

IMF WORKING PAPER

© 1990 International Monetary Fund

This is a working paper and the author would welcome any comments on the present text. Citations should refer to an unpublished manuscript, mentioning the author and the date of issuance by the International Monetary Fund. The views expressed are those of the author and do not necessarily represent those of the Fund.

WP/90/115

INTERNATIONAL MONETARY FUND

Research Department

Management of the Nominal Public Debt
Theory and Applications

Prepared by Guillermo A. Calvo and Pablo E. Guidotti*

December 1990

Abstract

Optimal management of the public debt is explored in a context where economic policy is continuously revised because, when the public debt is non-indexed, policy-makers are tempted to use inflation in order to reduce the real value of the public debt.

The model's implications are explored following two approaches. First, the effects of various exogenous disturbances are examined by means of numerical simulations. Secondly, the analysis explores—for Italy, Ireland, and the United States—if the model's implications concerning the maturity structure of government debt are consistent with actual experience.

JEL Classification Number:
320

*This paper was prepared for the symposium, "The Political Economy of Government Debt," held at the Dutch Central Bank on June 21–24, 1990. It will be forthcoming in The Political Economy of Government Debt, edited by Harrie Verbon and Frans van Winden, North-Holland. We thank Olivier Blanchard, Alex Cukierman, and Guido Tabellini for valuable comments and discussions offered at the symposium. We also thank Carmen Reinhart and Carlos Végh for their perceptive comments.

Table of Contents

I.	Introduction	1
II.	The Model	3
III.	Public Debt Management. Numerical Simulations	10
IV.	Debt Maturity in Italy, Ireland, and the United States	14
V.	Concluding Remarks	17
Figures	1. The Benchmark Case	18a
	2. Optimal Debt Maturity: The Benchmark Case	18b
	3. A Permanent and Unanticipated Increase in Government Expenditures	18c
	4. A Permanent and Anticipated Increase in Government Expenditure	18a
	5. A Temporary Increase in Government Expenditure	18b
	6. An Increase in the Initial Debt Stock	18c
	7. A Permanent Increase in A.	18d
	8. Level and Maturity Structure of the Domestic Public Debt	18e
	9. Actual and Predicted Maturity	18a
	10. Actual and Predicted Inflation	18b
	11. Index of Maturity with Time Varying A.	18c
References		18
Appendix: Data Sources and Definitions		18

I. Introduction

The theory of optimal taxation has provided a rich framework for understanding important aspects of public debt management. Accounting for tax distortions has meant moving away from a Ricardian world in which debt policies are totally irrelevant, to a world in which changes in the level of public debt matter. By introducing tax-collection costs, Barro (1979) develops a positive theory of public debt determination based on the principle of smoothing tax distortions over time. However, while changes in the level of public debt matter in Barro's (1979) model, the *composition* of the public debt—in particular its maturity structure or whether debt is nominal or indexed—continues to be irrelevant. Debt maturity, for instance, is totally irrelevant in this framework because, given the equilibrium interest rate path, all that matters is the present discounted value of the public debt. As long as the path of taxes is predetermined, *debt maturity is irrelevant because future governments are free to reschedule their obligations, thus generating the same flow of payments to the public.*

Despite Tobin's (1963) pioneering essay on public debt management, the issue of the optimal management of the maturity structure of government debt has remained largely neglected, until recently, by mainstream economics. Part of the reason for this surprising neglect is that the assumptions of policy exogeneity, full policy precommitment, and complete markets, have played a central role in macroeconomic analysis. Under these assumptions, many aspects of public debt management are deprived of a useful function.

The emergence and lasting popularity of the Rationality Hypothesis in macroeconomics and the growing interest in modeling policy-making as an endogenous phenomenon—at a time in which the evolution of the public debt has been at the center of policy discussions in several industrial and developing countries—have instilled new energy into the study of debt management policies, and of the optimal maturity structure of government debt in particular, which constitute the focus of this paper.

In a path-breaking contribution, Lucas and Stokey (1983) show that in a context where policy-makers are constantly trying to maximize social welfare—which, as is well known, lays the roots for time-inconsistency of optimal policy (e.g., Kydland and Prescott (1977) and Calvo (1978))—a careful management of debt maturity could attain the outcome of optimal policy with precommitment even though policy-makers are unable to commit all of their future actions. The conclusion that the maturity structure could be an essential component of the optimal policy contrasts sharply with the implications that could be drawn from a framework in which policy is exogenous or, alternatively, where policy-making occurs in a world of perfect markets and full precommitment (for instance, as in Barro (1979)).

Persson, Persson and Svensson (1987) extend the Lucas-Stokey theory to a monetary framework. They suggest that, even if precommitment is incomplete, the attainment of the full-precommitment optimum involves

This paper examines in greater detail the implications of Calvo and Guidotti's (1990c) model for the management of the nominal public debt. Two different approaches are used to examine the model's implications. On the one hand, we examine the effects of various exogenous disturbances—distinguishing between their unanticipated, anticipated, permanent, or transitory nature—by means of numerical simulations. These results characterize the response of the equilibrium policy regarding debt maturity, the path of taxes, debt, and inflation to the exogenous disturbances. On the other hand, we examine the extent to which the model's implications concerning debt maturity are consistent, at a qualitative level, with the actual evolution of the maturity structure of government debt in three industrial countries in which the evolution of the public debt has played an important role in policy-making: Italy, Ireland, and the United States. The analysis suggests that the model appears capable of explaining some of the main features of the evolution of the maturity structure of the public debt in the above-mentioned countries. The analysis also discusses the implications concerning inflation.

The outline of the paper is the following. Section II presents the basic model, following Calvo and Guidotti (1990c). Section III discusses the results of the numerical simulations, while section IV examines, in light of the analytical model, the evolution of debt maturity in Italy, Ireland, and the United States. Section V contains concluding remarks. The Appendix provides data sources and definitions.

II. The Model

This section considers the optimal taxation problem of a typical government (in period t) which has to choose how to finance an exogenous stream of expenditures and the servicing of its public debt by levying taxes and by issuing new debt of different maturities. All government debt obligations are assumed to be set in nominal terms (and the interest rate on the public debt is non-indexed), and taxes are assumed to be distortionary.

Government expenditure and tax revenue as a proportion of output in period t is denoted by g_t and x_t , respectively. ^{1/} The value (as a proportion of output) in period s of government bonds issued in period s maturing in period t is denoted by b_{st} , the nominal interest rate factor (i.e., one plus the interest rate) on those bonds is denoted by I_{st} . Thus,

^{1/} Hence, x_t may be thought of as the average tax rate.

taking period 0 as the initial period, the government flow budget constraint in period t is given by: 1/

$$x_t = \sum_{s=0}^{t-1} \left(\frac{P_s b_{st} I_{st}}{P_{t-1}} \right) \Pi_t + g_t - \sum_{s=t+1}^{\infty} b_{ts} \quad (1)$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the inflation factor (i.e., normalizing P_0 to unity, we have $P_t = \Pi_1 \Pi_2 \dots \Pi_t$ for $t > 0$). The first term on the right-hand side of equation (1) represents the output value of government debt obligations (i.e., amortization plus interest) which falls due in period t ; these obligations are issued *before* period t (that is, between periods 0 and $t-1$). The last term on the right-hand side of equation (1) equals the output value of government bonds of all maturities issued in period t .

The policy-maker's loss function in period t is given by

$$L_t = \sum_{s=t}^{\infty} \beta^{s-t} [V(x_s) + H(\Pi_s)] \quad (2)$$

where β is the inverse of the discount factor, and $V(\cdot)$ and $H(\cdot)$ are strictly increasing and convex functions. 2/ Imbedded in equation (2) is the assumption that taxes, as well as anticipated and unanticipated inflation, are socially costly (as, for instance, in Barro and Gordon (1983), Calvo and Guidotti (1989), and Persson, Persson, and Svensson (1989)).

To focus specifically on the role of maturity, it is assumed that the initial stock of government bonds, b , is given. Thus, the price level in period 0 is assumed to be a pre-determined variable. 3/ Hence, the only problem faced by government in period 0 (government 0, for short) is to choose the maturity structure of the given initial stock of public debt.

1/ We abstract from the inflation tax on real balances to focus sharply on government debt. This may be thought of as a situation in which the government rebates with lump-sum transfers any seigniorage on real cash balances. The inflation tax on real cash balances may be introduced as in Calvo and Guidotti (1990a). For a model that concentrates primarily on the inflation tax on real money balances, see Obstfeld (1989).

2/ In addition, it is assumed that $V(0)=V'(0)=0$ and $H(1)=H'(1)=0$, which implies that zero taxes and zero inflation achieve absolute bliss.

3/ In future periods, i.e., for $t > 0$, however, planners are allowed to choose their respective price levels.

Assuming that private agents are rational and that government bonds are pure assets in their portfolios, the Fisher equation (under perfect-foresight) must hold in equilibrium, i.e.,

$$I_{st} = R^{s-t} \Pi_{s+1} \dots \Pi_t, \quad s < t \quad (3)$$

where $R = (1+r)/(1+n)$, r and n denoting the (constant and exogenous) real interest and output growth rates, respectively. 1/

If governments are unable to precommit future policies, then the problem of choosing an optimal sequence (x_t, Π_t) is subject to time-inconsistency (see Calvo and Guidotti (1990a and c)). Such time-inconsistency arises here because policy-makers at any future date t treat the nominal value of government obligations as a predetermined variable because those obligations—in particular I_{st} —are fixed ex-post. Thus, policy-makers have the incentive to increase prices at time t by more than they would if I_{st} varied, ex-post, with Π_t in accordance with the Fisher equation. In equilibrium, of course, interest rates reflect point for point the opportunistic inflation which results from this incentive. Moreover, this implies that inflation collects no government revenue in equilibrium.

Under these conditions, the formulation of a time-consistent (incentive-compatible) equilibrium policy for a typical government at time t requires taking into account all future government's reaction functions with respect to present government's policies. Calvo and Guidotti (1990c) show that the equilibrium policy has three main properties: (i) governments resort to inflation even though, in equilibrium, no revenue can be obtained from that action; (ii) governments do not choose to completely smooth out conventional taxes over time when, in the absence of time-inconsistency, they would; and (iii) a careful management of the maturity structure of government debt is an essential part of the equilibrium policy. We now turn to a more formal characterization of the equilibrium policy and of these three properties.

