IMF WORKING PAPER

WP/90/9

INTERNATIONAL MONETARY FUND

Exchange and Trade Relations Department

The Evaluation of Debt Exchanges

Prepared by John Clark*

Authorized for Distribution by Mark Allen

February 1990

Abstract

An approach is presented for analyzing debt-for-debt exchanges from the perspective of the exchange's impact on the country's contractual obligations and from the perspective of the creditors whose participation is sought. A general model is developed for valuing partially guaranteed debt instruments and an intuitive motivation is suggested for upper and lower bounds on the valuation of instruments carrying specific as well as rolling guarantees. An appendix presents several easily employed rules of thumb which have been suggested in the literature for valuing collateralized instruments and for estimating the possible net debt reduction achievable through their issuance.

JEL Classification Numbers:
3131, 4330

*I wish to thank Mark Allen, Caroline Atkinson, Ajai Chopra, Charles Collyns, Augusto de la Torre, Michael Dooley, Mohamed El-Erian, Luis Valdivieso and Maxwell Watson for comments, suggestions and criticisms during various stages of this exercise. While the paper has hopefully benefitted from their input, all remaining errors are my own.

Contents

Summary

An approach is presented for analyzing debt-for-debt exchanges from the perspective of the exchange's impact on a country's contractual obligations and from the perspective of the creditors whose participation is sought. It is argued that a distinction must be maintained between guarantee structures which involve a onetime outflow on the part of the debtor on the lines of an insurance premium and those which allow for a possible refund to the debtor of escrowed resources. In cases where, following the discharge of the new contractual obligations, the resources used to fund any associated enhancements are returned to the debtor, the difference in debt stocks before and after the transaction (including any debt incurred to fund any associated enhancements) understates the relief the country has obtained. A meaningful comparison with buybacks must be based on a formulation that recognizes the net burden of completely repaying partially collateralized debt is lower than its absolute level.

In the section on valuation, claims on a sovereign debtor are presented as entitlements to share in the aggregate payments which the debtor is willing to make. In periods when the debtor's aggregate payments are insufficient to discharge all the contractual obligations falling due, it is assumed that holders of unguaranteed claims receive new claims which may be used to attempt to share in subsequent aggregate payments. The valuation of guarantees needs to contemplate not only the probability of the guarantee being triggered, but also, once the guarantee is triggered, the value of the new claim foregone in return for the full contractual payment. It is argued that the secondary market price does not provide sufficient information to value guaranteed debt instruments. The paper presents some rule of thumb valuation formulas previously suggested in the literature. To provide an indication as to the relative optimism or conservatism of these formulations, boundaries are presented for the highest and lowest possible valuations, consistent with the observable secondary market price and a very general model of creditors' expectations, of partially enhanced instruments.

Taken together, the two sections demonstrate that debt-for-debt exchanges are not necessarily equivalent to buybacks. Exchanges involving refundable guarantees are shown to be functionally equivalent to targeted buybacks where in addition to lowering obligations across the board as in a conventional buyback, certain obligations, e.g. the final principal payment, are effectively eliminated. Given that payment claims falling due later in time might be serviced less fully than claims falling due earlier, discounts may vary across claims. Since the debt relief obtained through a buyback is only a function of the average value of claims while that obtained through an exchange may also involve the value of claims at specific times (which may differ from the average), the amount of present discounted contractual relief per dollar of precommitted resources achievable through a debt exchange may be more or less than that achievable through a buyback.

I. Introduction

This paper develops a framework for analyzing debt and debt-service reduction operations involving debt for debt exchanges. In such exchanges, creditors of a country whose debt is discounted in the secondary market trade existing claims for new instruments which restructure and generally lower the country's payment obligations. Creditors may voluntarily agree to such transactions because the new instruments carry enhancements, such as partial guarantees, which cause them to be discounted less steeply than the previous debt contracts.

The aim of the paper is to explain a possible analytical approach for considering how best to tailor a debt restructuring exercise to meet the debtor's present and future needs while staying within the confines of a voluntary framework. Maintaining a voluntary framework generally involves offering creditors options which are no worse, from the creditor's point of view, than the competing alternatives.

It is argued that the evaluation of debt exchanges needs to be based on a rigorous examination of the details of the exchange which is being proposed. It is shown that alternative contractual structures can yield results which differ significantly in terms of the impact on the structure of a country's contractual obligations as well as on the willingness of creditors to agree to the proposed exchange on a voluntary basis.

Several different types of guarantee structures have been suggested or discussed in the literature; Section II suggests a useful dichotomy between possible guarantee structures and provides some easily employed formulas for calculating or estimating the contractual relief implied by a proposed debt exchange, once the issue of how the guarantee will be structured has been clarified. A crucial issue in assessing the contractual flow implications of a proposed debt exchange involves whether the guarantees applied to the new debt instruments entail an effective prepayment of future payment obligations on the new instrument or a nonrefundable premium payment, along the lines of an insurance policy. Given a well-defined contractual structure and a particular exchange ratio between new and old debt, the impact of a proposed exchange on a country's contractual obligations should be unambiguous.

On the other hand, in Section III, it is argued that the likely response of creditors cannot be predicted as precisely based on available information on secondary market prices. Many proposed transactions are likely to differ significantly from transactions currently taking place in the market, i.e., conventional cash sales. Therefore, information on the prices of existing transactions can only offer a partial guide to creditors' approaches to evaluating significantly different exchanges. In Section III, the paper presents a unifying perspective within which various heuristic pricing models which have been developed in the relevant literature can be placed. A set of formulas is presented which give upper and lower bounds for creditors' likely valuation under reasonable but broad assumptions and limited information about creditors' valuation models. Several reasonable rule-of-thumb formulas which have been suggested in the literature are then located within these boundaries.

Section IV discusses the concept of "equivalence" between debt exchanges and buybacks. A characterization of debt-for-collateralized-debt exchanges as being functionally equivalent to "targeted" buybacks is presented; it is this targeting which makes such exchanges fundamentally nonequivalent to conventional buybacks. It is suggested that the differences between various debt-for-debt exchanges in terms of their impact on a country's contractual obligations and in terms of how creditors view the transaction is a function of which obligations, out of the whole stream of obligations of the existing contractual obligations, are targeted for effective retirement. It is also argued that, in contrast, a debt-for-debt exchange involving new debt enhanced by non-refundable premium-based guarantees may be virtually equivalent to a buyback using the same resources which were used to fund the guarantee.

In the last section, several limitations inherent in the analysis are discussed and suggested directions for future research are presented. In an appendix, formulas which have been suggested in the text have been brought together for ease of reference.

II. The Implications for the Debtor's Contractual Obligations

A debt-for-debt exchange has an impact on the debtor country's flow of contractual obligations, which in turn has implications for the country's need for exceptional financing, its reserve position, or more generally, its balance of payments. The period-by-period impact of alternative transactions may differ significantly. Therefore any analysis of the consequences of a debt-for-debt exchange must begin with a careful identification on a period-by-period basis of the net contractual implications of the proposed transaction. The net impact must take into account not only the contractual obligations retired, but also all of the new obligations incurred, including the obligations on the new instrument which must be paid by the country, as well as the net obligations to the parties which lent money to the country to fund any associated enhancements. Also the net impact should reflect any use by the country of its own resources.

To induce creditors to voluntarily accept the exchange, it must generally be the case that the new debt instrument is enhanced in some way. Rather than covering the full range of possible enhancements, the analysis here concentrates on guarantees. Possible enhancements derived from differentiating new claims from old claims by credibly making the former senior to the latter are ruled out by assumption, as they are not easily captured by the simple framework proposed in Section II. The new instruments are assumed to be perceived as being of the same class as the previous security.

1. A typology of guarantees

As the provision of guarantees generally entails costs for the debtor, the structure of these attendant costs must be taken into account in calculating the contractual flow implications of a proposed debt-for-debt exchange. Several rather distinct options for the contractual constitution of guarantees have been proposed in the literature. It is useful in this respect to distinguish on the one hand between specific guarantees and rolling guarantees and, on the other hand, between collateralized guarantees and guarantees structured along the lines of insurance contracts with nonrefundable premia.

When a specific guarantee is attached to a given payment, that payment is assumed to become riskless from the creditor's perspective. If for any reason the debtor does not make the specified payment in full on the due date, the guarantor agrees to recompense the creditor fully for the difference between the payment due and the actual payment made. Normally, the receipt of the payment from the guarantor would extinguish any remaining claims which the creditor might have on the debtor associated with the obligation which the debtor did not honor. However, the guarantor may assume the claim on the debtor.

In essence, a rolling guarantee is equivalent to a specific guarantee whose coverage is automatically extended forward, to the extent that it has not already been triggered, hence the term "rolling." A typical rolling guarantee would require the guarantor to pay to the creditor the value of any shortfall in the discharge of the debtor's contractual interest obligations over a specified time period, which may be as long as the maturity of the insured instrument. However, the guarantor's liability is capped at less than the nominal value of the obligations covered. If for example the cap is set at two year's worth of interest payments, the guarantor will make no additional payment to cover any additional shortfall after the first two years in which he has had to fully cover the debtor's shortfall. A set of obligations covered by a rolling guarantee may still bear residual risk. A creditor can only be absolutely certain of receiving payments equal in value to the ceiling on the guarantor's liability.

In addition to the distinction between specific and rolling guarantees, it is also useful to maintain a distinction between two different types of guarantee structures: collateralized guarantees and fee-based or premia-based guarantees. Collateralized guarantees entail the placement of resources into an escrow account (which becomes the guarantor) which is beyond the control of the debtor. Under well-specified conditions, the resources in the account will be delivered to the creditors should the debtor be unable or unwilling to meet certain guaranteed obligations. This would then retire, without penalty, the debtor's obligation to make the covered payment. 1/ However, if the debtor does discharge the specified obligation, such as the final principal payment, the escrowed resources are then returned to the debtor.

Collateralized guarantees effectively amount to a form of prepayment. This is most clear in the case of a specific guarantee. Generally, the debtor is required to invest in risk-free securities to ensure that on the date the insured payment is due the accrued resources in the escrow account are sufficient to discharge in full the covered obligation. On the due date, either the debtor will discharge the obligation and be almost immediately compensated by the return of the escrowed resources, or the debtor will not make the payment and the trustee of escrow account will make the payment for the debtor. In either case, the net cost to the debtor on the due date is effectively nil. Per definition it is assumed that with rolling collateralized guarantees, if the debtor does not make a specific payment in full on a given date, the trust fund, if it is not yet exhausted, will pay creditors the difference between what the debtor actually pays and what is owed. Moreover it is assumed that at the end of the guarantee period, the debtor receives whatever funds are left over in the trust fund. Effectively then, such a rolling collateralized guarantee constitutes a prepayment of some of the obligations at the end of the guarantee period, or earlier if replenishment is not required and the guarantee is triggered.

Alternatively the guarantee may be structured along the lines of a conventional insurance contract under which the debtor's initial payment is viewed as a nonrefundable premium payment. 2/ Such a payment represents a one-time outflow on the part of the debtor; if the guarantee is never triggered, the debtor is not reimbursed. On the other hand if the guarantee is triggered, the debtor still has an obligation to honor the payment which is in default, i.e., the debtor

---

1/ While this will cancel one obligation, the debtor may incur a new obligation to reconstitute the trust fund, if replenishment is required.

