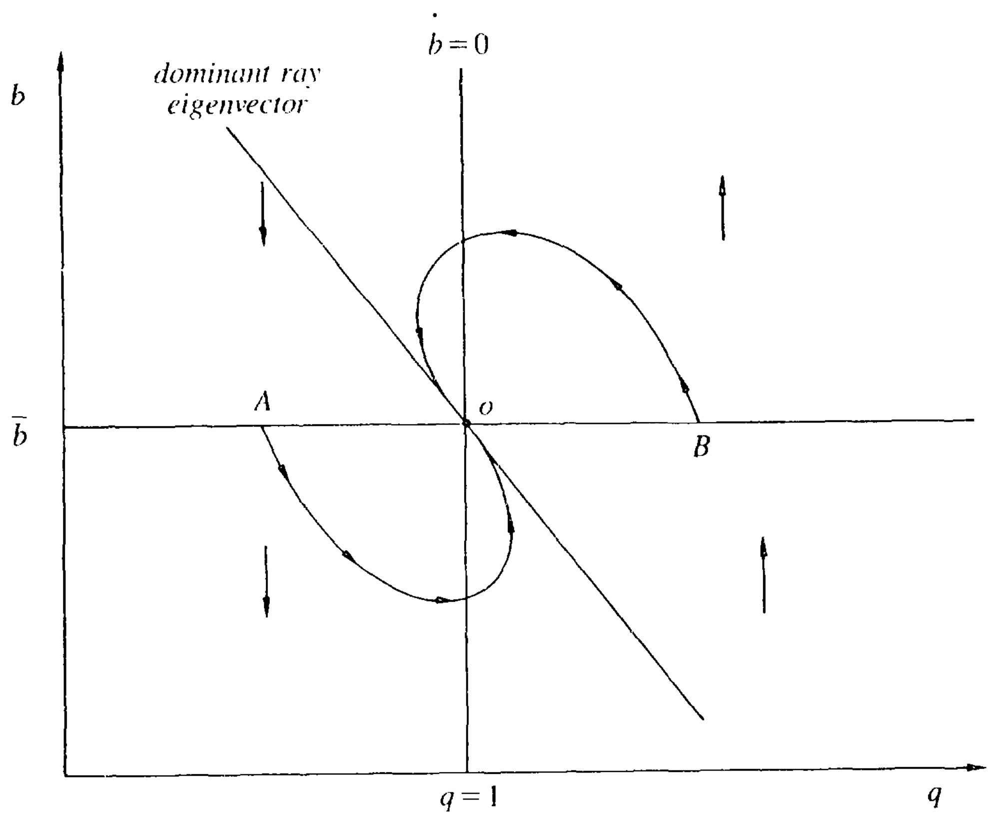
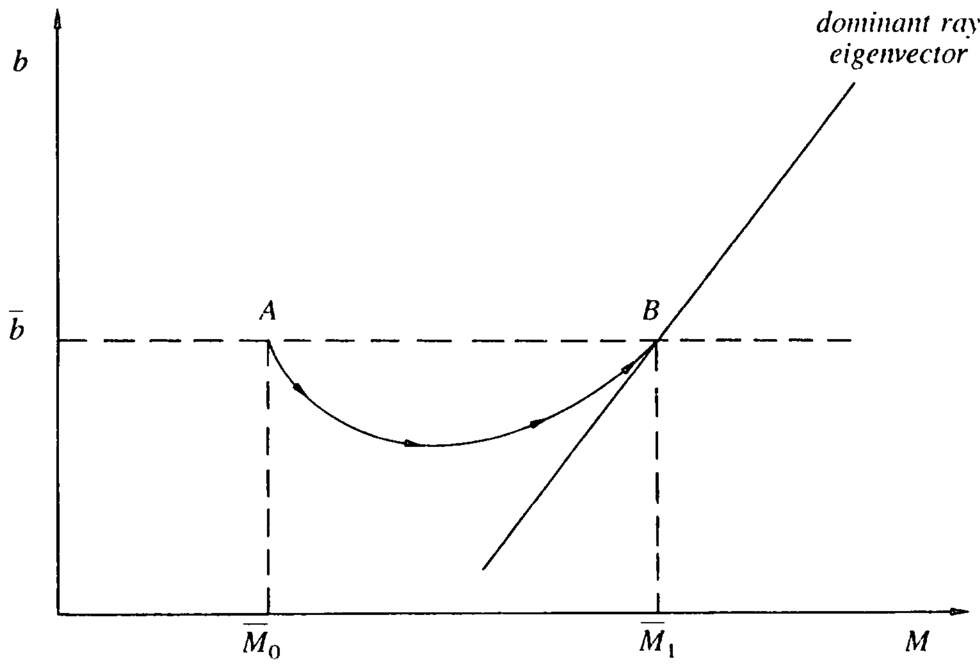
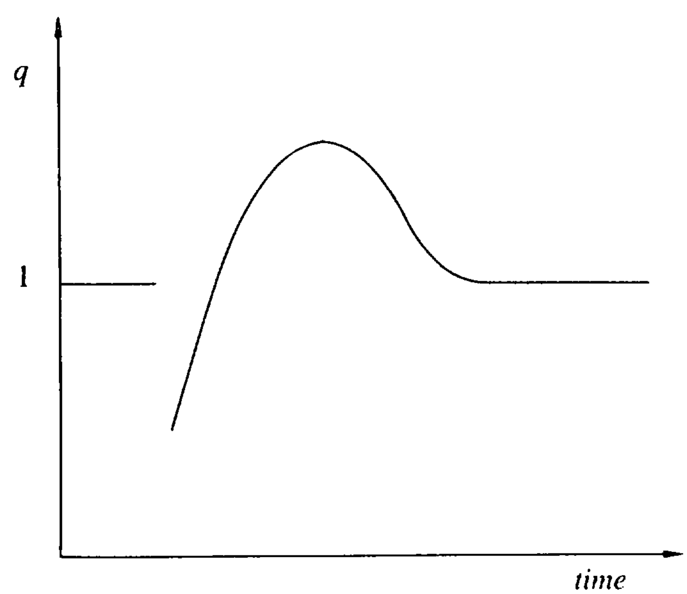


