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Foreign Borrowing and Export Promotion Policies

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Abstract

This paper considers the problem of allocation of investment for a debtor country that faces a ceiling on the amount of foreign debt it can accumulate. It shows that it is optimal for the debtor country to create a more open economy by favoring investment in the export sector over investment in the import-competing sector. The reason is that a more open economy is more sensitive to trade sanctions and therefore more creditworthy in international markets. Because international creditworthiness is basically an externality, there is a role for policy to provide higher returns to export producing activities.

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I. Introduction

Since 1982, a number of major debtor countries have faced weakened prospects for repayment of their debts and, consequently, very restricted access to foreign borrowing. The adjustment to that situation has brought about major changes in their economies, in particular, sharp drops in investment and large real depreciations of the exchange rates. This paper provides a formal framework to analyze one particular aspect of the optimal debtor response to this situation, namely that of determining the optimal level of productive investment and its composition in the face of external financing difficulties.

When a debtor country faces a situation of credit constraints, it needs to lower investment and/or raise domestic savings (or generate a smaller excess of investment over savings) to obtain the required improvement in its trade account. In the absence of policy intervention, the savings/investment adjustment would take place without consideration of the feedback of actions into the credit constraint itself because, for individual agents, these are nonexistent. Results available in the debt literature are that this market response to the credit constraint would involve an excessive reduction in investment. 1/ However, there is also another reason why investment may have a strong effect on the credit ceiling: the allocation of investment between export-producing and import-competing industries will affect the creditworthiness of the debtor country. The policy dilemma that the authorities face is the following: is it advisable to promote investment in the production of importables that would be valuable in the event of a default or is it preferable to invest in the production of exportables, which would help increase the availability of foreign financing? This paper shows that investment in export-producing activities contributes towards relaxing the foreign borrowing constraint, while investment in import-competing activities tightens the foreign borrowing constraint. Therefore, at least when terms of trade are exogenous to the debtor country, policy should be directed towards promoting investment in the export producing sector.2/

The framework of international credit markets that we adopt is one in which debtor countries may choose to repudiate their debts even in situations in which they have enough resources to repay them.3/ There are, of course, costs associated with a debt repudiation. At every point in time, a social planner computes the costs and benefits from the repayment and the repudiation alternatives, and decides which route to take. This option to repudiate foreign debt alters substantially the consumption/savings and resource allocation decisions even if the country

1/ For example, Aizenman (1987), Cohen and Sachs (1986), Krugman (1988).

2/ The consideration of endogenous terms of trade changes may modify the above recommendation. For a debt model with endogenous terms of trade, see Aizenman and Borensztein (1988).

3/ By contrast, another popular framework--adopted for example by Dooley (1987) and Krugman (1988)--is one in which the debtor countries pay back as much as they can afford given available resources.

never actually repudiates its debts. This framework is similar to the one studied by Eaton and Gersovitz (1981), and Cohen and Sachs (1986).

The nature of the sanctions that a debtor country may expect to be subjected to in case of repudiation is a critical factor in this framework. When a country repudiates its debt, it is obtaining debt relief at the cost of financial and economic sanctions. The exact penalties that a repudiating country would face are not explicit, and very difficult to determine. Although there has been some historical experience, it cannot be easily extrapolated. Given the critical role of sanctions in any debt model, we discuss the issue in detail below. We conclude that it is reasonable to assume that sanctions comprise a permanent exclusion from foreign borrowing and some trade-related measures that reduce the advantages of international trade for the debtor country.

In this paper, we consider the case of an economy producing two goods--an import and an export good. Then, in addition to the savings/investment balance, the adjustment to the more stringent foreign borrowing conditions implies a current account adjustment that usually requires large real exchange rate changes. This paper suggests that the optimal response, because it implies the need to promote export production relative to import-competing production, cannot be achieved through exchange rate policy alone. An increase in the production of exports helps to increase the amount of foreign borrowing available to the country. Consequently, the country needs to shift the pattern of investment towards the export sector.^{1/} Although the country invests less in the export sector than in the case of no credit ceiling, it will invest more than a decentralized economy. It is noteworthy that this policy response implies an increase in the returns to the production of exports relative to that of imports, and that an exchange rate policy is not a good instrument for that purpose. Even the existence of uncertainty about the future regime (default or repayment) does not alter the basic result of the convenience of export promotion, because investment in the export-producing sector shifts out the credit supply function for the country, improving its borrowing conditions.

The plan of the paper is as follows. Section II discusses the issue of costs and benefits of default, and justifies the assumptions adopted in this paper. Section III develops a two-period framework, in which precise results can be obtained. Section IV considers the infinite horizon case, which enables us to consider issues such as the cost resulting from exclusion from future borrowing. Section V extends the model by incorporating uncertainty. Section VI concludes with a summary of results and their possible policy implications. Two appendices detail the derivation of results in section III and the numerical solution method applied in simulating the model of section IV.

^{1/} This result is consistent with Aizenman's (1987) findings. In that paper, investment in the productive sectors with a higher component of imported inputs is favored because it helps improve borrowing conditions.

II. Debt Repudiation: Possible Sanctions

A key determinant of the implications of any debt model is the assumption about the resolution of a default situation. In this paper we assume that a country defaults whenever the expected benefits from default exceed the expected costs. This seems a more relevant criterion than determining the default decision by the country's ability to pay. Few countries are physically unable to meet their obligations (indeed sovereign lending was considered safe since true insolvency of a country is virtually impossible) but the costs of doing so may far exceed the benefits.^{1/}

Although the benefit to the debtor from debt repudiation is simply the avoidance of debt service, the costs of that action are even difficult to identify, let alone estimate. Although there exists a growing literature in this area,^{2/} a number of issues are unresolved, and some legal issues remain to be tested in the courts. Unlike an ordinary commercial borrower, a debtor country is protected from legal sanctions under the broad concept of "sovereign immunity." Since a foreign borrower cannot be brought to bankruptcy court, private creditors have limited recourse to legal sanctions.

The actual power of creditors to legally obtain compensation for unpaid sovereign debt is doubtful. One frequently cited possible line of defense is based on the *Foreign Sovereign Immunities Act* (1976) in the U.S. (and the corresponding legislation in the U.K.). However, the actual effectiveness of the FSIA is limited, because sovereign borrowers explicitly waive their immunities in the loan contracts, and because of the commercial activity exception that prevents the application of the FSIA when the act is connected with a commercial activity. Another, potentially more effective line of defense might be based on the "act of state doctrine," which originally applied to expropriation, but that recently has also been applied to foreign exchange controls. Although the act of state doctrine would not apply in the case of default on a specific contract, it could apply if, for example, a debtor country were to impose exchange controls that make impossible to service foreign debts in foreign currency.^{3/} The experience of private creditors (until the 1970s these were mainly bond holders) attempting to attach assets belong to the defaulting country has been varied, and, especially in the last century, none too successful. Eichengreen and Portes (1986), for example, have discussed the sanctions for default in the 1930s.

^{1/} Wriston, who headed Citibank during the 1970s, developed the theory of "sovereign risk hypothesis" where he stated (Lausanne, 1981) that "any country, however badly off, will 'own' more than it 'owes'." Lever and Huhne (1986).

^{2/} See, in particular, Eaton and Gersovitz (1981b), Enders and Mattione (1984), Kaletsky (1985) and Lever and Huhne (1985).

^{3/} See Zamora (1987).

Therefore, it appears likely that the main penalties for default will consist of non-judicial sanctions. Of these, the most often-cited penalty would be the country's exclusion from further participation in the capital markets.^{1/} Of course, a borrower contemplating default will be unlikely to obtain much long-term credit from the capital markets in any case, which means that a future exclusion from borrowing will not represent a severe cost. Kindleberger (1982), for example, argues that several defaults during the 1920s were prompted by the perception that financial markets were collapsing so that the reputation for being a "good" borrower became less valuable and default became more attractive. More conclusively, in a recent paper Bulow and Rogoff (1988) show that, if the country can hold foreign assets after default, it will *always* choose default on its debt at some point in time, which renders reputation an empty concept in international borrowing.^{2/}

A more important direct penalty is that the defaulting country is liable to lose its short-term trade finance, and the trade intermediation services provided by international banks that go along with it. Export credit flows themselves have grown rapidly in recent years, and constitute a major proportion of developing countries' external finance. In addition to the financial component, bank intermediation provides a number of ancillary services that may greatly facilitate international transactions for debtor countries. Should a country lose access to these lines of credit and the availability of attached services, the cost of international transactions may rise considerably. Enders and Matrone (1984), for example, estimate that the rise in the costs of imports, per unit of exports, may be in the order of 5 to 10 per cent. The loss of such credits and international bank services constitute perhaps the most severe penalty creditor banks may impose on a defaulting country.

Ultimately, of course, the penalties for default will depend upon a complex interaction of political and bargaining issues. Commercial banks will also have to consider the effects of their response to a default by one borrower on their reputations *vis à vis* all their other borrowers.^{3/} Obviously, a formal model cannot capture all, or indeed most, of these complex considerations. At a minimum, the model should incorporate the exclusion from future borrowing (which we term the "indirect" penalty) and some form of trade-related measures, or "direct" penalty. The latter is intended to capture the increased costs of trade without trade financing,

^{1/} However, historical evidence does not indicate a very long exclusion from international borrowing for defaulting countries. See Eichengreen and Portes (1986), and Lindert and Morton (1987).

^{2/} The reason is that there exists always a way of obtaining the same risk diversification through holding assets instead of debts, so that the inability to borrow (because of a past default) does not entail any real cost.

