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Stabilization Policy With Bands

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Abstract

This paper discusses stabilization policy in the presence of bands for the exchange rate. The bands are modelled in a probabilistic sense: monetary policy has to be such as to keep the probability, that the exchange rate stays within the bands, above a certain threshold. In contrast to other models of target zones, this formulation leads to a linear decision rule and implies sizeable intra-marginal interventions, which corresponds to the experience in the EMS. The extent to which short-run monetary policy is constraint by the bands depends on its own long-run components and on fiscal policy.

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1/ The author wishes to thank his colleagues in the Research Department for helpful discussions and comments. Daniel Gros was an economist in the Financial Studies Division of the Research Department at the time this paper was started and is now a staff member of the Centre for European Policy Studies and the Universite Catholique de Louvain.

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I. Introduction

It is a generally accepted principle that a small country that participates in a fixed exchange rate system looses the ability to conduct an independent national monetary policy. In a fixed exchange rate system with sizeable bands, like the EMS, this general principle has to be modified to take into account the additional room for maneuver created by the bands. Intuition suggests that the possibility for the exchange rate to fluctuate, in the short run, within the bands should not affect the loss of national monetary autonomy in the long-run. However, the short-run autonomy for national monetary policy might be used to contrast the effects of various shocks on the domestic economy. The question therefore arises to what extent the limited flexibility afforded by the bands can be used by the authorities in their attempt to stabilize income by offsetting the effects of various short-run shocks that affect all markets of the economy.

This paper argues that the bands do indeed give the authorities the opportunity to conduct short-run stabilization policy. If both fiscal and monetary policy can be used in the short-run to counteract the effects of various shocks on the economy as they affect the exchange rate, the authorities would have enough degrees of freedom to stabilize income and observe the exchange rate target. In this case, the target zone does not imply any cost in the sense that income can be stabilized to the same level with or without the target zones. However, if fiscal policy cannot be used in the short-run, a potential for a conflict between the aims of stabilizing the exchange rate and stabilizing income arises because in this situation, the bands reduce the margin of maneuver for the authorities to conduct stabilization policy. The degree to which monetary policy alone can be used depends on the width of the bands and other factors, such as the structure of the economy and the relative variances of the various shocks. Moreover, the paper argues that if the long-run component of monetary policy is not correctly aligned with the exchange rate target, this diminishes the ability of the authorities to stabilize the economy.

The paper also considers the interaction between the long-run or average components of monetary and fiscal policy by showing to what extent the authorities should take the stance of fiscal policy into account in setting the long-run and short-run components of monetary policy. The model used in this paper is a conventional one of a small open economy that takes the exchange rate (bands) as given, implicitly it is, therefore a model of the behavior of a "follower" in an asymmetric exchange rate system like the EMS.

The main innovation of the paper consists of the formulation of the constraint on policy that derives from the exchange rate bands. It is assumed here, that the authorities are only required to set their policy parameters such as to keep the probability, that the exchange rate stays inside the bands, above a certain threshold. Target zones are often represented by the assumption that there is no intervention at all as long as the exchange rate stays inside the bands and unlimited intervention as soon as the exchange rate is at one of the bands. However, such a representation is difficult to reconcile with the experience of the EMS since most interventions in the EMS occur while the exchange rates are well inside the bands. The probabilistic formulation used in this paper is compatible with the EMS experience since it predicts sizeable infra-marginal interventions.

The remainder of the paper is organized as follows: Section II describes briefly the simple and conventional macroeconomic model used in this paper. In this model monetary and fiscal policy are formulated as simple feedback rules since the shocks themselves are not directly observable; monetary and fiscal policy can therefore only react to the only variable that is currently observable, namely the exchange rate. Section III then solves the model for the variables that interest the authorities, namely aggregate activity and the exchange rate, in terms of the various shocks that affect the economy. Section IV then uses this solution to discuss optimal monetary policy and how it depends on the relative variances of the various shocks and parameters of the model. Section V finishes with some concluding remarks which apply the results of this model to some of the issues relevant for the EMS.

II. The Model

This section presents the formal model used for the discussion of monetary and fiscal policy in the EMS. It is divided into three building blocks, a real sector, a monetary sector, and two equations that describe the feedback rules for monetary and fiscal policy. The model is, as usual, in log linear form, all lower case latin letters refer to logarithms of the corresponding variables, all greek letters refer to constant coefficients, which are usually positive.

The first building block describes the real sector, its central equation is given by a standard surprise supply function for aggregate real output of the domestic good:

$$(1)\quad y_t^s = \lambda\ [p_t - E_{t-1}\ (p_t)] + w_t^s$$

where output supplied (of the home good), y_t^s, is measured in terms of deviations from a trend and w_t^s represents a stochastic shock to aggregate supply. The parameter λ measures the slope of the aggregate supply function. 1/

1/ This supply function can be justified, as usual, by assuming that wages are fixed each period and set to clear the labor market on average period.

