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General Equilibrium Under Shortage:
A Generalized Barro-Grossman Model

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ABSTRACT

In several recent articles, the Barro-Grossman model of general equilibrium under shortage has been modified to incorporate money demand and alternative retail sales mechanisms. This paper extends this work to allow for spillovers in deficit goods markets (modeled as feedback of black market prices on the real value of nominal money balances). Comparative statics analysis confirms the conventional view, recently challenged in the literature, that government expenditure in a shortage economy tends to reduce output. The conventional view associating shortage with higher savings is, however, substantially qualified. The model appears to be more consistent than previous models with the available empirical evidence, and offers insights into the consequences of price and monetary reform in shortage economies.

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Summary

This paper investigates a model of general equilibrium in a "shortage economy"--that is, an economy with significant excess demands at the officially controlled prices. Households are allowed to choose leisure and real money balances, taking wages, official prices, and rations and/or black-market prices as fixed. Household labor supply choices, in turn, affect supply. In equilibrium, households maximize welfare subject to constraints, while aggregate supply matches aggregate consumption.

The notion of applying general-equilibrium techniques to the study of a shortage economy is by no means original to this paper. Its main novelty is the incorporation of real-money-balance effects, specifically, the feedback on money demand of expected spending constraints. Claims, based on analysis of restricted models, made about equilibrium responses to various policy shifts are checked in a broader context. In the process, some aspects of the conventional wisdom are confirmed--for example, increased government spending, as measured by either direct government consumption or nominal money transfers to households, is likely to result in lower output. Other aspects of the conventional wisdom are qualified or overturned--for example, when shortages are expected to persist, increased government spending can be expected to cause real dissaving and, depending on the utility function, even nominal savings may decline.

The distinction between real and nominal saving trends may help to explain the high initial rates of inflation experienced by shortage economies undergoing price decontrol. Estimates of future price levels implicitly assume a certain level of real money holdings by households. To the extent that policymakers overestimate the latter, they would tend to underpredict the surge in the price level following the removal of controls.

The analysis also offers some insights into the distributional aspects of price and monetary reform. Often, price reform in shortage economies is postponed out of concern for consumer living standards or is enacted only together with extensive compensatory payments that hinder fiscal and monetary stabilization. The analysis in this paper suggests that the concern may be misplaced. Price reform may benefit households to such an extent that, apart from political considerations, only modest compensation, targeted to disadvantaged groups, need be considered.

Owing to the complexity of the model and the provision for general-utility and production functions, the technical analysis is unavoidably tedious. General equilibrium is characterized by six simultaneous equations whose differentials represent the equilibrium responses to small parameter shifts. Their signs cannot be ascertained through conventional maximization arguments alone, but some progress toward this end can be made by invoking the normality of household consumption and the local stability of equilibrium.

I. Introduction

This paper investigates a model of general equilibrium in a shortage economy. Here, "shortage economy" (alternatively, an economy with "suppressed inflation" or "repressed inflation") means an economy with significant excess demands at the officially-controlled prices, such as are thought to have characterized most centrally planned economies. 1/ Households are allowed to choose leisure and real money balances, taking wages, official prices, and rations and/or black market prices as fixed. Household labor supply choices in turn affect supply. In equilibrium, households maximize welfare subject to constraints, while aggregate supply matches aggregate consumption.

The notion of applying general equilibrium techniques to the study of a shortage economy is by no means original to this paper. The main novelty is the incorporation of real money balance effects, specifically, the feedback on money demand of expected spending constraints. Claims, based on analysis of restricted models, about equilibrium responses to various policy shifts are checked in a broader context. In the process, some aspects of the conventional wisdom are confirmed, e.g., increased government spending, as measured by either direct government consumption or nominal money transfers to households, is likely to result in lower output. Other aspects of the conventional wisdom are qualified or overturned, e.g., when shortages are expected to persist, increased government spending can be expected to cause real dissaving, and, depending on the utility function, even nominal savings may decline.

A brief review of the literature will highlight the issues addressed in this paper and motivate the modelling approach. A general equilibrium model was sketched in Hansen (1951), while Charlesworth (1956) explicitly connected shortages on goods markets with reductions in labor effort and output. Barro and Grossman (1971, 1974) provided the first rigorous treatment. In their so-called "disequilibrium" model, excess demand for present goods leads households to save more and work less than they would if all markets cleared at official prices. 2/ If the government increases its consumption of present goods, households tend to withhold more labor, which exacerbates excess demand, increases saving, and causes a further contraction of supply. Thus, in contrast to the standard Keynesian model, the so-called "supply multiplier" is negative.

1/ As Nuti (1986) has noted, technically the terms "suppressed inflation" and "repressed inflation" imply that excess demand is rising over time. Also, the concept of "shortage" allows the possibility that relative price adjustments could equilibrate markets without an aggregate price increase.