It can be shown that the optimal choice of taxes and inflation satisfies the following first-order condition: 2/

1/ A constant real interest rate would hold, for instance, in a small open economy with perfect capital mobility and facing constant foreign interest and inflation rates. In a closed economy this would hold if, for instance, consumer's utility is linear in consumption (like in Obstfeld (1989)).

2/ See Calvo and Guidotti (1990b) for a formal proof.

$$\frac{V'(x_t)w_t}{\Pi_t} = H'(\Pi_t) \quad (4)$$

where

$$w_t = \sum_{\tau=t}^{\infty} z_t(\tau) \quad (5)$$

equals the value of total (i.e., present plus future) government debt obligations as of period t (discounted at time t), and

$$z_t(\tau) = \sum_{s=0}^{t-1} b_{s\tau} R^{t-s} \quad (6)$$

equals the value (as of period t) of all obligations maturing in period τ which are issued before period t (i.e., between periods 0 and $t-1$). Equation (4), from which property (i) follows directly, indicates that, at the optimum, a typical government t equates the marginal cost of inflation, $H'(\Pi_t)$, to the reduction in the cost of taxes, $V'(x_t)$, associated with the perceived (however, illusory) reduction in the real value of the nominal public debt, w_t/Π_t . It is worth noting that, if the government had the ability to precommit future policy, then the optimal policy concerning inflation would be to set $\Pi_t=1$, for all t , because inflation raises no revenue in equilibrium and is socially costly.

First-order condition (4) implicitly defines a function

$$\Pi_t = \tilde{\Pi}(x_t, w_t), \quad (7)$$

where

$$\tilde{\Pi}_x = \frac{wV''(x)}{H''(\Pi)\Pi + H'(\Pi)} \quad \text{and} \quad \tilde{\Pi}_w = \frac{V'(x)}{H''(\Pi)\Pi + H'(\Pi)}. \quad (8)$$

If $w>0$, then $\tilde{\Pi}_x>0$, while $\tilde{\Pi}_w>0$ as long as $x>0$. Intuitively, if $w>0$, then an increase in taxes raises the policy-maker's perceived marginal gain of reducing the real value of nominal debt through inflation. Similarly, if $x>0$, then the higher the debt level the higher is the policy-maker's perceived marginal reduction of the real debt stock which follows an increase in inflation.

By using equations (3)-(6), flow constraint (1), in equilibrium, can be conveniently expressed in the following way:

$$w_{t+1} = R(w_t + g_t - x_t). \quad (9)$$

The following intertemporal government budget constraint can be obtained by solving (9) subject to the transversality condition $\lim_{s \rightarrow \infty} w_s R^{-(s-t)} = 0$:

$$w_t + \sum_{s=t}^{\infty} (g_s - x_s) R^{-(s-t)} = 0. \quad (10)$$

Equations (5), (6), and (10) clearly show that inflation raises no revenue in equilibrium.

By using equation (7), the policy-maker's loss function may be expressed as

$$L_t = \sum_{s=t}^{\infty} \beta^{s-t} \{V(x_s) + H[\tilde{\Pi}(x_s, w_s)]\} = \sum_{s=t}^{\infty} \beta^{s-t} C(x_s, w_s). \quad (11)$$

If $w > 0$ and $x > 0$, then $C(x, w)$ is increasing in both its arguments.

As shown by Calvo and Guidotti (1990c), the equilibrium policy can be characterized in two steps. The first step consists in finding the sequence $\{x_t, w_t\}$ that minimizes loss function (2) subject to constraint (10). The second step consists in finding the maturity structure of the public debt which ensures that the sequence $\{x_t, w_t\}$ obtained in the first step is incentive-compatible. Thus, debt and taxes can be shown to satisfy the following Euler equation:

$$C_x(x_t, w_t) = \beta R [C_x(x_{t+1}, w_{t+1}) + C_w(x_{t+1}, w_{t+1})], \quad (12)$$

where $C_i(x, w) \equiv \partial C / \partial i$, $i = x, w$. To understand equation (12), consider the following perturbation of the equilibrium. Suppose we decrease taxes at period t by dx_t units, borrowing to cover the shortfall in revenue. In period $t+1$, taxes are raised to repay the additional borrowing incurred because of the fall in x_t . This implies that $dx_{t+1} = -R dx_t$, so that the perturbation leaves unchanged w_s for $s \geq t+2$. At an optimum, this perturbation has no effect on welfare. Thus, the left-hand side of equation (12) represents the welfare effect of the initial change in x_t , while the left-hand side of the equation represents the welfare effect in period $t+1$, discounted back one period. The welfare effect in period $t+1$ is composed of the direct effect of changing taxes plus an indirect effect through a change in equilibrium inflation, induced—according to equation (7)—by the higher taxes and public debt.

Property (ii) follows directly from equation (12). Even in the case in which $\beta R=1$ the equilibrium policy generally does not imply a complete intertemporal smoothing of conventional taxes. This contrasts sharply with the case of full precommitment in which if $\beta R=1$, then the optimal policy would imply a constant x (and, hence, a constant w).

Calvo and Guidotti (1990c) show that if $\beta R=1$, then the optimal sequence $\{x_t, w_t\}$ which solves the dynamic system composed by equations (9) and (12) converges to a steady state where $(x, w)=(g, 0)$; i.e., at the steady state government expenditure is financed by conventional taxes and the public debt is repaid in its entirety. ^{1/} With a positive initial level of debt, convergence to the steady-state equilibrium implies falling public debt and taxes over time. Hence, relative to the precommitment solution, the equilibrium policy in the absence of precommitment exhibits "debt aversion" and a heavier tax revenue collection in the short-run than in the long-run. Intuitively, since nominal debt generates a negative externality because of the incentive it provides to resort to inflation, its "effective" cost to the policymaker exceeds the real interest rate. Hence, when $\beta R=1$, the policymaker finds it optimal to repay the public debt over time until the point in which the above-mentioned externality is reduced to zero, i.e., until when the public debt is repaid in its entirety. Debt repayment along the transition to the steady-state equilibrium implies that tax revenue collection is higher in the transition than at the steady-state.

Property (iii) follows from the fact that, as mentioned earlier, the management of the maturity structure of government debt is necessary to ensure the incentive-compatibility (or time-consistency) of the tax and debt policy described by equations (9) and (12). This contrasts sharply with the case of full precommitment where debt maturity is totally irrelevant. Formally, it can be shown (see Calvo and Guidotti (1990c)) that a typical government, say government i , faces the problem of minimizing

$$V(x_i) + H(\Pi_i) + \beta \sum_{t=i+1}^{\infty} \beta^{t-i-1} C(x_t, \tilde{w}_{t,i}), \quad (13)$$

where

$$\tilde{w}_{t,i} = \sum_{s=0}^{i-1} \left[\sum_{\tau=t}^{\infty} b_{s\tau} \left(\frac{I_{s\tau}}{\Pi_{s+1} \dots \Pi_{\tau}} \right) R^{s-\tau} \right] R^{t-s} + \sum_{s=i}^{t-1} \left[\sum_{\tau=t}^{\infty} b_{s\tau} \right] R^{t-s} \quad (14)$$

subject to the following flow constraint

^{1/} These dynamics are studied in a neighborhood of the steady state.

$$\tilde{w}_{t+1,i} = R(\tilde{w}_{t,i} + g_t - x_t), \quad (15)$$

and the transversality condition $\lim(t \rightarrow \infty) \tilde{w}_{t,i} R^{-(t-i)} = 0$. The variable $\tilde{w}_{t,i}$ denotes the value of total government debt obligations at period t , as perceived by government i ($< t$). The first term on the right-hand side of equation (14) reflects the fact that government i does not internalize Fisher equation (3) on obligations issued before period i (i.e., for $s < i$). However, from the perspective of government i , equation (3) is taken into account for debt issued for periods $s \geq i$, as indicated by the second term on the right-hand side of equation (14). In equilibrium, of course, $\tilde{w}_{t,i} = w_t$ and, hence, (13) and (14) boil down to (11) and (9), respectively.

In the present context, time-consistency means that the solution for $\{x_t, w_t\}$ obtained from the problem of minimizing (11) subject to (9) (and transversality condition $\lim(s \rightarrow \infty) w_s R^{-(s-t)} = 0$) is the same as that yielded by the problem of minimizing (13) subject to (14) (and transversality condition $\lim(t \rightarrow \infty) \tilde{w}_{t,i} R^{-(t-i)} = 0$). Calvo and Guidotti (1990c) show that the optimal maturity structure of government debt (i.e., that which ensures time-consistency) is fully characterized by the following general equation:

$$\frac{\sum_{\tau=s}^{\infty} \hat{z}_t(\tau)}{w_t} = \prod_{\tau=t}^{s-1} \frac{\tilde{\Pi}_x(x_{\tau}, w_{\tau}) \Pi_{\tau+1}}{R \Pi_{\tau} [\tilde{\Pi}_x(x_{\tau+1}, w_{\tau+1}) + \tilde{\Pi}_w(x_{\tau+1}, w_{\tau+1})]}, \quad s \geq t. \quad (16)$$

The left-hand side of equation (16) equals the value in period t of debt issued before period t with maturity in periods $\tau \geq s$ (for any $s \geq t+1$), as a proportion of the public debt outstanding at time t . For subsequent simulations it is convenient to note that, for $s = t+1$, equation (16) boils down to

$$\frac{\sum_{\tau=t+1}^{\infty} \hat{z}_t(\tau)}{w_t} = \frac{\tilde{\Pi}_x(x_t, w_t) \Pi_{t+1}}{R \Pi_t [\tilde{\Pi}_x(x_{t+1}, w_{t+1}) + \tilde{\Pi}_w(x_{t+1}, w_{t+1})]}, \quad (17)$$

where the left-hand side of the equation equals the proportion of total debt which has maturity longer than one period.