^{3/} Ghosh (1984) develops a model in which creditors maintain a reputation for being "tough" a la Kreps and Wilson (1982).

and to a lesser degree, the possible seizure of the debtor's goods in transit.^{1/}

Typical trade sanctions associated with default would be the lack of access to commercial credit and to bank trade intermediation, trade embargoes from some creditor countries--which could be avoided by trading through a third country--and the possibility of seizure of shipments of merchandise on international transport. It is not unreasonable to assume that all these actions would involve a cost that is proportionate to the dollar value of trade, and can therefore be represented as an increase in the unit cost of imports and a reduction in the unit return to exports, that is, simply a terms of trade deterioration. Equivalently, they could be thought of as increasing "transportation costs" or "transaction costs" which reduce export FOB prices and increase import CIF prices. Indeed, in their study, Enders and Mattione (1984), assume that "trade in both exports and imports is disrupted, and the costs of disruptions can be modeled as an X percent decrease in unit export earnings and an equivalent increase in unit import costs; these costs are due to foreign suppliers' attempts to insure themselves against new defaults, to creditors' attempts to attach goods and payments, and to the inefficiencies of administration such a scheme." Accordingly, we model the direct penalty for default as a permanent change in the country's terms of trade. In addition, we assume the existence of an indirect penalty of perennial exclusion from the capital markets.

III. Debt and Investment in a Two-period Model

We consider an economy with a two-period time horizon, producing one exportable and one importable good. The production of each good requires sector-specific capital. In this framework, three important results obtain:

1. The ceiling on foreign borrowing faced by this economy is an increasing function of the stock of capital in the export sector and a decreasing function of the stock of capital in the import sector.
2. The optimal response to the imposition of the credit ceiling is to reduce investment in both productive sectors (relative to the case of no risk of debt repudiation.)
3. The optimal amount of investment in the export sector is higher than the amount that would obtain in a decentralized economy. This means that the credit-rationed economy should follow an "export promotion" policy.

In the two-period case, the penalties for lack of repayment cannot include exclusion from future borrowing. Thus, penalties will consist entirely of trade sanctions. Consistently with the discussion of the

^{1/} This latter sanction is emphasized by Bulow and Rogoff (1987).

previous section, we assume that the effect of trade sanctions is equivalent to a permanent terms of trade deterioration for the debtor country. Specifically, in the event of external debt repudiation, a fraction ρ of exports are lost and the cost of imports raises by an equivalent proportion. Therefore, if the price of exports was unity, the net return to exports will become $1-\rho$ after default, and if the price of imports was P , the total cost of imports will become $P/(1-\rho)$ after default.^{1/} This means that the terms of trade--relative price of imports in terms of exports--will shift from P to θP , where $\theta = (1-\rho)^{-2}$.

As is known from several contributions to this literature in this type of framework creditor banks will set a ceiling on lending to sovereign countries in order to avoid the repudiation of their obligations.^{2/} This is because as foreign debt increases, the rewards to a repudiation of foreign debt also increase.^{3/} Consequently, creditor banks will never increase lending past the point in which repudiation becomes a more attractive program than repayment.

We are interested in finding out how the credit ceiling function is affected by investment in each productive sector. We start by obtaining the credit ceiling function as of the beginning of the second period. For this purpose, it is convenient to use the indirect utility functions that correspond to the repayment and default regimes. An indirect utility function gives the maximum utility obtained by the representative consumer, as a function of relative prices and income. In the case of repayment of foreign debt, the indirect utility function, V^R (the superscript R stands for repayment) will be obtained from:

$$(1) \quad V^R(P, y^* + Py - (1+r)D) = \max U(c^*, c)$$
$$\text{s.t. } y^* - c^* + P(y-c) - D(1+r) \geq 0$$

where P is the exogenous relative price of imports, y^* and y represent the the supply of the exportable and importable good, respectively, c^* and c represent consumption of each of the two goods, and D represents the level of foreign debt carried over from the previous period, which carries a (world) interest rate r . For the exportable and importable goods, production functions are given by:

1/ These prices are given in terms of an arbitrary common unit.

2/ See, for example, the survey by Eaton, Gersovitz and Stiglitz (1986).

3/ It is conceivable that the costs of repudiation also increase with foreign debt (creditors will spend more energy in trying to collect or imposing sanctions). However, only the case in which rewards to repudiation increase more than costs makes sense because otherwise the more indebted a country is, the safer debtor it becomes.

$$y^* = f(k^*), \text{ and}$$

$$y = f(k)$$

where k^* and k represent the capital stocks specific to the respective sector.

Let V^D represent the indirect utility function in case of repudiation (the superscript D standing for default). V^D is obtained from:

$$(2) \quad V^D(\theta P, y^* + \theta Py) = \max U(c^*, c)$$

$$\text{s.t.} \quad y^* - c^* + \theta P(y - c) \geq 0$$

where θP represent the (net) international terms of trade received by the debtor country as a consequence of trade sanctions. The credit ceiling is the maximum amount of debt that debtors can owe and still prefer the repayment option. Since V^R is a decreasing function of D , the credit ceiling is a quantity \bar{D} such that:

$$(3) \quad V^R(P, y^* + Py - (1+r)\bar{D}) = V^D(\theta P, y^* + \theta Py)$$

Note that this implies that the credit ceiling \bar{D} is the equivalent variation in income that compensates for a terms of trade deterioration in proportion θ . In terms of expenditure functions, \bar{D} can be obtained as:

$$\bar{D} = e(\theta P, V^D) - e(P, V^D)$$

where $e(\cdot)$ is the expenditure function, and V^D is the indirect utility function under default defined in (2). The determination of the credit ceiling can be illustrated with the aid of Figure 1. In Figure 1, the point E represents the pair (y, y^*) of output of the two goods. At the default terms of trade (θP) consumption would be at point D . At the undisturbed (free trade) terms of trade P , the same utility level could be achieved consuming at point R . It can be seen that, if the resources of the country were reduced by an amount $\bar{D}(1+r)$ (in export good units), the country would be indifferent between repaying or defaulting on its foreign debt. This implies that the credit ceiling is equal to \bar{D} .

Differentiating (3) we can investigate the dependency of the credit ceiling on the variables of interest. We first compute:

$$(4) \quad \frac{d\bar{D}}{dy^*} = \frac{V_I^R - V_I^D}{(1+r)V_I^R} = \frac{U_c^R - U_c^D}{(1+r)U_c^R}$$

where V_I represents the derivative of the indirect utility function with respect to income (its second argument). The second equality follows from a well-known envelope theorem that equates marginal utility of income to marginal utility of consumption at the optimal point.^{1/} Similarly, we have:

$$(5) \quad \frac{d\bar{D}}{dy} = \frac{PV_I^R - \theta PV_I^D}{(1+r)PV_I^R} = \frac{U_c^R - U_c^D}{(1+r)U_c^R}$$

where the superscripts R and D in the utility function indicate that the derivatives are computed at the consumption bundles corresponding to the repayment and default situations, respectively, and where subscripts indicate partial derivatives with respect to the variable in the subscript. In obtaining the second equality we have also used the conditions that, at an optimal point, $U_c^R = PU_c^{R*}$, and $U_c^D = \theta P U_c^{D*}$.

For fixed terms of trade, we can express the credit ceiling as a function of only the capital stock in each productive sector: $\bar{D} = h(k^*, k)$.^{2/} Noting that the marginal product of capital is the same in either the repayment or the default case because capital is predetermined, and because there is no further use for capital after period 1, the derivatives of the $h(\cdot)$ can be expressed simply as:

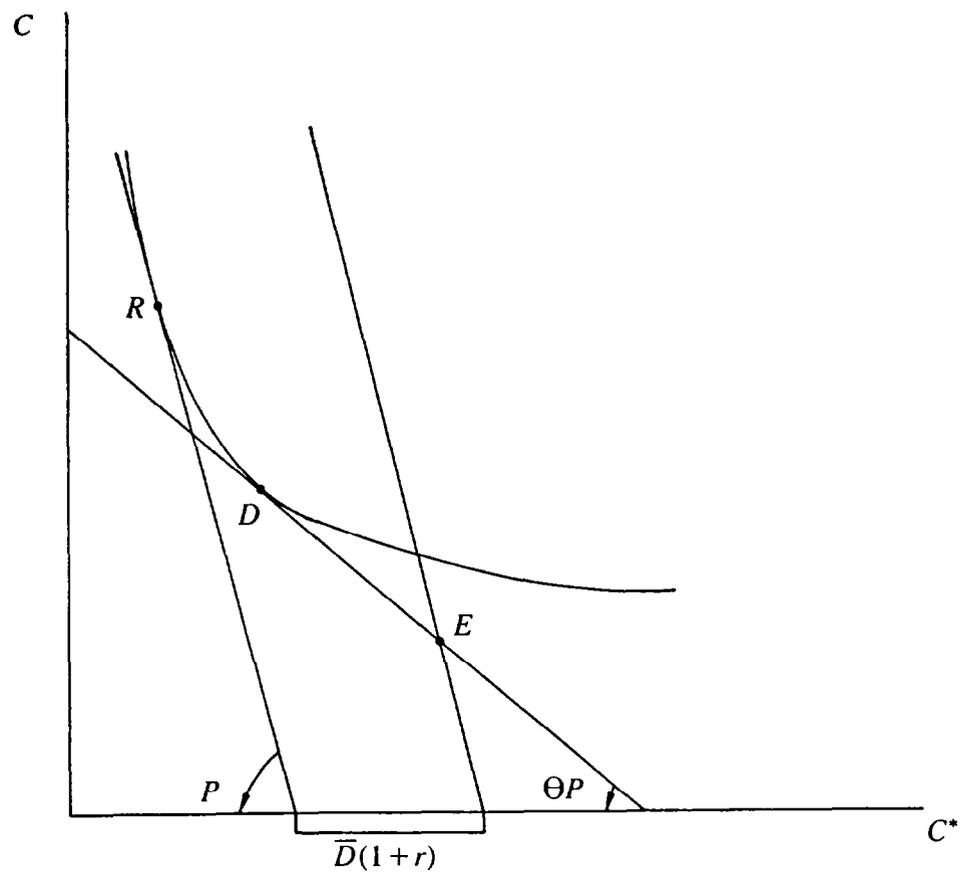
$$(6) \quad h_k' = \frac{U_c^R - U_c^D}{(1+r)U_c^R} f_k^* = \frac{f_k^*}{1+r} \left(1 - \frac{U_c^D}{U_c^R} \right) > 0$$

The sign of this derivative follows from $U_c^R > U_c^D$, which is proved in Appendix 1. The inequality means that the export good has a higher marginal utility under repayment than under default, when evaluated at the point in which the country is indifferent between repaying or defaulting. This is because when the country defaults, the export good becomes relatively cheaper (terms of trade deteriorate by a factor of θ), and the marginal utility of the export good decreases as its relative price falls and the utility level is kept constant.