Aggregate demand is assumed to depend on fiscal policy and the relative price of the domestically produced good in terms of the foreign good:

$$(2) \quad y_t^d = g_t + \phi^{-1} (s_t + p_t^* - p_t) + \bar{y}^* + w^d$$

Where g_t represents fiscal policy, this formulation implies that all government spending falls on the domestic good and that the private sector does not discount future tax liabilities. $(s_t + p_t^* - p_t)$ is the relative price of the domestic good in terms of the foreign good, given that s_t represents the nominal exchange rate (domestic currency per unit of foreign currency) and p_t^* represents the price of the foreign good. p_t^* is assumed to be given to this small economy and it is subject to stochastic shocks according to $p_t^* = \bar{p}^* + w_y^p$. The parameter ϕ measures the degree of product differentiation between domestic and foreign goods. A higher value of ϕ means that the two national goods are more differentiated, with $\phi = 0$ the two goods would be identical and purchasing power parity (PPP) would hold at each point in time. w_t^d represents a stochastic shock to aggregate demand, for the home good, which arises because of shocks to foreign spending.

The real sector is closed by the requirement that aggregate demand equals aggregate supply:

$$(3) \quad y_t^s = y_t^d$$

The second building block describes the monetary sector of the economy, it consists of a conventional money demand function:

$$(4) \quad m_t^d - p_t = \beta y_t - \delta \, [E_t(s_{t+1}) - s_t] + v_t^d$$

Where the terms $[E_t(s_{t+1}) - s_t]$ represents the domestic interest rate under the assumptions that the foreign interest rate is normalized to zero and that risk neutral speculators ensure, in the absence of capital controls, that the expected rate of depreciation is equal to the interest rate differential. The parameters β and δ represent the income elasticity and interest semi-elasticity of money demand, respectively.

As the goods market, the money market is always assumed to be in equilibrium:

$$(5) \quad m_t^d = m_t^s$$

The determinants of money supply, m_t^s, are discussed below.

The third building block of the model describes monetary and fiscal policy. Since the authorities have to keep the exchange rate within the bands (in the probabilistic sense) it is assumed that monetary policy is geared towards keeping the exchange rate close to a certain target value, $\bar{s}$, the money supply is, therefore, assumed to be governed by:

(6) $m_t^s = \bar{m} - \lambda(s_t - \bar{s}) + v_t^s$

The parameter λ reflects the degree to which the authorities pursue the exchange rate target, $\lambda = 0$ implies that the country has a freely floating exchange rate and $\lambda = \infty$ is equivalent to a fixed exchange rate. 1/ The optimal choice of λ and $\bar{m}$ by the authorities is discussed in Section IV. v_t^s represents a stochastic shock of money supply.

Given the long lags involved in adjusting fiscal policy it might be more realistic not to assume that this policy instrument is used to stabilize the exchange rate. However, since the literature on target zones has usually considered fiscal policy as one of the policy instruments, it is assumed here that fiscal policy can be adjusted in the short run and is also used to keep the exchange rate close to its target value $\bar{s}$.

(7) $g_t = \bar{g} + \theta(s_t - \bar{s}) + u_t$

Where u_t is again a stochastic shock.

All shocks are distributed independently with the same mean, equal to zero (i.e., E_{t-1} $(w_t{}^s, w_t{}^d, w_t{}^p, v_t{}^d, v_t{}^s, u_t) = 0$), but with the different variances. The nominal exchange rate, s_t, and the domestic nominal interest rate, $[E_t(s_{t+1}) - s_t]$, are the only variables that are currently observable.

The objective of the government is to minimize the deviation of real income from its equilibrium level 2/ and, at the same time, to keep the exchange rate within the bands. Since it cannot be expected that the exchange rate band is observed even for extreme realization of some shocks, it is assumed here that the commitment of the authorities to the exchange rate "target zone" or band is only such that they have to ensure that the probability that they will be able to keep the exchange rate inside the bands is above a certain threshold.

(8) $\text{Min } E\,(y_t - w_t^s)^2 \text{ s.t. } \Pr\{s' - b \le s_t \le s' + b\} \ge \overline{\Pr}$

1/ Given that the authorities know the domestic interest rate they could also have an interest rate target. However, it can be shown that in this framework exchange rate and interest rate targets are equivalent.

2/ The supply function (1) implies that minimizing the variance of income (minus the supply shock) is equivalent to minimizing the variance of unexpected inflation.

Where the minimization is done over the choice of λ, θ, and $\bar{m}$. s' denotes the central rate and b represents the margin of fluctuations allowed for by the bands. The target exchange rate, $\bar{s}$ is assumed to lie within the bands, i.e., $s' - b \leq \bar{s} \leq s' + b$, but the authorities do not necessarily have to aim for the middle, so that $s' \neq \bar{s}$ is possible. In the EMS, b would be equal to 2.25 percent for all member countries recent Spain for which it is 6 percent. With a "hard" target zone the authorities would have no choice, but to stabilize the exchange rate once it hits the bounds. However, such a nonlinear policy is difficult to treat analytically. Explicit solutions for the behavior of the exchange rate with "hard" target zones have recently been found by Krugman (1988) and Flood and Garber (1989), however, these solution techniques are limited to the continuous time case with forcing variables that follow a random walk. The formulation used here is closer in spirit to the original target zone literature (Currie and Wren-Lewis (1988), Edison, Miller and Williamson (1988), and Miller and Weller (1989) which just consider linear feedback rules; that is "soft" target zones.

