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Real Interest Rate Targeting: An Example From Brazil

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Abstract

This paper examines a real interest rate targeting procedure based on lagged inflation similar to the policy followed by the Brazilian monetary authorities during the period November 1986 to December 1988, focusing on the issue of the determinacy of the price level. For the specific model examined, the analysis suggests that such a targeting procedure would not suffer from the frequently noted defect of nominal interest rate targeting rules of leaving the conditional expectation of the next period price level undetermined.

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Summary

This paper examines a real interest rate targeting procedure based on lagged inflation similar to the policy followed by the Brazilian monetary authorities from November 1986 to December 1988. For the model examined, the analysis suggests that such a targeting procedure, unlike nominal interest rate targeting rules, would not leave undetermined the conditional expectation of the price level during the next period. The analysis also suggests, however, that such a policy might not necessarily give the monetary authorities much control over prices over the longer term.

This paper does not evaluate whether the real interest rate policy followed by the Brazilian monetary authorities successfully moderated the inflation rate nor does it attempt to answer the question of whether this was the optimal policy under the circumstances. The Brazilian economy at the time featured widespread indexation, but this is not taken into account in the model in this paper.

I. Introduction

The subject of conducting monetary policy by targeting a nominal interest rate has been well covered in the literature with recent works by McCallum (1986), Gagnon and Henderson (1988), Benavie and Froyen (1988), and Calvo and Végh (1990) providing good overviews of the controversy. The main question has been variously posed in the form as to whether a policy of targeting the nominal interest rate is "completely specified" or if the price level and/or inflation rate is determined with such a policy. The method of analysis of this question has undergone much change since the time of Wicksell with the more recent literature using models that incorporate rational expectations and focusing on the issue of what would provide the nominal anchor to pin down the price level if the monetary authorities pursued such a policy. The views on this question are not unanimous amongst economists but many authors, including Sargent and Wallace (1975), Canzoneri, Henderson, and Rogoff (1983), and McCallum (1986) have concluded that a policy of the monetary authorities being willing to buy or sell any amount of securities at the target interest rate would not determine the price level. However, in spite of the skepticism in the economic literature, as Barro (1989) notes, policy makers, in any case, appear "to talk mainly in terms of controlling or targeting interest rates (p.3)." Similarly, Calvo and Végh (1990) note that the nominal interest rate "is one of the most watched variables among the G7 countries (p.1)."

The interest rate targeting literature has largely concentrated on comparing nominal interest rate targeting rules with monetary growth rules while the subject of targeting real interest rates has been somewhat passed over with recent exceptions being observations in papers by Canzoneri, Henderson, and Rogoff (1983), McCallum (1986), and Barro (1989). Canzoneri, Henderson, and Rogoff (1983), for example, argue that stabilizing the real interest rate could be an optimal policy when goods market disturbances are relatively small. McCallum (1986) states that the monetary authority cannot literally peg the real interest rate under the assumptions of his model since it cannot observe without error the contemporaneous value of private agents' inflationary expectations. Barro (1989) asks rhetorically if "systematically and significantly influencing expected real interest rates ... is beyond the power of monetary authorities over periods of interesting length (p.4)." Although there is no clear consensus in the literature it seems that the most common view is that real interest rate targeting would likely not be a practical, or perhaps even feasible, strategy and that in any case it would not be as preferable as monetary aggregate targeting or some other type of policy such as targeting nominal GNP.

While the detailed analysis of real interest rate targeting has largely been passed over in the economic literature the use of such a policy has not been overlooked by economic policy makers either in industrial countries, as Barro (1989) notes, or in developing countries. The monetary authorities in Brazil, for example, used a version of such a policy from November 1986 to

December 1988.1/ This paper examines a real interest rate targeting procedure based on lagged inflation that on a conceptual level is similar to the monetary policy followed by the Brazilian monetary authorities during that period. It is shown that under specific assumptions such a policy would allow economic agents to calculate the conditional expectation of the next period price level. The policy would not, therefore, suffer from the indeterminacy problem defined by McCallum (1986) as "situations in which the model economy does not determine the value of *any* nominal magnitude (p.137)."2/ Under the real interest rate targeting rule examined anything which increases the expected price level would lead the authorities to increase the nominal interest rate which would tend to counteract the upward pressure on prices. The analysis also suggests, however, that this policy might not strictly pin down the price level in the sense that unforeseen changes in the price level in one period would be perpetuated and the price level would not converge towards any long-term equilibrium level.

The next section of this paper explains the interest rate targeting procedure followed by the Brazilian authorities, the third section formalizes the description of this targeting procedure, the fourth section develops a simple model to analyze this policy, and the last section offers some concluding observations.

It should be stressed that this paper does not evaluate whether the real interest rate policy followed by the Brazilian monetary authorities was a success in terms of its effects on the inflation rate nor does it attempt to address the issue of the optimal mix of fiscal, monetary, and external policies in stabilizing inflation in Brazil. The Brazilian economy at the time had many important features such as widespread indexation which are not dealt with in the analysis.

II. Interest Rate Targeting in Brazil

The Brazilian economy, which had been subject to inflationary pressure for some time, came under increasing pressure in the 1980's; inflation which had averaged 44 percent a year during 1970-81 increased to average 247 percent a year during 1982-88. In response to the increasing inflation the government of Brazil introduced a number of well-known economic adjustment programs. This section briefly summarizes the interest rate targeting policy that was followed by the monetary authorities of Brazil in the period

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1/ Similar policies were used during other periods but it was during this period that the targeting procedure was most explicit.

2/ Many alternative definitions of indeterminacy exist in the literature. Adams and Gros (1986), for example, define price indeterminacy as existing when an economic model fails to reveal the long-term price level. Under this definition the price level is indeterminate in the model examined in this paper.

between two of those plans: the Cruzado Plan of February 1986 and the Summer Plan of January 1989.

