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Exchange Rate Bands with Point Process Fundamentals

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Abstract

This note derives closed form solutions for exchange rates in terms of fundamentals within a fully credible band exchange rate regime when the fundamentals are driven by Brownian motion and multiple point processes. The inclusion of point processes allows one to relax quite substantially the distributional assumptions about exchange rates implicit in models based on Brownian motions alone, and should therefore prove of use in empirical applications. Models with discontinuous driving processes also differ from the Brownian motion model in that monetary authorities will be obliged periodically to intervene on a large scale in discrete amounts.

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1/ I should like to thank Lorenzo Figliuoli, Marcus Miller, Lars Svensson, and Bob Traa for useful discussions. Any errors remain my responsibility.

Table of Contents

List of Figures

Summary

This paper analyzes the behavior of the exchange rate within a band model of exchange rate determination when fundamentals are driven by Brownian motion and multiple point processes. It derives closed form expressions for the exchange rate as a function of fundamentals under two different assumptions about the intervention policy followed by the government.

The inclusion of discontinous components in the driving process is important, first, because it allows one to relax very considerably the stringent restrictions placed by existing models upon exchange rate distributions. This relaxation is likely to prove very useful in empirical work. Second, an important difference between the models developed in this paper and those based on continuous fundamentals is that the authorities will need to intervene periodically on a large scale to prevent realignments.

I. Introduction

The growing literature on band models of exchange rate determination[1] has relied almost exclusively on continuous processes to drive the fundamentals. The only exceptions are papers by Svensson (1989) and (1990) in which a Poisson component is used to model periodic realignments in exchange rate bands. Although the presence of devaluation risk introduces an extra premium into the relationship between fundamentals and exchange rates, it does not significantly change the analysis thanks to Svensson's assumptions, first, that the Poisson process generating the shocks is independent of the Brownian motion term in the fundamentals, and, second, that the band for fundamentals jumps up by exactly the same amount as the fundamentals themselves.

In this paper, we investigate the implications of introducing jump components into the fundamentals while assuming that the band itself is fully credible. Assuming mixed Brownian motion-Poisson process fundamentals, we examine two forms of intervention by the monetary authorities. Under the first, the authorities 'truncate' jumps that would otherwise take fundamentals outside their prescribed band. Under the second, the authorities 'reflect' the discrete jumps in fundamentals back into the band interior. As we argue in Section 4 of the paper, the form of the relation between exchange rates and fundamentals is determined by the 'instantaneous uncertainty' in the driving process. If the rate of jump of a point process is bounded and right continuous, then even if it evolves stochastically over time, this will not contribute to the instantaneous uncertainty associated with the fundamentals. This fact enables us to generalize our basic results for Poisson processes to a very wide class of point processes.

The importance of the results in this paper is two-fold. First, existing models of exchange rate bands place stringent restrictions upon the distributions of discrete time increments in exchange rates. While empirical work in this area is just begining, it is likely that these distributional restrictions will rapidly prove a handicap in the development of models that have a reasonable chance of fitting the data. Statistical studies of financial return distributions in other areas have consistently shown that simple normal distributions fit return data quite poorly.[2] Section 6 of this paper reports the results of estimating a mixed Brownian-Poisson process using weekly data on the sterling-deutschemark exchange rate between 1980 and 1990. Likelihood ratio tests strongly reject the restriction of this model to a purely Brownian motion forcing process. A second contribution of this paper is that it shows how the authorities may be obliged to employ reserves in non-negligible amounts to defend the intervention parities. If news

[1] See, for example, Krugman (1987), Krugman and Rotemberg (1990), Froot and Obstfeld (1990).

[2] See, for example, the classic studies of stock return distributions by Fama (1965) and Mandelbrot (1963). More recent work by Akgiray and Booth (1988) on freely floating exchange rates, Marsh and Rosenfeld (1983) on bond prices and Jarrow and Rosenfeld (1984), Ball and Torous (1985) and Ho, Perraudin and Sorensen (1989) on stock returns suggest that mixed Brownian-Poisson processes provide a better statistical model of financial returns than does Brownian motion with drift.

affecting fundamentals is 'lumpy' or discontinuous then the authorities may have to carry out large-scale interventions if the band is to be maintained.

The empirical results reported in the last Section of this paper suggest interesting results concerning the tradeoff between exchange rate and fundamentals with and without jump processes. From an estimation of the free float sterling-deutschemark exchange rate, it is possible to identify all but one of the parameters of the relationship that would hold between exchange rate and fundamentals in the presence of a credible band. A reasonable value for the remaining free parameter may be chosen on the basis of past empirical work on money demand. Solving the systems of equations characterizing the exchange rate as a function of fundamentals, I show that, at least for this parametrization, the inclusion of jump components improves the tradeoff since the authorities' commitment to offset jumps that would push fundamentals outside their prescribed band serves to stabilize the exchange rate even well into the interior of the target zone

II. The Free Float Model

The basic equation of exchange rate determination may be written as:

$$e_t \;=\; k_t \;+\; \alpha \frac{d}{dt} E e_t \tag{1}$$

where e_t and k_t are the log exchange rate and the fundamentals respectively at time t, where α is a constant, and where the expression $\frac{d}{dt} E_t e_t$ denotes the quantity $\lim_{\delta \to 0^+} E_t \frac{e_{t+\delta} - e_t}{\delta}$.[1] Now, suppose that fundamentals are the solution to the following stochastic differential equation:

$$dk_t \;=\; \beta_0 dt \;+\; \beta_1 dW_t \;+\; \beta_2 dN_{1t} \;+\; \beta_3 dN_{2t} \tag{2}$$

