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Pricing Floating-Rate Debt and Related Interest-Rate Options

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Abstract

Most developing country debt is denominated in U.S. dollars and has a floating interest rate. The pricing of floating rate debt and related interest rate options are examined in this paper. Formulas for pricing ceilings and floors on floating rate debt are derived for several different models of interest rate variability. A framework for pricing risky debt and loan guarantees is presented, and the implications of the debtor country's default option are analyzed. The elimination of large principal repayments, by collateralizing the principal, serves to reduce the debtor country's incentive to use its default option.

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1/ The author has benefited from comments by participants at the Research Department's seminar of the International Monetary Fund.

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Summary

This paper reexamines the valuation of floating-rate debt. It finds that the value of a cap or a floor on the floating rate can be determined by regarding these features as effectively being options on default-free discount bonds. Several formulas can compute values for these bond options. The paper also outlines a general method for valuing those cases for which no closed-form solutions exist.

The paper also discusses some of the complexities of pricing floating-rate debt with default risk. It notes that default risk can be reduced if lenders structure the loans so that there are no large single repayments of principal. The borrower's incentive to default is minimized when the repayment of principal is spread over the life of the loan.

Spreading the principal repayment over the life of the loan can be accomplished either by amortization or by requiring collateral for the repayment of principal. If the secondary market price is used as the value of the payments that the debtor country is expected to make, the value of a loan guarantee can also be determined.

The use of floating or variable rate debt instruments has increased steadily throughout the 1970s and 1980s. More recently, financial institutions have written options so that borrowers can hedge some of the risks associated with interest rate fluctuations. In addition, some of the floating rate debt instruments have extra features, such as ceilings and floors, which are effectively options on the underlying interest rate. Most of the external debt of developing countries is denominated in U.S. dollars and carries a floating interest rate that is tied to the London interbank offer rate (LIBOR). Determining the value of these debt claims and their related options is an important issue because there is a secondary market and valuation plays a crucial role when debtors and creditors renegotiate.

Because the developing country debt is denominated in dollars, standard results on floating rate debt can be applied. The one major difference concerns default, which is much more complicated in the case of a foreign government. In this paper, we analyze the pricing of floating rate debt and related interest rate options and we extend some of the results contained in previous papers, namely Cox, Ingersoll, and Ross (1980) and Ramaswamy and Sundaresan (1986). We also present a framework, without explicit solutions, for valuing risky debt of developing countries. Another important issue concerns the value of loan guarantees; our approach is to restate this problem in terms of valuing risky debt by observing that the value of the defaultable debt plus the value of the guarantee must equal the value of a corresponding default-free debt instrument. 1/

I. The Pricing Model

The analysis is presented within the framework of Cox, Ingersoll, and Ross (1985a,b), hereafter CIR, a continuous-time model which can be used to value cashflows that occur at discrete points in time. Most of their results apply to real interest rates and real cashflows, but in their Section 7, CIR (1985b) show that their methods can be applied to nominal interest rates and nominal cashflows to determine nominal prices. We begin with a brief review of the valuation results of this model.

The model has a set of state variables, Y, which determine changes in the nominal interest rate and the investment opportunity set, and r is used to denote the instantaneous nominal interest rate. The state variables Y and the interest rate r are diffusion processes and have stochastic differentials. We follow CIR and assume that the intertemporal utility function is logarithmic so that wealth does not directly influence our price solutions. The value today (time 0) of a single cashflow, C_t, that occurs at time t is determined as follows:

1/ Here we are assuming that there are no tax features which might alter this value additive relation.

$$V_0 = \hat{E}_0[\exp(-\int_0^t r(u)\, du)\, C_t],$$

where, $\hat{E}_0$ is the risk-adjusted expectation operator, conditioned on information at time 0. The risk-adjusted expectation operator works like the regular expectation operator, but we perform a risk adjustment on the state variables of our system before taking the expectation by subtracting a risk premium from the mean part of the diffusion process for each state variable:

$$dY_i = (\mu_i(Y) - \lambda_i)\, dt + \sigma_i(Y)\, dz_i.$$

Each risk premium is determined by the covariance of the state variable with marginal utility of nominal wealth, but for much of our analysis we will not need to determine explicitly these risk premia. These risk premia will be reflected in market prices. If a state variable is the nominal price of a traded asset, its risk-adjusted mean becomes the nominal interest rate r. The value of a financial claim with more than one cashflow is simply the sum of the values of the separate cashflows.