2/ Future markets do clear, so that eventually all demands are satisfied. Thus, the appellation "disequilibrium" is inappropriate, except as a shorthand for "quantity-constrained general equilibrium". Unfortunately, the disequilibrium term stuck, contributing to some confusion in the literature.

At first glance, the Barro-Grossman result appears to hinge on households' inability in the model to buy more than their ration of present goods. If private sector production and/or trade were allowed, then all household income would become spendable, which in turn would seem to make cash earnings more alluring. Such reasoning led Nuti (1986) to conjecture that inclusion of a private sector turns the supply multiplier positive. Attempting to explore Nuti's conjecture, Hare (1987) and Bennett and Phelps (1988) found that incorporating private production into the Barro-Grossman model changes the magnitude of the overall supply multiplier and possibly its sign, but neither suggested that a positive supply multiplier is typical. In contrast, when Bennett (1990) modified the Barro-Grossman model to allow costless resale of rationed goods at market-clearing prices, he claimed to find that the supply multiplier typically becomes positive.

On reflection, Nuti's conjecture is not so appealing, which makes Bennett's claim all the more intriguing. The direct impact of costless resale is simply to equalize marginal rates of substitution across households. If all households have identical utility functions and receive identical rations, marginal rates of substitution will equalize even without resale. Alternatively, consider that income can only be spent at black market prices, which adjust to reconcile demand with supply. In fact, the market-clearing prices would be identical to the shadow prices implicit in the Barro-Grossman model. From either perspective, it is difficult to see why the possibility of resale should affect household behavior.

As will be shown later, the apparent mystery turns out to have a simple resolution. There is an algebraic error in Bennett's analysis. For the information Bennett provides, the sign of the multiplier is in fact indeterminate.

Notwithstanding this mistake, Bennett's model marks a conceptual advance over that of Barro and Grossman. For Barro and Grossman's principal analysis, utility is separable in present consumption, future consumption, and leisure. Present consumption is constrained at official prices, but future consumption is not, while money is simply the vehicle to transfer income from present to future. In Bennett's model, money holdings substitute for future goods, normality is asserted for goods, money, and leisure, and otherwise the form of the utility function is left unspecified. In this respect the Barro-Grossman model can be viewed as a special case of Bennett's.

Moreover, Bennett's model can potentially address one of the thorniest criticisms of the Barro-Grossman model: namely, the treatment of present goods as a single aggregate category. Many authors (e.g., Kornai (1982, p. 94), Podkaminer (1988), Kemme (1989), Shu-ki (1990)) argue that by ignoring forced substitution from deficit to surplus present goods, aggregation misses an important feature of shortage economies and severely understates macroeconomic dislocation. Other authors (e.g., Portes and Winter (1980), Burkett (1988), Portes (1989)) argue that aggregation is a legitimate and indeed indispensable tool of macroeconomics. They suggest

that the degree of forced substitution actually occurring in socialist economies has been exaggerated. Since money can be interpreted as a proxy for either future goods or present surplus goods, Bennett's approach would appear to accommodate either perspective.

In fact, Bennett's model better captures intratemporal substitution (i.e., substitution from present deficit to present surplus goods) than intertemporal substitution. In his formulation, the usefulness of nominal money balances to households is unaffected by present official and black market prices for deficit goods. This might be reasonable if all money is channelled to purchases of present surplus goods. If money is being held in part to purchase deficit goods, however, the assumption is not so reasonable. Presumably the household will require extra money to meet transaction demands on black markets or queue-constrained official markets, or expects future official markets to clear only at higher prices. ^{1/} In such cases it would appear more realistic for money to enter utility in the form of real balances, that is, with nominal money deflated by an aggregate index of official and black market prices.

In both the Bennett and Barro-Grossman models, shortages are strictly confined. That is, in the Barro-Grossman model, shortages are expected to cease after T years, with T exogenously set, while in the Bennett model, money implicitly can always be spent in a fixed-price market that is immune to deficit. Allowing for real balance effects would better capture the notion that shortages in some markets spill over into others.

The next section of the paper reformulates Bennett's model to allow for "real balance" effects. For simplicity, households are assumed to be identical. General equilibrium is characterized by six simultaneous equations, whose differentials determine the equilibrium response to small parameter shifts.

Section III attempts to "sign" (i.e., to determine the sign of) these responses. As will be seen, conventional maximization arguments do not suffice to sign the Jacobian determinant of the system. Even when the determinant sign is known, the comparative statics calculations are complicated, and often the net response is indeterminate. Nevertheless, some progress can be made by invoking normality of household consumption and local stability of the equilibrium.