In Brazil a key interest rate is the rate in the overnight repurchase market for government securities that is set by the Central Bank. Every business day the Bank, while acting as the residual supplier of liquidity to the market, would buy or sell any amount of securities at the chosen interest rate. In November 1986 the Central Bank of Brazil began to conduct its open market operations so that the cumulative overnight interest rate from the 15th day of one month to the 14th day of the following month was equal to the inflation rate for the first month (on a calendar basis) as indicated by the Consumer Price Index (IPC).1/ In effect the authorities were maintaining the "real" interest rate, as defined using lagged inflation, at about zero.

In June 1987 the authorities adopted a new anti-inflation program which became known as the Bresser Plan. Under this plan the authorities, as a means of improving the flexibility of monetary policy, dropped their commitment to keep the monthly overnight rate strictly aligned with inflation.2/ However, in practice, the overnight interest rate and the inflation rate continued to move in tandem because--as the exchange rate was still adjusted on a daily basis in line with domestic inflation as determined by the IPC--the overnight interest rate could not fall below inflation without triggering a movement of funds from the government securities market into assets denominated in foreign exchange. Consequently ex-post the "real" interest rate defined in terms of lagged inflation was about as stable during the period October 1987 to December 1988 as during the earlier period when the monetary authorities had been explicitly targeting the "real" interest rate. News reports suggest that during the latter period many market participants were under the impression that the

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1/ Since the IPC was computed on a calendar month basis it was centered on the 15th day of the month and thus it was approximately measuring inflation from the middle of the previous month to the middle of the current month, assuming inflation is linearly distributed. The interest rate target, however, was targeting the cumulative daily interest rate from the 15th of the current month to the 14th of the following month. Thus the nominal interest rate of any given month was made to match the inflation of the previous month.

2/ The authorities also changed the calculation of the price index and established that the IPC would measure the average price level from the 15th day of one month until the 14th day of the second month. The interest rate targeting period was also moved back by two weeks at that time. Thus the same comparison can be made between the IPC and the cumulative overnight interest rate as before with the period shifted two weeks earlier. Developments associated with these changes distort the calculation of the real interest rate in June and July 1987.

monetary authorities were still targeting the "real" interest rate.1/ As can be seen in Chart 1 (top), the "real" interest rate defined in terms of lagged inflation was, in fact, very stable from November 1986 to December 1988, with the exception of June/September 1987 during the implementation of the Bresser Plan.2/

The ex-ante real interest rate cannot be measured without some indicator of market inflationary expectations. It is possible, however, to measure the ex-post real interest rate by deflating the nominal interest rate by actual contemporaneous inflation which, in this case given how the IPC was calculated, requires deflating the current month's interest rate by the IPC of the following month (Chart 1-bottom). From Chart 1 it is clear that although the authorities were largely able to target the real interest rate as defined using lagged inflation, ex-post the real interest rate, defined using contemporaneous inflation, was more volatile and slightly negative on average during this period.

III. A "Real" Interest Rate Target Rule

To simplify the formal analysis it is assumed that the "real" interest rate targeting policy under consideration is to set the nominal interest rate in each period by being ready to buy and sell any quantity of securities at that rate according to:

$$r_t = E_{t-1}p_t - p_{t-1} + R \qquad (1)$$

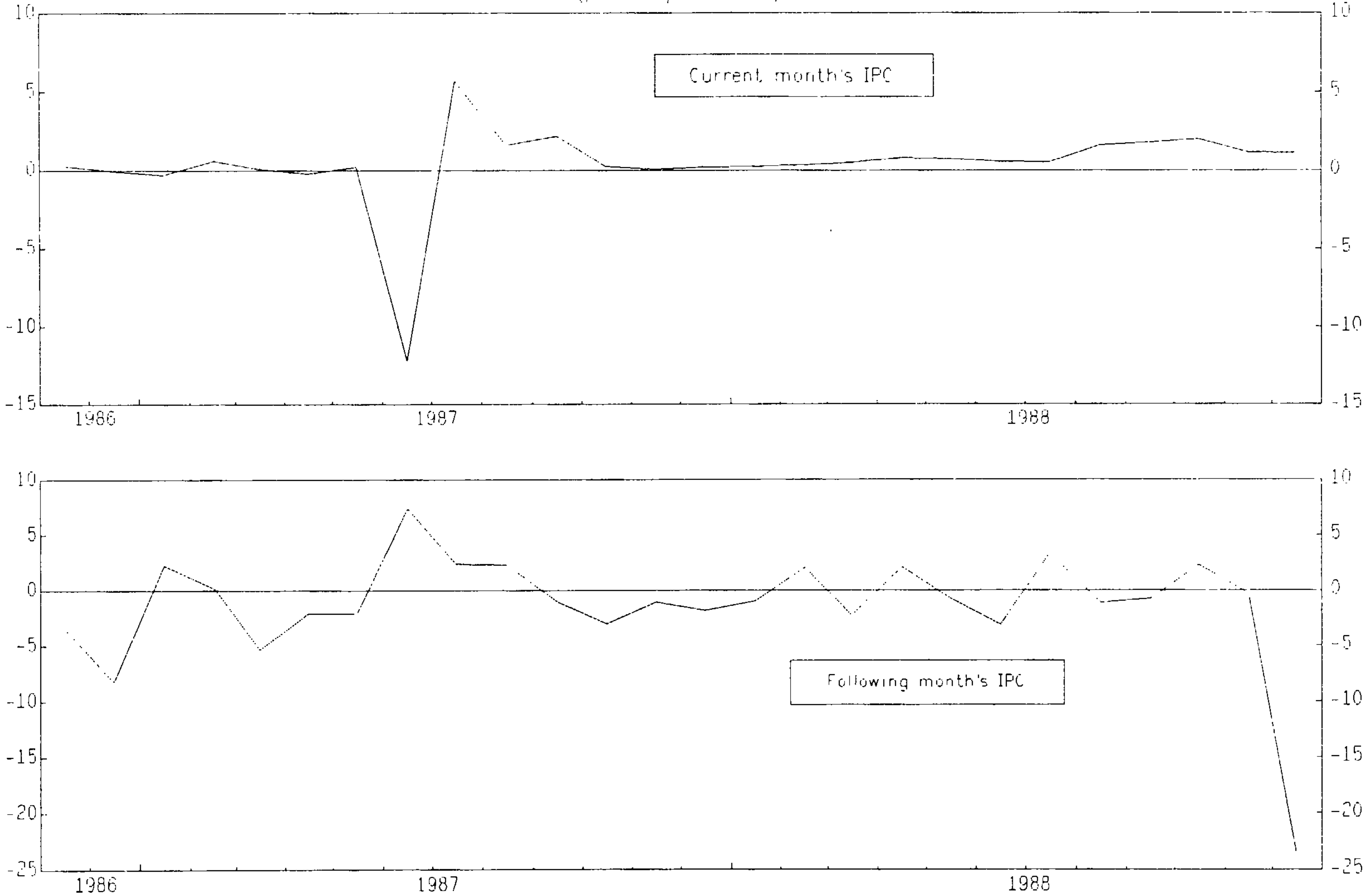
where r_t is one plus the nominal interest rate, p_t is the price level, and R is one plus the real interest rate target (all as natural logarithms).3/

