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Currency Bands, Target Zones, and Cash Limits:
Thresholds for Monetary and Fiscal Policy

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Abstract

Exchange rate behavior is analyzed in the context of a stochastic rational expectations model in which there are random shocks to the price setting mechanism and in which the authorities choose to impose either nominal or real exchange rate bands. Results are compared to those which emerge from a simple monetary model subject to velocity shocks. The effects of a realignment of the Band, and of fiscal policy used in conjunction with monetary policy to defend the band, are also examined.

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Summary

This paper discusses how the commitment to keep a currency within prescribed limits in relation to a hard currency affects the conduct of monetary policy. It also examines how adjustments to fiscal policy may reduce the strain on the credibility of strict exchange rate commitments posed by inflationary surges in the domestic economy.

The techniques involved assume that foreign exchange markets are rational and that the authorities' commitment to maintaining the exchange rate within the band is fully credible. Briefly, they consist, first, in seeing how the exchange rate will logically relate to economic fundamentals and, then, in determining what rules for monetary policy are consistent with keeping the currency within bands.

The paper begins with a brief survey of the literature that studies the implications of a random walk in velocity for the conduct of policy in a flexible-price, full-employment, monetary model, where the exchange rate is always at its purchasing power parity level. In such models, monetary policy must accommodate velocity shocks beyond a certain level.

The main body of the paper considers how currency bands affect the conduct of monetary policy when prices are not fully flexible and price setting is subject to random supply side shocks. Specifically, the implications for monetary policy of pursuing nominal currency bands in a stochastic version of a well-known "exchange rate dynamics" model are examined.

As the target zones for exchange rates that have recently been advocated are for real, not nominal exchange rates, real currency bands, which are indexed to the price level, are also examined. The implications of announcing credible rules for the accommodation of cumulative price movements by discrete realignments are also briefly discussed.

The same framework is employed to determine the effects of adjustments to fiscal policy (by imposing cash limits on public spending) when the trade deficit goes beyond a critical threshold, either when the rate is floating or when it is stationary at the edge of a band. The conclusion discusses some of the limitations of the analysis, including, in particular, the assumption that financial markets are efficient and that the bands are fully credible.

I. Introduction

Since 1979 macroeconomic policy in many European countries has been conducted within the constraints imposed by the exchange rate commitments of the EMS. As recourse to realignments has been sharply reduced and the pace of capital market integration deliberately accelerated, so the loss of monetary autonomy this involves for countries such as France and Italy has become ever more apparent. (It is not a loss than is much lamented, however.) Currently the Delors Report has posed the question of how far fiscal policy also may be constrained by membership. At the global level, G7 countries have been trying to live within currency bands against the dollar ever since the Louvre Accord of 1987: but the extent of policy coordination that this involves has been (like the bands themselves) much less explicit.

In this paper, we discuss what the commitment to keep one's currency within prescribed limits against a hard currency implies for macroeconomic policy, monetary policy in particular. We also see how adjustments to the stance of fiscal policy may reduce the strain on the credibility of strict exchange rate commitments posed by inflationary surges in the domestic economy. The techniques involved assume that foreign exchange markets are rational and that the commitments are fully credible. Briefly they consist in first seeing how the exchange rate will logically relate to economic fundamentals (where the latter involve both stochastic disturbances to the economy and the stance of monetary policy itself); then in determining what rules for monetary policy are consistent with keeping the currency within bands.

These techniques were first applied for this purpose by Paul Krugman in 1987, in a context where the stochastic disturbances specified were those affecting the velocity of money. We begin with a brief survey of those papers, by Krugman and others, which study the implications of a random walk in velocity for the conduct of policy in a flex-price, full-employment "monetary" model where the exchange rate is always at PPP. (Basically monetary policy must accommodate velocity shocks beyond a certain level.)

In the main body of the paper, however, we see what currency bands imply for the conduct of monetary policy when prices are not fully flexible and the price-setting process is subject to random "supply side" shocks. Specifically we study the implications for monetary policy of pursuing nominal currency bands in a stochastic Dornbusch model. As the "target zones" for exchange rates advocated since 1983 by John Williamson are for real, not nominal exchange rates, we also consider real currency bands, which are indexed to the price level. The implications of announcing credible rules for the accommodation of cumulative price movements by discrete realignments are also briefly discussed.

The same framework is employed to see what the effects might be of adjustments to fiscal policy (by imposing cash limits on public spending) when the trade deficit goes beyond a critical threshold, either when the rate is floating or when it is stationary at the edge of a band.

In conclusion, we discuss some of the limitations of this analysis including in particular the assumption that financial markets are efficient and that the bands are credible.

II. Monetary Models

The monetary models of exchange rate determination used in Krugman (1989), Froot and Obstfeld (1989) and Flood and Garber (1989) are all variants of the model outlined in the following equations:

$$(1) \quad m - p = \kappa \bar{y} - \lambda i - v$$

$$(2) \quad s = p - p^*$$

$$(3) \quad E(ds) = (i - i^*)dt$$

$$(4) \quad dv = \mu dt + \sigma dz$$

The first equation describes the equilibrium condition for the domestic money market, where m is the log of the domestic money supply, p the log of the domestic price level, \bar{y} is the log of "full employment" GNP, and i is the nominal interest rate. Shocks to velocity cumulate in the variable v , which is assumed, in (4), to follow a "white noise" process with drift coefficient μ and variance σ^2 per unit of time. This is just the continuous time equivalent of a random walk. Equation (2) states that purchasing power parity always holds, where s is the log of the exchange rate (domestic price of foreign currency) and an asterisk indicates a variable in the "rest of the world." Equation (3) is a risk-neutral arbitrage condition, in which expected depreciation of the exchange rate is set equal to the interest differential.

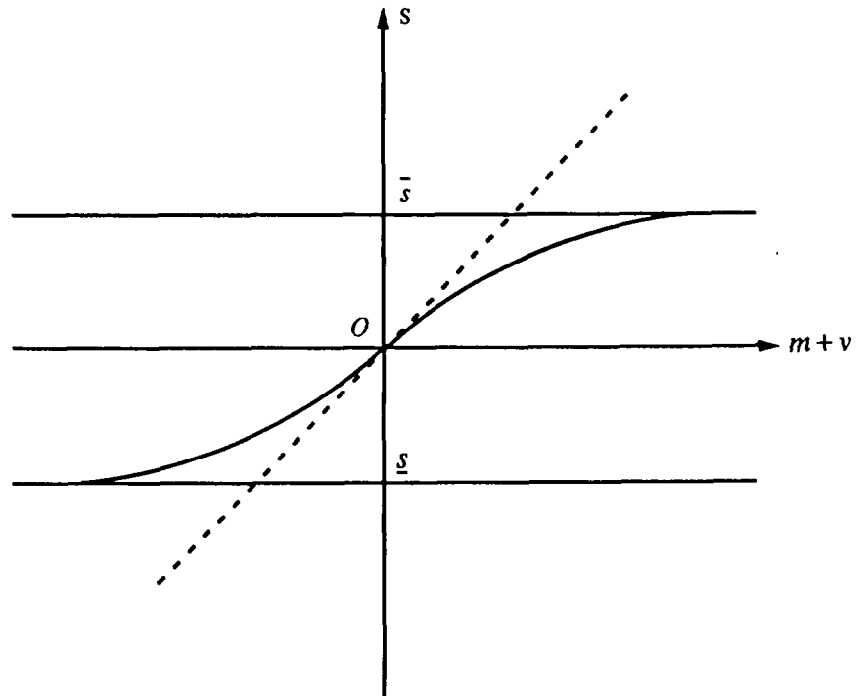
Domestic output is exogenous, as are all variables in the rest of the world, so that we may subsume them in the velocity shock term to obtain:

$$(5) \quad s = m + v + \lambda E(ds)/dt$$

This provides us with a single equation to describe the evolution of the exchange rate. Its simplicity has been achieved at the cost of assuming, in effect, that the domestic economy is always at full employment, and that the real exchange rate is constant. Its great advantage is that, as Krugman has shown, one can obtain simple analytical expressions to describe the path of the exchange rate within a currency band.

