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Index Number Biases During Price Liberalization

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Abstract

When a formerly centrally-planned economy frees prices and allows or compels producers to respond to market signals, conventional measures tend to severely overstate short-run output decline and inflation. In part the overstatement stems from neglect of private sector activity, or from belated recognition of inflation previously disguised as quality improvements. Even when individual prices and outputs are correctly measured, however, shifts in relative prices consequent to price decontrol create a serious aggregation problem. Moreover, the standard indices ignore the deflationary trends in black markets. Superior growth and inflation indices are devised using a combination of official and black market prices.

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* This paper elaborates on ideas initially suggested to the author by Tim Lane. Kasper Bartholdy and Peter Wickham provided useful comments, while Catherine Fleck assisted with editing.

Contents

Summary

When a formerly centrally planned economy frees prices and allows or compels producers to respond to market signals, conventional measures may severely overstate the short-run surge in prices and decline in output. In part, the overstatement stems from miscounting. For example, the size of the fast-growing private sector tends to be drastically understated. Also, reappraisal of product quality may cause past inflation, previously unrecorded, to be erroneously attributed to liberalization.

Even when individual prices and outputs are correctly measured, however, shifts in relative prices consequent to price decontrol create a serious aggregation problem. In free-market economies, Laspeyres (base-period weighted) and Paasche (current-period weighted) indices normally tend to bracket the ideal utility-based measures of inflation and growth. For a economy undergoing massive liberalization, however, both Paasche and Laspeyres indices tend to be biased upward for price changes and downward for quantity changes. Paradoxically, the more flexibly that producers respond to the post-liberalization incentives, the worse the distortions may be. Distortions exceeding 5 percentage points for growth or 10 percentage points for inflation do not appear unlikely.

Alternative indices can be formed using black market prices. Relative black market prices better reflect relative scarcities than do official prices, while average black market prices reflect a purchasing power swollen by "shortage rent," as the wedge between official and shadow prices may be described. The more official prices are distorted and the broader the spectrum of deficit goods, the higher average black market prices will be, and the more they can be expected to fall after liberalization. Thus, insofar as black market exchange rates reflect purchasing power parity, liberalization will tend to cause the domestic currency to appreciate.

Information on the full spectrum of black market prices can be useful in adjusting official indices. The Laspeyres growth index formed using black market prices as weights puts an upper limit on the constant base-period utility index of growth. If this Laspeyres index is divided into the nominal value ratio, the resulting pseudo-Paasche index puts a lower limit on the constant current-period utility price index. To approximate constant-utility indices, it is recommended to average the black-market-derived Laspeyres growth index with the standard Paasche growth index, or the pseudo-Paasche price index with the standard Laspeyres price index. In trial calculations, the margin of error was less than 1 percentage point.

I. Introduction

As the centrally-planned economies of Eastern Europe and the Soviet Union attempt to restructure along free-market lines, measures of price inflation and output contraction have reached alarming proportions. Several authors, including Lipton and Sachs (1990), Kemme (1990), Lane (1991), and Williamson (1991), have suggested that some of the reported deterioration is spurious. On the one hand, the size of the fast-growing private sector may be drastically understated, 1/ as the Eastern European and Soviet statistical services are unaccustomed to dealing with the private sector. Also, many private entrepreneurs purposely conceal output and assets in order to evade current taxes or out of fear of possible future expropriation. On the other hand, enterprises that, when subject to price controls, disguised inflationary price hikes as quality improvements, may no longer feel a necessity to dissemble after liberalization, so that some of the inflation reported in current statistics could reflect understatement of prior price levels.

This paper analyzes another way in which the deterioration can be overstated. Even when individual prices and outputs are correctly measured, shifts in relative prices and output proportions consequent to liberalization create serious index number problems. 2/ Both Laspeyres (base-year weighted) and Paasche (current-year weighted) indices tend to be biased upward for price changes and downward for quantity changes. Distortions of over 5 percentage points for growth measures and over 10 percentage points for inflation measures are not unlikely.

In saying this, the intent is not to deny that the current crises are severe. Even if the real changes are only half the reported levels, the shock would be substantial. Moreover, the arguments presented here cannot account for stagflation persisting long after prices have been freed. Indeed, insofar as the initial shock of liberalization has been overstated, the ensuing stagflation appears even more worrisome, for part of the deterioration it allegedly arrested never occurred.

Another by-product of the analysis is the suggestion that black market prices, including black market exchange rates, are likely to decline with liberalization. The greater the relative price distortion prior to decontrol, the greater the deflation in black market prices will tend to be.

The paper is organized as follows. The next section reviews standard price and output indices and their relation to ideal indices based on principles of utility maximization. Section III explains how and why standard indices tend to overstate the inflation in official prices and the

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1/ According to Kolodko (1991), Polish industrial output during 1990 fell 25 percent in the state sector and rose 8.5 percent in the private sector. With the private sector accounting for an average 13.4 percent of total industry, aggregate industrial output fell 20.5 percent. If in fact the size of the private sector had been understated by half, the aggregate contraction would be 17 percent, a difference of 3.5 percentage points.

2/ The problem is noted in Lane (1991).

contraction in output attendant to liberalization. In Section IV, indices of black market prices are examined, and the expected deflation is related to money demand parameters and to relative price distortions prior to decontrol. Section V shows how black market prices can, in principle, be used to impute characteristics of household utility and adjust standard indices. The main conclusions are summarized in Section VI.

II. Standard and Utility-Based Indices

This section reviews basic index number theory and introduces some useful analytic techniques. The comparisons of index numbers under free-market conditions provide a benchmark for later reference, so that the index number peculiarities of economies undergoing large-scale decontrol, explored in later sections, might be better appreciated.

For a collection of goods, let P and Q denote the vectors of prices and quantities respectively. The nominal value of goods Y equals the product PQ. The time periods before and after decontrol will be indicated by the superscripts ^{0} and ^{1}.

To form aggregate indices of the impact of liberalization, individual prices and quantities have to be weighted. Laspeyres price [quantity] indices are weighted by base-period quantities [prices], while Paasche indices are weighted by current-period values. In the context of this paper, it is natural to consider pre-decontrol as the base period and post-decontrol as the current period. Hence, if π_L, π_P, g_L, and g_P denote, respectively, the Laspeyres-measured inflation rate, the Paasche-measured inflation rate, the Laspeyres-measured output growth rate, and the Paasche-measured output growth rate, we have:

$$1 + \pi_L = \frac{P^1Q^0}{P^0Q^0} \quad ; \tag{1}$$

$$1 + \pi_P = \frac{P^1Q^1}{P^0Q^1} \quad ; \tag{2}$$

$$1 + g_L = \frac{P^0Q^1}{P^0Q^0} \quad ; \tag{3}$$

$$1 + g_P = \frac{P^1Q^1}{P^1Q^0} \quad . \tag{4}$$

The price or quantity indices are given by 1 plus the corresponding inflation or growth rates; that is, the values of equations (1) through (4). Note that the Laspeyres price index times the Paasche quantity index equals V^1/V^0, the ratio of nominal aggregate value of goods, as does the Paasche price index times the Laspeyres quantity index.

