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Alternative Dual Exchange Market Regimes:
Some Steady State Comparisons

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Abstract

This paper compares two alternative dual exchange market regimes. In one of them, the official market clears through changes in international reserves, while in the other regime, the central bank implements a rationing scheme so as to keep international reserves constant. The paper discusses the effects on the rate of inflation, the balance of payments, the real exchange rate, and the spread between the free and the official exchange rate, of various economic policies, including exchange rate policy, fiscal policy, and unification of the exchange markets. It concludes that the steady state effects for most of these policies are qualitatively the same under both regimes.

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<u>Table of Contents</u>		<u>Page</u>
I.	Introduction	1
II.	Alternative Dual Exchange Market Regimes	3
	1. Reserves adjustment model	3
	2. Reserves rationing model	7
III.	Economic Policy Under Dual Exchange Market Regimes	9
	1. Reserves adjustment model	9
	2. Reserves rationing model	10
	3. Comparison across regimes	11
IV.	Unification of the Foreign Exchange Markets	15
V.	Conclusions	20
	Appendix: The Unified Exchange Rate Model	21
	References	24

I. Introduction

A large number of developing countries operate dual exchange market regimes. 1/ The typical arrangement involves an official market in which the exchange rate is determined by the authorities, and a second market in which the exchange rate is determined by market forces. The official market usually channels all the transactions of the public sector and some selected transactions of the private sector, while the second market channels the remaining transactions.

The specific operation the official market varies among countries, particularly with respect to the principles governing central bank sales of foreign exchange to the private sector. The rules for central bank purchases of foreign exchange from the private sector are similar for most countries. The authorities usually specify some type of exports whose proceeds must be surrendered at the official exchange rate, and the central bank buys all the foreign exchange that is offered at that price. In contrast, the rules for central bank sales of foreign exchange to the private sector differ among countries, usually obeying one of the following two basic arrangements. Under one of the arrangements, the authorities specify the type of imports that have access to foreign exchange at the official exchange rate, and the central bank is committed to sell all the foreign exchange that is demanded at that price. In this case, the official market clears through changes in international reserves. Under the alternative arrangement, the authorities protect international reserves by implementing a rationing scheme in the official market. Under this scheme, the central bank first allocates to the public sector the foreign exchange needed for its imports, and then sells to the private sector the remaining proceeds from exports surrendered in the official market. In this way, international reserves remain constant.

The purpose of this paper is to discuss the similarities and differences between these two arrangements with respect to the effects of various economic policies, including the unification of the exchange markets. Although both exchange arrangements have been examined in the literature, sometimes using similar models, their implications have not been explicitly compared. 2/ Furthermore, a transparent and comprehensive comparison based on existing results is difficult, since previous models generally used different assumptions and discussed different sets of

1/ For a description of the exchange regime of a particular country see International Monetary Fund (1990).

2/ Most models assume that the official market clears through changes in international reserves. However, Nowak (1984) argued that in some cases it is more appropriate to assume a rationing mechanism in the official market. This assumption was later used by Pinto (1986, 1990) to examine the experience of African countries, and by Kharas and Pinto (1989) to discuss the Bolivian experience.

policies. Using a uniform model for examining both arrangements allows for a direct comparison of results for a broad range of policies, a comparison that is free from effects due to differences in assumptions.

The discussion in this paper focuses on the effects of various policies on the rate of inflation, on the spread between the free and the official exchange rate, on the real exchange rate, and on the balance of payments, under the two alternative exchange regimes. It also examines the inflationary consequences of unifying the exchange markets, and the consequences for the exchange rate spread of adopting a rationing scheme in an economy that was operating a dual exchange market without rationing. In all these cases, the discussion refers to steady state effects, that is, to effects that take place in the long run, after all real variables have adjusted to their new stationary levels. Consequently, there is no discussion of the dynamics between steady states, which depend crucially on assumptions about the formation of expectations, and on the speed of adjustment in the various markets. The model in this paper is based on similar models that have been used to analyze these exchange arrangements. 1/ The structure of the model is kept as simple as possible, while still allowing for the discussion of the issues under consideration. 2/

The results of the paper indicate that the qualitative effects on inflation, on the spread between the exchange rates, and on the real exchange rate, are the same for both exchange regimes for most of the policies considered. They also indicate that the effect of unification on

1/ There is a variety of models of dual exchange markets in the literature, with differing implications regarding the effects of various economic policies. The discussion in this paper is confined to one particular type of model. For a survey of theoretical results from different models see Lizondo (1990).

2/ The basic model used here was developed in Lizondo (1987a, 1987b) for examining a dual exchange market economy with an official market that clears through changes in international reserves. This model was later modified and extended to examine an economy with an official market subject to rationing by Pinto (1986, 1990) and Kharas and Pinto (1989). The analysis here replicates various of the results presented in those papers, but it also expands the discussion in some directions. One of the basic differences is that the present paper includes private sector and public sector expenditure on both traded and nontraded goods. In contrast, Lizondo (1987a, 1987b) and Pinto (1986) assume that all goods are traded, while Pinto (1990) and Kharas and Pinto (1989) assume that the private sector consumes only nontraded goods and the public sector spends only on traded goods. The assumption in this paper, besides allowing for a discussion of the effects of changes in public sector expenditure on nontraded goods, has some implications that differ from those in the other papers, as will be noted where relevant. Also, this paper examines the effects of adopting a rationing scheme in a dual economy that operated without rationing.

the rate of inflation depends on the spread between the exchange rates for the "reserves rationing" regime, and on the balance of payments and the spread between the exchange rates for the "reserves adjustment" regime. The elasticity of the demand for money is also relevant in both cases. The effect on the spread of switching from a reserves adjustment regime to a reserves rationing regime depends on the balance of payments prior to the switch.

