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Capital Controls and International Portfolio Theory:  
A Microeconomic Approach

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Abstract

This paper examines the effects of capital controls on asset prices. A closed-form valuation model by Eun and Janakiramanan (1986) is extended to analyze the impact of three restrictions on international portfolio investment: a percentage quantity constraint on the amount of foreign securities a domestic resident may hold in her portfolio; a constraint on the absolute amount of foreign securities a domestic resident may hold; and a percentage tax on the domestic purchase price of a foreign security. Comparative statics and numerical analysis are used to reveal the effects of these distortions on domestic and world equilibrium prices.

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Table of Contents

	<u>Page</u>
Summary	iii
I. Introduction	1
II. Key Assumptions of the World Capital Market Model	3
III. The Investor's Choice Problem in a Perfectly Integrated World Capital Market	4
1. The domestic investor	4
2. The foreign investor	6
3. Equilibrium asset pricing	6
IV. The Investor's Choice Problem with Barriers to International Investment	7
1. A percentage portfolio constraint on foreign investment	7
2. An absolute quantity constraint on foreign portfolio investment	10
3. A proportional tax on domestic residents' foreign investment	12
V. Numerical Analysis	14
VI. Conclusions	17
 <u>Tables</u>	
1. Description of Model Economy	15
2. Equilibrium Asset Pricing in the World Capital Market	16
References	18

"Capital Controls and International Portfolio Theory:  
A Microeconomic Approach" by Majorie B. Rose

Restrictions on domestic residents' investments in foreign financial markets are a prevalent means of controlling capital flows between countries. Governments in both developed and developing nations have frequently used quantitative controls and differential taxation to influence the amount of foreign financial assets that a domestic resident may hold in his or her portfolio.

This paper investigates the impact of government policies that restrict international portfolio diversification on the prices of risky domestic and foreign assets. A closed-form evaluation model is extended to analyze and compare the effects of three types of restrictions on foreign portfolio investment: a percentage quantity constraint on the amount of foreign assets a domestic resident may hold; an absolute quantity constraint on the amount of foreign assets a domestic resident may hold; and an ad valorem tax on the domestic purchase price of a foreign security. Comparative statistics reveal that both the absolute and percentage quantity restrictions may lead to a premium on the domestic price of foreign securities, since domestic investors are willing to pay more in order to diversify away some of their portfolio risk. The effect of the ad valorem tax on relative asset prices is ambiguous, with an absolute discount on the price of domestic securities and a proportional discount on the price of foreign securities. A numerical analysis simulates the effects of the restrictions on asset prices under several different sets of assumptions.



## I. Introduction

Restrictions on international portfolio investment are a prevalent means of controlling capital flows between countries. These barriers cause segmented capital markets in which individuals face a more limited opportunity set in which to invest their wealth. Despite the proliferation of these barriers in both the industrialized and developing world, they have been the subject of little scrutiny in the international portfolio theory literature.

International portfolio theory has evolved from the analysis of risk-averse investors' choices of assets in a closed economy to models of portfolio decisions in a world with completely integrated capital markets. Two central issues in international portfolio diversification are the problems of exchange risk and segmented capital markets. The effect of exchange risk has been tackled by numerous studies of equilibrium asset pricing, such as Levy and Sarnat (1975), Fama and Farber (1979), and Adler and Dumas (1983). By contrast, the treatment of market segmentation has been rather sparse. In particular, the case of a partially integrated capital market in the presence of barriers to international portfolio investment has been first studied only recently.

Segmented capital markets may develop from several types of barriers to international investment. Discriminatory taxation, exchange restrictions on foreign capital transactions, and explicit limitations on ownership of foreign securities are examples of direct legal barriers that preclude international capital market integration. Imperfections such as differences in foreign accounting procedures, transactions costs, or a greater risk aversion to investing abroad are some indirect factors that may account for some market segmentation.

Attempts to model the effects of barriers to international market integration on equilibrium conditions have generally isolated a single type of imperfection. Black (1973) and Stulz (1981) show that the world market portfolio will be efficient for neither foreign nor domestic investors in the presence of differential taxation on foreign investments. Their use of proportional taxes on foreign securities can be extended to represent a variety of costs to international diversification (for example, transaction or information costs).

A one-way barrier precluding domestic agents from investing in foreign assets, but allowing foreign agents to freely invest in domestic markets, is the focus of a study by Errunza and Losq (1985). This restriction results in a higher return, or "super risk premium," on foreign securities by foreign investors over the unrestricted equilibrium return. Eun and Janakiramanan (1986) investigate the impact of a legal restriction by the government that constrains the fraction of equities of local firms that can be owned by foreigners. Noting the divergence between the supply and demand of securities resulting from the constraint, they adjust the price of foreign securities to reflect a premium for domestic investors and a discount for foreign investors.

Explicit barriers to investors diversifying their portfolios internationally have been a popular means of restricting capital flows for both developing and industrialized nations. For example, Korea requires government approval for foreign investments in Korea that exceed 50 percent of the ownership of a company and/or US\$3,000,000. Chile permits capital inflows, but restricts domestic residents from investing abroad. Until recently, France maintained quantitative controls on foreign portfolio investment as a means of restricting movements of the franc exchange rate within the EMS target. Canadian pension funds are limited to 20 percent of total investments in foreign countries. Prior to 1981, Japanese investors were completely precluded from holding foreign equities.