1/ As indicated in various issues of the Brazilian business daily newspaper, the Gazeta Mercantil, during this period.

2/ From November 1986 to June 1987 the overnight interest rate in Chart 1 is measured from the 15th of one month to the 14th of the next month. From July 1987 to December 1988 the overnight interest rate is measured on a calendar month basis.

3/ There is some controversy as to whether such a formulation of a monetary policy is completely specified. McCallum (1986) contends that a "pure (nominal) interest rate peg," which he defines as a policy of standing ready to buy or sell any quantity of securities at a given interest rate, does not constitute a well-formulated monetary policy since it does not indicate if the monetary authorities will permit base drift of the money supply. Benavie and Froyen (1988) argue that if the monetary authorities are not going to permit base drift this is another element of the policy which would not be feasible because if they simultaneously sought to eliminate base drift they could not peg the interest rate completely. Walsh (1986) and Goodfriend (1987) have also pointed out that base drift is necessary for price level stationarity. In this paper it is assumed that the authorities are not concerned about base drift.

CHART 1
BRAZIL
REAL INTEREST RATE 1/
November 1986-December 1988
(percent per month)



1/Overnight interest rate deflated by the Consumer Price Index (IPC) of the same month (top) and the following month (bottom).

The assumption is that expectations are rational, i.e., that E_{t-1} refers to the mathematical expectation of the indicated variable conditional on all information available through time t-1, where the equations of the model (specified below) are assumed by both policy makers and other economic agents to represent the true structure of the economy and are therefore used to form expectations.

Rule (1) can be contrasted with one that would target the actual ex-post real interest rate,

$$r_t = E_{t-1}p_{t+1} - E_{t-1}p_t + R \quad . \qquad (2)$$

McCallum (1986) states that a rule such as (2) would be difficult to implement in practice because the monetary authorities cannot observe without error economic agents' inflationary expectations. As a practical matter in rule (1) all that is necessary is that the monetary authorities and other economic agents have the same expectations concerning the current price level and not both the current and future price levels as in rule (2). As a technical matter, under the assumption of rational expectations and with the identified model the same as the true model which can, as is shown below, be uniquely solved for $E_{t-1}p_t$ all agents would have the same conditional expectation of the current price level.

To return to the Brazilian experience for a moment, Chart 1 (top) shows the implementation of a rule such as (1) during the period November 1986 to May 1987. Given that economic agents knew that the authorities' policy was to set $R \approx 0$ and the value of p_{t-1}, while r_t was revealed on a daily basis, the calculation of the authorities' $E_{t-1}p_t$ by economic agents would have been an easy task to the extent that they were thinking along the lines sketched in this paper.

IV. Model and Analysis

A. Basic model

The model to analyze the interest rate rule (1) is developed along the lines of the method of undetermined coefficients originally developed by Lucas (1972) and extended by McCallum (1978,1981,1986). The model starts with three basic equations:

$$y^d{}_t = b_0 + b_1[r_t - (E_{t-1}p_{t+1} - E_{t-1}p_t)] + b_2(m_t - p_t) + v_t \qquad (3)$$

where $b_1<0$ and $b_2>0$;

$$m_t - p_t = c_0 + c_1r_t + c_2y_t + \eta_t \qquad (4)$$

where $c_1<0$ and $c_2>0$; and,

$$y^s_t = a_0 + a_1[p_t - E_{t-1}p_t] + a_2 y_{t-1} + u_t \qquad (5)$$

where $a_1>0$ and $1>a_2>0$, y_t is the logarithm of real output (with superscripts referring to demand and supply), m_t is the logarithm of the money supply (M1), and v_t, η_t, and u_t are independently distributed white noise disturbances. Equations (3) and (4) are therefore traditional IS-LM curves and (5) is a natural rate Phillips curve. This model has been commonly used in the literature on interest rate targeting because of its simplicity and orthodox character.