2/ The recent paper by Symansky and Tryon would appear to assume this sort of guarantee structure.

cannot use the resources of the guarantee fund to effectively discharge its obligations. 1/

In cases where the cost of a proposed guarantee is not fully spelled out in a proposal, an estimate will have to be constructed. This is a fairly straightforward exercise for the case of collateralized specific guarantees. Assuming the escrowed resources will grow at the risk-free rate of interest r, if the debtor puts

$$P \;/\; (1+r)^t \tag{1}$$

into escrow, where P is the amount of the insured payment, then at time t the escrow account will be able to fully cover the insured payment. Similarly, for rolling collateralized guarantees, the initial contribution will be equal to the value of the maximum liability of the escrow account, discounted to the present at the rate r. In the case of premium-based guarantees structured along the lines of insurance contracts with nonrefundable premia, the cost of the guarantee may be lower. This is because the guarantor may only charge the debtor his expected costs in covering for the debtor's shortfall. The cost of this premium therefore will be a function of the guarantor's perception of his expected loss on the contract. As this is a subjective calculation, it may be difficult to predict with precision.

To this point, most proposals for debt-for-debt swaps which have been brought to fruition (specifically the Mexico-Morgan transaction), agreed in principle (the recent agreement between Mexico and the steering committee of its commercial bank creditors) or have been the subject of serious negotiations between bank creditors and debtor countries have involved the exclusive use of collateralized guarantees. While guarantees structured along the lines of insurance contracts with nonrefundable premia cannot be ruled out a priori for the future, there is yet to be a case where the use of this contractual structure has received active consideration.

1/ The purchase of real estate in the U.S. offers an example where often both collateralized and nonrefundable premium-based guarantees are in existence simultaneously. A loan may be secured by a lien on the property being purchased. If the mortgage is discharged, the title of the property is transferred in full to the borrower. If the borrower falls into default, the property may be sold to a third party; the lender however, may only deduct from the proceeds sufficient funds to cover the remaining principal amount on the loan and a small penalty. In addition to securing a loan with a lien, the lender may require the borrower to purchase mortgage insurance. In the case of the mortgage insurance, if the borrower honors all of his obligations, he does not receive any asset in return. On the other hand if the borrower defaults, the insurer makes payments to the holder of the mortgage and attempts to collect from the borrower.

2. Net debt reduction

Once the implications of the proposed restructuring for the country's contractual obligations on a period-by-period basis are understood, summary statistics may be computed which conflate these changes to a single number, i.e., the net debt reduction. The net debt reduction is merely the sum of the net changes in each period, discounted to the present at some appropriate discount rate. This paper adopts the convention of using a risk-free rate, e.g., LIBOR or the long-term yield on U.S. Treasury obligations, although the formulas can be easily adapted to the use of alternatives; since the country faces significant credit constraints, it may be appropriate in some cases to adopt a higher discount rate reflecting the scarcity of funds. The calculation of net debt reduction allows a shorthand method of comparing various proposed transactions with each other and with such benchmarks as a buyback at the prevailing secondary market price. 1/

In certain cases, the calculation of the net debt reduction achieved through a given exchange reduces to the application of some rather simple formulas. In general, the amount of net debt reduction is equal to the present value of the old obligations retired minus that of the new costs incurred. The latter would include the present value of the contractual obligations on the new debt as well as the cost of any enhancements, less an adjustment for the present value of any obligations on the new debt which are effectively prepaid.

The present value of the obligations retired can be expressed as:

$$D_o \frac{I_o}{I_{rf}} \tag{2}$$

where D_o is the amount of old debt to be retired, I_{rf} is the risk free rate of interest (either LIBOR or the yield on risk-free government

---

1/ It is important to emphasize that this measure of net debt reduction is not necessarily an appropriate measure of the expected improvement in the country's welfare resulting from the debt exchange. A measure of the welfare impact, which would be more difficult to construct, would take into account the country's own expectations about its payments, the transactions costs associated with continual renegotiations and the country's opportunity cost of capital. A country that is credit constrained may prefer options which front-load debt relief even if the net debt reduction (using a discount rate which is lower than the country's opportunity cost of funds) is lower. Also, to give an example, a buyback would have an adverse welfare impact if the country expects to pay less than current secondary market prices would indicate.

securities) and I_o is the interest rate on the old debt which may include a spread over the risk-free rate. 1/

If the new debt has a bullet maturity, the present value of its associated contractual obligations can be taken as the sum of the present value of the final principal payment and the value of a perpetual interest stream which begins next year, minus the value of a perpetual stream which begins after the final principal payment is due, i.e., the value of the new obligations is given as:

$$D_n\left[\frac{1}{(1+I_{rf})^t}\right] + D_n \frac{I_n}{I_{rf}} - D_n \frac{I_n}{I_{rf}} \left[\frac{1}{(1 + I_{rf})^t}\right] \qquad (3)$$

where D_n is the principal amount of the new debt, t is the year when the principal is due and I_n is the interest charge on the new debt.

The cost of any enhancements needs to be factored in as well. This would be essentially the cost of funding any guarantees; assuming that the country can borrow to fund the guarantees at the risk-free rate, it makes no difference whether one explicitly accounts for when the country has to actually pay for the guarantees, i.e., whether the payment is upfront or over a period of time.

Finally, any payments which are explicitly or effectively prepaid need to be subtracted off from the costs. Hence in the case of collateralized guarantees, the country would either expect to receive the escrowed resources in return for honoring its obligations or would see some of its obligations retired directly by the resources in the escrow account. In cases where the guarantee structure follows the insurance premium model, there would be no rebate of paid-in premiums so this latter adjustment would be zero. Equations (4) and (5) present the full formulas for the net debt reduction achieved through a debt-for-debt swap under alternative assumptions that the guarantees are funded through escrow accounts (equation 4) or through the payment of nonrefundable premia (equation 5).

$$D_o \frac{I_o}{I_{rf}} - D_n \frac{I_n}{I_{rf}} \left[1 - \frac{1}{(1+I_{rf})^t}\right] - \frac{D_n}{(1+I_{rf})^t} \qquad (4)$$

1/ Strictly, this formula assumes that the old debt is effectively structured as a perpetuity; hence it must be viewed as something of an approximation. In making this calculation it is important to maintain consistency with regard to the rate of discount. If the old debt carried a spread over Libor and the chosen discount rate is the lower US Treasury bill rate, then the spread should be the spread over LIBOR plus the average spread between LIBOR and the Treasury bill rate.

$$D_o \frac{I_o}{I_{rf}} - D_n \frac{I_n}{I_{rf}} \left[1 - \frac{1}{(1+I_{rf})^t}\right] - \frac{D_n}{(1 + I_{rf})^t} - G \qquad (5)$$

It can be seen that the two expressions differ only by the exclusion of the term G, representing the cost of the guarantees, in equation 1. This is because this item nets out in the case of prepayment through collateralized guarantees.

III. The Analysis from the Creditors' Perspective

In the introduction it was noted that the intention of this paper was to present a framework for analyzing transactions which were feasible within the confines of a voluntary framework. 1/ In this section, a method is presented for estimating the cash value to a representative creditor of a new debt instrument enhanced by the provision of guarantees. It is argued that a proposed transaction will be feasible if the cash value of the new instruments offered per unit of old debt is greater than or equal to the cash value of the debt being retired. If this is not the case, the creditor could make himself better off by selling his current claims on the secondary market and purchasing the new claims through the secondary market; in that way he would be able to obtain more of the new instruments than the country is offering. 2/

The structure of this section is as follows: a general framework is exposited for valuing claims on a sovereign debtor. It is then shown how this framework can be applied to the valuation of a debt instrument enhanced through partial guarantees; specific guarantees are studied first and then the analysis turns to the complications related to rolling guarantees. It is explained that within the general framework it is not possible to construct precise estimates of the incremental contribution which guarantees make to the value of a debt instrument

---

1/ It must be recognized that there are important ambiguities involved in constructing a meaningful definition of "voluntary" and "feasible" in this context. For example, if all creditors believe with certainty that the country can and will make payments whose present value is equal to exactly one half of the present stock of debt, they would regard themselves as being no worse off if they all agreed to an across the board halving of the debt. Yet, given the behavior of the other creditors, any individual creditor might perceive it as being to his advantage not to agree to the proposed reduction. Thus while the the proposed reduction, is feasible if all creditors volunteer, it would not be feasible if individual creditors have the option not to volunteer.

2/ Assuming the efficiency of financial markets, the current secondary market price would reflect the market's current expectations about the most profitable use of the existing claims, including holding out for some more favorable debt exchange offer.

carrying sovereign risk. While more precise estimates are feasible given additional assumptions, the validity of the assumptions may be open to doubt. Therefore, boundary conditions are presented within which reasonable conjectures may be placed.

1. The valuation of payment claims carrying sovereign risk 1/

Individual creditors are assumed to value financial instruments (bonds or loans, including both the old and the new debt instrument) by reasoning along the following lines. Financial instruments give the creditor claims on the debtor which fall due at different points in time. Each claim is settled either in cash or through the issuance of new claims (which may be in the form of arrears). These new claims in turn give rise to cash payments and new claims. The present value of each original claim is ultimately the discounted value of expected associated cash payments, including the value of the cash payments on the new claims associated with that claim. The expected present value of the whole bundle (the instrument) is equal to the sum of the expected present values of each of the individual payment claims.

It is assumed that, due to sharing clauses in syndicated loan agreements and/or the absence of seniority for new claims, actual payments in any period made in respect to unguaranteed claims, are distributed among bank creditors according to their share in the total claims of that type. For example, if the payment due in a given period on the new bond represents 1 percent of the total commercial bank claims coming due in a given period, it assumed that whatever aggregate payment the debtor makes, the holder of the new bond will receive 1 percent of it. 2/ If on a given date in a given sequence, the debtor only pays half of the aggregate obligations and issues new instruments (such as new bonds, capitalized interest or arrears) for the remainder, then the bondholder will get 1 percent of that aggregate one half and receive

1/ The model here builds on the work Dooley, Symansky, and Tryon in constructing a nonstochastic dynamic model of payment claims on sovereign debtor. The analysis is extended by recognizing that different payment sequences may be possible and by taking into account the model's implications for the value of claims maturing at different points in times. As shown below, this extension is crucial to the analysis of collateralized debt exchanges as opposed to the conventional buybacks studied by these authors.

2/ It is of course possible that debtors may attempt to establish legal or de facto seniority on claims associated with new or old debt. If this is in fact feasible, this would further complicate the already ambiguous results presented below. In order to maintain some tractability, seniority is assumed away. However, it is worth noting that if new instruments which are exchanged for old debt are perceived by the market as carrying some seniority, then the valuation formulas suggested below would only provide lower bounds, since the enhancement value of the seniority of a new instrument is not factored in.

1 percent of the new claims. The total value of the bondholders' claim on that specific date will be the value of the expected cash payment plus the value of the new instruments. Those new instruments can then be evaluated by examining the value, claim by claim, of their new calls on the debtors' resources.

The aggregate payments are dependent on many factors, both political and economic, including of course the aggregate stock of claims in question, the initial wealth position of the debtor and competing financial claims which will be made on the debtor's income. Other factors could be the prices of key commodity imports and exports. Exact knowledge of how all the the political and economic factors would unfold and how they would affect the willingness of the debtor to pay, would lead to exact estimates of what the debtor will pay. However, creditors do not know what sequence of events will obtain. Rather they may imagine possibly many sequences. For each imaginable sequence, (e.g., the price of oil follows such and such a path, the economy grows at such and such a rate, political developments follow such and such path), the creditor can derive the payments which he would expect to receive. The creditor is then assumed to calculate a probability-weighted average of the value of the payments he would expect to receive under each of the imaginable payment sequences. This weighted average would be the value of the bond to the creditor.

