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Equilibria with Unemployment in Segmented Labor Markets

Prepared by Dimitri G. Demekas*

Authorized for Distribution by Malcolm D. Knight

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Abstract

The paper proves four theorems in an n-sector model of a segmented labor market, with search costs, and a continuum of workers with different reservation wages, who can apply to any number of sectors. The main conclusions are that: (i) an equilibrium with unemployment always exists; and (ii) some of the unemployment is involuntary, in the sense that it consists of workers with reservation wages below the equilibrium wage in the secondary market. These conclusions hold in the case of both separate and non-separate markets.

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Contents

Summary

Traditional models of segmented labor markets usually make one or more of three restrictive assumptions: first, that the total supply of labor is fixed, or that all workers have identical reservation wages; second, that there is only one primary and one secondary market for labor; and, third, that the primary and secondary markets are separate, in the sense that workers cannot search for jobs simultaneously in both. These assumptions restrict the generality of the results of segmented market models.

This paper removes these assumptions and develops a general n-sector model of the segmented labor market, with search costs, and a continuum of workers with different reservation wages, who can apply to any number of sectors. The optimality conditions are derived and existence of equilibrium is demonstrated in the case of both separate and nonseparate markets.

The main conclusions are these. First, an equilibrium with unemployment always exists, second, some of the unemployment is involuntary, in the sense that it consists of workers with reservation wages below the equilibrium wage in the secondary market. These conclusions hold in the case of both separate and nonseparate markets.

I. Introduction

The term "labor market segmentation" is used here to characterize a particular kind of dichotomy in the labor market. In one part, usually referred to as the primary market or the regulated sector, the real wage is inflexible at above market-clearing levels, and jobs are rationed. In the other, the secondary market or the free sector, the real wage moves to clear the market. Empirical evidence from developing economies (Berry & Sabot 1978; Fields 1980; Squire 1981; Heckman & Hotz 1985; Johnson 1986) as well as industrialized countries (Osterman 1975; Carnoy & Rumberger 1980; Reich 1984; Dickens & Lang 1985) establishes segmentation as a prevalent stylized fact of the labor market.

Several hypotheses that explain the non-competitive wage determination mechanism in the primary market have been suggested. They range from "job competition" models (Thurow 1979), to "internal labor market" theories (Doeringer & Piore 1971; Berger & Piore 1980), to the "insider-outsider" approach (Lindbeck & Snower 1986a and 1986b), to the family of "efficiency wage" models (see the review in Akerloff & Yellen 1986). In the presence of such non-competitive structures in the primary labor market, even if the labor force is homogeneous, some queuing unemployment will persist at equilibrium, despite the existence of a free secondary market. 1/ The unemployment thus endemic in segmented markets has usually been characterized in the literature as "involuntary" because, as the unemployed workers have skills and reservation wages identical to those who work, unemployment does not arise as a voluntary phenomenon on the supply side of the labor market (Akerloff & Yellen 1986, p. 11; Hall 1975, p. 303; Blanchard & Summers 1986, pp. 43-4; Bulow & Summers 1986).

Most of the aforementioned contributions use a more or less similar analytical framework and derive their conclusions based on three restrictive assumptions. First, they postulate a fixed total supply of identical workers. This has two implications: (i) participation decisions and the "discouraged worker" effect (when some workers do not participate in the labor force because they believe their chances of finding employment to be small) cannot be addressed; and (ii) the conclusion that unemployment is "involuntary" is trivial, since all workers are assumed from the outset to have zero reservation wages.

In contrast, Harberger (1974) and Mincer (1976) have developed models with elastic labor supply. In this context, the former proposed a criterion for characterizing unemployment in a segmented market. According to this criterion, those among the jobless who decided *ex ante* to queue for primary jobs because their supply price is higher than the

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1/ This conclusion implicitly assumes away the possibility of on-the-job queuing. In that trivial case, of course, there would be no unemployment, as everyone would accept work in the secondary market while queuing for primary market jobs.

going secondary wage are clearly voluntarily unemployed; there are jobs available in the economy, but their reservation wage is too high. The nature of unemployment, in other words, depends on the supply prices of the unemployed relative to the wage in the free secondary market. Applying this criterion, Harberger (op. cit., p. 168) and A. Edwards (1984 and 1986) conclude that all unemployment in segmented markets is voluntary. Although it has been shown that this conclusion is not accurate (Demekas 1987), Harberger's criterion for the characterization of unemployment is more operational in the segmented market framework.