Here,the β_i $i = 0, 1, ., 3$ are fixed constants, $\{W_t\}_{t=0}^{\infty}$ is a standardized Brownian motion and suppose initially that the $\{N_{it}\}_{t=0}^{\infty}$ $i = 1, 2$ are Poisson processes with rates of jump λ_i $i = 1, 2$. We shall discuss in a later section what are the implications if the

[1] This equation may be derived from a simple monetary model of exchange rate determination. Suppose that the logarithms of money demand in the domestic economy and the rest of the world are equal to: $m_t = p_t + \gamma_1 y_t + \gamma_2 i_t - v_t$ and $m_t^* = p_t + \gamma_1 y_t^* + \gamma_2 i_t^* - v_t^*$, where p_t and y_t are the logs of the price level and output in the domestic country, i_t is the nominal interest rate and v_t represents velocity. Also, γ_i $i = 1, 2$ are fixed parameters, and $*$ denotes the corresponding variable for the rest of the world. Suppose that Purchasing Power Parity (P.P.P.) holds, i.e. $p_t = e_t + p_t^*$. Uncovered interest rate parity implies that $i_t - i_t^* = \frac{d}{dt} Ee_t$. Substituting in this last expression for the interest rate differential and using the P.P.P condition, one may derive equation 1 in the text where the 'fundamentals' k_t are defined as: $k_t \equiv \gamma_1(y_t^* - y_t) + m_t - m_t^* + v_t - v_t^*$, and where $\alpha = -\gamma_2$.

$\{N_{it}\}_{t=0}^{\infty}$ $i = 1,2$ are more general point processes. By a generalized version of Ito's lemma (see Elliott (1982) Theorem 12.19):

$$\begin{aligned} de_t \equiv dG(k) &= \beta_0 G'(k_t)dt + \beta_1 G'(k_t)dW_t + \frac{\beta_1^2}{2}G''(k_t)dt + \\ &\alpha[G(k_t+\beta_2) - G(k_t)]dN_{1t} + \alpha[G(k_t+\beta_3) - G(k_t)]dN_{2t} \end{aligned} \tag{3}$$

Taking expectations and substituting in equation (1) yields:

$$G(k_t) = k_t + \alpha\beta_0 G'(k_t) + \alpha\frac{\beta_1^2}{2}G''(k_t) + \alpha[G(k_t+\beta_2)-G(k_t)]\lambda_1 + \alpha[G(k_t+\beta_3)-G(k_t)]\lambda_2 \tag{4}$$

The difference between this equation and the corresponding result without jump components is the presence of the difference terms on the right hand side. In many stochastic process applications, the difference terms that result from the inclusion of jump components mean that only numerical solutions are possible.[1] In this case, however, the fact that the solution for the continuous case includes just linear and exponential terms means that a simple analytical solution is possible. To see this, guess the solution:

$$G(k_t) = A_0 k_t + A_1 + A_2 \exp(\xi_1 k_t) + A_3 \exp(\xi_2 k_t) \tag{5}$$

Taking derivatives of $G(.)$ and substituting in equation (4) yields a linear expression involving constants and terms in k_t, and $\exp(\xi_i k_t)$ $i = 1,2$. Equating coefficients leads to the system:

$$A_0 = 1 \tag{6}$$

$$A_1 = \alpha A_0(\beta_0 + \beta_2\lambda_1 + \beta_3\lambda_2) \tag{7}$$

$$A_2\left(\frac{1}{\alpha} + \lambda_1 + \lambda_2\right) = A_2\left(\lambda_1\exp(\xi_1\beta_2) + \lambda_2\exp(\xi_1\beta_3) + \beta_0\xi_1 + \frac{\beta_1^2}{2}\xi_1^2\right) \tag{8}$$

$$A_3\left(\frac{1}{\alpha} + \lambda_1 + \lambda_2\right) = A_3\left(\lambda_1\exp(\xi_2\beta_2) + \lambda_2\exp(\xi_2\beta_3) + \beta_0\xi_2 + \frac{\beta_1^2}{2}\xi_2^2\right) \tag{9}$$

These equations determine the parameters A_0, A_1, ξ_1 and ξ_2. If one takes the solution that implies no bubbles for the free float exchange rate, one has A_0 at unity, A_1 equaling the total drift in the process including the mean increase in fundamentals due to the jumps, and $A_2 = A_3 = 0$. Thus, the logarithm of the free float exchange rate itself is equal to:

[1] For example, solving the Fokker-Plank equation for the conditional density of a process is often more difficult when the underlying forcing process has jump components.

$$e_t = k_t + \alpha(\beta_0 + \beta_2\lambda_1 + \beta_3\lambda_2) \tag{10}$$

The only difference between this expression and the standard formula for the free float exchange rate with Brownian motion fundamentals is the intercept term which here includes not just the drift parameter, β_0, but also the per period rates of jump weighted by their respective jump sizes. Together, the bracketed terms on the right hand side comprise the total conditionally expected growth in the fundamentals per unit of time. One might note that this free float solution corresponds to the free float solution obtained by Svenson (1989). In Svensson's study Poisson components are used to model the idea of periodic realignments of the exchange rate. Svensson assumes that both fundamentals and intervention levels jump up or down at the same instant. By contrast, in our model, jumps represent shifts in the position of fundamentals *relative to* the intervention levels. Not surprisingly, in the free float case without bubbles, the two approaches are identical. As we show in the next section, however, when bands are present the solutions obtained differ substantially.

