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Valuing Interest Payment Guarantees On Developing Country Debt

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Abstract

This paper develops a technique to value guarantees on interest payments on developing-country debt, and provides some preliminary estimates of the cost of such guarantees. The cost of interest payment guarantees is not directly observable because a guarantee is a contingent obligation that becomes effective only if the debtor fails to make a certain payment. The strategy adopted in this paper is to estimate the market price that an interest payment guarantee would have if such a contract existed and were traded in financial markets. Using results from option pricing theory it is possible to calculate the price that an "interest guarantee contract" would carry in financial markets on the basis of the price of developing-country debt in secondary markets.

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Contents

I. Introduction

In many of the proposals designed to solve the foreign debt problem of developing countries, one of the elements present is some kind of contract in which part of the repayment to creditor banks is insured by a third party, such as a donor country or a multilateral organization. Evaluating the cost of issuing these guarantees is more complicated than that of other schemes (such as, for example, cash buybacks) in which the actual cost is directly observable because it involves a cash payment. By contrast, a guarantee on future payments is a contingent obligation that will only become effective if a certain condition is met--i.e., the debtor fails to make a certain payment.1/ The purpose of this paper is to develop a technique to value those guarantees and to provide some preliminary estimates of their cost.

The strategy that we adopt is to estimate the market price that an interest payment guarantee would have if such contract were traded in financial markets. Many other contingent liabilities with similar characteristics are traded in financial markets, most notably options, which means that we can make use of a number of results in finance theory. We show that an interest payment guarantee can be modeled as a portfolio of two put options; its price is therefore derived from the theoretical prices of those two options.

The main problem faced when attempting to price a guarantee is the specification of the random structure of the debtor country's payments. In the case of a "problem" debtor, in particular, net payments to the private banking system become difficult to predict, and are probably the result of a complicated bargaining process, which is itself affected by factors such as terms of trade changes, the economic and political evolution of the debtor country, lending policies of official creditors, etc. Without loss of generality, we assume that there exists an unobservable state variable that determines debtor country repayments to private banks. This state variable expresses the result of the bargaining process just described. We also make the relatively nonrestrictive assumption that that this variable can be modeled as a random variable that follows a certain stable stochastic time-series process. We further assume that this process would not be altered by the issuance of a guarantee on payments. This means that we are abstracting from "moral hazard" problems--both from the point of view of countries and banks--that might reduce actual payments once a guarantee has been issued, or the conceivable opposite case in which payments on guaranteed debt are higher because of the possibly stronger bargaining power of the donor country or the multilateral organization involved.

1/ The cost of a guarantee should not be confused with the maximum potential liability of the guarantor; even if that amount is required to be set aside as collateral, the true value of the guarantee is a smaller amount given by the economic cost of providing the contingent payments that might be made out of the collateral.

Under the above assumptions, we are able to identify the stochastic process followed by the state variable (and therefore to determine the theoretical market price of the guarantee) by using available observations on the price of developing country debt in secondary markets. This procedure minimizes the number of specific assumptions that need to be made regarding the nature of the random structure of payments to creditor banks, or regarding what kind of variables determine those payments. In studying this or related problems, previous work has been based on alternative approaches. Dooley and Simansky (1988), and Lamdany (1989) implicitely assume a give probability structure for debt repayments, and Claessens and van Wijnbergen (1989) assume that the price of oil is perfectly correlated with Mexico's external debt payments. 1/ The advantage of the technique followed in this paper is that no special assumptions of the kind made in these other studies are necessary in order to arrive at the price of the guarantee.

In addition, we incorporate another major source of risk to the issuance of an interest guarantee on floating rate securities, namely interest rate volatility. To evaluate this risk along with the risk associated with the debtor country's uncertain repayment, we use an option model with stochastic interest rates inspired by Merton (1973). As a first step, we estimate a model of the term structure of interest rates based on Vasicek (1977) using data on U.S. Treasury bills. We then use the estimated parameters that describe the stochastic processes followed by interest rates at different maturities together with the parameters that describe the stochastic process of the market value of debt to obtain all the moments necessary for the option price equation.

The results of the estimation indicate that the cost of hypothetical interest payment guarantees for four years would fluctuate between close to the full value of interest payments for most countries with a market price of debt at or below 30 cents on the dollar to nearly half that amount for countries whose debt sells at about 60 cents on the dollar. Loosely speaking, the estimates indicate that the cost of guarantees are high because debt prices are low and do not have a large enough variance; therefore, there is little hope that payments would be high enough to avoid a substantial use of guarantee money, unless one imposes the assumption that things are going to improve sharply in the near future.

The remainder of the paper proceeds as follows: Section II warns about the risks of using a simple back-of-the-envelope calculation for pricing guarantees. Section III derives a mathematical formula for pricing guarantees, inspired by option pricing theory. Section IV discusses the technique for the estimation of guarantee values on the basis of the prices of debt in secondary markets alone. Section V presents the estimation results, and finally, Section VI contains some concluding remarks.

1/ Other papers that provide useful insights into the pricing of guarantees are Nocera (1989), and Clark (1990).