The second common restrictive assumption in segmented market models is that there is only one primary and one secondary market. The models then cannot explain what happens when, instead of a uniform fixed real wage, the primary market is subject to other common forms of wage inflexibility, like pay scales, seniority systems, etc. Moreover, they cannot explain situations where the free wage in the secondary market appears to be higher than the entry wage for equally qualified workers in the primary market.

Third, all segmented market literature is based on the implicit assumption that the primary and secondary markets are separate. The workers, in other words, have to decide from the outset where they will supply their labor and cannot apply for jobs to both the secondary and the primary market. This assumption reflects the origins of segmented market theory. Development economists were analyzing situations where the primary and secondary markets were geographically apart (hence the early nomenclature in Todaro (1969) and Corden & Findlay (1975), with "urban"/"rural" instead of regulated/free sectors). A general model of segmentation, however, cannot ignore the case where the markets are not separate and workers can search simultaneously for both primary and secondary jobs. In this case, of course, introducing different reservation wages and search costs is not just desirable for analytical completeness, but necessary. Otherwise the results are trivial: if all workers are identical and can apply costlessly to all sectors, no unemployment will ever arise; those who do not get primary jobs will simply accept employment in the secondary market.

This paper removes these restrictive assumptions: a general n-sector model of a segmented market is developed, with search costs, and a continuum of workers with different reservation wages, who can apply to any number of sectors. The optimality conditions are derived and existence of equilibrium with unemployment is proved. Then the nature of unemployment is examined using Harberger's criterion, distinguishing between the case where the primary and secondary markets are separate and the case where they are not. Surprisingly, part of the unemployment is always involuntary, in the sense that it consists of workers with reservation wages below the free wage, even when the markets are non-separate.

The first part of the paper analyzes the market equilibrium and the nature of unemployment in the case where the primary and secondary markets are separate, and the second part the alternative case. The summary and conclusions are in the last part and the proofs of the theorems in Appendix I.

II. The Model

1. Separate markets

The demand side of the labor market consists of n sectors, where $n > 2$. Sector 1 is the secondary market. The other n-1 sectors constitute the primary market, offering n-1 different fixed real wages. Firms in all sectors maximize profits subject to well-defined production functions and fixed output prices. This implies that the fixed wage w_j determines the number of vacancies $L_j(w_j)$ in each sector j of the primary market ($2 \leq j \leq n$), and that there exists a continuous labor demand function $L_1(w_1)$ in the secondary market, where the wage w_1 is flexible.

On the supply side there is a interval T of workers t, each with an individual utility function defined over income and leisure, which, for simplicity, is assumed to be of the form:

$$U_t = w + b_t \ell \tag{1}$$

where w is wage income, ℓ is leisure out of a unitary endowment of time and b_t is a constant. This utility function implies risk neutrality and linear indifference curves. 1/ In other words, the worker chooses either all work, with utility $U_t = w$, or all play, with $U_t = b_t$. b_t can be naturally interpreted in this case as the reservation wage of worker t. Reservation wages differ between workers; workers are distributed over the interval T with a density function b(t).

The following assumptions about demand and supply are going to be utilized throughout the paper:

(A1) $w_n > w_{n-1} > w_{n-2} > \ldots > w_2$

(A2) b(t) is continuous with range $[0, b_o]$, where $b_o > 0$

(A3) $L_1\left[\min(w_n, b_o)\right] < m(\text{all } t\text{: } 0 < b_t \leq \min\{w_n, b_o\})$

(A4) applying to any sector j requires a fixed fee $F > 0$.

1/ Linear utility functions are widely used in contract theory. Their implications for the total supply of labor are discussed in detail in Rosen (1985).

$m(Z)$ is the measure of a set Z. (A1) specifies the ordering of the fixed wages in the primary market. (A2) rules out "gaps" and negative values in the distribution of reservation wages over the interval T. (A3) is essentially a non-triviality condition; it means that the free sector can never, at equilibrium, absorb the entire market. 1/ (A4) establishes the search cost F, set, for simplicity, equal for all sectors.