Figure 1

KRUGMAN'S "SMOOTH PASTING" SOLUTION



Suppose that the authorities wish to contain s within some specified range $[\underline{s}, \bar{s}]$. Now it is immediately obvious that s could be perfectly stabilized if the money stock m were to be continuously adjusted so that all velocity shocks were fully accommodated. So the assumption which makes the problem interesting is that there is no accommodation so long as the exchange rate lies strictly within the range $[\underline{s}, \bar{s}]$. Only when the exchange rate hits the edge of the band is m adjusted so as to neutralize the shocks to v .

In the absence of a band, and with m held constant at \bar{m} , it seems reasonable to assume that the free float solution is:

$$(6) \quad s = \bar{m} + v + \lambda\mu$$

This is the only solution to (5) which does not involve ever-widening interest differentials as velocity shocks cumulate. The free float interest rate is constant: thus $i = i^* + \mu$, and as v increases, weakening money demand puts upward pressure on prices. Since PPP is always assumed to hold, the currency steadily depreciates.

In the presence of a fully credible band, the path for the exchange rate must incorporate the expectations of market participants that there will be a change in monetary policy at the edges of the band, designed to prevent the rate from straying outside its predetermined limits.

Krugman shows that the general solution to this problem when the velocity shock has no trend ($\mu = 0$) and we normalize by centering the band around $s = 0$, takes the form

$$(7) \quad s = m + v + A [e^{\rho v} - e^{-\rho v}]$$

where $\rho = (2/\lambda\sigma^2)^{1/2}$, and A is a constant to be determined by suitable boundary conditions.

One of the most important features of this solution is the absence of any explicit dependence upon time. One can characterize the behavior of the exchange rate as a simple deterministic function of fundamentals, m and v .

The expression in (7) is the sum of the free float value, $m + v$, and an additional term $A[e^{\rho v} - e^{-\rho v}]$, which incorporates the effect of the anticipated check to s at the edges of the band. But A is as yet an undetermined constant, and needs to be pinned down to identify a unique solution for s in any given band. The necessary boundary condition is that the solution path be tangent to the edges of the band, the so-called "smooth pasting" condition. One then obtains the solution illustrated in Figure 1.

The intuitive justification for this tangency condition is as follows. Suppose that s has just reached its upper limit \bar{s} . If the path for s cuts the upper edge, then if v were to jump upwards by a small amount, then without intervention s would be expected to rise above \bar{s} . This expectation by construction would be the only one consistent with the arbitrage condition in (3), given current interest rates. Intervention to hold s at \bar{s} if v increases, but not to prevent a decline in s if v decreases, will imply that the current level of domestic interest rates, which must lie below foreign levels, will still be too high to be consistent with arbitrage. In other words, any policy intervention designed to prevent s from rising above \bar{s} will reduce $E(ds)$ at that point, with no corresponding impact on domestic interest rates. Only if the exchange rate path is smoothly tangent to each edge of the band will arbitrage be satisfied.

Let us now turn to considering more explicitly the impact of this policy on events in the money market. We can solve for the demand for nominal cash balances m^d as

$$m^d = s(v) - \lambda i - v$$

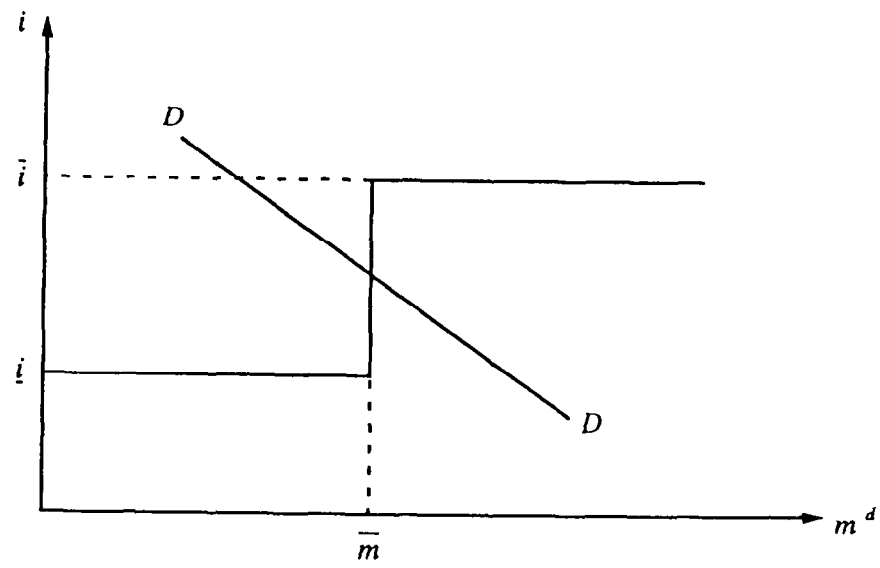
where $s(v)$ denotes the solution for s in (7). Money market equilibrium can then be depicted as in Figure 2. The shocks to velocity cause the demand curve DD to shift, with negative shocks moving the curve to the right and vice versa. So long as the nominal interest rate i lies within the range $[\underline{i}, \bar{i}]$, the money supply is held fixed at \bar{m} . However, as soon as v has fallen to the point at which $i = \bar{i}$, and any further decline would cause i to rise above \bar{i} , then m is adjusted as necessary to accommodate such a shock, and to prevent any further rise in i . But if, after an increase in m , the demand for money starts to drift back downwards, m is held fixed at its new level, and interest rates immediately start to fall, and the exchange rate moves back inside its band.

A more formal justification of the "smooth pasting" boundary condition is provided in the papers by Froot and Obstfeld (1989) and Flood and Garber (1989). They observe that the problem studied by Krugman is equivalent to that of regulating a Brownian motion process. They appeal to the results of Harrison (1985) and the simplified discrete-time argument of Dixit (1988). The interpretation in terms of regulated Brownian motion is of considerable theoretical interest and has already served to provide a link between the original pioneering work of Flood and Garber (1983) on stochastic regime switching and the more recent currency band literature (see Froot and Obstfeld (1989), Flood and Garber (1989)). Since the approach is conceptually quite distinct from that which we have chosen to take in the work we describe in the next section, it is worth emphasizing the important economic features which emerge.

First, in order to be able to obtain the simple and elegant analytical solutions of Krugman, and of Froot and Obstfeld, the velocity

Figure 2

DEMAND AND SUPPLY OF MONEY



shock variable v must be assumed to follow a Brownian motion process (with or without an exogenous trend component μ). This implies that v has infinite asymptotic variance, although s will have a well-defined limiting distribution. If one introduces any form of mean-reversion into the process driving v , one must forgo simple analytical solutions.

Second, the amount of time that the exchange rate is actually observed at the limits of its range is "short." This can be explained quite straightforwardly in economic terms. The monetary intervention at the edges of the band is not designed to wipe out the interest differential that exists between the domestic economy and the rest of the world. Its effect will be to prevent the current velocity shock from feeding into interest rates, and so to stabilize interest rates at current levels. Suppose now that the exchange rate were expected to remain on the edge of the band for more than an instant. This implies that over that period of time it is expected to be stationary, or $E(ds) = 0$. But then arbitrage would require that the interest differential be equal to zero, contrary to hypothesis. Thus the adjustment to the money stock designed to defend the band will always be "small," and, most importantly, will not produce any jump in interest rates.

Recently, Flood and Garber (1989) have shown how "large" interventions can be incorporated into the above analysis. Their arguments are of particular interest since their formal results have an exact parallel within the "sticky price" model examined in the next section, although the economic implications are rather different.