III. The Case with Band Intervention

Now, suppose that the authorities intervene to defend upper and lower levels for fundamentals denoted by $\overline{k}$ and $\underline{k}$. The question immediately poses itself: how do the authorities react to jumps in the fundamentals that without intervention would take them outside the range $[\underline{k}, \overline{k}]$? Various approaches are possible. In the next section, we shall consider the case in which fundamentals are 'reflected back' into the interior of the band by the authorities' actions. From an analytical point of view, that approach turns out to have important advantages. In this section, however, we concentrate on the case in which the authorities 'truncate' the jumps. In other words, when fundamentals are less than β_2 in distance from $\overline{k}$ then, in the event of a jump in N_{1t}, the authorities intervene so that the jump size after intervention is $\overline{k} - \beta_2$. The authorities perform a similar truncation of downward jumps near the $\underline{k}$ barrier. In this case, the fundamentals process after the discrete interventions associated with fundamental jumps may be described as follows:

$$\begin{aligned}
\text{Case H} : dk_t &= \beta_0 dt + \beta_1 dW_t + (\overline{k} - k_{t-})dN_{1t} + \beta_3 dN_{2t} \quad , \; k_{t-} \in [\overline{k} - \beta_2, \overline{k}] && (11)\\
\text{Case M} : dk_t &= \beta_0 dt + \beta_1 dW_t + \beta_2 dN_{1t} + \beta_3 dN_{2t} \quad , \; k_{t-} \in [\underline{k} - \beta_3, \overline{k} - \beta_2] && (12)\\
\text{Case L} : dk_t &= \beta_0 dt + \beta_1 dW_t + \beta_2 dN_{1t} + (\underline{k} - k_{t-})dN_{2t} \quad , \; k_{t-} \in [\underline{k}, \underline{k} - \beta_3] && (13)
\end{aligned}$$

where the superscripts h, m and l indicate high, middle and low ranges within the normal band $[\underline{k}, \overline{k}]$. Note that we assume here that $\beta_2 > 0$, $\beta_3 < 0$ and that $\beta_2 - \beta_3 > \overline{k} - \underline{k}$. It is straightforward to alter the analysis to treat the case in which both Poisson processes jump up. For example, if $\beta_i > 0\; i = 1,2$ and, without loss of generality, $\beta_1 \geq \beta_2$, then there are again three ranges but this time given by $[\underline{k}, \overline{k} - \beta_1]$, $[\overline{k} - \beta_1, \overline{k} - \beta_2]$, and $[\overline{k} - \beta_2, \overline{k}]$. A system resembling that of the free float solution, i.e. equations (6)-(9), holds in the low range. In the middle range, jumps in N_{1t} are truncated at the edge of the band, $\overline{k}$, while in the upper range, jumps in either N_{1t} or N_{2t} are truncated. Introducing additional jump processes, $\{N_{it}\}_{t=0}^{\infty}\; i = 3,4,..,I$ with positive or negative coefficients, $\beta_i\; i = 3,4,..,I$, simply results in $I-2$ additional ranges, each with its own system of equations. Returning to the case with a single positive and a single negative jump, one obtains the following three systems. For Case H, we have:

$$
\begin{aligned}
A_0^h &= 1 - \alpha\lambda_1 A_0^h && (14)\\
A_1^h &= \alpha(\; A_0^h\beta_0 + A_0^h\overline{k}\lambda_1 + A_0^h\beta_3\lambda_2 + A_2^h\exp(\xi_1^h\overline{k})\lambda_1 + A_3^h\exp(\xi_2^h\overline{k})\lambda_1\;) && (15)\\
A_2\frac{1}{\alpha} &= A_2\left(-\lambda_1 - \lambda_2 + \lambda_2\exp(\xi_1^h\beta_3) + \beta_0\xi_1^h + \frac{\beta_1^2}{2}(\xi_1^h)^2\right) && (16)\\
A_3\frac{1}{\alpha} &= A_3\left(-\lambda_1 - \lambda_2 + \lambda_2\exp(\xi_2^h\beta_3) + \beta_0\xi_2^h + \frac{\beta_1^2}{2}(\xi_2^h)^2\right) && (17)
\end{aligned}
$$

For case M, the solution is of the same form as the system of free float equations (6)-(9), (i.e. in (6)-(9) one may replace A_0, A_1, ξ_1 and ξ_2 by A_0^m, A_1^m, ξ_1^m and ξ_2^m). For Case L, the solution is:

$$
\begin{aligned}
A_0^l &= 1 - \alpha\lambda_2 A_0^l && (18)\\
A_1^l &= \alpha(\; A_0^l\beta_0 + A_0^l\underline{k}\lambda_2 + A_0^l\beta_2\lambda_1 + A_2^l\exp(\xi_1^l\underline{k})\lambda_2 + A_3^l\exp(\xi_2^l\underline{k})\lambda_2\;) && (19)\\
A_2\frac{1}{\alpha} &= A_2\left(-\lambda_1 - \lambda_2 + \lambda_1\exp(\xi_1^l\beta_2) + \beta_0\xi_1^l + \frac{\beta_1^2}{2}(\xi_1^l)^2\right) && (20)\\
A_3\frac{1}{\alpha} &= A_3\left(-\lambda_1 - \lambda_2 + \lambda_1\exp(\xi_2^l\beta_2) + \beta_0\xi_2^l + \frac{\beta_1^2}{2}(\xi_2^l)^2\right) && (21)
\end{aligned}
$$

The 3 systems of equations corresponding to the three ranges determine the parameters A_0^j, A_1^j, ξ_1^j, and ξ_2^j for $j = h,m,l$. However, this still leaves us with six undetermined parameters, namely A_2^j, A_3^j for $j = h,m,l$. To see how these parameters are tied down, consider first the behavior of the system in range H.