In addition, Calvo and Guidotti (1990c) show that, in equilibrium,

$$\frac{\sum_{\tau=t}^{\infty} b_{0\tau}}{b - \sum_{\tau=1}^{\infty} b_{0\tau}} = \frac{\sum_{\tau=t}^{\infty} b_{i\tau}}{\sum_{\tau=i+1}^{\infty} b_{i\tau}} = \frac{\sum_{\tau=t+1}^{\infty} z_{i+1}(\tau)}{w_{i+1}}, \quad i \geq 0, \tau \geq t, t \geq i+1, \quad (18)$$

that is, each successive government $i > 0$ is provided with the incentive to follow the effective maturity structure chosen by government 0—note that the left-hand side of (18) shows the proportion of debt issued by government 0 due to mature in periods $\tau \geq t$ as a proportion of the stock of debt issued by government 0 which remains outstanding at period i , while the term in the middle of (18) shows the proportion of debt issued by government i due to mature in periods $\tau \geq t$ as a proportion of the total debt issued by government i . Equation (18) shows an important property of the equilibrium policy: the maturity structure of the outstanding debt along the entire equilibrium path can be computed from the maturity structure of the initial debt.

III. Public Debt Management. Numerical Simulations

This section examines, by means of numerical simulations of a thirty-period version of the model presented in section II, the effects of a number of exogenous disturbances. These simulations provide a richer characterization of the equilibrium policy discussed in the previous section. The policy-maker's objective function is assumed to take the following quadratic form:

$$L_t = \frac{1}{2} \sum_{s=t}^{30} \beta^{s-t} [A_s(x_s)^2 + (\Pi_s - 1)^2], \quad (19)$$

where A_s is an exogenous (possibly time-dependent) parameter. In equilibrium, inflation is related to taxes and the debt level by the following equation (which is the form equation (7) takes under the quadratic specification):

$$\Pi_t = \frac{1}{2} \left[1 + \sqrt{1 + 4A_t x_t w_t} \right]. \quad (20)$$

The transition equation (9) holds for all $t < 30$. In the last period, we assume that $x_{30} = g_{30} + w_{30}$, i.e., the initial public debt is repaid in full by the last period. 1/

1/ Recall that, when $\beta R = 1$, $(x, w) = (g, 0)$ in the steady-state equilibrium of the infinite-horizon model. Hence, the simulation of the 30-period version of the model generates a path $\{x_t, w_t\}$ which is qualitatively similar to that of the infinite-horizon model.

For the purpose the ensuing discussion, we define debt with maturity longer than one period as long-term maturity debt. Thus, the specific form of equation (17) is given by

$$m_t = \frac{w_t \sqrt{1 + 4A_{t+1}x_{t+1}w_{t+1}} \left(1 + \sqrt{1 + 4A_{t+1}x_{t+1}w_{t+1}} \right)}{R(x_{t+1} + w_{t+1}) \sqrt{1 + 4A_t x_t w_t} \left(1 + \sqrt{1 + 4A_t x_t w_t} \right)}, \quad (21)$$

where m_t denotes the proportion of the outstanding debt which has long-term maturity.

A benchmark case for the numerical simulations was defined by the following set of parameter values. The parameter A was set constant over time and equal to unity, the difference between the real interest rate and the rate of output growth was set equal to 3 percent per period (i.e., $R=1.03$), the discount factor β was set equal to $1/R$. Government expenditure was set equal to 30 percent of GDP (i.e., $g=0.3$), while the initial stock of debt was set equal to 50 percent of GDP (i.e., $b=0.5$). This parametrization makes it plausible to assume that each period lasts for one year.

Figure 1 shows the optimal path of x , w , and Π , and the optimal maturity structure of the initial debt for the benchmark case. The time-path of taxes, debt, and inflation is monotonic and decreasing. By year 15, the three variables have converged to values very close to their last period values, where $(x_{30}, w_{30}, \Pi_{30}) \approx (.3, 0, 1)$. Thus, the simulations for the benchmark case show a high degree of debt aversion because the public debt is repaid early in the simulation period by high taxes in the initial years of the optimal plan. As a consequence of early debt repayment, inflation falls rapidly towards zero from an initial level of 18 percent.

The maturity structure of the initial debt in the benchmark case—as shown by the time-path of b_{0t} —is downward sloping; 44 percent of the initial debt has one-year maturity, 53 percent has maturity between one and 5 years, and the remaining 3 percent has maturity longer than 5 years.

Figure 2 shows the evolution of m_t —that is, the share of long-term debt in the total debt—during the first 10 years of the optimal plan. As both the debt level and the average tax rate falls over time, debt maturity shortens. As w_t falls from 50 percent of GDP to 5 percent of GDP by the 10th year, m_t falls from an initial level of 56 percent to 14 percent by the 10th year.

The remainder of this section reports the results of a number of exercises in which the value (or time-path) of one exogenous variable is modified while maintaining the remaining exogenous variables at the level of the benchmark case. The effects of changing government expenditure, the initial debt level, and the parameter A of the policy-maker's objective function are examined. For expositional purposes we examine the effects of

quantitatively large changes in the exogenous variables. The emphasis of the analysis is, however, on the qualitative properties of the equilibrium policy rather than on the quantitative effects.

A permanent increase in government expenditure

The effects of a permanent and unanticipated increase in government expenditure are summarized by Figure 3. Relative to the benchmark case, the increase in g (from 0.3 to 0.5) induces higher "debt aversion"—in the sense of accelerating the repayment of the initial debt—it shortens the maturity structure of the initial debt, and generates an increase of the inflation rate in the short-run and a decrease in the long-run.

Intuitively, the increase in government expenditure, by raising the required taxes, induces, *ceteris paribus*, an increase in inflation in each period. Consequently, it is optimal to accelerate the repayment of the initial stock of public debt in order to reduce the inflationary effect of the increase in government expenditure. Because at each point in time policy-makers exhibit debt aversion *only with respect to the new debt they issue* (see Calvo and Guidotti (1990a)), it is optimal to shorten the maturity structure of the initial debt in order to induce future governments to implement a faster debt repayment. ^{1/} The proportion of the initial debt with long-term maturity (i.e., m_0) falls from 56 percent to 42 percent following the increase in government expenditure.

Compared to the benchmark case, the permanent increase in government expenditure generates an increase in inflation during the first three periods while it reduces inflation in later periods. Inflation increases initially—from 17 to 26 percent—because the increase in taxes required to finance the higher government expenditure implies a higher marginal cost of raising conventional taxes. By equation (4), the higher marginal cost of taxation induces policy-makers to rely more heavily on inflation. The later fall in inflation—relative to the benchmark case—reflects the increase in "debt aversion", by which the repayment of the public debt repayment is anticipated. Repaying faster the public debt has the effect of reducing at a faster rate the base of the inflation tax (towards zero) and, hence, the policy-maker's incentive to resort to inflation. This explains why, after the third period, inflation falls—compared to the benchmark case—following the increase in government expenditure.

We next consider the effects of a permanent and anticipated increase in government expenditure. In this exercise, $g_t=0.3$ for $t=1,2$, while $g_t=0.5$ for

^{1/} Calvo and Guidotti (1990a) discuss in detail the phenomenon of debt aversion. In particular, they show that governments experience debt aversion—that is, they internalize the cost imposed by time-inconsistency—only with respect to the new debt they have to issue for tax smoothing purposes. The inherited debt does not provide *per se* an incentive to anticipate tax revenue collection.