The rest of the paper is organized as follows. Section II presents the model of the economy, and obtains the conditions for steady state equilibrium for both exchange regimes. Section III derives the effects of a variety of policies, and compares those effects across dual exchange market regimes. Section IV examines the inflationary consequences of unifying the exchange markets. It also discusses the effects of adopting a rationing scheme in a dual exchange market economy whose official market was clearing through changes in international reserves. Finally, section V summarizes the main conclusions of the paper.

II. Alternative Dual Exchange Market Regimes

The structure of the economy is essentially the same for both dual exchange market regimes, the only difference being the rule governing central bank sales of foreign exchange to the private sector. This section discusses first the reserves adjustment model, presenting simultaneously the basic structure of the economy, and then the reserves rationing model.

1. Reserves adjustment model

There are two foreign exchange markets: the official market, which channels public sector transactions and some private sector trade transactions, and the free market, which channels the remaining transactions. The exchange rate in the official market is denoted by e , and is depreciated by the authorities at a constant rate π , while the exchange rate in the free market is denoted by b , and is determined by market forces. ^{1/}

The private sector produces and consumes traded and nontraded goods. Production of nontraded goods, denoted by y_n , is fixed. Production of traded goods, denoted by y_t , is fixed and evenly distributed across a continuum of goods indexed from zero to unity. Following the official allocation of transactions between markets, goods indexed from zero to v can be exported through the free market, while the rest of traded goods must be

^{1/} The exchange rate is defined as the amount of domestic currency equivalent to one unit of foreign currency.

exported through the official market. 1/ Private sector preferences are defined by a Cobb-Douglas utility function, with a share α of expenditure devoted to traded goods, and a share $(1-\alpha)$ devoted to nontraded goods. Private sector expenditure on traded goods is distributed evenly across a continuum of traded goods, indexed from zero to unity. All traded goods consumed by the private sector are imported. Following the official allocation of transactions between markets, goods indexed from zero to u must be imported through the free market, while the rest of traded goods can be imported through the official market. The public sector consumes fixed quantities of traded and nontraded goods.

There are two noninterest bearing assets, domestic money, denoted by M and foreign money, denoted by f . The demand for domestic money, henceforth referred to as the demand for money, depends on the expected rate of depreciation of the free market exchange rate 2/

$$(1) \quad m = \lambda(b/b) = \lambda[(s/s)+\pi] \quad \lambda' < 0$$

where $m=(M/P)$ is the real quantity of money, P is the price level, and $s=(b/e)$ is equal to one plus the spread between the free and the official exchange rates. 3/ Assuming that the country is small in the world market for traded goods, and that foreign inflation is nil, the foreign currency price of traded goods can be set equal to unity. As a result, the domestic currency price of traded goods is equal to the official exchange rate for goods imported through the official market, and to the free exchange rate for other traded goods. The domestic price level is thus given by

$$(2) \quad P = p_n^{1-\alpha} e^{\alpha(1-u)} b^{\alpha u} = e p^{1-\alpha} s^{\alpha u}$$

1/ The model does not include overinvoicing and underinvoicing of transactions, so the private sector is assumed to comply with the official allocation between markets.

2/ The implications of the model would be essentially the same if the demand for money is assumed to depend also on the level of wealth, as in Lizondo (1987a), Pinto (1986, 1990), and Kharas and Pinto (1989). However, the simpler formulation used in this paper permits a sharper comparison across regimes.

3/ Expected and actual rates of depreciation are taken to be equivalent since the discussion is centered on the steady state results.

where p_n denotes the price of nontraded goods and $p = (p_n/e)$ is the real exchange rate, defined as the relative price between nontraded goods and traded goods channeled through the official market. 1/

Private sector total real expenditure is assumed to be a constant fraction ω of real wealth, and so nominal expenditure is a constant fraction ω of nominal wealth. As a result, the condition for equilibrium in the market for nontraded goods, is given by

$$(3) \quad y_n = g_n + (1-\alpha) \omega (m p^{-\alpha} s^{\alpha u} + f p^{-1} s)$$

where g_n is public sector consumption of nontraded goods, and the second term on the right hand side of (3) is private sector consumption of nontraded goods. 2/

While output must equal demand in the case of nontraded goods, this condition needs not hold for traded goods, in any of the foreign exchange markets. The difference between traded goods exported and imported through the free market is equal to private sector accumulation of foreign money.

$$(4) \quad \dot{f} = v y_t - \alpha u \omega (m p^{1-\alpha} s^{\alpha u-1} + f)$$

The difference between traded goods exported and imported through the official market is equal to the accumulation of official reserves.

$$(5) \quad \dot{r} = (1-v) y_t - g_t - \alpha (1-u) \omega (m p^{1-\alpha} s^{\alpha u} + f s)$$

where r denotes official reserves in terms of foreign currency, and g_t denotes public sector consumption of traded goods.

1/ With the real exchange rate defined this way, an increase in p indicates a real appreciation, while a decline in p indicates a real depreciation, of the official exchange rate. The discussion focuses on the behavior of the real exchange rate defined by using the nominal official exchange rate because this is one of the variables typically examined when assessing the adequacy of official exchange rate policy. In any case, the behavior of the real exchange rate defined by using the nominal free exchange rate $(p_n/b) = p s^{-1}$, is also indicated in the paper.

2/ The expression for private sector consumption of nontraded goods is obtained by dividing nominal private sector expenditure on nontraded goods $(1-\alpha) \omega (M+bf)$, on the price of nontraded goods, and rearranging terms. Similar reasoning applies to private sector consumption of traded goods.