The advantage of the "stochastic" target zone used here is that it leads to a standard linear problem. Moreover, it could also be argued that in reality target zones would not be maintained in the face of extreme realizations of the shocks. In the EMS in particular a really large shock would probably lead to a realignment. The formulation of the exchange rate commitment in (8) is not quite satisfactory since it does not take into account that the authorities would have to stop following the policy rules embedded in (6) and (7) once the exchange rate is at the upper or lower band. However, equation (8) does seem to reflect the implicit rule in the EMS that realignments are possible if really large shocks occur. It is an attempt to describe a world in which it is taken as desirable to limit exchange rate fluctuations but in which it is also recognized that in the face of unusually large shocks it might be preferable to adjust the exchange rate (band) instead of cutting short the efforts of national authorities to stabilize economic activity.

III. The Solution

This section provides a sketch for the solution for the exchange rate and real income in terms of the policy parameters and the stochastic shocks. Details of the calculations are provided in Appendix I. A solution for the exchange rate in the form of a stochastic difference equation can be found by containing equations (2), and (4)-(7). This yields:

$$(9) \quad (s_t - \bar{s})\,(1+\lambda+\delta+\phi\theta) = (\phi-\beta)y_t - \phi(w^d_t + u_t) - w^p_t - \phi\bar{g}$$
$$- \phi\bar{y}^* - \bar{p}^* + \bar{m} - \bar{s} + v_t + \delta E_t\,(s_{t+1}-\bar{s})$$

Where the term $v_t = v_t^s - v_t^d$ represents the "net" shock to money supply. The term $(\phi-\beta)y_t$ on the R.H.S. of equation (9) indicates that, ceteris paribus, income has two effects on the exchange rate (or, equivalently, on domestic prices). Equilibrium in the international market for domestic output requires that a higher domestic output can be sold only if the real exchange rate depreciates by the amount ϕy_t. But since an increase in y_t also raises domestic money demand, this reduces the domestic price level and therefore causes a real appreciation of βy_t even at an unchanged nominal exchange rate.

Equation (9) can be further simplified by grouping the terms on the R.H.S. into two parts, one consisting of a linear combination of various shocks, denoted by A_t and another one that takes into account the constant terms and is denoted by $\bar{h}$. This is done in the following definitions:

$$(10)\quad A_t = y_t(\phi-\beta) - z_t + v_t$$

$$(11)\quad z_t = [\phi(u_t+w_t^d) + w_t^p]$$

$$(12)\quad \bar{h} = \bar{m} - \bar{s} - \phi(\bar{g}+\bar{y}^*) - \bar{p}^*$$

The exchange rate can then be written as a simple first order stochastic difference equation:

$$(13)\quad (1+\lambda+\delta+\phi\theta)\ (\bar{s}_t-s) = A_t + \bar{h} + \delta[E_t(s_{t+1})-\bar{s}]$$

Using the conventional transversality condition, the solution for this unstable difference equation can be shown to be:

$$(14)\quad (s_t-\bar{s}) = \frac{A_t}{1+\lambda+\delta+\phi\theta} + \bar{h}\,\frac{1}{1+\lambda+\phi\theta}$$

This solution implies that the unconditional expectation for s_{t+1} is equal to the constant, $\bar{h}/(1+\lambda+\phi\theta)$. However, equation (14) still contains the endogenous variable y_t (if $\phi \neq \beta$, see the definition of A_t); it does not constitute therefore the required solution for the exchange rate in terms of the shocks if $\phi \neq \beta$. A solution for y_t can be obtained by using the money demand equation in which the exchange rate has been eliminated using equation (14). Taking expectations and using the supply equation (1) yields finally:

$$(15)\ y_t - w_t^s = \Omega_g^{-1}\ \gamma((\lambda+\delta)z_t + (1+\phi\theta)v_t - [\beta(1+\lambda+\delta+\phi\theta) + (\lambda+\delta)(\phi-\beta)]\ w_t^s)$$

with

$$(16)\ \Omega_g = (1+\lambda+\delta+\phi\theta)(1+\beta\gamma) + \gamma(\lambda+\delta)(\phi-\beta)$$

The composite parameter Ω_g has the subscript g to distinguish the general case from an interesting special case which occurs when the income elasticity of demand for money is equal to one (i.e., if $\beta = 1$) and if preferences are Cobb Douglas (i.e., $\phi = 1$). If this case, the terms in $(\phi-\beta)$ cancel and the solution is considerably simplified:

$$(17)\ y_t - w_t^s = \Omega^{-1}\gamma[(\lambda+\delta)z_t + (1+\theta)v_t - (1+\lambda+\delta+\theta)\ w_t^s]$$

where Ω is defined by:

$$(18)\ \Omega = (1+\lambda+\delta+\theta)(1+\gamma)$$

This result can be used to obtain a reduced form for the exchange rate by substituting it into equation (9). This yields:

$$(19)\ s_t - \bar{s} = \Omega_g^{-1}[-(1+\gamma\beta)z_t + (1+\gamma\phi)v_t + (\phi-\beta)w_t^s] + \bar{h}\ \frac{1}{1+\lambda+\phi\theta}$$

This solution implies that the expected value of s_t is equal to $\bar{s}$ only if $\bar{h}$ is equal to zero. Since $\bar{h}$ is given by $\bar{m} - \bar{s} - \phi(\bar{g} + \bar{y}^*) - \bar{p}^*$ it follows that the expected exchange rate is equal to $\bar{s}$ only if $\bar{m} = \bar{s} + \phi(\bar{g} + \bar{y}^*) + \bar{p}^*$, that is if the government sets the mean component of the money supply taking into account the exchange rate target, the effects of fiscal policy and foreign demand on the real exchange rate, and the foreign price level. It is apparent from this solution that if $\phi = \beta = 1$ supply shocks do not affect the exchange rate.