For most of the analysis it will be assumed that all goods are intended for direct household consumption. Thus, intermediate goods will be ignored, and investment goods will be treated only as proxies for future consumption goods. Moreover, market outcomes will be assumed to reflect the choices of a representative household with utility function U. That is, the ratio of marginal utility to price will be equal across goods. Prior to decontrol, this equality will generally not hold at official prices. However, if households can freely resell and buy on black markets, the ratio of marginal utility to black market price should equilibrate across goods regardless of the distortion on official markets.

Ideally, we would like to find utility-based measures of inflation and output growth. A utility-based measure of inflation would indicate the extra nominal expense of attaining a given level of welfare, typically either the base-period level of welfare or the current-period level. A utility-based measure of output growth would indicate the excess of current output over that assuring the base-period level of welfare, or alternatively, the percentage growth of base-period output necessary to obtain current-period welfare. Even with market-clearing prices, such indices are generally not independent of the reference utility level; the only exception occurs when preferences are homothetic, or equivalently, when all income elasticities of demand equal 1. 1/

Since utility-based measures require knowledge of the unobserved utility function, it is helpful to know how their values relate to more conventional indices. Some classic results of index number theory (e.g., Allen (1975), Pollak (1989)) indicate that, when prices are market-clearing, the Laspeyres inflation index is an upper bound to a constant-base-period-utility measure of inflation, while the Paasche inflation index is a lower bound to a constant-current-period-utility measure of inflation. Hence, when preferences are homothetic, the Paasche and Laspeyres measures of inflation bracket, from below and above respectively, the constant-utility measure of inflation. The same relationship holds for growth measures.

To strengthen the reader's intuition for these results, and to introduce some techniques that will be used again later, geometric proofs are given below for the case of two goods. In Figure 1, the horizontal and vertical axes indicate the quantities of the two goods x and y respectively. The curved line U^0 represents the indifference curve through Q^0, which is assumed to have the standard convex-to-the-origin shape. At current-period prices P^1, Q^* is the least-cost bundle assuring base-period welfare, and the tangent to U^0 at Q^* satisfies the equation $P^1Q = P^1Q^*$. At all points to the right of this tangent, P^1Q strictly exceeds P^1Q^*. In particular, P^1Q^0 exceeds P^1Q^*, so that the Laspeyres price index P^1Q^0/P^0Q^0 exceeds the constant base-period utility price index P^1Q^*/P^0Q^0.

1/ For a thorough discussion in the context of cost-of-living (inflation) indices, see Pollak (1989).

To prove the Paasche price index result, simply redefine Q^0 in Figure 1 as Q^1, U^0 as the indifference curve U^1 through Q^1, P^1 as P^0, and Q^* as the least-cost bundle Q^{**} under prices P^0 of obtaining utility U^1. It follows immediately that P^1Q^1/P^0Q^1 exceeds the constant current-period utility price index P^1Q^1/P^0Q^{**}.

In Figure 2, the axes again indicate the quantities of the two goods, and U^0 again indicates the indifference curve through Q^0. Point B is given by the intersection of U^0 with the ray from the origin through Q^1, and represents the multiple of current-period output needed to attain base-period welfare. Hence, the constant base-period utility quantity index equals $L(Q^1)/L(B)$, where $L(\cdot)$ denotes length from the origin. If base-period prices are market-clearing, U^0 will be tangent at Q^0 to the budget line $P^0Q = P^0Q^0$, which intersects at A with the ray from the origin through Q^1. The dotted line passing through Q^1, defined by $P^0Q = P^0Q^1$, is parallel to the budget line. It follows from the properties of similar triangles that P^0Q^1/P^0Q^0 equals $L(Q^1)/L(A)$, so that the Laspeyres quantity index exceeds the constant base-period utility quantity index. Switching the base- and current-period designations as above confirms that the constant current-period utility quantity index exceeds the Paasche quantity index.

III. Aggregation Biases Under Shortage

When base-period prices are not market-clearing, some of the standard index number relationships cease to apply. The following example illustrates what can happen, and helps to motivate a more general treatment.

A mathematical example

Let there be two goods x and y, produced in quantities Q_x and Q_y, respectively, and yielding utility Q_xQ_y. Note that in market-clearing equilibrium, the budget shares of the two goods should be equal. Suppose that prior to decontrol $Q^0 = [2\ 6]$ and $P^0 = [1\ 1]$, so that demand for good x at official prices exceeds supply. After decontrol, let $Q^1 = [3\ 4]$, while $P^1 = [2\ 1.5]$. Thus, the output of the previously deficit good rises, the output of good y falls, and both goods become more expensive at official prices.

Note that total expenditure on goods rises from 8 to 12, while utility remains unchanged at 12. Hence, a utility-based measure suggests an inflation of 50 percent. 1/ Yet both the Laspeyres and the Paasche indices register considerably more inflation: π_L is 63 percent, and π_P is 71 percent. The average Laspeyres-Paasche index of 67 percent overstates

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1/ The utility-based measures in this section ignore the savings in shopping time when prices become market-clearing, just as standard Paasche and Laspeyres measures ignore changes in the price and quantity of leisure. We will return to this issue in Section V.