Flood and Garber suppose that the authorities announce an intervention rule which specifies both the upper and lower limits of the fundamental v at which intervention will occur, and the magnitude of the intervention at each limit. They show that this will lead to an exchange rate path of the kind illustrated in Figure 3a. Here we have chosen for the purpose of later comparison to assume that there is no time trend in v , and to work with intervention points equidistant from the origin, and an intervention rule which exactly accommodates the accumulated velocity shock. Thus $L = -U$. The curved path is, as before, a particular solution to (5). However, the boundary conditions are determined differently as a result of the different intervention rule. Suppose that initially $m = 0$. Then if v hits the upper limit U , the authorities immediately tighten the money supply, setting $m = -U$. But since this intervention is fully anticipated, arbitrage imposes the requirement that there be no discontinuous jump in the exchange rate. This means that the system will jump from U to 0 .

There are a number of rather curious and interesting implications of this argument. First, any given currency band can be defended by any one of an infinite number of (fully credible) intervention rules. The same path, for example, would be picked out by an intervention of size $u' - U'$ promised at U' (together with its symmetric counterpart). Second, any discrete intervention will occur at a point when the exchange rate lies strictly within the interior of the band it is designed to support. In

the special case of Figure 3a, intervention occurs only when s lies in the center of the band. Third, it will occur in a range of "paradoxical" response of exchange rate to fundamental. Thus, as interest rates continue to decline as positive shocks to v reduce money demand, the currency will actually be appreciating. Of course, there is nothing very mysterious about this, since what the market is doing is partially discounting the large rise in interest rates anticipated when v hits its upper limit at U and the substantial monetary squeeze occurs.

The final point which turns out to be of central importance in differentiating discrete intervention of the sort described by Flood and Garber from the apparently similar rules we discuss below relates to their reversibility. If an intervention occurs in the Flood and Garber analysis, it will never be reversed after only a "short" period of time. In particular, if an intervention is provoked by a positive shock to the fundamental, then an intervention of the same size but opposite sign will not occur if there is an immediately subsequent negative shock. This turns out to be formally very similar to the analysis of rules for realigning currency bands, which we have discussed elsewhere (Miller and Weller (1989a)). It produces precisely the same form of "hysteresis" in exchange rate behavior which we described there, shown here in Figure 3b. If we view the exchange rate as a function of v alone, the intervention rule produces a family of curves, and as v passes through the point U , one can think of the exchange rate switching from the solid to the dashed path.

III. A Stochastic Dornbusch Model

In the literature so far surveyed, Brownian motion processes are introduced as disturbances to a flex-price, full employment model of an economy which has no inherent dynamics of adjustment. In the absence of exogenous shocks, the economy would necessarily be at full employment with stable prices and an exchange rate that satisfies purchasing power parity. The purpose of this part of the paper is to show how the stochastic analysis of currency bands can be applied when there are endogenous lags of adjustment.

Specifically we examine the case where prices are less than fully flexible, and the shocks are "supply side" disturbances to the process of wage/price setting. Formally the innovation of the Brownian motion process introduces "noise" into the equation describing the evolution of the price level.

Though financial markets are taken to be forward-looking throughout our analysis, we focus in this paper on the simple case where the process of price adjustment is not. Specifically we use a stochastic version of the popular Dornbusch model (1976) where the process of price setting is a simple Phillips curve. (The procedures we use can be extended to include forward-looking labor contracts, but we do not pursue this here.)

Figure 3A

FLOOD AND GARBER'S DISCRETE REGULATION

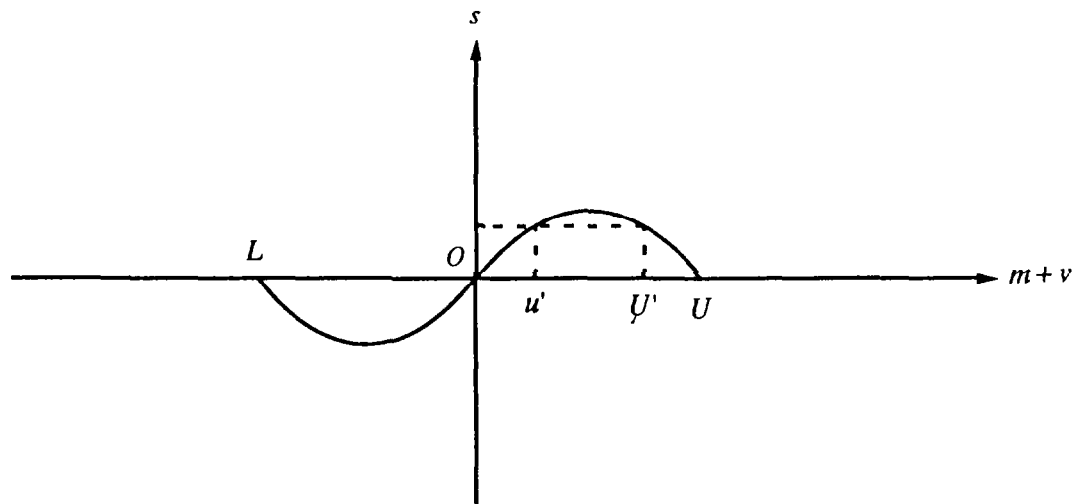
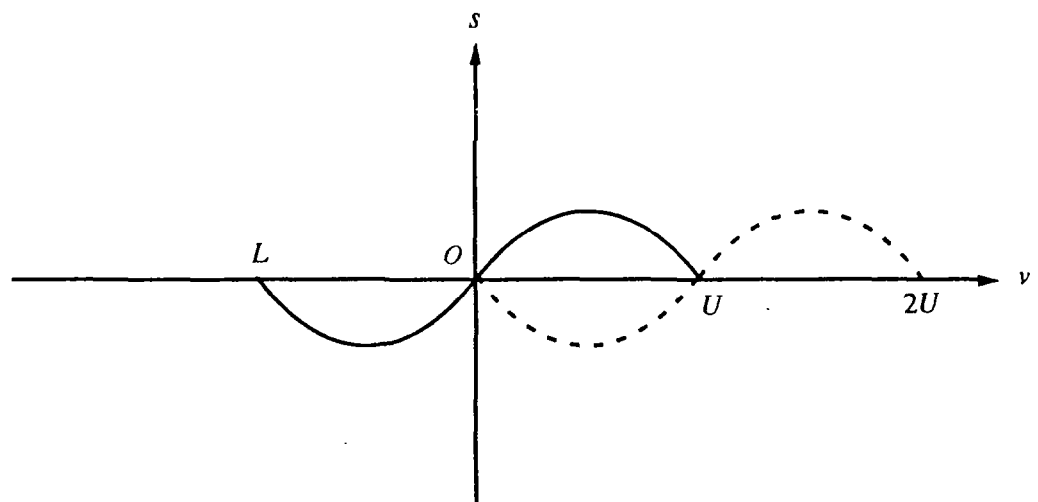


Figure 3B

DISCRETE REGULATION IN THE MIDDLE OF THE BAND



The equations to be used in this section are as follows:

(8)	$m - p = \kappa y - \lambda i$	Money Market
(9)	$y = -\gamma i + \eta(s - p)$	Goods Market
(10)	$E(ds) = (i - i^*)dt$	Currency Arbitrage
(11)	$dp = \phi(y - \bar{y})dt + \sigma dz$	Price Adjustment

where the variables are as defined in the last section.