Assuming that changes in domestic credit are used to finance the public sector budget deficit, and that tax revenues are fixed in real terms

$$(6) \quad \dot{D} = e \dot{g}_t + p_n \dot{g}_n - P \dot{t}$$

where D denotes domestic credit in nominal terms, and t denotes tax revenues in real terms. Assuming for simplicity that the banking system is composed only of a central bank, the change in domestic money is equal to

$$(7) \quad \dot{M} = e \dot{r} + \dot{D}$$

Using equations (5), (6), and (7), the change in the real quantity of domestic money is given by

$$(8) \quad \dot{m} = (1-v) y_t p^{\alpha-1} s^{-\alpha u} - \alpha(1-u) \omega (m+f p^{\alpha-1} s^{1-\alpha u}) + g_n p^\alpha s^{-\alpha u} - t - (P/P) \dot{m}$$

In steady state $\dot{m} = \dot{f} = \dot{s} = 0$, and all prices are increasing at the rate of crawl of the official exchange rate, π . From equations (1), (4), and (8), the conditions for steady state are equation (3), and the following equations

$$(9) \quad m = \lambda(\pi)$$

$$(10) \quad v y_t = \alpha u \omega (m p^{1-\alpha} s^{\alpha u-1} + f)$$

$$(11) \quad (1-v) y_t p^{\alpha-1} s^{-\alpha u} = \alpha(1-u) \omega (m+f p^{\alpha-1} s^{1-\alpha u}) - g_n p^\alpha s^{-\alpha u} + t + \pi m$$

Equation (3) indicates equilibrium in the nontraded goods market. Equation (9) indicates equilibrium in the money market when relative prices are constant and the rate of inflation is equal to the rate of crawl of the official exchange rate. Equation (10) indicates that private sector exports and imports through the free market must be equal for private sector holdings of foreign exchange to remain constant. Equation (11) indicates that the factors leading to monetary expansion must be offset by those leading to monetary contraction for private sector real holdings of domestic money to remain constant. Monetary expansion originates from exports channeled through the official market, and from public sector expenditure on

nontraded goods. Monetary contraction arises from private sector imports channeled through the official market, from private sector tax payments, and from the depreciation of the real stock of money due to inflation.

Using (5) and (11), the steady state balance of payments is given by

$$(12) \dot{r} = (m\pi + \tau) p^{1-\alpha} s^{\alpha u} - g_t - g_n p$$

Using (2), this equation can be rewritten as:

$$(13) \dot{r} (e/P) = m\pi + \tau - g_t (e/P) - g_n (p_n/P)$$

so that the steady state balance of payments will be in surplus or deficit depending on whether the steady state inflation tax is higher or lower than the "official" public sector budget deficit, where g_t is valued at the official exchange rate. 1/

2. Reserves rationing model

Under a reserves rationing regime, prices of all traded goods are likely to reflect the exchange rate in the free market. 2/ Since private importers are not facing a perfectly elastic supply of foreign exchange from the central bank for the importation of certain goods through the official market, there is no competition between potential importers that would constrain the domestic price of those goods to equal the official exchange rate. Therefore, those who obtain foreign exchange at the official exchange rate receive a rent that they can collect by selling the imported goods at the free exchange rate, which is the real cost for other potential competitors. The price level is thus given by

$$(14) P = p_n^{1-\alpha} b^\alpha = e p^{1-\alpha} s^\alpha$$

1/ The definition of steady state equilibrium must be interpreted in a broad sense in the reserves adjustment model. A situation of balance of payments deficit is not sustainable in the long run, and thus cannot be a steady state equilibrium in the strict sense. Policies will eventually have to be modified, or the exchange rate system will have to be abandoned. However, a situation in which all real variables other than international reserves remain constant, seems the closest possible approximation to the strict steady state equilibrium used as base for comparison in the other exchange regimes. This qualification must be kept in mind when interpreting the results regarding the "steady state" effects of the various policies.

2/ This is the assumption made in Pinto (1986, 1990), and Kharas and Pinto (1989).

The condition for equilibrium in the nontraded goods market becomes

$$(15) \quad y_n = g_n + (1-\alpha) \omega (m p^{-\alpha} s^{\alpha} + f p^{-1} s)$$

International reserves are constant, since the central bank, after setting aside foreign exchange for public sector imports, sells back to the private sector the remaining exports proceeds surrendered at the official market. Therefore, the change in the real quantity of money is given by

$$(16) \quad \dot{m} = g_t p^{\alpha-1} s^{-\alpha} + g_n p^{\alpha} s^{-\alpha} - t - (P/P) m$$

Since international reserves are constant, the accumulation of foreign exchange by the private sector is equal to total production of traded goods minus total consumption of traded goods.

$$(17) \quad \dot{f} = y_t - g_t - \alpha \omega (m p^{1-\alpha} s^{\alpha-1} + f)$$

In steady state, equation (9), representing equilibrium in the money market, and equation (15), representing equilibrium in the nontraded goods market, must hold. In addition, equations (17) and (16) imply

$$(18) \quad y_t = g_t + \alpha \omega (m p^{1-\alpha} s^{\alpha-1} + f)$$

$$(19) \quad m \pi + t = g_t p^{\alpha-1} s^{-\alpha} + g_n p^{\alpha} s^{-\alpha}$$

Equation (18) indicates that output of traded goods must be equal to private sector plus public sector demand for traded goods, for private sector holdings of foreign exchange to remain constant. Equation (19), which by (14) can be rewritten as

$$(20) \quad m\pi + t = g_t (e/P) + g_n (p_n/P)$$

indicates that the factors leading to monetary expansion, public sector expenditure on traded goods valued at the official exchange rate and public sector expenditure on nontraded goods, must be offset by factors leading to monetary contraction, the inflation tax and other taxes, for the real stock

of domestic money to remain constant. 1/ In other words, in steady state, the "official" public sector deficit is entirely financed by the inflation tax.