Figure IV: Effects of a devaluation under incomplete market separation





stock of bonds, however, exhibits non-monotonic behavior. 1/ It decreases initially (i.e., there are capital inflows), then reaches a stationary point, whereafter it begins to increase (capital outflows) and reaches the same initial value. The lower panel of Figure IV illustrates the path of q which follows from Figure III. On impact, q falls as it also does in the no-leakage case (recall Figure I). In the presence of leakage, however, as long as q remains below unity, the private sector has an incentive to overinvoice exports thus causing a decrease in the stock of bonds. The illegal reduction in the stock of bonds (i.e., capital inflow) continues until q reaches unity. Because the stock of bonds has to return to its original level (to equate the rate of return on bonds to the utility rate of time preference), q must overshoot its long run value (unity) in order to induce underinvoicing of exports and accumulation of bonds (i.e., capital outflows) which occurs until q returns to unity.

The path of consumption mirrors the behavior of real money balances and falls on impact while increasing over time to its new steady state. The domestic real interest rate, as was the case before, must rise on impact and remain above its steady-state level to induce this consumption path (see the Appendix for details). The behavior of M, c, and ρ is therefore qualitatively the same as in the no-leakage case illustrated in Figure I.

2. Decrease in government spending

Consider now an unexpected and permanent decrease in government spending. Figure IV also applies in this case. The new level of steady state nominal money balances is higher. Nominal money balances increase throughout the adjustment path (from A to B along the arrowed curve), i.e., the economy runs current account surpluses. There are capital inflows in the first phase of the adjustment followed by capital outflows, which return the stock of bonds to its initial level. For reasons analogous to the devaluation case, q jumps downward on impact, overshoots its long-run value, and then returns to unity. The behavior of consumption and the domestic real interest rate is the same as in the complete separation case.

1/ In principle, there are many possible paths from A to B in the (b,M) plane but, based on Figure III, we can infer that the path depicted in Figure IV is in fact the actual adjustment path. Figure III shows that b can follow only two possible paths. Since, when $\phi = 1$, this model degenerates into the complete separation model studied in the previous section, in which case a devaluation causes a fall in q, it should be clear that, at least for high values of ϕ, the same happens under incomplete market separation. Therefore, in the case of a devaluation, the path in Figure III, going from A to O, is the relevant one.

3. Change in administrative regulations

Consider an unanticipated and permanent increase in β. This reflects a greater ability from the part of the authorities to seize and confiscate bond holdings, a portion of which have been illegally acquired. From equation (25), it follows that steady-state nominal money balances and bond holdings fall. 1/ The decrease in bond holdings reflects the fact that a higher β decreases the real rate of return on bonds for a given level of bond holdings. Therefore, the consumer needs to reduce his bond holdings to equate their real return to the rate of time preference. This, in turn, decreases the interest received by the economy in the steady state which reduces nominal (and real) money balances. Proceeding as before, it is simple to show that the adjustment path will now involve a monotonic decrease not only of money balances (as has always been the case) but also of bond holdings. This implies, in turn, that unlike the previous cases, there is no overshooting of q. On impact, the private sector tries to get rid of its bonds holdings which provokes a fall in their real price, q. The fall in q below unity induces, over time, a capital inflow through underinvoicing of exports until q returns to its original level, unity, at which point the private sector has achieved its desired stock of foreign bonds.