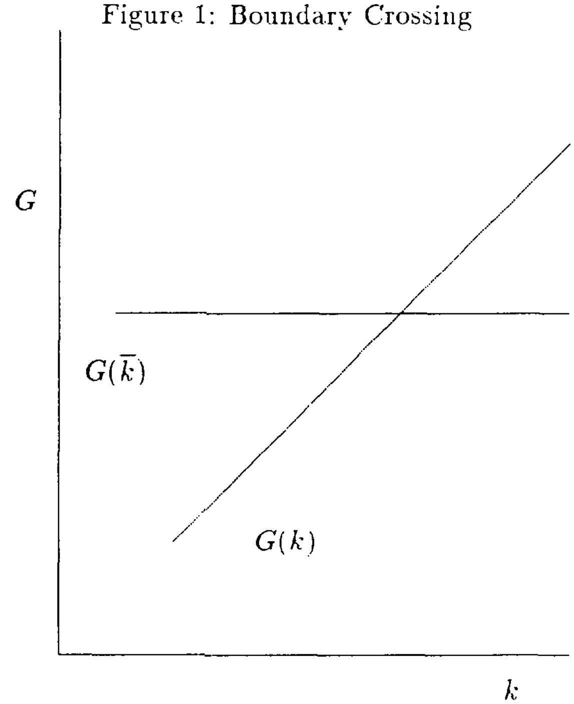
Suppose that fundamentals follow the process given by equation (11) over the whole domain. This is a somewhat odd 'fundamentals' process since it already includes the authorities' 'truncation' of the jumps in the actual fundamentals. Also, if the process holds over the whole real line, then the discontinuous part consisting of jumps back to $\overline{k}$ is quite unrealistic for any plausible fundamentals. However, what is important in this context is that the solution of equation (1) with *hypothetical* fundamentals obeying equation (11) in the presence of a reflecting barrier at $\overline{k}$, is a special case of solutions to the same equation in the absense of such a barrier.

Now, equation (1) may be written as:

$$G(k_t)\delta t \;=\; k_t\delta t \;+\; \alpha E_t(\,G(k_{t+\delta t}) - G(k_t)\,) \;+\; o(\delta t) \tag{22}$$

Suppose that $G'(\overline{k}) > 0$, so that locally, it may be represented as in Figure 1, that is as a (locally) straight line crossing the horizontal at $\overline{k}$.

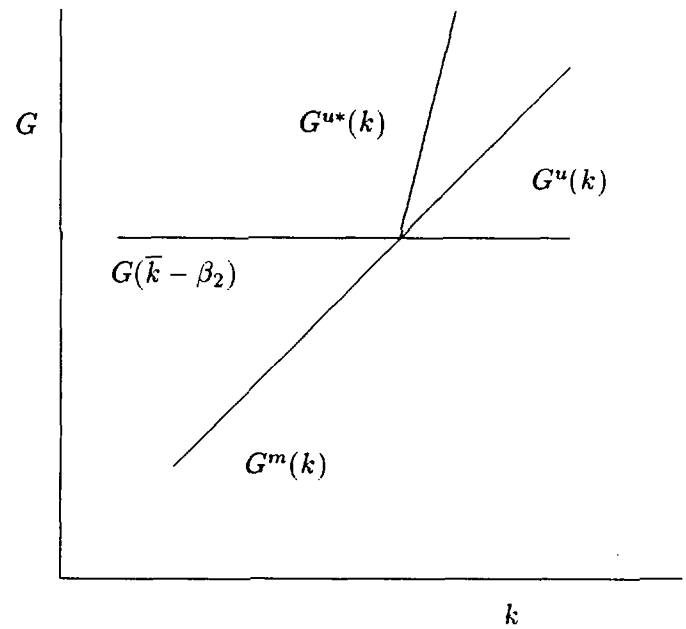
Figure 1: Boundary Crossing



It is clear that $G(\overline{k})$ cannot be an equilibrium in *both* the free float (locally linear) case *and* in the presence of the band, since in the latter case there is zero probability of fundamentals rising above $\overline{k}$. Equilibrium can only hold if $G'(\overline{k}) = 0$. A similar argument holds for $G'(\underline{k})$. We, therefore, have the boundary conditions $G'(\overline{k}) = 0$ and $G'(\underline{k}) = 0$.

Of the four remaining parameters, two are required to maintain continuity of $G(.)$ at $\overline{k} - \beta_2$ and $\underline{k} - \beta_3$, and two are needed to ensure the continuity of $G'(.)$ at the same two points. The fact that G' must be continuous at the boundaries between the three ranges may be seen from Figure 2.