Setting the supply of output equal to demand in (3) and (5), and solving for r_t gives

$$\begin{aligned} r_t = {} & d_0 + d_1[p_t - E_{t-1}p_t] + d_2 y_{t-1} + d_3(m_t - p_t) \\ & + E_{t-1}p_{t+1} - E_{t-1}p_t + u_t - v_t \end{aligned} \qquad (6)$$

where $d_1<0$, $d_2<0$, and $d_3>0$. The equation set (4), (5), and (6) is used as the basic model with r_t replaced in (4) and (6) by rule (1).1/ This system has m_t, p_t, and y_t as unknowns, r_t as the control variable set by the authorities, v_t, η_t, and u_t as stochastic disturbances, and y_{t-1} and p_{t-1} as predetermined variables.

The method of undetermined coefficients uses the linearity of the model and the white noise character of the disturbances to derive reduced-form equations with the unknowns expressed as functions of the predetermined variables and the disturbances (and implicitly the fixed control, R). These reduced-form equations are

$$p_t = \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1} + \pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t \qquad (7)$$

$$m_t = \pi_{20} + \pi_{21}y_{t-1} + \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t \qquad (8)$$

$$y_t = \pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t + \pi_{34}\eta_t + \pi_{35}v_t \ . \qquad (9)$$

Using the method of undetermined coefficients this system can be solved for a bubble-free solution for the π_{ii} as functions of the parameters of the structural equations (see Appendix A for details on the solution procedure). The solution values (for brevity leaving those for the disturbance terms in the Appendix and using Π_{ii} for solved values) are

$$\Pi_{10} = [1/d_3c_1]\{R(1-d_3c_1) - d_0 - d_3(c_0+c_2a_0) - [a_0(d_2+d_3c_2a_2)/(1-d_3c_1-a_2)]\}$$

$$\Pi_{11} = (d_2+d_3c_2a_2)/(1-d_3c_1-a_2) \qquad \Pi_{12} = 1 \qquad (10)$$

1/ Strictly u_t and v_t in equation (6) should have $(1/b_1)$ as coefficients but since it does not affect the analysis these coefficients will be dropped for notational simplicity.

$$\Pi_{20} = (1+c_1)[[1/d_3c_1]\{R(1-d_3c_1)-d_0-d_3(c_0+c_2a_0)-[a_0(d_2+d_3c_2a_2)/(1-d_3c_1-a_2)]\}] +c_0+c_1R+c_2a_0$$

$$\Pi_{21} = [(1+c_1)(d_2+d_3c_2a_2)/(1-d_3c_1-a_2)]+c_2a_2$$

$$\Pi_{22} = 1 \qquad \Pi_{30} = a_0 \qquad \Pi_{31} = a_2 \qquad \Pi_{32} = 0.$$

Using (10) and substituting into equations (7), (8), and (9) it follows that

$$p_t - p_{t-1} = \Pi_{10} + \Pi_{11}y_{t-1} + \text{disturbances} \tag{11}$$

$$m_t - p_t = [\Pi_{20} - \Pi_{10}] + [\Pi_{21} - \Pi_{11}]y_{t-1} + \text{disturbances} \tag{12}$$

and

$$y_t = a_0 + a_2y_{t-1} + \text{disturbances.} \tag{13}$$

The signs of Π_{11} and Π_{21} are not fixed by the assumptions of the model. But from the expression for Π_{10} and equation (11) it can be seen that the expected rate of inflation is inversely related to the value of the real interest rate peg and that for a given y_{t-1} a value of R exists that would give an expected value of zero to inflation. The coefficient of the p_{t-1} term in (11) is equal to unity which implies that any positive shock, given a fixed R, will cause a permanent increase in the price level compared to what it otherwise would have been. From (12) and the expressions for Π_{10}, Π_{20}, Π_{11} and Π_{21} it can be seen that the ex-post real money stock is positively related to growth and the level of the real interest rate. Not much should be made of the relation between the level of the real interest rate and output as the supply function in this model is particularly simplistic.

The important point is that the expected price level is determined, i.e., conditional on p_{t-1} the model allows economic agents to calculate the expected value of p_t. If R has been chosen so as to eliminate expected inflation then this implies that $E_{t-1}p_t = p_{t-1}$ and that r_t=R. The policy rule (1), however, does not allow an unconditional expectation of the price level to be calculated, i.e., even if conditionally expected inflation is set equal to zero through the judicious choice of R, the price level will follow a random walk over the long run. The price level will simply adjust to whatever shocks affect the economy.

To compare this to the discussion of Sargent and Wallace (1975, p.250) on the issue of nominal interest rate targeting, under the rule (1) the public will rationally expect that a foreseen increase in p_t will be met by an increase in r_t. From the LM curve (4) the increase in r_t will lead to a decrease in the demand for real balances which from the IS curve (3) will have a depressing effect on the demand for real output. Thus, as could be expected, the determinacy of the price level in this model is dependent on the inclusion of the real balance term in the IS curve.1/ While the real balance effect is necessary for the price level to be determined in this model, as is also known, its presence is not sufficient to determine the price level with a simple nominal interest rate peg, r_t=R.2/

With rule (1), other things being equal, economic agents would expect no change in the price level from p_{t-1} to p_t since they would know that the monetary authorities would react to a foreseen disturbance by adjusting nominal interest rates to maintain the given level of the real interest rate. But under rule (1) unexpected shocks at time t would not cause an adjustment in the nominal interest rate and thus the real interest rate ex-post would change and the price level would not return to its previous level. *In Chart 1 it can be seen that although the authorities held the* target interest rate relatively constant the ex-post real interest rate fluctuated widely. Thus under rule (1) the price level would follow a random walk since only expected shocks (and not unexpected shocks) would be offset by movements in the nominal interest rate. Therefore it is only in this limited sense that the price level is determined in this model.