The objective of this paper is to construct a closed form valuation model in a two-country setting to analyze and compare the effects of several types of legal barriers to international portfolio diversification. The three barriers analyzed include a restriction of the percentage of foreign securities that may be held by domestic residents in their portfolio, a restriction on the absolute amount of foreign securities that may be held in a domestic resident's portfolio, and a proportional tax on the price of foreign securities for domestic residents.

An extension of the model developed by Eun and Janakiramanan (1986) is used to evaluate portfolio choice when the investment in foreign assets is explicitly restricted. Several important conclusions emerge from the model. First, depending upon the nature of the covariance structure between domestic and foreign securities, the binding constraint will lead to a premium on the internal domestic price of foreign securities and a discount on the internal price of domestic securities. This result is intuitively appealing because it supports the notion that agents are willing to pay more to diversify away some of the risk associated with holding an exclusively domestic portfolio. Second, the effect of the barrier on world market equilibrium prices will depend upon the importance of the domestic countries' share in the world market. In particular, if a country has a relatively large share of the world's capital market, equilibrium prices in both the constrained domestic country and the unconstrained foreign country will be distorted in proportion to its relative market shares. The case of the proportional tax on foreign securities is ambiguous. The price of domestic securities is lowered by an absolute amount, while the price of foreign securities falls in proportion to the tax rate.

The remainder of the paper is as follows. Section 2 offers a brief description of the assumptions underlying our model of the world capital market. Section 3 presents the model in a world with completely integrated capital markets. In Section 4, the derivation of the equilibrium asset demands and prices from the investor's choice problem is presented for each of the three types of restrictions. Comparative statistics reveal the differences between the various barriers. A numerical analysis simulates portfolio choices within the context of the two-country model

under several different sets of assumptions in Section 5. Conclusions and a summary are provided in Section 6.

## II. Key Assumptions of the World Capital Market Model

In our simplified world, only two countries exist--the domestic country, D, and the foreign country, F. The domestic country is assumed to be small relative to the foreign country. In this respect, the foreign country can be thought to represent the rest of the world's capital market, of which the domestic country has a relatively minor share.

In order to concentrate on the specific problem of market segmentation and abstract from the concept of exchange risk, a further simplifying assumption of a fixed exchange rate regime is made. <sup>1/</sup> In addition, we make the following simplifying assumptions, which are widely used by most theories of capital market equilibrium:

- (A1) Perfect competition in each countries' capital market.
- (A2) Investors have homogeneous expectations of securities' risk and return.
- (A3) Security prices are distributed jointly normal.

The following notation will be used in the remainder of the chapter:

- $N_d$  - the vector of the number of shares in the domestic country.
- $N_f$  - the vector of the number of shares in the foreign country.
- $n_{kd}, n_{kf}$  - vectors of the number of shares of domestic and foreign securities held by the kth individual, respectively.
- $P_d, P_f$  - vectors of the random end-of-period prices of domestic and foreign securities, respectively.
- $\mu_d, \mu_f$  - vectors of the expected values of end-of-period domestic and foreign prices, respectively.
- $\Gamma_d, \Gamma_f$  - covariance matrices of the prices of domestic and foreign securities, respectively.
- $\Gamma_{df}$  - covariance matrix of the prices of domestic securities and the prices of foreign securities.

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<sup>1/</sup> Another approach would be to use a logarithmic form of the investor's utility function, which is invariant to relative price changes (Stehle 1977, Jorion and Schwartz 1986).

$W_k^*$  = initial wealth of individual  $k$  at time 0.

$W_k$  = random end-of-period wealth of individual  $k$ .

### III. The Investor's Choice Problem in a Perfectly Integrated World Capital Market

#### 1. The domestic investor

The investor's choice problem is straightforward. Each investor maximizes the expected utility of [his or] her end-of-period wealth. To do this, each individual  $k$  chooses to invest [his or] her initial wealth  $W$  between the risk-free asset and the risky foreign and domestic assets,  $n_f$  and  $n_d$ , respectively. Investors in either country can borrow and lend at an identical risk-free rate ( $r$ ). If the investor has a constant measure of absolute risk aversion, the utility function may be represented by:

$$U_k(W_k) = -\exp [-A_k W_k] \quad (1)$$

where  $A_k = -U''/U'$ , the Pratt-Arrow measure of risk aversion.

Given the assumptions of jointly normal security returns and exponential utility, portfolio separation obtains and thus:

$$E [U_k(W_k)] = -\exp [-A_k \bar{W}_k - A_k/2 \text{Var}(W_k)] \quad (2)$$

where the optimal investment choice can be seen to depend upon the first two moments of the end-of-period wealth. The objective is, therefore, to maximize the difference between the individual's expected wealth and the risk premium, i.e., the level of wealth the individual would accept with certainty if the riskiness were removed, subject to the budget constraint. The individual maximizes [his or] her certainty equivalent of wealth given the set of market prices for the securities ( $P_i$ ) over the possible vector of the number of shares held for each security ( $n_{ki}$ ). This certainty equivalent of end-of-period wealth is given by:

$$CEW_k = \bar{W}_k - (A_k/2) \text{Var}(W_k) \quad (3)$$

and the budget constraint is the initial wealth of the investor which is allocated between the risk-free asset and the two risky assets:

$$W_k^* = n_{kd}'P_d + n_{kf}'P_f + F_k. \quad (4)$$

where  $F_k$  is the amount of wealth invested in the risk free asset.