Default-free discount bonds play an important role in the analysis; here the price of a default-free discount bond that pays $1 at time t is given by

$$P_0(t) = \hat{E}_0[\exp(-\int_0^t r\, du)]$$

These bond prices involve expectations of an integral of future interest rates, but there is a risk adjustment on the stochastic processes that determine the interest rate. The bond prices can also be interpreted as discount factors for future cashflows that are nominally risk-free. For the U.S. market (and for dollar denominated claims), one can easily compute these prices from prices in the Treasury market. This approach to valuation is similar in spirit to the certainty equivalent method in which we reduce expected cashflows to a certainty equivalent and discount at riskless interest rates.

II. Pricing Default-Free Floating Rate Debt

We first examine the pricing of default-free floating rate debt. Most floating rate bonds and loans have rates that are tied to either the Treasury bill rate or LIBOR. Ramaswamy and Sundaresan (1986) include a description of the features common to floating rate debt issued in the U.S. To value debt claims tied to LIBOR, we need to add a state variable for the difference between LIBOR and the T-bill rate. To simplify our analysis, we assume that the difference between the T-bill rate and LIBOR is constant, so that we are effectively pricing debt claims tied to the T-bill rate.

Let the floating rate debt extend for N periods and have a markup over T-bills equal to s. To simplify the analysis we assume that the interest *rate is set at the beginning of each period and paid at the end of the* period. 1/ The bond equivalent yield on one-period T-bills is used as the base rate:

$$R_t = 1/P_{t-1}(1) - 1$$

In our notation $P_t(k)$ is the price at time t of a default-free discount bond that pays \$1 at time t+k; (k) is omitted when k=1. Let F equal the face amount (or par value) of the debt and the interest payment each period is

$$C_t = F\ (1/P_{t-1} - 1 + s).$$

At maturity we receive the interest plus par:

$$C_N = F\ (1/P_{N-1} + s)$$

The value of the floating rate debt instrument is then

$$V_0 = \sum_{t=1}^{N} \hat{E}_0[\exp(-\int_0^t r\ du)\ C_t]$$

$$= F \sum_{t=1}^{N} \hat{E}_0[\exp(-\int_0^t r\ du)\ (1/P_{t-1} - 1 + s)] + F\ \hat{E}_0[\exp(-\int_0^N r\ du)]$$

By applying the law of iterated expectations, 2/ we make the following observation for a typical cashflow:

1/ Ramaswamy and Sundaresan note that many of the floating rate instruments use an average of rates during the month that the rate is reset.

2/ The law of iterated expectations is also known as the law of total probability for conditional expectations (see Karlin and Taylor (1975), pages 239 and 246).

$$\hat{E}_0[(1/P_{t-1})\ \exp(-\int_0^t r\ du)] = \hat{E}_0\{\hat{E}_{t-1}[\exp(-\int_0^t r\ du)/\hat{E}_{t-1}\ (\exp(\int_{t-1}^t r\ du))]\}$$

$$= \hat{E}_0[\exp(-\int_0^{t-1} r\ du)] = P_0(t-1).$$

$P_0(t-1)$ is the discount factor today for a dollar that is received at time t-1. The valuation formula simplifies to

$$V = F\ [1 + s \sum_{t=1}^{N} P_0(t)]$$

A similar result was previously derived in CIR (1980). If the markup s is zero, then the value is equal to the face amount on the payment dates, but as CIR note, the value can fluctuate between payment dates. This valuation formula can be used to determine the value of a floating rate loan that has no default risk and carries a markup over the default-free rate. This calculation would be important for an institution that is considering a guarantee on the payments of an existing risky loan.