Figure 1. The Benchmark Case

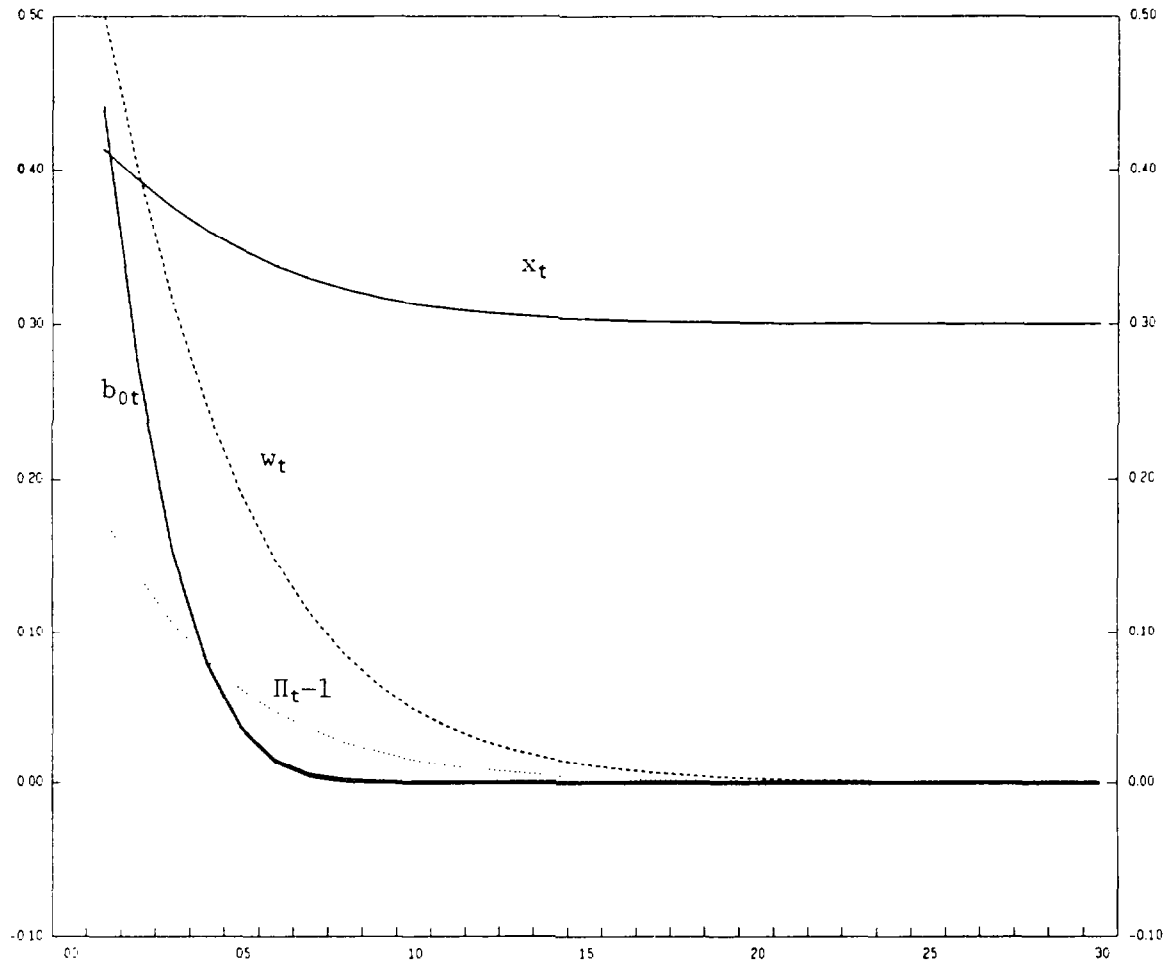


Figure 2. Optimal Debt Maturity
The Benchmark Case

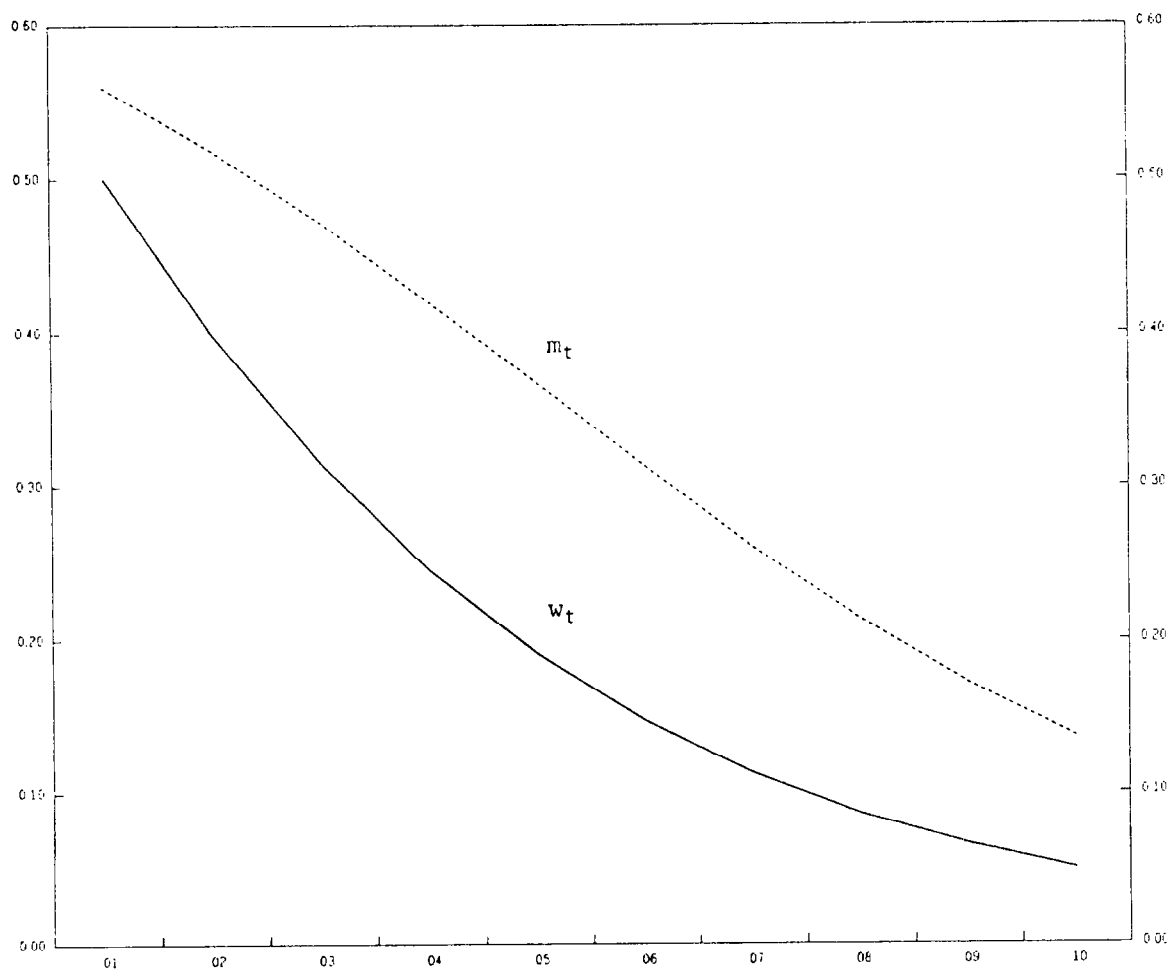
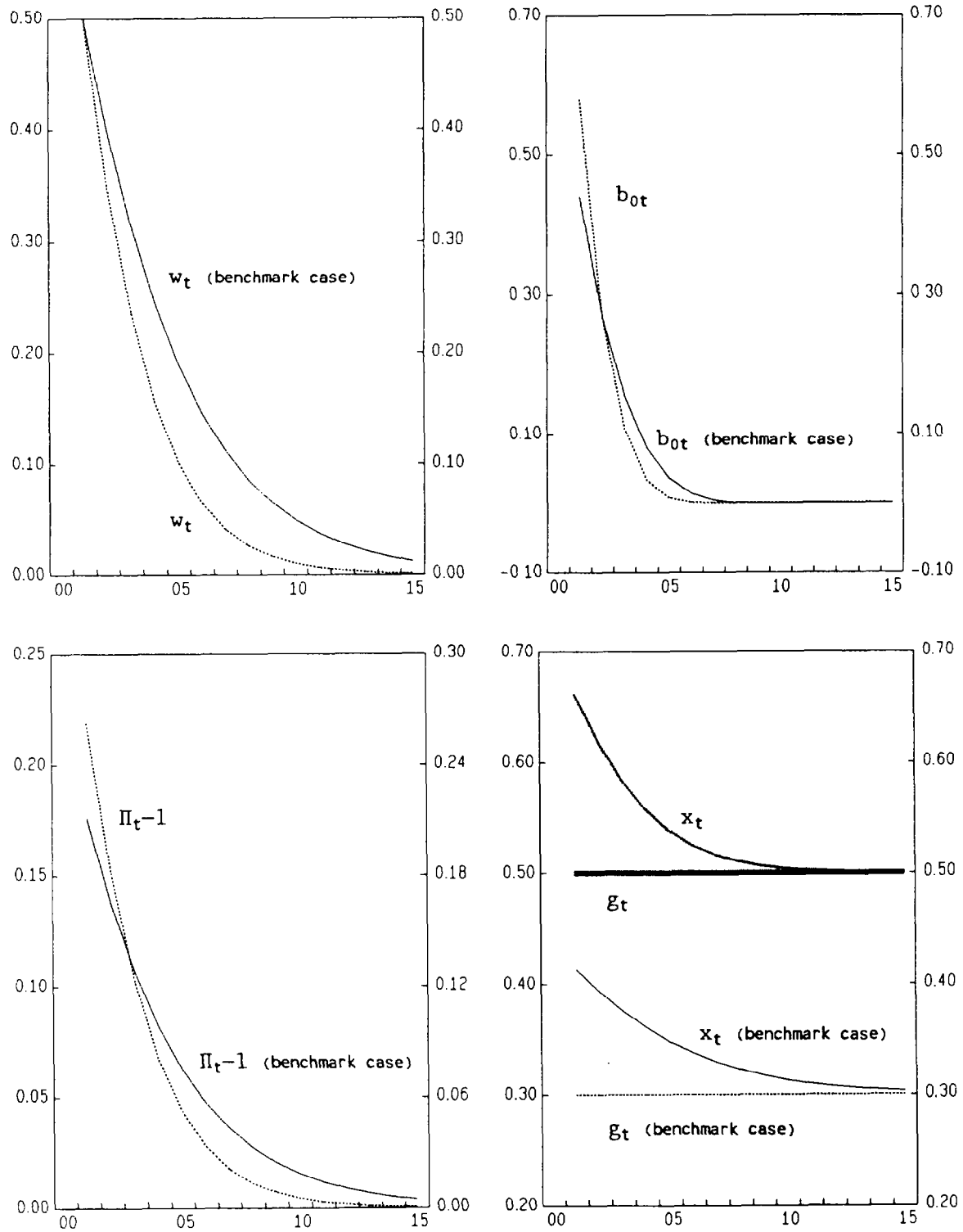


Figure 3. A Permanent and Unanticipated Increase
in Government Expenditure



$t > 2$. As can be observed in Figure 4, the anticipation of an increase in government expenditure speeds up dramatically debt repayment in the periods preceding the increase in expenditure. Initially, when government expenditure is low, taxes are increased to a level which is *higher* than that which taxes take during the periods of high g . As a result, w falls from 50 percent of GDP to 7 percent of GDP in the first two periods, and continues falling thereafter at a slower pace. The effect of an anticipated increase in government expenditure on the maturity structure of initial debt is qualitatively similar to that of an unanticipated increase, though it is quantitatively smaller—the anticipated increase in g induces a fall in m_0 from 56 percent to 50 percent. Inflation increases—relative to the benchmark case—only in the first period, while it falls in all of the remaining periods. The fall in inflation reflects, once again, the drastic reduction in the public debt taking place in the first two periods of the optimal plan.

A temporary and anticipated increase in government expenditure

To study the effects of a temporary and anticipated increase in government expenditure it was assumed that $g_t = .3$ for $t = 1, 2$, $g_t = .5$ for $t = 3, 4, 5, 6, 7$, and $g_t = .3$ for $t > 7$. Figure 5 summarizes the results of this exercise.

The anticipated increase in government expenditure induces a drastic increase in taxes during the periods *preceding* the policy change. The purpose of the increase in taxes is to achieve a drastic reduction in the debt level during the periods of low government expenditure. In period 4 (i.e., soon after government expenditure increases), a period of debt accumulation starts—following a standard tax smoothing argument. Debt increases until government expenditure falls back to its initial level, after which the dynamics of x and w are once again qualitatively the same as those of the benchmark case. Inflation follows the path of taxes and debt: it falls sharply in the periods preceding the increase in expenditure, it increases during the periods of high expenditure while the public debt builds-up, and falls thereafter. The maturity structure of the public debt follows the changes in the debt level: it shortens as the debt falls and it lengthens as the debt increases.