Figure 1. Paasche Price Index with Free Prices

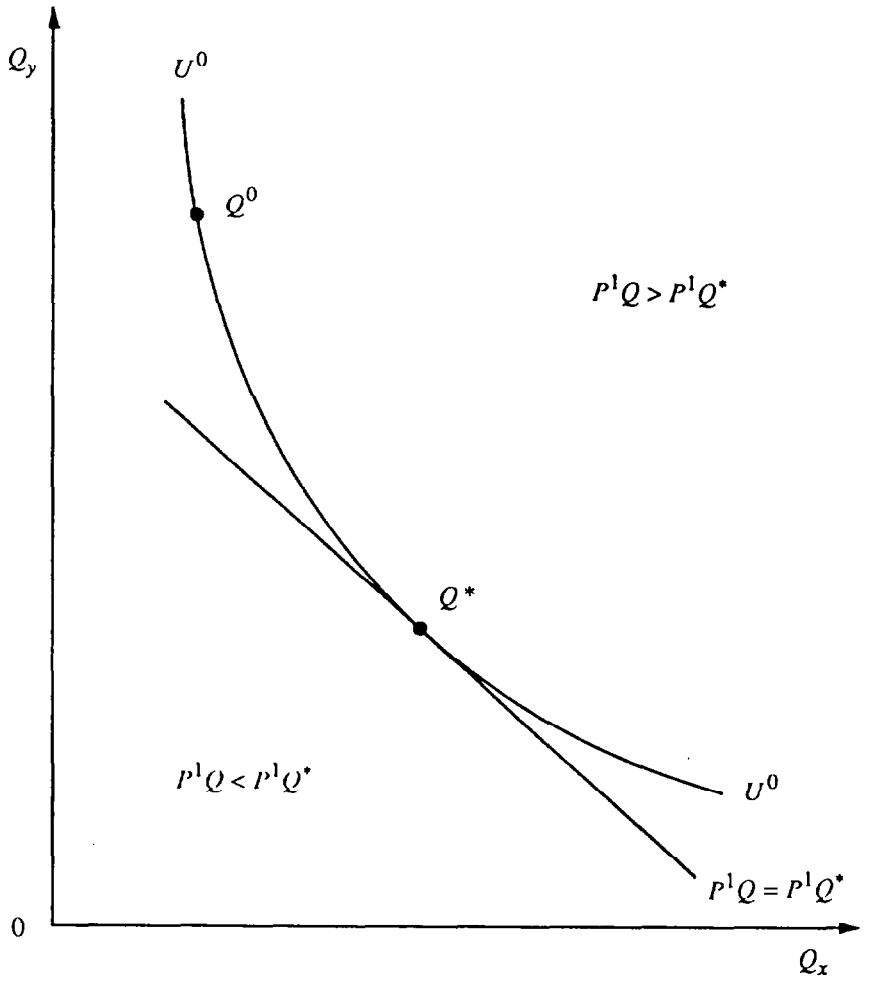
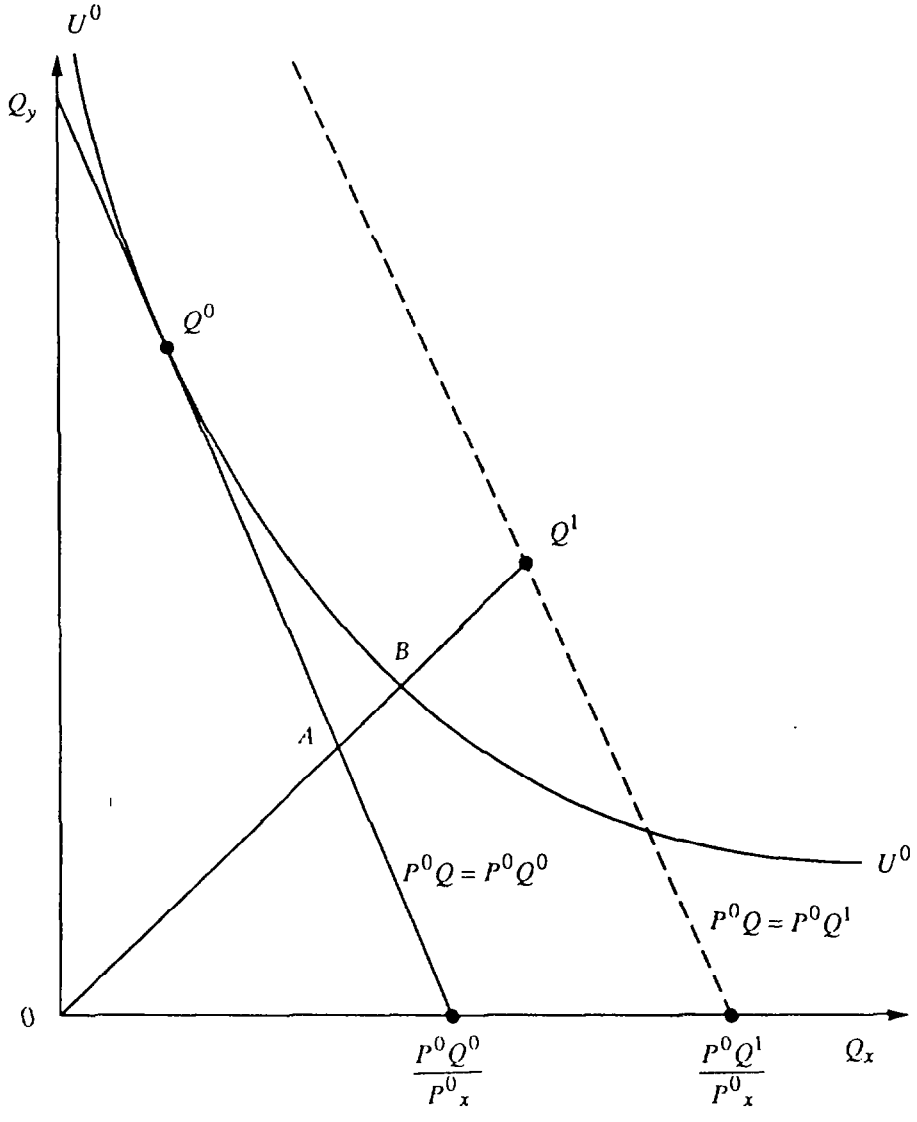


Figure 2. Laspeyres Quarterly Index with Free Prices



the utility-based inflation measure by a third. Turning to output indices, growth is zero according to a utility-based measure, but g_L is -13 percent and g_P is -8 percent. Thus, the Laspeyres and Paasche indices overstate both the inflation and the decline in output pursuant to decontrol.

To appreciate the role of price control in generating these results, it is helpful to imagine an intermediate stage of decontrol, where quantity and aggregate nominal value remain unchanged but the prices adjust from P^0 to P^{0*} to clear markets. In this example, P^{0*} is [2 2/3]. From the initial to the intermediate stage, the Paasche, Laspeyres, and utility-based indices all record zero inflation and zero growth. From the intermediate to the final stage, inflation is 38 percent according to the Paasche method and 63 percent according to Laspeyres. Output growth from the intermediate to the final stage is -13 percent by Laspeyres and +8 percent by Paasche. Hence, the distortion caused by price controls falls entirely on the Laspeyres price index and the Paasche quantity index.

A generalization

Now, let us try to generalize the preceding results. First, note that insofar as quantities and nominal aggregate value are fixed, price controls will not affect either the Laspeyres price index or the Paasche quantity index. To sign the impact of price controls on the remaining indices, we shall need two additional assumptions.

Assumption I: If firms were allowed to maximize value added at the official prices prior to decontrol, output of the deficit good would stay the same or decrease.

Assumption II: Firms respond to price decontrol by producing relatively more of the previously deficit good.

Assumption I says that when planners intervene in enterprise output decisions prior to decontrol, they do so mainly to increase the production of deficit goods. Assumption II says that once firms are allowed to respond directly to market demand, output of the previously deficit good immediately rises. In practice, there are not two types of goods but millions (even in a shortage economy), and doubtless, for some of these goods, either or both of the assumptions will be violated. However, if the surplus/deficit goods distinction is identified not with particular sectors but with "low quality" versus "high quality" products, both assumptions seem reasonable. Because "high quality" is an amalgam of many difficult-to-measure characteristics, because price controllers can process only a limited amount of information, and because producers subject to price controls try to pass off dubious quality improvements as genuine, price controls invariably shortchange quality. With so little premium at official prices for genuinely high quality products, firms have to be prodded administratively to produce better goods. Price decontrol allows firms more scope to profit from quality work, partly by raising the prices chargeable for it and partly—as

East German firms abruptly discovered—by decreasing the saleability of inferior wares.

Assumptions I and II are summarized geometrically in Figure 3. The horizontal axis indicates the amount of the deficit good, the vertical axis the amount of the surplus good. The curved line FF passing through Q^0 represents the production frontier, given the normal operating efficiency of the pre-decontrol economy. 1/ By Assumption I, the frontier is steeper at Q^0 than the budget line $P^0Q = P^0Q^0$. Even steeper is the indifference curve $U = U^0$ passing through Q^0, whose slope defines relative free-market prices if output were fixed. In actuality, the decontrol of prices provides an incentive to produce more of the deficit good, moving production to, say, Q^{1*}. However, the shock of the transition (modeled here as proportionately equal across sectors) may keep the economy from reaching Q^{1*}, so that output temporarily falls to Q^1.