IV. Optimal Policy

As mentioned above, the objective of the authorities is to minimize the variance $y_t - w_t^s$ subject to the constraint that the exchange rate stays inside the bounds with a certain probability. Using the solutions for $y_t - w_t^s$ in equation (17) and the assumption that all the shocks are independently distributed, the total social loss defined in equation (8) is given by:

$$(20)\ E(y_t - w_t^s)^2 = [\lambda/(1+\gamma)]^2 (\sigma_{ws}^2 + (1+\lambda+\delta+\phi)^{-2}\ [(\lambda+\delta)^2\ \sigma_z^2 + (1+\theta)^2\ \sigma_v^2])$$

It is apparent from equation (24) that the choice of the intervention parameter λ, does not affect the direct impact of a supply shock on real activity, 1/ changing λ affects, therefore, the variance of y_t only through the variance of the combined real shock z_t and the monetary shock v_t. (The variance of the combined real shock is given by: $\sigma_z^2 = \phi^2\sigma_u^2 + \phi^2\sigma_{wd}^2 + \sigma_{wp}^2$.) The optimal value of λ in the absence of any exchange rate commitment can be found formally by setting:

$$(21)\ \frac{\partial E(y_t - w_t^s)^2}{\partial \lambda} = 0 = 2[\gamma/(1+\gamma)]^2$$

$$\{-(1+\lambda+\delta+\theta)^{-3}\ [(\lambda+\delta)^2\sigma_z^2 + (1+\theta)^2\sigma_v^2] + (\lambda+\sigma)\sigma_z^2\ (1+\lambda+\delta+\theta)^{-2}\}$$

$$= -2[\gamma/(1+\gamma)]^2\ \Omega^{-3}$$

$$\{[(\lambda+\delta)^2 - (1+\lambda+\delta+\theta)\ (\lambda+\delta)]\sigma_z^2 + (1+\theta)^2\ \sigma_v^2\}$$

This expression can be solved for the optimal degree of exchange rate flexibility in terms of the value of λ that would minimize the variance of real activity in the absence of a target zone for the exchange rate:

$$(22)\ \lambda^* = (1+\theta)\ \frac{\sigma_v^2}{\sigma_z^2} - \delta$$

The optimal intervention parameter thus depends on the variance of the monetary shocks, σ_v^2, relative to σ_z^2 which represents the variance of the combined real shocks to aggregate demand, fiscal policy, and the foreign price level. If $\phi \neq \beta \neq 1$ there is an additional term on the R.H.S. of equation (22), however, this terms would not affect the results that are discussed below.

The result (22) implies that the optimal intervention parameter could be negative, if the variance of the monetary shocks (relative to that of the real shocks) is small enough. It is apparent from equation (22) that the

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1/ This result is a special feature of the case $\phi = \beta = 1$, see equation (27) below.

optimal choice for the intervention policy parameter is influenced by θ, which reflects the extent to which fiscal policy is adjusted as a function of the exchange rate. Equation (22) implies that fiscal and monetary policy are complements, in the sense that a higher value of θ leads to higher desired value of λ.

However, the constraint that the probability of the exchange rate staying inside the bands must exceed a certain threshold, might not allow the government to actually use the optimal intervention parameter because it could lead to a too high degree of exchange rate variability. Formally, this can be analyzed by assuming that all the disturbances are normally distributed. 1/ In this case the exchange rate constrain implies:

$$(23)\ \overline{PR} = \Pr\{s' - b \le s_t \le s' + b\} =$$

$$[(2\pi)^{-(1/2)}][\int_{\infty}^{x_u} \exp\ (-\ \tfrac{x}{2})^2\ dx - \int_{-\infty}^{x_1} \exp\ (-\ \tfrac{x}{2})^2 dx]$$

where x is the standardized normal, given in this case by:

$$(24)\ x_t = [s_t - \bar{s} - \bar{h}/(1+\lambda)]/\sigma_s$$

The upper and lower limits of integration are therefore given by:

$$(24a)\ x_u = [s' + b - \bar{s} - \bar{h}/(1+\lambda)]/\sigma_s$$

$$(24b)\ x_1 = [s' - b - \bar{s} - \bar{h}/(1+\lambda)]/\sigma_s$$

The standard deviation of the exchange rate, σ_s, can be calculated directly from equation (19), for the special case $\phi = \beta = 1$ it is given by:

$$(25)\ \sigma_s = (1+\lambda+\delta+\theta)^{-1}(1+\gamma)\ [\sigma_z^2 + \sigma_v^2]^{1/2}$$

As will be shown, the R.H.S. of equation (23) is a decreasing function of both intervention parameters, intuitively this expresses just the idea that the more the money supply reacts to the exchange rate, the less variable becomes the exchange rate. Formally, this is implied by the result that an