Given the rate of return on the risk-free asset, the expected end-of-period wealth and variance become:

$$W_k = n_{kd}'(\mu_d - P_d r) + n_{kf}'(\mu_f - P_f r) + W_k^* r \quad (5)$$

$$\text{Var}(W_k) = n_{kd}'\Gamma_d n_{kd} + 2n_{kd}'\Gamma_{df} n_{kf} + n_{kf}'\Gamma_f n_{kf} \quad (6)$$

Substituting (5) and (6) into (3), the unconstrained maximization problem becomes:

$$\text{Maximize}_{(n_{kd}, n_{kf})} \quad CEW_k = n_{kd}'(\mu_d - P_d r) + n_{kf}'(\mu_f - P_f r) + W_k^* r \quad (7)$$

$$-A_k/2(n_{kd}'\Gamma_d n_{kd} + 2n_{kd}'\Gamma_{df} n_{kf} + n_{kf}'\Gamma_f n_{kf}) .$$

To solve for the demand for domestic and foreign securities by a domestic resident, the first order conditions are obtained:

$$\frac{d(CEW_k)}{d(n_{kd})} = (\mu_d - P_d r) - A_k(\Gamma_d' n_{kd} + \Gamma_{df}' n_{kf}) = 0 \quad (8)$$

$$\frac{d(CEW_k)}{d(n_{kf})} = (\mu_f - P_f r) - A_k(\Gamma_f' n_{kf} + \Gamma_{df}' n_{kd}) = 0 \quad (9)$$

and the resulting demands are:

$$n_{kd} = \frac{\Gamma_d'(\mu_d - P_d r) - \Gamma_{df}'(\mu_f - P_f r)}{A_k [\Gamma_d \Gamma_f - \Gamma_{df}' \Gamma_{df}]} \quad (10)$$

$$n_{kf} = \frac{\Gamma_f'(\mu_f - P_f r) - \Gamma_{df}'(\mu_d - P_d r)}{A_k [\Gamma_d \Gamma_f - \Gamma_{df}' \Gamma_{df}]} \quad (11)$$

Aggregating the individual demands for domestic and foreign securities:

$$n_{dd} = \sum n_{kd} , \quad n_{df} = \sum n_{kf} , \quad 1/A_d = \sum 1/A_k$$

permits a solution for domestic prices:

$$P_{dd} = (1/r) (\mu_d - A_d \Gamma_{df} n_{df} - A_d \Gamma_d n_{dd}) \quad (12)$$

$$P_{df} = (1/r) (\mu_f - A_d \Gamma_{df} n_{dd} - A_d \Gamma_f n_{df}) \quad (13)$$

where  $P_{dd}$  is the domestic price of domestic securities and  $P_{df}$  is the domestic price of foreign securities.

## 2. The foreign investor

The demand by the residents,  $q$ , of the foreign country is derived from a similar choice problem:

$$n_{qd} = \frac{\Gamma_d' (\mu_d - P_d r) - \Gamma_{df}' (\mu_f - P_f r)}{A_q (\Gamma_d \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (14)$$

$$n_{qf} = \frac{\Gamma_f' (\mu_f - P_f r) - \Gamma_{df}' (\mu_d - P_d r)}{A_q (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (15)$$

Aggregating the individual foreigner's demands for domestic and foreign assets in matrix form:

$$\begin{bmatrix} n_{fd} \\ n_{ff} \end{bmatrix} = 1/A_f \begin{bmatrix} \Gamma_d & \Gamma_{df} \\ \Gamma_{df}' & \Gamma_f \end{bmatrix} \begin{bmatrix} (\mu_d - P_d r) \\ (\mu_f - P_f r) \end{bmatrix} \quad (16)$$

where the aggregate demand by foreign residents is the sum of the individual demands:

$$n_{fd} = \sum n_{qd}, \quad n_{ff} = \sum n_{qf}, \quad 1/A_f = \sum 1/A_q.$$

## 3. Equilibrium asset pricing

Equilibrium asset pricing for the two securities can be obtained by summing the aggregate demands across countries, where the market demands equal the market supplies. Summing the set of equations, (10), (11) and (16) across both countries, the world demand conditions for the domestic and foreign assets can be expressed as:

$$N_d = \frac{\Gamma_d' (\mu_d - P_d r) - \Gamma_{df}' (\mu_f - P_f r)}{(A_d + A_f) (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (17)$$

$$N_f = \frac{\Gamma_f' (\mu_f - P_f r) - \Gamma_{df}' (\mu_d - P_d r)}{(A_d + A_f) (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (18)$$

The security demands may be solved in terms of prices:

$$P_d = 1/r [\mu_d - (A_d + A_f) \Gamma_d' N_d - (A_d + A_f) \Gamma_{df}' N_f] \quad (19)$$

$$P_f = 1/r [\mu_f - (A_d + A_f) \Gamma_{df}' N_d - (A_d + A_f) \Gamma_f' N_f] \quad (20)$$

#### IV. The Investor's Choice Problem with Barriers to International Investment

Equations (1) through (20) describe the investor's choice problem in an unrestricted investment environment. Each country has a number of risky assets-- $N_d$  in the domestic country and  $N_f$  in the foreign country. Foreign residents may invest freely in securities from either home or abroad. However, if there are restrictions on capital outflows in the domestic country, domestic residents may be precluded from investing freely in foreign securities. This section investigates the effects of three types of investment barriers on the quantities demanded and the prices of securities.