Note that the movement from Q^0 to Q^{1*} represents a clear increase in output according to a utility-based measure. A Laspeyres quantity index based on market-clearing prices P^{0*} would show an even greater improvement. However, in terms of pre-decontrol prices actually in effect, the Laspeyres quantity index declines. Indeed, the more responsive production is to the shift in relative prices (i.e., the greater the shift down the production frontier), the more the Laspeyres quantity index declines. As for the shift from Q^{1*} to Q^1, the decline will be correctly measured by any quantity index, since the relative contraction is equal across sectors. Putting the two effects together we see that the standard Laspeyres quantity index overstates the net decline in output.

Figure 4 demonstrates that, under Assumptions I and II, the Paasche price index tends to overstate inflation. The indifference curve $U = U^1$ passes through Q^1. By Assumption II, the minimum output needed to assure a utility of U^1 prior to decontrol is given by the intersection Q^* of the indifference curve with the quantity constraint $Q_x = \bar{Q}_x$. Assumptions I and II together imply that the budget line $P^0Q = P^0Q^*$ passes above Q^1. Hence, P^1Q^1/P^0Q^1 is strictly less than the constant post-decontrol utility price index P^1Q^1/P^0Q^*.

Measurement biases are compounded by inefficiencies in the pre-decontrol distribution system. Excess demand for deficit goods creates a wedge, or "shortage rent", between the official price and the higher shadow price, accruing to whomever is fortunate enough to buy at the official

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1/ Of course, under a reformed economic system the normal operating efficiency would presumably improve, so that Q^0 would lie strictly behind the frontier. These improvements take time, however, and in the short term appear to be overwhelmed by the negative shock of transition. The shape of the frontier otherwise reflects the standard assumption of a convex production possibility set; i.e., that the average of two feasible output hurdles is also feasible.

Figure 3. Laspeyres Quarterly Index Under Shortage

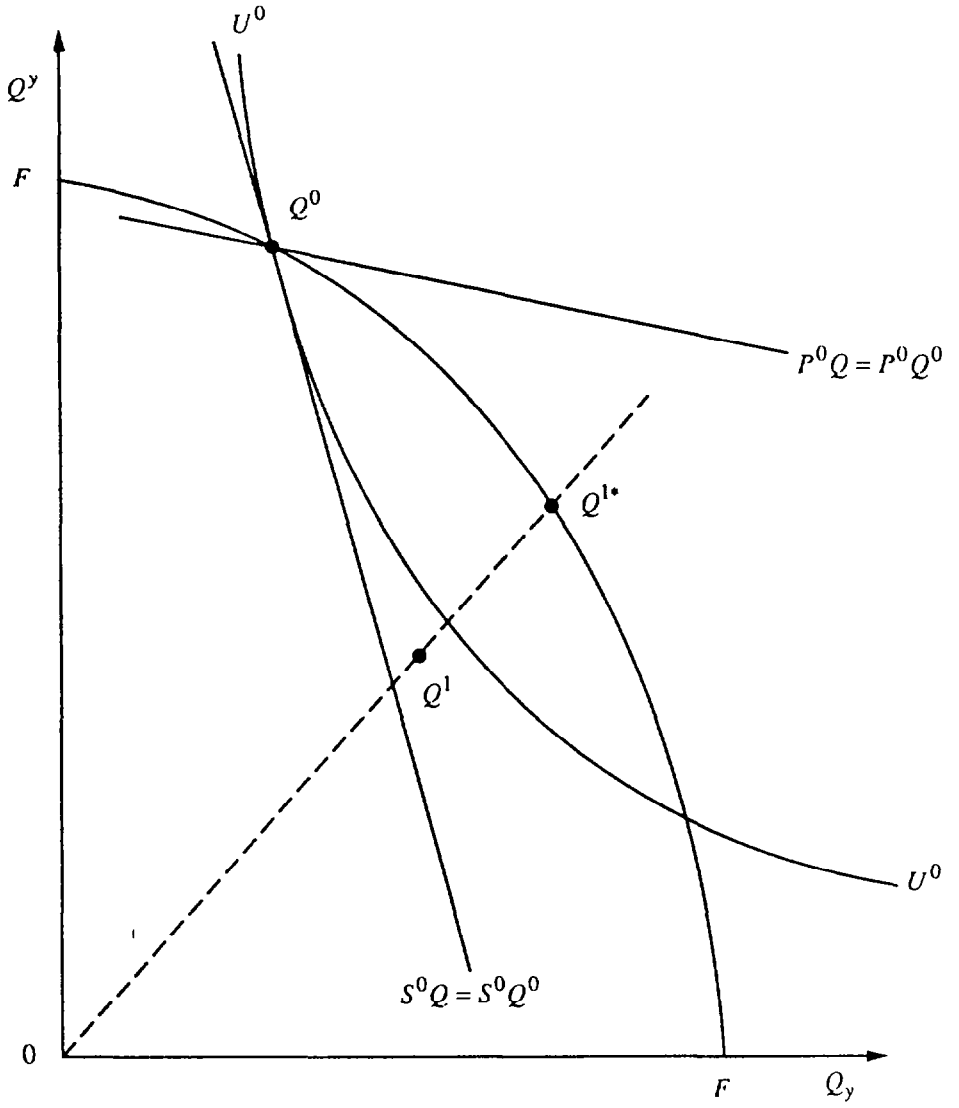
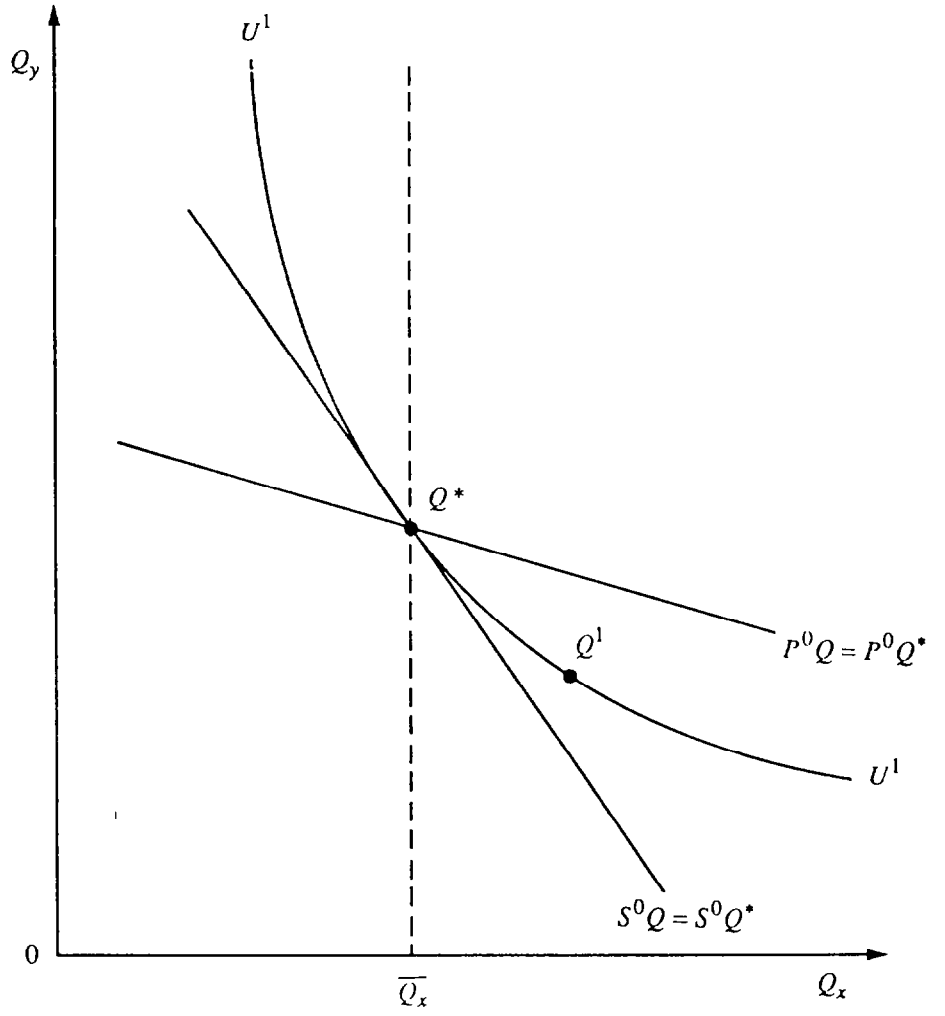