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increase in either θ or λ lowers the variance of the exchange rate, σ_s^2, as can be seen by inspection of equation (25). Since the variance of the exchange rate, σ_s^2, is a decreasing function of both θ and λ, it is clear that there are many combinations of θ and λ that can lead to the same variance of the exchange rate. The combinations of θ and λ that keep the variance of the exchange rate at a certain fixed level can be calculated analytically from the requirement that these combinations must lead Ω unchanged. Inspection of the definition of Ω shows that for the special case $\phi = 1$ this implies: 1/

$$(26)\quad \left.\frac{d\lambda}{d\theta}\right|_{\sigma_s^2 = \text{constant}} = -1$$

The two relationships between λ and θ in equations (22) and (26) can be represented diagrammatically as in Figure 1. In this figure, the negatively sloped line indicates the points in the θ, λ plane that satisfy the requirement that the exchange rate has to stay inside the bands with a certain probability. All combinations of θ and λ to the right of this line satisfy the target zone requirement. The positively sloped line shows the relationship between λ and θ that is derived from the minimization of the variance of income. It is clear from these two relationships that there is an infinite number of combinations of λ and θ that would deliver the same result: minimization of the variance of income subject to the target zone constraint. The result is, of course, a consequence of the fact that with two policy instruments the authorities should be able to attain the two targets income and exchange rate variability.

If the authorities have enough instruments, a target zone does not involve therefore any cost in terms of higher income variability. However, given the framework of various political pressures in which fiscal policy is actually formulated, it appears unrealistic to assume for most countries that the stance of fiscal policy could be adjusted quarterly or even annually to reflect an exchange rate target. If fiscal policy cannot be used, that is if $\theta \equiv 0$ in this model, the possibility arises that the optimal value of λ (for the purpose of stabilizing income is lower, for any given value of $\bar{m}$ (and thus $\bar{h}$) than the value of λ that satisfies equation (23). In this case, λ^* is determined by equation (28) instead of equation (22) (both with $\theta \equiv 0$). 2/

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2/ Even if fiscal policy cannot be used as a policy instrument in the short-run, the variance of the fiscal policy shock, u_t (which enters σ_z^2) still influences the optimal intervention parameter. Even with $\theta \equiv 0$, λ^* is a decreasing function of σ_z^2, this implies that the less controllable fiscal policy is (the higher σ_u^2) the higher is the probability that the target zone becomes constraining because the optimal λ from equation (22) does not satisfy the target zone requirement, equation (23).

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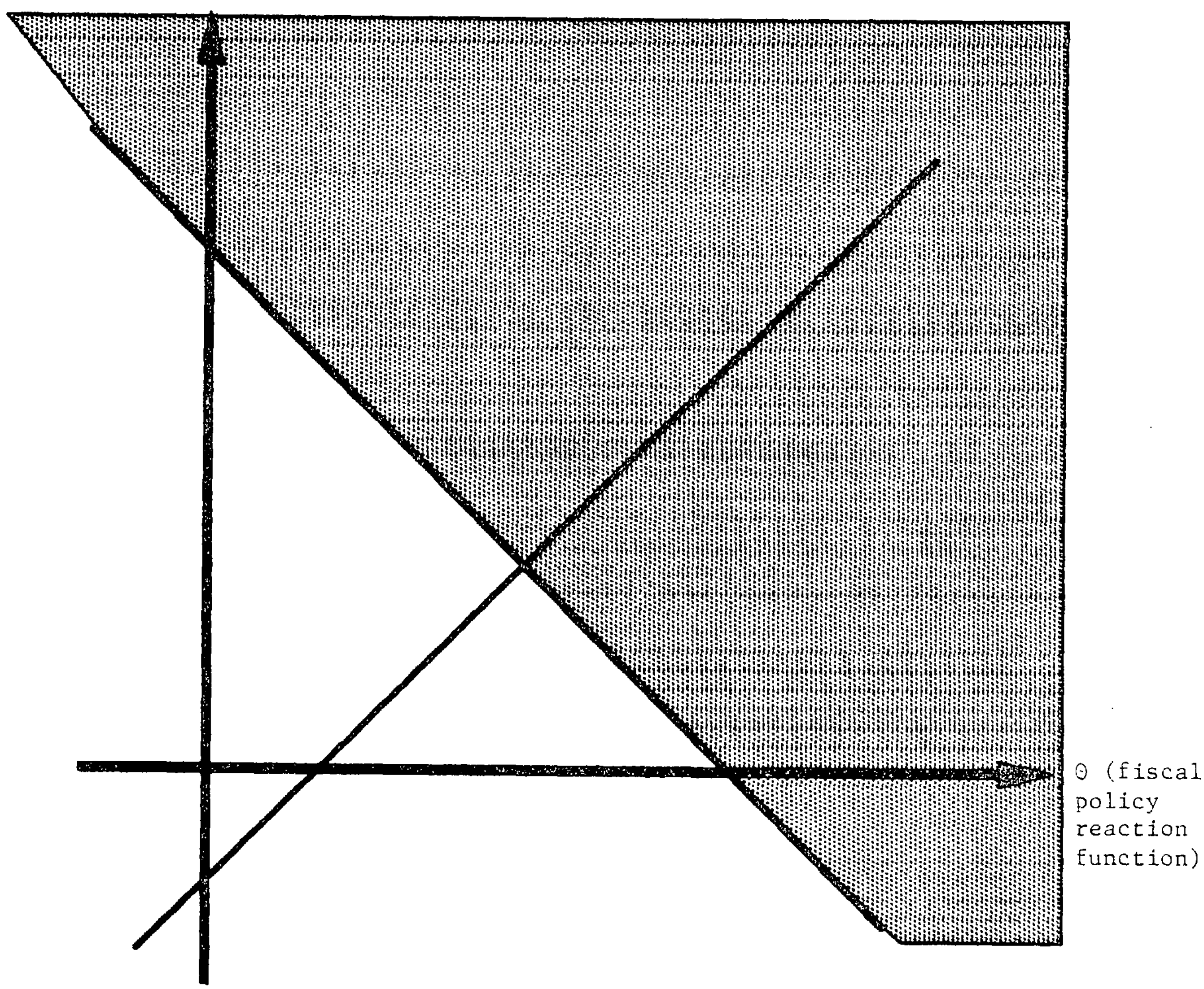
Figure 1. The Relationship Between Monetary and Fiscal Stabilization Parameters Under the EMS Constraint