Any application of the above analysis to the actual Brazilian case would obviously require a great number of caveats. Nonetheless, the essentially passive character of the monetary policy in the model raises questions about the efficacy such a policy would have in combating inflation.

B. An alternative rule

The monetary policy rule (1) is based on lagged inflation. It is interesting to ask what if the monetary authorities tried to target contemporaneous inflation. As a practical matter this would be a much more formidable task but as a technical question in the context of the current model, rule (2) can be introduced since it is assumed that equations (3), (4), and (5) represent the true structure of the economy of which both the monetary authorities and other economic agents are aware. Initially it would seem that upon examining the structural equations (2), (4), (5), and (6), p_{t-1} would not be included as a predetermined variable in the reduced-form equations. In that case it would not be possible to solve for π_{10} or π_{20} and the system would not be consistent unless R took a specific value

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1/ The real balance term could take a different form such as m_{t-1}-p_t. McCallum (1986) argues that the more appropriate form of the real balance term would be m_{t-1}-p_t given the budget constraint. In the present model if the real balance term in equation (3) were m_{t-1}-p_t it would complicate the procedure for arriving at a solution because m_{t-1} would be added to the system as a predetermined variable; however, the expected price level would still be determined.

2/ See McCallum (1981).

determined by the structural coefficients (see Appendix B, part 1, for details).

If p_{t-1} were alternatively included in the reduced-form equations the values for the coefficients could be derived in a similar way as in the basic model (see Appendix B, part 2). Given the linearity of the model it is not surprising that while the exact quantitative reduced-form solution would be different, qualitatively the solution would be the same as in the basic model.

Because it can be shown that expected inflation, $E_{t-1}(p_{t+1}-p_t)$, would be the same in the model with rule (2) whether or not p_{t-1} is included in the reduced-form equations as a predetermined variable (see Appendix B, part 3) it might be argued that it would only be "rational" for economic agents to use p_{t-1} in forming their expectations since it would allow them to calculate $E_{t-1}p_t$ and so, even though p_{t-1} does not appear in the structural equations it should appear in the reduced-form equations. But, with rule (2) economic agents would not know if they should consider p_{t-1} or, alternatively, m_{t-1} when forming their expectations of p_t. The problem arises because while $E_{t-1}(p_{t+1}-p_t)$ would be equal in both cases it is not the case that $E_{t-1}p_t$ in both versions would be equal because p_{t-1} and m_{t-1} would incorporate the disturbances that occurred in t-1 in slightly different ways, thus the expected value of p_t would be different if one agent based his expectation on p_{t-1} and another on m_{t-1} (see Appendix B, part 4, for details).

With rule (1) the monetary authorities, in contrast, are explicitly providing information on what economic agents should use to form their expectations, i.e., agents would know the authorities' targeting procedure and would be able to calculate the implicit value of R. Thus the use of lagged inflation in rule (1) not only makes the targeting strategy easier to implement on a practical basis, it also serves to peg down $E_{t-1}p_t$ by telling all agents what to use in forming their expectations.1/

V. Concluding Observations

The analysis in this paper suggests that a real interest rate targeting rule similar to the one used by the Brazilian monetary authorities during the period November 1986 to December 1988 would enable economic agents to calculate the conditional expectation of the next period price level. Such a rule would not suffer from the frequently noted defect of nominal interest rate targeting rules of leaving the expected value of the next period price level undetermined. However, the above analysis also suggests that such a

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1/ Note that if under rule (2) the real balance effect were included in the IS curve as $m_{t-1}-p_t$ then m_{t-1} would be a predetermined variable and the price level would be determined since once again the structural model would indicate the variable to be used as the basis of the formation of expectations.

policy might not necessarily allow the monetary authorities to exercise control over prices over the longer term, i.e., it suggests that the price level would follow a random walk over the long run. This result is consistent with the findings of Adams and Gros (1986) in regard to the instability of real exchange rate rules (defined as rules where the nominal exchange rate is adjusted in line with the differential between domestic and foreign inflation). They found that such rules are "essentially a policy of full monetary accommodation (p. 471)."

While the rule (1) analyzed in this paper is similar to that used by the Brazilian monetary authorities for much of the period November 1986 to December 1988, the intention here is not to suggest that the analysis explains why economic policy in Brazil was unable to halt the increase in inflation during this time. There are many unanswered questions when one tries to apply the analysis contained herein to the Brazilian experience, in part because at the time the Brazilian economy featured widespread indexation which is omitted from the analysis.

APPENDIX

A. Basic Model

This appendix provides details for the calculations of the solutions (10). Using equations (7), (8), and (9) and taking expectations gives

$$E_{t-1}p_t = \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1} \quad \text{and}$$

$$\begin{aligned} E_{t-1}p_{t+1} = \pi_{10} &+ \pi_{11}[\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1}] \\ &+ \pi_{12}[\pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1}]. \end{aligned} \tag{A.1}$$

Substituting into equation (5) using (7), (8), (9), and (A.1) gives

$$\begin{aligned} &\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t + \pi_{34}\eta_t + \pi_{35}v_t = \\ &a_0 + a_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] + a_2y_{t-1} + u_t. \end{aligned} \tag{A.2}$$

Substituting into equation (6) using (1), (7), (8), (9), and (A.1) gives

$$\begin{aligned} \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1} - p_{t-1} + R = d_0 + d_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] \\ + d_2y_{t-1} + d_3[\pi_{20} + \pi_{21}y_{t-1} + \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t \\ - \pi_{10} - \pi_{11}y_{t-1} - \pi_{12}p_{t-1} - \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t] \\ + \pi_{11}[\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1}] + \pi_{12}[\pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1}] \\ - \pi_{11}y_{t-1} - \pi_{12}p_{t-1} + u_t - v_t. \end{aligned} \tag{A.3}$$