Figure 2: Internal Boundary Crossing



Here, G^u is the differentiable and hence locally-linear extension of G^m into the range above $\bar{k} - \beta_2$. Given Jenson's inequality, it is not possible for $G^m(\bar{k})$ to be a particular solution to the exchange rate equation both *with* and *without* the shift in the stochastic regime unless the derivatives $G^{u'}$ and $G^{m'}$ are equal at $\bar{k} - \beta_2$. Otherwise, G would be kinked at $\bar{k} - \beta_2$ which would imply that it would be convex even locally at this point. A similar argument may be applied to G at $\underline{k} - \beta_3$.

IV. Reflected Jumps

An alternative assumption about the authorities' policy in the event of a jump in fundamentals is to suppose that, instead of truncating the jump at the edge of the band, they actually reflect fundamentals back in towards the center. Such a policy resembles to some degree the discrete interventions investigated by Flood and Garber (1989).

In Flood and Garber, fundamentals are driven by a Brownian motion but when the exchange rate hits the edge of the target zone, the authorities shift fundamentals so the exchange rate jumps to some predetermined point within the zone. Here we shall assume that the authorities' intervention reflects the fixed size jumps so that if the process is at some fraction, q, of its fixed jump size β_1 from the upper boundary, then after the jump occurs, it will be at a point $(1-q)\beta_1$ from the edge of the zone.

We adopt this assumption partly because of its analytical tractability, but one may justify it by the argument that the authorities react differently to jumps that take place close to the boundary compared with those occuring further away. Thus, the closer fundamentals are to the edge of the zone, the further they will push them into the interior of the band.

When the authorities' intervention policy follows this pattern, the process for fundamentals after the intervention associated with jumps has taken place, will be of the form:

$$\begin{aligned}
\text{Case H} : dk_t &= \beta_0 dt + \beta_1 dW_t + (2\overline{k} - 2k_{t-} - \beta_2)dN_{1t} + \beta_3 dN_{2t}\,,\ k_{t-} \in [\overline{k} - \beta_2, \overline{k}] && (23)\\
\text{Case M} : dk_t &= \beta_0 dt + \beta_1 dW_t + \beta_2 dN_{1t} + \beta_3 dN_{2t}\,,\ k_{t-} \in [\underline{k} - \beta_3, \overline{k} - \beta_2] && (24)\\
\text{Case L} : dk_t &= \beta_0 dt + \beta_1 dW_t + \beta_2 dN_{1t} + (2\underline{k} - 2k_{t-} - \beta_3)dN_{2t}\,,\ k_{t-} \in [\underline{k}, \underline{k} - \beta_3] && (25)
\end{aligned}$$

it is a straightforward exercise to derive the systems of equations for the parameters A_i^j, ξ_k^j for $i = 0,1,2,3$, $k = 1,2$ and $j = h, m, l$ that this system implies given equation (4), guessing a solution of the form of equation (5), and then equating coefficients and using value matching and smooth pasting conditions.

V. More General Point Processes

This section shows how one can extend the above results to the case in which fundamentals contain much more general point processes than the Poisson processes employed above. It turns out that the form of the solution in band models of this kind depends almost entirely upon the continuous part of the driving process. Hence, one may, for example, introduce autocorrelation and heteroskedasticity into the fundamentals process by way of a discontinuous component.

It is helpful to adopt a more precise terminology than that employed up to now. Define a probability space by the triple $(\Omega, \mathcal{F}, P)$. Suppose that fundamentals are driven as before by a Brownian motion $\{W_t\}_{t=0}^{\infty}$ and two point processes adapted to a filtration $\mathcal{F}_t$. The point processes are defined as sequences of random variables $\{T_i\,,\ i = 1,2,..\}$ on $(\Omega, \mathcal{F}, P)$ such that $T_i(\omega) < T_{i+1}(\omega)$ for all $\omega \in \Omega$, $i = 1,2,...$ The counting processes associated with these sequences may be written as: $N_{it} \equiv \sum_{n\geq 1} 1(T_n \leq t)$, $i = 1,2$. Assume that these point processes are non-explosive (i.e. they cannot jump an infinite number of times within a finite period).

Assume that each point process has an $\mathcal{F}_t$-progressive stochastic intensity, i.e. a nonnegative $\mathcal{F}_t$-progressive process λ_{it}, such that, for all $n \geq 1$, $N_{i,t\wedge T_n} - \int_0^{t\wedge T_n} \lambda_{is} ds$ $i = 1, 2$ is a martingale.

Given these definitions, it is easy to repeat the arguments of Section 2. Theorem 12.19 of Elliott again yields an equation of the form of (3). Given that $\int[G(k_s + \beta_{1+i}) - G(k_s)](dN_{is} - \lambda_{is}ds)$, $i = 1, 2$ are martingales (see Bremaud (1981), Theorem T9), and if one assumes that λ_{it}, $i = 1, 2$ are bounded and right continuous, one may use the Lebesgue averaging and dominated convergence theorems to show (see Bremaud (1981) page 28) that equation (4) holds as before.

Clearly, if the intensities, λ_i $i = 1, 2$ just have to be bounded, right continuous $\mathcal{F}_t$-progressive processes to yield equations of the form of (4), this leaves considerable flexibility for empirical applications. For example, in a study using daily data, one could allow λ_{it} $i = 1, 2$ to be functions, say, of the change in the previous day's exchange rate. Alternatively, using longer frequency data, one could think of λ_{it} $i = 1, 2$ as being driven by other observable processes such as trading volume, real economic indicators or time.