##### 1. A percentage portfolio constraint on foreign investment

The domestic investors choice problem will change if domestic residents are prohibited from investing more than a fraction,  $\delta$ , of their portfolios in the foreign country. In this case, the maximization problem after the substitution of the ownership constraint is given by:

$$\text{Maximize } CEW_k = \delta n_{kd}(\mu_d - P_d r) + (1-\delta)n_{kf}'(\mu_f - P_f r) + W_k r \quad (21)$$

$$(n_{kd}, n_{kf})$$

$$- A_k/2[\delta n_{kd}' \Gamma_d n_{kd} + 2\delta(1-\delta)n_{kd}' \Gamma_{df} n_{kf} + (1-\delta) n_{kf}' \Gamma_f n_{kf}]$$

where the ownership restriction of foreign securities by domestic residents is a fraction,  $\delta$ , of their total desired domestic holdings,  $n_{kf} = \delta n_{kf}^*$  and the remainder is invested in domestic securities,  $n_{kd} = (1-\delta)n_{kd}^*$  ('\*' represents the unconstrained holdings).

The first-order conditions are consequently:

$$\frac{d(CEW_k)}{d(n_{kd})} = \delta(\mu_d - P_d r) - A_k[\delta \Gamma_d' n_{kd} + \delta(1-\delta) \Gamma_{df}' n_{kf}] = 0 \quad (22)$$

$$\frac{d(CEW_k)}{d(n_{kf})} = (1-\delta)(\mu_f - P_f r) - A_k[\delta(1-\delta) \Gamma_{df}' n_{kd} + (1-\delta) \Gamma_f' n_{kf}] = 0 \quad (23)$$

Solving equations (15) and (16) for the demand for assets by domestic residents yields:

$$n_{kd} = \frac{\Gamma_d' (\mu_d - P_d r) - \Gamma_{df}' (\mu_f - P_f r)}{(1-\delta)A_k (\Gamma_d \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (24)$$

$$n_{kf} = \frac{\Gamma_f' (\mu_f - P_f r) - \Gamma_{df}' (\mu_d - P_d r)}{\delta A_k (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (25)$$

The total demand for domestic and foreign assets by domestic residents can be written in matrix notation:

$$\begin{bmatrix} n_{dd} \\ n_{df} \end{bmatrix} = 1/A_d \begin{bmatrix} 0 & (1-\delta) \\ \delta & 0 \end{bmatrix} \begin{bmatrix} \Gamma_d & \Gamma_{df} \\ \Gamma_{df}' & \Gamma_f \end{bmatrix} \begin{bmatrix} (\mu_d - P_{dr}) \\ (\mu_f - P_{fr}) \end{bmatrix} \quad (26)$$

The ramifications of the restriction on foreign financial investment in the domestic country are readily apparent in the internal domestic pricing structure. Rewriting the domestic demand for assets from home and abroad in equation (26) in terms of prices becomes:

$$P_{dd} = (1/r) [\mu_d - \delta A_d \Gamma_{df}' n_{df} - (1-\delta) A_d \Gamma_d' n_{dd}] \quad (27)$$

$$P_{df} = (1/r) [\mu_f - (1-\delta) A_d \Gamma_{df}' n_{dd} - \delta A_d \Gamma_f' n_{df}] \quad (28)$$

If the constraint on foreign investment is binding,  $n_{df} < \delta n_{df}^*$ , actual holdings are less than desired holdings. Note that any arbitrage opportunities are also assumed to be restricted, i.e., a foreign investor cannot trade a foreign security purchased on the world market at a lower price and resell it to a domestic investor at a higher price. The internal price of securities will differ from the unrestricted price by the amount:

$$- ((1-\delta) A_d \Gamma_{df} n_{df} + \delta A_d \Gamma_d n_{dd})$$

By contrast, the actual domestic holdings will be greater than the desired amount i.e.,  $n_{dd} > (1-\delta) n_{dd}^*$  and will likewise possess a price that is greater or less than the unrestricted price, depending on the covariance structure of the securities:

$$- (\delta A_d \Gamma_{df}' n_{dd} + (1-\delta) A_d \Gamma_f' n_{df})$$

A reasonable assumption is that the covariance of securities within countries is generally greater than the covariance of securities between countries. Differentials in financial and economic structure are the basis for portfolio diversification into international capital markets. Risk averse investors desire to hold assets that will not be highly correlated. If this is the case, then the internal price of domestic securities will be at a discount, and the price of foreign assets will be at a premium:

$$\frac{d(P_{dd})}{d(\delta)} = (1/r) A_d (\Gamma_{df}' n_{df} - \Gamma_d' n_{dd}) < 0 \quad (29)$$

$$\frac{d(P_{df})}{d(\delta)} = -A_d \Gamma_{df}' n_{dd} + A_d \Gamma_f' n_{df} > 0 \quad (30)$$

Logically, if the domestic investors desire to hold more foreign assets than is permitted, then they will be willing to pay more than the unconstrained price of foreign securities. This premium can be seen to reflect the amount the risk averse investor is willing to pay to avoid the diversification loss imposed by the regulation.