IV. Conclusion

This paper has constructed and analyzed a model of multiple exchange rates characterized by the presence of fraudulent cross-transactions between the two exchange markets. Unlike previous analyses that have incorporated such "leakage", the present model is a fully optimizing one. In addition, we are able to treat the costs of engaging in illicit activity more comprehensively than prior work, in that both direct and indirect transactions costs are explicitly recognized.

The model is used to examine the effects of certain domestic disturbances such as devaluation of the commercial exchange rate, real government expenditure disturbances, and a change in administrative regulations relating to the authorities' ability to seize and confiscate illegally acquired assets. A new and interesting result is that following such disturbances, the behavior of at least some domestic variables, such as the exchange rate spread, is oscillatory. Specifically, upon devaluation of the commercial rate, for example, the financial rate is, in the first instance, relatively appreciated vis-a-vis the commercial rate. With the passage of time the discount diminishes progressively and turns into a premium with the financial rate being depreciated vis-a-vis the commercial rate. The final phase of adjustment involves a declining premium until the steady-state with a zero spread is ultimately restored.

1/ Naturally, in the particular case where $r=\delta$, a change in β does not affect the steady state because $b=0$ is the only level of bond holdings that is consistent with that parameter configuration.

As regards the stock of bonds, similar non-monotonic behavior is predicted. On impact, since the financial rate becomes relatively appreciated relative to the commercial rate, a process of reduction of the stock of bonds (i.e., capital inflows) takes place until the spread reaches zero. Afterwards, however, the financial rate becomes relatively depreciated which causes an illegal accumulation of bonds (capital outflows). These variations in the stock of bonds occur, in the aggregate, through underinvoicing/overinvoicing of exports. These patterns of adjustment appear to be in conformity with several actual experiences for the period following commercial devaluation. Previous models dealing with dual exchange markets have not been capable of explaining this empirically observed behavior.

The results of this paper may have important policy implications. For example, commercial devaluation, if undertaken in an effort to re-unify exchange rates and with no change in other policies, is seen to result during the transitional phase, in an increasing exchange rate spread and may thus appear counter-productive. Similarly, our results indicate that devaluation will involve periods of both capital outflows and inflows. We recognize of course, the fact that the behavior of speculative capital movements is also substantially influenced by factors such as "credibility"; however, the effects of the latter factors are in addition to those discussed in the text and insofar as conflicting effects may be involved, the net effect of devaluation upon the capital account may be difficult to predict.

This Appendix proves the following:

Proposition: For any initial values of q and b that satisfy (25b) and (25d),

$$\lim_{t\to\infty} \frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{h_{23}}{h_{24}}$$

Proof: (a) Suppose that $\frac{q_o-\bar{q}}{b_o-\bar{b}} \neq \frac{h_{13}}{h_{14}}$ (where a subscript "o" denotes the initial value--after the jump in the case of q--of a variable). Then, if $\omega_2 \neq 0$,

$$\frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{\omega_1 h_{13}e^{\mu_1 t} + \omega_2 h_{23}e^{\mu_2 t}}{\omega_1 h_{14}e^{\mu_1 t} + \omega_2 h_{24}e^{\mu_2 t}}$$

$$= \frac{\omega_1 h_{13}\, e^{(\mu_1-\mu_2)t} + \omega_2 h_{23}}{\omega_1 h_{14}\, e^{(\mu_1-\mu_2)t} + \omega_2 h_{24}}$$

which implies, given that $\mu_1 < \mu_2$, that

$$\lim \frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{h_{23}}{h_{24}}$$

(b) Suppose $\frac{q_o-\bar{q}}{b_o-\bar{b}} = \frac{h_{13}}{h_{14}}$, then if $\omega_2 = 0$

$$\frac{q_t-\bar{q}}{b_t-\bar{b}} = \frac{h_{13}}{h_{14}} < 0$$

The Case (b) can be dismissed for the three dynamic exercises conducted in the text because of the following:

(1) In case of devaluation or a decrease in government spending, case (b) is inconsistent with b being constant across steady states.

(2) In the case of a change in β, given that $\frac{h_{11}}{h_{14}} = \frac{\bar{E}(\mu_1 - r)}{[-\frac{1}{\alpha} - r) - \mu_1]} < 0$, M and b would move in opposite directions which is inconsistent with their new steady-state values.

We now show how the initial jump in ρ can be established. Recall (21) given by:

$$\dot{q} = \rho q - r + 2\beta b q^2 \tag{21}$$

Consider the case of a devaluation. We have already established that q falls on impact which means that the last term on the right-hand-side of (21) falls on impact. On the other hand, $\dot{q}$ becomes positive on impact which implies that ρq must rise on impact. Since q falls, ρ must necessarily increase. Given the initial fall in ρ, proceeding as before, it can be established that the dominant ray eigenvector in the (b,ρ) plane slopes upward, which in turn implies that the adjustment path of ρ is monotonic.

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