VI. An Application

To investigate further the implications of incorporating jump processes into a band model, one may solve the systems of equations characterizing the solution, given in Section 3. To make the exercise more interesting, the model was parametrized by estimating a stochstic differential equation of the form:

$$de_t = \beta_0 dt + \beta_1 dW_t + \beta_2 dN_{1t} + \beta_3 dN_{2t} \tag{26}$$

where $\{W_t\}_{t=0}^{\infty}$ and $\{N_{it}\}_{t=0}^{\infty}$ $i = 1, 2$ indicate, as before, a standard Brownian motion and Poisson processes with constant rates of jump, λ_i $i = 1, 2$. The data used was the sterling-deutschemark exchange rate at each Wednesday closing from January 1980 to July 1990.The number of observations was 551.

Assuming that this data is well-described by a free-float model with fundamentals driven by a Brownian motion and two Poisson processes, one can identify all the parameters of a credible band model like that of Section 3, with the exception of α. In other words, one can determine from the behavior of the free float exchange rate, how the exchange rate would behave if the British authorities introduced a band against the deutschemark that the markets found credible. The one free parameter, α, may be thought of as minus the interest elasticity of money demand. Choosing a reasonable value for α (the literature on money demand functions suggests such a reasonable figure would be around 9)[1], and employing equations (6)-(9), (14)-(21) and the smooth pasting and value matching boundary conditions, one can obtain a function $G(.)$.

The results of the maximum likelihood estimation are given in Table 1. As one may see, the drift parameter is close to zero so that while its standard error is in line with that of other parameters, the t-statistic is not significant. Most of the mean change in the series is due to the jump components. While the estimated jump sizes are close to each other in absolute size the positive jump tends to occur more frequently. The estimated rates of jump suggest that jumps occur every 2 1/2 to 3 weeks. Finally, the instantaneous standard deviation parameter on the Brownian motion is large and very significantly different from zero.[2]

Table 1 also provides likelihood ratio statistics for the presence of one or two Poisson components in the free float exchange rate. The restriction of the two jump process model to a one jump process model is overwhelmingly rejected, as is the restriction from a one jump to a no jump model.[3] These tests show the importance of generalizing the model from the basic single Brownian motion driving process of, for example, Krugman (1987) if one is to do serious empirical work in this area.

Figure (4) shows the solutions for the base assumption of $\alpha = 9$ in cases with and without jump components. The solution without jump components supposes that the other parameters (i.e. the drift term and the coefficient on the Brownian motion) are as estimated in the model with jumps. Figures (3) and (5) give the corresponding solutions for the cases in which $\alpha = 4.5$ and $\alpha = 13.5$ respectively.

A consistent pattern in these plots is that the trade off between exchange rate and fundamentals in the presence of the band becomes more favourable when one includes jump terms and as money demand becomes more elastic. To explain this phenomenon, recall, firstly, that $G(k) = A_0k + A_1 + A_2\exp(\xi_1 k) + A_3\exp(\xi_2 k)$, and, secondly, that $A_0 = 1$ in the free float and in the middle zone of the band solution, while $A_0 = 1/(1+\alpha\lambda_1)\,,\, 1/(1+\alpha\lambda_2)$ in the upper and lower zones. A large value for α in the presence of jump components thus improves the 'direct' linear trade off between e and k.

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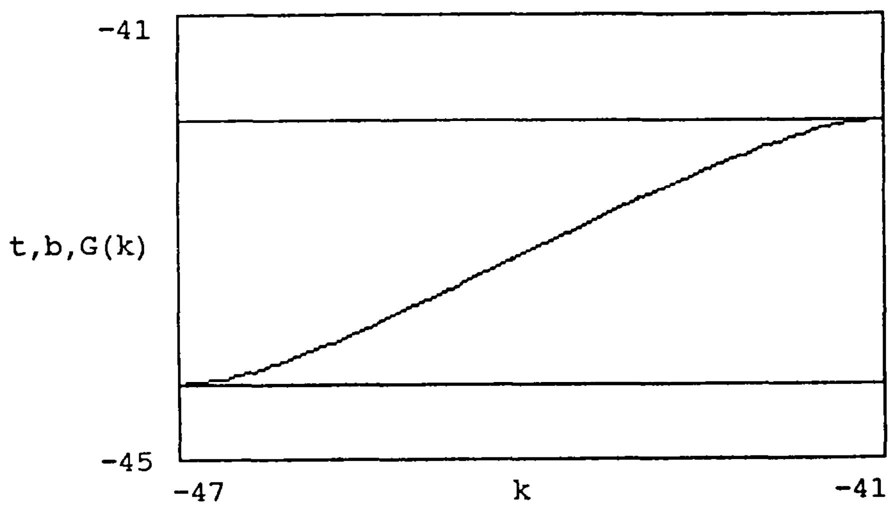
[1] See Boughton (1990). Boughton estimates a semi-elasticity of demand for UK M1, with respect to short term interest rates of 0.09. Given the use of the uncovered interest arbitrage formula in the derivation of the basic exchange rate equation in footnote 3 above, it is necessary to express interest rates so that, for example, a 10% rate equals 0.1. In this case, Boughton's figure becomes 9

[2] These results resemble those of other studies that have modeled financial return data with mixed Brownian-Poisson processes. Drift parameters are generally difficult to estimate precisely, while coefficients on Brownian terms tend to be large and highly significant (see Ho, Perraudin and Sørensen (1989) and the references cited therein).