λ (monetary policy reaction function)

$$\lambda = \frac{(1+\phi\theta)[\sigma^2_{ws}(\phi-\beta) + \sigma^2_v \gamma(1+\gamma\phi)]}{\sigma^2_z(1+\beta\gamma) - (\phi-\beta)\sigma^2_{ws}} - \delta$$



$$\left.\frac{\partial\lambda}{\partial\theta}\right|_{\sigma^2_s = \text{constant}} = -\phi\left(\frac{1+\beta\gamma}{1+\phi\gamma}\right)$$

The shaded area to the right satisfies target zone constraint.

A different special case, in which monetary policy would become the only usable policy instrument occurs when purchasing power parity obtains continuously because the foreign and the domestic good are perfect substitutes. If the two goods are perfects substitutes, an increased demand for the home good by the home government has no effect on the exchange rate or domestic output since consumers can just substitute foreign for domestic goods. Formally, this idea is expressed in this model by the fact that the parameter θ enters the solution for real income, equation (17) always multiplied by ϕ. With continuous PPP, that is if $\phi = 0$, θ has no effect on the solution for the exchange rate or income. In this case, the optimum intervention parameter is given by: 1/

$$(27)\quad \lambda^{*}\Big|_{\phi = 0} = \frac{(1+\phi\theta)\ \sigma_{v}^{2} + \beta^{2}\sigma_{ws}^{2}}{(1+\beta\gamma\delta)\sigma_{wp}^{2}} - \delta$$

Where σ_{wp}^2 denotes the variance of the shocks to PPP.

The result (27) implies that when PPP holds continuously (i.e., when the variance of the PPP shock goes to zero), λ goes to infinity which implies that fixed exchange rates become the optimal policy. The Literature on optimum currency areas (Mundell (1968), Chapter 12) stresses the importance of wage flexibility (or labor mobility) to offset the effects of intra-regional real shocks. In the present model where nominal wages are fixed for each period (and later immobile) an optimum currency area arises if there are no interregional real shocks (i.e., no shocks to PPP) although there might still be real and nominal shocks in a given region.

Will PPP, as opposed to the case where $\phi = \beta = 1$, the optimal intervention parameter with the variance of the real shock, σ_{ws}^2, does influence the optimal intervention parameter which becomes an increasing function of σ_{ws}^2 if $\phi - 0$.

It can also be shown that the R.H.S. of equation (23) is, for any given value of λ and $\bar{h}$, an increasing function of b (the wider the bands the more probable it is that the exchange rate stays inside), this implies that a target zone could always be made wide enough so that it does not constrain short-run stabilization policies. 2/ How wide the bands would have to be

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1/ Notice that with $\phi = 0$ the variance of the real shock is reduced to the variance of the deviations from PPP: $\sigma_z^2 = \sigma_{wp}^2$.

2/ The derivative of the R.M.S. of equation (29) with respect to b is equal to:

$$\{[(2\pi)^{(-1/2)}]/\sigma\}\{[\exp\left(\frac{s'+b-\bar{s}-\bar{h}/(1+\lambda)}{\sigma s}\right)^{2} + \exp\left(\frac{s'+b-\bar{s}-\bar{h}/(1+\lambda)}{\sigma s}\right)^{2}]\} > 0$$

to cease to be constraining depends on the structure of the economy and the relative variances, that determine λ^*, as well as the absolute level of the variances, that determine σ_s (and the threshold portability).

It is apparent from equation (17) that the choice of $\bar{m}$ does not affect the variance of y_t; however, the choice of $\bar{m}$ will affect the probability that the exchange rate, s_t, hits one of the bands, $s' \pm b$. Assuming that the constraint on λ is binding, the authorities would like to maximize the R.H.S. of equation (23), i.e., the probability that exchange rate stays within the bounds. Maximizing the R.H.S. of equation (29) over the choice of $\bar{m}$ would allow the authorities to have more freedom in choosing λ. Formally, this implies:

$$(28)\quad \frac{\partial Pr}{\partial \bar{m}} = 0 = (2\pi)^{-\frac{1}{2}} \frac{1}{(1+\lambda)\sigma_s} [\exp (\frac{s' + b - \bar{s} - \bar{h}/(1+\lambda)}{\sigma_s})^2 - \exp (\frac{s' - b - \bar{s} - \bar{h}/(1+\lambda)}{\sigma_s})^2]$$