II. Is the Market Price a Sufficient Statistic?

It is tempting to adopt a simple, back-of-the-envelope calculation to estimate the cost of guarantees on debt payments, which would use the market price of debt as the estimated probability that the country makes *each* payment. This calculation implies, then, setting the value of the guarantee equal to one minus the market price of debt times the value of the guaranteed payments. However, this simple calculation might, in some cases, be far off the true theoretical price of the guarantee. Although the guarantee does not represent an independent risk in the sense that all the risk derives only from the randomness of the payments of the original debt, the random distributions of all the future cash flows of the debt contract are in principle necessary to determine the value of the guarantee. In particular, knowledge of the price at which debt is transacted is by no means sufficient to determine (or even rank) the cost of guarantees, although it is an essential piece of information; in fact, our pricing of the guarantee will be based as much as possible on the market price of debt as opposed to using other variables.

Let us consider an example to illustrate why the market price of debt does not convey sufficient information to price the guarantee. Consider a two-period obligation, with equal payments due each period. Abstracting from issues of asset pricing or risk valuation, let us assume that the (risk-adjusted) interest rate is fixed and equal to zero. The face value of the debt is 100, and therefore the two payments are equal to 50. Suppose that the market price of debt, in this case equal to the expected value of payments, is equal to 40. There are, however, many different random structures for the debt payments that may support the same market price for debt, but different prices for the guarantees. Let us consider two of them. Country A makes, with certainty, a fraction p of every payment that is due. It is clear that the market price of claims on country A is going to be equal to p, (and p=40 in our example.) The expected cost of a guarantee on the first period debt payment for country A is going to be equal to:

$$E(G^A) = (1-p)50 = 30$$

with certainty, because a fraction 1-p will have to be paid by the guarantor.

Country B's debt has a different payoff structure. In the first period, payments are going to be equal to 0 with certainty. In the second period, payment is going to be determined with the following probability distribution:

$$= 50 \quad \text{with probability } \Pi$$

$$= 0 \quad \text{with probability } 1-\Pi$$

where $\Pi=2p$, that is $\Pi=80$ in our example. It is clear that the cost of a guarantee on the first period debt payment of country B is for certain:

$$E(G^{B}) = 50$$

Therefore, in this example, the cost of the guarantee is 66 percent higher for country B than for country A despite the fact that they both have the same market price of debt, which should cast some doubt on simple back-of-the-envelope calculations. The divergence could be greater if the maturity of the debt were longer because there would be more potential for different probability distributions of guaranteed payments. There is, however, one case in which the guarantee could be easily priced on the basis of the market price of debt, which is when the guarantee is full; that is, when it covers all of the promised payments of a debt contract. If the guarantee were a full guarantee, its pricing would be straightforward: the guarantee would convert the risky debt into a default-free one and its value would therefore be the difference between the price of a default-free bond and the market price of the risky debt. But in practice, although this insight is useful, it is not sufficient to price contracts that imply partial insurance.

Besides, the market value of a guarantee contract will in general differ from its expected value by a risk premium. In the above example the cost of the guarantees can be determined exactly, without any uncertainty, but in general, this would not be the case. Therefore, a proper valuation of a guarantee contract cannot be made only in terms of expected payments but must also include the risk associated with possible guarantee payments. In other words, expected guarantee payments must be discounted by the appropriately risk-adjusted interest rates, which increases the potential for divergence in the value of guarantees for different countries with the same market price of debt but different random structure of payments.

III. A Framework for Pricing Guarantees

We will assume that all debt takes the form of floating rate infinite maturity contracts, i.e., floating rate perpetuities.1/ We will assume that there exists a random variable S(t) that represents the state of nature and that determines the amount paid by the debtor country to the holders of its foreign debt. Each contractual payment that becomes due at time j is given by $i_j D$, where i_j is the interest rate applicable to the time j payment and D is the contractual value (principal) of the debt.

1/ While developing country loans have fixed contractual maturities, in practice principal repayments have tended to be rescheduled. Besides, prices of long term bonds (30 years or so) do not differ much from perpetuities.

Without loss of generality, we assume that the actual payment, V_j , made by the debtor country is determined according to the following schedule:

$$
\begin{aligned}
V_j &= i_j D & &\text{if } S(j) \geq D(1+i_j) \\
&= S(j) - D & &\text{if } D \leq S(j) \leq D(1+i_j) \\
&= 0 & &\text{if } S(j) \leq D
\end{aligned}
$$

Consider now a contract that guarantees a single interest payment, $i_j D$. At the maturity date, j, the payoff of this contract, G_j, will be:

$$
\begin{aligned}
G_j &= 0 & &\text{if } S(j) \geq D(1+i_j) \\
&= D(1+i_j) - S(j) & &\text{if } D \leq S(j) \leq D(1+i_j) \\
&= i_j D & &\text{if } S(j) \leq D
\end{aligned}
$$

This means that the value of the contract at maturity can be written as:

$$(1) \quad G_j = \mathrm{Max}[0, D(1+i_j) - S(j)] - \mathrm{Max}[0, D - S(j)]$$

which is equivalent to a portfolio of two put options written on the underlying variable S: a long position on a put with exercise price $D(1+i_j)$ and a short position on a put with exercise price D.