When the markets open, each worker has three options: stay at home; apply to the secondary market; or apply to some sectors in the primary market. When the markets close (i.e. job offers are made), those who had applied to the secondary market work there and those who had applied to some regulated sectors accept the highest wage offer. Those to whom no offers are made are the unemployed.

For the rigorous analysis of the workers' optimizing decision, the following nomenclature is necessary.

Definition 1: Define $a(t)$ as the decision vector of worker t, consisting of binary elements $a_j(t)$, $j = 1, \ldots, n$, where $a_j(t) \in \{0, 1\}$; $a_j(t)$ is 1 if the worker t applies to sector j, zero otherwise. Different $a(t)$'s composed from zeros and ones represent different strategies of worker t. A secondary market strategy $a(t)$ is a vector with $a_1 = 1$, and a primary market strategy is a vector with some $a_j = 1$, $j \geq 2$. The hypothesis of separate markets, which is removed in the second part of the paper, implies that primary and secondary market strategies are mutually exclusive.

For worker t the expected utility of each of the three options is the following:

(a) stay at home: $U_t = b_t$ (2.a)

(b) apply to the secondary market: $U_t = w_1 - F$ (2.b)

(c) apply to the primary market:

1/ More precisely, the mathematical formulation of (A3) rules out the following cases:
(a) if $b_0 > w_n$, that at the free sector equilibrium: $w_1 > w_n$;
(b) if $b_0 < w_n$, that at the free sector equilibrium: $w_1 > b_0$.
Both equilibria are trivial. In the first, the secondary market is so large that it can pay more than the highest regulated wage and still employ everybody who cares to apply. In the second, the secondary market is so large that it absorbs the entire labor force. (A3), however, does not rule out equilibria where the free wage is higher than some regulated wages, or where some regulated sectors receive no applications.

$$\begin{aligned}
U_t(a) &= p_n a_n(t) U_t(w_n, 0) + \\
&+ [1 - p_n a_n(t)] p_{n-1} a_{n-1}(t) U_t(w_{n-1}, 0) + \\
&+ [1 - p_n a_n(t)][1 - p_{n-1} a_{n-1}(t)] p_{n-2} a_{n-2}(t) U_t(w_{n-2}, 0) + \\
&+ \ldots + \\
&+ [1 - p_n a_n(t)][1 - p_{n-1} a_{n-1}(t)] \ldots [1 - p_3 a_3(t)] p_2 a_2(t) U(w_z, 0) + \\
&+ [1 - p_n a_n(t)][1 - p_{n-1} a_{n-1}(t)] \ldots [1 - p_3 a_3(t)][1 - p_2 a_2(t)] b(t) - \\
&- \sum_{2}^{n} a_j(t) F \qquad (2.c)
\end{aligned}$$

p_j is the objective probability of finding employment in sector j, defined as:

$$p_j = \min\left\{\frac{L_j(w_j)}{\int_T a_j(t)dt}, 1\right\} \qquad (3)$$

where the numerator is the demand and the denominator is the supply of labor to j. In the case of zero or negative excess supply in sector j the probability of finding employment there is unity.

<u>Definition 2</u>: Define

$$q_k = \prod_{j=k+1}^{n} (1 - p_j a_j) p_k \text{ for all } k \geq 1 \qquad (4.a)$$

also define

$$q_o = \prod_{j=1}^{n} (1 - p_j a_j) \qquad (4.b)$$

q_k can be interpreted as the probability that sector k is the highest wage offer to worker t. In other words, q_k is the probability that he is offered a job in sector k, provided that he has lost in all the sectors with wage higher than w_k to which he had applied. Obviously $q_n = p_n$. Similarly, q_o is the probability that worker t loses in all the sectors he had applied.

Using expressions (1) and (4), (2.c) can be rewritten as:

$$U_t(a) = \sum_{2}^{n} q_j a_j(t) w_j + q_o b_t - \sum_{2}^{n} a_j(t) F =$$

$$= q_o b_t + \sum_{2}^{n} a_j(t)(q_j w_j - F) \qquad (2.d)$$

Each worker t can be thought as choosing his optimal strategy in two stages. In the first stage, he calculates the primary strategy that maximizes (2.d). The following Theorem shows the conditions for maximization.