[3] Note that the correct distribution for the likelihood ratio statistic when parameters are not identified under the null is hard to determine (see Ho, Sørensen and Perraudin (1989) for a discussion of this issue in the context of ML estimation of mixed Brownian-Poisson processes). A reasonably safe expedient, however, is to take a χ^2 distribution with degrees of freedom equal to the total number of parameters associated with the jump components.

Figure 3. The Exchange Rate Solution for α=4.5

(With jumps)



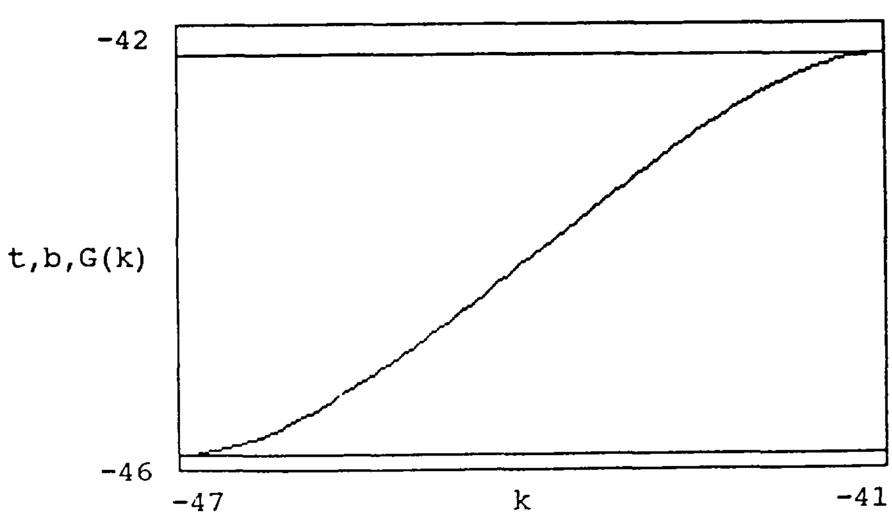
G(ktg) = -41.952

G(ktg - b2) = -42.903

G(kbg - b3) = -43.367

G(kbg) = -44.321

(Without jumps)



G(ktg) = -42.275

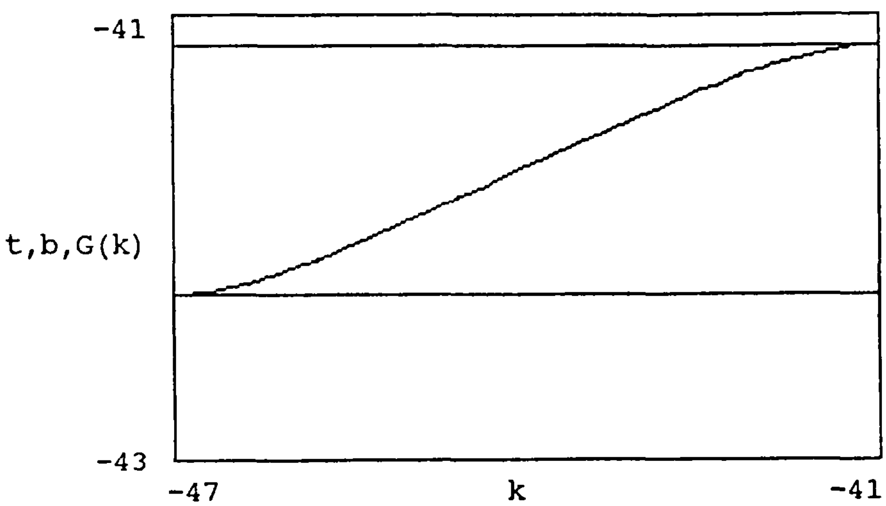
G(ktg - b2) = -43.73

G(kbg - b3) = -44.487

G(kbg) = -45.854

Figure 4. The Exchange Rate Solution for α=13.5

(With jumps)



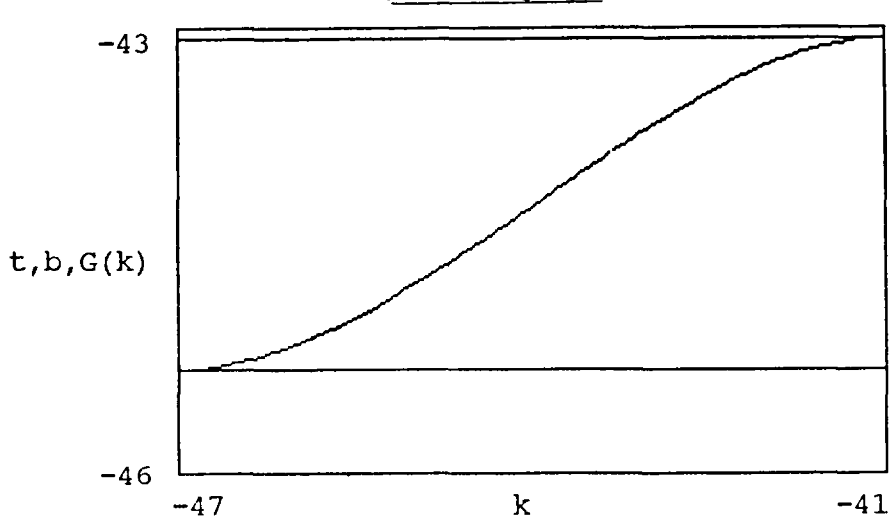
G(ktg) = -41.139

G(ktg - b2) = -41.584

G(kbg - b3) = -41.807

G(kbg) = -42.257

(Without jumps)