If the guarantee covered only a fraction α of the interest due, the exercise price of the first put would be $D(1+\alpha i_j)$. In the case of a partial interest guarantee, however, a better contract would be one that covers a fraction α of the shortfall in interest payments, that is, $\alpha(i_j D - V_j)$. While incentive considerations are beyond the scope of this paper, guaranteeing a fraction of the shortfall would likely reduce the effects of moral hazard. In this case, when the debtor country makes one dollar of payment, it "loses" a fraction α from the guarantee contract, while in the previous case it "loses" one full dollar of potential guarantee money. In the case in which the guarantee covers a fraction α of the shortfall, it is straightforward to see that its value will be equal to α times the value of the full interest guarantee; all estimations were, therefore, done for the case of full interest guarantee, since the value of partial guarantees can be computed easily from that basis.

Expression (1) above suggests the possibility of using option pricing theory to obtain the value of the guarantee G. However, the problem is more complicated than the standard Black-Scholes case because the floating rate feature makes the applicable interest rate a stochastic variable (until the time at which it is set for the next payment.) This means that the exercise price itself will be a stochastic variable. However, using results by Merton (1973) on option pricing with stochastic interest rates and by Vasicek (1977) on the term structure of interest rates, it is possible to derive a formula for the first and second put options in (1). The details are provided in Section 1 of the Appendix. The derivation requires the assumptions that the rate of change of state variable S follows a continuous-time process with a constant variance per unit of time, and that the instantaneous interest rate follows an Ornstein-Uhlenbeck process. 1/

IV. Measuring the Characteristics of the State Variable

The state variable S, which represents the repayment prospects by the debtor country, will, in general, depend on a number of variables. First, it will depend on variables affecting the debtor country's economic situation such as random shocks affecting GDP, terms of trade changes, government policies, etc. Second, S will be affected by policies adopted by creditor countries or international initiatives such as the tax and regulatory environment for banks or proposals to deal with the debt situation. Finally, S will depend on variables affecting the outcome of the bargaining process between the country and its creditor banks such as, for example, the state of negotiations between creditor banks and other debtor countries.

The measurement of S thus poses a significant problem. Our strategy, following the methodology developed by Marcus and Shaked (1984) and Pennacchi (1987), is to obtain a measurement of S using data on the secondary market prices for debt. We start by noting that the value of a single interest payment on developing country debt in secondary markets equals the value of a default-free payment minus the value of a contract that guarantees full payment of interest, that is:

$$(2) \quad V_j(t) = F_j(t) - G_j(t)$$

where $V_j(t)$ indicates the time t value of the debtor country's interest

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1/ This means that the rate of return on S follows the continuous-time analogy of a random walk with a possibly stochastic drift and the short-term interest rate follows the continuous-time analogy of a first-order autoregressive process.

payment that is contracted to be paid at date j, $F_j(t)$ is the time t value of a default-free floating rate payment of Di_j received at time j, and $G_j(t)$ is the time t value of a guarantee on this interest payment at date j, which we described previously.1/ Assuming that developing country debt can be modelled as a perpetuity, we can write the total value of this debt, V(t), as:

$$(3)\quad V(t) = \sum_{j=1}^{\infty} V_j(t)$$

$$= \sum_{j=1}^{\infty} P_j(t)F_j(t) - \sum_{j=1}^{\infty} G_j(t)$$

Given that we have a solution for the value of a guarantee for each payment, G_j, and the value of a default-free payment, F_j, then equation (3) represents a formula for the market value of developing country debt. Since each guarantee contract G_j is equal to the value of the portfolio of two put options given in (1), V(t) will be a (non-linear) function of S, the standard deviation of the rate of change in S, and the correlation between S and the instantaneous interest rate (see equations (A.11) and (A.12) of the Appendix).2/ In the context of deposit insurance valuation, Pennacchi (1987) has shown that it is possible to estimate those three values by solving a three-equation system: the option price formula, the expression for the variance of the price of debt, and the expression for the correlation of the price of debt and default-free bond prices. This procedure is outlined in Section 3 of the Appendix.

V. Estimation Results

The general thrust of the estimation results is that, based on the information derived from secondary markets for developing country debt, the cost of guarantees would be pretty high. The reason is that debt of heavily-indebted countries carries low prices with relatively low volatility (including volatility arising from the floating interest rate

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1/ Because of interest rate uncertainty, i_j will, in general, be random. The formula for $F_j(t)$ is given implicitely in equation (A.11) of the Appendix.

2/ V(t) will also depend on the difference between the rate of return on a marketable asset with the same risk as S(t) and the expected rate of change of S(t). This variable is denoted as c in the Appendix. Estimates of interest guarantees are carried out under alternative assumptions regarding the value of c.

feature of the contracts.) This means that the situation looks bleak, as indicated by prices of debt, and the volatility of debt prices is not large enough to suggest that a sufficiently large improvement is likely. In Table 1 we present descriptive statistics for secondary market prices of foreign debt for ten major highly-indebted countries.