Theorem 1: Define q_{ko} as follows:

$$q_{ko} = \prod_{\substack{j=1 \\ j \neq k}}^{n} (1 - p_j a_j) \qquad (5)$$

Maximizing the expected utility (2.d) implies the following rule for each element of a(t):

$$\underset{k \geq 2}{a_k(t)} = \begin{cases} 1 & \text{iff } b_t \leq w_k \text{ and } q_{ko} p_k b_t > F \\ 0 & \text{otherwise} \end{cases} \qquad (6)$$

(for proof see Appendix I).

The condition $b_t \leq w_k$ reflects the fact that nobody would apply to a sector with a wage lower than his supply price. The condition $q_k w_k - q_{ko} p_k b_t > F$, has a very straightforward intuitive explanation. It says that the net expected benefit of having $a_k = 1$, which is the expected payoff of sector k being the highest wage offer minus the corresponding loss of leisure, must be worth the application fee. The two inequalities must be satisfied independently.

In the second stage of the optimization process, after determining the primary strategy that maximizes (2.d), the worker t compares its expected value with (2.a) and (2.b) and picks his optimal strategy.

If all workers optimize, the interval T is partitioned into four sets, which can be described as follows:

T1, the set of workers who apply to the secondary market, is:

$$T1 = \{\text{all } t: \; bt \leq w_1 - F \text{ and } q_o b_t + \sum_2^n (q_j w_j - F)a_j(t) < w_1 - F,$$

for all primary market strategies}

T2, the set of workers who could apply to both, but choose to apply to the primary market, is:

$$T2 = \{\text{all } t: \; b_t \leq w_1 - F < q_o b_t + \sum_2^n (q_j w_j - F)a_j(t),$$

for at least one primary market strategy}

T3, the set of workers who apply only to the primary market, is:

$$T3 = \{\text{all } t: \; w_1 - F < b_t \leq q_o b_t + \sum_2^n (q_j w_j - F)a_j(t),$$

for at least one primary market strategy}

T4, the set of workers who cannot apply to any market, is:

$$T4 = \{\text{all } t: b_t > w_1 - F \text{ and } b_t > q_o b_t + \sum_2^n (q_j w_j - F)a_j(t),$$

for all primary market strategies}

The intersection of sets T1 to T4 is null and their union is T by construction. The total supply of labor to the primary market is m(T2) + m(T3), namely all the workers for whom there exists at least one primary market strategy a(t) with expected utility higher than both b_t and $w_1 - F$. The total supply of labor to the secondary market is m(T1), namely all workers for whom no primary market strategy dominates $w_1 - F$. Workers in T4, for whom no strategy dominates b_t, do not participate in the market.

After the workers optimize and the markets close, an equilibrium in this segmented market is a wage w_1 that clears the secondary market. The following Theorem establishes existence.

<u>Theorem 2</u>: In a segmented market which satisfies (A1) through (A4) and where, furthermore, the primary and secondary markets are separate, there exists a stable equilibrium at a wage $w_1 < w_n$ that clears the free sector (for proof see Appendix I).

Theorem 2 proves the existence of a stable segmented market equilibrium and guarantees that w_1 is less than at least one of the fixed primary market wages. It follows then immediately that at least one primary market sector faces excess supply conditions and, therefore, unemployment will be positive (if excess supply were nonpositive in all j with $w_j > w_1$, then it would pay anyone who participates in the free sector to switch to a primary market strategy).

The equilibrium free wage is determined by the equation:

$$w_1 = f\,[\,m(T1)\,]\ ,\ \text{or}\ f\left[\int_{T1} a_1(t)dt\right]\ , \tag{7}$$

where f is the marginal product of labor in the free sector, namely the inverse of the labor demand function $L_1(w_1)$. Unemployment is simply:

$$\Phi = m(T2) + m(T3) - \sum_{2}^{n} L_j(w_j) \tag{8}$$

The unemployed now have different reservation wages. Regarding the distinction between voluntary and involuntary unemployment according to Harberger's criterion discussed in the Introduction, the following theorem holds.