III. Economic Policy Under Dual Exchange Market Regimes

This section first derives the qualitative effects of a number of policies for both exchange market regimes, and then compares those effects across regimes.

1. Reserves adjustment model

Using equations (3) and (10), obtain,

$$(21) p = A s \quad \text{where } A = (1-\alpha) v y_t / \alpha u (y_n - g_n)$$

Using (10) and (21) in (11), and rearranging terms,

$$(22) m \pi + t = (1-v)y_t A^{\alpha-1} s^{\alpha(1-u)-1} - \{[(1-u)/u] v y_t A^{\alpha-1} - g_n A^\alpha\} s^{\alpha(1-u)}$$

where the right hand side expression is decreasing in s . 2/ From (20) and (21), it follows that in steady state 3/

1/ Under the reserves rationing model, public sector expenditure on traded goods contributes to monetary expansion because the central bank must buy the necessary foreign exchange from the private sector in exchange for domestic money. In contrast, under the reserves adjustment model there is no monetary expansion because the central bank provides the necessary foreign exchange from its own reserves.

2/ This assumes $y_n > [1 + (1-\alpha)/(1-u)\alpha] g_n$. Other things constant, an increase in the spread produces two opposing effects on the real value of monetary expansion. On the one hand, monetary expansion declines due to a decline in the real value of exports, and an increase in the real value of private sector imports channeled through the official market. On the other hand, monetary expansion increases due to an increase in the real value of public sector expenditure on nontraded goods. It is assumed that the two factors leading to a lower monetary expansion dominate.

3/ In all the discussion it is assumed that $s \geq 1$.

$$\begin{aligned}
 (23) \quad (ds/d\pi) &\geq 0 & \text{and} & \quad (dp/d\pi) &\geq 0 & \quad \text{depending on } \eta &\geq 1 \\
 (ds/dt) &< 0 & \text{and} & \quad (dp/dt) &< 0 \\
 (ds/dg_t) &= 0 & \text{and} & \quad (dp/dg_t) &= 0 \\
 (ds/dg_n) &\geq 0 \text{ 1/} & \text{and} & \quad (dp/dg_n) &> 0
 \end{aligned}$$

where η denotes the elasticity of the demand for money, defined as $\eta = -(\lambda'\pi/\lambda)$. 2/ 3/ Using these results, together with equations (3) and (5), it is possible to derive the following effects on the steady state balance of payments

$$\begin{aligned}
 (24) \quad (dr/d\pi) &\geq 0 & \text{depending on } \eta &\leq 1 \\
 (dr/dt) &> 0 \\
 (dr/dg_t) &< 0 \\
 (dr/dg_n) &\leq 0 & \text{depending on } (ds/dg_n) &\geq 0
 \end{aligned}$$

2. Reserves rationing model

Using equations (15) and (18), obtain

$$(25) \quad p = B s \quad \text{where } B = (1-\alpha) (y_t - g_t)/\alpha (y_n - g_n)$$

Since $(s/p) = (b/p_n)$, equation (25) implies that the relative price between traded and nontraded goods is equal to B^{-1} . Using (25) in (19),

1/ The condition for $(ds/dg_n) > 0$ is $v(1+\alpha) > u\alpha$.

2/ The real exchange rate defined by using the nominal free exchange rate is equal to $ps^{-1} = A$. Therefore, among the policies examined above, the only one to affect the real exchange rate thus defined is a change in public sector expenditure on nontraded goods. An increase in this type of expenditure causes a real appreciation.

3/ The reserves adjustment model in Lizondo (1987a) assumes all goods to be traded, so there is no discussion about effects on the real exchange rate.

$$(26) \quad m \pi + \tau = g_t B^{\alpha-1} s^{-1} + g_n B^\alpha$$

From (26) and (25), it follows that,

$$(27) \quad (ds/d\pi) \geq 0 \quad \text{and} \quad (dp/d\pi) \geq 0 \quad \text{depending on } \eta \geq 1$$

$$(ds/dt) < 0 \quad \text{and} \quad (dp/dt) < 0$$

$$(ds/dg_t) \geq 0 \quad \text{and} \quad (dp/dg_t) \geq 0 \quad \underline{1/}$$

$$(ds/dg_n) \geq 0 \quad \underline{2/} \quad \text{and} \quad (dp/dg_n) > 0$$

in steady state. 3/ 4/

3. Comparison across regimes

The results in the previous sections can be used to derive a number of implications regarding the effects of various policies for both exchange market regimes. We first discuss exchange rate policy, and then proceed with other policies.

A once-and-for-all devaluation of the official exchange rate has no long run effect on the rate of inflation, the spread, the real exchange rate, or the balance of payments, in any of the two regimes. Since the

1/ The condition for $(ds/dg_t) > 0$ is $y_t > \alpha (g_t + p g_n)$, while the condition $(dp/dg_t) > 0$ is $y_t > g_t + \alpha (g_t + p g_n)$. Therefore, although both variables may move in either direction, it is not possible for the spread to decline and the real exchange rate to appreciate at the same time.

2/ The condition for $(ds/dg_n) > 0$ is $(y_n - g_n) (y_t - g_t) + \alpha g_n y_t > \alpha g_t y_n$.

3/ The real exchange rate defined by using the nominal free exchange rate is equal to B . Therefore, an increase in public sector expenditures on nontraded goods, or a reduction in public sector expenditure on traded goods, causes the real exchange rate thus defined to appreciate.

4/ In contrast to the results derived here, in Pinto (1990), and in Kharas and Pinto (1989), the real exchange rate defined by using the official exchange rate always moves in the same direction as the spread while the real exchange rate defined by using free exchange rate always moves in the opposite direction. Those results are due to some particular assumptions regarding production and smuggling technology, demand conditions play no role. In contrast, the results in this paper are mainly due to demand factors, since output is assumed to be fixed and smuggling nonexistent.

level of the official exchange rate does not appear in any of the steady state conditions, an official devaluation can have only transitory effects on those variables.