It can be seen that the expected present value of the cash payments made in association with claims with equal present discounted contractual values but maturing at different points in time may be different. This is because, in addition to variations in expected aggregate payments across periods, there may be endogenous changes in the number of competing claims. This can be illustrated through a simple example. Suppose that a debtor owes $1 to each of one hundred creditors; the debt is structured as a perpetuity paying a 10 percent coupon. Thus, each year the debtor is obligated to pay $10 in the aggregate. Suppose further that the debtor is only willing to pay $5 per period for all periods, that he does so with certainty and that he issues new securities (at 10 percent) for the remainder of the obligations falling due each period. Then over time the debt will grow without bound and the price of the debt will fall toward zero. However the aggregate market value of the debt stock will under these assumptions stay constant.

Suppose now that one of these creditors agrees to sell to another creditor the right to all of the actual cash payments he expects to receive from the first years interest claim. 1/ The creditor selling the claim will lose 5 cents in the first period, and will also forego the interest on the new claim associated with the other half of the

---

1/ Equivalently, the creditor might sell his claims and repurchase them in one year. Hence he would forego the interest paid in the first year as well as any new claims issued in lieu of cash payments.

first years' interest claim. As Table 1 demonstrates, present value of the loss associated with the interest foregone on the new money claim would be 2.2 cents, which is just the discounted value of 5 cents times the secondary market price in the second period. Thus the creditor would have to be paid 6.7 cents upfront (or 7.4 cents after one year) to accept the exchange; given that the present value of the contractual obligation is 9.1 cents, the implicit price on the first year's interest payment would be about 74 cents on the dollar.

Suppose instead the creditor agreed to sell the right to all of the cash payments received in associations with the original contractual interest claims maturing from year twenty onwards. 1/ The reduction in discounted contractual obligations implied by such an exchange would be the present value of a consol which begins paying in twenty years, i.e., $1 times (1+r) raised to the power of minus nineteen or 16.4 cents. However, from the creditor's point of view, in terms of weakening his ability to collect his share of the aggregate payments, the effect is much smaller. As demonstrated in Table 2, by year twenty, obligations on new loans (in which the creditor in question shared) will outnumber obligations on the initial debt by over 2.5 to 1. Hence, by virtue of these claims, the creditor will expect to share in some of the aggregate payments made from year twenty onwards. Since the present value of the expected aggregate payments made during period 20 and afterwards would be $8.20, and a unit of original debt is expected to account for 1/355.8 of the total debt stock, the creditor would only have to be paid 2.3 cents, i.e., $8.20/355.8, this year to give up claims whose present discounted contractual value is over 16 cents.

2. Applying the model to debt-for-debt exchanges

In valuing a new instrument, enhanced by either specific or rolling guarantees, a creditor is assumed to reason along the following lines. First he values the instrument, payment claim by payment claim and sequence by sequence as suggested in the preceeding section, in the absence of any guarantees, i.e., assuming that all the payments carry

1/ Equivalently, the creditor might engage in a forward sale of the original perpetuity. The creditor would keep all the new claims he accumulated prior to the delivery date.

Table 1. The Value of a Contractual Interest Claim in Year One 1/

Year	Total Payments Due	of which: Payments Due on Orig. Debt	Payments Corresponding to Year 1 Interest Obligation	Payments Corresponding to Other Years Interest Obligations	Aggregate Payment Made	New Money (per period)	Payment per $ due (aggregate)	PV of Payments lost due to sacrifice of claim on Year 1 Interest	Cumulative Present Value of 1 percent of Total Payments
1	10.000	10	10.000		5	5.000	0.5000	0.04545	0.045
2	10.500	10	0.500	10.000	5	5.500	0.4762	0.00197	0.087
3	11.050	10	0.526	10.524	5	6.050	0.4525	0.00179	0.124
4	11.655	10	0.555	11.100	5	6.655	0.4290	0.00163	0.158
5	12.321	10	0.587	11.734	5	7.321	0.4058	0.00148	0.190
6	13.053	10	0.622	12.431	5	8.053	0.3831	0.00134	0.218
7	13.858	10	0.660	13.198	5	8.858	0.3608	0.00122	0.243
8	14.744	10	0.702	14.042	5	9.744	0.3391	0.00111	0.267
9	15.718	10	0.748	14.969	5	10.718	0.3181	0.00101	0.288
10	16.790	10	0.800	15.990	5	11.790	0.2978	0.00092	0.307
11	17.969	10	0.856	17.113	5	12.969	0.2783	0.00083	0.325
12	19.266	10	0.917	18.348	5	14.266	0.2595	0.00076	0.341
13	20.692	10	0.985	19.707	5	15.692	0.2416	0.00069	0.355
14	22.261	10	1.060	21.201	5	17.261	0.2246	0.00063	0.368
15	23.987	10	1.142	22.845	5	18.987	0.2084	0.00057	0.380
16	25.886	10	1.233	24.654	5	20.886	0.1932	0.00052	0.391
17	27.975	10	1.332	26.643	5	22.975	0.1787	0.00047	0.401
18	30.272	10	1.442	28.831	5	25.272	1.1652	0.00043	0.410
19	32.800	10	1.562	31.238	5	27.800	0.1524	0.00039	0.418
20	35.580	10	1.694	33.885	5	30.580	0.1405	0.00035	0.426
21	38.637	10	1.840	36.798	5	33.637	0.1294	0.00032	0.432
22	42.001	10	2.000	40.001	5	37.001	0.1190	0.00029	0.439
23	45.701	10	2.176	43.525	5	40.701	0.1094	0.00027	0.444
24	49.772	10	2.370	47.401	5	44.772	0.1005	0.00024	0.449
25	54.249	10	2.583	51.665	5	49.249	0.0922	0.00022	0.454
26	59.174	10	2.818	56.356	5	54.174	0.0845	0.00020	0.458
27	64.591	10	3.076	61.515	5	59.591	0.0774	0.00018	0.462
28	70.550	10	3.360	67.190	5	65.550	0.0709	0.00017	0.465
29	77.105	10	3.672	73.433	5	72.105	0.0648	0.00015	0.468
30	84.315	10	4.015	80.300	5	79.315	0.0593	0.00014	0.471
31	92.247	10	4.393	87.854	5	87.247	0.0542	0.00012	0.474
32	100.972	10	4.808	96.164	5	95.972	0.0495	0.00011	0.476
33	110.569	10	5.265	105.304	5	105.569	0.0452	0.00010	0.478
34	121.126	10	5.768	115.358	5	116.126	0.0413	0.00009	0.480
35	132.738	10	6.321	126.417	5	127.738	0.0377	0.00008	0.482
36	145.512	10	6.929	138.583	5	140.512	0.0344	0.00008	0.484
37	159.563	10	7.598	151.965	5	154.563	0.0313	0.00007	0.485
38	175.020	10	8.334	166.685	5	170.020	0.0286	0.00006	0.487
39	192.022	10	9.144	182.878	5	187.022	0.0260	0.00006	0.488
40	210.724	10	10.034	200.689	5	205.724	0.0237	0.00005	0.489
41	231.296	10	11.014	220.282	5	226.296	0.0216	0.00005	0.490
42	253.926	10	12.092	241.834	5	248.926	0.0197	0.00004	0.491
43	278.818	10	13.277	265.541	5	273.818	0.0179	0.00004	0.492
44	306.200	10	14.581	291.619	5	301.200	0.0163	0.00004	0.492
45	336.320	10	16.015	320.305	5	331.320	0.0149	0.00003	0.493
46	369.452	10	17.593	351.859	5	364.452	0.0135	0.00003	0.494
47	405.898	10	19.328	386.569	5	400.898	0.0123	0.00003	0.494
48	445.987	10	21.237	424.750	5	440.987	0.0112	0.00002	0.495
49	490.086	10	23.337	466.749	5	485.086	0.0102	0.00002	0.495
50	538.595	10	25.647	512.947	5	533.595	0.0093	0.00002	0.496
51	591.954	10	28.188	563.766	5	586.954	0.0084	0.00002	0.496
52	650.650	10	30.983	619.666	5	645.650	0.0077	0.00002	0.496
53	715.215	10	34.058	681.157	5	710.215	0.0070	0.00002	0.497
54	786.234	10	37.440	748.796	5	781.236	0.0064	0.00001	0.497
55	864.360	10	41.160	823.200	5	859.360	0.0058	0.00001	0.497
56	950.296	10	45.252	905.044	5	945.296	0.0053	0.00001	0.498
57	1,044.825	10	49.754	995.072	5	1,039.825	0.0048	0.00001	0.498
58	1,148.808	10	54.705	1,094.103	5	1,114.808	0.0044	0.00001	0.498
59	1,263.189	10	60.152	1,203.037	5	1,258.189	0.0040	0.00001	0.498
60	1,389.007	10	66.143	1,322.864	5	1,384.007	0.0036	0.00001	0.498

1/ This table tracks the income foregone by a creditor who sells to another creditor his year one interest claim. It is assumed that there are 100 creditors holding perpetuities entitling them to a payment of $.10 per year. The debtor is assumed to be willing to make aggregate payments of $5 per year which is divided among creditors in proportion to the number of claims which they are holding; column eight details the rate at which current claims are cancelled with cash payments each period. To the extent creditors' claims are unpaid each period, it is assumed that they receive new claims carrying an interest charge of 10 percent. The seventh column tracks the number of such new claims issued each period. Column nine shows the present value of the cash foregone each period by the creditor assenting to the transaction. The cumulative present value of the per-period losses is equal to 6.7 cents.

Table 2. The Value of Contractual Interest Claims Maturing After Year Nineteen 1/