An increase in the initial stock of debt

The effects of doubling the initial debt stock from 50 percent of GDP to 100 percent of GDP are considered. Figure 6 summarizes the main effects of this shock.

The main qualitative effects of an increase in the initial stock of debt is the lengthening of the maturity structure of the public debt. For instance, the proportion of the initial debt with maturity longer than one period increases—compared to the benchmark case—from 56 percent to 66

percent. As far as taxes and inflation are concerned, the increase in the initial debt is associated with a higher debt aversion, so that the largest increase in taxes and inflation is concentrated in the initial periods.

A permanent and unanticipated increase in the parameter A

Figure 7 summarizes the effects of increasing the parameter A of the policy-maker's loss function from a value of 1 to a value of 2. The increase in A represents, *ceteris paribus*, an increase in the marginal cost of taxes relative to the marginal cost of inflation. Thus, it is not surprising that the main effect of an increase in A is a sharp increase in inflation. For instance, inflation in the first period increases from 18 percent to 33 percent. The change in the parameter A induces a moderate shortening of optimal debt maturity. Although the effect on debt maturity is quantitatively small, it provides a rationale for a possible negative relationship between inflation and debt maturity.

IV. Debt Maturity in Italy, Ireland, and the United States

This section explores the extent to which the implications of model presented in earlier sections are consistent, at a qualitative level, with the evolution of the maturity structure of the public debt in three industrial countries: Italy, Ireland, and the United States. These three countries provide interesting cases to examine the model's implications because, besides the availability of data for a sufficiently long sample period, these countries have experienced significant changes--both cyclically and secularly--in the level as well as the maturity composition of their domestic public debt over the past decades. (Definitions and sources for the data used in this section are provided in the Appendix.)

As Figure 8 illustrates, after a period of relative stability during the 1960's, Italy has experienced a significant build-up--particularly marked during the 1980's--in its domestic public debt reaching levels of nearly 100 percent of GDP. After being relatively stable during the 1960's, the maturity composition of Italy's domestic (privately-held) public debt shortened sharply during the 1969-83 period and lengthened thereafter.

The level and maturity composition of Ireland's domestic (privately-held) public debt has evolved cyclically over the 1970's and 1980's, exhibiting a positive co-movement between the two variables, as an increase in the public debt is accompanied by a lengthening of its maturity. Despite this cyclical pattern, however, Ireland's domestic public debt has grown--particularly during the 1980's--by over 20 percentage points of GDP from the beginning of the 1970's to the end of the 1980's.

The domestic (privately held) public debt in the United States has experienced a sustained decline before the mid-1970's and an increase thereafter. As in the case of Ireland, the maturity structure of the public

Figure 4. A Permanent and Anticipated Increase
in Government Expenditure

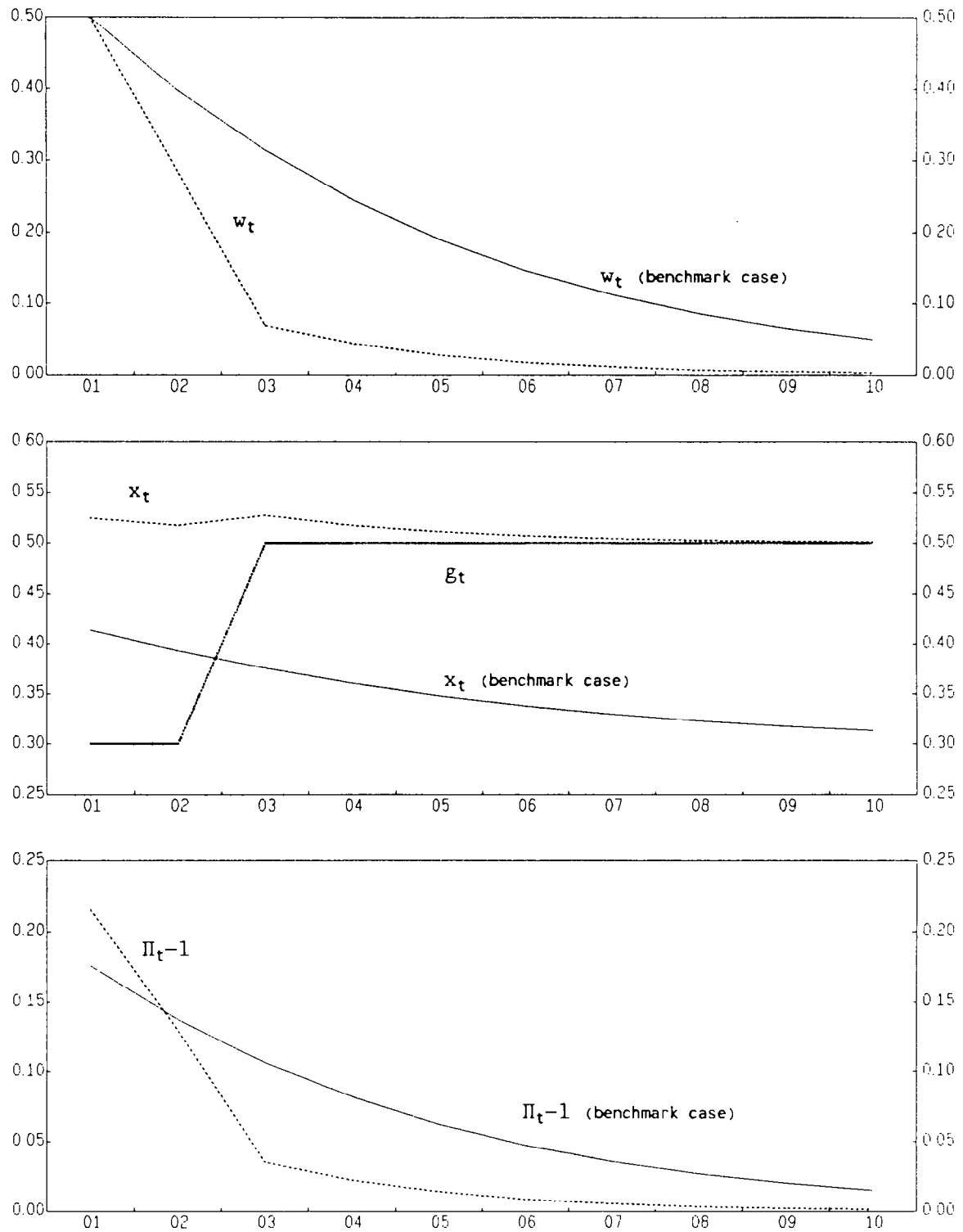


Figure 5. A Temporary Increase in Government Expenditure.

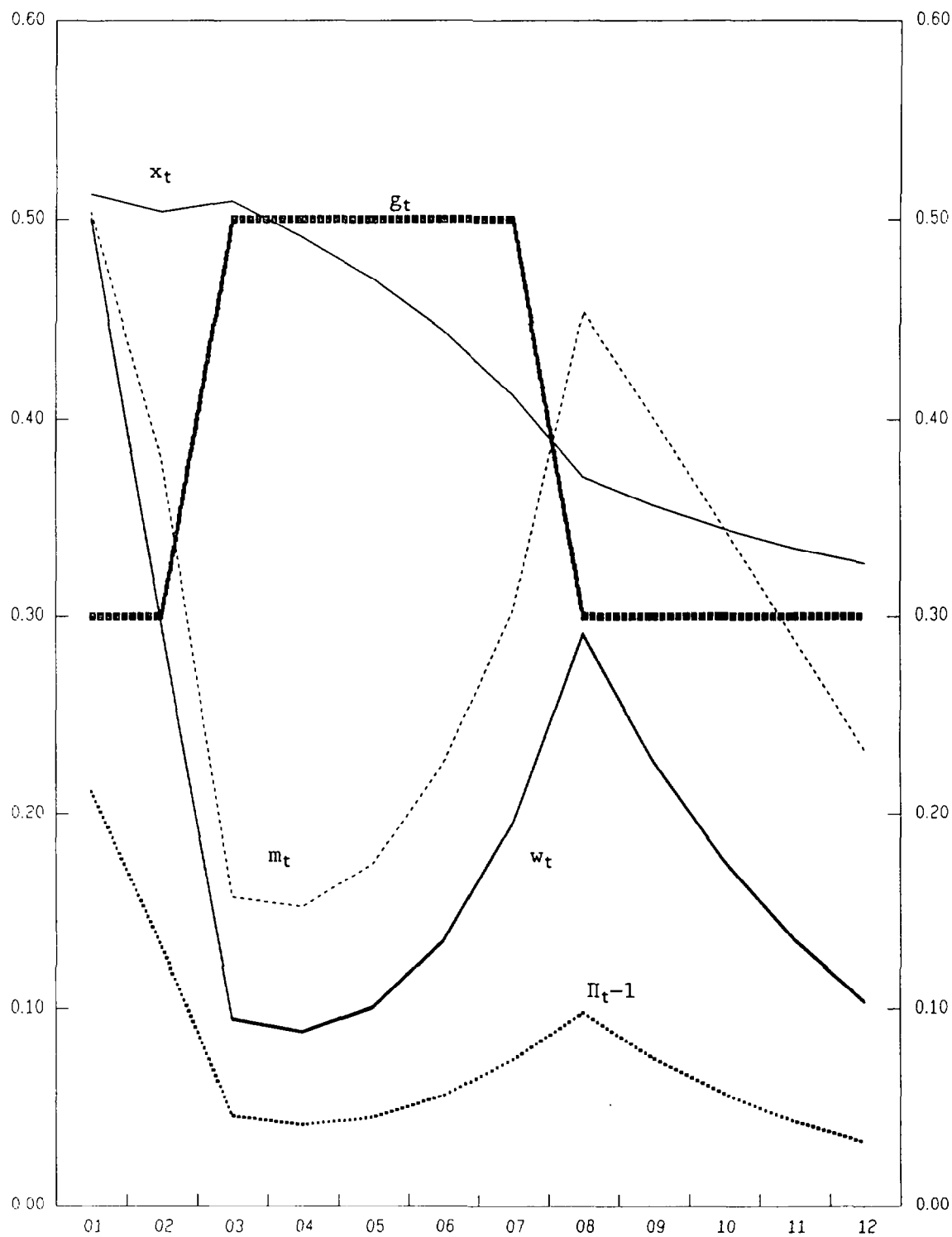


Figure 6. An Increase in the Initial Debt Stock.