Figure 4. Paasche Price Index Under Shortage



price. Shortage rents in turn prompt queuing or extra inventory accumulation, as long as the marginal expected gain from shopping exceeds the marginal cost in foregone leisure. Yet, while extra queuing or inventory accumulation may be rational from an individual perspective, it is waste to society as a whole. Therefore, utility measures based simply on the supply of goods tend to overstate pre-decontrol welfare relative to post-decontrol welfare. If utility were truly held constant, the inflation would be less than it appears, while growth would be greater, thereby making the standard Paasche or Laspeyres indices even more misleading.

Even if there were no social waste from queuing or hoarding of deficit goods, either because rationing was perfectly efficient or because corruption was complete, high prices for surplus goods could create another problem: excess unsold goods. In this category should also be included the portion of investment representing construction that will never be finished. Since such products do not enter social consumption, it is misleading to treat them as useful output. Again, adjustment would serve to lower constant-utility estimates of inflation and increase constant-utility estimates of growth. The adjusted Paasche and Laspeyres growth indices would be higher and the adjusted Laspeyres price index would be lower, while the adjusted Paasche price index would not change (since Q^0 does not enter the calculation).

IV. Indices of Black Market Prices

All measures considered until now have been based on official prices. However, in a shortage economy there is another set of prices, relating to transactions on unofficial or "black" markets. Prior to decontrol, black market prices should normally exceed official prices for deficit goods and be equal to official prices for surplus goods. 1/ After decontrol, the black market should disappear (apart from tax evasion and sale of contraband goods), or alternatively, the black market and official prices should equalize.

This section explores the impact of liberalization on black market prices, assuming transaction costs on black markets are negligible. Since foreign currency is often sold on black markets, the inquiry may offer some insights about the impact of decontrol on exchange rates, at least so far as exchange rates reflect current purchasing power. Another topic of interest is whether information on black market prices prior to decontrol can be used to improve estimates of post-decontrol prices.

1/ If the black market price equilibrates significantly below the official price, transaction costs for buyers must be significantly higher on black markets than on official markets and/or no sales occur at official prices. The latter possibilities, which admittedly have some practical bearing, are ignored in the subsequent discussion.

To better appreciate the influences on black market prices, it is helpful to begin with a simple example. Suppose that:

(a) prior to decontrol, supply and demand exactly match for the (weakly) surplus good;

(b) decontrol leaves quantities unchanged; and

(c) decontrol leaves money velocity, as measured in terms of official prices, unchanged.

For example, the Laspeyres, Paasche, and constant-utility indices will all indicate a growth rate of zero and an inflation rate equal to the percentage change in the money stock. Since the black market price of the surplus good equals the official price, the black market price of the deficit good prior to decontrol, S_x, can be readily calculated:

$$S_x = P_y^0 \frac{\partial U(Q_x^0, Q_y^0)/\partial Q_x^0}{\partial U(Q_x^0, Q_y^0)/\partial Q_y^0} \equiv P_y^0 \frac{U_x^0}{U_y^0} \quad , \tag{5}$$

where U is the utility function. The aggregate value of goods in black market prices prior to decontrol will equal the official aggregate value or P^0Q^0, plus the aggregate shortage rent $(S^0_x - P^0_x)Q^0_x$.

Since decontrol does not change the marginal rate of substitution U_x/U_y between goods, the relative price of the two goods after decontrol should match the relative black market price prior to decontrol. Thus, decontrol will cause a uniform deflation in black market prices, with aggregate black market value dropping by the amount of shortage rent, or by the fraction $\eta/(1+\eta)$, where η denotes the ratio of aggregate shortage rent R^0 to aggregate official value Y^0. The greater is the volume of deficit goods, and the larger the relative price distortion prior to decontrol, the greater will be the deflation in black market prices.

While extreme, the preceding example illustrates several important points, which are valid more generally. First, black market prices prior to decontrol, assuming costless resale, are determined by official prices for surplus goods and marginal rates of substitution. Second, the elimination of shortage rents after decontrol tends to reduce black market prices. Third, the amount of deflation on the black market—and implicitly, the appreciation of the exchange rate, insofar as the exchange rate reflects current purchasing power—cannot be forecast from macro-level variables alone, but depends crucially on micro-level relative price distortions.

Now let assumption (c) be relaxed, while retaining assumptions (a) and (b). Specifically, suppose that the required money supply is linear in the sum of the official value of goods Y and a fraction of the shortage rent R; i.e.,

$$M = k[PQ + \delta(S-P)Q] = k(Y+\delta R) \quad , \tag{6}$$

where M is the nominal money supply, S is the vector of black market prices, and k and δ are constants. Essentially, equation (6) says that velocity 1/k is constant, provided that prices are measured as a weighted average of black market and official prices. If the weight δ on black market prices is 0, we are back to assumption (c). If the weight δ is 1, households gauge money needs as if all transactions occurred at black market prices. It is tempting, but misleading, to identify δ with the share of black market transactions in total transactions. In fact, δ will typically exceed this share, since even for transactions at official prices, difficulties in locating and obtaining goods may slow money turnover. Such difficulties tend to rise with the gap between official and shadow prices; indeed, the gap is the proximate cause of the difficulties. Hence, δ should be viewed as reflecting both direct transactions at black market prices and frictions in transactions at official prices.