The models in this paper can also be used to derive implications referred to countries that have, besides the free market, an official market with two different exchange rates, one for private sector transactions and another one for public sector transactions. The dynamics and the steady state solutions are the same as those found in the models examined above, with only one official exchange rate, provided that e in the various equations is interpreted as the official rate for private sector transactions. 1/ This implies that all the effects of policy changes derived in this paper also apply to a two-official-exchange-rate economy. For this equivalence to hold, the term "official exchange rate" in the discussion of models with one official rate, must be interpreted as "official exchange rate for private sector transactions" when examining economies with two official rates.

The only policy change that refers specifically to economies with two official exchange rates is a devaluation of the official rate for public sector transactions. For those economies, however, the level of the official rate for public sector transactions is irrelevant for the determination of the rate of inflation, the spread, the real exchange rate, or the balance of payments, even in the short run. As mentioned above, the level of this exchange rate does not affect the dynamics of the economy. Therefore, a devaluation of this exchange rate, unless accompanied by other policy changes, has no effect at all on the behavior of the economy. This exchange rate is only relevant for the determination of the "accounting" deficit of the public sector, in which public sector imports are valued at the official rate for public sector transactions. The size of this deficit,

1/ For the reserves adjustment model, the evolution of the real quantity of money is given by (8). If the central bank were to charge e^* for foreign exchange sold to the rest of the public sector (different from e , charged to the private sector) equation (6) would be replaced by (6') $D = e^* g_t + p_n g_n - P_t$. Equation (7) would also be modified to account for the fact that the domestic currency value of the change in reserves has to reflect e^* , instead of e , for the transactions of the public sector. These two modifications leave equation (8) unchanged. The same reasoning applies for the reserves rationing model, where the evolution of the real quantity of money is given by (16). If the central bank were to charge e^* to the rest of the public sector, the change in domestic credit would be given by (6'). In addition, although international reserves remain constant in terms of foreign exchange, they would change by $g_t (e - e^*)$ in terms of domestic currency, since the central bank would be buying g_t of foreign exchange at e , and selling it at e^* . These modifications in the behavior of domestic credit and reserves offset each other, so that equation (16) remains valid.

however, has no implications for the behavior of the economy, unless it prompts changes in other policies. 1/

An increase in the rate of crawl of the official exchange rate increases the long run rate of inflation by the same amount. For both dual regimes, the steady state rate of inflation is equal to the official rate of crawl, so changes in the rate of crawl have a one-to-one impact on inflation. Furthermore, this is the only policy that affects the long run rate of inflation.

An increase in the rate of crawl of the official exchange rate also has an impact on the other variables under consideration, but the direction of these effects depend on the elasticity of the demand for money with respect to the rate of depreciation of the free exchange rate. If the elasticity is lower than unity, the spread between the exchange rates declines and the real exchange rate depreciates, for both dual regimes. In addition, for the reserves adjustment regime the balance of payments improves. However, if the elasticity is higher than unity, all these effects work in the opposite direction. This implies that there is a limit to the desirable results that can be obtained by increasing the rate of crawl. Beyond that limit, the spread increases, the real exchange rate appreciates, and the balance of payments worsens.

An increase in taxes reduces the spread between the exchange rates and depreciates the real exchange rate under both dual regimes. In addition, the balance of payments improves under the reserves adjustment regime. In this model, the inflation tax and other taxes play a similar role. This is the reason why the direction of the effects of an increase in the rate of crawl depends on the elasticity of the demand for money. If this elasticity is lower than unity, an increase in the rate of crawl increases the revenue from the inflation tax, and thus has the same effects as an increase in other taxes. In contrast, if the elasticity is higher than unity, an increase in the rate of crawl reduces the revenue from the inflation tax, and thus is equivalent to a reduction in other taxes.

1/ For economies with one official exchange rate, for both private and public sector transactions, the "accounting" deficit mentioned above is the same as the "official" deficit included in equations (13) and (20). Clearly, in this case the accounting deficit has implications for the behavior of the economy. However, for economies with two official exchange rates, one for private and another one for public sector transactions, the "accounting" deficit mentioned above must be distinguished from the deficit included in equations (13) and (20). The latter, which is the relevant one for the behavior of the economy, must be interpreted in this case as the deficit calculated by valuing public sector expenditure on traded goods at the official exchange rate applied to private sector transactions.

In contrast to the policies examined above, an increase in public sector expenditure on traded goods have different effects, on both the spread and the real exchange rate, depending on the type of dual regime. Under the reserves adjustment regime, the central bank provides the foreign exchange needed to increase public sector imports from its own reserves, without altering its transactions with the private sector. Therefore, the balance of payments worsens by an amount equal to the increase in public sector expenditure, but neither the spread nor the real exchange rate are affected. 1/

In contrast, under a reserves rationing regime the central bank has to obtain from the private sector the additional foreign exchange needed for public sector imports, resulting in an ambiguous effect on the spread and on the real exchange rate. The direction of the change in the spread depends on the effect of the increase in public sector imports on the public sector deficit, for a given spread. 2/ An increase in public sector imports has a positive direct effect on the deficit. However, it also has an ambiguous indirect effect, caused by an increase in the relative price of traded goods. 3/ This change in relative price may have a positive or a negative effect on the public sector deficit depending on the composition of public sector expenditure. If, as it is likely, the overall effect on the deficit, for a given spread, is positive, the spread increases. 4/ In this way, the public sector deficit, with imports valued at the official exchange rate, is brought back to the level consistent with the unchanged source of financing, the inflation tax. The real exchange rate is also likely to appreciate.