Some floating rate debt instruments have ceilings and floors on the interest rate, and borrowers who do not have ceilings on their floating rate loans can purchase interest rate caps from financial institutions. The ceiling or the cap can be viewed as a sequence of options for the borrower, and each period the option payoff is equal to $\max(R_t+s-U,\ 0)$ times the face amount. U is the ceiling rate and L will be the floor rate. The cap is a call option on the floating rate (R_t+s) held by the borrower. In the case of a floating rate loan with a ceiling, the lender (or bond-holder) has sold call options on the floating rate and the value of the bond is equal to the value of a straight floating rate bond without any extra features minus the value of the call options on the rate. A floor represents a sequence of put options on the floating rate that is held by the lender; the payoff for the lender each period is $\max[L-(R_t+s),\ 0]$ times the face amount. The value of a floating rate loan with a ceiling and a floor is equal to the value of the straight floating rate loan plus the value of the put options on the floating rate minus the value of the call options on the floating rate.

All of these options are European options and the payoffs can be rewritten in terms of one-period bond prices. The value of the cap is

$$V(\text{Cap}) = \sum_{t=2}^{N} F\ \hat{E}_0\{\exp(-\int_0^t r\,du)\ \max\ [1/P_{t-1} - 1 + s - U,\ 0]\}$$

and the value of the floor is

$$V(\text{Floor}) = \sum_{t=2}^{N} F\ \hat{E}_0\{\exp(-\int_0^t r\,du)\ \max\ [L - (1/P_{t-1} - 1 + s),\ 0]\}$$

Here we assume that L and U are set so that the option for period 1 has no value: $L < R_1 < U$. *To simplify the problem further, we focus on* the individual terms in the valuation of the cap.

$$\hat{E}_0\{\exp(-\int_0^t r\,du)\ \max\ [(1/P_{t-1}) - 1 + s - U, 0]\}$$

$$= \hat{E}_0\{\exp(-\int_0^{t-1} r\,du - \int_{t-1}^{t} r\,du)\ \max[1/P_{t-1} - 1 + s - U, 0]\}$$

Apply the law of iterated expectations.

$$\hat{E}_0\{\hat{E}_{t-1}\{\exp(-\int_0^{t-1} r\,du)\ \exp(-\int_{t-1}^{t} r\,du)\ \max[(1/P_{t-1}) - 1 + s - U,\ 0]\}\}$$

$$= \hat{E}_0\{\exp(-\int_0^{t-1} r\,du)\ P_{t-1}\ \max\ [(1/P_{t-1}) - 1 + s - U,\ 0]\}$$

$$= (1+U-s)\ \hat{E}_0\{\exp(-\int_0^{t-1} r\,du)\ \max\ [1/(1+U-s)\ -\ P_{t-1},\ 0]\}.$$

The last term represents the value of a claim that pays at time (t-1) a cashflow equal to max $[1/(1+U-s) - P_{t-1}, 0]$. Here we have effectively turned the European call option on the floating rate into a European put option on the discount bond that is used to set the floating rate. This put has a strike price equal to 1/(1+U-s) and we must multiply the value of the put by (1+U-s).

To value this interest rate option, one must formally specify the dynamics for the state variables that determine changes in the interest rate r; the general structure is

$$dr = \mu_r(Y)\ dt + \sigma_r(Y)\ dz$$

$$dY = \mu(Y)\ dt + \sigma(Y)\ dw,$$

where dz and dw represent Brownian motion processes. Closed form solutions are available for two classes of interest rate processes, and we show here the results for models in which the interest rate is determined by a single state variable. For a normal mean reverting process, $dr = k(\Theta - r)\ dt + \sigma\ dz$, Jamshidian (1989) has derived a formula for a Europan call option on a discount bond. By using the put-call parity theorem for European options, one can deduce the corresponding formula for the European put:

$$\text{Put} = (1+U-s)\ \hat{E}_0\{\exp(-\int_0^{t-1} r\ du)\ \max\ [1/(1+U-s) - P_{t-1},\ 0]\}$$

$$= P_0(t-1)\ [1-N(h-v)] - (1+U-s)\ P_0(t)\ [1-N(h)]$$

where

$$h = \ell n\ [(1+U-s)\ (P_0(t)/P_0(t-1))]/v + v/2$$

and

$$v = \left[\frac{\sigma^2\ (1 - e^{-2k(t-1)})}{2k} \right]^{1/2} (1 - e^{-k})/k$$