This equation can be solved for the optimal value of $\bar{h}$ which is equal to: 1/

$$(29)\quad \bar{h}^* = (s' - \bar{s})(1+\lambda)$$

Using the definition of $\bar{h}$ this implies that the optimal monetary target is:

$$(30)\quad \bar{m}^* = (s' - \bar{s})(1+\lambda) + \bar{s} + \phi(\bar{g} + \bar{y}^*) + \bar{p}^*$$

This shows that if the authorities aim with their intervention policy at the middle of the band, i.e., if $s' = \bar{s}$, the money supply target that maximize the probability of staying inside the bands is determined by just that exchange rate and the stance of fiscal policy on the real exchange rate. If $\bar{s}$ can be chosen without constraint, it would of course be optimal to aim at the middle of the band since, for a given variance of the shocks and a given intervention parameter, this minimizes the probability that the exchange rate reaches the upper or the lower band.

1/ The R.H.S. of equation (33) is equal to zero if the two exponents are equal, taking the negative root of the smaller exponents yields the result (35).

V. Conclusions

This paper has analyzed the scope for stabilization policy within an exchange rate target zone. The target zone was formulated in a probabilistic sense since it was assumed to require that the probability that the exchange rate stays within the band has to exceed a certain level. The discussion was conducted in terms of the optimal rule for fiscal and monetary stabilization policy for a given exchange rate band. The main result seems to be that if both policy instruments are available, the existence of the exchange rate commitment does not reduce the ability of the authorities to stabilize income. However, a potential for a conflict for such a policy goal exists if only one policy instrument can be used. This would be the case if monetary policy is the only available instrument to influence output, either because fiscal policy cannot be adjusted in the short-run or because PPP holds continuously and fiscal policy can no longer affect output. The target zone becomes more constraining if the variability of the exogenous disturbances to fiscal policy increases, or if the long-run component of monetary policy is not correctly aligned with the exchange rate target.

Applying the discussion to the EMS and turning it around in the sense of choosing the minimum nonconstraining width of the bands (in terms of the features of the economy discussed above) these results imply that wider margins of fluctuations are preferable for those countries that have difficulties in controlling fiscal policy and the long-run component of monetary policy.

The Solution for the Exchange Rate

This appendix provides details of the calculations required for the solution of the model. To obtain the required solution for y_t, it is necessary to first solve for the exchange rate. To determine the exchange rate it is convenient to solve equation (2) in the text for the exchange rate:

$$s_t = \phi\,[y_t - \bar{y}^* - g_t - w_t^d] + p_t - \bar{p}^* - w_t^p \tag{A.1}$$

In this equation, p_t can be eliminated by using the money market equilibrium conditions; combining equations (4)-(6) yields:

$$p_t = \bar{m} - \lambda(s_t - \bar{s}) + v_t^s - v_t^d - \beta_{yt} - \delta[E_t\,(s_{t+1}) - s_t] \tag{A.2}$$

Using this equation in (A.1) and simplifying leads to the following expression for the deviation of the exchange rate from its target value:

$$(s_t - \bar{s})(1+\lambda+\delta) = (\phi-\beta)y_t - \phi(g_t + w_t^d) - p_t^* + \bar{m} - \phi\bar{y}^* - \bar{p}^* - \bar{s} \tag{A.3}$$

$$+ v_t + \delta E(s_{t+1} - \bar{s})$$

Using this equation, fiscal policy, g_t has to be eliminated using the policy function (7), this yields equation (9) in the text.

$$p_t = \bar{m} - \frac{(\lambda+\delta)}{1+\lambda+\delta+\phi\theta} A_t - \frac{\lambda}{1+\lambda}\bar{h} + v_t - \beta y_t \tag{A.4}$$

The price level expected as of period t-1 is therefore equal to:

$$E_{t-1}\,(p_t) = \bar{m} - \frac{\lambda}{1+\lambda}\bar{h} \tag{A.5}$$

Equation (A.5) can be subtracted from equation (A.4) to calculate the unexpected component in the price level, using equation (1) this yields an expression for real income in terms of the shocks:

$$(y_t^s - w_t^s)(1+\lambda+\delta+\phi\theta) = -\gamma(\lambda+\delta)A_t + \gamma(v_t - \beta y_t)(1+\lambda+\delta+\phi\theta) \tag{A.6}$$

After inserting the value for A_t from its definition (10) in the text, and some simplifications, this equation can be transformed to yield the solution for y_t - w_t^s in the text. To obtain a reduced form the exchange rate, the result (20) has to be substituted into the definition of A_t (equation (10) to yield:

$$(A.7) \qquad y_t = \Omega_g^{-1} \{[(\phi-\beta)\gamma(\lambda+\delta) - \Omega_g]z_t + [(\phi-\beta)\gamma + \Omega_g] v_t$$

$$- (\phi-\beta)[\Omega_g \ \gamma[\beta(1+\gamma+\delta+\phi\theta)+(\lambda+\gamma)(\phi-\beta)]]w_t^s)$$

After some simplifications, this leads to the expression for the exchange rate in the text.