An increase in public sector expenditure on nontraded goods appreciates the real exchange rate, but it may either increase or reduce the spread, for both dual regimes. For the reserves rationing regime, the ambiguity regarding the effect on the spread is due to the same reasons that were mentioned when discussing an increase in public sector imports. There is a positive direct effect and an ambiguous indirect effect on the public sector deficit, for a given spread. If, as it is likely, the overall effect is positive, the spread increases. For the reserves adjustment regime, the

1/ Since the spread and the real exchange rate are not affected, the one-to-one effect of a change in public sector expenditure on traded goods on the balance of payments can be derived directly from equation (12).

2/ This discussion is based on equation (20), which can also be written as (20') $m\pi = g_t (p_t/P) s^{-1} + g_n (p_n/P) - t$, where p_t denotes the domestic currency price of traded goods, which in the rationing regime is equal to the free exchange rate b .

3/ From (25), an increase in public sector imports increases B^{-1} , which is the relative price of traded with respect to nontraded goods.

4/ The possibility of an overall negative effect on the deficit, and therefore a resulting decline in the spread, does not arise in Kharas and Pinto (1989) due to their assumptions that the public sector spends only on traded goods.

ambiguity arises because of opposing effects on the real value of monetary expansion. For a constant spread, monetary expansion increases directly due to the increase in public sector expenditure on nontraded goods, and indirectly due to a reduction in the real value of imports channeled through the official market, which are valued at the official exchange rate. Monetary expansion declines indirectly due to a reduction in the real value of exports channeled through the official market, which are also valued at the official exchange rate. If, as it is likely, the positive effects dominate, the spread increases. In this case, the balance of payments worsens.

IV. Unification of the Foreign Exchange Markets

This section examines the long run inflationary effects of unifying the exchange markets. To do so, the appendix solves the model for a unified floating exchange rate, and for a unified crawling peg exchange rate. As can be expected, in the crawling peg system the long run rate of inflation is equal to the rate of crawl. Therefore, for this unified system the long run inflationary effects of unification depend on the rate of crawl, which in principle can be chosen arbitrarily by the authorities. However, if the condition of long run balance of payments equilibrium is imposed on the crawling peg system, the rate of crawl must be equal to the steady state rate of depreciation from a floating system. ^{1/} As a result, both unified systems imply the same steady state solution. Therefore, in the discussion about unification below, both unified systems are taken to be equivalent. While under the floating system the effects on inflation must be interpreted as originating endogenously from an exchange market without central bank intervention, under the crawling peg system it must be interpreted as originating from a rule of intervention consistent with long run balance of payments equilibrium.

For the purposes of the discussion, it is convenient to rewrite equations (A.8), from the appendix, and equations (20) and (13). Each of these equations provides a condition that must hold in one of the exchange market regimes. For the unified regime, equation (A.8) is equivalent to

$$(28) \quad \lambda(\pi) \pi = g_t (p_t/P) + g_n (p_n/P) - \tau$$

where p_t , the domestic currency price of traded goods, is equal to the uniform exchange rate. For the reserves rationing regime, equation (20) is equivalent to

$$(29) \quad \lambda(\pi) \pi + g_t [(b-e)/P] = g_t (p_t/P) + g_n (p_n/P) - \tau$$

^{1/} See the appendix.

where p_t is equal to the free exchange rate b . For the reserves adjustment regime, equation (13) is equivalent to

$$(30) \lambda(\pi) \pi + g_t [(e^{1-u} b^u - e)/P] - \dot{r} (e/P) = g_t (p_t/P) + g_n (p_n/P) - t$$

where p_t is equal to the average exchange rate paid for imports by the private sector, $e^{1-u} b^u$.

The right hand side in each of these equations shows the "actual" public sector deficit, where public sector expenditure on traded goods is valued at the market price for imports, this being equal to the uniform exchange rate in the unified regime, the free exchange rate in the reserves rationing regime, and the average exchange rate in the reserves adjustment regime. As long as relative prices are the same in the three regimes, the actual budget deficit is also the same. As shown in the appendix, relative prices are the same for the unified regime and the reserves rationing regime. Relative prices in the reserves adjustment regime, on the other hand, are not necessarily equal to those in the other two regimes. However, in the discussion below they are assumed to be equal in order to facilitate comparison.

The left hand side of equations (28), (29), and (30) show the financing of the actual public sector deficit for each of the exchange regimes. In the unified regime, the actual budget deficit is entirely financed by the inflation tax. The rate of inflation is the endogenous variable that adjusts so as to make equation (28) hold. 1/ If the elasticity of the demand for money increases from levels below unity to levels above unity as the rate of inflation increases, there will be usually two rates of inflation that satisfy this equation.

In the reserves rationing regime, the actual budget deficit is financed by the inflation tax, and by an implicit exchange tax. The exchange tax arises because the foreign exchange used for public sector imports are purchased by the central bank at the official exchange rate, instead of the free exchange rate reflected in the price of imports paid by private sector consumers. The revenue from the inflation tax is exogenous because the rate of inflation is determined by the authorities official depreciation rule. In this regime, the endogenous variable that makes (29) hold is the spread between the exchange rates. The spread increases until the part of the actual deficit that is not financed by the inflation tax is financed by the implicit exchange tax. Unifying the markets of a reserves rationing regime in which there is a positive spread implies that the revenue from the inflation tax must increase to replace the revenue from the implicit

1/ It is assumed that the size of the deficit is such that it can be financed by the inflation tax at some finite rate of inflation. Otherwise there is no steady state solution.

exchange tax that is lost with the unification. 1/ Whether this requires an increase or a decline in the rate of inflation, depends on the elasticity of the demand for money. If this elasticity is lower than unity, the rate of inflation rises, while if it is higher than unity, the rate of inflation falls.