N( ) is the standard normal distribution function. 1/

For a mean reverting square root process, $dr = k(\Theta - r)\ dt + \sigma\sqrt{r}\ dz$, CIR (1985b) present a formula for the European call on a discount bond. The corresponding formula for our put option is

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1/ The risk premium for the short rate becomes buried in the price of the bond in this model.

$$\text{Put} = P_0(t-1)\left[1-\chi^2\left[2r^*[\phi+\Psi];\ \frac{4k\theta}{\sigma^2},\ \frac{2\phi^2 r_0 e^{v(t-1)}}{\phi+\Psi}\right]\right]$$

$$-(1+U-s)P_0(t)\left[1-\chi^2\left[2r^*[\phi+\Psi+B(t-1,t)];\ \frac{4k\theta}{\sigma^2},\ \frac{2\phi^2 r_0 e^{v(t-1)}}{\phi+\Psi+B(t-1,t)}\right]\right],$$

where χ^2 is the noncentral Chi-squared distribution function, r_0 is the current value of the interest rate,

$$r^* = \ell n[(1+U-s)\ A(t-1,t)]/B(t-1,t)$$

$$A(t-1,t) = \left[\frac{2v\ \exp[(\lambda+k+v)/2]}{(\lambda+k+v)\ (e^v-1) + 2v}\right]^{2k\theta/\sigma^2}$$

$$B(t-1,t) = \frac{2(e^v-1)}{(\lambda+k+v)\ (e^v-1) + 2v}$$

$$v = [(k+\lambda)^2 + 2\sigma^2]^{1/2}$$

$$\phi = \frac{2v}{\sigma^2(e^{v(t-1)}-1)}$$

$$\Psi = (k+\lambda+v)/\sigma^2$$

The square root process produces a more complicated formula, but it has the attractive feature that the interest rate cannot become negative.

By a similar set of arguments, we can show that the value of the floor is determined by call options on the one-period discount bonds:

$$\hat{E}_0\{\exp(-\int_0^t r\ du)\ \max[L-(1/P_{t-1}\ -1+s),0]\}$$

$$= (1+L-s)\hat{E}_0\{\exp(-\int_0^{t-1} r\ du)\ \max[P_{t-1}-(1/(1+L-s)),0]\},$$

where the strike price of the call is 1/(1+L-s). 1/ For the interest rate processes discussed above, one can directly apply the formulas in Jamshidian or CIR to value the call options.

Richer, more realistic models for the interest rate movements can be developed by using more than one state variable in the determination of the interest rate and the term structure. Formulas for option prices on discount bonds with multiple state variables can be found in Chaplin (1987), Sharp (1988), and Chen and Scott (1990). Chaplin presents an option pricing formula for a model in which the term structure is determined by two state variables: $r = y_1 + y_2$, where y_1 and y_2 are driven by mean reverting normal processes. Sharp extends the results for a model in which the term structure is determined by n state variables which follow mean reverting normal processes. Chen and Scott present an option pricing formula for a two-state variable version of the CIR model with mean reverting square root processes. If closed form solutions for the options are not available in more complex models, one can use Monte Carlo simulation to calculate the values. To summarize, the value of the cap equals the face amount times the sum of the values of the put options on discount bonds. The value of the floor equals the face amount times the sum of the values of the call options on discount bonds.