An identifying assumption is needed in order to proceed to the estimation. The reason is that the state variable S is not really a traded asset whose rate of return is determined in accordance with asset market equilibrium. Instead, it is a state variable whose expected rate of change may differ from that of an asset with the same systematic risk, because it is not an asset that would be held in an investor's portfolio. Therefore, the estimation requires an assumption regarding the expected rate of change of S, or more precisely the difference between the rate of return on an asset with the same risk as S and the true expected rate of change in S. This parameter is denoted as c. 1/ It is important to note that the effect of the assumed value of c over the estimated cost of guarantees is somewhat weaker that it might appear. The reason is that there is some tradeoff between the assumed value for c and the estimated value for S. If one assumes a higher expected rate of growth for S--lower c--the estimate of S will be lower, partially offsetting the effect on the valuation of the guarantee.

The estimation, reported in Tables 2 to 5, was carried out for two values of c: 0 and 0.09. These are the two more natural assumptions of values for c. A value of 0 for c, as in Tables 2 and 4, implies that the expected expected rate of change of S(t) equals the (unknown) expected rate of return on a marketable asset with the same risk as S(t). Thus, in this case, the value of the interest guarantee can be interpreted as the difference between two put options written on an asset. On the other hand, a value of 0.09 for c, as in Tables 3 and 5, is an approximation for the case in which the expected rate of growth of S is zero. Although a value of 0.09 for c only means that the expected rate of change of S(t) equals that of a marketable asset with the same risk as S(t) less a 9 percent annual rate of change, 9 percent being approximately equal the risk-free interest rate. The expected rate of return on an asset with the same risk as S could be higher or lower than that, but in the case of S representing a risk uncorrelated with other assets in the market, or in the case of a risk neutral economy, a value of 0.09 for c would represent an expected rate of change in S(t) of approximately zero.

In addition, the estimation was carried out with data corresponding to a date just before the announcement of Secretary Brady's initiative (in April 1989), and after it. The reason is that the Brady plan itself, by affecting the expected return on debt, may have had a major impact for certain countries, thus distorting our estimate of the actual payment capacity of debtor countries. Therefore, Tables 2 and 3 give estimates of the value of interest payment guarantees based on market prices of debt observed just before the announcement of the Brady plan, and Tables 4 and 5

1/ Alternative identifying assumptions are possible. Below we also report the results from fixing the value of the volatility of S.

Table 1

Secondary Market Prices of Debt

	Market Price 3/2/89	Market Price 1/18/90	Standard Deviation 1/	Covariance with 6 Month Bond 2/
Argentina	17.25	11.75	0.3294	-1.402E-05
Brazil	26.75	27.50	0.3030	2.081E-05
Chile	55.25	64.25	0.1628	1.270E-05
Colombia	50.00	59.75	0.1425	2.034E-07
Costa Rica	13.50	18.75	0.3816	-1.597E-05
Ecuador	12.00	14.50	0.2847	6.335E-06
Mexico	33.00	38.00	0.1871	-6.407E-06
Philippines	36.00	48.00	0.1534	2.121E-05
Uruguay	57.00	50.00	0.0758	2.974E-06
Venezuela	27.25	35.25	0.2456	8.690E-06

Source: Salomon Brothers Inc, and IMF, International Financial Statistics.

1/ Annualized standard deviation of the rate of return on the market price of debt.

2/ Covariance between the rate of return on the market price of debt and the change in the log price of 6 month Treasury Bills.

Table 2

Value of Interest Payment Guarantees 1/

Pre-Brady Plan Announcement, 4/7/87 to 3/2/89, with c = 0

	Market Price of Debt	S	σ_S	ρ_{sr}	Value of Guarantee
Argentina	17.25	23.90	0.3369	-0.0300	36.39
Brazil	26.75	35.92	0.3125	-0.0480	35.27
Chile	55.25	61.41	0.1680	-0.0577	30.76
Colombia	50.00	54.07	0.1455	-0.0356	33.77
Costa Rica	13.50	20.45	0.3908	-0.0325	36.43
Ecuador	12.00	15.28	0.2881	-0.0166	36.79
Mexico	33.00	37.24	0.1903	-0.0233	36.19
Philippines	36.00	39.12	0.1555	-0.0270	36.32
Uruguay	57.00	58.36	0.0764	-0.0229	33.85
Venezuela	27.25	33.24	0.2509	-0.0324	36.10

1/ Guarantee refers to a four year guarantee on a floating rate perpetuity with semi-annual payments tied to the yield on the 6-month U.S. Treasury bill rate. The initial 6-month T-Bill yield was 9.0625 percent.

S is the current level of the state variable, c is the difference between the expected rate of growth of an asset with the same risk as S and that of S, σ_S is the standard deviation of the rate of growth of S, and ρ_{Sr} is the correlation of the rate of growth of S and the short term (instantaneous) U.S. rate of interest.