In the reserves adjustment regime, the actual budget deficit is financed by the inflation tax, by an implicit exchange tax, and by a loss of international reserves. The revenue from the inflation tax is exogenous, because the rate of inflation is determined by the authorities official depreciation rule. In contrast, the other two sources of financing, the implicit exchange tax and the loss of international reserves, are endogenous. The shares of the deficit not covered by the inflation tax that are financed by each of these two additional sources depend on the specific allocation of transactions between the two exchange markets. 2/ The effect of unification on the rate of inflation depends on the spread and the balance of payments under the dual regime. A positive spread, and a balance of payments deficit, imply that some source of financing is lost with unification, thus requiring a higher inflation tax revenue. 3/ Once again, this implies an increase in the rate of inflation if the elasticity of the demand for money is lower than unity, and a fall in the rate of inflation if the elasticity is higher than unity.

The sources of financing for the actual public sector deficit can also be expressed by using the concepts of central bank exchange profits and losses arising from the dual system. In a dual system, the central bank buys and sells foreign exchange to the private sector at the official exchange rate. If the amounts bought and sold differ, the central bank is a net buyer or seller of foreign exchange at a price that is lower than the "market" price. Therefore, if the central bank is a net buyer, there is a

1/ This trade-off between the implicit exchange tax and the inflation tax was first stressed by Pinto (1986).

2/ The values of u and v .

3/ This relationship between the spread and the required inflation tax revenue does not necessarily hold in models where all current transactions are channeled through the official market. In those models, the spread is partly determined by private sector holdings of foreign money, which remains constant at the level outstanding at the time of adoption of the dual system because private sector net capital flows are necessarily zero. See Kiguel and Lizondo (1990).

profit, while if it is a net seller, there is a loss. The profit (or loss) is equal to net purchases (or net sales) times the difference between the "market" exchange rate and the official exchange rate. 1/

In the reserves rationing regime, the "market" exchange rate can be represented by the free exchange rate, since this is the rate reflected in the domestic price of traded goods, and thus in the price level. In this regime, the central bank is a net buyer of foreign exchange from the private sector, by an amount equal to public sector expenditure on traded goods. Therefore,

$$(31) \text{ CBP} = g_t [(b-e)/P]$$

where CBP represents central bank profits in real terms. Using (31), equation (29) can be rewritten as,

$$(32) \lambda(\pi) \pi + \text{CBP} = g_t (p_t/P) + g_n (p_n/P) - t$$

Therefore, the second component of the financing of the actual public sector deficit can be interpreted alternatively as an implicit exchange tax, or as central bank profits from the dual system.

In the reserves adjustment regime, the "market" exchange rate can be represented by the weighted average of the official and the free market exchange rate, as reflected in the domestic price of traded goods. In this case, central bank net purchases (or sales if negative) of foreign exchange are equal to the sum of public sector expenditure on traded goods and reserves accumulation. Therefore,

$$(33) \text{ CBP} = (g_t + r) [(e^{1-u} b^u - e)/P]$$

Using (33), equation (30) can be rewritten as

1/ If the central bank were to buy and sell foreign exchange in both markets, the calculation of profits and losses must take into account the average buying and the average selling exchange rate. Denoting those rates by e_b and e_s , respectively, and the market rate by e_m , central bank profits are equal to $X(e_m - e_b) + Z(e_s - e_m)$, where X and Z represent respectively central bank purchases and sales of foreign exchange to the private sector. If the central bank intervenes only in the official market, $e_b = e_s = e$, thereby resulting in central bank profits equal to $(X-Z)(e_m - e)$, which is the definition used above.

$$(34) \lambda(\pi) \pi + CBP - r (e^{1-u} b^u/P) = g_t (p_t/P) + g_n (p_n/P) - t$$

Therefore, the second component of the financing of the actual public sector deficit can be interpreted alternatively as an implicit exchange tax, or as central bank profits from the dual system. However, the amount financed by the second component varies according to the interpretation that is chosen, unless the balance of payments is in equilibrium. The implicit exchange tax includes only central bank purchases of foreign exchange used for public sector imports. In contrast, central bank profits from the dual system include central bank net purchases from all its transactions with the private sector. These two magnitudes differ, unless the balance of payments is in equilibrium. As a consequence of this distinction, the valuation for the third source of financing, changes in international reserves, also depends on the chosen interpretation for the second component. Under the interpretation of an implicit exchange tax, changes in reserves must be valued at the official exchange rate, while under the alternative interpretation, changes in reserves must be valued at the "market" exchange rate. 1/

Another implication that can be derived from equations (29) and (30) refers to the effect on the spread of switching from a reserves adjustment regime to a reserves rationing regime. The result depends on the balance of payments situation under the initial regime. If under the reserves adjustment regime the balance of payments is in equilibrium, switching to a reserves rationing regime produces a decline in the spread. 2/ This follows because under this condition both regimes require the same exchange tax. The exchange tax depends on the difference between the exchange rate reflected in the domestic price of imports paid by the private sector and the official exchange rate. In the new regime the domestic price of all imports reflect the free exchange rate, in contrast to the initial regime in which the price of some imports reflected the official exchange rate. Therefore, in order to produce the same exchange tax, the new regime requires a lower spread between the free and the official exchange rate.

If there is an initial balance of payments surplus, switching regimes results in a larger decline of the spread than in the case of balance of payments equilibrium. An initial balance of payments deficit, on the other hand, has the opposite effect. Thus, for a sufficiently large initial deficit, adopting a rationing scheme produces an increase in the spread.

1/ See equations (30) and (34).

2/ From (29) and (30), and remembering the assumption that relative prices are the same for both regimes, the condition $\dot{r}=0$ implies that $s_r = s_a^u$, where s_a is the spread in the reserves adjustment regime, and s_r is the resulting spread in the reserves rationing regime.