The effect on the quantity of domestic and foreign securities demanded by domestic residents can be seen by the total differentiation of the demand conditions in equation (26). An increase in the constraint,  $\delta$ , i.e., permitting domestic residents to hold a greater portfolio share of foreign securities, will tend to decrease the quantity of domestic assets and increase the number of foreign assets demanded, ceteris paribus:

$$\frac{d(n_{dd})}{d(\delta)} = (1/\delta) [\Gamma_{df}'(\mu_f - P_f r) - \Gamma_d(\mu_d - P_d r)] < 0 \quad (31)$$

As expected, relaxing the constraint will have the opposite effect on the quantity of foreign securities demanded as investors shift into foreign assets in order to diversify their portfolios:

$$\frac{d(n_{df})}{d(\delta)} = [1/(1-\delta)] [\Gamma_f(\mu_f - P_f r) - \Gamma_{df}'(\mu_d - P_d r)] > 0 \quad (32)$$

The foreign equations will be unaffected by the imposition of the constraint in the domestic country. However, aggregating both the domestic and foreign demands for the assets permits an analysis of the effects on the world market prices in the case of such a quantitative investment barrier:

$$P_d = 1/r \{ \mu_d - (\delta A_d + A_f) \Gamma_{df}' N_f - [(1-\delta)A_d + A_f] \Gamma_d' N_d \} \quad (33)$$

$$P_f = 1/r \{ \mu_f - [(1-\delta)A_d + A_f] \Gamma_{df}' N_d - (\delta A_d + A_f) \Gamma_f' N_f \} \quad (34)$$

Taking derivatives with respect to the constraint reveals that the equilibrium prices of domestic assets will rise with the imposition of the constraint and the equilibrium foreign price will decline. The effect of domestic prices arises mainly due to the greater demand from domestic residents, while lower foreign prices reflect the relatively greater supply in the non-domestic market of the foreign assets. The degree of the distortion is weighted by the aggregate level of risk aversion in the domestic country,  $A_d$ :

$$\frac{dP_d}{d\delta} = (A_d/r) (\Gamma_d' N_d - \Gamma_{df}' N_f) > 0 \quad (35)$$

$$\frac{dP_f}{d\delta} = (A_d/r) (\Gamma_{df}' N_d - \Gamma_f' N_f) < 0 \quad (36)$$

Because the size of the level of aggregate risk aversion is determined by both the number of participants in the market and their respective level of risk aversion, the degree of the distortion will depend heavily on how large the country is relative to the rest of the world.

2. An absolute quantity constraint on foreign portfolio investment

Another constraint on domestic foreign investment is a legal restriction on the absolute amount of foreign assets that a domestic resident is permitted to own. This type of barrier will become more binding as the amount of wealth increases. If domestic residents are limited to an amount of foreign investment,  $\bar{n}_{df}$ , such that  $\bar{n}_{df} < n_{df}^*$ , a Lagrangian function may be used resulting in the modified objective function, where  $\theta$  represents the Lagrangian multiplier:

$$\begin{aligned} \text{Maximize } CEW_k &= n_{kd}'(\mu_d - P_d r) + n_{kf}'(\mu_f - P_f r) + W_k^* r \\ & (n_{kd}, n_{kf}) \\ & - A_k/2(n_{kd}'\Gamma_d n_{kd} + 2n_{kd}'\Gamma_{df} n_{kf} + n_{kf}'\Gamma_f n_{kf}) \\ & + \theta (n_{kf} - \bar{n}_{kf}) \end{aligned} \quad (37)$$

The following set of first order conditions represent a system of simultaneous equations:

$$\frac{d(CEW_k)}{d(n_{kd})} = (\mu_d - P_d r) - A_k(\Gamma_d' n_{kd} + \Gamma_{df} n_{kf}) = 0 \quad (38)$$

$$\frac{d(CEW_k)}{d(n_{kf})} = (\mu_f - P_f r) - A_k(\Gamma_f' n_{kf} + \Gamma_{df}' n_{kd}) + \theta = 0 \quad (39)$$

$$\frac{d(CEW_k)}{d(\theta)} = n_{kf} - \bar{n}_{kf} = 0 \quad (40)$$

Assuming the constraint is binding, the domestic demand for foreign securities is given by the constraint and is used to solve for the domestic securities demand. Aggregating the quantities across all individuals gives the following demand conditions:

$$n_{df} = \bar{n}_{df} \quad (41)$$

$$n_{dd} = \frac{\mu_d - P_d r - \Gamma_{df} \bar{n}_{df}}{A_d \Gamma_d} \quad (42)$$

$$\theta = (\mu_f - P_f r) - \frac{\Gamma_{df}(\mu_d - P_d r)}{A_d \Gamma_d} + \frac{(\Gamma_{df}' \Gamma_{df} - A_d \Gamma_f) \bar{n}_{df}}{A_d \Gamma_d} \quad (43)$$