Two of these behavioral equations are much as before, namely (8) the LM curve defining equilibrium in the money market and (10) the riskless arbitrage condition for the foreign exchange market. Note that disturbances to the velocity of money are omitted here, however. Instead, it is the price adjustment equation--where prices rise if GNP is high, and vice versa--which is disturbed by "white noise" shocks. With prices evolving in this way there is no guarantee that output will remain at any given level, nor that the exchange rate will remain at PPP. The log of the level of output, y , is therefore taken to be demand determined, where the level of demand depends on the real exchange rate ($s - p$) and on the interest rate i (equation (9)). ^{1/}

The essential idea is, of course, that the stance of monetary policy be switched at some point so as to stabilize the exchange rate. For cumulative velocity shocks, for example, we have seen how Krugman describes a switch from keeping the money supply constant inside the band to one of active "intervention" at the edge where money supply is adjusted to offset any shocks which would take the exchange rate outside the band.

When there are supply side shocks to the price level, however, a variety of policy responses seem worth considering, depending firstly on whether the money supply is kept constant inside the band (or whether it accommodates the price shocks) and secondly on whether the bands triggering the policy switch are for nominal or real exchange rates. These alternatives generate four polar cases, shown in Table 1.

We do not, in this paper, formally examine either of the cases of monetary accommodation inside the band and which appear in the second row of the table. One of these, however, that in the lower left of the

^{1/} The choice of nominal interest rate here as the influence on output is for simplicity only. Nothing of substance in our analysis changes if we work with the real interest rate.

table, may be of interest as it would represent the case where monetary policy was essentially highly permissive, except for the commitment to the band. This might be a better characterization of those countries who seek to adopt currency targets so as to ensure monetary discipline, than is the maintained assumption of monetary discipline within the band in what follows. (The principles developed below are easy to apply to this case, too.)

Table 1. Varieties of Policy Response

		Type of Currency Band	
		Nominal	Real
Policy within band	Fixed money supply	Nonaccommodation subject to currency bands	Williamson's target zones
	Monetary accommodation	Full accommodation limited by currency bands	Fully indexed monetary policy

The policy assumptions indicated in the upper left corner involve a switch of monetary policy from a fixed nominal target within the band to one of nominal exchange rate stabilization at the edges (it is assumed that prices are stable in the foreign country). We have analyzed the consequences of the overriding of monetary targets by currency commitments in some detail elsewhere (Miller and Weller (1988, 1989)) by introducing currency bands into a stochastic Dornbusch model, and the main results are presented immediately below.

In the second column the exchange rate commitment is to bands on the real exchange rate. If a fixed nominal money supply target is pursued within the band, this then approximates the target zone idea proposed by John Williamson (1983, 1985).

1. Nominal currency bands

The dynamics of the system can be summarized in the equation below:

$$(12) \begin{bmatrix} dp \\ E(ds) \end{bmatrix} = A \begin{bmatrix} pdt \\ sdt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where p and s are now measured as deviations from long-run equilibrium, and A is a matrix of parameters (see Appendix I).

The economic implications of this model are, not surprisingly, rather different from those of the monetary model already discussed. Because the fundamental in the model, namely the price level, follows an autoregressive process whose trend is endogenously determined, there are no closed form solutions for the function $s = f(p)$, describing how the exchange rate responds to fluctuations in the price level. However, we show in Appendix I how to derive the differential equation characterizing $f(p)$.

The absence of closed form solutions is no serious drawback in this case, since a complete qualitative characterization of the solutions we wish to consider is available (see Miller and Weller (1989a)). These are illustrated in Figure 4.

There are two linear solutions to the system passing through the origin, corresponding to the stable and unstable saddlepaths of the deterministic model. The stable saddlepath is assumed to be the free float solution, for the same reasons as in the monetary model. It may have either a positive or, as shown in the figure, a negative slope, in which case the exchange rate "overshoots."

When a currency band is imposed, the appropriate solution is identified by imposing the "smooth pasting" condition as for the monetary model. The reason for imposing the condition is, however, rather different. There is no economic justification for supposing that the impact of price shocks can be "regulated" in the presence of a nominal band in the same way as velocity shocks. What is necessary to defend the band is a discrete but reversible change in money supply designed to equalize domestic and foreign interest rates. This implies that the exchange rate will lie temporarily on the edge of the band, with any further divergent price shocks being partially accommodated in order to hold domestic interest rates constant. The necessary monetary adjustment is shown in Figure 5 for the overshooting case. The equilibrium level of m and p are normalized at zero. As prices rise above their long-run equilibrium level, the interest rate rises, so causing the currency to strengthen within the band, despite the fact that the economy is in recession. Eventually, s will hit the bottom of its permitted range of variation when p has risen to \bar{p} , and interest rates immediately have to be lowered to world levels to prevent further appreciation. So there is a sudden upward jump in m . If there are further positive shocks to p , they are partially accommodated, but the market is aware that when p drifts back to \bar{p} , the original adjustment to m will be reversed, at which point the exchange rate moves off the edge of the band. It is easy to confirm that the adjustment to m required to set $i = i^*$ will still imply that $y < \bar{y}$, so that downward pressure on prices is maintained throughout the period when the exchange rate is held on the edge of the band.

In terms of concrete policy it is probably most realistic to suppose that reversion to the original regime of floating within the band will be triggered by an index of competitiveness, which in the fixed-rate regime at the edge of the band is equivalent to the domestic price index.

2. Target zones

Williamson has argued forcefully for a sharp distinction to be drawn between nominal and real currency bands. The target zone proposal (1983, 1985) is explicitly a proposal for real currency bands. The analysis of the monetary model we have already seen is limited by the fact that nominal and real bands are equivalent, since PPP is assumed always to hold. This is not the case in the stochastic Dornbusch model, and a number of important differences between a nominal currency band and a target zone emerge.

It is convenient to redefine variables in the model laid out in (8)-(11), introducing the real exchange rate $c = s - p$, and real balances $l = m - p$. The transformed model can then be written as:

$$\begin{bmatrix} dl \\ Edc \end{bmatrix} = A' \begin{bmatrix} ldt \\ cdt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where again the variables are expressed as deviations from long-run equilibrium. The arbitrage condition now imposes the requirement that the real interest rate differential be equal to the expected depreciation of the real exchange rate.

The system is formally identical to that in (12), except that the matrix A' differs from A . The only significant qualitative difference is that the stable saddlepath now always has a positive slope.

The authorities, in order to enforce a target zone, must now concentrate upon the relationship between real balances and the real exchange rate. We assume, as before, that m is held fixed in the interior of the zone. But now a willingness to accommodate changes in the price level can sensitively regulate the fundamental l , precisely as was true of the composite fundamental $x + v$ in the monetary model. What this means is that infinitesimal intervention at the limits of a target zone will lead to a solution path such as AA for the real exchange rate as depicted in Figure 6. The real exchange rate will never spend more than an instant at the edge of the target zone, but will be gently reflected at the limits of its range of variation. The effect of the monetary intervention will be to check the real interest rate.

In order to show what the effect of regulating real balances is on nominal variables, we need to consider the target zone illustrated in Figure 7. Its limits run parallel to the PPP line, and the path for the