Combining equation (6) with assumptions (a) and (b), the average inflation rate π in official prices can be calculated as:

$$\pi = \frac{Y^1}{Y^0} - 1 = \frac{M^1}{M^0}\frac{Y^0+\delta R^0}{Y^0} - 1 = (1+m)(1+\delta\eta) - 1 = \delta\eta + m(1+\delta\eta) \quad , \tag{7}$$

where m is the percentage growth in nominal money supply. The average inflation rate π^B in black market prices is:

$$\begin{aligned} \pi^B &= \frac{Y^1}{Y^0+R^0} - 1 = \frac{M^1}{M^0}\frac{Y^0+\delta R^0}{Y^0+R^0} - 1 = (1+m)\frac{1+\delta\eta}{1+\eta} - 1 \\ &= -\frac{(1-\delta)\eta}{1+\eta} + m\frac{1+\delta\eta}{1+\eta} = \frac{\pi-\eta}{1+\eta}. \end{aligned} \tag{8}$$

If the measurement period is very short, the money supply is unlikely to change much, in which case π^B will be negative. It is readily verified that the post-decontrol surge in official prices is strictly increasing in the growth rate of money supply m, in the relative weight η of shortage rent prior to decontrol, and in the influence δ of black market prices on money demand. The post-decontrol deflation in black market prices is strictly decreasing (that is, the inflation rate, typically negative, is increasing) in the growth rate of money supply and in the influence of black market prices on money demand, but is increasing in the relative weight prior to decontrol of shortage rent. Table 1 indicates the inflation/deflation rates associated with various values of δ and η, for money supply and quantities held constant.

Table 1. Dependence of Inflation Rates on δ and η, Assuming Fixed Money Supply and Quantities

		Percentage Inflation in Official Prices					
		η					
		0.2	0.4	0.6	0.8	1.0	1.5
δ	0	0	0	0	0	0	0
	0.25	5	10	15	20	25	38
	0.50	10	20	30	40	50	75
	0.75	15	30	45	60	75	113
	1	20	40	60	80	100	150

		Percentage Inflation in Black Market Prices					
		η					
		0.2	0.4	0.6	0.8	1.0	1.5
δ	0	-17	-29	-38	-44	-50	-60
	0.25	-13	-21	-28	-33	-38	-45
	0.50	-8	-14	-19	-22	-25	-30
	0.75	-4	-7	-9	-11	-13	-15
	1	0	0	0	0	0	0

Equations (7) and (8) can also be used to infer the underlying values of δ and η from the observed inflation rates and changes in money supply.

From (8),

$$\eta = \frac{\pi - \pi^B}{1 + \pi^B} \quad . \tag{9}$$

Substitution into (7) and rearrangement of terms yields:

$$\delta = \frac{\pi - m}{(1+m)\eta} = \frac{(\pi-m)(1+\pi^B)}{(1+m)(\pi-\pi^B)} \quad . \tag{10}$$

The greater is the surge in official prices, the higher are the inferred values of η and δ. The greater is the deflation in black market prices, the higher is the inferred η and the lower is the inferred δ. Higher estimates of money supply growth, for π and π^B unchanged, serve to reduce the estimated δ but leave η unchanged. Table 2 indicates the values of δ and η implied by various combinations of official and black market inflation rates, assuming money supply and quantities are fixed.

Table 2. Values of δ and η Implied by π and π^B, Assuming Fixed Money Supply and Quantities

		Implied value of η (by π)					
		20%	40%	60%	80%	100%	150%
π^B	0	0.20	0.40	0.60	0.80	1.00	1.50
	-10%	0.33	0.56	0.78	1.00	1.22	1.78
	-20%	0.50	0.75	1.00	1.25	1.50	2.13
	-30%	0.71	1.00	1.29	1.57	1.86	2.57
	-50%	1.40	1.80	2.20	2.60	3.00	4.00

		Implied value of δ (by π)					
		0.2	0.4	0.6	0.8	1.0	1.5
π^B	0	1.00	1.00	1.00	1.00	1.00	1.00
	-10%	0.60	0.72	0.77	0.80	0.82	0.84
	-20%	0.40	0.53	0.60	0.64	0.67	0.71
	-30%	0.28	0.40	0.47	0.51	0.54	0.58
	-50%	0.14	0.22	0.27	0.31	0.33	0.38

When quantities adjust, equations (7) through (10) do not fully describe the relationship between money demand and inflation, even when equation (6) applies. Nevertheless, the preceding analysis continues to be useful. Recall the earlier mental construct of an intermediate stage of decontrol, in which quantities are fixed but prices adjust to clear markets. Assuming that equation (6) holds, equations (7) and (8) indicate price level changes in the first phase, while the second phase can be analyzed as an ordinary problem of inflation in free markets.

The analysis is further complicated to the extent that, prior to decontrol, there are excess inventories of surplus goods. Here excess inventories include not only consumer goods that are unsaleable at current prices, but also, in line with our treatment of investment as a proxy for future consumer goods, construction projects that will never be finished. Ideally, such goods should be excluded from quantity estimates, since they do not improve social welfare. When goods will be sold or construction projects finished only after long delays, a discount factor reflecting the delay should be applied to the nominal value. In practice, such adjustments are rarely made, as identification of excess inventories and/or appropriate discount factors is extremely difficult. As a result, the understatement of growth by Paasche and Laspeyres quantity indices is exacerbated.

The Paasche price index should be unaffected by the misstatement of initial period quantities. As for the Laspeyres price index, at first glance its overstatement of inflation seems to be mitigated by the artificially high weight of Q^0_y on the slower-rising surplus goods prices. There is, however, a countervailing effect. If the surplus good is worthless, output should cease after liberalization, and the existing stocks given away or discarded. Properly, the good should be included in P^1 with a price of zero, and should be weighted in the Laspeyres price index according to pre-decontrol output. Unfortunately, goods with zero price are customarily excluded from the index, or are included at the last recorded price of sale, i.e., at the pre-decontrol price.

As an illustration, suppose that prior to liberalization, an ambitious construction project was started that would never be finished, or if finished, would produce items no one wanted. After liberalization, the project is abandoned and the remains sold for scrap. Ideally, this should be recorded as a slight increase in useful output and a decrease in price. Instead, the standard quantity indices record a decline in output, while the standard price indices ignore the fall in price.

V. Using Black Market Prices to Adjust Standard Indices

This section examines how, in principle, information on black market prices might be used to adjust standard indices of the quantity and price impact of liberalization.

The fact that black market prices are market-clearing suggests an obvious adjustment for quantity indices: use black market prices instead of official prices for weights. The substitution will not affect the Paasche measure of growth, since black market prices after decontrol equal official prices. The substitution will raise the Laspeyres measure of growth, however. Provided excess inventories are small, the Laspeyres growth measure based on black market prices, g^*_L, will set an upper bound for a constant pre-decontrol utility measure of growth. If excess inventories are large, even the black-market-based Laspeyres measure may understate growth, but at least the understatement will be less than if the official-price Laspeyres measure were used.

For inflation on black markets, Paasche and Laspeyres measures tend to bracket the constant-utility measures. However, in using black-market price indices to estimate changes in real household wealth, one must be careful to use black-market-based measures of nominal income. If the elimination of shortage rent is ignored, deflating nominal income by black market price indices tends to overstate real increases or understate real decreases, unless shortage rents prior to liberalization were completely dissipated by queuing.