Table 3

Value of Interest Payment Guarantees 1/

Pre-Brady Plan Announcement, 4/7/87 to 3/2/89, with c = .09

	Market Price of Debt	S	σ_S	ρ_{sr}	Value of Guarantee
Argentina	17.25	54.41	0.0543	-0.5059	36.76
Brazil	26.75	67.15	0.0624	-0.5303	35.91
Chile	55.25	92.59	0.0536	-0.7476	24.01
Colombia	50.00	88.92	0.0503	-0.7928	27.35
Costa Rica	13.50	48.24	0.0542	-0.4544	36.81
Ecuador	12.00	45.63	0.0411	-0.5720	36.82
Mexico	33.00	74.12	0.0501	-0.7115	34.75
Philippines	36.00	77.11	0.0475	-0.7787	33.89
Uruguay	57.00	93.58	0.0434	-0.9263	23.20
Venezuela	27.25	67.78	0.0543	-0.6134	35.93

1/ Guarantee refers to a four year guarantee on a floating rate perpetuity with semi-annual payments tied to the yield on the 6-month U.S. Treasury bill rate. The initial 6-month T-Bill yield was 9.0625 percent.

S is the current level of the state variable, c is the difference between the expected rate of growth of an asset with the same risk as S and that of S, σ_S is the standard deviation of the rate of growth of S, and ρ_{Sr} is the correlation of the rate of growth of S and the short term (instantaneous) U.S. rate of interest.

Table 4

Value of Interest Payment Guarantees 1/

Post-Brady Plan Announcement, 4/7/87 to 1/18/90, with c = 0

	Market Price of Debt	S	σ_S	ρ_{sr}	Value of Guarantee
Argentina	11.75	18.51	0.4142	-0.0328	36.48
Brazil	27.50	41.04	0.3722	-0.0643	33.89
Chile	64.25	71.48	0.1644	-0.0780	26.11
Colombia	59.75	65.57	0.1549	-0.0604	29.16
Costa Rica	18.75	27.60	0.3709	-0.0423	35.91
Ecuador	14.50	20.21	0.3427	-0.0312	36.58
Mexico	38.00	45.26	0.2290	-0.0413	34.55
Philippines	48.00	54.96	0.1953	-0.0548	32.58
Uruguay	50.00	51.29	0.0813	-0.0178	35.68
Venezuela	35.25	45.44	0.2825	-0.0555	33.85

1/ Guarantee refers to a four year guarantee on a floating rate perpetuity with semi-annual payments tied to the yield on the 6-month U.S. Treasury bill rate. The initial 6-month T-Bill yield was 9.0625 percent.

S is the current level of the state variable, c is the difference between the expected rate of growth of an asset with the same risk as S and that of S, σ_S is the standard deviation of the rate of growth of S, and ρ_{Sr} is the correlation of the rate of growth of S and the short term (instantaneous) U.S. rate of interest.

Table 5

Value of Interest Payment Guarantees 1/

Post-Brady Plan Announcement, 4/7/87 to 1/18/90, with c = 0.09

	Market Price of Debt	S	σ_S	ρ_{Sr}	Value of Guarantee
Argentina	11.75	45.04	0.0532	-0.4356	36.82
Brazil	27.50	67.99	0.0718	-0.4632	35.65
Chile	64.25	97.89	0.0527	-0.7497	17.75
Colombia	59.75	95.34	0.0515	-0.7721	21.05
Costa Rica	18.75	56.60	0.0607	-0.4702	36.70
Ecuador	14.50	49.99	0.0509	-0.5051	36.81
Mexico	38.00	79.03	0.0580	-0.6413	32.94
Philippines	48.00	87.48	0.0568	-0.6983	28.26
Uruguay	50.00	88.88	0.0436	-0.9186	27.56
Venezuela	35.25	76.38	0.0646	-0.5634	33.75

1/ Guarantee refers to a four year guarantee on a floating rate perpetuity with semi-annual payments tied to the yield on the 6-month U.S. Treasury bill rate. The initial 6-month T-Bill yield was 9.0625 percent.

S is the current level of the state variable, c is the difference between the expected rate of growth of an asset with the same risk as S and that of S, σ_S is the standard deviation of the rate of growth of S, and ρ_{Sr} is the correlation of the rate of growth of S and the short term (instantaneous) U.S. rate of interest.

give estimates that make use of all available data. In each table, we have assumed a debt principal level of D = 100 and an initial U.S. 6-month default-free interest rate of 9.0625 percent 1/ Ten debtor countries with relatively more active secondary markets were selected for estimation. The first columns of each table gives secondary market prices of debt quoted by Salomon Brothers. The following three columns give the parameter estimates that were inferred from the market prices of this debt, the variance of the rate of return on this debt, and the correlation of the return on this debt with U.S. interest rates. Finally, the fourth column uses these parameter estimates to calculate the value of a four-year guarantee on semi-annual floating rate interest payments, where the floating rate is equal to the yield on a six month default-free U.S. discount bond issued six months prior to the interest payment date.