Derivation of the Optimal Intervention Parameter

In the general case, when $\beta \neq \phi \neq 1$, the optimal intervention parameter, λ^* can be found by setting:

$$\text{(A.7)} \qquad \frac{\partial \mathrm{Var}\ (y^s_t - w_t)}{\partial \lambda} = \frac{\partial \gamma^2}{\partial \gamma} \Omega_g^{-2} \ [[(\delta+\lambda)^2\ \sigma_z^2 + (1+\phi\theta)^2\ \sigma_v^2$$

$$+ [\beta(1+\lambda+\delta+\phi\theta) + (\lambda+\delta)(\phi-\beta)]^2\ \sigma_{ws}^2]$$

equal to zero. The calculations can be simplified by using the result that:

$$\text{(A.9)} \qquad \frac{\partial}{\partial\lambda}\left(\frac{A+B\lambda}{C+D\lambda}\right)^2 = \{\left(\frac{A+B\lambda}{C+D\lambda}\right) \frac{(C+D\lambda)B - (A+B\lambda)D}{(C+D\lambda)^2}$$

$$\text{(A.10)} \qquad = 2\ \frac{(A+B\lambda)}{(C+D\lambda)^3}\ [CB-AD]$$

In this specific case, the values for A, B, C, and D are:

$$\text{(A.11)} \qquad C = (1+\delta+\phi\theta)(1+\beta\lambda) + \gamma(\phi-\beta)$$

$$\text{(A.12)} \qquad D = (1+\beta\gamma) + \gamma(\phi-\beta) = 1 + \gamma\phi$$

$$\text{(A.13)} \qquad \sigma_z^2:\ \ A = \delta,\ \beta = 1$$

$$\text{(A.14)} \qquad \sigma_v^2:\ \ A = (1+\phi\theta),\ B = 0$$

$$\text{(A.15)} \qquad \sigma_{ws}^2:\ \ A = \beta(1+\phi\theta) + \sigma\phi \qquad B = \phi$$

The F.O.C. equation (A.1) can therefore be written as:

$$\text{(A.16)} \qquad \frac{\partial \mathrm{Var}\ (y_t - w^s_t)}{\partial \lambda} = 2\gamma^2 \Omega_g^{-3}$$

$$\{(\lambda+\delta)\ [(1+\delta+\phi\theta)(1+\beta\lambda) + \gamma(\phi-\beta) - (1+\gamma\phi)\delta]\ \sigma_z^2$$

$$-\ (1+\phi\theta)(1+\phi\theta)(1+\gamma\phi)\sigma_v^2$$

$$+\ [\beta(1+\phi\theta) + \delta\phi + \phi\lambda][\phi(1+\delta+\phi\theta)\ (1+\beta\gamma)$$

$$+\ \phi\gamma\ (\phi-\beta) - \beta(1+\phi\theta)(1+\gamma\phi) - \delta\phi(1+\gamma\phi)]\}\sigma_{ws}^2$$

Setting this equal to zero and simplifying yields:

$$\text{(A.17)}\quad 0 = (\lambda+\delta)\ [(1+\phi\theta)(1+\beta\gamma) + \gamma(\phi-\beta)(1-\delta)]\sigma_z^2 - [(1+\phi\theta)^2(1+\gamma\phi)]\sigma_v^2$$

$$+ [\beta(1+\phi\theta) + \delta\phi + \lambda\phi][\phi-\beta][(1+\phi\theta) + \phi\gamma(1-\delta)]\sigma_{ws}^2$$

This can be written in a more compact notation as:

$$\text{(A.18)}\qquad \lambda^*\Sigma = \Gamma,$$

Where Γ and Σ are two composite parameters, defined by:

$$\text{(A.19)}\quad \Sigma = [(1+\phi\theta)(1+\beta\gamma) + \gamma(\phi-\beta)(1-\delta)]\sigma_z^2 + \phi[\phi-\beta][(1+\phi\theta) + \phi\gamma(1-\delta)]\sigma_{ws}^2$$

$$\text{(A.20)}\quad \Gamma = -\delta[(1+\phi\theta)(1+\beta\gamma) + \gamma(\phi-\beta)(1-\delta)]\sigma_z^2 + (1+\phi\theta)^2\ (1+\gamma\theta)\sigma_v^2$$

$$- [\beta(1+\phi\theta) + \delta\phi]\ [\phi-\beta]\ [(1+\phi\theta) + \phi\gamma(1-\delta)]\sigma_{ws}^2$$

Since $\lambda^* = \Gamma/\Sigma$ and since in both Γ and Σ σ_{ws}^2 is multiplied by $(\phi-\beta)$ it is apparent that for $\phi = \beta = 1$ the variance of the real shock, σ_{w2}^2 becomes irrelevant and this solution gives equation (22) in the text.

For the special case of stochastic PPP, that is when $\phi = 0$, the optimal intervention parameter is given by:

$$\text{(A.21)}\qquad \lambda^* = \frac{-\ \delta(1+\beta\gamma\delta)\sigma_z^2 + (1+\gamma\theta)\sigma_v^2 + \beta^2\sigma_{ws}^2}{(1+\beta\gamma\delta)\sigma_z^2}$$

Since $\phi = 0$ also implies $\sigma_z^2 = \sigma_{wp}^2$ this result can easily be transformed into equation (27) in the text.

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