Adjustment of official price indices is less straightforward. If quantities do not change, all standard price indices will give the same

results. Yet this unanimity can be misleading, for none of the standard indices take into account the post-decontrol improvements in shopping "technology", i.e., the reduced need for queuing and money balances. If nominal income is deflated by the index of official prices, declines in real income will be overstated. When quantities change, as has been demonstrated, standard indices are even more likely to overstate inflation on official markets.

It is tempting to correct official price indices by including a measure of black market prices, but this begs an appropriate choice of weights. Weighting black market and official prices by the volume of transactions on each market, as in Holzman (1960), may provide a more reasonable approximation of official prices but does not indicate real opportunity costs. Fortunately, if quantities, official prices, and black market prices are all known, a pseudo-Paasche price index π^*_P can easily be constructed. Simply divide the ratio of nominal values Y^1/Y^0 by $1+g^*_L$, the black market variant of the Laspeyres quantity index:

$$1 + \pi^*_P = \frac{Y^1/Y^0}{1 + g^*_L} = \frac{P^1Q^1/P^0Q^0}{S^0Q^1/S^0Q^0} = \frac{P^1Q^1}{S^0Q^1} \cdot \frac{S^0Q^0}{P^0Q^0} \quad . \tag{11}$$

Like the Paasche inflation index in a normal market-clearing economy, the pseudo-Paasche inflation index will understate the constant-utility measure of inflation, provided excess inventories are not large. To verify this, note that equation (11) can be interpreted as a standard Paasche index, given an initial price of $(P^0Q^0/S^0Q^0)S^0$ and a final price of P^1. Since each price vector reflects the relative scarcities at the time, the standard Paasche relationship applies.

To approximate a constant-utility measure of inflation, average the Laspeyres and pseudo-Paasche inflation indices. To approximate a constant-utility measure of growth, average the Paasche and black-market-based Laspeyres growth indices. Of course, the precise constant-utility measures of inflation or growth will vary with the utility function.

The case of CES utility

In principle, the shape of the utility function can be inferred given sufficient micro-level information on prices and quantities. To illustrate the potential uses of the latter, and to gauge the accuracy of various approximations, constant-utility measures of inflation and growth for a special case are worked out below. There will be one deficit and one surplus good, with no excess inventories of the latter. Demand for each good is assumed to be unit-elastic in income, with a constant (though unknown) elasticity of substitution between goods, i.e., utility is CES, of

the form $[\alpha Q_x^{\rho}+(1-\alpha)Q_y^{\rho}]^{1/\rho}$, with $\rho \leq 1$. 1/ Black markets are assumed to clear without transaction costs.

To simplify the notation, let prices and quantities be measured so that $P^0_x = P^0_y = 1 = Q^0_x+Q^0_y$, and define $X = Q^0_x$, $Y = Q^0_y$, $A = Q^1_x/X$, and $B = Q^1_y/Y$. Note that X and Y represent budget shares at official prices for deficit and surplus goods, respectively, while A exceeds B by Assumption II.

Since marginal rates of substitution equal relative prices on the black market, we see that:

$$\frac{\alpha X^{\rho-1}}{(1-\alpha)Y^{\rho-1}} = \frac{S^0_x}{I} = S^0_x \quad , \tag{12}$$

before decontrol, while:

$$\frac{\alpha A^{\rho-1}X^{\rho-1}}{(1-\alpha)B^{\rho-1}Y^{\rho-1}} = \frac{P^1_x}{P^1_y} \quad , \tag{13}$$

after decontrol. To solve for ρ in terms of A, B, and black market prices, divide (13) by (12) and take logarithms:

$$\rho - 1 = \frac{\ln P^1_x - \ln P^1_y - \ln S^0_x}{\ln A - \ln B} \quad . \tag{14}$$

Equation (12) can then be solved for α:

$$\alpha = \frac{S^0_x Y^{\rho-1}}{X^{\rho-1} + S^0_x Y^{\rho-1}} \quad . \tag{15}$$

In the form specified, the utility function exhibits constant returns to scale. Hence, the constant-utility growth rate g_U is given by:

$$1 + g_U = \left[\frac{\alpha A^{\rho}X^{\rho} + (1-\alpha)B^{\rho}Y^{\rho}}{\alpha X^{\rho} + (1-\alpha)Y^{\rho}}\right]^{\frac{1}{\rho}} \quad . \tag{16}$$

The constant-utility price index will be proportional to the average cost per unit of utility, so that the constant-utility inflation rate π_U—disregarding the increased ease of shopping after decontrol—is given by:

1/ The elasticity of substitution equals $1/(1-\rho)$. As ρ approaches 0 the CES function converges smoothly to $Q_x^{\alpha}Q_y^{1-\alpha}$, i.e., Cobb-Douglas with weights α and $1-\alpha$.

$$1 + \pi_U = \frac{P_X^1 AX + P_Y^1 BY}{[\alpha A^\rho X^\rho + (1-\alpha) B^\rho Y^\rho]^{1/\rho}} \cdot \frac{[\alpha X^\rho + (1-\alpha) Y^\rho]^{1/\rho}}{X+Y} = \frac{P_X^1 AX + P_Y^1 BY}{1 + g_U} \quad . \qquad (17)$$

Note from (17) that the constant-utility quantity index times the constant-utility price index equals the ratio of nominal aggregate values after and before liberalization, the same as the Paasche quantity index times the Laspeyres price index or the Laspeyres quantity index times the Paasche price index. Hence, given the ratio of nominal aggregate values, all of the price indices are, in a sense, symmetrical with the quantity indices, and either can be derived from the other. In practice, however, it is customary to think of distortions in terms of absolute or relative gaps in rates of change—i.e., $\pi_P - \pi_U$ or π_P / π_U, instead of $(1+\pi_P)/(1+\pi_U)$—in which case the standard inflation and growth measures may not appear equally biased. Typically, inflation rates are much higher than the percentage declines in output, so that distortions will appear to fall relatively more heavily on growth than inflation.

Sample calculations are recorded in Table 3. In the baseline case, listed on the first row, deficit goods account for 40 percent of sales at official prices prior to decontrol, with a black market price of 3 times the official price of 1. After decontrol the output of deficit goods rises by 10 percent, while the output of surplus goods fall by 30 percent. Prices after decontrol are 2 for the formerly deficit good and 1 (unchanged) for the formerly surplus good. Assuming the utility function is CES, the implied ρ and α are 0.10 (i.e., elasticity of substitution 1.11) and 0.68 respectively. The standard Paasche and Laspeyres indices suggest that the real economy contracted between 7 and 14 percent. Using black market prices, however, the Laspeyres method estimates only a 3 percent decline. Indeed, the constant-utility measure of growth registers -5 percent, halfway between the Paasche and black-market Laspeyres measures. Similarly, standard methodology suggests an inflation in official markets of between 40 and 51 percent, but according to the pseudo-Paasche index, the inflation is only 34 percent. In fact, the constant-utility measure of inflation is 37 percent, the average of the Laspeyres and pseudo-Paasche measures. The last two columns indicate the deflation in black market prices: 25 percent according to the Paasche index (denoted π^B_P) and 22 percent according to the Laspeyres index (denoted π^B_L).