Figure 4

STOCHASTIC SOLUTIONS THROUGH THE ORIGIN

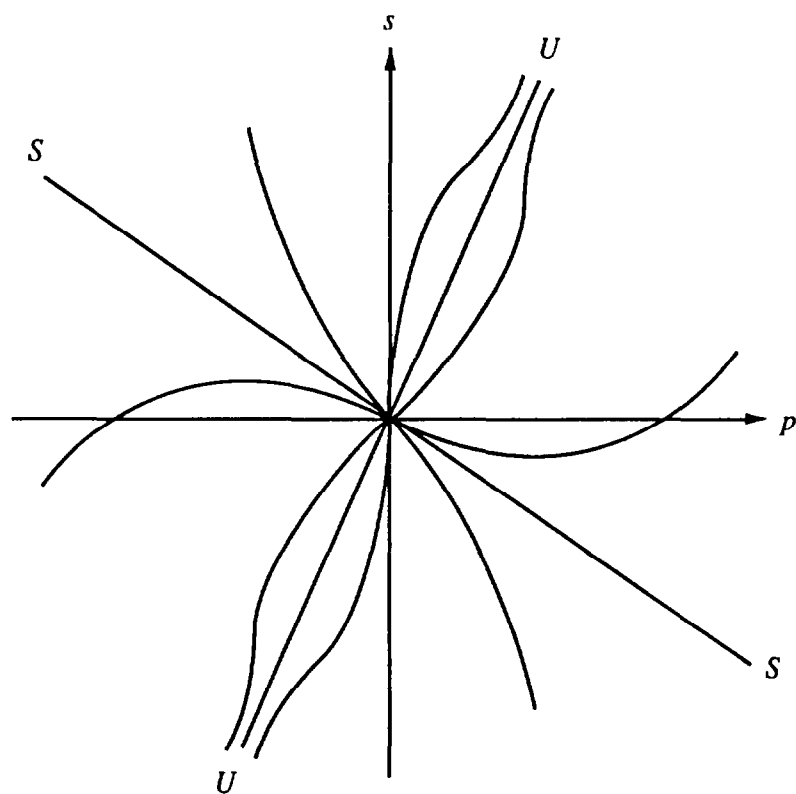


Figure 5

MONETARY POLICY TO DEFEND A CURRENCY BAND

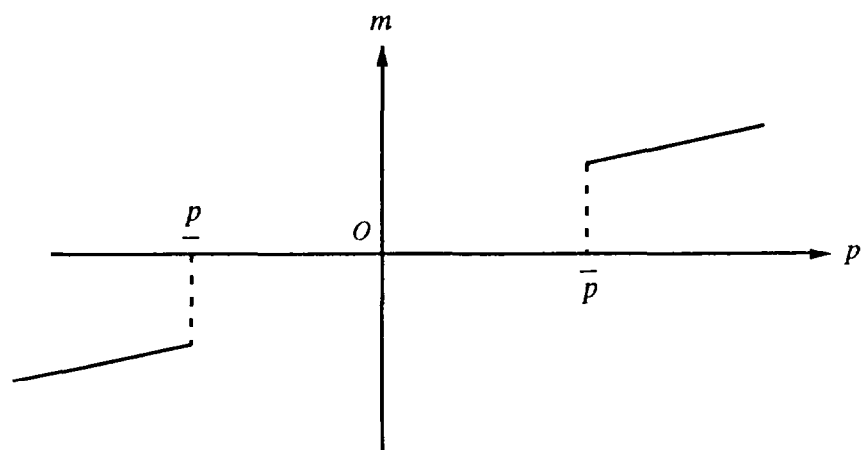


Figure 6

A "TARGET ZONE "

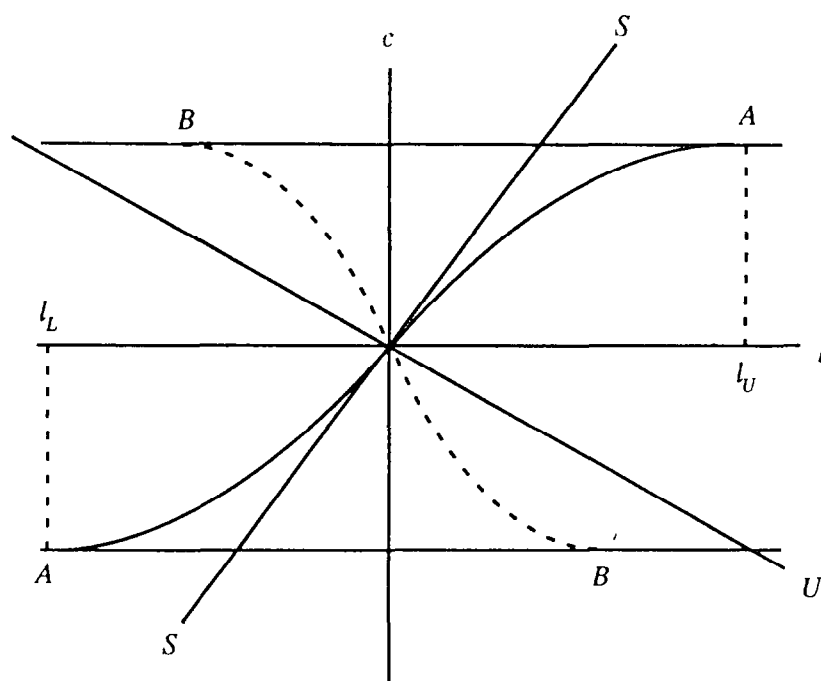
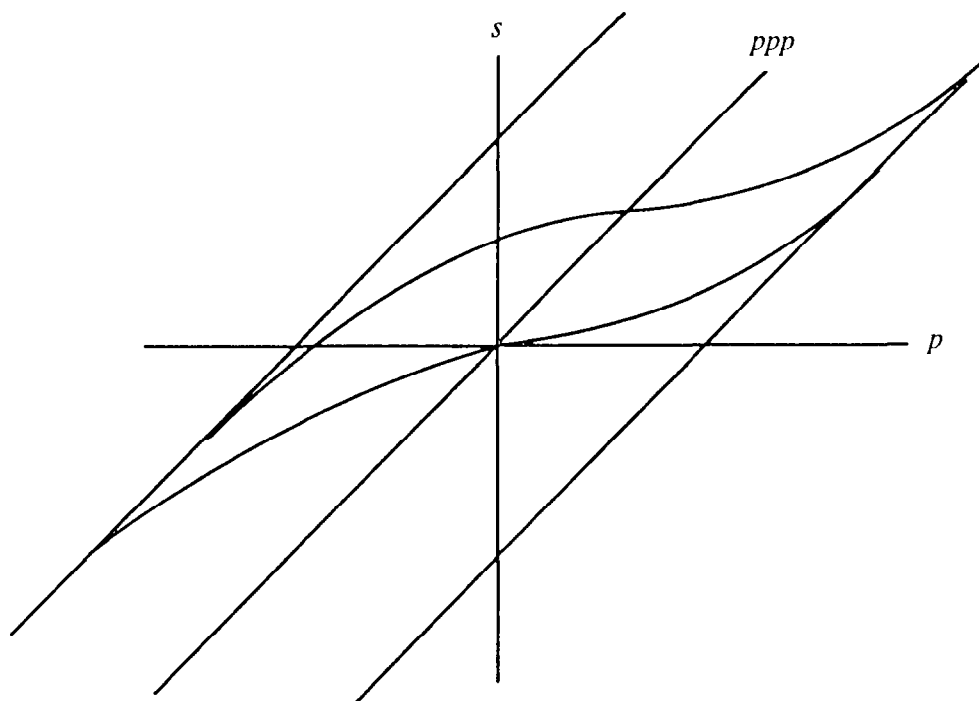


Figure 7

A "TARGET ZONE" AS AN INDEXED CURRENCY BAND



nominal exchange rate within the zone will drift up and down as the money stock is adjusted from time to time to defend the zone.

It is immediately clear that the behavior of the nominal exchange rate is strikingly different within a target zone defended by perfect accommodation of shocks to prices at the edges, than it is within a nominal currency band, where defence must involve reversible switches of regime. What the figure reveals is that the impact of a target zone is exactly the same as a nominal currency band whose limits are indexed to changes in the money stock, when those occur each time the exchange rate hits the edge of the band.

In the long run, both the money stock and the price level will follow a random walk under a target zone arrangement. However, both will display short-run stability. This is in contrast to what happens to money and prices in the case of a nominal currency band, where both variables display a global stability in the sense that they are always expected to adjust in the direction of long-run equilibrium.

This discussion is subject to one important qualification. We have shown that in the case of a target zone, depending upon parameter values there may exist a second "smooth pasting" solution, BB in Figure 6 (see Miller and Weller (1989a)). This solution is unstable, and raises a potentially serious problem. If the intervention rule is formed in terms of what will happen when the exchange rate, rather than the fundamental, reaches a particular level, as seems desirable, then the market will have no means of distinguishing between the two possible trajectories. Both will solve an "inverse regulation" problem for the given target zone. This suggests the need for policy to be directed towards deterring the market from embarking upon the unstable path.