Year	Total Payments Due	of which: Payments Due on Orig. Debt	Payments Corresponding to Interest Obligations After Year 19	Payments Corresponding to Interest Obligations Before Year 20	Aggregate Payment Made	New Money (per period)	Payment per $ due (aggregate)	PV of Payments lost due to sacrifice of interest claims After Year 19	Cumulative Present Value of 1 percent of Total Payments
1	10.000	10	—	10.000	5	5.000	0.5000	—	0.045
2	10.500	10	—	10.500	5	5.500	0.4762	—	0.087
3	11.050	10	—	11.050	5	6.050	0.4525	—	0.124
4	11.655	10	—	11.655	5	6.655	0.4290	—	0.158
5	12.321	10	—	12.321	5	7.321	0.4058	—	0.190
6	13.053	10	—	13.053	5	8.053	0.3831	—	0.218
7	13.858	10	—	13.858	5	8.858	0.3608	—	0.243
8	14.744	10	—	14.744	5	9.744	0.3391	—	0.267
9	15.718	10	—	15.718	5	10.718	0.3181	—	0.288
10	16.790	10	—	16.790	5	11.790	0.2978	—	0.307
11	17.969	10	—	17.969	5	12.969	0.2783	—	0.325
12	19.266	10	—	19.266	5	14.266	0.2595	—	0.341
13	20.692	10	—	20.692	5	15.692	0.2416	—	0.355
14	22.261	10	—	22.261	5	17.261	0.2246	—	0.368
15	23.987	10	—	23.987	5	18.987	0.2084	—	0.380
16	25.886	10	—	25.886	5	20.886	0.1932	—	0.391
17	27.975	10	—	27.975	5	22.975	0.1787	—	0.401
18	30.272	10	—	30.272	5	25.272	0.1652	—	0.410
19	32.800	10	—	32.800	5	27.800	0.1524	—	0.418
20	35.580	10	10.000	25.580	5	30.580	0.1405	0.00209	0.426
21	38.637	10	10.859	27.778	5	33.637	0.1294	0.00190	0.432
22	42.001	10	11.805	30.196	5	37.001	0.1190	0.00173	0.439
23	45.701	10	12.845	32.857	5	40.701	0.1094	0.00157	0.444
24	49.772	10	13.989	35.783	5	44.772	0.1005	0.00143	0.449
25	54.249	10	15.247	39.002	5	49.249	0.0922	0.00130	0.454
26	59.174	10	16.631	42.542	5	54.174	0.0845	0.00118	0.458
27	64.591	10	18.154	46.437	5	59.591	0.0774	0.00107	0.462
28	70.550	10	19.829	50.721	5	65.550	0.0709	0.00097	0.465
29	77.105	10	21.671	55.434	5	72.105	0.0648	0.00089	0.468
30	84.315	10	25.927	60.618	5	79.315	0.0593	0.00081	0.471
31	92.247	10	28.379	66.320	5	87.247	0.0542	0.00073	0.474
32	100.972	10	31.077	72.593	5	95.972	0.0495	0.00067	0.476
33	110.569	10	34.044	79.492	5	105.569	0.0452	0.00061	0.478
34	121.126	10	37.307	87.082	5	116.126	0.0413	0.00055	0.480
35	132.738	10	40.898	95.431	5	127.738	0.0377	0.00050	0.482
36	145.512	10	44.897	104.614	5	140.512	0.0344	0.00045	0.484
37	159.563	10	49.191	114.716	5	154.563	0.0313	0.00041	0.485
38	175.020	10	53.970	125.829	5	170.020	0.0286	0.00038	0.487
39	192.022	10	59.226	138.052	5	187.022	0.0260	0.00034	0.488
40	210.724	10	65.008	151.498	5	205.724	0.0237	0.00031	0.489
41	231.296	10	71.369	166.288	5	226.296	0.0216	0.00028	0.490
42	253.926	10	78.365	182.557	5	248.926	0.0197	0.00026	0.491
43	278.818	10	86.061	200.140	5	273.818	0.0179	0.00023	0.492
44	306.200	10	94.526	220.140	5	301.200	0.0163	0.00021	0.492
45	336.320	10	103.838	241.794	5	331.320	0.0149	0.00019	0.493
46	369.452	10	114.082	265.614	5	364.452	0.0135	0.00018	0.494
47	405.898	10	125.349	291.816	5	400.898	0.0123	0.00016	0.494
48	445.987	10	137.744	320.638	5	440.987	0.0112	0.00014	0.495
49	490.086	10	151.378	352.342	5	485.086	0.0102	0.00013	0.495
50	538.595	10	166.375	387.217	5	533.595	0.0093	0.00012	0.496
51	591.954	10	182.872	425.579	5	586.954	0.0084	0.00010	0.496
52	650.650	10	201.018	467.778	5	645.650	0.0077	0.00009	0.496
53	715.215	10	220.980	514.196	5	710.215	0.0070	0.00008	0.497
54	786.234	10	242.937	565.256	5	781.236	0.0064	0.00007	0.497
55	864.360	10	267.090	621.422	5	859.360	0.0058	0.00007	0.497
56	950.296	10	293.659	683.205	5	945.296	0.0053	0.00006	0.498
57	1,044.825	10	322.884	751.166	5	1,039.825	0.0048	0.00006	0.498
58	1,148.808	10	355.032	825.923	5	1,114.808	0.0044	0.00005	0.498
59	1,263.189	10	390.395	908.156	5	1,258.189	0.0040	0.00005	0.498
60	1,389.007	10	429.294	998.613	5	1,384.007	0.0036	0.00004	0.498

1/ This table tracks the income foregone by a creditor who sells to another creditor all of his original interest claims falling due from year twenty onwards; any new claims which the creditor receives in association with previous partial payments are assumed to be retained by the creditor. It is assumed that there are 100 creditors holding perpetuities entitling them to a payment of $.10 per year. The debtor is assumed to be willing to make aggregate payments of $5 per year which is divided among creditors in proportion to the number of claims which they are holding; column eight details the rate at which current claims are cancelled with cash payments each period. To the extent creditors' claims are unpaid each period, it is assumed that they receive new claims carrying an interest charge of 10 percent. The seventh column tracks the number of such new claims issued each period. Column nine shows the present value of the cash foregone each period by the creditor assenting to the transaction. The cumulative present value of the per-period losses is equal to 2.3 cents.

country risk. Then the creditor will add on the incremental value of the payments which he expects to receive from the guarantor. 1/

It is very important to distinguish between incremental value and payments actually made by the guarantor, which may be larger. In the absence of guarantees, if the debtor is unable to discharge his contractual obligations according to the specified schedule the creditor receives some new claim instead. Sometimes the provision of this claim is on a firm contractual basis, i.e., creditors make a concerted new money loan to allow the debtor to pay his interest obligations, and sometimes it is less regularized, i.e., the debtor incurs arrears. In contrast, if a guarantee is triggered and the creditor receives a payment (or effective payment) 2/ from the guarantor, the creditor normally does not get a new claim. The incremental value of a guarantee

---

1/ Here we ignore any costs experienced by the guarantor as well as any payments made to the guarantor, as the guarantor is assumed to be some third party, e.g., a trust account, whose welfare is of no consequence to the debtor. However, Symansky and Tryon analyze a contractual structure whereby the creditor is effectively assumed to play the role of the guarantor, since, even if the guarantee is only partially triggered, the creditor is assumed to be able to keep all the resources committed to fund the guarantee. In that case the value of payments received by the creditor just offsets his costs in acting as the guarantor so that the incremental value of the guarantee being triggered is zero. Since the payments for funding the guarantee are effectively made to the creditor, the incremental value of the guarantee is always exactly equal to the initial amount of money contributed to fund the guarantee. Under Symansky and Tryon's assumptions, the valuation exercise reduces to taking the sum of the risk-adjusted value of the instrument without a guarantee plus the cost to the debtor of the guarantee. The transaction then is almost identical, both in terms of its contractual flow impact and its valuation by creditors, to a combination of buyback and a par exchange of unguaranteed debt. While this structure has certain advantages for analytical tractability, it is unlike most of the exchanges currently under active consideration in the market.

2/ Collateralized guarantees may never be formally triggered. Since the debtor receives the escrowed funds in return for making the covered payment, he may be in a position to make the payment directly, since it implies no net cost. However, in that case it still would be appropriate to say that the guarantee was effectively triggered since the return of the escrowed resources might allow the debtor to make a payment which otherwise would not be made.

is then the payment actually received from the guarantor less the value of the new claim foregone. 1/

Implicit in this approach is an assumption that the addition of a guarantee is not expected to change the behavior of the debtor towards the covered or the uncovered payments. In part this reflects an assumption that the debtor does not accord any more or less seniority to the new debt than to the old debt.

a. Specific guarantees

Under these conditions, the valuation of the contribution of a specific guarantee would be in a certain sense be fairly straightforward. In the absence of the guarantee, the creditor would have expected to receive, in association with the payment to be guaranteed, some cash payment on the due date as well as some new claim which would have some value greater than or equal to zero. As a result of adding the specific guarantee, the payment in question is no longer risky, but under the circumstances which would be expected to cause the debtor to miss the payment in question, the creditor would not expect to receive a new claim. Hence the incremental value of guaranteeing any specific payment is clearly the difference between the full present value of the contractual obligation and the value of the payment obligation in the absence of the guarantee. Thus the present value of the guaranteed instrument can be expressed as the risk adjusted present value of the instrument in the absence of the guarantee plus the full present value of the payment obligation to be guaranteed less the risk adjusted value of the payment obligation to be guaranteed.

Alternatively and equivalently, the value of the guaranteed instrument can be expressed as the sum of the full present value of the riskless guaranteed payments, e.g. the amortization obligation in the case of the Mexico-Morgan exchange, plus the present value of the unguaranteed payments, e.g. the interest obligations, adjusted to take into account their riskiness. The argument is reprised in Table 3.

Figuring out how the creditor will evaluate the remaining risky payments is not a straightforward matter. Unfortunately, the secondary market price may not necessarily be a very helpful guide. One problem arises in that the average discount on the value of currently outstanding claims reflects the aggregate payments the debtor is expected

1/ To illustrate the difference, consider the case where the debtor is expected with certainty to miss the first year's interest payment. In the absence of the guarantee, the creditor expects with certainty to receive in return for the unpaid interest a new claim whose value is near the current secondary market price for the country's debt. In the presence of the guarantee, the creditor would receive the full interest payment but not the new claim. Hence the incremental value may be significantly less then the cash payment made by the guarantor.

Table 3. The Enhancement Value of a Specific Guarantee

Value of Guaranteed Instrument =

Risk adjusted value of identical instrument without guarantees

+ Incremental value of payments received from the guarantor

= Risk adjusted value of identical instrument without guarantees
+ Expected value of payments from the guarantor

- Expected value of claims foregone in return for payment from guarantor

= Risk adjusted value of identical instrument without guarantees

+ Present value of contractual obligation to be guaranteed

- Expected value of cash payments which would be made on claims which are to be covered by guarantees

- Expected value of claims foregone in return for payment from guarantor

= Risk adjusted value of identical instrument without guarantees

+ Present value of contractual obligation to be guaranteed

- Risk adjusted value of payment obligations to be guaranteed

= Risk adjusted value of payment obligations not covered by guarantees

+ Risk adjusted value of payment obligations to be guaranteed

+ Present value of contractual obligation to be guaranteed

- Risk adjusted value of payment obligations to be guaranteed

= Risk adjusted value of payment obligations not covered by guarantees

+ Present value of contractual obligation to be guaranteed to make and the number of claims to share in those aggregate payments. When a country engages in a self-financed debt reduction operation, it both reduces the number of competing bank claims and its available resources which enable it to make aggregate payments (this assumes that if the operation is financed through borrowed resources, that the loans to finance the enhancements are serviced ahead of the bank loans). On the assumption that the creditor in question has a small share of the country's total bank debt, he would regard his exchange as having a negligible impact on both the stock of competing claims and the country's capacity to repay. Therefore, he might appropriately use the pre-exchange period-by-period value of claims to guide his post-exchange valuation of the new claims. Nonetheless, such a procedure would be open to question if simultaneously many other marginal creditors were also engaging in debt exchanges.

This difficulty arises of course in the case of buybacks as well. Analysts have suggested conditions under which a self-financed buyback could cause the price of the remaining debt to either rise or fall. 1/ In the absence of further assumptions one can only say that the secondary market price should be a good predictor of the cost of purchasing a marginal amount of debt. However, under the assumption that the expenditure of resources to buyback debt lowers the expected present value of the subsequent aggregate payments by the debtor to the banks by an equivalent amount, a self-financed buyback would have no impact on the secondary market price, regardless of the proportion of debt repurchased.

A similar convention will be adopted here. It is assumed that the transaction in question is marginal (in which case there are no significant price effects under any assumptions) or that following an asset exchange in which the debtor issues new enhanced debt instruments, the expected present value of the debtor's aggregate payments to banks will decline by the net expenditure which the debtor has made to fund the new enhancements. The net expenditure is defined as the initial cost expended by the debtor less any resources which the creditors

---

1/ For an overview of the conditions under which a buyback might cause the price of the debt to rise or fall, see Diwan and Claessens, "An Analysis of Debt-Reduction Schemes Initiated by Debtor Countries," PPR Working Paper WPS 153 (Washington: International Bank for Reconstruction and Development, March 1989).

expect the debtor to receive back from the collateral fund in return for honoring certain guaranteed obligations. 1/

Unfortunately, even under these not necessarily uncontroversial assumptions, the secondary market price of the old debt may not be a very helpful guide as it gives only an average value of all the claims arising from the old debt. 2/ If the new debt had uninsured payments falling due on all the same dates as the old debt and in the same proportions (i.e., if the instrument's uninsured obligations were just a reduced proportion of the old uninsured obligations), then it would be natural to apply the secondary market price to the uninsured payments on the new debt. However, the new instrument may in fact reallocate the profile of risky payments, moving them forward or backwards in time. For example, if the old debt is structured as a perpetuity, then the contractual repayment obligations are constant over time. However, the new instrument may insure the first three years of interest. Then the risky payments are all concentrated in year four onwards.