^{1/} Note that we are using the exportable good as numeraire.

^{2/} At this point, we are abstracting from temporal inconsistencies or monitoring problems that may arise in the borrowing/investment process. In our model, foreign borrowing and investment take place essentially at the same time, while in practice there is some lag between the loan and the execution of investment that might be problematic because the debtor country would have incentives to change the investment plan after drawing the loan.

Figure 1
The 2-Period Model



- D: Default consumption
- R: Repay consumption (when Debt = Debt ceiling)
- E: Endowment point (y, y^*)
- \bar{D} : Debt ceiling

The derivative of the credit ceiling function with respect to capital installed in the import good industry is:

$$(7) \quad h'_k = \frac{U_c^R - U_c^D}{(1+r)U_c^R} f_k = \frac{f_k}{1+r} \left(1 - \frac{U_c^D}{U_c^R} \right) < 0$$

Similarly, the sign of (7) is determined by the fact that $U_c^D > U_c^R$, also proved in Appendix 1. In other words, the marginal utility of the import good increases as its relative price goes up and the utility level is kept constant.

Consider now the problem of the optimal composition of investment in the debtor country given its foreign borrowing constraints. The "social planner" of the debtor economy, must decide how much to allocate to consumption and investment on each industry, given the credit ceiling function $h(\cdot)$. That decision is reached by solving:

$$(8) \quad V^P(y_0^*, y_0) = \max U(c_0^*, c_0) + \beta U(c_1^*, c_1)$$

$$\text{s.t.} \quad y_0^* - c_0^* - I^* + P(y_0 - c_0 - I) +$$

$$+ \frac{1}{1+r} \left[f(I^*) - c_1^* + P(f(I) - c_1) \right] \geq 0;$$

$$\text{and} \quad h(I^*, I) \geq \left[c_0^* + I^* - y_0^* \right] + P \left[c_0 + I - y_0 \right].$$

where we have assumed a 100 percent depreciation rate, i.e. $k_1 = I$ in both sectors, and that there is no initial debt. The first order conditions for this problem are:

$$(9a) \quad U_{c_0^*} = \lambda + \mu$$

$$(9b) \quad U_{c_0} = P(\lambda + \mu)$$

$$(9c) \quad \beta U_{c_1^*} = \frac{\lambda}{1+r}$$

$$(9d) \quad \beta U_{c_1} = P \frac{\lambda}{1+r}$$

$$(9e) \quad \lambda \left[\frac{f_{k^*}}{1+r} - 1 \right] = -\mu \left[h_{k^*} - 1 \right]$$

$$(9f) \quad \lambda P \left[\frac{f_k}{1+r} - 1 \right] = -\mu \left[h_k - P \right]$$

where λ and μ are the Lagrange multipliers associated with the constraints in (8). Investment in the export sector is implicitly given by (9f). Using (6) one obtains:

$$(10) \quad \frac{f_{k_1}^*}{1+r} = \frac{\lambda + \mu}{\lambda + \mu \left[1 - \frac{U_{c_1}^D}{U_{c_1}^R} \right]} > 1$$

This means that investment in the export sector will be less than the level that would obtain if the country did not have the option to repudiate its debt. With no risk of repudiation, the domestic interest rate would be equal to the world interest rate r and the marginal product of capital would be equal to both. The existence of risk of debt repudiation and the consequent credit ceiling increases the actual cost of borrowing for the debtor country; the optimal domestic interest rate (the one that generates the optimal amount of investment) exceeds the world interest rate by a factor--in the right-hand side of (10)--that expresses how the choice between default or repayment is affected by additional units of capital in the export sector. Investment in the import sector can be obtained using (7) and (9f):

$$(11) \quad \frac{f_{k_1}}{1+r} = \frac{\lambda + \mu}{\lambda + \mu \left[1 - \frac{U_{c_1}^D}{U_{c_1}^R} \right]} > 1$$

Thus, investment in the import sector also declines relative to the level that would prevail if there were no option to repudiate foreign debt. The decline in investment in the import sector is larger than the decline in the export sector, in the sense that the optimal domestic cost of capital for the import sector is larger than for the export sector. This is clear from (6) and (7).

The reason for the differential effect on investment in the two sectors is that investment in the export sector has a beneficial effect on the credit ceiling. Because the relative price of exports falls in the event of repudiation, investment in the export sector makes repudiation more costly to the debtor country and permits a higher safe limit of indebtedness, easing the credit constraint. Then, the response to the existence of a credit constraint by an optimal planner should be a generalized reduction in investment, but with a change in its composition into export production (relative to the levels prevailing in the absence of the repudiation option).

For policy purposes, however, the relevant comparison is between the social planner decisions and those which would obtain in a (partially) decentralized economy, because this indicates the precise nature of intervention in the form of taxes or subsidies that is needed to achieve the social optimum. In this context, an economy can only be partially decentralized because the decision to repudiate foreign debt must be a coordinated one. Then, we consider an economy in which there is a central authority whose main function is to decide, at each point in time, if the country should repudiate its foreign debt. In addition, the central authority is a financial intermediary between foreign lenders and domestic borrowers. We assume the central authority conducts this function in such a way as to allow domestic prices and interest rates to be determined in competitive markets, and that it refunds to the private sector, in a lump-sum way, all profits resulting from this intermediation activity.

The first thing to note about such a decentralized economy is that it will face exactly the same credit ceiling *function* as does the socially planned economy. This is because the aggregate costs and benefits of repudiation are the same. In case of repudiation, the decentralized economy will face the same implicit terms of trade deterioration, and benefit from full debt relief, just as the planned economy. Thus, from the point of view of a representative consumer, the point at which default becomes optimal is the same for the two economies. This is certainly a special feature of the two-period model; in a multiperiod setup, the change in the domestic interest rate following default would be different in the two economies, and this factor alone would make the value of the repudiation option different.

The mechanics of the decentralized economy are the following: a central authority does all foreign borrowing and repayments. The foreign resources thus obtained are passed on to the public at a market-clearing interest rate. If the country is credit-constrained, the domestic interest rate will exceed the international interest rate, and the central authority will make a profit, which we assume is returned to the private sector as a lump-sum payment at the beginning of the second period.

The private sector behavior in the decentralized economy will be determined by the solution to the following optimization problem (where the superscript *c* identifies the decentralized economy problem):

$$(12) \quad V^c(y_0^*, y_0) = \text{Max} \quad U(c_0^*, c_0) + \beta U(c_1^*, c_1)$$

$$\text{s.t.} \quad y_0^* - c_0^* - I^* + P(y_0 - c_0 - I) +$$

$$+ \frac{1}{1+r^D} \left[f(I^*) - c_1^* + P(f(I) - c_1) \right] + \frac{r^D - r}{1+r^D} h(I^*, I) \geq 0$$

where r^D is the domestic interest rate. The term $\frac{r^D - r}{1+r^D} h(I_0^*, I_0)$ represents profits made by the central authority by borrowing abroad at the rate r , and competitively allocating those funds among domestic borrowers at rate

r^D . If the country is credit constrained, profits will always be positive. Although profits are immediately reimbursed to the private sector, agents know that their individual actions have an insignificant impact on the central authority's profit distribution and correspondingly ignore any such repercussion. In consequence, the first-order conditions for the private sector's problem are:

$$(13a) \quad U_{c_0}^* = \lambda$$

$$(13b) \quad U_{c_0} = P\lambda$$

$$(13c) \quad \beta U_{c_1}^* = \frac{\lambda}{1+r^D}$$

$$(13d) \quad \beta U_{c_1} = P \frac{\lambda}{1+r^D}$$

$$(13e) \quad \frac{f_k^*}{1+r^D} = 1$$

$$(13f) \quad \frac{f_k}{1+r^D} = 1$$

These first-order conditions are quite standard. Rates of growth of marginal utility and marginal products of capital in each sector are equalized to the domestic interest rate r^D . There are also two market clearing conditions that, assuming that the economy is credit constrained, are given by:

$$(14) \quad h(I^*, I) = c_0^* + I^* - y^* + P(c_0 + I - y)$$

$$(15) \quad (1+r)h(I^*, I) = f(I^*) - c_1^* + P(f(I) - c_1)$$

These two conditions represent the current account balance in periods 1 and 2. Of course, one of these conditions is redundant by application of the budget constraints.