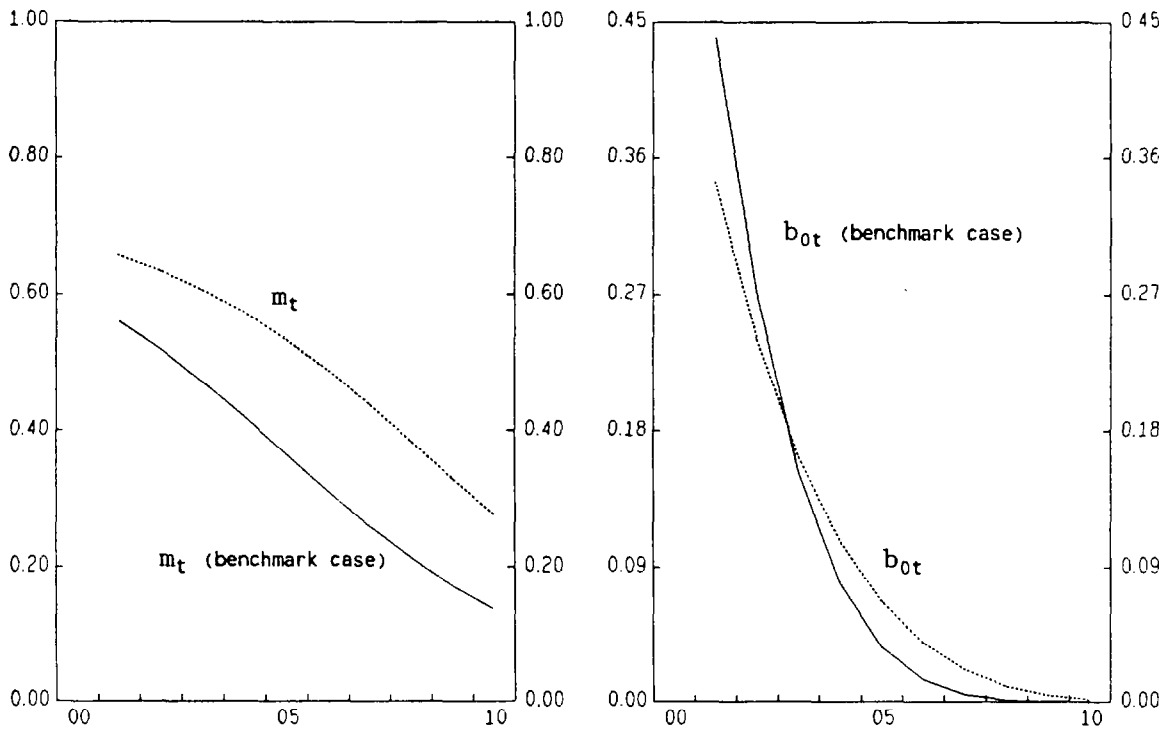
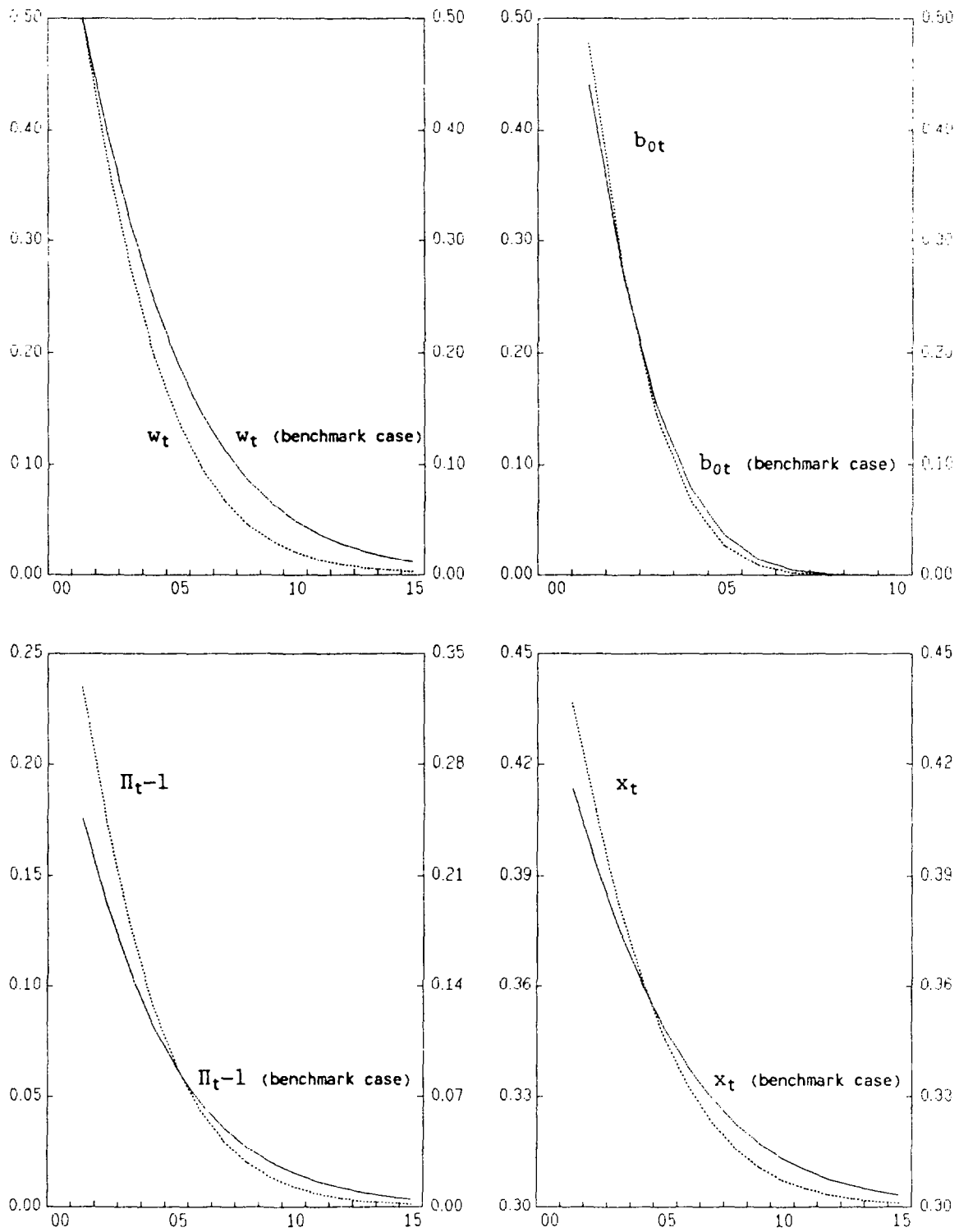


Figure 7. A Permanent Increase in A.



THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO

debt shortened in periods when the debt level was declining and lengthened after the mid-1970's, when public debt began to grow rapidly.

The qualitative implications of the theoretical model for the evolution of debt maturity in these countries were explored by using equation (21). Employing data on domestic public debt and tax revenue (both as percent of GDP) for variables w_t and x_t , respectively, an index of predicted maturity was constructed for each country by using the right-hand side of equation (21). As in the simulations, the real interest factor net of growth was given a value of 1.03 for the entire sample period. 1/ Parameter A was initially assumed to be time-invariant with a sample value which equates the average inflation rate predicted by equation (20) with the average actual inflation rate over the sample period. Thus, A equals 0.88 in the case of Italy, 1.01 in the case of Ireland, and 1.65 in the case of the United States. 2/ Figure 9 compares, for each of the three countries, the index of predicted maturity, denoted by im^p_t , to the index of actual maturity, denoted by im_t . Reflecting availability of data, the sample period for Ireland 1969-87, while for Italy and the United States the sample period is 1960-88.

In the case of Italy, the model fails to explain the sharp shortening of debt maturity that occurred during the 1970's (in particular between the periods 1970-73 and 1976-79), while the evolution of im^p_t appears to be broadly consistent with the actual behavior of debt maturity during the 1960's and 1980's. The predicted maturity index, however, appears to conform rather well with the evolution of debt maturity in both Ireland and the United States. In the case of Ireland, the model is able to capture the cyclical pattern followed by debt maturity over the past two decades predicting turning points reasonably well. In the case of the United States, the model tracks closely the shortening of debt maturity in the period preceding the mid-seventies as well as its subsequent

1/ Given that we construct an index for maturity, the value given to R is irrelevant since, as can be seen from equation (21), changes in R only change the scale of m_t .

2/ From the theoretical model we know that a higher value of parameter A means *ceteris paribus* a higher marginal cost of taxes relative to inflation. We also know that *ceteris paribus* a higher A means higher equilibrium inflation. Thus, it may appear quite surprising that comparing the values of A for Italy and the U.S., we find that Italy--the country with higher inflation during the sample--turns out with the lowest value of A. This puzzle is resolved by noting that, by equation (20), equilibrium inflation is also a function of the level of debt and taxes, and the sample mean of these variables is much higher in Italy than in the U.S.. It is worth noting also that part of the difference between the debt levels of the two countries results from the fact that, in the case of the U.S., we include as debt only the portion of the Federal debt which is marketable (see Appendix) underestimating, therefore, the total public debt level.

lengthening. ^{1/}

As Figure 9 suggests, the implications of model presented in section II appear to be broadly consistent with the main qualitative characteristics of the actual evolution of debt maturity in the three countries in our sample. Given that the model also has implications for the inflation rate, summarized by equation (20), we explored the extent to which these are qualitatively consistent with the actual evolution of inflation. Thus, the predicted inflation rate was obtained from the right-hand side of equation (20) using data for the debt level and tax revenue and the above-mentioned values of A. Figure 10 compares the evolution of actual and predicted inflation.