Confronted with the need to construct a reasonable conjecture for how market participants will evaluate the residual risk in a new instrument, a number of approaches may be considered. One possible solution would be to construct bounds on the average value of the uninsured payments (or equivalently on the incremental value of the payment guarantees). Under this approach, the new instruments are valued under assumptions that the unguaranteed payments are as risky or as riskless as can be imagined. The only constraints which are imposed are that the perception of riskiness of the unguaranteed payments not be inconsistent with the prevailing secondary market price for unguaranteed obligations. For example, if the current secondary market price is 50 cents on the dollar, and a new instrument is to be created which is identical in payment profile with the previous debt instrument except that 40 percent of its obligations are fully guaranteed, it would be inconsistent with a

---

1/ For example, suppose the debtor was expected to initially be willing to make aggregate payments to the banks equal to $100. In an exchange the debtor retires old debt for new debt with $20 of enhancements, say in rolling interest guarantees. If the debtor, for arguments sake, is expected to keep up with his obligations for a while, he might be expected to receive $5 back from the guarantee fund while the creditor receives the remaining $15. This assumption would require that creditors expect that after the exchange, the debtor's total payments to banks would be equal to $85, i.e., $100 - $20 + $5. Hence, both before and after the exchange, the creditors expectation of the total net cash flow from the debtor to the banks, including what the creditor expects to receive from the guarantee, would be $100.

2/ The secondary market prices for different tranches of Brazil's or Mexico's debt are differentiated, reflecting different maturity profiles and possibly degrees of perceived seniority. This further suggests that equal claims maturing at different points in time have different values.

50 percent discount to assume that the risk adjusted aggregate value of the unguaranteed payments was equal to zero; the aggregate value of the unguaranteed payments (with a contractual value of 60 cents) would have to be at least 10 cents and no more than 50 cents. 1/

Unfortunately, as this example shows, the maximum theoretical bounds can be very wide. Also the bounds will be time dependent; the bounds for the incremental value of a guarantee on a payment due in year one will differ from the bounds on a payment due at the end of the contractual period. Under the most extreme assumptions, the incremental value of a guarantee on a payment due in the very near future cannot be greater than the present value of the contractual obligation times one minus the current secondary market price. However, it could be zero, if the payment was expected to be made anyway. In contrast, in the context of the implicit model suggested in Section III.1. which relates payments to the volume of outstanding claims, it would be inconsistent to assume that later payments can be made in full while earlier payments are defaulted upon completely; if cash payments are only made after a certain point in time, some of the cash must be ascribable to the new instruments issued in lieu of earlier cash payments. However the opposite could be true; the debtor may make payments for a while and then default. Thus the theoretical bounds, upper and lower, on the incremental value of a specific guarantee tend to rise as the guarantee is shifted to payments which come due later in the profile of contractual obligations.

1/ The value of the instrument in the absence of any guarantees is by definition equal to MVG + MVU, where MVG is the market value of the payment claims to be guaranteed and MVU is the market value of the payment claims to be left unguaranteed. Therefore it must be the case that so long as the proportion of the present discounted contractual value which is guaranteed is less than the secondary market price, i.e., that

$$PVG/(PVG+PVU) < p,$$

where p is the secondary market price of the entire instrument in the absence of any guarantee, and PVG and PVU represent the present value of the nominal contractual obligations to be guaranteed and to be left unguaranteed respectively, that the market value (or the expected present value of the cash payments made in association with these claims) of the unguaranteed claims (MVU) must be less than or equal to the market value of the entire debt stock and greater than or equal to the market value of the debt stock minus the present value of the contractual obligations to be guaranteed, i.e.,

$$p(PVU+PVG) \geq MVU \geq p(PVG+PVU) - PVG$$

This follows from the fact that MVU + MVG = p[PVU + PVG].

It is beyond the scope of the paper project to present a formal proof of these results. 1/ However, below in equations (6) through (9) upper and lower bounds are presented for an instrument carrying a specific guarantee; it is assumed that the new instrument is structured identically to the previous instrument, i.e., that the payment obligations are shrunk equiproportionately. The first two equations present the bounds assuming that all payments coming due before a certain date are guaranteed while the second two equations assume that all payments coming due after a certain date are guaranteed.

$$\text{Upper Bound} = \rho .(PVG + PVU) + (1- \rho).PVG \tag{6}$$

$$\text{Lower Bound} = \rho .(PVG + PVU) \tag{7}$$

$$\text{Upper Bound} = \rho .(PVG + PVU) + PVG \tag{8}$$

$$\text{Lower Bound} = \rho .(PVG + PVU) + (1- \rho).PVG \tag{9}$$

PVG corresponds to the present value of the contractual obligations to be guaranteed and PVU corresponds to the present value of the

---

1/ The intuition of course is quite straightforward. In each case one needs to ask how valuable the guaranteed payments would be if there were no guarantee. If all the payments up to a certain point are guaranteed, it is not inconceivable that the guarantee is of no use, since the payments are expected to be made anyway, hence equation (7) which implies that the guarantee has no incremental value. However, if payment obligations after a certain point are expected to be met, then, because of the assumed sharing sale, previous payment obligations must be ultimately honored as well. Given the secondary market discount, the probability of that happening must be circumscribed. This leads to (9), which tells us that probability can be no more than ρ . At the other extreme, the debtor may be expected to miss all of the guaranteed payments, yet he must be expected to make some payments, otherwise the debt would be worthless. If the aggregate payments begin at a later stage, the new claims associated with the missed payments will be serviced at the same rate as the outstanding later maturing original claims, i.e., all claims would be serviced at the rate ρ . This leads to (6). However, if all the aggregate payments are expected to take place earlier in time, holders of later maturing claims cannot expect to share in them; hence, in the absence of the guarantee the later maturing claims would be valueless. This leads to (8), which implies that the incremental value of the guarantee is equal to the value of the guaranteed payment.

contractual obligations not covered by guarantees. 1/ The sum of PVG and PVU thus equals the present value of the entire stream of contractual obligations. ρ is equal to the ratio of current market value of the entire debt stock of the same class as the debt in question to the present discounted contractual obligations on that debt, and is thus equal to the conventionally quoted secondary market price with an adjustment to take into account the fact that the interest rate on the existing debt may be higher than the risk-free rate, i.e.,

$$\rho = p(I_{rf}/I_o) \tag{10}$$

where p is the conventionally defined secondary market price.

Given the width of these bounds it would also be useful to construct appropriately understood point estimates or conjectures. Several rules of thumb have been developed by various observers and market participants. One easily employed rule-of-thumb assumes that all payments are equally risky. In that case, the current secondary market price would be an appropriate risk factor by which to multiply the present value of the uninsured payments; equivalently the incremental value of a guarantee is just one minus the current secondary market price times the value of the payment to be guaranteed. In equation form, this rule would result in the following expressions:

$$\begin{aligned} \text{Value of guaranteed instrument} &= \rho\,(PVU) + PVG \qquad (11) \\ &= \rho\,(PVU + PVG) + (1-\rho)(PVG) \end{aligned}$$

It is interesting to note that while this approach yields a valuation corresponding to the theoretical upper bound when the guaranteed payments all fall due at the beginning of the payment period, it offers a conservative approach to valuing instruments where the guarantees are extended to payments due at the end of the payment period.

1/ The present value of contractual obligation due at time t is given by

$$P/(1 + I_{rf})^t$$

where P is the payment obligation.

The present value of a constant stream of interest payments between time t1 and time t2 is given by

$$\frac{D.I_c}{I_{rf}}\left[\frac{1}{(1 + I_{rf})^{t1}} - \frac{1}{(1 + I_{rf})^{t2}}\right]$$

where D is the principal amount, I_c is the contractual rate and I_{rf} is the risk free rate of interest.

An alternative approach builds from market participants' apparent fondness for thinking in terms of spreads over the risk-free rate needed to compensate them for risky transactions. This implies that due to compounding, the longer is the waiting period for payment, the higher is the risk premium. One can invert the secondary market price to derive an implicit "spread". For example, if the outstanding debt is a perpetuity and trades for 50 cents on the dollar, this would imply that creditors are evaluating its contractual claims by discounting them at a rate equal to twice the contractual rate of interest. Using this sort of model, the uninsured payments on the new bond would be valued by discounting them to the present at this risk-adjusted discount rate. If the risk-free rate is ten percent and the risk-adjusted rate is twenty, then the risk-adjusted present value of a payment maturing in one year would be (1+0.1)/(1+0.2) or 91.7 percent of the present value of that contractual obligation discounted to the present at the risk free rate. Similarly the risk-adjusted present value of a payment maturing in year two would be 84 percent of the unadjusted present value and so on. Using this formulation, the value of an instrument with a bullet maturity which carries a guarantee on the final principal payment would be:

$$\rho\ (PVG+PVU) + PVG[1 - \frac{(1 + I_{rf})^t}{(1 + d)^t}] \qquad (12)$$

where d would be the risk-adjusted discount rate and t is the maturity date. If the old debt is effectively viewed as a consol, then

$$d = I_o/p \qquad (13)$$

The differences between the two models and their relation to the boundaries can be illustrated by examination of Table 4 which gives the cash value to creditors of a proposed collateralized debt instrument. In the case where the new debt is enhanced by collateralizing the final principal payment, due in 20 years (or equivalently, all interest payments from year twenty onwards) and when the secondary market is equal to 50 cents, the cash value of the bond is 4.9 cents less per dollar of new debt for participants using the equal risk as opposed to the exponential risk formula. As a result, in a debt-for-debt exchange, participants using the equal risk model would offer 9.8 cents less old debt in exchange for a dollar of new debt. In contrast, if instead the first year's interest payment is guaranteed, the cash value would be 3.7 cents higher for participants using the equal risk formula.

b. Rolling guarantees

The provision of rolling guarantees is a newer feature in the menu of enhancement options. As a result, fewer published analyses have been made offering rules of thumb for estimating how the market will evaluate enhancements of this type. Also, the lack of experience with this type of instrument does not allow the comparison of any model's predictions

Table 4: Alternative Valuations of a Debt Instrument Enhanced Through a Specific Guarantee

Case I: Guarantee Applied to Interest Payment in Year One

Secondary Market Price 1/	Cash Value of $100 of Enhanced Debt 2/3/			
	Lower Bound	Equal Risk Rule	Exponential Risk Rule	Upper Bound
0.60	60.0	63.6	60.5	63.6
0.50	50.0	54.5	50.8	54.5
0.33	33.3	39.4	34.7	39.4

Case II: Guarantee Applied to Principal Payment in Year Twenty

Secondary Market Price 1/	Cash Value of $100 of Enhanced Debt 2/3/			
	Lower Bound	Equal Risk Rule	Exponential Risk Rule	Upper Bound
0.60	65.9	65.9	70.3	74.9
0.50	57.4	57.4	62.3	64.9
0.33	43.2	43.2	47.7	48.2

1/ Ratio of market value to value of present discounted value of contractual obligations on unguaranteed debt.

2/ Calculations assume that debt carries an interest charge of 10 percent, equal to the risk-free rate of interest.

3/ If enhancement is through a collateralized guarantee, the enhancement cost would be 9.1 cents on the dollar.

4/ If enhancement is through a collateralized guarantee, the enhancement cost would be 14.9 cents on the dollar.

with experience. Moreover, it would appear that there is not a firm consensus on precisely how these guarantees are likely to be structured; as some of the ambiguities may have important consequences for the valuation of instruments carrying these enhancements, they will need to be kept in mind.