Substituting into equation (4) using (1), (7), (8), (9), and (A.1) gives

$$\begin{aligned} \pi_{20} + \pi_{21}y_{t-1} + \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1} \\ - \pi_{12}p_{t-1} - \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t = c_0 + c_1[\pi_{10} + \pi_{11}y_{t-1} \\ + \pi_{12}p_{t-1} - p_{t-1} + R] + c_2[\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t \\ + \pi_{34}\eta_t + \pi_{35}v_t] + \eta_t. \end{aligned} \tag{A.4}$$

The method of undetermined coefficients derives expressions for the coefficients in terms of the structural parameters by noting that in each equation (A.2), (A.3), and (A.4) the left- and right-hand side expressions must be identical irrespective of particular time-series values of the predetermined variables and the disturbance terms. Using this method the coefficients on the disturbance terms and π_{30}, π_{31}, and

π_{32} can be determined easily. Using equations (A.3) and (A.4) it can be determined that

$$0 = (\pi_{12})^2 + \pi_{12}[d_3c_1 - 2] + [1 - d_3c_1]. \tag{A.5}$$

With the positive root π_{12} is equal to 1 and with the negative root π_{12} is equal to $1-d_3c_1$ which is greater than one which from equation (7) would imply explosive price behavior. If such explosive price behavior is excluded then $\pi_{12}=1$ is the unique solution which allows the other coefficients to be solved as given in (10) by straightforward substitution. The coefficients for the disturbance terms are as follows

$$\begin{array}{ll}
\Pi_{13} = -(1+d_3c_2)/\Phi & \Pi_{14} = -d_3/\Phi \\
\Pi_{15} = 1/\Phi & \Pi_{23} = (c_2(d_1-a_1-d_3)-1)/\Phi \\
\Pi_{24} = (d_1-d_3)/\Phi & \Pi_{25} = (1+c_2a_1)/\Phi \\
\Pi_{33} = (d_1-a_1)/\Phi & \Pi_{34} = -d_3a_1/\Phi \\
\Pi_{35} = a_1/\Phi &
\end{array} \tag{A.6}$$

where $\Phi = (d_1+d_3c_2a_1)$.

B. An alternative rule

1. Without p_{t-1} as a predetermined variable

In this case the reduced-form equations are (7), (8), and (9) but with all the π_{i2} set equal to zero since p_{t-1} is not included as a predetermined variable. The solution values for the coefficients on the disturbance terms and π_{30} and π_{31} can be determined exactly as with the basic model and take the same values. Substituting into (6) using (2) and the reduced-form equations gives

$$\begin{aligned}
R = {} & d_0 + d_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] + d_2y_{t-1} + \\
& + d_3[\pi_{20} + \pi_{21}y_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t \\
& - \pi_{10} - \pi_{11}y_{t-1} - \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t] \\
& + u_t - v_t
\end{aligned} \tag{B.1}$$

and substituting into equation (4) gives

$$\pi_{20} + \pi_{21}y_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1}$$
$$- \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t = c_0 + c_1[\pi_{11}(\pi_{30} + \pi_{31}y_{t-1})$$
$$-\pi_{11}y_{t-1} + R] + c_2[\pi_{30} + \pi_{31}y_{t-1} + \pi_{33}u_t \tag{B.2}$$
$$+ \pi_{34}\eta_t + \pi_{35}v_t] + \eta_t.$$

By substitution it can be determined that (using Λ_{ii} for solution values)

$$\Lambda_{11} = (d_2+d_3c_2a_2)/(d_3c_1(1-a_2))$$

$$\Lambda_{21} = \Lambda_{11} - (d_2/d_3) \qquad \Lambda_{20}-\Lambda_{10}= (R-d_0)/d_3 \tag{B.3}$$

and $R = [1/(1-c_1d_3)][d_3(\Lambda_{11}c_1a_0+c_0+c_2a_0)+d_0]$

for consistency.

2. <u>With p_{t-1} as a predetermined variable</u>

With p_{t-1} included as a predetermined variable and with (2) as the monetary policy rule, the reduced-form equations for the model are again (7), (8), and (9) and the solution procedure is as with the basic model. Taking expectations gives the equations (A.1) again while substituting into (5) using equations (7), (8), and (9) gives

$$\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t + \pi_{34}\eta_t + \pi_{35}v_t =$$
$$a_0 + a_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] + a_2y_{t-1} + u_t \tag{B.4}$$

and substituting into (6) using equations (2), (7), (8), and (9) gives

$$R = d_0 + d_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] + d_2y_{t-1} + d_3[\pi_{20} + \pi_{21}y_{t-1}$$
$$+ \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1} - \pi_{12}p_{t-1}$$
$$- \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t] + u_t - v_t \tag{B.5}$$

and substituting into (4) using equations (2), (7), (8), and (9) gives

$$\pi_{20} + \pi_{21}y_{t-1} + \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1}$$
$$- \pi_{12}p_{t-1} - \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t = c_0 + c_1[\pi_{11}[\pi_{30} + \pi_{31}y_{t-1}$$
$$+ \pi_{32}p_{t-1}] + \pi_{12}[\pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1}] - \pi_{11}y_{t-1} \tag{B.6}$$
$$- \pi_{12}p_{t-1} + R] + c_2[\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t$$
$$+ \pi_{34}\eta_t + \pi_{35}v_t] + \eta_t.$$