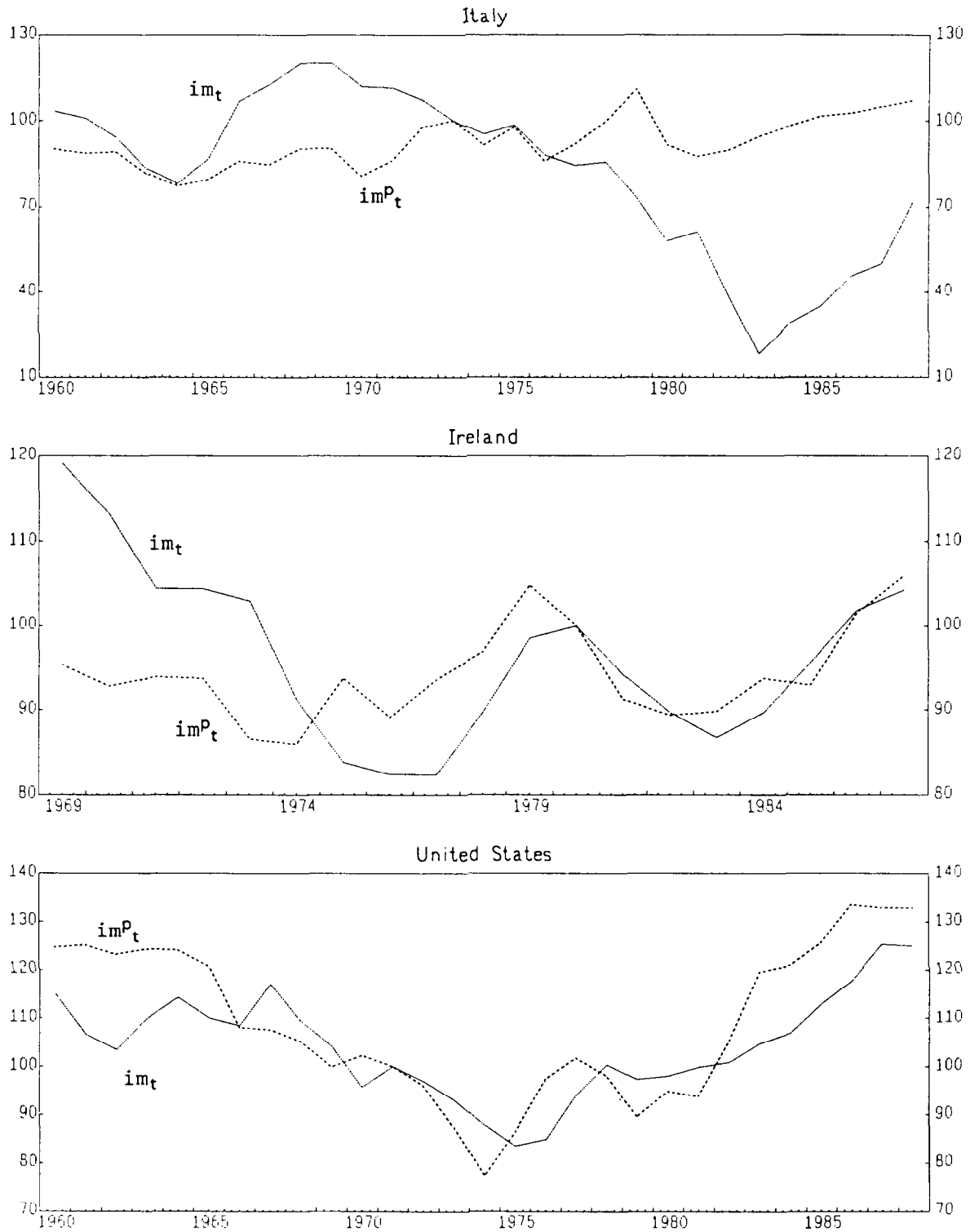
In broad terms, the predictions of the model fail to capture the increase in inflation that occurred during the 1970's, and the disinflation process of the 1980's. This is in part not surprising given the stylized nature of the model and given that the inflation process *per se* is not the focus of the analysis. The model, in fact, isolates only one aspect of the inflation process; namely, that which is associated with the incentives provided to the policy-maker by the presence of nominal debt. What is of relevance to the issue of maturity, however, is the extent to which the predictions concerning debt maturity are affected by the fact that the model's implications concerning inflation differ significantly from the actual evolution of inflation.

To address this issue, we examined how the index of predicted maturity would behave *if the model had exactly predicted actual inflation*. Thus, the parameter A was assumed to vary over time and its value was calculated for each period so as to obtain the *actual* inflation rate from the right-hand side of equation (20). This, of course, required—relative to the constant values used earlier—larger values for A_t during the 1970's and smaller values for A_t in the 1980's. This time-profile for A_t has a plausible interpretation in the context of the theoretical model: the policy-maker's aversion to inflation was lower—possibly on account of political-economy considerations—during the 1970's than during the 1980's. The values of A_t obtained in this fashion were used to construct a second index of predicted maturity—denoted by im_t^A —from the right-hand side of equation (21). Figure 11 compares this new index of maturity to the index of actual maturity.

The qualitative characteristics of Figure 11 do not differ from those of Figure 9. This is, in part, not surprising given that the numerical simulations presented in section III suggested that changes in the value of A have only a moderate influence on optimal debt maturity. In the case of Italy for the periods 1960–70 and 1979–88, as well as in the case of Ireland, the new index appears to show somewhat more sharply the patterns

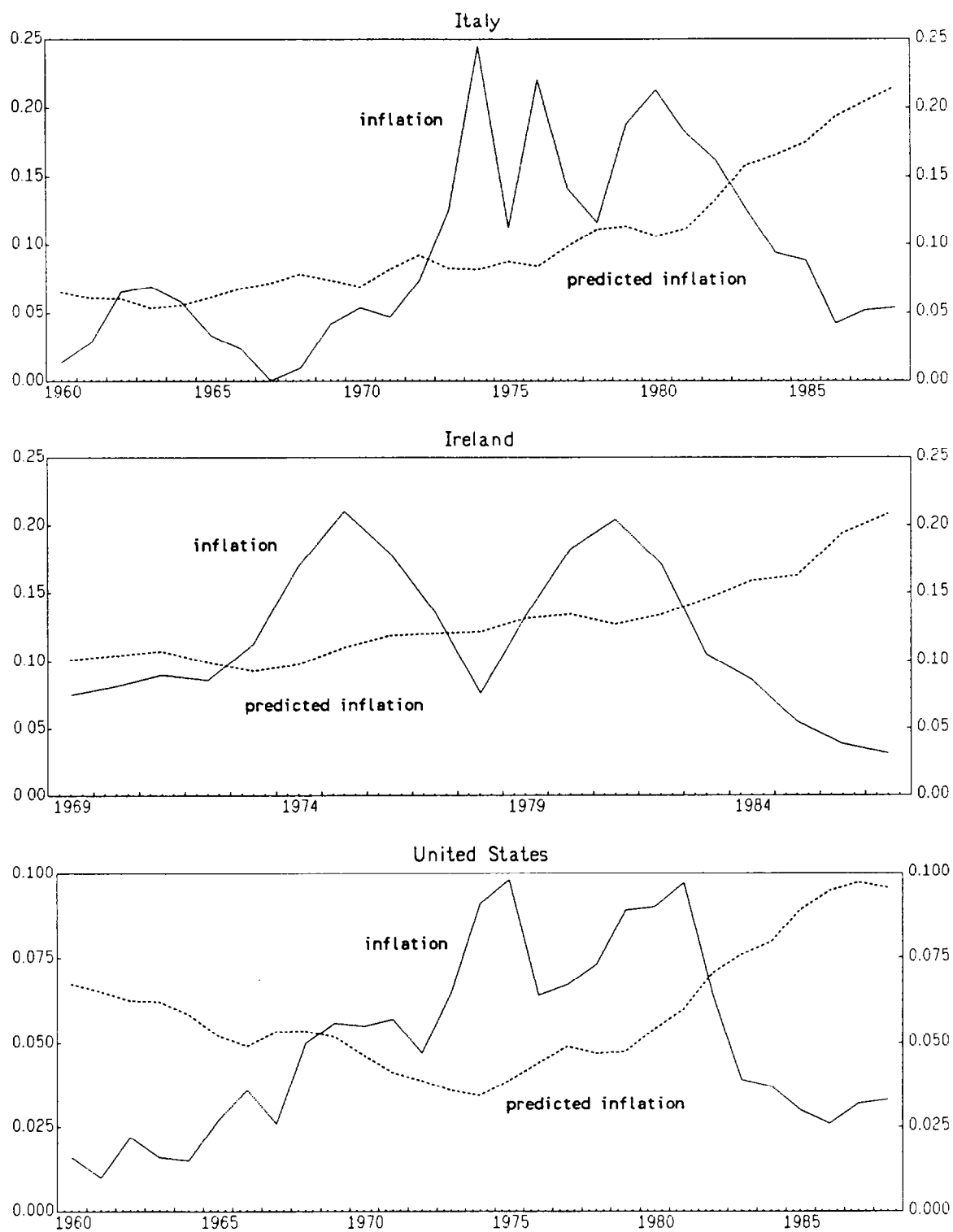
^{1/} Calvo, Guidotti and Leiderman (1990) provide a rigorous empirical test of the model for the United States during the post-WWII period.

Figure 9. Actual and Predicted Maturity.