The comparison of the planned economy and the decentralized economy can be done in the following way. We are going to obtain the effect on utility of a representative consumer resulting from a marginal increase in investment in each sector: dV^C/dI^* and dV^C/dI . Since we know that the planned economy achieves maximum utility, it follows that if $dV^C/dI^* > 0$, the planned economy invests more in the export sector than the decentralized economy, and if $dV^C/dI < 0$, the planned economy invests less in the import sector than the decentralized economy.

$$(16) \quad \frac{dV^C}{dI^*} = U_c^* \left[-1 + \frac{f_k^*}{1+r^D} + \frac{r^D-r}{1+r^D} h_k'^* + \frac{h}{1+r^D} \frac{dr^D}{dI^*} - \right. \\ \left. - \frac{1}{(1+r^D)^2} \left[h(1+r) + (r^D-r)h \right] \frac{dr^D}{dI^*} \right]$$

Applying first-order and equilibrium conditions, the expression reduces to:

$$(16) \quad \frac{dV^C}{dI^*} = U_c^* \frac{r^D-r}{1+r^D} h_k'^* > 0$$

because $h_k'^* > 0$. The effect on utility of the change in the domestic interest rate vanishes because the private sector is in a balanced position with respect to r^D with any increase in the cost of borrowing refunded by the central authority. Symmetrically:

$$(17) \quad \frac{dV^C}{dI} = U_{c0} \frac{r^D-r}{1+r^D} h_k' < 0$$

In summary, in the two-period horizon case, the optimal policy response involves a subsidy for investment in the export sector and a tax on investment in the import-competing sector. We will now study the extension of this result to the infinite horizon model.

IV. Debt and Investment in an Infinite-Horizon Model

The addition of the indirect penalty in the infinite-horizon model does not alter the results of the previous section in a significant way. Although we have been unable to show that the optimal policy involves a subsidy to investment in the export-producing sector and a tax on investment in the import-competing sector *at every point in time*, there is a strong presumption that this is in fact the case. If the derivatives of the credit ceiling function have the same signs--as we conjecture--investment in the export sector will be subsidized in the steady state position. With adjustment costs to investment, it then seems natural that investment in the export sector will always be subsidized. Furthermore, we provide a numerical example that is in accordance to this argument.

As before, the credit ceiling function faced by this economy is obtained from a comparison of the value functions under the default and repayment alternatives. The determination of the credit ceiling in this model is extremely complicated. Recall that one of the benefits of repayment is the continued access to capital markets; the more the country will use the capital markets in the future, the greater its incentives to repay today. Therefore, the higher the credit ceiling the country expects

tomorrow the larger the debts it can safely owe today. But the credit ceiling tomorrow will be an increasing function of the credit ceiling the period after as well. Clearly, the credit ceiling is infinitely recursive. This suggests that the credit ceiling function can be solved for by using dynamic programming techniques to compute the utility value for the debtor country of the repayment and default alternatives.

Let $V^D(k, k^*)$ denote the present discounted utility (i.e. the value function) if the country decides to default. This value function is computed from:

$$(18) \quad V^D(k_t^*, k_t^*) = \max U(c_t^*, c_t^*) + \beta V^D(k_{t+1}^*, k_{t+1}^*)$$

$$\text{s.t.} \quad k_{t+1}^* = (1-\delta)k_t^* + I_t^*$$

$$k_{t+1} = (1-\delta)k_t + I_t$$

$$f(k_t^*) - c_t^* - I_t^* = \theta P_t \left[c_t + I_t - f(k_t) \right]$$

where δ is the depreciation rate of capital.

Let $V^R(k, k^*, D)$ denote the value function under repayment. This value function is computed from:

$$(19) \quad V^R(k_t^*, k_t^*, D_t) = \max U(c_t^*, c_t^*) + \beta V^R(k_{t+1}^*, k_{t+1}^*, D_{t+1})$$

$$\text{s.t.} \quad k_{t+1}^* = (1-\delta)k_t^* + I_t^*$$

$$k_{t+1} = (1-\delta)k_t + I_t$$

$$D_{t+1} = D_t(1+r) + c_t^* + I_t^* - f(k_t^*) + P_t \left[c_t + I_t - f(k_t) \right]$$

$$D_{t+1} \leq \bar{D}_{t+1}$$

where \bar{D} represents the ceiling on foreign debt faced by the country. As before, the credit ceiling is obtained from:

$$(20) \quad V^D(k, k^*) = V^R(k, k^*, \bar{D}) \Leftrightarrow \bar{D} = h(k, k^*)$$

From (18)-(20) we can compute the effects that capital accumulation in each sector has on the country's credit ceiling. These effects are given by:

$$(21) \quad h'_{k^*} = \frac{V^R_{k^*} - V^D_{k^*}}{-V^R_D}, \quad \text{and}$$

$$(22) \quad h'_k = \frac{V_k^R - V_k^D}{-V_D^R}$$

Note that V_k^* and V_k are the marginal utility values of a unit of capital in the corresponding productive sector.^{1/} This means that, in each sector, the credit ceiling will be an increasing function of the capital stock if and only if the marginal value of capital is larger under the repayment option than under the default option. (Note that V_D^R is negative). In other words, creditors will be more willing to extend credit if the country invests in projects that make the repayment alternative more valuable than the default alternative.

An analytical derivation of the signs of the derivatives of the credit ceiling functions is, in fact, quite complex. It involves comparing the relative sizes of investment q 's in two different optimization problems (the repayment and the default problems) which display different interest rates (marginal rates of substitution in consumption). However, there is a strong presumption that--as in the two-period case--the credit ceiling is increasing in capital in the export sector and decreasing in capital in the import-competing sector. Consider the effect of an additional unit of investment in the import-competing sector. In the event of default the country is assumed to suffer a terms of trade deterioration, therefore, the value (measured in terms of the true CPI) of its capital stock in the import competing sector rises by a discrete jump θ . If the country repays, however, the value of its import-competing capital remains constant. This suggests that the gain from an additional unit of capital in the import competing sector is larger in the case of default than it is under repayment. Exactly the opposite holds for capital in the export sector. The larger the export sector, the larger the loss resulting from the terms of trade deterioration should the country default. Hence the utility value of export capital is greater under repayment than it is under default. Thus we conjecture:

$$(23) \quad V_k^D(k, k^*) > V_k^R(k, k^*, D), \text{ and } V_{k^*}^R(k, k^*, D) > V_{k^*}^D(k, k^*)$$

and the credit ceiling function displays:

$$h_{k^*}(k, k^*) > 0 \text{ and } h_k(k, k^*) < 0$$

that is, the credit ceiling is an increasing function of capital in the export sector, but a decreasing function of capital in the import-competing sector.

^{1/} In the numerical simulations, we will introduce adjustment costs to investment, and V_k^* and V_k will be equal to Tobin q 's, measured in units of marginal utility of the respective good.

Once again, debt repudiation will never take place in this framework. This is because rational creditors foresee the debtor country's incentives to repudiate its foreign debt and restrict credit in such a way that the debtor country always finds repayment preferable to default. Consider now the optimal plan for the debtor economy given the existence of the credit ceiling function $h(\cdot)$ described above. The first-order conditions for that problem are given by:

$$(24a) \quad U_c^* = -\beta V_{D_{t+1}}^R$$

$$(24b) \quad U_{c_t} / P_t = -\beta V_{D_{t+1}}^R$$

$$(24c) \quad V_{k_{t+1}}^R = -V_{D_{t+1}}^R$$

$$(24d) \quad V_{k_{t+1}}^R / P_t = -V_{D_{t+1}}^R$$

where $V_{D_{t+1}}^R$ is a shorthand for $V_D^R(k_{t+1}^*, k_{t+1}, D_{t+1})$, etc. Also, the following envelope theorems can be proved:

$$(24e) \quad V_{k_{t+1}}^R = U_c^* \left[f_{k_{t+1}}^* + (1-\delta) \right] + \mu_{t+1} h_{k_{t+1}}^*$$

$$(24f) \quad V_{k_t}^R = U_c^* \left[f_{k_t} + (1-\delta) \right] + \mu_{t+1} h_{k_{t+1}}$$

$$(24g) \quad V_{D_t}^R = U_c^* (1+r) - \mu_{t+1}$$

where μ is the Lagrange multiplier on the external credit ceiling for the economy. While the credit ceiling is *not* binding, μ is equal to zero, and the above conditions imply the standard optimal rules for consumption and investment:

$$(25a) \quad U_c^* = \beta(1+r) U_{c_{t+1}}^*$$

$$(25b) \quad U_{c_t} = \beta(1+r) \frac{P_t}{P_{t+1}} U_{c_{t+1}}$$

$$(25c) \quad f_{k_{t+1}}^* = r + \delta$$

$$(25d) \quad f_{k_t} = r + \delta$$

That is, the marginal utility of consumption grows at a rate inverse to the world interest rate, and the marginal product of capital equals the interest plus depreciation rates in each sector.