Adams and Gros (1986) have criticized real exchange rate rules for being inflationary. But target zones are only contingent real rate rules: inside the band strict monetary control applies. Are they nevertheless liable to the same critique? If the parameter values are such as to give a unique "smooth pasting" solution (e.g., if the demand for money is interest inelastic), then the answer is no. Along the unique path leading to the edge of the band prices are essentially stable, and even if shocks take the system to the edge of the band, these infinitesimal visits are not enough to let inflation expectations build up. But if the parameters allow for a second solution then the critique seems to be valid as the second solution involves an unstable price level. So the system will be thrown against the indexed edges of the band and inflationary expectations quite likely to be established. It can, however, be shown that this second solution will be ruled out by a "credible threat" to adjust monetary policy within the band, if ever the market were to move in the perverse fashion required for this solution to apply.

3. Discrete realignment of a currency band

We have already shown that defending a target zone by infinitesimal adjustment to the money supply at the edges of the zone is equivalent to a policy in which a nominal band is realigned by small amounts each time it is defended. But what if the rule for realignment involves a discrete shift in the position of the band? This is a question we have analyzed at length elsewhere (Miller and Weller (1989b)) so our treatment here will be brief.

Suppose for the sake of argument that the authorities announce a rule of the following form: whenever the exchange rate hits the top of the band, the band will be shifted upwards by an amount equal to half the total width of the band: and this realignment will be validated by a change in money supply designed to shift the long-run equilibrium exchange rate to the center of the new band. Then we find that the exchange rate will follow the path depicted in Figure 8. In the original band, it will follow the solid line, and when it hits the top edge of the band at point A, an upwards realignment is triggered. By construction, A is now the long-run equilibrium for the realigned band, and the rate now moves on the dotted trajectory. If subsequent price shocks drive the system back to point O, the realignment is reversed and the money stock is returned to its original level.

Note that there is no smooth transition from one regime to another. The reasons for this are (1) the presence of discrete intervention; and (2) the locally irreversible nature of the regime shift. If we illustrate the impact of this realignment rule in our equivalent "real" model (see Figure 8), we find that the analysis corresponds exactly to that of Flood and Garber (1989) for the monetary model (see Figure 3a). Of course the interpretation of the policy objective is considerably different, but the parallel is of interest, and indicates that our analysis of realignment rules is formally equivalent to the treatment of discrete regulation of Brownian motion by Harrison (1985).

A case of some theoretical interest arises when one considers the effect of a realignment rule in which the top of the old band becomes the bottom of the new one. This is the greatest realignment which the market could anticipate with certainty, contingent upon the exchange rate hitting the edge of the band. Any larger shift in the band would force a fully anticipated jump in the exchange rate, which would be inconsistent with arbitrage. In the case of bands which are always centered around long-run equilibrium, intervention in Figure 9 will be of size $U-L$ at U , and of size $L-U$ at L . But this means that there is movement between one band and the other in either direction. This is formally analogous to the switch of regime which occurs at the edge of a nominal currency band, and as we have already argued, requires "smooth pasting" to be satisfied. Here too, smooth pasting will be satisfied, since the slopes of the curve LOU at its endpoints are equal. In Figure 8, the dotted path slides up the PPP line until its lower endpoint reaches A.

Figure 8

REALIGNMENT OF A CURRENCY BAND

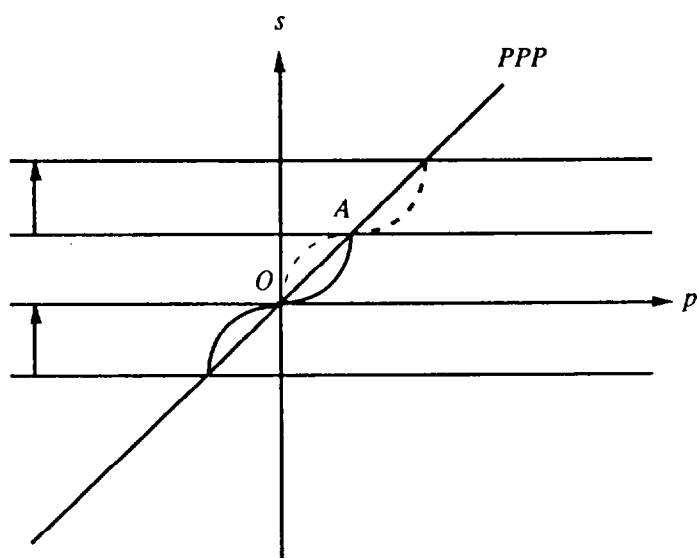
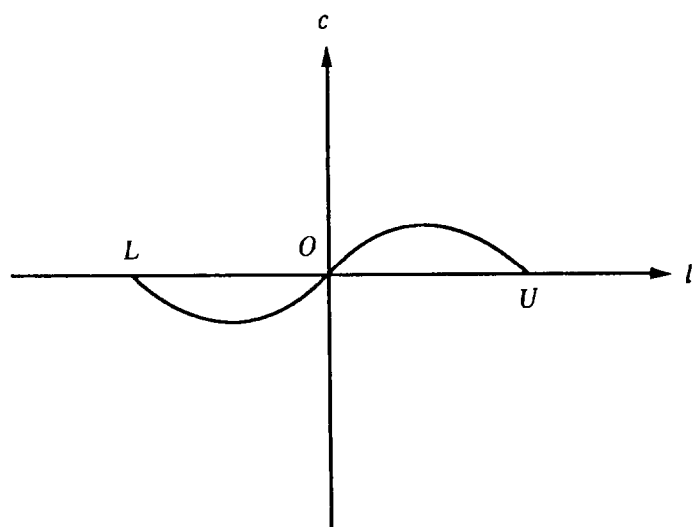


Figure 9

EFFECTS OF REALIGNMENT ON REAL VARIABLES



IV. Cash Limits on Public Spending when the Trade Balance is in Serious Deficit: An Example of "State Contingent" Fiscal Policy

The exclusive use of monetary policy to combat supply-side price shocks in an open economy may well lead to substantial fluctuations in the exchange rate. This is the message emphasized by "overshooting" models of the exchange rate. The reason is, of course, that in an open economy under floating rates the channels through which monetary control affects aggregate demand include the trade balance as well as interest sensitive components of domestic demand.

One may cite current experience in the U.K. as a relevant example. The tight money policy presently in force to check wage/price pressure has already trimmed economic growth and is expected to bring down inflation-- but it has been associated with an overvalued exchange rate and alarming deficit on the current account.

To take some of the strain off monetary policy in such circumstances, the idea of making some fiscal adjustment has obvious attractions. Indeed, in his latest (1989) budget, the British Chancellor postponed long-planned structural changes to the tax system as necessary to help cope with the cyclical situation: specifically, he postponed cuts to the basic rate of income tax and aimed for a substantial budget surplus instead.

The policy adjustment made by the Chancellor was reluctant and ad hoc, but it seems worth asking what would happen if there were a systematic strategy of activating fiscal policy at the point when the pressures borne by monetary policy become too great. Our treatment of this question departs from the British experience in two (further) respects. Firstly, we assume that the policy change consists in the imposition of "cash limits" on (some) public spending; second we assume that the trigger for such activism is explicitly the external deficit (and not the general cyclical situation).

Let public expenditure in real terms be specified by $g_0 - \min [\beta(p - \hat{p}), 0]$ where \hat{p} is the price level at which the trade deficit reaches a critical level, and β is a measure of how widely the ensuing cash limits are imposed: if $\beta = 1$, all public spending is cash limited. (Note that \hat{p} will depend on the exchange rate and has to be determined endogenously, see below.) We assume no adjustments to monetary policy: so a fixed money supply target is pursued and the exchange rate allowed to float freely.