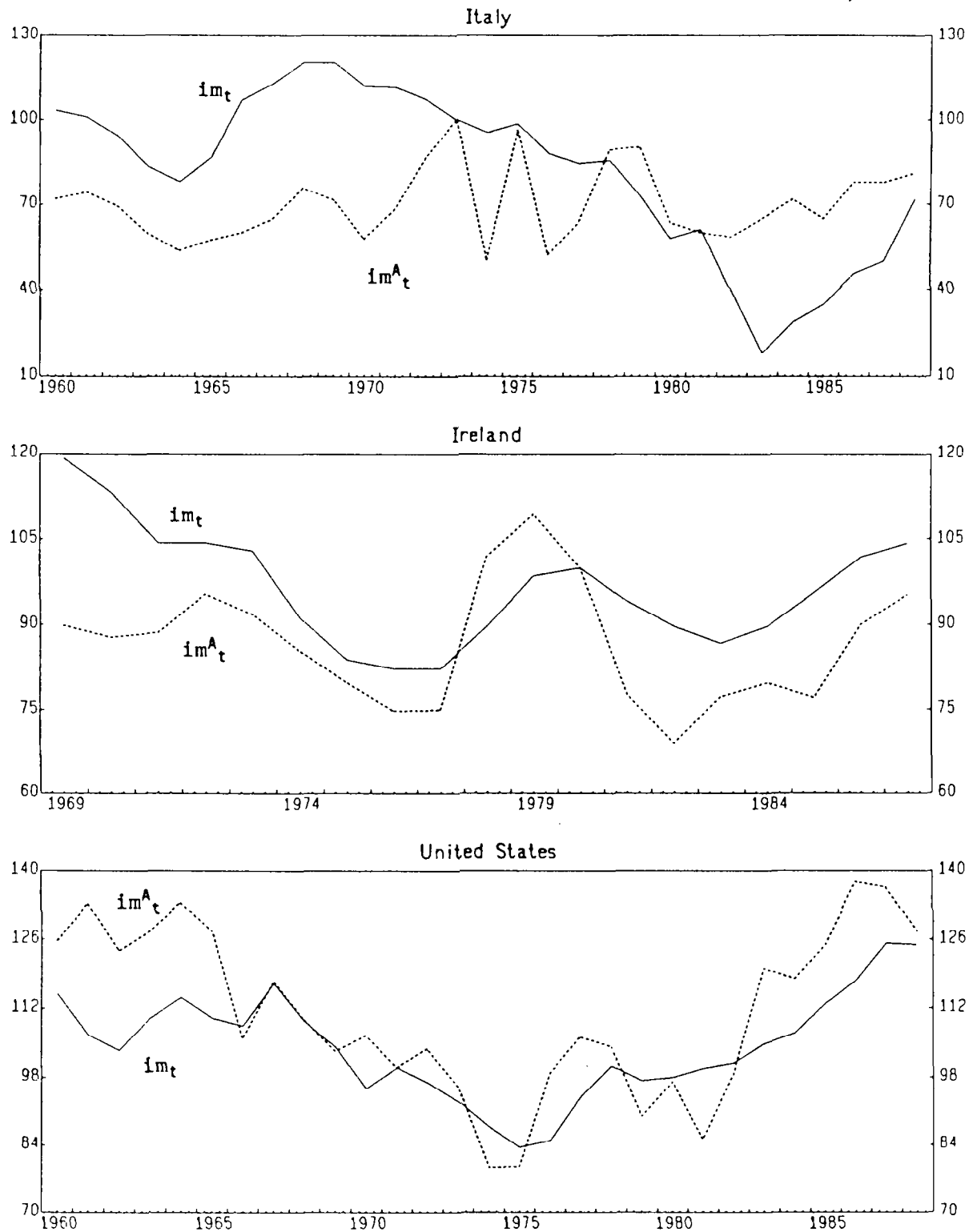
Source: See Appendix.

Figure 10. Actual and Predicted Inflation.



Source: See Appendix.

Figure 11. Index of Maturity with Time Varying A.



Source: See Appendix.

exhibited by the evolution of actual maturity. In the case of the U.S., the new index appears to perform slightly worse.

This exercise, therefore, suggests that the model's implications concerning debt maturity are not very sensitive to the failure in accounting for the actual inflation experience, provided that this failure is related to considerations which can be safely interpreted as affecting the time-path of the parameter A_t . One particular consideration which could have a bearing on the time-path of A_t is the issue of credibility. In principle, one could associate a strengthening of credibility with an increase in the value of A , that is, with an increase in the relative cost of inflation to the policy-maker. In this vein, the simulations suggest that, while affecting substantially equilibrium inflation, changes in credibility would not necessarily lead to significant changes in the observed maturity structure of the public debt. An entirely different way of thinking about credibility, however, is to associate a strengthening of credibility with moving to a world in which forms of partial policy precommitment are possible. In this alternative view, as shown by Calvo and Guidotti (1990a), changes in credibility may lead to significant changes in the optimal maturity structure of the public debt. ^{1/}

V. Concluding Remarks

Accounting for the potential incentive problems associated with the presence of nominal government debt provides the basis for formulating a theory of optimal public debt management in which both changes in the level and in the maturity composition of the public debt matter. In a world of incomplete markets and less than full policy precommitment, the equilibrium time-consistent policy does not replicate the optimal policy which obtains when policy-makers have the ability to fully precommit their (and their successors') future actions.

Despite its stylized nature, the model appears to be capable of accounting for some important aspects of actual public debt management. Particularly, it can explain the observed co-movement of maturity and the stock of public debt as well as produce implications which are qualitatively consistent with the evolution of actual maturity. Extensions to this work would take explicit account of random shocks so as to be able to generate estimable stochastic Euler equations. Also, it appears important at this point to explore the microfoundations of the model a little further, particularly to improve our understanding of the costs of unanticipated inflation (see Calvo and Guidotti (1989) for a discussion).

Finally, to improve the model's ability to explain actual government behavior, it appears important to endogenize the real interest rate and to

^{1/} Of course, in the extreme case of full precommitment debt maturity becomes irrelevant.

Appendix: Data Sources and Definitions

Italy:

Data on the privately-held domestic public debt stock and its maturity composition was obtained from the Relazione Annuale, Banca d'Italia, various years, and L'Indebitamento Pubblico in Italia, (Roma: Camera dei Deputati), 1985. Long-term debt comprises "Titoli a medio e lungo termine sul mercato" (see Relazione Annuale, Banca d'Italia) but excludes from this category debt which is indexed to the short-term interest rate (i.e., it excludes "Buoni Ordinari del Tesoro" (see Relazione Annuale, Banca d'Italia)). Data on tax revenue was obtained from Revenue Statistics of OECD Member Countries, OECD, various years, and from the Government Finance Statistics, International Monetary Fund. Data for CPI inflation and GDP was obtained from the International Financial Statistics, International Monetary Fund.

Ireland:

Domestic public debt data was obtained from the Quarterly Bulletin and the Annual Report of the Bank of Ireland. The domestic public debt stock comprises "Marketable Irish Government Securities" (see Quarterly Bulletin, Bank of Ireland) but excludes debt held by the Central Bank. Long-term debt is defined as the portion of the outstanding debt with maturity of 3 or more years (see Quarterly Bulletin, Bank of Ireland). Data on tax revenue was obtained from Revenue Statistics of OECD Member Countries, OECD, various years, and from the Government Finance Statistics, International Monetary Fund. Data for CPI inflation and GDP was obtained from the International Financial Statistics, International Monetary Fund.

United States:

Domestic public debt data and its maturity composition was obtained from the Economic Report of the President, various years. The domestic public debt stock is measured by the stock of "Marketable interest-bearing public debt securities held by private investors" (see the Economic Report of the President). Long-term debt is defined as the proportion of the outstanding debt with maturity longer than one year. Data of tax revenue, CPI inflation, and GDP was obtained from the International Financial Statistics, International Monetary Fund.

References

- Alesina, Alberto, and Guido Tabellini, "A Positive Theory of Fiscal Deficits and Government Debt," Review of Economic Studies, Vol. 57 (1990), pp. 403-414.
- Barro, Robert J., "On the Determination of the Public Debt," Journal of Political Economy, Vol. 87 (October 1979), pp. 940-971.
- Barro, Robert J., and David B. Gordon, "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy, Vol. 91 (August 1983), pp. 589-610.
- Calvo, Guillermo A., "On the Time Consistency of Optimal Policy in a Monetary Economy," Econometrica, Vol. 46 (November 1978), pp. 1411-28.
- Calvo, Guillermo A., and Maurice Obstfeld, "Time Consistency of Fiscal and Monetary Policy: A Comment," Institute for International Economic Studies, Seminar Paper No. 427 (Stockholm, Sweden, January 1989). Forthcoming in Econometrica.
- Calvo, Guillermo A., and Pablo E. Guidotti, "Level and Variability of the Optimal Inflation Tax," Unpublished, October 1989.
- and ———, "Credibility and Nominal Debt: Exploring the Role of Maturity in Managing Inflation," IMF Staff Papers, Vol. 37 (September 1990a), pp. 612-35.
- and ———, "Indexation and Maturity of Government Bonds: An Exploratory Model," in Public Debt Management: Theory and History, edited by Rudiger Dornbusch and Mario Draghi, Cambridge University Press, 1990b.
- and ———, "Optimal Maturity of Nominal Government Debt: An Infinite-Horizon Model," Unpublished, August 1990c.
- Calvo, Guillermo A., Pablo E. Guidotti and Leonardo Leiderman, "Optimal Maturity of Nominal Government Debt: The First Tests," August 1990. Forthcoming Economic Letters.
- Giavazzi, Francesco, and Marco Pagano, "Confidence Crises and Public Debt Management," in Public Debt Management: Theory and History, edited by Rudiger Dornbusch and Mario Draghi, Cambridge University Press, 1990.
- Kydland, Finn E., and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy, Vol. 85 (1977).

Lucas, Robert E., Jr., and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy Without Capital," Journal of Monetary Economics, Vol. 12 (July 1983), pp. 55-94.

Obstfeld, Maurice, "Dynamic Seigniorage Theory: An Exploration," NBER Working Paper 2869 (February 1989).

Persson, Mats, Torsten Persson, and Lars E.O. Svensson, "Time Consistency of Fiscal and Monetary Policy," Econometrica, Vol. 55 (November 1987), pp. 1419-32.

———, ———, and ———, "Time Consistency of Fiscal and Monetary Policy: A Reply," Seminar Paper No. 427, Institute for International Economic Studies (Stockholm, Sweden, 1989).

Persson, Torsten, and Lars E.O. Svensson, "Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences," Quarterly Journal of Economics, Vol. 104 (1989), pp. 325-346.

Persson, Torsten, and Guido Tabellini, Macroeconomic Policy, Credibility, and Politics, (London: Harwood) 1990.

Tabellini, Guido, "The Politics of Intergenerational Distribution", 1989. Forthcoming in the Journal of Political Economy.

Tobin, James, "An Essay on the Principles of Debt Management," in Fiscal and Debt Management Policies, reprinted in J. Tobin, Essays in Economics, Vol. 1, Prentice-Hall 1963 (North-Holland, Amsterdam, 1971).