G(ktg) = -43.066

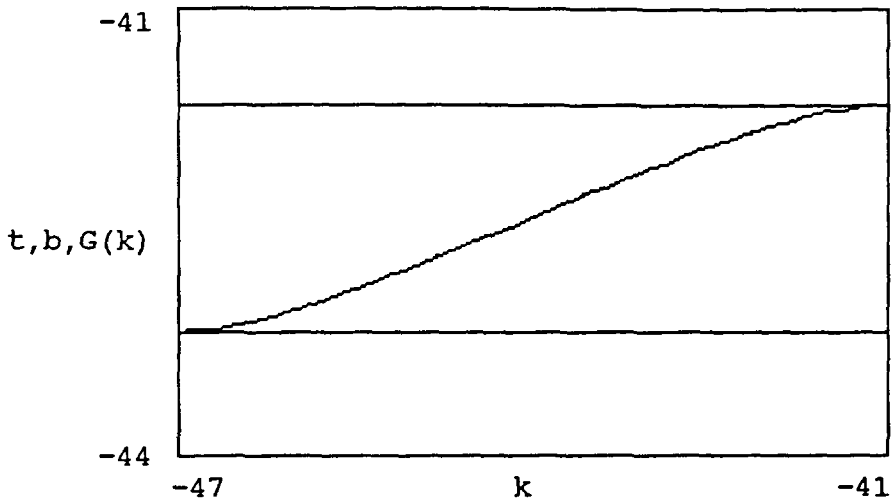
G(ktg - b2) = -43.962

G(kbg - b3) = -44.455

G(kbg) = -45.289

Figure 5. The Exchange Rate Solution for α=9

(With jumps)



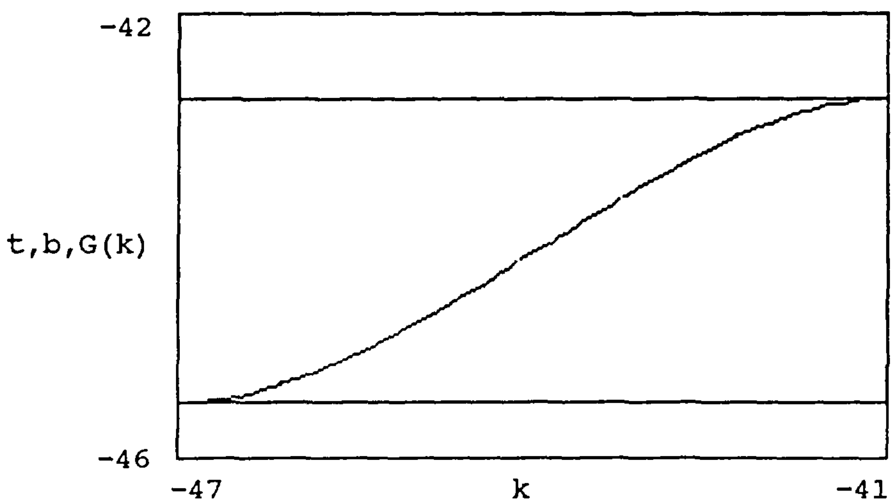
G(ktg) = -41.642

G(ktg - b2) = -42.248

G(kbg - b3) = -42.55

G(kbg) = -43.16

(Without jumps)



G(ktg) = -42.757

G(ktg - b2) = -43.863

G(kbg - b3) = -44.463

G(kbg) = -45.498

This effect is in addition to the nonlinearities close to the edge of the band required by smooth pasting. The intuition behind these effects is clearly that, even within the strict interior of the band, the authorities' truncation of jumps affects expected movements in the exchange rate in a stabilizing fashion.

Table 1: ML Estimation Results

Parameter	Value	T-Statistic
Two Jump Process Model		
β_0	-0.01	-0.14
β_1	0.82	21.14
β_2	2.58	7.18
β_3	-2.51	-5.43
λ_1	0.20	5.03
λ_2	0.15	3.55
One Jump Process Model		
β_0	-0.03	-0.76
β_1	0.92	27.69
β_2	2.86	7.26
λ_1	0.03	2.38
Model Without Jump Processes		
β_0	0.05	1.08
β_1	1.04	33.20

Likelihood Ratio Statistics	Value
H0:no jumps, H1:2 jumps	46.76*
H0:1 jump, H1:2 jumps	17.92**
H0:no jumps, H1:1 jump	28.81**

Notes: log exchange rate changes were scaled by 100 to facilitate estimation.
*The relevant 1% significance level for $\chi^2(4)$ is 13.28.
**The relevant 1% significance level for $\chi^2(2)$ is 9.21.

VII. Conclusion

This note has extended existing models of exchange rate determination within a target zone to the case in which the driving process for fundamentals includes discontinuous point process components. Models of this kind have rather different implications for the behavior of the authorities' foreign exchange reserves than those with continuous fundamentals since, in the presence of jump fundamentals, the central bank may have to intervene in large quantities to defend the target zone.

An important field for future research is the empirical estimation and testing of target zone models. The empirical evidence advanced in Section 6 suggests that the stringent restrictions placed upon exchange rate distributions by, for example, the simple Krugman-type Brownian motion exchange rate model are difficult to square with the data. The introduction of Poisson or other point process components may partially remedy this problem since it allows one to derive models in which discretely sampled exchange rate data has quite general distributions.

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