To gauge the sensitivity of the results to the parameters, the remaining rows of Table 3 allow one baseline value at a time to change. For example, as the initial shadow price of the deficit good rises from 3 to 6, none of the standard Paasche and Laspeyres measures change, but the utility-weighted decline in output shrinks from 5 to 2 percent, while the utility-weighted inflation in official prices falls from 37 to 33 percent. In black market prices, deflation increases to about 55 percent.

As the contraction in output of the surplus good becomes more severe (i.e., B declines), all growth indices decline and the proportional gap

between them narrows, but in absolute terms the gap widens. For B = 0.4, the aggregate decline in output is 20 percent by the Paasche method and 32 percent by the Laspeyres method, but only 16 percent in constant-utility terms. As for measures of inflation in official prices, the gaps between them grow in both relative and absolute terms as B declines. While the Laspeyres index of inflation stays the same and the Paasche index of official inflation rises, inflation in official prices actually declines modestly in constant-utility terms, because the imputed utility weight on the deficit good rises along with its imputed ability to substitute for the surplus good. The Paasche index of deflation in black market prices rises slightly.

The lower is the initial budget share at official prices of the deficit good, the greater is the aggregate decline in output and the lower is inflation. The three growth measures tend to converge both absolutely and relatively. The gaps between the three measures of official inflation narrow in absolute terms but widen in relative terms. At X = 0.1, the growth rates range from -21 to -26 percent, while the official inflation rates range from 8 to 15 percent. The deflation in black market prices falls to the 10 percent range.

As the post-decontrol price of the deficit good rises, inflation in official prices rises by all three measures and the gaps between them narrow. The Laspeyres measure of growth is unchanged, while both the Paasche and constant-utility measures of growth register further declines. However, the gap between the Paasche and constant-utility growth rates widens. At $P^1_y = 1.6$, the contraction in output is 14 percent by the Laspeyres method and 12 percent by the Paasche method, compared to 7 percent in constant-utility terms. Deflation in black market prices again decreases, but the disparity between the Paasche and Laspeyres measures of its value increases.

Perhaps the most encouraging result of Table 3 is the good performance of the approximate growth and inflation measures recommended above. In all but one case, the average of g_P and g^*_L—the Paasche and Laspeyres' growth indices using black market prices as weights—lies within half of a percentage point of the constant-utility measure of growth, and the outlier remains within 1 percentage point of the constant-utility measure. For inflation, the average of the Laspeyres and pseudo-Paasche indices also lie *consistently within 1 percentage point of the constant-utility measure.*

VI. Conclusions

The transition from a centrally-planned economy to a market-driven economy is inevitably traumatic. Even if the administrative restrictions on markets are lifted quickly, and even if households and firms are fully forward-looking, the legal, financial, and informational frameworks of a developed market economy cannot immediately be put into place. Disruptions

Table 3. Sample Growth and Inflation Rates When Utility is CES.

Observed values						CES weights		Growth in percent				Inflation in percent					
X	s^0_X	p^1_X	p^1_Y	A	B	ρ	α	g_U	g_P	g_L	g^*_L	π_U	π_P	π_L	π^*_P	π^B_P	π^B_L
.4	3	2	1	1.1	.7	0.10	.68	-5	-7	-14	-3	37	51	40	34	-25	-22
.4	4	2	1	1.1	.7	-0.53	.68	-4	-7	-14	-1	35	51	40	31	-40	-36
.4	5	2	1	1.1	.7	-1.03	.69	-3	-7	-14	1	34	51	40	29	-50	-46
.4	6	2	1	1.1	.7	-1.43	.69	-2	-7	-14	2	33	51	40	27	-58	-53
.4	3	2	1	1.1	.6	0.33	.70	-9	-11	-20	-7	36	55	40	33	-26	-22
.4	3	2	1	1.1	.5	0.49	.71	-13	-16	-26	-10	35	59	40	31	-27	-22
.4	3	2	1	1.1	.4	0.60	.72	-16	-20	-32	-13	34	65	40	29	-28	-22
.3	3	2	1	1.1	.7	0.10	.58	-10	-12	-18	-7	27	40	30	24	-22	-19
.2	3	2	1	1.1	.7	0.10	.46	-15	-17	-22	-13	17	28	20	15	-18	-14
.1	3	2	1	1.1	.7	0.10	.29	-21	-23	-26	-20	8	15	10	6	-11	-8
.4	3	2	1.2	1.1	.7	-0.30	.64	-6	-9	-14	-3	47	61	52	43	-20	-16
.4	3	2	1.4	1.1	.7	-0.64	.61	-7	-10	-14	-3	57	71	64	52	-16	-9
.4	3	2	1.6	1.1	.7	-0.94	.58	-7	-12	-14	-3	68	80	76	61	-11	-2

are bound to occur. Inflation is likely to surge when prices are decontrolled, on account of "monetary overhang" inherited from the old regime, weak non-price controls for limiting the emission of money, and/or political and social pressures to increase wages and benefits. Output in the short run is likely to contract, on account of adjustment costs, monopolistic behavior, credit constraints, tapping of previously hoarded inventories, and/or uncertainty about the new environment.

In the statistical record, the real disruption tends to be compounded by measurement biases. Much of the private sector, where growth is most rapid, is hidden from statistical view. Also, some of the inflation attributed to liberalization may in fact reflect price increases incurred earlier, but which at the time were disguised as quality improvements in order to evade price controls.

Even when individual prices and quantities are correctly measured, however, liberalization poses a serious index number problem. Whereas in a free-market economy, Paasche and Laspeyres indices tend to bracket the theoretically more appealing constant-utility indices, during liberalization both Paasche and Laspeyres indices tend to understate growth and overstate inflation. Distortions exceeding 5 percentage points for growth or 10 percentage points for inflation do not appear unlikely. Paradoxically, the more flexibly producers respond to the post-liberalization incentives, the worse the distortions may be.

Black market prices prior to liberalization are a misleading guide to the black market/official price level after liberalization, because the former reflect an income inclusive of shortage rent. As shortage rent disappears, average black market prices are likely to fall unless the money supply grows very rapidly. The expected degree of deflation varies both with the influence of shortage on money demand and with the weight of shortage rent relative to official income. Thus, the movement of black market prices—including black market exchange rates, to the extent that foreign currency is held for black market purchases—cannot be predicted in terms of macro-level variables alone, but depends crucially on relative price distortions.

Information on the full spectrum of black market prices can also be useful in adjusting official indices. The Laspeyres growth index formed using black market prices as weights bounds from above the constant base-period utility index of growth. If this black-market-weighted Laspeyres index is divided into the nominal value ratio, the resulting pseudo-Paasche index ($P^1Q^1S^0Q^0/P^0Q^0S^0Q^1$, where P, S, and Q are vectors of official prices, black market prices, and quantities respectively and the superscripts ^{0} and ^{1} indicate pre- and post-control measurements) bounds from below the constant current-period utility price index. To approximate the constant-utility growth or price indices, just average the black-market-derived Laspeyres growth index with the Paasche growth index, or the pseudo-Paasche price index with the Laspeyres price index. In sample calculations assuming a

constant elasticity of substitution, the margin of error was less than one percentage point.

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