When the expected rate of change in S(t) is assumed to equal that of an asset with the same risk as S(t), i.e. c=0, the estimated value of the guarantee is in fact close to its upper bound of 36.82 (the full amount of their promised interest payments) with the exception of the three countries with higher price of debt (Chile, Colombia and Uruguay.) When c is assumed to be equal to 9 percent, there is not a significant difference for countries with low secondary market prices of debt, but in the three above cases the cost of guarantees falls significantly, as the larger estimated value for S makes them more likely to service their debt.

In general, the estimates of interest guarantees are somewhat lower when based on secondary debt prices observed after the Brady plan announcement than when based on these prices observed prior to the Brady plan announcement. For most debtor countries, debt prices are currently higher than prior to the Brady plan announcements and the estimated standard deviation of prices is also a little higher. (The exceptions are Argentina and Uruguay). In many cases, however, despite a sharp increase in market prices of debt, the value of the guarantee has not shown a significant decrease (for example, Costa Rica, Mexico, Philippines and Venezuela.) The reason is that when the level of the state variable S is sufficiently low, increases in its estimated value do not generate a significant increase in the probability of debt service by the debtor country and thus do not translate into much lower guarantee values.

To check the robustness of the results, a different specification was estimated by using a fixed value for the standard deviation of the state variable S, and estimating both the level and expected rate of change of the state variable S. The standard deviation of the state variable was made equal to the standard deviation of GNP of each debtor country, which was generally higher than the estimated variance of S in the previous exercise. The results, which are reported in Tables 6 and 7, do not show a significant difference with those obtained using the above specification.

1/ As described in the Appendix, our estimates used the Vasicek (1977) model of the term structure, which assumes that the instantaneous (short term) rate of interest follows a mean-reverting process. This interest rate was estimated to have a long run mean of 8.89 percent

Table 6

Value of Interest Payment Guarantees 1/

Pre-Brady Plan Announcement, 4/7/87 to 3/2/89, with σ_S=volatility of GNP

	σ_S	S	c	ρ_{Sr}	Value of Guarantee
Argentina	0.393	23.80	-0.0104	-0.0196	36.12
Brazil	0.125	48.12	0.0529	-0.2676	36.35
Chile	0.226	57.66	-0.0254	0.0311	30.13
Colombia	0.086	67.11	0.0408	-0.3890	32.71
Costa Rica	0.177	25.33	0.0390	-0.1444	36.81
Ecuador	0.168	19.12	0.0259	-0.1278	36.82
Mexico	0.200	36.61	0.0033	-0.0072	36.15
Philippines	0.129	42.45	0.0124	-0.1205	36.35
Uruguay	0.213	43.62	-0.0832	0.4800	33.46
Venezuela	0.136	41.88	0.0364	-0.2133	36.52

1/ Guarantee refers to a four year guarantee on a floating rate perpetuity with semi-annual payments tied to the yield on the 6-month U.S. Treasury bill rate. The initial 6-month T-Bill yield was 9.0625 percent.

S is the current level of the state variable, c is the difference between the expected rate of growth of an asset with the same risk as S and that of S, σ_S is the standard deviation of the rate of growth of S, and ρ_{Sr} is the correlation of the rate of growth of S and the short term (instantaneous) U.S. rate of interest.

Table 7

Value of Interest Payment Guarantees 1/

Post-Brady Plan Announcement, 4/7/87 to 1/18/90, with σ_S-volatility of GNP

	σ_S	S	c	ρ_{Sr}	Value of Guarantee
Argentina	0.393	18.48	0.0032	-0.0350	36.55
Brazil	0.125	53.44	0.0621	-0.2753	36.01
Chile	0.226	67.80	-0.0301	0.0125	25.58
Colombia	0.086	79.01	0.0474	-0.4236	27.25
Costa Rica	0.177	33.12	0.0398	-0.1568	36.70
Ecuador	0.168	25.57	0.0358	-0.1550	36.81
Mexico	0.200	46.88	0.0094	-0.0765	34.83
Philippines	0.129	62.40	0.0301	-0.2225	32.79
Uruguay	0.213	37.97	-0.0712	0.4648	35.26
Venezuela	0.136	55.66	0.0464	-0.2471	35.12

1/ Guarantee refers to a four year guarantee on a floating rate perpetuity with semi-annual payments tied to the yield on the 6-month U.S. Treasury bill rate. The initial 6-month T-Bill yield was 9.0625 percent.

S is the current level of the state variable, c is the difference between the expected rate of growth of an asset with the same risk as S and that of S, σ_S is the standard deviation of the rate of growth of S, and ρ_{Sr} is the correlation of the rate of growth of S and the short term (instantaneous) U.S. rate of interest.

The estimated values of c range between -3 percent and +6 percent and the differences with respect to previous estimates of the value of guarantees are therefore small.

VI. Conclusions

This paper has outlined a framework for valuing guarantees on developing countries' floating rate debt payments. The main advantage of this method is that it derives the current level and the parameters of the stochastic process that determine repayments by utilizing data on market value of debt and interest rates only. Therefore, it requires no assumptions regarding the economic or political determinants of repayments which, as mentioned above, could be of a quite diverse nature and hard to measure in any comprehensive way.