A possibly important issue for both the valuation of a rolling guarantee and the analysis of its contractual flow implications involves whether the guarantee is specified in nominal terms or effectively in present value terms. The latter could occur if the guarantee were implemented through the creation of collateral accounts; if the collateral account's interest earnings are retained in the account, the nominal coverage of the guarantee can be extended so long as the guarantee does not get triggered. However, in present value terms the coverage would be held constant as the nominal coverage would be growing at the same rate as the discount factor. Given an initial nominal coverage, banks naturally would prefer to receive a guarantee specified in present value rather than nominal terms. They can be expected to retire fewer of the old obligations in exchange for the new debt instruments if they are sure that the expected present value of payments from a given escrow account will be lower because of the skimming off of interest earnings. On the other hand, if the guarantee is established through the maintenance of a collateral account, debtors may prefer to receive the interest concurrently as they are keeping current on their obligations rather than receiving relief at a later stage.

In the one case to date where an agreement involving rolling guarantees is close to finalization, the agreement has been structured through the establishment of collateral accounts. So long as the guarantee does not get triggered, the interest earnings accrue to the debtor and not to the escrow account. 1/ Moreover, the obligation to replenish the collateral fund should the guarantee be triggered, which had at one point been part of the creditors' proposal, is not explicitly required by the finalized term sheet.

As a general matter, the valuation of an instrument carrying a rolling guarantee would begin, as in the case of a specific guarantee, with an examination of its incremental contribution to the value of the instrument to be guaranteed. Because of the possibly high probability that a rolling guarantee would actually be triggered, it is very important in this context to be aware of the distinction between the

---

1/ If the guarantee is triggered and the escrow account is depleted, the contract may require the channeling of the collateral fund's interest earnings into the replenishment of the account or may allow the debtor to continue to receive whatever interest is earned by the escrowed resources. So long as the percentage of covered interest oblgations is small, the pricing implications of these alternative structures should be small.

probability the guarantee gets triggered and the value of its incremental contribution. 1/

As with a specific guarantee, the incremental contribution which a rolling guarantee would make under any imaginable sequence of payments would be the value of the cash payment received from the guarantor (or the escrow account) less the value of a new claim on the debtor which would be foregone as a consequence of the payment. Hence, in order to assess the incremental contribution, a judgement is needed as to the likelihood that the guarantee is triggered, and then a judgement as to the likely value of new claims on the debtor at the time the guarantee gets triggered. If the rolling guarantee extends to all payments (as we shall assume throughout), then the probability of it being triggered must be at least $1-\rho$; otherwise the probability of full payment on the debt would be greater than ρ which would mean that the expected value of payments made associated with the original debt divided by the contractual value of the payments would be greater than ρ . Furthermore, if creditors imagine that triggering is relatively unlikely, i.e., that the probability is close to $1-\rho$, then they must imagine that the value of new claims on the debtor, conditional on the guarantee being triggered, must be close to zero. Any other assumption would lead to a contradiction along the lines that creditors were expecting an average level of payments greater than the current discount in the secondary market would imply. Hence in the case where creditors expect the debtor to either always make payments or never make payments, the incremental contribution of the guarantee must be equal to $(1-\rho)$ times the amount of money set aside to fund the guarantee.

1/ It is possible to have a high probability of triggering and a low contribution of incremental value. For example even a creditworthy borrower may have need of exceptional financing from time to time. If a guarantee were triggered by such a borrower encountering temporary difficulties, the creditor would get little incremental value because in receiving a payment from the collateral fund, he would forego making a low-risk new money loan. Moreover, the possibility of moral hazard exists; if the debtor refuses to make a payment which he would have otherwise made as a result of the guarantee, the creditor might receive no more money than he would have expected to get if the guarantee did not exist.

If alternatively there is a replenishment obligation, if the guarantee is triggered the debtor will be required to reconstitute the escrow account. Thus, the debtor cannot use the guarantee fund to pay current obligations at his discretion. Instead, in states of the world where the debtor needs new money to meet his current obligations, the creditor will first receive the funds he is being asked to refinance; thus the bargaining position is shifted in the creditors' favor.

Similarly, it is possible that creditors may imagine that with certainty the guarantee eventually will be triggered, although considerable uncertainty may remain about exactly when the guarantee would be triggered. If the guarantor's liability is specified in present value terms this would not cause problems for the evaluation of the expected present value of the cash payment received, since it would be the same regardless of when the guarantee is triggered. However, the value of the foregone claim will be sensitive to when the guarantee is expected to be triggered. For example, if in all imaginable sequences, the debtor is expected to make full payments for a period (which may vary across imaginable sequences) and then cease making any payments, the value of a new claim received at precisely the moment when default begins would be nil. In that case the present value of the incremental contribution of the guarantee would be exactly equal to the amount of money required to finance the guarantee. Thus, the guarantee would attain its theoretical upper bound; a perpetuity enhanced by a continuously rolling interest guarantee would be worth

$$\rho\, D_n \cdot \frac{I_n}{I_{rf}} + CI \tag{14}$$

where CI is the initial contribution to the interest guarantee account.

An intuitive approach to constructing a lower bound on the incremental value of a rolling guarantee specified in present value terms builds from the realization that market efficiency would require that the discount on the debt should not be expected to fall over time: if it is expected to fall tomorrow it should fall today. It can however, be expected to rise. This implies that the incremental value of a guarantee established in present value terms would be at least one minus the current spread adjusted secondary market price multiplied by the initial amount contributed to the collateral fund (i.e., $(1-\rho)CI$, where CI is equal to the present value of the first n payments whose coverage would exhaust the guarantee). This is due to the fact that if the guarantee is fully triggered the creditor will get the full present value of the payments made to establish the escrow accounts less the value of the new claims he would be foregoing. The latter would be expected to be discounted by no less than the current secondary market discount. If it is expected that the guarantee will not be triggered until a later point in time when the discount is steeper, then its incremental contribution would be expected to be higher.

Difficulties arise in valuing guarantees specified in nominal terms because if the guarantee is fixed in nominal terms, then the later the guarantee is expected to be triggered, the less is the present value of the cash payment to be received. This insight has moved some analysts to conclude that less net debt reduction can be achieved through rolling guarantees specified in nominal terms than through conventional buybacks. However, such a view would not appear warranted. In considering the incremental value of the guarantee, account must also be given of the expected behavior of the discount on the country's debt. A discount exists on the debt because the country is expected to eventually miss some payments and not to make them up later. Hence, to the extent that creditors expect the debtor to initially make full payments on the debt, they must also expect the discount on the debt to rise sharply. When the guarantee is ultimately triggered, the expected cash payment will be worth less due to discounting; however, the foregone new claim will be worth even less. It can be shown that under well specified conditions these two effects would exactly offset each other: as the one factor lowers the incremental value of the guarantee the later it gets triggered, the second factor raises the value of the guarantee. Hence, regardless of the period when the guarantee is triggered, its incremental value is the same: the value of the nominal guarantee times one minus the current secondary market price. Generalizing the analysis, it can be shown that $(1-\rho)$ CI, shown to give the lower bound for the incremental contribution of a guarantee specified in present value terms, also provides the lower bound for rolling guarantees specified in nominal terms, provided that the guarantee is applied to the entire stream of contractual obligations, i.e., the value of a perpetuity carrying rolling interest guarantee, where the interest reverts to the debtor would be:

$$\rho D_n \frac{I_n}{I_{rf}} + (1-\rho)\, CI \tag{15}$$

where ρ is the risk-adjusted discount rate as before, CI is the amount of cash put into the interest guarantee account, and D_n and I_n are the principal amount and interest rate of the enhanced instrument.

An alternative explanation for seeing why a formulation like (15) would be appropriate can be arrived at by reasoning along the following lines. Consider two debtors: one of whom is holding D units of debt, partially enhanced by a collateralized guarantee of one year's interest, and the other of whom is holding D-C units of unenhanced debt, where C is equal to D.I/(1+I) and I is the risk-free interest rate and also the rate of interest charged on both the enhanced and unenhanced debt. Now if we can figure out how much more money the first creditor expects to get than the second creditor, we can figure out how to value the enhanced debt instrument. This is because we know the second creditor expects to earn ρ.(D-C) on his unenhanced instruments. Thus the first creditor would expect to receive ρ.(D-C) plus that extra increment.

Consider the money the first creditor is paid so long as the guarantee does not get triggered: he receives D.I which is C.I more than the second creditor receives. In the period when the guarantee gets triggered, the first creditor would still get D.I, which would then be C.(1+I) more than the second creditor would receive. Subsequently, the first creditor would receive no more money from the collateral fund, as it will be exhausted. He will, however, share in whatever aggregate payments the debtor makes in proportion to the ratio of D.I to the total claims falling due (note that this creditor was paid in the first default period out of the collateral fund and therefore did not receive any new claim on the debtor). The second debtor will have received some new claims as a consequence of his having received nothing in the period when the guarantee was triggered. In fact his claims will rise from D-C to (D-C).(1+I). A little algebra will show that the second debtor would then have the same number of unguaranteed claims as the first debtor. Hence after the guarantee is exhausted both creditors will be in the identical positions and there should be no differences in the amount of money they receive. The difference in the cash which the two debtors expect to get under any imaginable payment sequence therefore can be expressed as being equal to:

$$\frac{C}{(1+I)^t} + \sum_{n=1}^{t} \frac{C.I}{(1+I)^n} \tag{16}$$

where t is the period when the guarantee is triggered. Since the value of this sum is always C, regardless of t, we see that regardless of the actual sequence of behavior by the debtor, the enhanced instrument is worth exactly C more than the specified quantity of unenhanced instruments. Hence the value of the D units of enhanced debt would be:

$$\rho(D - C) + C \tag{17}$$

which is equivalent to (15).

IV. Comparisons with Buybacks

In this section, the analysis of Sections II and III is brought together to shed light on the issue of "equivalence" between buybacks and debt-for-debt exchanges. It is shown that debt-for-debt exchanges may have similar consequences to a buyback, but this depends on the structure of the transaction being proposed.

A collateralized debt-for-debt exchange can be viewed as a combination of a conventional buyback and a targeted buyback. For example, in the Mexico-Morgan exchange, 1/ Mexico reduced its interest payment obligations by $75 million per year for twenty years. In addition, since the principal obligations on the new bonds was prepaid, a total of $3.7 billion in principal obligations was also eliminated. 2/ A similar outcome could have been achieved if the country had engaged in a conventional buyback to purchase $860 million of debt (which would have

1/ In that exchange Mexico issued $2.6 billion in new debt, with an interest rate of 9.56 percent for $3.7 billion in old debt carrying an interest charge of 8.75 percent.

2/ Equivalently, one could say that from year 21 onwards the interest savings increased to $325 million.

reduced its interest obligations by $75 million) and then, in addition, had engaged in a forward purchase of $2.8 billion in debt, deliverable in 20 years. If the conventional buyback had taken place at the prevailing secondary market price of 50 cents on the dollar (and hence cost $430 million), Mexico would have had to have made the forward purchase at a discount of around 90 percent, 1/ in order to have achieved identical results using the same amount of reserves.

To take the analogy further, in a debt for debt swap involving rolling collateralized guarantees where the interest accrues to the collateral account, the debtor achieves principal and interest relief defined by the difference between the old debt-service obligations retired and new debt-service obligations created. This relief may be similar in its temporal distribution to that obtained through a conventional buyback. However, in addition, if the debtor requires exceptional financing to discharge his interest obligations, the guarantee will be triggered, providing relief for the debtor without increasing the debtor's debt-service obligations. Hence, in order to achieve similar contractual relief, the debtor would have to engage in both a conventional buyback and a buyback of the first x dollars in present discounted new money obligations to achieve the same contractual profile across all imaginable sequences. 2/

In the case where the debtor receives the interest from the collateral account, so long as the guarantee has not been triggered the corresponding equivalent transaction would involve, in addition, a buyback of some of the debtor's interest obligations in the period before the exceptional financing is to be sought. Since in this case after the guarantee is triggered, the debtor will have more net payment obligations (as he no longer receives the interest from the collateral fund), the net impact of triggering the guarantee will be the same as if there were initially less debt but no guarantee and as a result of missing some contractual obligations the debtor had to service a new debt. Thus rolling guarantees where the interest accrues to the debtor appear to correspond more closely to buybacks in their contractual flow implications across sequences.