III. Floating Rate Debt with Default Risk

Valuing debt with default risk is considerably more complicated. To analyze the problem, we use the following relationship between the value of risky debt and the value of debt guarantees: if there are no tax effects on valuation, then

$$\text{Value}\begin{bmatrix}\text{Bond}\\ \text{with Default}\\ \text{Risk}\end{bmatrix} + \text{Value}\begin{bmatrix}\text{Guarantee}\\ \text{on}\\ \text{Bond}\end{bmatrix} = \text{Value}\begin{bmatrix}\text{Default}\\ \text{Free}\\ \text{Bond}\end{bmatrix}.$$

One can attack the problem of valuing defaultable debt by (1) valuing the debt directly or (2) valuing a guarantee on the interest and principal repayments. Because the default mechanism and the sequence of events which trigger default are important, we begin with an analysis of default.

There are well-developed models of default for securities like corporate bonds and residential mortgages. 2/ In the theoretical models for corporate debt, default occurs if the value of the firm is less than the face amount of the debt at maturity. In the case of a coupon bond, we

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1/ Note: if the markup s is greater than the floor rate L, this option has no value.

2/ See Merton (1974) for a model of default on corporate bonds.

can modify this rule as follows: each period, make the payment if the value of the firm is greater than the payment due. The payoff to the bond-holder each period, if default has not previously occurred, is

$$\begin{array}{ll} PMT_t & \text{if } A_t > PMT_t \\ A_t & \text{if } A_t < PMT_t, \end{array}$$

where PMT_t is the promised bond payment due and A_t is the value of the firm's assets (including cash). For interim periods if $PMT_t < A_t < F$, the firm's owners have an incentive to make the payment and to continue operating the firm. If the debt has a large payment (principal) at maturity, then default is more likely to occur at the end.

The typical rule for default in models for valuing residential mortgages is to assume that default occurs if the value of the house drops below the face amount or the principal balance of the mortgage. A better rule for rational mortgage default is the following: make the current payment due

$$\text{if } PMT_t < RENT_t$$

or

$$\text{if } H_t > F + INT_t,$$

where H_t is the current value or price of the house, INT_t is the current interest due, and $RENT_t$ represents either the rent one can receive or the current value of living in the house. Note that if $H_t < F$ and $PMT_t < RENT_t$, the borrower has an incentive to make the payment. If $PMT_t > RENT_t$ and $H_t > F + INT_t$, the borrower has an incentive to make the payment and preserve the equity claim on the house. Here there is an incentive to sell the house and pay off the loan early.

In the case of sovereign debt with no collateral and little or no legal recourse for creditors, the problem is more complicated. In the examples above, there is a transfer of assets from the borrower to the lender. If a country defaults on its foreign debt, there is no immediate transfer of assets, but the country does lose its access to world credit markets. Bulow and Rogoff (1989) mention that the creditors may also be able to impose sanctions or penalties on debtor countries that default. A simple rule for default would be to make the current payment if it is less than the value of maintaining access to world credit markets plus the costs associated with default, otherwise default and pay nothing. The value of a country's access to world credit markets would depend on its ability to obtain new loans and what those proceeds would be able to produce. These would, in turn, depend on export earnings, the country's ability to generate foreign reserves, productivity of new investment, etc. A country can be current on its debt payments, but if it approaches its upper limit on debt capacity, the value of its access to world credit markets would become quite low. For this reason, lenders who want to avoid default have an incentive not to lend up to the upper limit of a country's ability to service debt. There is also an incentive for lenders to structure the loans so that the principal or face amount is spread evenly over the life of the loan. In the corporate bond market for example, sinking fund provisions are frequently included for borrowers who issue low-quality debt. Another possibility is to require the borrower to collateralize the principal repayments at the end by purchasing safe (default-free) zero coupon bonds and depositing them with a third party.