Theorem 3: At equilibrium the set T2 has positive measure (for proof see Appendix I).

Theorem 3 states that there are always some workers with reservation wages $b_t \leq w_1 - F$ and, therefore, less than the free wage, whose optimal strategy $a(t)$ has at least some $a_j(t) = 1$ for $j \geq 2$. Those of them who do not win jobs in the primary market are involuntarily unemployed. Although they would accept to take existing jobs at the free wage, they find it optimal to queue. The general, n-sector analysis of the segmented market confirms the basic result derived in Demekas (1987), in a model with only one primary market sector and no search costs.

2. Non-separate markets

In the case of non-separate markets the restriction: if $a_1(t) = 1$, then $a_j(t) = 0$ for all $j \geq 2$, does not apply. Primary and secondary market strategies are not mutually exclusive. Everything else in the setup of the problem, including (A1) through (A4), remains the same.

It follows immediately that Theorem 1 holds now for all j. In other words, the optimality conditions (6) can be rewritten as:

$$a_k(t) = \begin{cases} 1 & \text{iff } b_t \leq w_k \text{ and } q_k w_k - q_{ko} p_k b_t > F \\ 0 & \text{otherwise,} \end{cases} \tag{6.a}$$

Rule (6.a) is now enough to determine the optimal strategy of each worker, without the two-step optimization process described in the previous section being necessary.

In order to characterize the equilibrium, equation (7) must be rewritten. The supply to the secondary market now consists of two parts: those who apply to sector 1 only, plus those who apply to other sectors as well but lose, and w_1 is their highest wage offer. Recalling the definition of q's, this can be written as:

$$w_1 = f\left[\int_T q_1 a_1(t)dt\right] \tag{9}$$

Existence of equilibrium can be easily shown along the lines of Theorem 1, using the integral in (9) for the supply function. The same properties of equilibrium w_1 are guaranteed by the non-triviality assumption (A3). Supply to the primary market is again all t for which some $a_j = 1$, for $j \geq 2$, or:

$$S = \sum_2^n \left[\int_T a_j(t)dt\right] \tag{10}$$

This can be broken down to two parts: those who applied only to the primary market ($a_1 = 0$), and those who applied to both ($a_1 = 1$). In other words:

$$S = \sum_2^n \left[\int_T [1 - a_1(t)]a_j(t)dt\right] + \sum_2^n \left[\int_T a_1(t)a_j(t)dt\right] \tag{10.a}$$

The unlucky ones in the first group, who do not get any primary job offers, remain unemployed. The unlucky ones in the second group, however, have the free sector jobs as their highest wage offer. Expressing unemployment as the difference between total supply and demand, as in the case of separate markets, leads now to overestimation of the true number of jobless workers. Turning now to the nature of unemployment, and using the same criterion as before, it can be shown that involuntary unemployment will still arise.

Theorem 4: In a segmented market where the primary and secondary sectors are non-separate, some workers with reservation wages below the free wage will apply only for primary sector jobs (for proof see Appendix I).

This Theorem says that, despite the fact that primary and secondary strategies are not mutually exclusive, there will be workers who maximize their expected utility by applying only to the primary sector. Those among them who get no job offers are, therefore, involuntarily unemployed.

The persistence of involuntary unemployment even in the case of non-separate markets is due to the application fee F that has to be paid in all sectors (including the free). If search is costless ($F = 0$) in the secondary market, then it is shown in the Appendix that only voluntary unemployment arises. The intuition behind this is obvious: if applying to the free sector costs nothing, then everybody who would not mind employment at the going free wage will apply, "securing" himself against the probability of not getting a primary job. This is equivalent to allowing workers who had applied for primary jobs to "return" to the secondary market if they get no offers.

III. Summary and Conclusions

In this paper a general n-sector model of a segmented labor market has been developed and solved in two cases. The first one, that of separate primary and secondary markets, covers the existing segmented market models in the literature as special cases. Existence of equilibrium has been demonstrated in both the case of separate, and that of non-separate markets, and simple expressions for the endogenous free sector wage and unemployment have been derived. Using, in particular, the definition of involuntary unemployment first employed by Harberger (1974) in the context of segmented markets, it has been shown that segmentation always causes some involuntary unemployment, even in the case of non-separate markets.