Since a collateralized exchange would be equivalent in its contractual implications and pricing to a combination of a conventional buyback and a targeted buyback, the associated net debt reduction per dollar of precommitted resources would be more, less or equal to that

---

1/ The present value, discounted at the risk-free rate, prevailing in March of 1988, of a dollar delivered in 20 years would be around 20 cents. Since Mexico would be purchasing $2.8 billion worth of debt, worth $0.6 billion in present value terms, the discount would need to be around 90 percent.

2/ If the interest earnings do not accrue to the collateral account, then the debtor would be buying back the first x dollars of new money obligations.

achievable through a straight buyback as the discount on the targeted obligations is deeper, shallower or the same as that on all obligations. However, even in the case where the discount is the same, i.e., all payments are equally risky, and hence the net debt reduction is the same, the exchange may not be "equivalent" to a buyback, as the contractual relief may be front, back or stochastically distributed depending on which obligations are prepaid.

This highlights the need for caution in employing a summary statistic, which conflates the cross-period impact of the alternative transactions, i.e., the net debt reduction to compare the net contractual relief achieved through a given transaction and a buyback.

This point is further illustrated through Table 5, which presents the contractual flow implications of various debt and debt-service reduction operations. In all four columns, the debt retired is assumed to be structured as a twenty year bond with a bullet maturity and the operation is assumed to cost 50 cents, financed out of the debtor's own resources. In column A, the period-by-period net impact of a buyback at 33.3 cents on the dollar is shown while column B shows the period by period impact of a buyback at 50 cents. Columns C and D show the net impact of collateralized debt exchange where the final principal payment in year twenty is secured by a specific guarantee. In both columns the amount of resources committed to fund the guarantee are assumed to be equivalent to the resources used for the buybacks in columns A and B. It is assumed in these cases that creditors exchange 3.5 units of old debt, carrying an interest charge of 8.4 percent for 2.5 units of enhanced new debt with an identical interest and maturity structure. In column C it is assumed that the guarantee takes the form of a refundable escrow account. Hence the net cost to the debtor of making the final principal payment is nil. In column D it is assumed that the guarantee is structured along the lines of an insurance contract with a nonrefundable premium. At the bottom of each of the lettered columns, the net debt reduction which would be achieved by the hypothetical exchange is given.

As can be seen, an equivalent amount of net reduction in contractual obligations would be achieved through examples A and C; however, the implications for the flow of contractual obligations is quite different. It is interesting to note that, up until year 20, the stream of contractual obligations achieved through example C is identical to that which would be achieved through a buyback at 50 cents. In contrast, the contractual flow implications of the nonrefundable premium structure shown in D are identical to that of a buyback at 50 cents. While this example highlights one of the important ways in which debt-for-collateralized-debt swaps differ from buybacks, i.e., the manner in which principal prepayment results in a backloading of the contractual relief, it can be noted the differences between exchanges involving collateralized guarantees and buybacks can be more diverse. For example, a rolling collateralized guarantee which expires after five years would result in the debtor's receiving the escrowed resources back

Table 5: The Contractual Flow Implications of Alternative Debt and Debt-Service Reduction Operations

Year	Buybacks		Debt-for-Debt Swaps 1/				
	Net Reduction in Payment Obligations		Payment Obligations on Debt Retired	Payment Obligations on New Debt	Payment Covered Escrow Account	Net Reduction in Payment Obligations: Collateralized Guarantee	Net Reduction in Payment Obligations: Nonrefundable Premium
	A. Price equal one third	B. Price equal one half					
0	-0.500	-0.500	0.000	0.000	-0.500	-0.500	-0.500
1	0 126	0.084	0.293	0.210		0.084	0.084
2	0.126	0.084	0.293	0.210		0.084	0.084
3	0.126	0.084	0.293	0.210		0.084	0.084
4	0.126	0.084	0.293	0.210		0.084	0.084
5	0.126	0.084	0.293	0.210		0.084	0.084
6	0.126	0.084	0.293	0.210		0.084	0.084
7	0.126	0.084	0.293	0.210		0.084	0.084
8	0.126	0.084	0.293	0.210		0.084	0.084
9	0.126	0.084	0.293	0.210		0.084	0.084
10	0.126	0.084	0.293	0.210		0.084	0.084
11	0.126	0.084	0.293	0.210		0.084	0.084
12	0.126	0.084	0.293	0.210		0.084	0.084
13	0.126	0.084	0.293	0.210		0.084	0.084
14	0.126	0.084	0.293	0.210		0.084	0.084
15	0.126	0.084	0.293	0.210		0.084	0.084
16	0.126	0.084	0.293	0.210		0.084	0.084
17	0.126	0.084	0.293	0.210		0.084	0.084
18	0.126	0.084	0.293	0.210		0.084	0.084
19	0.126	0.084	0.293	0.210		0.084	0.084
20	1.626	1.084	3.792	2.710	2.500	3.584	1.084
Net Debt Reduction: 2/	1.000	0.500				1.000	0.500

1/ A debt for debt exchange in which 3.5 units of old debt are retired in exchange for 2.5 units of new debt. The bullet principal payment on the new debt is guaranteed either by the commitment of 0.5 units into an escrow account which is refunded to the debtor after the payment is made or through the payment of a nonrefundable insurance premium.

2/ The net debt reduction is defined as the present discounted sum of the period by period net changes in the debtor's payment obligations. The calculations here use a discount rate of 8.4 percent, which is assumed to be equivalent to the interest rate charged on old and new debt.

in five years if not sooner (i.e., if the guarantee gets triggered earlier and effectively discharges some of the debtor's obligations).

It is generally more meaningful to say that a debt exchange involving nonrefundable premium-based guarantees is "equivalent" to a buyback, as long as the structure of payment obligations on the new debt is essentially the same as the structure on the old debt. This is because, as was discussed in Section II, when guarantees take this latter form, the debtor is obligated to pay all of the obligations on the new debt; the payment burden is never eased by the return of any escrowed resources.

For example, in the case of the Mexico-Morgan exchange, if the principal guarantee had taken the form of a non-refundable premium payment and the same exchange ratios had been observed, then Mexico would have only attained a $1.1 billion reduction in its amortization obligations along with the $75 million per period reduction in interest obligations. In comparison, if instead the authorities had dedicated the roughly $500 million which was used to enhance the new bonds to engage in a conventional buyback, the country's amortization obligations would have declined by $1 billion and its interest obligations, prior to the due date of the amortizations, by around $87 million. Hence if the guarantee had been structured along the lines of an insurance contract rather than a prepayment, the transaction would have much more nearly "equivalent" to a buyback at the then prevailing secondary market price.

V. Limitations and Directions for Future Research

The exposition above, especially in Section III, has largely eschewed formal exposition of the underlying analytical model. Intuitive explanations have in several instances been substituted for formal proofs of propositions regarding boundary conditions on possible valuations. The essential implication of the analysis is that in any case precision is not feasible; therefore a prudent strategy would involve rules of thumb together with an understanding of the possible directions and margins of error likely to be encountered.

Nonetheless further work is required to present a formal statement of the underlying model and a rigorous exposition of the conditions under which the propositions presented in the text regarding boundary conditions would hold. Such work is currently underway and interested readers are encouraged to contact the author.

The approach taken here also ignores any feedback effects on the behavior or performance of the debtor country. Comprehensive debt restructurings may lead to increased output in debtor countries as a result of reduced uncertainty about the distribution of any gains from enhanced economic performance, and therefore increase the expected servicing of the remaining debt. While such effects may be significant, a consensus on their likely magnitude and their operational mechanism has not emerged. However, given a well-specified model of such feedback effects, it should be feasible to merge such a model into the framework suggested here.

Alternative Formulas for Analyzing Debt Exchanges

This appendix presents some easily employed formulas for calculating the net debt reduction achieved through a specified debt-for-debt exchange as well as formulas for estimating the cash value which creditors will attach to partially guaranteed debt instruments. Formulas are also provided for designing hypothetical exchanges under the condition that creditors use one of the suggested valuation models. These formulas assume that the debt to be retired is structured as a perpetuity and the new debt is amortized in a single fully guaranteed payment in t years. The following notation is used:

Notation

D_o = old debt to be retired

D_n = new debt to be created

I_{rf} = risk-free interest rate

I_o = interest rate on old debt

I_n = interest rate on new debt

p = prevailing secondary market price

ρ = spread adjusted secondary market price (defined below)

CI = initial contribution to escrow fund to finance rolling guarantee of interest payments

CP = initial contribution to escrow fund to specific guarantee of principal payment

1. Net debt reduction

The net debt reduction is defined as the value, discounted to the present, of the net reduction in contractual obligations which the country must pay. It is equal to the present value of the obligations on the old debt retired (PVO) plus any resources the debtor expects to receive from the escrow account (PVE) minus the total payment obligations on the new debt (PVN) and minus the cost of funding any guarantees (G).

$$NDR = PVO + PVE - PVN - G \tag{1}$$

$$PVO = \frac{D_o I_o}{I_{rf}} \tag{2}$$

$$PVN = D_n \frac{I_n}{I_{rf}} [1 - \frac{1}{(1 + I_{rf})^t}] + \frac{D_n}{(1 + I_{rf})^t} \tag{3}$$

a. Refundable collateralized guarantee

If the triggering of the guarantee results in the retirement of the obligations without the debtor being obliged to still make the payment and if any resources which are not paid to creditors out of the collateral account are ultimately returned to the debtor then:

$$PVE = G \tag{4}$$

Therefore

$$NDR = PVO - PVN \tag{5}$$

b. Nonrefundable premium-based guarantee

If the debtor is obligated to repay to the guarantor any payments which the guarantor makes on the debtor's behalf and if the guarantor does not return to the debtor any resources that he does not ultimately disburse, then:

$$PVE = 0 \tag{6}$$

Therefore

$$NDR = PVO - PVN - G \tag{7}$$

The model of Symansky and Tryon implicitly has this structure. In their model G is equal to CI + CP as developed in Section 2.

c. Buyback

The initial cash payment is similar to a nonrefundable premium. In a conventional buyback

$$D_o = C/p \tag{8}$$

where C is the amount of cash dedicated to the buyback.