Rewriting the demands in terms of the internal price for domestic and foreign securities under this type of quantity constraint yields:

$$P_{dd} = \frac{\mu_d - A_d \Gamma_d n_{dd} - \Gamma_{df} \bar{n}_{df}}{r} \quad (44)$$

$$P_{df} = \frac{\mu_f}{r} - \frac{\theta}{r} - \frac{A_d \Gamma_f \bar{n}_{df}}{r} - \frac{2 \Gamma_{df} \mu_d}{A_d \Gamma_d r} + \frac{\Gamma_{df} n_{dd}}{r} \quad (45)$$

As expected, an increase in the amount of foreign securities a domestic resident is permitted to purchase,  $\bar{n}_{df}$ , will increase the amount of foreign securities purchased and will lower the quantity demanded of domestic securities:

$$\frac{d(n_{dd})}{d(\bar{n}_{df})} = \frac{-\Gamma_{df}}{A_d \Gamma_d} < 0 \quad (46)$$

$$\frac{d(n_{df})}{d(\bar{n}_{df})} = 1 > 0 \quad (47)$$

Differentiation of the domestic price equations reveals that the domestic price for domestic securities will fall with an increase in the amount of foreign securities permitted to be purchased as domestic residents shift their demand from domestic to foreign securities:

$$\frac{d(P_{dd})}{d(\bar{n}_{df})} = \frac{-\Gamma_{df}}{r} < 0 \quad (48)$$

Similarly, the domestic price of foreign prices is expected to fall as the supply curve of foreign securities shifts out:

$$\frac{d(P_{df})}{d(\bar{n}_{df})} = \frac{-A_d \Gamma_f}{r} < 0 \quad (49)$$

Aggregating the total domestic and foreign demand for the two securities yields:

$$N_d = \frac{\Gamma_f'(\mu_d - P_d r) - \Gamma_{df}'(\mu_f - P_f r)}{A_f(\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} + \frac{\mu_d - P_d r - \Gamma_{df} \bar{n}_{df}}{A_d \Gamma_d} \quad (50)$$

$$N_f = \frac{\bar{n}_{df} + \frac{\Gamma_d (\mu_f - P_f r) - \Gamma_{df} (\mu_d - P_d r)}{A_f(\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})}}{\Gamma_d} \quad (51)$$

These two equations can be rearranged in terms of world prices:

$$P_d = \frac{1}{r A_w} (\mu_d - [\Gamma_{df} (1 - A_d)] \bar{n}_{df} - \Gamma_{df} N_f - \Gamma_d N_d) \quad (52)$$

$$P_f = \frac{1}{r} \frac{[A_w A_d \Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df}] \bar{n}_{df} + [A_f(\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df}) + \Gamma_{df}' \Gamma_{df}] N_f}{A_w A_d \Gamma_d} + \mu_f - \frac{\Gamma_{df}}{A_w} N_d \quad (53)$$

where  $A_w = A_d + A_f$

The effect of such a quantity constraint on equilibrium prices can be seen in the following derivatives:

$$\frac{d(P_d)}{d(\bar{n}_{df})} = \frac{-\Gamma_{df} (1 - A_d)}{r A_w} < 0 \quad (54)$$

$$\frac{d(P_f)}{d(\bar{n}_{df})} = \frac{A_w A_d \Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df}}{r A_w A_d \Gamma_d} > 0 \quad (55)$$

where the excess supply of domestic securities results in a fall in the world price in order to induce risk averse agents to hold a relatively greater share of the world market. An opposite effect on world foreign price results from the imposed fall in the supply for foreign securities.

### 3. A proportional tax on domestic residents' foreign investment

In the case where the domestic residents are required to pay a tax,  $t$ , on the purchase price of foreign securities the maximization problem can be expressed as:

$$\begin{aligned} \text{Maximize } CEW_k &= n_{kd}(\mu_d - P_d r) + n_{kf}(\mu_f - P_f(1+t)r) + W_k^* r \\ (n_{kd}, n_{kf}) & \\ & - A_k/2 \{n_{kd}' \Gamma_d n_{kd} + 2n_{kd}' \Gamma_{df} n_{kf} + n_{kf}' \Gamma_f n_{kf}\} \end{aligned} \quad (56)$$

whose first order conditions:

$$\frac{d(CEW_k)}{d(n_{kd})} = (\mu_d - P_d r) - A_k(\Gamma_d n_{kd} + \Gamma_{df} n_{kf}) = 0 \quad (57)$$

$$\frac{d(CEW_k)}{d(n_{kf})} = [\mu_f - P_f(1+t)r] - A_k[\Gamma_f n_{kf} + \Gamma_{df} n_{kd}] = 0 \quad (58)$$

result in the following demand equations:

$$n_{kd} = \frac{\Gamma_f (\mu_d - P_d r) - \Gamma_{df} [\mu_f - P_f(1+t)r]}{A_k [\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df}]} \quad (59)$$

$$n_{kf} = \frac{\Gamma_f [\mu_f - P_f(1+t)r] - \Gamma_{df} (\mu_d - P_d r)}{A_k (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} \quad (60)$$

Aggregating the individual demands for domestic and foreign securities:

$$n_{dd} = \sum n_{kd}, \quad n_{df} = \sum n_{kf}, \quad 1/A_d = \sum 1/A_k$$

permits a solution for securities prices in the domestic country:

$$P_{dd} = (1/r) [\mu_d - A_d \Gamma_{df} n_{df} - A_d \Gamma_d n_{dd}] \quad (61)$$

$$P_{df} = (1/(1+t)r) [\mu_f - A_d \Gamma_{df} n_{dd} - A_d \Gamma_f n_{df}] \quad (62)$$