Once the credit constraint is binding, however, μ is positive, and the optimal policies must be modified:

$$(26a) \quad U_{c_t}^* = \beta(1+r) U_{c_{t+1}}^* + \beta\mu_{t+2}$$

$$(26b) \quad U_{c_t} = \beta(1+r) \frac{P_t}{P_{t+1}} U_{c_{t+1}} + \beta\mu_{t+2}$$

$$(26c) \quad f_{k_t}^* = (1-h_{k_{t+1}}^*) S_t + h_{k_{t+1}}^* (1+r) - 1 + \delta$$

$$(26d) \quad f_{k_t} = (1-h_{k_{t+1}}) S_t + h_{k_{t+1}} (1+r) - 1 + \delta$$

where S_t is equal to $U_{c_t}^* / \beta U_{c_{t+1}}^*$, which we will refer to as the implicit

domestic interest rate. Therefore, when the credit constraint is binding, the marginal product of capital in each sector is set equal to a weighted average of the implicit domestic interest rate and the international interest rate, with weights $h_{k_t}^*$ and h_{k_t} , respectively. It can be shown that this policy implies a subsidy to investment in the export sector relative to the import-substitution sector in the following way:

$$(27) \quad f_{k_t}^* - f_{k_t} = [S_t - (1+r)] [h_{k_t} - h_{k_t}^*]$$

Since the interest rate in the credit-constrained economy must be greater than the international interest rate, the first term in square brackets is positive and since $h_{k_t}^* > 0$ and $h_{k_t} < 0$, the second term in square brackets is negative. This implies that there is a bias towards greater investment in the export sector relative to the import sector. The reason is, once again, that altering the investment mix of the economy can relax the credit ceiling, which increases welfare in a credit constrained country.

1. The Decentralized Economy

The decentralized economy is organized in the same way as the decentralized economy in the previous section. The actions of consumers and firms will be equivalent to those that result from maximizing utility in the following problem:

$$\begin{aligned}
 (28) \quad V^C(k_t^*, k_t, \bar{D}_t) &= \text{Max} \sum_{t=0}^{\infty} \beta^t U(c_t^*, c_t) \\
 V^C(k_t^*, k_t, \bar{D}_t) &= \text{max} U(c_t^*, c_t) + \beta V^C(k_{t+1}^*, k_{t+1}, \bar{D}_{t+1}) \\
 \text{s.t.} \quad k_{t+1}^* &= (1-\delta)k_t^* + I_t^* \\
 k_{t+1} &= (1-\delta)k_t + I_t \\
 D_{t+1} &= D_t(1+r^D) + c_t^* + I_t^* - f(k_t^*) + P_t \left[c_t + I_t - f(k_t) \right] \\
 &\quad - (r^D - r)\bar{D}_t \\
 D_{t+1} &\leq \bar{D}_{t+1}
 \end{aligned}$$

As before, the constraint \bar{D} and the lump-sum rebate of financial intermediation profits are exogenous to the atomistic agent, whose decisions are made according to the domestic interest rate r^D . Therefore, the consumption and investment rules in the decentralized economy are standard:

$$(29a) \quad U_{c_t}^* = \beta(1+r^D) U_{c_{t+1}}^*$$

$$(29b) \quad U_{c_t} = \beta(1+r^D) \frac{P_t}{P_{t+1}} U_{c_{t+1}}$$

$$(29c) \quad f_k^* = r^D + \delta$$

$$(29d) \quad f_k = r^D + \delta$$

It is evident that the decentralized economy does not bias investment towards the export sector and away from the import competing sector.

Comparing the investment rules of the planned economy to that of the decentralized economy is very difficult, because the implicit domestic interest rates (which we have defined as equal to the intertemporal marginal rates of substitution of consumption) are different.^{1/} However, in the steady state, the implicit domestic interest rate is equal to β for both economies, and this makes the comparison easier. In a steady-state position, investment in the planned economy will be carried out according to:

^{1/} However, because the planned economy will be "less credit-constrained", it appears that its implicit domestic interest rate should be lower.

$$(30) \quad f_k^R = (1-h_k^*)\beta^{-1} + h_k^*(1+r) - 1 + \delta$$

and in the decentralized economy:

$$(31) \quad f_k^C = \beta^{-1} - 1 + \delta$$

which means that:

$$(32) \quad f_k^R - f_k^C = (1+r-\beta^{-1}) h_k^* < 0 \text{ if } h_k^* > 0$$

and symmetrically:

$$(33) \quad f_k^R - f_k^C = (1+r-\beta^{-1}) h_k > 0 \text{ if } h_k < 0.$$

The decentralized economy fails to take into account the effects of investment policies on the country's credit ceiling. Accordingly, it fails to recognize the additional benefits of investment in the export sector and the negative externalities of investment in the import-competing sector, and it undertakes too little investment in the export sector and too much in the import-substitution sector.

2. Simulation Results

In order to confirm that the credit ceiling is indeed an increasing function of capital in the export sector but a decreasing function of capital in the import competing sector, we undertook some numerical simulations. In essence, the simulation algorithm follows the steps outlined in Appendix 2: starting with the steady-state value functions the repay and default value functions are recursively computed until stationary investment and borrowing rules are obtained. These policy rules are calculated as functions of the inherited state variables--that is, on a grid of possible (k, k, D) combinations. The numerical grid used consisted of 8000 $(20 \times 20 \times 20)$ points, and 5 iterations were performed to obtain convergence of the functions. In total, therefore, some 8000 non-linear optimization problems had to be solved for each iteration. Given initial capital stocks and debt level (and assuming that the initial debt is not so high that the country defaults immediately) the stationary policy rules trace the dynamic path of the economy.

The parameterization of the model was done in the following way. Production functions were taken to be Cobb-Douglas; the utility function was chosen to be logarithmic; investment was assumed to be subject to quadratic costs of adjustment (Abel (1979), Hayashi (1982)) so that I units of investment cost $I(1+\psi/2(I/k))$ units of the good. The values of the parameters are indicated in Table 1.

Table 1. Simulation Model

Utility function:	$\sum_{t=0}^{\infty} \beta^t \{ \alpha \ln(c_t) + (1-\alpha) \ln(c_t^*) \}$
Production functions:	$f(k) = aK^\gamma$ $g(k^*) = a^*k^{*\gamma}$
Costs of investment installation:	$I(1 + \psi/2(I/k))$ $I^*(1 + \psi^*/2(I^*/k^*))$
Direct penalty for default:	repay terms of trade P default terms of trade θP
World interest rate:	$(1+r)$

Parameter Values

$\beta = 0.87$	$a = 0.35$	$\gamma = 0.85$
$\psi = 2.0$	$\delta = 0.05$	$a^* = 0.35$
$\gamma^* = 0.85$	$\psi^* = 2.0$	$\delta^* = 0.05$
$r = 0.05$	$\alpha = 0.5$	$P = 0.1$
	$\theta = 1.2$	

Initial Conditions: $d_0 = 0$; $k_0 = 4.00$; $k_0^* = 4.00$

Figure 2 shows the time path of each capital stock in both the planner's economy and the decentralized economy. As is evident, the latter undertakes too much investment in the import competing sector and too little in the export sector. Figure 3 graphs the credit ceiling $h(\cdot)$ as function of the capital stock in each sector: it is an increasing function of the export sector capital, k , and a decreasing function of the import sector capital k^* .^{1/}

It is remarkable that the difference between the optimal capital stocks and those chosen by the decentralized economy are quite substantial despite the fact that the default penalty involves only a 20 percent deterioration in terms of trade. The capital stock in the export sector of the optimally planned economy is 50 per cent higher than the capital stock in the import competing sector; by contrast, the decentralized economy chooses the same capital stock in each sector.^{2/} As a result, steady-state GDP is almost 17 per cent higher for the optimally planned economy and the maximum debt level is 20 per cent greater. The simulation of the two sector model thus shows that the optimal policy intervention requires a subsidy to investment in the export sector, and a tax on investment in the import-competing sector.

V. Debt and Investment in a Model with Uncertainty

Are the above results robust to the existence of uncertainty? The question is a relevant one because, with uncertainty, the debtor country does not know whether it may eventually choose to repudiate its debts. Then, it might not seem to be wise to embark in a strategy of export promotion that would hurt the debtor country if default actually takes place.

The results in this section will show that the results extend very well to the case of uncertainty. With uncertainty, there does not exist a credit ceiling in the form of a fixed amount beyond which additional borrowing is not possible. Instead, the debtor country faces a credit supply function which is upward sloping in some range but becomes backward-bending at some point.^{3/} An export promotion policy achieves a

^{1/} The different gradients along k and k^* reflect the relative price of the two goods, with the price of k^* , P , being much lower than that of k , the numeraire. However, a unit of investment in k^* costs only P units of k so that \$1 of investment in the export sector tends to *increase* the credit ceiling by the same amount that \$1 of investment in the import competing sector *decreases* the credit ceiling.

^{2/} This is a consequence of the symmetry in the production functions. Since it is the capital stocks in the planner economy *relative* to those in the decentralized economy that define the targets for optimal policies, the assumption of complete symmetry contributes to expositional clarity.

^{3/} Because of the risk of debt repudiation. See Aizenman (1987), Kletzer (1984), and O'Connell (1988).

relaxation of the foreign borrowing constraint in the sense of shifting outwards the credit supply function.