V. Conclusions

Most of the policies analyzed in this paper have the same steady state qualitative effects under both alternative dual exchange market regimes. A devaluation of an official exchange rate that applies to private sector transactions, or to both private sector and public sector transactions, does not affect the steady state equilibrium. A devaluation of an official exchange rate that applies only to public sector transactions does not affect even the short-run equilibrium. An increase in the rate of crawl of the official exchange rate reduces the spread and depreciates the real exchange rate in both dual regimes, and improves the balance of payments in a reserves adjustment regime, if the elasticity of the demand for money is lower than unity. If this elasticity is higher than unity, those effects work in the opposite direction. An increase in taxes reduces the spread and depreciates the real exchange rate in both dual regimes, and improves the balance of payments in a reserves adjustment regime. An increase in public sector expenditure on nontraded goods appreciates the real exchange rate. The spread may change in either direction, but it is likely to increase, for both regimes. For the reserves adjustment regime, the balance of payments is more likely to worsen. The effects of an increase in public sector expenditure on traded goods differ across regimes. For the reserves adjustment regime, the spread and the real exchange rate remain constant, while the balance of payments worsens. For the reserves rationing regime, the effects on the spread and the real exchange rate are ambiguous, although the spread is likely to increase and the real exchange rate is likely to appreciate.

The effects of unification on the rate of inflation depend on the spread under the reserves rationing regime, and on the spread and the balance of payments under the reserves adjustment regime. A positive spread, and a balance of payments deficit, imply the need for higher inflation tax revenue under a unified exchange market. Whether this requires an increase or a decline in the rate of inflation depends on whether the elasticity of the demand for money is lower or higher than unity.

Finally, the effects on the spread of adopting a rationing scheme in the official market of a dual market economy that was functioning without rationing, depend on the balance of payments prior to the adoption of rationing. If the balance of payments was in surplus, or in equilibrium, the spread declines. If the balance of payments was in deficit, the spread may increase.

The Unified Floating Exchange Rate Model

Under a unified system the price of traded goods reflect the uniform exchange rate, denoted by x . The price level is thus given by

$$(A.1) \quad P = p_n^{1-\alpha} x^\alpha$$

The condition for equilibrium in the nontraded goods market becomes

$$(A.2) \quad y_n = g_n + (1-\alpha) \omega [m (x/p_n)^\alpha + f (x/p_n)]$$

Since international reserves are constant under a uniform floating exchange rate, the change in the real quantity of domestic money is given by

$$(A.3) \quad \dot{m} = g_t (x/p_n)^{1-\alpha} + g_n (x/p_n)^{-\alpha} - t - (\dot{P}/P) m$$

The change in private sector holdings of foreign money is equal to

$$(A.4) \quad \dot{f} = y_t - g_t - \alpha \omega [m (x/p_n)^{\alpha-1} + f]$$

In steady state, equations (A.3) and (A.4) imply

$$(A.5) \quad m \pi + t = g_t (x/p_n)^{1-\alpha} + g_n (x/p_n)^{-\alpha}$$

$$(A.6) \quad y_t = g_t + \alpha \omega [m (x/p_n)^{\alpha-1} + f]$$

The other conditions for steady state equilibrium are indicated by equations (9), representing money market equilibrium, and (A.2), representing nontraded goods market equilibrium. The steady state relative price between traded and nontraded goods can be obtained from equations (A.2) and (A.6),

$$(A.7) \quad (x/p_n) = \alpha (y_n - g_n) / (1-\alpha) (y_t - g_t) = B^{-1}$$

Therefore, the relative price between traded and nontraded goods is the same as in the reserves rationing model. Using (A.7) in equation (A.6), gives

$$(A.8) \quad m \pi + \tau = g_t B^{\alpha-1} + g_n B^\alpha$$

The unified crawling peg exchange rate model

Under a unified crawling peg exchange rate system, the price level is given by (A.1), and the condition for equilibrium in the nontraded goods market is given by (A.2), where x now represents the crawling official exchange rate. The change in international reserves is given by

$$(A.9) \quad \dot{r} = (y_t - g_t - \alpha \omega [m (x/p_n)^{\alpha-1} + f]) - \dot{f}$$

where the first term on the right hand side represents the current account, and f represents the capital account. The change in domestic credit is equal to

$$(A.10) \quad \dot{D} = x g_t + p_n g_n - P \tau$$

Using (A.9) and (A.10),

$$(A.11) \quad \dot{m} = y_t (x/p_n)^{1-\alpha} - \alpha \omega [m + f (x/p_n)^{1-\alpha}] - \dot{f} (x/p_n)^{1-\alpha} + g_n (x/p_n)^{-\alpha} - \tau - (P/P) m$$

In steady state $\dot{m} = \dot{f} = 0$, and $(P/P) = \pi$, so (A.11) implies

$$(A.12) \quad y_t = \alpha \omega [m (x/p_n)^{\alpha-1} + f] - g_n (x/p_n)^{-1} + \tau (x/p_n)^{\alpha-1} + \pi m (x/p_n)^{\alpha-1}$$

Using (A.12) in (A.9), in steady state

$$(A.13) \quad \dot{r} = (\tau + \pi m) (x/p_n)^{\alpha-1} - g_t - g_n (x/p_n)^{-1}$$

Thus, the steady state balance of payments is in surplus or deficit depending on whether the steady state inflation tax is higher or lower than the public sector deficit. If the rate of crawl is chosen to be consistent with steady state balance of payments equilibrium, equations (A.13) and

(A.12) become equivalent respectively to equations (A.5) and (A.6) of the unified floating exchange rate model. Since the conditions for equilibrium in the money market and in the nontraded goods market are also the same for both unified systems, it follows that in this case both systems provide the same steady state solution.

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