This analysis of default on sovereign debt needs to be extended one more step. The lenders have an incentive to negotiate for a smaller payment from the debtor who elects to use the default option because a smaller payment is better than no payment. The result is a complex sequence of negotiations which has been analyzed recently by Bulow and Rogoff (1989), Fernandez and Rosenthal (1988), and Grossman and Van Huyck (1988). Our primary interest here is to value the sequence of payments that result from the negotiations between creditors and the debtor country. The future cashflows will be outcomes of repeated negotiations and these cashflows will not necessarily be related to the loan balance or future interest rates. This form of risky debt is more like an equity claim with an upper limit determined by the contractual obligations of the debt. One approach to pricing defaulted debt would be to analyze the country's ability to make future payments and discount the expected cashflows at an appropriate rate. Relevant economic variables would include export earnings and the growth potential of the economy. Many of these debt issues actually trade in secondary markets and we have market prices which reflect the value of the payments that the debtor country is expected to make.

With the market price reflecting the value of what the country might pay, we can calculate the value of a full guarantee by subtracting the market value of the risky debt from the value of the corresponding default-free floating rate debt. 1/ The formula in Section II can be used to determine the value of the default-free floating rate loan and we noted that this value is greater than the loan balance if the floating interest rate contains a markup over the default-free rate. We should note that this analysis assumes that the presence of a guarantee does not alter the debtor country's willingness to pay. If the third party that provides the guarantee does not negotiate as aggressively as the original lenders then the value of the risky debt component decreases and the value of the guarantee increases. Partial guarantees would be more difficult to price because their values will depend directly on the future payments that determine the value of the risky debt. The indirect approach can be used to price only the full guarantee. An alternative to an interest guarantee would be for the third party to pay the interest above a certain rate; this arrangement would be effectively a cap on the interest rate and would eliminate the interest rate risk for the debtor country. The models

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1/ We are assuming that there is no default risk with the third party that guarantees the loan.

described in Section II could be used to value this cap. If the cap, the interest rate ceiling, is set low enough it could raise the value of the risky debt.

In the last part of this section we outline a model which takes a different approach to pricing defaultable debt. To make the model tractable, we consider the case in which missed interest payments are lost and there is no increase in principal. Define x_t to be the proportion of the contractual payment actually made in period t. For the floating rate loan with principal repayment at the end we have

$$V = F \sum_{t=1}^{N} \hat{E}_0\{\exp(-\int_0^t r\,du)\,(1/P_{t-1} - 1 + s)\,x_t\} + F\,\hat{E}_0\,(\exp(-\int_0^N r\,du)\,x_N)$$

The value of this defaultable floating rate loan is calculated by taking the risk-adjusted expectation of these cashflows with respect to a set of dynamic equations for r, Y, and the additional state variables needed to describe default or payment each period. In the absence of analytical solutions, one can use Monte Carlo techniques to value this defaultable debt by simulating risk-adjusted processes for the interest rate, the state variables, and default.

A simple model can be developed if we are willing to assume that x_t is independent of interest rate movements. What we need is the condition that the proportion of the payment made be uncorrelated with interest rate movements. One weak rationale for this approach is the following. These are nominal interest rates; if the real interest rate is constant and the country's ability and willingness to pay are based on real economic variables and politics, then the correlation with nominal interest rates is zero. The result is

$$V = F \sum_{t=1}^{N} [P_0(t-1) + (s-1)\,P_0(t)]\,\hat{E}_0(x_t) + F\,P_0(N)\,\hat{E}_0(x_N).$$

If $\hat{E}_0(x_t)$ is the same for all periods, $\hat{E}_0(x_t) = \bar{x}$, we get

$$V = \bar{x}\,F\,[\,1 + s \sum_{t=1}^{N} P_0(t)],$$

where $F\,[1+s \sum_{t=1}^{N} P_0(t)]$ is the value of the corresponding default-free floating rate loan. In Table 1, we present some calculations of this last formula for different values of $\bar{x}$, N, and the spread; the values as a percentage of par are a little higher than the expected proportion of the payment.