A formal treatment is provided in Appendix II, but Figure 10 suffices to show the main points of the analysis. The locus SS shows the track followed by the exchange rate (s) as a function of the price level (p) in the absence of any fiscal adjustment. Unlike the PPP line, this locus slopes downward to the right, which signifies that there is exchange rate "overshooting." The schedule labelled CC shows where the trade deficit

becomes critical and implicitly defines the function $\hat{p}(s)$, capturing how \hat{p} depends on s . (It slopes up to the right but by less than 45° because the loss of demand associated with a move to the North East is assumed to reduce imports, requiring some move from the PPP line to compensate for the effect this has on the trade balance.)

We show in the Appendix that the effect of introducing cash limits is to reduce the extent of exchange rate overshooting. In the diagram, indeed, we make the convenient assumption that the overshooting is precisely checked by the cash limits; so the rate will neither rise nor fall as p rises above \hat{p} .

When will the cash limits be triggered? In a nonstochastic environment one could easily identify the critical point by seeing where the floating rate path crosses the locus CC (at point A in the figure). But in a stochastic environment this is not the appropriate solution because a reversible regime switch calls for "smooth pasting" of exchange rates on either side of the switch point; and an exchange rate that switches at A from the locus SS to a constant value to the right of A fails to satisfy this condition. The point is that, in a stochastic environment, the possibility that price shocks will push the trade balance into a critical region (leading to a tighter fiscal stance and lower nominal interest rates than otherwise) will tend to affect the floating rate even before the critical trade deficit is reached.

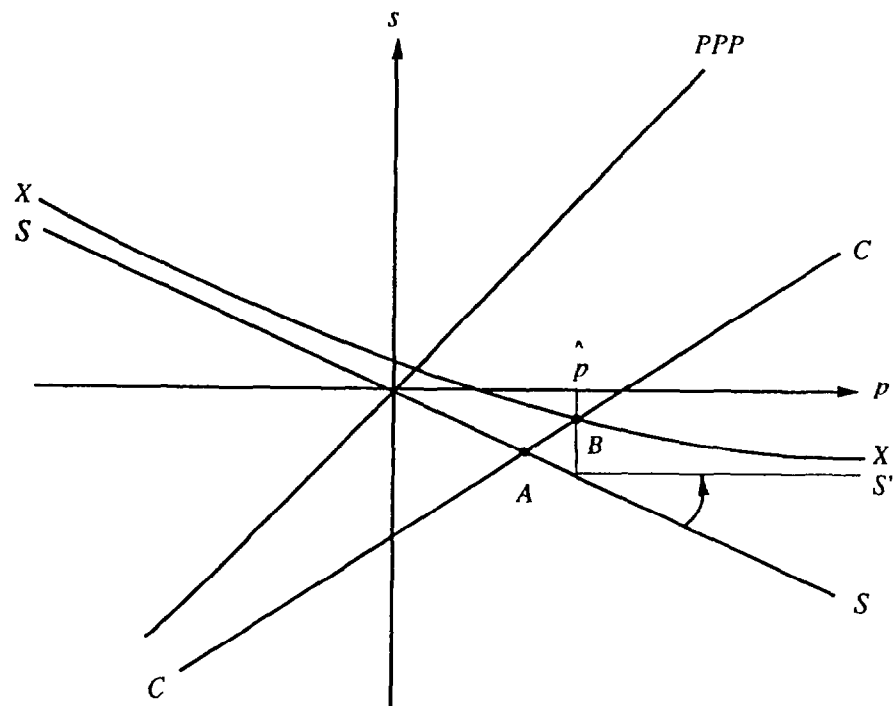
Though determining the exact solution calls for the use of numerical methods, its qualitative properties are obvious from the figure. It is shown, labeled XX, crossing the locus CC at point B. From this crossing point, the solution approaches asymptotically, in either direction, the stable manifold appropriate to the regime in question (SS to the left and the horizontal line labelled S' to the right). And there is no change in the slope of XX as it passes through B.

The "stochastic smoothing" indicated by this solution means that the price level can go higher before it pushes the trade balance to its critical level than would have been true without cash limits. Whether this effect on the exchange rate--analogous to the "honeymoon effect" described by Krugman (1987)--will actually postpone the expected hitting time for reaching the critical trade deficit is a moot point. This is because here the exchange rate "feeds back" on the economy and in particular a weakening of the rate (a rise in s) adds to inflationary pressure. So the price level is less effectively stabilized on XX than on SS.

What is clear, however, is that when the policy is activated it will both increase the downward pressure on prices, see Appendix II, and improve the trade balance. (As the expected total of borrowing required to handle the crisis should fall, it is the sort of policy a foreign lender might be expected to approve of!)

Figure 10

CASH LIMITS AND EXCHANGE RATES



The policy assignment implicit in the above example, where the money supply is kept constant to stabilize prices and fiscal policy is activated to help achieve external balance is not unlike that recommended recently in Boughton (1989) in his proposal for somewhat flexible exchange rates. Our analysis confirms that this assignment does help to avoid overshooting of the floating rate: it also suggests how his proposal could be modified to avoid the charge that it involves fiscal fine tuning.

It is of course possible to link the ideas of a state-contingent switch in fiscal policy with a similar switch in monetary policy. Consider for example cash limits together with currency bands. In Figure 11, we show the case where there are currency bands and the exchange rate smooth pastes onto the band at \bar{B} and \underline{B} respectively. If the fiscal policy switch only occurs on the edge of the band (so the locus showing the fiscal switch lies to the right of \underline{B}) then this does not affect the rate within the band, but it will clearly affect the length of time the rate has to be held along the bottom edge. If cash limits are imposed inside the band (so the line CC falls to the left of \underline{B}) then one has the added effect of the rate moving up within the band.

What these two examples suggest is that the use of such state-contingent fiscal rules might help either to reduce the pressure on monetary policy under a free float, or the credibility of policy where the float is limited by currency bands.

V. Summary and Conclusions

Though the technical details may seem esoteric, the main ideas and results of this emerging literature on currency bands are reasonably accessible. As is only to be expected, the policy implications depend very much on the model used and the way in which the shocks are introduced. We have looked in some detail only at a "monetary" model with velocity shocks and the "Dornbusch" model with price shocks.

Though the monetary model is characterized by flexible prices and full employment it is nevertheless profoundly non-monetarist in its conclusions. Far from inducing price stability, a policy of controlling the money supply will lead to a random walk in the price level and the exchange rate! It is only by intervening at some point to fully offset shocks to velocity that limits can be set to these potentially infinite fluctuations in nominal values. But, as the analysis shows, the expectation that such intervention will occur is sufficient to induce a good deal of stability even before that--a phenomenon Krugman has referred to as the "honeymoon effect."

If prices are not fully flexible, and the postulated process of price setting is affected by the level of economic activity, then fixing the money supply does ensure long-run price stability, but only if the exchange rate is free to fluctuate widely with interest rates.

Stabilizing the latter within bands requires a switch of monetary policy at some point; and as before the expectation of such a switch induces exchange rate stabilization in advance.

If it is nominal rates that are stabilized, then as we have indicated, the system may spend some time at the edge of the band (which is why we also consider what happens if rules for realignment are implemented instead.)

If, as in Williamson's target zones, it is the real exchange rate that is to be kept within bounds, then the system spends very little time at the edge of the band. This makes it easier to support, but as a consequence (as with the realignment rules) the price level, though "locally" stable, essentially reverts to a random walk.

We are able to provide a description of the different policies required to defend a nominal currency band as opposed to a target zone because of the technical distinction we draw between regulation of a diffusion process and diffusion across a boundary. The technique for analyzing the latter phenomenon technique in particular, promises to have substantially wider application than to the modelling of exchange rate regimes.