Comparative static exercises for the case of taxes reveal similar results to that of the quantity constraints on the demand for the securities by domestic residents:

$$\frac{d(n_{dd})}{d(t)} = \frac{\Gamma_{df} P_f r}{A_d (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} > 0, \quad \Gamma_d' \Gamma_f > \Gamma_{df}' \Gamma_{df} \quad (63)$$

$$\frac{d(n_{df})}{d(t)} = \frac{-\Gamma_f P_f r}{A_d (\Gamma_d' \Gamma_f - \Gamma_{df}' \Gamma_{df})} < 0, \quad \Gamma_d' \Gamma_f > \Gamma_{df}' \Gamma_{df} \quad (63)$$

Domestic prices for home securities are unaffected by the tax, however, the domestic price for foreign assets will increase with an increase in the tax as reflected in the derivative:

$$\frac{d(P_{dd})}{d(t)} = 0 \quad (65)$$

$$\frac{d(P_{df})}{d(t)} = \frac{-1}{(1+t)^2 r} (\mu_f - A_d \Gamma_{df} n_{dd} - A_d \Gamma_f n_{df}) < 0 \quad (66)$$

The effects on world prices of the securities can be seen as the demands are aggregated and solved in terms of prices:

$$P_d = \frac{\mu + \Gamma_{df} (r + A_w) \mu_f}{r \Gamma_f r} \frac{A_w \Gamma_d N_d}{r} - \frac{A_w \Gamma_{df} N_f}{r} \quad (67)$$

$$P_f = (1/rb) (\mu_f - \Gamma_{df} N_d - \Gamma_f N_f) \quad (68)$$

where  $b = (1/A_w + t/A_d)$ .

#### V. Numerical Analysis

A numerical simulation of the effects of the various constraints would serve two useful purposes. First, it will provide a basis of comparison for the results found in the earlier work of Eun and Janakiramanan (1986). Second, it will illustrate the differences in the impact between the type of capital controls applied. In addition to the solutions for the world prices of domestic and foreign securities in the presence of investment barriers, the completely unrestricted as well as the completely restricted cases are provided.

The exogenous parameters in the model economy are identical to that of two earlier papers, Stapleton and Subrahmanyam (1977) and Eun and Janakiramanan (1986) and are presented in Table 1. There are eight firms; firms one through four are domestic, and firms five through eight are foreign. Each firm has one thousand shares with possess an expected end-of-period price of \$100 resulting in an expected value of each firm of \$100,000. The correlations and the standard deviations of the expected values for each firm are also provided. There are 20 investors, 10 domestic and 10 foreign, whose measures of constant absolute risk aversion (CARA) are also given in Table 1. The risk-free rate is set at eight percent.

The calculations for equilibrium world prices for the following cases are presented in Table 2:

- (i) complete integration, where there are no constraints on international portfolio investment;
- (ii) a percentage portfolio constraint where domestic investors are restricted from holding more than a fraction,  $\delta$ , of their portfolio in the form of foreign securities;

Table 1. Description of Model Economy

Firms	No. of Shares $N_i$	$V_i = \mu_i i$ \$	$\sigma_{vi} = \sigma_i i$	Correlation Matrix							
1	1000	100,000	18,000	1.0	0.7	0.9	0.7	0.1	0.1	0.1	0.1
2	1000	100,000	22,000		1.0	0.7	0.9	0.1	0.1	0.1	0.1
3	1000	100,000	18,000			1.0	0.9	0.1	0.1	0.1	0.1
4	1000	100,000	22,000				1.0	0.1	0.1	0.1	0.1
5	1000	100,000	25,000					1.0	0.7	0.9	0.7
6	1000	100,000	30,000						1.0	0.7	0.9
7	1000	100,000	25,000							1.0	0.7
8	1000	100,000	30,000								1.0

Domestic Investors	CARA Parameter	Foreign Investor	CARA Parameter
1	7600	11	6000
2	7800	12	6200
3	8000	13	6400
4	8200	14	6500
5	8400	15	6600
6	8500	16	6800
7	8800	17	7000
8	9000	18	7200
9	9500	19	7400
10	10000	20	7500

Note: The risk-free interest rate is 8 percent.

Table 2. Equilibrium Asset Pricing in the World Capital Market

Asset	Complete Integration	Percentage Quantity Constraint $\delta=20\%$	$\delta=60\%$	Absolute Quantity Constraint		Proportional Tax	
				$\bar{n}_{df}=12$	$n_{df}=10$	$t=0.10$	$t=0.25$
1	84.34	98.38	101.68	86.51	88.89	75.41	75.41
2	82.23	99.98	104.11	84.49	88.40	71.10	71.10
3	84.34	98.24	101.58	85.64	88.19	74.19	74.19
4	82.24	99.86	104.02	84.75	87.80	72.27	72.27
5	77.90	69.49	76.99	84.33	84.06	71.36	69.67
6	74.50	70.59	77.96	82.24	81.97	72.31	70.11
7	77.90	71.11	78.20	84.26	84.01	73.12	71.37
8	74.50	66.51	75.10	82.17	81.87	70.93	68.96

(iii) an absolute quantity constraint where domestic residents are precluded to invest more than a fixed number of shares in foreign securities; and

(iv) a proportional tax on the domestic price of foreign securities.