As a final example we consider a loan in which the principal is repaid at the end and there is a possible default on the final payment only (principal plus interest).

$$V = F\,\{ \sum_{t=1}^{N} [P_0(t-1) + (s-1)\,P_0(t)] + [P_0(N-1) + sP_0(N)]\,\hat{E}_0(x_N)$$

$$+ \hat{COV}_0[\exp(-\int_0^N r\,du)\,(1/P_{N-1} + s),\ x_N]\},$$

where $\hat{COV}_0$ is the conditional covariance using the risk-adjusted process. The covariance term can be rewritten as follows

$$\hat{COV}_0 = s\,\hat{COV}_0[\exp(-\int_0^N r\,du_0),\ x_N]$$

$$+ \hat{COV}_0[\exp(-\int_0^{N-1} r\,du - \int_{N-1}^{N} r\,du)/\hat{E}_{N-1}(\exp(-\int_{N-1}^{N} r\,du)),\ x_N].$$

If we assume a negative correlation between r and x_N, then we have a positive covariance between x_N and terms like

$$\exp(-\int_0^N r\,du).$$

From this we conjecture that this covariance term is positive and adds value to the defaultable floating rate loan. Consider a loan in which the spread is set so that the initial value equals the face value. By rearranging the valuation formula with V = F, we get

Table 1. Valuation of Floating Rate Debt with a Simple Default Variable

Five Year Maturity		
Spread, s′	Expected proportion of payment $\hat{E}(x)=\bar{x}$	Value of debt (Face=$100)
0.005	1.0	102.03
0.005	0.8	81.62
0.005	0.6	61.22
0.005	0.4	40.81
0.01	1.0	104.06
0.01	0.8	83.24
0.01	0.6	62.43
0.01	0.4	41.62
0.02	1.0	108.11
0.02	0.8	86.49
0.02	0.6	64.87
0.02	0.4	43.24

Ten Year Maturity		
Spread, s′	Expected proportion of payment $\hat{E}(x)=\bar{x}$	Value of debt (Face=$100)
0.005	1.0	103.40
0.005	0.8	82.72
0.005	0.6	62.04
0.005	0.4	41.36
0.01	1.0	106.80
0.01	0.8	85.44
0.01	0.6	64.08
0.01	0.4	42.72
0.02	1.0	113.59
0.02	0.8	90.87
0.02	0.6	68.15
0.02	0.4	45.44

Note: The floating rate debt is set up with semiannual payments. A spread of 0.01 is equal to 1 percent or 100 basis points. For these calculations, we use a flat term structure for default-free bonds with a yield of 8 percent at all maturities.

$$s = \frac{[(1-\hat{E}_0(x_N))P_0(N-1)] - \hat{COV}_0[\exp(-\int_0^N r\, du)/\hat{E}_0(\exp(-\int_{N-1}^N r\, du)),\ x_N]}{\sum_{t=1}^{N-1} P_0(t) + P_0(N)\ \hat{E}_0(x_N) + \hat{COV}_0[\exp(-\int_0^N r\, du),\ x_N]}$$

Consider a simple calculation for the markup that ignores the covariance terms:

$$s' = \frac{(1-\hat{E}_0(x_N))\ P_0(N-1)}{\sum_{t=1}^{N-1} P_0(t) + P_0(N)\ \hat{E}_0(x_N)}$$

If the covariances terms are positive, then $s' > s$ and the simple calculation tends to overstate the necessary markup. In Table 2, we present some calculations of s' for different maturities and different values of $\hat{E}(x_N)$.

Table 2. Spread Calculation with Default on Last Payment Only

Maturity (Years)	Expected proportion of payment (Last payment)	Spread, s' (in percent)
5	1.0	0.00
5	0.9	0.81
5	0.8	1.62
5	0.7	2.46
5	0.6	3.30
5	0.5	4.16
5	0.4	5.03
10	1.0	0.00
10	0.9	0.34
10	0.8	0.68
10	0.7	1.02
10	0.6	1.37
10	0.5	1.72
10	0.4	2.07
20	1.0	0.00
20	0.9	0.11
20	0.8	0.22
20	0.7	0.33
20	0.6	0.44
20	0.5	0.54
20	0.4	0.65

Note: The floating rate debt is set up with semi-annual payments. For these calculations, we use a flat term structure for default-free bonds with a yield of 8 percent at all maturities.

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