The model is useful not only because it captures an important stylized fact of labor markets in developing and industrialized countries alike, but also because, with a minimum of initial assumptions, it yields simple equilibrium conditions. Specific functional forms for the demand and supply substituted in equations (7) and (8) or (9) and (10) can be used to estimate the precise effects of policies such as the reduction of wage differentials in the primary market on the free wage and unemployment. Previous segmented market models with only one primary market sector and/or fixed labor supply were unable to answer such questions. The paper also provides the basic framework for dynamic modeling of segmented markets.

The model can also be used to estimate the impact of different forms of wage regulation on participation rates. Since participation decisions here depend not only on wages but also on search costs and probabilities, the model can accommodate the empirical evidence showing negative effects of minimum wages on participation (see the discussion in Mincer 1976).

Proof of Theorem 1: To decide whether $a_k(t)$ should be zero or one, the worker t compares the expected value of a primary market strategy with $a_k = 1$ to that of a strategy with $a_k = 0$. He compares, in other words:

$$U_t(a)\Big|_{a_k=1} = \prod_{\substack{j=2\\j\neq k}}^{n} (1 - p_j a_j)(1 - p_k)b_t + \sum_{\substack{j=2\\j\neq k}}^{n} a_j(t)(q_j w_j - F)$$

$$+ (q_k w_k - F), \text{ and}$$

$$U_t(a)\Big|_{a_k=0} = \prod_{\substack{j=2\\j\neq k}}^{n} (1 - p_j a_j)b_t + \sum_{\substack{j=2\\j\neq k}}^{n} a_j(t)(q_j w_j - F)$$

For $a_k = 1$ it must be:

$$U_t(a)\Big|_{a_k=1} > U_t(a)\Big|_{a_k=0} \quad \text{or} \quad U_t(a)\Big|_{a_k=1} - U_t(a)\Big|_{a_k=0} > 0 \qquad \text{(P.1)}$$

Substitute the expressions above for $U_t(a)\Big|_{a_k=1}$ and $U_t(a)\Big|_{a_k=0}$ in (P.1)

and, after standard manipulations, the condition for $a_k = 1$ can be written:

$$- \prod_{\substack{j=2\\j\neq k}}^{n} (1 - p_j a_j)p_k b_t + q_k w_k - F > 0 \qquad \text{(P.2)}$$

Finally, using the definition of q_{ko} from the paper, the condition can be expressed as:

$$\underset{k \geq 2}{a_k(t)} = \begin{cases} 1 & \text{iff } b_t \leq w_k \text{ and } q_k w_k - q_{ko} p_k b_t > F \\ 0 & \text{otherwise} \end{cases}$$

Proof of Theorem 2: Labor demand in the secondary market $L_1(w_1)$ is, by assumption, well-defined and continuous. Supply to the secondary market is m(T1). Since p_1 is unity, it follows from the definition of q's in (4.a) and (4.b) that $q_1 = q_o$. The set T1 can then be rewritten as:

$$T1 = \{\text{all } t\text{:} \quad w_1 - F > q_1 b_t + (q_1 w_1 - F)\sum_2^n (q_j w_j - F) \text{ and}$$

$$w_1 - F \geq b_t, \text{ for all } a(t) \text{ with } a_1(t) = 0\} \qquad \text{(P.3)}$$

The supply to the secondary market, m(T1), is a function of w_1 described by the two inequalities in (P.3). Since all q_j's and w_j's for $j \geq 2$, as well as F are independent of w_1, and b(t) is by assumption (A2) a continuous function of t, both inequalities in T1 are continuous in t. T1, then, consists of a finite number of continuous functions of t and, therefore, the supply m(T1) as a function of the wage is also continuous. Consequently, the excess demand function $H(w_1)$, defined as $L_1(w_1)$ - m(T1), is continuous.