We have shown in this paper, for example, that it may be used to examine the effects of a change in fiscal policy triggered by the value of some economic indicator here the trade balance. We have also used the same approach to study the impact of a partial debt default triggered by a critical debt-servicing burden (Miller, Skidelsky and Weller (1989)).

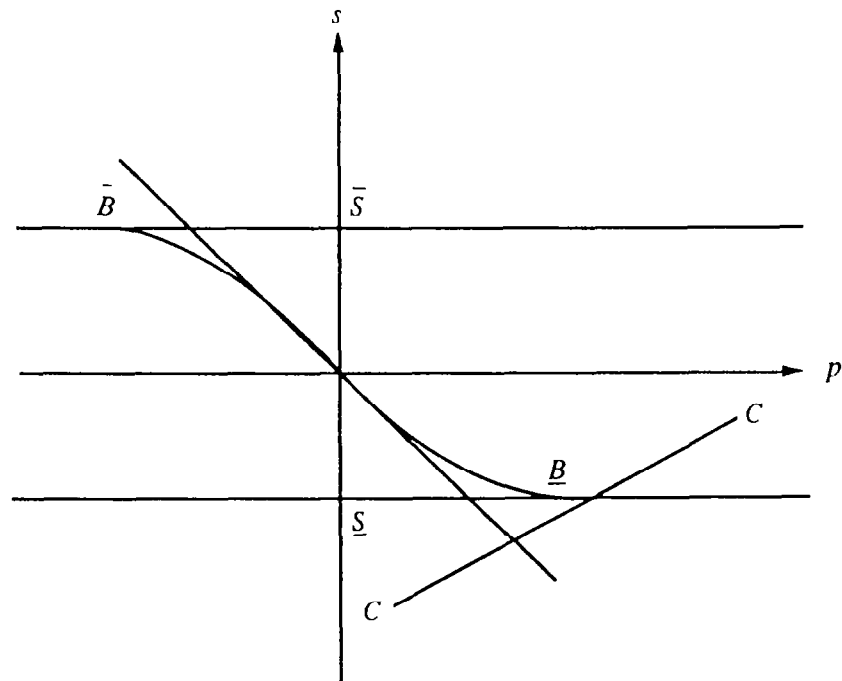
Two topics which have been explored, elsewhere, but are not discussed in this paper are first what happens if there are inefficiencies in the foreign exchange market--bubbles or fads. Some of the possible consequences--and the role of currency bands and/or target zones in avoiding them--is discussed in Miller and Weller (1989c) and Miller, Weller and Williamson (1989).

Second is the question of the use of reserves to help maintain the credibility of a currency band. This issue is addressed in Krugman (1989).

Although the literature we have described on the subject of currency bands makes implicit appeal to costs (of market intervention, policy "fine tuning" etc.) and benefits (of exchange rate stabilization), so far an explicit welfare analysis is lacking. It is clear that if one wishes to address questions such as "How wide should the band be?" or "Is a nominal band preferable to a target zone,?" such as analysis, if not essential, would certainly be highly desirable. We hope to be able to tackle this in future work.

Figure 11

CASH LIMITS AND CURRENCY BANDS



Monetary Targets Subject to Nominal Bands

The evolution of the price level, p , and the exchange rates, s , determined by the equations (8)-(11) can be summarized in terms of two simultaneous stochastic differential equations (after elimination of y and i and normalizing by setting $\bar{y} = 0$) as

$$(A1) \quad \begin{bmatrix} dp \\ E(ds) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -\phi (\gamma + \lambda \eta) & \phi \lambda \eta \\ 1 - \kappa \eta & \kappa \eta \end{bmatrix} \begin{bmatrix} p dt \\ s dt \end{bmatrix} +$$

$$\frac{1}{\Delta} \begin{bmatrix} \phi \gamma & \phi \lambda \eta & 0 \\ -1 & \kappa \eta & -\Delta \end{bmatrix} \begin{bmatrix} m dt \\ p^* dt \\ i^* dt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where $\Delta = \kappa \gamma + \lambda$. Alternatively, if we redefine variables s and p to denote deviations from long-run equilibrium, we may rewrite (A1) as

$$(A2) \quad \begin{bmatrix} dp \\ E(ds) \end{bmatrix} = A \begin{bmatrix} p dt \\ s dt \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where A is the matrix of coefficients multiplying the vector of endogenous variables on the right hand side of (A1).

In order to obtain solutions to the system (A2), one begins by postulating a deterministic functional relationship $s = f(p)$. Following the rules for stochastic differentiation one obtains

$$ds = f'(p)dp + \frac{\sigma^2}{2} f''(p) dt$$

from which it follows that

$$E(ds) = f'(p) E(dp) + \frac{\sigma^2}{2} f''(p) dt.$$

Substituting for $E(dp)$ and $E(ds)$ from (A2) we obtain

$$\frac{\sigma^2}{2} f''(p) + (a_{11}p + a_{12} f(p)) f'(p) - (a_{12}s + a_{22}f(p)) = 0$$

where a_{ij} denotes the appropriate element of the matrix A . This second-order, nonlinear differential equation has no closed form solutions in general, but we have shown that it is possible to characterize completely the qualitative features of the relevant solutions (see Miller and Weller (1989)).

Cash Limits and Exchange Rate Overshooting

The effect of a (partial) cash limit on public spending is proxied by adding the term $g_0 - \beta p$ to the equation determining aggregate demand. (g_0 is a measure of the demand effect of public expenditure without the limit, and β is an index of how hard the limit "bites".)

It is easiest to calculate the effect of keeping such a limit permanently in operation before analyzing the "state contingent" use of such a device. Assuming that there is a fixed money supply target and that the exchange rate floats freely, one has the following equations to solve:

$$B(1) \quad \bar{m} - p = \kappa y - \lambda i$$

$$B(2) \quad y = g_0 - \beta p - \gamma i + \eta(s - p)$$

$$B(3) \quad dp = \phi y + \sigma dz$$

$$B(4) \quad E ds = i - i^*$$

which, after substitution, can be written

$$B(5) \quad \begin{bmatrix} dp \\ E ds \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -\phi(\gamma + \lambda(\eta + \beta)) & \phi\lambda\eta \\ 1 - \kappa(\eta + \beta) & \eta\kappa \end{bmatrix} \begin{bmatrix} p \\ s \end{bmatrix} + \begin{bmatrix} \sigma dz \\ 0 \end{bmatrix}$$

where $i^* = \bar{m} = g_0 = 0$ for convenience, and $\Delta = \kappa\gamma + \lambda$ as before.

Note that the locus of stationarity for the exchange rate (where $E ds = 0$), now has to satisfy the condition that

$$(1 - \kappa(\eta + \beta))dp + \eta\lambda ds = 0.$$

Hence, given cash limits, the condition for "no overshooting" becomes $1 - \kappa\eta \leq \kappa\beta$; instead of $1 - \kappa\eta \leq 0$ as is true otherwise (i.e., the effect of $\beta > 0$ is to reduce overshooting).

Consider, for concreteness, the case where potential overshooting is just reduced to zero, so

$$\kappa(\eta + \beta) = 1, \beta > 0$$

and so $\frac{ds}{dp} = 0$ along the locus of stationarity.

In this special case, the stable manifold happens to coincide with the locus of stationarity, and in the absence of nonlinearities, it is appropriate to restrict the system to the stable manifold. So, with the exchange rate stationary at its equilibrium value, the solution is simply

$$dp = \frac{-\phi(\gamma + \lambda(\eta + \beta))}{\Delta} + \sigma dz.$$

In general, of course, the eigenvector will not have zero slope. But the slope will be less negative as a result of the cash limits.

To establish that cash limits enhance the stability of the system, it is sufficient to observe that raising β above zero has no effect on the determinant of the matrix in B(5), while evidently increasing the trace in absolute value: so the size of the stable root must increase with β .

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