We consider a two-period model, in which the source of uncertainty is a productivity shock on the export production function. That is, the production of exportables is given by:

$$(34) \quad y^* = \xi f(k^*)$$

where ξ is a random variable that, for simplicity, can assume only two values:

$$\begin{aligned} \xi &= \xi_0 \quad \text{with probability } .5, \text{ and} \\ \xi &= \xi_1 \quad \text{with probability } .5, \text{ and where} \\ \xi_0 &< \xi_1 \end{aligned}$$

At the beginning of the second period, the value of ξ is revealed and the debtor country makes the decision regarding its foreign debt. It will repay its foreign debt whenever:

$$V^R(P, \xi y^* + Py - (1+r)D) \geq V^D(\theta P, \xi y^* + \theta Py)$$

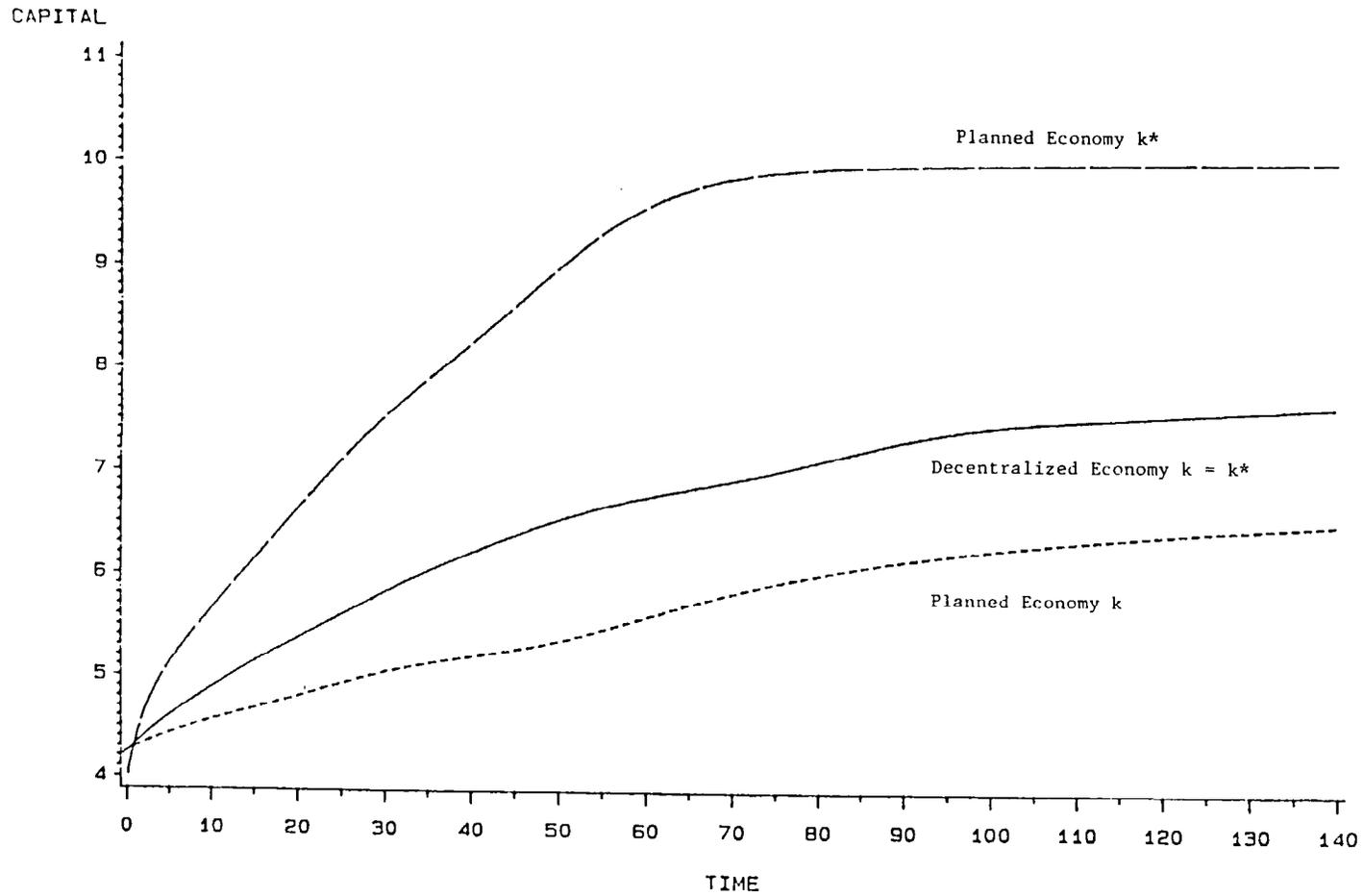
and repudiate its foreign debt otherwise. Note that, since the export good is relatively more expensive under repayment than under default because of the terms of trade deterioration, a larger positive productivity shock reduces the likelihood of default. That is, there are situations in which the debtor country will default if the value of the productivity shock is ξ_0 but not if it is ξ_1 , but the converse is never possible.

On the creditors side we assume that, by risk neutrality (or zero correlation of the country's debt with all other market securities), plus some perfect competition mechanism, the expected rate of return on debt is equal always to a fixed rate \bar{r} . The expected rate of return is:

$$(35) \quad \Omega = \Pi (1+r), \quad \text{where} \\ \Pi = \Pr(V^R > V^D)$$

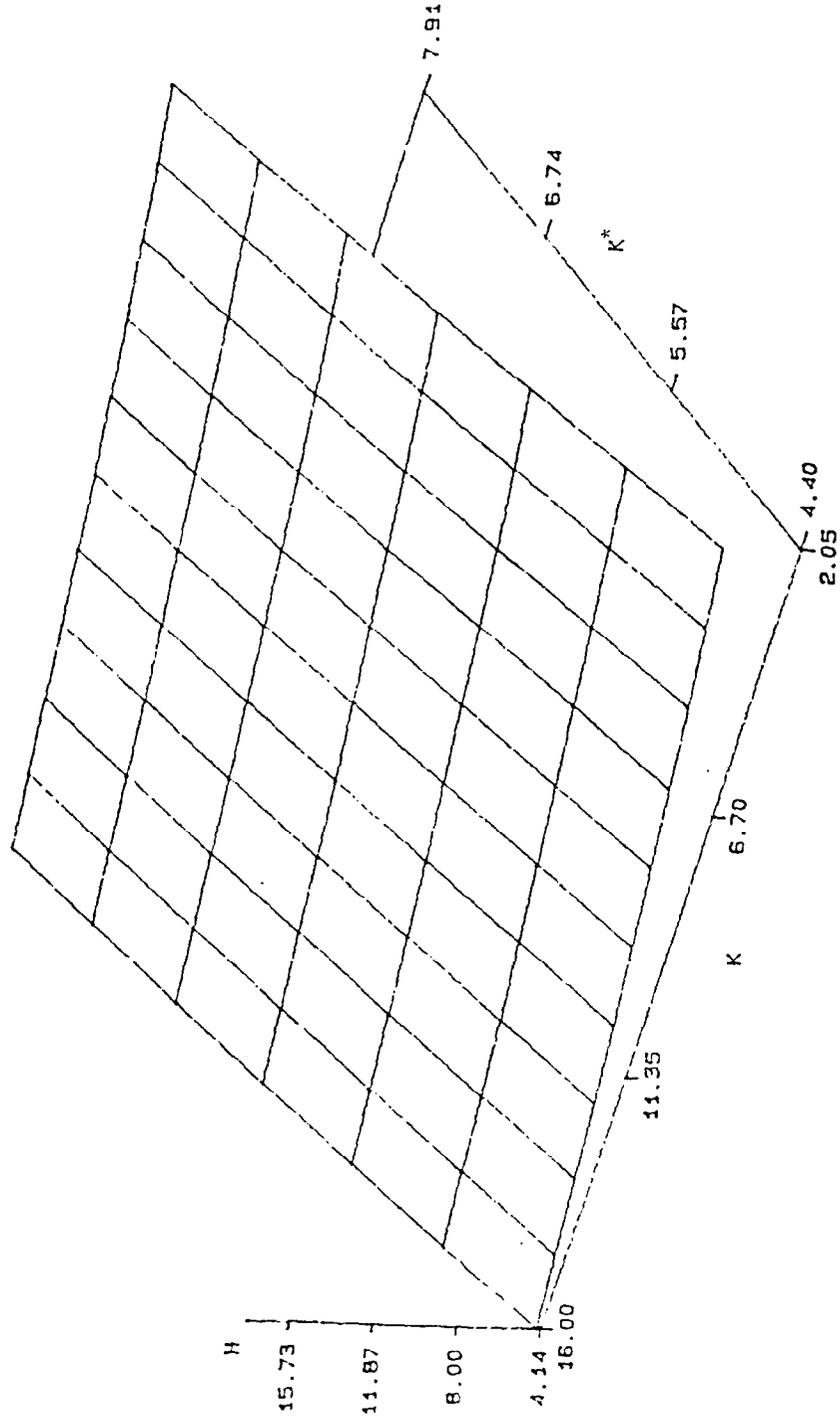
Thus a "credit supply function" is derived, which is the set of all pairs (D, r) such that $\Omega = 1+\bar{r}$. The credit supply function is very simple in this case. First consider very low values of D . For that range of values, the debtor country will never default, and loans will carry the interest rate \bar{r} . Recalling that V^R is a decreasing function of D , it can be seen that the maximum debt level for which the country will never default, D^0 , is given by:

FIG. 2. Path of the Capital Stock



k^* Capital stock in the export sector.
 k Capital stock in the import-competing sector.

FIG. 3. Credit Ceiling as a Function of Export and Import-Competing Sector Capital Stocks



As a Function of Import and Export Sector Capital

k^* Capital stock in the export sector.
 k Capital stock in the import-competing sector.

$$(36) \quad V^R(P, \xi_0 y^* + Py - (1+r)D^0) = V^D(\theta P, \xi_0 y^* + \theta Py)$$

The next tranche of the supply function comprises values of debt such that the debtor country would default in state 0 but not in state 1. Thus, the probability of default is .5, and the contractual interest rate is $2(1+r)$. The maximum debt level within this range, D^1 is given by:

$$(37) \quad V^R(P, \xi_1 y^* + Py - (1+r)D^1) = V^D(\theta P, \xi_1 y^* + \theta Py)$$

In analogy with the certainty case, the credit supply function includes a "credit ceiling" which is given by D^1 . For values of debt larger than D^1 the debtor country will default in either state of nature, and so there is not interest rate that could compensate for that. The credit supply function is plotted in Figure 4. If the state space were included more discrete states, the credit supply function would have more "steps". In the limit, a continuous state space would generate an upward-sloping curve in certain range.