The main caveat of the technique applied in this paper is the necessity of making an identifying assumption about the expected rate of growth of the unobservable variable that determines payments to banks. The problem arises because it is not possible to measure all the random factors affecting the payments made by the debtor country, and there does not exist a traded financial asset which is perfectly correlated with the unobservable state variable that determines a debtor country's payments. However, it is possible to compute the value of guarantees for reasonable boundaries for the value of the unknown parameter. In any event, the assumptions required by the methodology applied in this paper are certainly much weaker than those that are implicit in the application of a rule of thumb of the type discussed in Section II.

The estimated values for the guarantee contracts may perhaps appear to be on the high side, especially for the six or seven debtor countries for which the prices of debt are lower. This is merely a reflection of the low market valuation of the debt of these countries. It opens, however, the question of whether the estimated specification of the process followed by debt prices is consistent with the record of payments by debtor countries or whether it might appear too pessimistic relative to that record, possibly reflecting fears of dramatic bad news for payments to banks, arising perhaps from political or institutional considerations.

1. Derivation of the Interest Payment Guarantee Formula

The stochastic process for the state variable, $S(t)$, is assumed to take the form:

$$\text{(A.1)} \quad \frac{dS(t)}{S(t)} = \alpha_s(t)dt + \sigma_s dz$$

where $\alpha_s(t)$ is the (possibly time varying) instantaneous expected rate of change of $S(t)$, dz is a standard Wiener process, and σ_s, the instantaneous standard deviation of $S(t)$, is assumed to be a constant. We will also assume that $\alpha_s(t)$ may differ from the expected rate of return on a marketable asset with the same risk as $S(t)$. The difference between the rate of return on this marketable asset and $\alpha_s(t)$ is assumed to equal a constant, c.

Let the price at time t of a default-free discount bond that pays $1 at time $t+\tau$ be given by $P(t,\tau)$. As in Merton (1973), we assume this bond price has the following dynamics:

$$\text{(A.2)} \quad \frac{dP(t,\tau)}{P(t,\tau)} = \alpha_p(t)dt + \sigma_p(\tau)dq\,, \qquad dzdq = \rho dt$$

where $\sigma_p(t)$ is only a function of the bond's time to maturity.

We further assume that interest rates are described by the model of Vasicek (1977), which is consistent with the assumed dynamics for bond prices given in (A.2.) This implies that the instantaneous nominal interest rate, $r(t)$, follows the process:

$$\text{(A.3)} \quad dr(t) = \alpha\big(\gamma - r(t)\big)dt - \sigma dq$$

The parameter γ is the steady state mean of $r(t)$, while σ^2 represents its instantaneous variance. The parameter α measures the magnitude of mean reversion in the short term interest rate.

Given (A.3), and assuming the market price of interest rate risk is a constant, ϕ, Vasicek (1977) shows that the equilibrium price of a default -free discount bond is of the form:

$$(A.4)\quad P(r(t),\tau) = A(\tau)\exp\left(-\frac{1}{\alpha}(1 - e^{\alpha\tau})r(t)^2\right)$$

where

$$A(\tau) = \exp\left[\left(\frac{1-e^{-\alpha\tau}}{\alpha} - \tau\right)\left(\gamma + \frac{\sigma\phi}{\alpha} - \frac{\sigma^2}{2\alpha^2}\right) - \frac{\sigma^2}{4\alpha^3}(1-e^{-\alpha\tau})\right]$$

Using Ito's lemma, the standard deviation of this bond's rate of return is given by:

$$(A.5)\quad \sigma_p(\tau) = \frac{\sigma}{\alpha}(1 - e^{-\alpha\tau})$$

Now consider the value of a guarantee on a single floating-rate debt interest payment, where the debt interest payment is tied to the yield on a default-free discount bond of maturity τ_r. It is assumed that the floating-rate debt's interest reset date is also exactly τ_r periods prior to the interest payment date. More specifically, the debt interest payment equals a spread, s, plus the yield on a default-free bond of maturity τ_r that was issued τ_r periods prior to the interest payment date. Under these assumptions, the maturity value of the interest guarantee is given by equation (1) in the text where the promised interest payment is:

$$(A.6)\quad 1 + i_{t+\tau} = \frac{e^{s\tau_r}}{P\left(t+\tau-\tau_r,\tau_r\right)}$$

Using a straightforward extension of the work of Merton (1973) to value options when interest rates are stochastic, the value of the guarantee, G(t) is given by:

$$(A.7)\quad G(t,\tau) = e^{s\tau_r}\, DP(t,\tau-\tau_r)\, N(-d_{12}) - e^{-c\tau}\, S(t)\, N(-d_{11})$$
$$- DP(t,\tau)\, N(-d_{22}) + e^{-c\tau}\, S(t)\, N(-d_{21})$$

where:

$$d_{11} = \ln\left(\frac{e^{-c\tau} S(t)}{e^{s\tau_r} DP(t,\tau-\tau_r)}\right) / T_1^{1/2} + \frac{1}{2} T_1^{1/2}$$

$$d_{12} = d_{11} - T_1^{1/2}$$

$$d_{21} = \ln\left(\frac{e^{-c\tau} S(t)}{DP(t,\tau)}\right) / T_2^{1/2} + \frac{1}{2} T_2^{1/2}$$

$$d_{22} = d_{21} - T_2^{1/2}$$

and where:

$$T_1 = \int_0^{\tau_r} \sigma_s^2 + \sigma_p^2(\omega) - 2\rho\sigma_s\sigma_p(\omega)\, d\omega + \int_0^{\tau-\tau_r} \sigma_s^2 + \sigma_p^2(\omega) - 2\rho\sigma_s\sigma_p(\omega)\, d\omega$$

$$T_2 = \int_0^{\tau} \sigma_s^2 + \sigma_p^2(\omega) - 2\rho\sigma_s\sigma_p(\omega)\, d\omega$$