Calculate the value of $H(w_1)$ for two extreme values of w_1: when $w_1 = 0$ and when $w_1 = w_n$. Since b_t cannot be negative by assumption (A2), T1 is empty when $w_1 = 0$, because there are no t's that satisfy the second inequality. m(T1) is, then, zero, and:

$$H(0) = L_1(0) - 0 > 0 \qquad \text{(P.4)}$$

When $w_1 = w_n$, there are two cases for $H(w_n)$:

(i) If $b_o > w_n$, then $H(w_n) < 0$, by the non-triviality assumption (A3).
(ii) If $b_o < w_n$, then the non-triviality assumption imposes $H(b_o) < 0$. But this implies a fortiori that $H(w_n) < 0$, since $L_1(w_n) < L_1(b_o)$ and $\{T1, \text{ for } w_1 = w_n\} = \{T1, \text{ for } w_1 = b_o\} = \{T\}$. In both cases (i) and (ii), then: $H(w_n) < 0$ (P.5)

(P.4), (P.5) and the continuity of $H(w_1)$ satisfy the requirements of a simple fixed point theorem. Hence, a stable equilibrium w_1 exists in the segment $[0, w_n]$.

Continuity of b(t) is sufficient but not necessary for the proof. A discontinuous b(t) satisfying the requirements of a more general fixed point theorem can also guarantee existence.

<u>Proof of Theorem 3:</u>

The fact that the equilibrium w_1 is less than at least one regulated wage means that the supply to the primary market cannot be zero. In other words: m(T2) + m(T3) is positive. Therefore, either:

(i) m(T3) is zero and then m(T2) is necessarily positive, q.e.d., or:
(ii) m(T3) is positive.

It must be shown that m(T2) is positive in case (ii) as well.

Consider the contradiction hypothesis: m(T2) is zero when m(T3) is positive. In other words:

$$\sum_{2}^{n} (q_j w_j - F) a_j(t) + q_o b_t < w_1 - F,$$

for all primary strategies of all $t \in T2$ (P.6)

(P.6) holds for all t's with $b_t \le w_1 - F$, hence also for t's with $b_t = w_1 - F$. Substitute in (P.6) and:

$$\sum_{2}^{n} (q_j w_j - F) a_j(t) < (1 - q_o)(w_1 - F), \text{ for all primary strategies} \quad (P.7)$$

(P.7) is independent of b_t, so if it holds for some t it holds for all of t's in T. Consider now a t in T3, with $b_t > w_1 - F$. Since (P.7) holds for all t, it is <u>a fortiori</u> true that:

$$\sum_{2}^{n} (q_j w_j - F) a_j(t) < (1 - q_o) b_t \iff$$

$$\sum_{2}^{n} (q_j w_j - F) a_j(t) + q_o b_t < b_t, \text{ for all primary strategies} \quad (P.8)$$

But in that case the reservation wage of all t's in T3 dominates every possible primary market strategy. Every t with $b_t > w_1 - F$, then, would prefer to stay at home and T3 would have zero measure. This is by construction impossible in case (ii) and, therefore, the contradiction hypothesis does not hold; T2, in other words, has positive measure in case (ii) as well.

Proof of Theorem 4: Consider the contradiction hypothesis: all workers with reservation wages below $w_1 - F$ have $a_1 = 1$, or, in other words, $b_t \leq w_1 - F$ is sufficient for $a_1 = 1$.

From the optimality conditions (6.a) it follows that the rule for $a_1 = 1$ is: $q_1 w_1 - q_{1o} p_1 b_t > F$ (P.9)

For workers with $b_t \leq w_1 - F$ the other inequality in (6.a) is not binding. Given that $p_1 \doteq 1$, it can be seen from the definition of q's that $q_1 = q_{1o}$. (P.9) then collapses to:

$$q_1(w_1 - b_t) > F, \text{ or}$$

$$w_1 - \frac{F}{q_1} > b_t \qquad \text{(P.10)}$$

Since q_1 is generally less than unity, clearly $w_1 - F \geq b_t$ is not sufficient for (P.10) to be satisfied. The fact that a worker t has a reservation wage below w_1 -F is not enough to guarantee that he will apply to the secondary market. Since b_t is continuous, there will be some workers such that:

$$w_1 - \frac{F}{q_1} < b_t < w_1 - F$$

who apply only to the primary market.

If F = 0 for secondary market jobs, however, then the conditions $w_1 \geq b_t$ and (P.10) are equivalent. This means that in this case having a reservation wage below w_1 is sufficient to make the worker apply to the free sector. These workers, then, will never remain unemployed.

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