The calculations for the percentage quantity constraint were computed for two different values of  $\delta$ , 20 percent and 60 percent. In the case of  $\delta=20$  percent, the prices for both domestic and foreign firms increase. However, foreign securities' prices increase more than domestic securities' in both absolute and relative terms. As the constraint becomes less binding, in the case of  $\delta=60$  percent, both sets of prices increase, but increases in foreign prices are lower than their domestic counterparts in absolute terms.

When the absolute quantity constraint is imposed on domestic investors, the price for both sets of securities is lowered relative to the unrestricted case. The more binding the constraint (i.e., the lower the  $\bar{n}_{df}$ ) the lower the relative price of foreign to domestic securities. The threshold level for the constraint to become binding is  $n_{df}=13.9$ . The two cases,  $\bar{n}_{df}=12$  and  $\bar{n}_{df}=10$  illustrate the greater impact on foreign versus domestic prices.

In the case of a proportional tax on the domestic price of foreign securities, domestic prices decrease compared to the unrestricted case, but appear to be unaffected by changes in the tax rate. Prices may be lower overall because there are fewer transactions in this case. One intuitive reason why taxes do not appear to affect  $P_d$  is that foreigners can still buy into the domestic market, and therefore support prices even though the converse is not true for domestic residents. By contrast, foreign securities' prices decrease in proportion to increases in the tax rate.

Overall, the degree for each of the three distortions will generally affect the equilibrium prices in a proportionate manner. However, it appears that the different regulatory schemes are not comparable on a price basis.

## VI. Conclusions

In this paper, a closed form model of the investor's choice problem using a finance theoretical framework was presented to investigate the impact of capital controls on security demands and prices. A legal barrier that restricted the percentage of the total portfolio that a domestic investor may hold in the form of foreign securities was seen to result in a premium on the domestic price of foreign securities and a discount on domestic securities. The effect on world prices is dependent on the share of the domestic holdings in the world market. The degree of the distortions to both the internal and world prices of securities is affected by the severity of the constraint on domestic ownership of foreign securities,  $\delta$ , and the significance of the covariance structure between domestic and foreign securities,  $\Gamma_{df}$ . If the correlation between the two sets of assets is insignificantly positive or inversely related, the domestic investors will pay a premium to avoid the diversification loss on the foreign securities.

### References

- Black, F., "International Capital Market Equilibrium with Investment Barriers," Journal of Financial Economics (December 1974), vol. 1, pp. 337-52.
- Brennan, M., and E. Schwartz, "Asset Pricing in a Small Economy: A Test of the Omitted Assets Model," in Capital Market Equilibria, ed. by G. Bamberg and K. Spreman (New York: Springer-Verlag, 1984), pp. 164-88.
- Errunza V., and E. Losq, "International Asset Pricing under Mild Segmentation: Theory and Test," Journal of Finance, vol. 40 (March 1986), pp. 105-24.
- Eun, C. S., and S. Janakiraman, "A Model of International Asset Pricing with a Constraint on the Foreign Equity Ownership," Journal of Finance, vol. 41 (September 1986), pp. 897-914.
- Grauer, Frederick L.A., Robert H. Litzenberger and Richard E. Stehle, "Sharing Rules and Equilibrium in an International Capital Market Under Uncertainty," Journal of Financial Economics, vol. 3 (June 1976), pp. 233-36.
- Gultekin, M. N., N.B. Gultekin and A. Penati, "Capital Controls and International Capital Markets Segmentation: The Evidence from the Japanese and American Stock Markets," Journal of Finance, vol. 44 (September 1989), pp. 849-70.
- Hietala, Pekka T., "Asset Pricing in Partially Segmented Markets: Evidence from the Finnish Market," Journal of Finance, vol. 44. No. 3 (July 1989), pp. 697-718.
- Levy, H., and M. Sarnat, "Devaluation Risk and the Portfolio Analysis of International Investment," in International Capital Markets, ed. by E. J. Elton and M.J. Gruber (Amsterdam: North Holland, 1975), pp. 177-206.
- Solnik, Bruno H., "An Equilibrium Model of the International Capital Market," Journal of Economic Theory, vol. 8 (August 1974), pp. 500-24.
- \_\_\_\_\_, "An International Market Model of Security Price Behaviour," Journal of Financial and Quantitative Analysis, vol. 9 (September 1974), pp. 537-54.
- Stambaugh, R., "On the Exclusion of Assets from Tests of the Two Parameter Model," Journal of Financial Economics, vol. 10 (November 1982), pp. 237-68.

Stapleton, R. G., and M. G. Subrahmanyam, "Market Imperfections, Capital Market Equilibrium and Corporation Finance," Journal of Finance vol. 32 (May 1977), pp. 307-21.

Stehle, R., "An Empirical Test of National and International Pricing of Risky Assets," Journal of Finance, vol. 32 (May 1977), pp. 493-502.

Stulz, R. M., "On the Effects of Barriers to International Investment," Journal of Finance, vol. 36 (September 1981), pp. 923-34.