Hence:

$$NDR = \frac{(C)(I_o)}{(p)(I_{rf})} - C \tag{9}$$

$$= \frac{C(1-\rho)}{\rho}$$

where ρ is the spread adjusted secondary market price:

$$\rho = \frac{p.I_{rf}}{I_o} \tag{10}$$

[Note that $\frac{V_o}{PVO} = \frac{p.D_o}{\frac{D_o.I_o}{I_{rf}}} = \rho$]

2. Initial resource requirement for collateralized guarantee

a. Specific guarantee of principal

$$CP = D_n/(1 + I_{rf})^t \tag{11}$$

where t is the due date for the principal payment.

b. Rolling guarantee of two years' interest

$$CI = D_n.I_n.[\frac{1}{(1 + I_{rf})} + \frac{1}{(1 + I_{rf})}^2] \tag{12}$$

3. The cash value of a guaranteed debt instrument (V_n)

Several alternatives for valuing the new instrument are presented below. As discussed in the text, the first three models assume third party guarantees, e.g., refundable collateralized guarantees, while the fourth assumes a nonrefundable premium-based structure.

a. Equal risk model

This approach assumes that all payment obligations are equally likely to be serviced.

$$V_n = \rho.PVN + (1-\rho)(CI + CP) \tag{13}$$

As discussed in the text, if the interest guarantee is continuously rolled forward, so long as it is not triggered, this corresponds to the theoretical lower bound for the cash value of the new instrument, regardless of whether the interest earnings on the interest guarantee account are retained in the account or given to the debtor.

b. Exponential risk model

This inequality provides a lower bound for the cash value ascribed to a new debt instrument by participants using the exponential risk model, i.e., by creditors who believe that the risk of nonpayment grows exponentially.

$$V_n \geq \rho.PVN + (1-\rho)CI + CP \left[1 - \frac{(1 + I_{rf})^t}{(1 + d)^t}\right] \qquad (14)$$

where d is the risk adjusted discount rate:

$$d = I_o/p \qquad (15)$$

c. Theoretical upper bound

Assuming that the interest earnings on the escrow account to guarantee interest payments accrue to the account, the guarantee is fully triggered with certainty and that after the guarantee is triggered the country is not expected to make any further payments, the value would be:

$$V_n = \rho.PVN + CI + CP \qquad (16)$$

d. The Symansky-Tryon model

This model assumes that the guarantee is structured such that the creditor receives the escrowed resources regardless of whether the debtor discharges the insured obligations.

$$V_n = \rho.PVN + (CI + CP) \qquad (17)$$

While (17) is equivalent to (16), the two models do not have identical implications. While (16) would be paired with (5), (17) would be paired with (7). Hence the associated net debt reduction would be lower with the Symansky-Tryon model.

4. Estimating the net debt reduction (NDR)

For an exchange to be feasible, it must be the case that the market or cash value of the old debt to be retired, (V_o) is less than or equal to the market or cash value of the new instrument, (V_n) i.e., $V_o \leq V_n$ With a well functioning secondary market, V_o is given by

$$V_o = p.D_o \tag{18}$$

Given a hypothetical instrument and its associated degree of enhancement, its value can be projected using any of the four models presented in Section C. The amount of old debt which would be exchanged for new debt can be derived by setting.

$$p.D_o = V_n \tag{19}$$

Given D_o and the details of whether the guarantee is refundable or not, the associated net debt reduction can be calculated by employing either equation (5) or (7), depending on the guarantee structure. For equations (13), (14), and (16) the appropriate formula would be (5) while for the Symansky-Tryon model (equation (17)) the appropriate formula would be (7); the projected net debt reduction in each case would be given by:

a. Equal risk model

$$NDR = \frac{(CI + CP)(1-\rho)}{\rho} \tag{20}$$

b. Exponential risk model

$$NDR \geq \frac{CI\ (1-\rho)}{\rho} + \frac{CP}{\rho} \cdot [1 - \frac{(1 + I_{rf})^t}{(1 + d)^t}] \tag{21}$$

c. Upper bound

$$NDR = \frac{CI + CP}{\rho} \tag{22}$$

d. Symansky-Tryon model

$$NDR = \frac{(CI + CP)(1-\rho)}{\rho} \tag{23}$$

While (21) and (22) imply more net debt reduction per dollar of enhancement, this needs to be interpreted with caution. The greater relief is a function of its being backloaded, i.e., much of the additional relief only comes into play when the amortization payments would come due.

5. Buyback equivalent price (P^e)

A meaningful comparison between a buyback and a debt exchange could involve the ratio of the projected net debt reduction to the amount of precommitted resources. Alternatively, given the amount of net debt reduction implied by a given exchange, one might ask, at what price would a buyback have to be realized in order to achieve the same amount net debt reduction with the same amount of precommitted resources. That price could be called the buyback equivalent price (P^e). Since in a buyback:

$$NDR/C = (1-\rho)/\rho \tag{24}$$

the relationship between the transaction price and NDR/C is given by

$$p = \frac{I_o}{I_{rf}} \cdot \frac{C}{C + NDR} \tag{25}$$

Hence for a given exchange proposal one can substitute the implied net debt reduction and the total cost of enhancements into (25) to derive a buyback equivalent price. If the terms of the exchange, e.g., the exchange ratio or the ratio of the interest rate on the new and old debt, are set to equate the market value of the new and old debt, and where the market value of the new debt is given by one of the four formulas from Section (C), the projected buyback equivalent prices would be:

a. Equal risk formula

$$p^e = \frac{I_o}{I_{rf}} \cdot \frac{(CI + CP)}{(CI + CP) + (CI + CP)((1-\rho)/\rho)} \tag{26}$$

$$= \frac{I_o}{I_{rf}} \cdot \rho$$

$$= p$$

b. Exponential risk model

$$p^e = \frac{CI + CP}{CI + CP(1 + \rho - \rho_2)} \, p \tag{27}$$

where

$$\rho_2 = \frac{(1 + I_{rf})^t}{(1 + d)^t} \tag{28}$$

c. Upper bound

$$p^e = p \cdot \frac{1}{(1 + \rho)} \tag{29}$$

d. Symansky-Tryon model

$$p^e = p \tag{30}$$

6. Designing hypothetical exchanges

Using the valuation formulas proposed in Section (3), one can construct hypothetical exchanges which would be consistent with the old and new debt instruments having equal market value. For example, one could solve for the amount of debt which would be retired in exchange for a given quantity of new debt, where the degree of enhancement, maturity and interest charge are predetermined. Alternatively, the exchange ratio, and degree of enhancement could be fixed and one could solve for the required interest rate on the new debt. Presented below are several general formulas for the exchange ratio, the required degree of collateralization and the required interest rate. The starting point for each derivation is the arbitrage condition that the market value of the old debt equal the market value of the new. When combined with the specific valuation models from Section (C) this implies that:

$$p.D_o = \rho PVN + (1-\rho_1)\ CI + (1-\rho_2)\ CP \tag{31}$$

where

$$\rho_1 = \rho_2 = \rho \quad \text{for the \underline{Equal risk model};} \tag{32}$$

$$\rho_1 = \rho,\ \rho_2 = \frac{(1 + I_{rf})^t}{(1 + d)^t} \quad \text{for the \underline{Exponential risk model};} \tag{33}$$

and

$$\rho_1 = \rho_2 = 0 \quad \text{for both the \underline{Upper bound} and the \underline{Symansky-Tryon model}} \tag{34}$$

a. The exchange ratio (R)

$$R = D_n/D_o \tag{35}$$

Since:

$$p.D_o = \rho D_n \left[\frac{I_n}{I_{rf}} (1 - \tau) + \tau\right] + (1 - \rho_1)\ CI + (1 - \rho_2)\ CP \tag{36}$$

$$\text{where} \quad \tau = 1/(1 + I_{rf})^t \tag{37}$$

we have that:

$$R = \frac{\rho\frac{I_o}{I_{rf}} - (1 - \rho_1)\frac{CI}{D_o} - (1 - \rho_2)\frac{CP}{D_o}}{\rho\frac{I_n}{I_{rf}}(1 - \tau) + \tau\rho} \tag{38}$$

Note that if $CI = CP = \tau = 0$, then $R = I_o/I_n$.

b. The interest ratio (I)

$$I = I_n/I_o \tag{39}$$

$$I = \frac{1}{(1 - \tau)}\left[\frac{D_o}{D_n} - \tau\frac{I_{rf}}{I_o} - \frac{CI}{D_n}\cdot\frac{(1 - \rho_1)}{\rho}\cdot\frac{I_{rf}}{I_o} - \frac{CP}{D_n}\cdot\frac{(1 - \rho_2)}{\rho}\cdot\frac{I_{rf}}{I_o}\right] \tag{40}$$

c. The required degree of interest collateralization (δ)

$$\delta = \frac{CI}{D_n I_n} \tag{41}$$

It is assumed that the principal is fully collateralized, hence

$$CP = \tau D_n \tag{42}$$

therefore:

$$\delta = \frac{1}{I_{rf}}\cdot\frac{\rho}{(1 - \rho_1)}\left[\frac{I_o D_o}{I_n D_n} - (1 - \tau) - \frac{\tau I_{rf}}{I_n} - \frac{(1 - \rho_2)}{\rho}\,\frac{\tau I_{rf}}{I_n}\right] \tag{43}$$

References

Borensztein, E. and G. Pennacchi, "Valuing Interest Payment Guarantees", (Washington: unpublished manuscript, July 13, 1989).

Brainard, L.J. and A. Ginsburg "Valuation of the Mexican Debt Reduction Bonds" (New York: unpublished manuscript, November, 1989).

Bulow, Jeremy, and Kenneth Rogoff, "The Buyback Boondoggle," Brookings Papers on Economic Activity (1989), The Brookings Institution (Washington).

Claessens, S. and S. van Wijnbergen, "Secondary Market Prices under Alternative Debt Reduction Strategies: an Option Pricing Approach, with an Application to Mexico" (Washington: unpublished manuscript, September 1989).

Diwan, I. and S. Claessens, "An Analysis of Debt-Reduction Schemes Initiated by Debtor Countries", PPR Working Paper WPS 153 (Washington: International Bank for Reconstruction and Development, March 1989).

Dooley, Michael P., "Buy-Backs and Market Valuation of External Debt," Staff Papers, International Monetary Fund (Washington), Vol. 35 (June 1988), pp.215-29.

Dooley, Michael P., and Steven Symansky, "Comparing Menu Items," in Analytical Issues in Debt, ed. by Jacob A. Frenkel (Washington: International Monetary Fund, 1989).

Dooley, Michael P., "Self Financed Buy-Backs and Asset Exchanges," Staff Papers, International Monetary Fund (Washington), Vol. 35 (1988) No. 4, pp.714-723.

Dooley, M.P., S. Symansky and R.W. Tryon, "A Dynamic Model of Buybacks" IMF Working Paper WP/89/56, (Washington: International Monetary Fund, July, 12, 1989).

Folkerts-Landau, David and Carlos Rodriguez, "The Mexican Debt Exchange," in Analytical Issues in Debt, ed. by Jacob A. Frenkel (Washington: International Monetary Fund, 1988).

Helpman, E., "Voluntary Debt Reduction: Incentives and Welfare" in Analytical Issues in Debt ed. by Jacob Frenkel (Washington: International Monetary Fund, 1989).

Krugman, Paul, "Market Based Debt Reduction Schemes," in Analytical Issues in Debt, by Jacob A. Frenkel (Washington: International Monetary Fund, 1988).

Nocera, Simon E., "Pricing an Interest Payment Guarantee--A Contribution to Debt Reduction Techniques", IMF Working Paper WP/89/65, (Washington: International Monetary Fund, August 21, 1989.

Purcell, J.F.H. and D.J. Orlanski, "Developing Country Loans: Refinements to the Relative Value Model fro the Secondary Market", Solomon Brothers: International Loan Trading Analysis, June 8, 1989.

Rodriguez, Carlos, "Strategy of Debt Buy-backs: A Theoretical Analysis of the Competitive Case," in Analytical Issues in Debt, ed. by Jacob A. Frenkel (Washington: International Monetary Fund, 1988).

Symansky, S.A. and R.W. Tryon, "Valuation of Menu Items in Debt Restructuring", IMF Working Paper WP/89/69, (Washington: International Monetary Fund, September 8, 1989).

Telljohann, K.L., J.F.H. Purcell and R.H. Buckholz, "Evaluating the Mexican Brady Bonds: Principal Reduction or Interest Reduction", Solomon Brothers: Memorandum to Portfolio Managers, (New York: November 2, 1989).

Telljohann, K.L. and J.F.H. Purcell,"Implementing the Brady Plan: Rolling Interest Guarantees", Solomon Brothers: Memorandum to Portfolio Managers, (New York: June 28, 1989).