The key issue is how does the credit supply curve depend on the stocks of capital in the export and the import-competing sectors. If an increase in the capital stock of the export sector shifts the supply curve outwards (and an increase in the capital stock of the import-competing sector shifts it inwards) the basic results in this paper will extend in a straightforward manner to the uncertainty case. The shift in the supply curve involves only shifts in the values D^0 and D^1 . Differentiating (36) and (37) we have:

$$(38) \quad \frac{dD^0}{dk^*} = \frac{\xi_0 V_I^R - \xi_0 V_I^D}{(1+r)V_I^R} f_k^*, \text{ and}$$

$$(39) \quad \frac{dD^1}{dk^*} = \frac{\xi_1 V_I^R - \xi_1 V_I^D}{2(1+r)V_I^R} f_k^*$$

These expressions are entirely analogous to (4), and their sign is positive for precisely the same reason that $\frac{d\bar{D}}{dy^*}$ in (4) is positive.

Regarding changes in the stock of capital in the import-competing sector, the shifts in D^0 and D^1 are given by:

$$(40) \quad \frac{dD^0}{dk} = \frac{PV_I^R - \theta PV_I^D}{(1+r)PV_I^R} f_k$$

$$(41) \quad \frac{dD^1}{dk} = \frac{PV_I^R - \theta PV_I^D}{2(1+\bar{r})PV_I^R} f_k$$

which are proportional to (5), and therefore are also negative.

VI. Conclusions

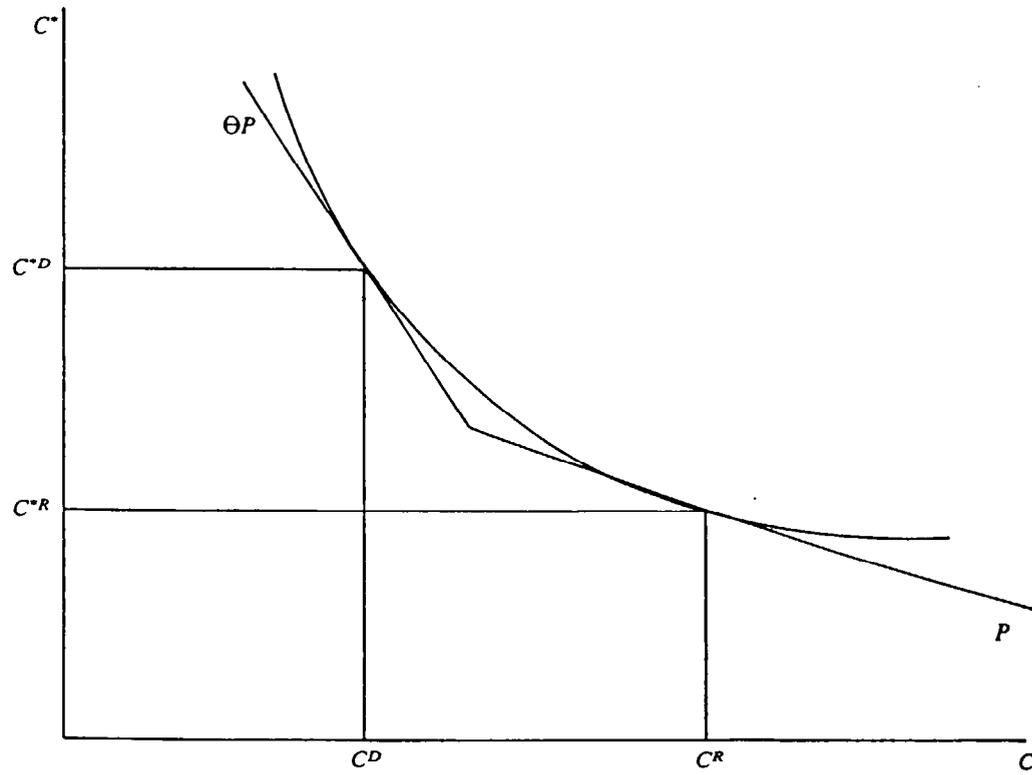
In this paper we have studied the choice of allocation of investment for a debtor country facing a ceiling on the amount of foreign debt it can accumulate. This ceiling is imposed by creditors to prevent default; it is the highest level of foreign debt at which the debtor country prefers to repay its obligations rather than commit default and suffer the ensuing economic sanctions. We have showed, in a two-period framework, that it is optimal for the debtor country to create a more open economy by favoring investment in the export sector over investment in the import-competing sector. The reason is that a more open economy is more sensitive to trade sanctions and therefore less likely to default on its foreign debt. Rational creditors recognize this fact, which brings about a higher credit ceiling to the country, providing an additional return to the reallocation of investment towards export-producing activities.

We later extended the basic result to an infinite-horizon model and to a model with uncertainty. The extension to the infinite-horizon case is somewhat limited because the conclusions are based only on a plausible conjecture and a numerical simulation because of the complexity of the problem. In the case of uncertainty, the debtor country does not face a single credit ceiling but instead a credit supply function. In this case, we have showed that a more open economy generates an outward shift in the credit supply function, providing for more favorable borrowing terms.

Some fairly direct policy implications can be extracted from the above result. Given that country risk (in terms of the credit supply available to the country) is basically an externality, individual investors would not benefit from contributing to a more open economy. Therefore, they would undertake investment in such a way as to equalize marginal returns in both productive sectors. This sets too stringent credit constraints on the country. Policy should therefore contribute to expand the export sector because lower returns to it are compensated by a larger credit availability. This could be achieved by either subsidizing capital in the export sector or taxing capital in the import-competing sector, and providing the corresponding lump-sum compensations.

Another implication of this paper refers to a long-standing debate within development economics concerning the relative merits of export promotion and import substitution policies, that is, those of outward or inward orientation of a developing economy. Considering the foreign debt problem adds a new dimension to that debate, one that strengthens the case for a more open economy. We have shown that investment in the export sector generates a more ample limit on the amount of foreign financing that

Figure 4
Comparison of Default and Repay Utilities



rational creditors will be willing to extend to a developing country. This result, therefore, brings support to an export promotion strategy on the grounds of achieving higher growth and more resiliency to adverse shocks on foreign credit availability.

Proof of the Relevant Inequalities of Section III.

In this Appendix we prove the two inequalities: $U_c^R > U_c^D$, and $U_c^D > U_c^R$. We start by noting that the derivatives in the above inequalities are evaluated at the credit ceiling function, which means that the utility level is the same for the repay and the default programs. Therefore, we can pose the problem as that of comparing the marginal utility of a good in two points on the same indifference curve (in a two-good world). In the case of the export good, its consumption is higher under default and in the case of the import good, its consumption is lower under default. Then, what we want to prove in both cases is that marginal utility is always lower at the point with higher consumption of the good.

The situation is represented in Figure 5, where the relative price of imports in terms of exports is equal to P under repayment and is equal to θP under default.

Along an indifference curve $u(c^*, c) = \bar{u}$, it must be true that:

$$(A1.1) \quad u_c^*(c^*, c) dc^* = -u_c(c^*, c) dc, \quad \text{or:}$$

$$(A1.2) \quad \frac{dc}{dc^*} = - \frac{u_c^*(c^*, c)}{u_c(c^*, c)}$$

Using the above equation, we can write the marginal utility of the export good as a function of its consumption level (Along the indifference curve). Call that function $f(c^*)$. The derivative of $f(c^*)$ is equal to:

$$(A1.3) \quad \frac{df}{dc^*} = u_{c^*c^*} - u_{c^*c} \frac{u_c^*}{u_c}$$

All we need to show is that (A1.3) is negative. We will show that that is true as long as the import-competing good is not an inferior good, which is always the case in a two-good world. Consider the first-order conditions for the two-good consumer problem:

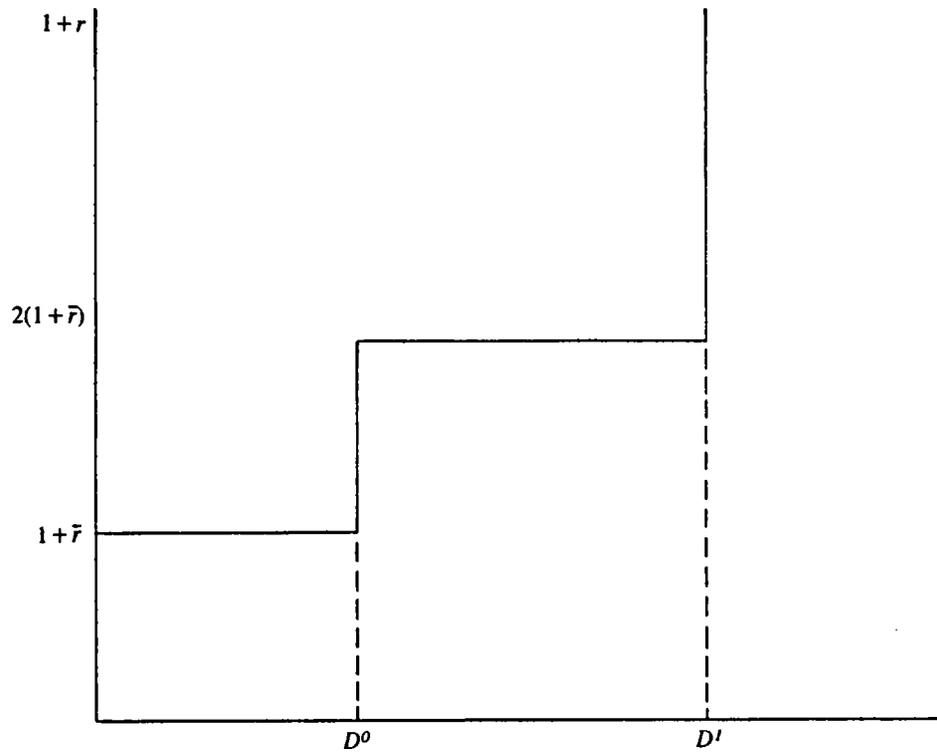
$$(A1.4) \quad u_c = \lambda P$$

$$(A1.5) \quad u_c^* = P$$

$$(A1.6) \quad c^* + Pc = I$$

where λ is the Lagrange multiplier associated with the budget constraint and I is the level of income and expenditure. Differentiation of (A1.4)-(A1.6) produces:

Figure 5
Credit Supply Function Under Uncertainty



$$(A1.7) \quad \begin{bmatrix} u_{cc} & u_{cc^*} & -P \\ u_{c^*c} & u_{c^*c^*} & -1 \\ P & 1 & 0 \end{bmatrix} \begin{bmatrix} dc \\ dc^* \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ dI \end{bmatrix}$$

Now, from (A1.7) we can compute $\frac{dc}{dy}$, which we know is positive:

$$(A1.8) \quad \frac{dc}{dy} = P(-u_{cc^*} + Pu_{c^*c^*}) > 0 \Rightarrow -\frac{u_{c^*}}{u_c} u_{cc^*} + u_{c^*c^*} > 0$$

which implies that (A1.4) is negative, as we wanted to show. The proof that $U_c^D > U_c^R$ follows from the non-inferiority of the export good and is entirely symmetrical.

Numerical Simulation Algorithm

This appendix describes how dynamic programming may be used to solve for the time consistent sequence of borrowing and investment in each sector.^{1/} Only the solution of the optimally planned economy is described in detail since the decentralized economy is simply a special case of the planner's problem. Normal dynamic optimization cannot be used to solve the model since there exists a simultaneity problem: the credit ceiling is determined by equating the repay-value function $V^R(\cdot)$ to the default value function, $V^D(\cdot)$, but to compute the repay-value function the credit ceiling must be known.

The infinite horizon model is solved by first considering its finite period analogue and then letting the length of the time horizon, T , go to infinity. Suppose, therefore, that the economy has attained its steady state capital and debt stocks by period T :

$$k_{T+i} = k_T \quad k_{T+i}^* = k_T^* \quad D_{T+i} = D_T \quad \forall i \geq 0$$

Since the investment decisions are trivial (consisting only of investment to cover depreciation) the value function and the associated investment rules, should the country decide to default are:

$$(A2.1) \quad i_T^{*D}(k_T, k_T^*) = \delta k_T^* \quad i_T^D(k_T, k_T^*) = \delta k_T$$

and:

$$(A2.2) \quad V^D(k_T, k_T^*) = \text{Max } u(c, c^*) / (1-\beta)$$

$$\text{s.t. } f(k_T^*) + \theta P f(k_T) - \delta k_T^* - \delta \theta P k_T = c_T^* + \theta P c_T$$

which is a purely static optimization problem. By contrast, if the country decides to service its debts, its stationary policies are to replace capital stocks, to make interest payments on its constant external debt, and to choose consumption optimally:

$$(A2.3) \quad i_T^{*R}(k_T, k_T^*, D_T) = \delta k_T^* \quad i_T^R(k_T, k_T^*, D_T) = \delta k_T \quad D_{+1T}^R(k_T, k_T^*, D_T) = r D_T$$

and

^{1/} This algorithm is a straightforward extension of that developed for the one-sector non-linear model of Cohen & Sachs (1986) and Ghosh (1985), which in turn are based on Bellman (1957) and Bertsekas (1976).

$$(A2.4) \quad V^R(k_T, k_T^*, D_T) = \text{Max } u(c, c^*) / (1-\beta)$$

$$\text{s.t. } f(k_T^*) + Pf(k_T) - rD_T - \delta k_T^* - P\delta k_T = c_T^* + Pc_T$$

Equating the two value functions determines the credit ceiling for T:

$$(A2.5) \quad V_T^R(k_T, k_T^*, h_T(k_T, k_T^*)) = V_T^D(k_T, k_T^*)$$

Therefore, if and only if D_T exceeds $h_T(k_T, k_T^*)$ will the country default. It is simple to prove, using standard revealed preference arguments (and the assumption that $u(\cdot)$ is strictly increasing) that $V^R(\cdot)$ is always strictly monotonically decreasing in D . Hence the repay value function can always be inverted to obtain the credit ceiling.

Now consider a hypothetical period T-1. The default optimization problem is:

$$(A2.6) \quad V_{T-1}^D(k_{T-1}, k_{T-1}^*) = \text{Max } u(c_{T-1}, c_{T-1}^*) + \beta V_T^D(k_T, k_T^*)$$

$$\text{s.t. } k_T = (1-\delta)k_{T-1} + i_{T-1}$$

$$k_T^* = (1-\delta)k_{T-1}^* + i_{T-1}^*$$

$$c_{T-1}^* + \theta Pc_{T-1} + i_{T-1}^* + \theta Pi_{T-1} = f(k_{T-1}^*) + \theta Pf(k_{T-1})$$

While the repay case is given by:

$$(A2.7) \quad V_{T-1}^R(k_{T-1}, k_{T-1}^*, d_{T-1}) = \text{Max } u(c_{T-1}, c_{T-1}^*) + \beta V_T^R(k_T, k_T^*, d_T)$$

$$\text{s.t. } k_T = (1-\delta)k_{T-1} + i_{T-1}$$

$$k_T^* = (1-\delta)k_{T-1}^* + i_{T-1}^*$$

$$D_T = (1+r)D_{T-1} + c_{T-1}^* + Pc_{T-1} + i_{T-1}^* + Pi_{T-1} - f(k_{T-1}^*) - Pf(k_{T-1})$$

$$D_T \leq h_T(k_T, k_T^*)$$

The crucial constraint is the last one, the country's credit ceiling. Since the capital market imposes this ceiling, the country's inherited debt is always low enough to make repayment the preferred alternative, and the country never defaults.

The value functions for period T-1 are then equated to obtain the credit ceiling which is imposed on borrowing in period T-2:

$$(A2.8) \quad V_{T-1}^R(k_{T-1}, k_{T-1}^*, h_{T-1}(k_{T-1}, k_{T-1}^*)) = V_{T-1}^D(k_{T-1}, k_{T-1}^*)$$

The process is repeated recursively for periods $T-2, T-3, \dots$ until the value functions and optimal policy functions converge to stationary functions: $\{V^D(\cdot), V^R(\cdot), i^D(\cdot), i^{*D}(\cdot), i^R(\cdot), i^{*R}(\cdot), D_{+1}^R(\cdot), h(\cdot)\}$.^{1/} These stationary functions are then used to determine the investment and debt policies that solve the infinite horizon model.

The algorithm we used obtains a numerical solution for the stationary functions being sought. The functions need to be calculated over a two-dimensional grid in the default case (k, k^*) , and over a three-dimensional grid in the repay case (k, k^*, D) . For each point on these grids the non-linear optimization problem of choosing k_{+1}, k_{+1}^*, d_{+1} must be solved. The maximization was done using a simple numerical search technique. The grid sizes chosen in the simulation were 20 units per dimension so that in the repay case some 8,000 optimization problems had to be solved for each iteration $T, T-1, \dots$ until convergence.

After obtaining the policy functions via backward recursion, the time path of the economy is simulated forward using these converged policy functions to generate the dynamics. Consider the repay case. Given initial capital and debt k_0, k_0^* and D_0 , the first period dynamics of the economy are:

$$\begin{aligned} k_1 &= (1-\delta)k_0 + i^R(k_0, k_0^*, D_0) \\ k_1^* &= (1-\delta)k_0^* + i^{*R}(k_0, k_0^*, D_0) \\ D_1 &= D_{+1}^R(k_0, k_0^*, D_0) \end{aligned}$$

The values for k_1, k_1^*, D_1 are then fed back into the policy functions to obtain:

$$\begin{aligned} k_2 &= (1-\delta)k_1 + i^R(k_1, k_1^*, D_1) \\ k_2^* &= (1-\delta)k_1^* + i^{*R}(k_1, k_1^*, D_1) \\ D_2 &= D_{+1}^R(k_1, k_1^*, D_1) \end{aligned}$$

In this manner the entire time series path $\{k_t, k_t^*, D_t\}$ is generated; this is the optimal path that the economy will follow and it is plotted in Figure 2. By construction, since the appropriate credit ceiling has been imposed, the default value function never exceeds the repay value function. In equilibrium the only possibility for default occurs if the initial debt

^{1/} Conditions for convergence of this algorithm are given, for example, in Blackwell (1965).

stock D_0 exceeds the credit ceiling $h(k_0, k_0^*)$ in which case the country would default in the first period.

The dynamics of the decentralized economy are obtained in a similar manner except that the credit ceiling is taken to be a fixed number while performing the optimization. Since agents have rational expectations about the level of the credit ceiling in the decentralized economy the fixed credit ceiling must represent the maximum debt allowable at the optimally chosen investments. In practice this was done by first conjecturing a credit ceiling $\bar{h}_{\tau+1}$ (in period τ), and then solving for the optimal investment choices and checking that $V_{\tau+1}^D(k_{\tau+1}, k_{\tau+1}^*) = V_{\tau+1}^R(k_{\tau+1}, k_{\tau+1}^*, \bar{h}_{\tau+1})$. If the implied credit ceiling at the optimal policies differed from $\bar{h}_{\tau+1}$ then the conjectured credit ceiling was revised and the optimization problem was solved again. The process is repeated until the actual and conjectured credit ceiling coincide.

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