Using equations (B.4), (B.5), and (B.6) the coefficients on the disturbance terms and π_{30}, π_{31}, and π_{32}, which are all the same as in the basic model, can be determined easily. Using equations (B.5) and (B.6) it can be determined that π_{12} can be equal to 0 or 1. With $\pi_{12}=0$ it is also true that $\pi_{22}=0$. Since $\pi_{32}=0$ also, this would be the same as not including p_{t-1} as a predetermined variable, therefore it is assumed that $\pi_{12}=1$ is the relevant solution. With this result (B.5) and (B.6) can easily be solved for the other coefficients by straightforward substitution. The solutions are (using Ψ_{ii} for solved values)

$$\Psi_{10} = [1/d_3c_1]\{R(1-d_3c_1)-d_0-d_3(c_0+c_2a_0)+d_3c_1a_0[(d_2+d_3c_2a_2)/(d_3c_1a_2)]\}$$

$$\Psi_{11} = -(d_2+d_3c_2a_2)/(d_3c_1a_2) \qquad \Psi_{12} = 1$$

$$\Psi_{20} = (R-d_0)/d_3 + \Psi_{10} \tag{B.7}$$

$$\Psi_{21} = (-d_2-d_3c_2a_2-d_2c_1a_2)/(d_3c_1a_2)$$

$$\Psi_{22} = 1 \qquad \Psi_{30} = a_0 \qquad \Psi_{31} = a_2 \qquad \Psi_{32} = 0.$$

These values can be substituted into equations (7), (8), and (9) to give

$$p_t - p_{t-1} = \Psi_{10} + \Psi_{11}y_{t-1} + \text{disturbances} \tag{B.8}$$

$$m_t - p_t = [\Psi_{20} - \Psi_{10}] + [\Psi_{21} - \Psi_{11}]y_{t-1} + \text{disturbances} \tag{B.9}$$

and

$$y_t = a_0 + a_2y_{t-1} + \text{disturbances} \tag{B.10}$$

which are similar to equations (11), (12), and (13) although the exact quantitative values are different.

3. Expected inflation with or without p_{t-1} as predetermined

When p_{t-1} is not included as a predetermined variable expected inflation can be calculated using (B.3) as

$$E_{t-1}(p_{t+1}-p_t) = \Lambda_{11}(E_{t-1}y_t - y_{t-1})$$

$$= \Lambda_{11}(a_0 + (a_2-1)y_{t-1})$$

or given the required value of R as

$$E_{t-1}(p_{t+1}-p_t) =[[((R-d_0)/d_3)-c_0-c_2a_0-c_1R]/c_1a_0][a_0+(a_2-1)y_{t-1}] \ . \tag{B.11}$$

From the solution to the model when p_{t-1} is included as a predetermined variable, and using (B.8), it follows that

$$\pi_{20} + \pi_{21}y_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1}$$
$$- \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t = c_0 + c_1[\pi_{11}(\pi_{30} + \pi_{31}y_{t-1})$$
$$-\pi_{11}y_{t-1} + R] + c_2[\pi_{30} + \pi_{31}y_{t-1} + \pi_{33}u_t \tag{B.2}$$
$$+ \pi_{34}\eta_t + \pi_{35}v_t] + \eta_t.$$

By substitution it can be determined that (using Λ_{ii} for solution values)

$$\Lambda_{11} = (d_2+d_3c_2a_2)/(d_3c_1(1-a_2))$$
$$\Lambda_{21} = \Lambda_{11} - (d_2/d_3) \qquad \Lambda_{20}-\Lambda_{10}= (R-d_0)/d_3 \tag{B.3}$$

and $\qquad R = [1/(1-c_1d_3)][d_3(\Lambda_{11}c_1a_0+c_0+c_2a_0)+d_0]$

for consistency.

2. With p_{t-1} as a predetermined variable

With p_{t-1} included as a predetermined variable and with (2) as the monetary policy rule, the reduced-form equations for the model are again (7), (8), and (9) and the solution procedure is as with the basic model. Taking expectations gives the equations (A.1) again while substituting into (5) using equations (7), (8), and (9) gives

$$\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t + \pi_{34}\eta_t + \pi_{35}v_t =$$
$$a_0 + a_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] + a_2y_{t-1} + u_t \tag{B.4}$$

and substituting into (6) using equations (2), (7), (8), and (9) gives

$$R = d_0 + d_1[\pi_{13}u_t + \pi_{14}\eta_t + \pi_{15}v_t] + d_2y_{t-1} + d_3[\pi_{20} + \pi_{21}y_{t-1}$$
$$+ \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1} - \pi_{12}p_{t-1}$$
$$- \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t] + u_t - v_t \tag{B.5}$$

and substituting into (4) using equations (2), (7), (8), and (9) gives

$$\pi_{20} + \pi_{21}y_{t-1} + \pi_{22}p_{t-1} + \pi_{23}u_t + \pi_{24}\eta_t + \pi_{25}v_t - \pi_{10} - \pi_{11}y_{t-1}$$
$$- \pi_{12}p_{t-1} - \pi_{13}u_t - \pi_{14}\eta_t - \pi_{15}v_t = c_0 + c_1[\pi_{11}[\pi_{30} + \pi_{31}y_{t-1}$$
$$+ \pi_{32}p_{t-1}] + \pi_{12}[\pi_{10} + \pi_{11}y_{t-1} + \pi_{12}p_{t-1}] - \pi_{11}y_{t-1} \tag{B.6}$$
$$- \pi_{12}p_{t-1} + R] + c_2[\pi_{30} + \pi_{31}y_{t-1} + \pi_{32}p_{t-1} + \pi_{33}u_t$$
$$+ \pi_{34}\eta_t + \pi_{35}v_t] + \eta_t.$$