Using the expression for $\sigma_p(\omega)$ from (A.5), the integration in the formulas for T_1 and T_2 can be easily carried out.

2. Estimation of the Parameters of the Term Structure

Let $B(t) = \ln P(r(t),\tau)$, i.e., the log of a given maturity bond price. Then, using (A.4) and Ito's lemma, one can show that B(t), given a constant maturity τ, will follow the process:

$$\text{(A.8)} \quad dB(t) = \left(\alpha\ln A(\tau) - \gamma(1-e^{-\alpha\tau}) - \alpha B(t)\right)dt + \frac{\sigma}{\alpha}(1-e^{-\alpha\tau})dq$$

$$= \left(K(\tau) - \alpha B(t)\right)dt + \frac{\sigma}{\alpha}(1-e^{-\alpha\tau})dq$$

This continuous time process has a discrete time AR(1) representation of the form:

$$\text{(A.9)} \quad B(t+\delta) = K'(\delta) + e^{-\alpha\delta}B(t) + v_t(\delta)$$

where v_t is normally distributed with mean zero and variance equal to:

$$\text{(A.10)} \quad \mathrm{Var}\big(v_t(\delta)\big) = \frac{\sigma^2}{2\alpha^3}(1-e^{-\alpha\tau})^2(1-e^{-2\alpha\delta})$$

Using (A.9) and (A.10), maximum likelihood estimation of the parameters α, σ, γ, and ϕ can be carried out. This was done using end of month prices of 30, 90, 180, and 345 day Treasury bills over the period 1970 through 1986. The estimates and standard errors are:

α	σ	γ	ϕ
.1961	.0452	.0889	.3146
(.1210)	(.0032)	(.0525)	(1.1329)

3. Estimation of the Parameters of the State Variable S(t)

In this section we describe a technique that allows us to estimate the level of S(t), its rate of return variance, σ_s^2, and the correlation parameter ρ. We do this using data on secondary market prices of developing country debt. The developing country debt is assumed to be equal to a floating-rate perpetuity. Let $V(t,\tau)$ equal the value of time t of a single floating-rate payment to be received in t periods that is subject to default risk, i.e., it is not guaranteed. Then its value must equal the value of a default-free floating-rate payment less the value of the guarantee on this floating-rate payment:

$$\begin{aligned}\text{(A.11)} \quad V(t,\tau) &= e^{s\tau_r} DP(t,\tau-\tau_r) - DP(t,\tau) - G(t,\tau) \\ &= D\big(e^{s\tau_r} P(t,\tau-\tau_r)\, N(d_{12}) - P(t,\tau)\, N(d_{22})\big) \\ &\quad + e^{-c\tau} S(t) \big(N(-d_{11}) - N(-d_{21})\big)\end{aligned}$$

Therefore, the market value of this floating-rate perpetuity, V, is given by

$$\text{(A.12)} \quad V(t) = \sum_{\tau_i}^{\infty} V(t,\tau_i)$$

Using Ito's lemma, we can solve for the instantaneous variance of V(t) as well as its covariance with the rate of return on a t period discount bond.

$$\text{(A.13)} \quad \sigma_V^2 = \left(\frac{\partial V}{\partial S}\sigma_s \frac{S}{V}\right)^2 + \left(\frac{\partial V}{\partial r}\frac{\sigma}{V}\right)^2 + 2\rho \frac{\partial V}{\partial S}\frac{\partial V}{\partial r}\sigma_s \sigma \frac{S}{V^2}$$

$$\text{(A.14)} \quad \sigma_{Vr} = -\frac{\partial V}{\partial S}\rho\sigma_s \frac{\sigma}{\alpha}(1-e^{-\alpha\tau})\frac{S}{V} - \frac{\partial V}{\partial r}\frac{\sigma^2}{\alpha}(1-e^{\alpha\tau})\frac{1}{V}$$

where $\frac{\partial V}{\partial S}$ and $\frac{\partial V}{\partial r}$ are evaluated in a straightforward (but lengthy) manner using (A.12) and (A.11). By using secondary market prices of developing country debt as well as prices of U.S. Treasury bills, we can observe V(t) as well as estimate σ_V^2 and σ_{Vr}. Given these estimates, then equations (A.12), (A.13), and (A.14) are a system of three non-linear equations in the three unknowns, S(t), σ_s, and ρ. Numerical methods can then be used to solve this system.

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