Using equations (B.4), (B.5), and (B.6) the coefficients on the disturbance terms and π_{30}, π_{31}, and π_{32}, which are all the same as in the basic model, can be determined easily. Using equations (B.5) and (B.6) it can be determined that π_{12} can be equal to 0 or 1. With $\pi_{12}=0$ it is also true that $\pi_{22}=0$. Since $\pi_{32}=0$ also, this would be the same as not including p_{t-1} as a predetermined variable, therefore it is assumed that $\pi_{12}=1$ is the relevant solution. With this result (B.5) and (B.6) can easily be solved for the other coefficients by straightforward substitution. The solutions are (using Ψ_{ii} for solved values)

$$\Psi_{10} = [1/d_3c_1]\{R(1-d_3c_1)-d_0-d_3(c_0+c_2a_0)+d_3c_1a_0[(d_2+d_3c_2a_2)/(d_3c_1a_2)]\}$$

$$\Psi_{11} = -(d_2+d_3c_2a_2)/(d_3c_1a_2) \qquad \Psi_{12} = 1$$

$$\Psi_{20} = (R-d_0)/d_3 + \Psi_{10} \tag{B.7}$$

$$\Psi_{21} = (-d_2-d_3c_2a_2-d_2c_1a_2)/(d_3c_1a_2)$$

$$\Psi_{22} = 1 \qquad \Psi_{30} = a_0 \qquad \Psi_{31} = a_2 \qquad \Psi_{32} = 0.$$

These values can be substituted into equations (7), (8), and (9) to give

$$p_t - p_{t-1} = \Psi_{10} + \Psi_{11}y_{t-1} + \text{disturbances} \tag{B.8}$$

$$m_t - p_t = [\Psi_{20} - \Psi_{10}] + [\Psi_{21} - \Psi_{11}]y_{t-1} + \text{disturbances} \tag{B.9}$$

and

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which are similar to equations (11), (12), and (13) although the exact quantitative values are different.

3. Expected inflation with or without p_{t-1} as predetermined

When p_{t-1} is not included as a predetermined variable expected inflation can be calculated using (B.3) as

$$E_{t-1}(p_{t+1}-p_t) = \Lambda_{11}(E_{t-1}y_t - y_{t-1})$$

$$= \Lambda_{11}(a_0 + (a_2-1)y_{t-1})$$

or given the required value of R as

$$E_{t-1}(p_{t+1}-p_t) =[[((R-d_0)/d_3)-c_0-c_2a_0-c_1R]/c_1a_0][a_0+(a_2-1)y_{t-1}] \ . \tag{B.11}$$

From the solution to the model when p_{t-1} is included as a predetermined variable, and using (B.8), it follows that

$$E_{t-1}(p_{t+1} - p_t) = \Psi_{10} + \Psi_{11}[(a_0 + a_2)y_{t-1}]$$

$$= [1/d_3c_1][R(1-d_3c_1)-d_0-d_3(c_0+c_2a_0)]$$

$$- [(d_2+d_3c_2a_2)/(d_3c_1)]y_{t-1}$$

which by manipulation can be shown to be the same as equation (B.11) when R is determined by (B.3).

4. With m_{t-1} as a predetermined variable

With m_{t-1} included as a predetermined variable, m_{t-1} would replace p_{t-1} in the reduced-form equations (7), (8), and (9) thus the i2 subscript below refers to m_{t-1}. Proceeding exactly as before the coefficients can be determined as (using γ_{ii} for solution values)

$$\gamma_{10} = [1/d_3c_1]\{R(1-d_3c_1)-d_0-d_3(c_0+c_2a_0)$$

$$-d_3c_1[((R-d_0)/d_3)-a_0[(d_2+d_3c_2a_2-c_1d_2)/(d_3c_1a_2)]]\}$$

$$\gamma_{11} = -(d_2+d_3c_2a_2-c_1d_2)/(d_3c_1a_2) \qquad \gamma_{12} = 1$$

$$\gamma_{20} = (R-d_0)/d_3 + \gamma_{10} \tag{B.12}$$

$$\gamma_{21} = (-d_2-d_3c_2a_2-d_2c_1a_2+c_1d_2)/(d_3c_1a_2)$$

$$\gamma_{22} = 1 \qquad \gamma_{30} = a_0 \qquad \gamma_{31} = a_2 \qquad \gamma_{32} = 0.$$

These values can be substituted into equations (7), (8), and (9) to give

$$p_t - m_{t-1} = \gamma_{10} + \gamma_{11}y_{t-1} + \text{disturbances} \tag{B.13}$$

$$m_t - p_t = [\gamma_{20} - \gamma_{10}] + [\gamma_{21} - \gamma_{11}]y_{t-1} + \text{disturbances} \tag{B.14}$$

and
$$y_t = a_0 + a_2y_{t-1} + \text{disturbances} \tag{B.15}$$

If $E_{t-1}p_t$ is to be identical when p_{t-1} or m_{t-1} is included as a predetermined variable then it must be true that

$$\Psi_{10} + \Psi_{11}y_{t-1} + p_{t-1} = \gamma_{10} + \gamma_{11}y_{t-1} + m_{t-1} \tag{B.16}$$

or rearranging terms and substituting from (B.7) or (B.12) that

$$[(a_0d_2/d_3a_2)+(R-d_0)/d_3)]-[d_2/d_3a_2]y_{t-1}=m_{t-1}-p_{t-1} \tag{B.17}$$

would have to hold in both versions of the model with rule (2), i.e., with p_{t-1} or with m_{t-1} as the predetermined variable.

Consider the case when p_{t-1} is included as a predetermined variable, using equations (B.7), and (A.6) for the disturbance terms, and taking in consideration that (B.17) would have to hold for all ex-post values of y_t, m_t, and p_t, by straightforward substitution it can be shown that the disturbance terms would not cancel out of (B.17). Therefore (B.16) does not hold and $E_{t-1}p_t$ is not the same when p_{t-1} is included as a predetermined variable as when m_{t-1} is included.

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