The assumption that all deficit goods are rationed is relaxed in Section IV. As in Bennett's followup article (1991), unrationed deficit goods will be assumed to be allocated via queuing, and the amount of queuing will be determined endogenously to equalize the full marginal shopping costs across markets. Again, the main difference with Bennett is the explicit

^{1/} In principle the aggregate price level could remain the same after price decontrol, but sticky nominal wages, government fiscal needs, and/or release of pent-up demand make this outcome extremely unlikely.

modelling of real balance effects. In Section V, equilibrium responses are signed where possible.

Section VI summarizes the main results. The typically negative sign predicted for the supply multiplier confirms the relevance of the Barro-Grossman result and refutes the revisionist claims of Nuti (1986) and Bennett (1990). However, the Barro-Grossman claim associating shortage with higher saving is not confirmed for the broader model, as the response depends on real balance effects. When households hold money chiefly for black market transactions, higher government spending will precipitate real dissaving, and even nominal savings may fall. Such effects call conventional notions of "monetary overhang" into question, and offer a new perspective on the distributional aspects of price and monetary reform. The paper concludes with some remarks on methodology and suggestions for future research.

Due to the complexity of the model and the provision for general utility and production functions, the technical analysis is unavoidably tedious. Local stability calculations are relegated to an appendix (pp. 35-41). Readers chiefly interested in the results are advised to focus on the model description in Section II, Tables 1 and 2 summarizing the typical equilibrium responses (pp. 7-13 and 20 and 29, respectively), or to proceed directly to the discussion in Sections VI and VII (pp. 30-34).

II. A Model with Rationing and Resale

In the stylized economy we will examine, a large number of identical households and derive utility U from leisure h , goods x , and real money balances m . Typically U is assumed to be strictly concave with h , x , and m normal, although weaker assumptions will be considered as well. At the official price p_0 for goods, demand exceeds supply, prompting the government to impose an upper purchase limit or "ration" of y per household. All goods channelled to household consumption are sold initially through the official market. However, there is a black market (less pejoratively, a "parallel" or "unofficial" market) on which rationed goods can be resold free of transaction costs at a market-clearing price p greater than p_0 . Conceptually, each household can be thought of as buying its entire ration y at price p_0 , reselling it for profit $(p-p_0)y$ on the black market, and then purchasing from the black market its desired consumption x .

Each household initially holds money M_0 and can also earn a wage w per unit of labor. Labor and leisure are perfect substitutes and by convention sum to 1. It follows that end-period nominal money M equals "full" wealth $M_0 + w + (p-p_0)y$, denoted as Z , less "full" consumption $wh + px$. To convert nominal money to real money, the former is deflated by a composite price level S , i.e., $m = M/S$. It seems reasonable to suppose that S is neither less than the official price nor greater than the black market price, and that if official and black market prices both increase by the same

percentage then so will S . These properties can be summarized mathematically as:

$$S = S(p_0, p) = p_0 s \left(\frac{p}{p_0} \right); \quad s(1) = 1; \quad 0 \leq s' \leq \frac{p_0 s}{p}. \quad (1)$$

The geometric average of official and black market prices is an example of an S function satisfying (1), as is an arithmetic average.

One possible interpretation of S is that it represents the expected price of future goods, for which purchases money is being held. In this case a low s' would reflect expectations that shortage will ease soon at official prices, while a high s' would reflect expectations of protracted shortage or of official prices rising to near black-market levels. To the extent that money is held for present transaction needs, however, s' might be high regardless of future prices.

An alternative interpretation is that money is being used to purchase "notionally surplus" present goods at average price S . "Notionally surplus", means that supply exceeds demand at official prices, barring spillover effects from other markets. In that case s' indicates the degree to which demands diverted from one deficit good market create deficits elsewhere. If inventories of notionally surplus goods are large and elasticities of substitution between goods are small, s' will be low. If inventories are thin and goods are ready substitutes, s' will be high. The model below allows for any s' , not necessarily constant, between 0 and 1. 1/

Households choose x , h , and m to maximize U subject to their wealth constraint. First-order conditions, which for U strictly quasi-concave are also sufficient, require that

$$\mathcal{L}_x = U_x - \lambda p = 0 \quad (2)$$

$$\mathcal{L}_h = U_h - \lambda w = 0 \quad (3)$$

$$\mathcal{L}_m = U_m - \lambda S = 0 \quad (4)$$

$$\mathcal{L}_\lambda = M_0 + w + (p-p_0)y - px - wh - Sm = 0, \quad (5)$$

where the subscripts on U denote partial derivatives and λ is the marginal utility of wealth in the Lagrangean \mathcal{L} .

1/ This is not to say that all feasible values of s' are equally relevant to economic policy. In section V, I suggest that s' is likely to be high.

Two additional conditions are needed for general equilibrium. First, the per capita ration y must equal per capita production, assumed to be an increasing differentiable function f of labor $1-h$, less per capita government consumption g . Second, average consumption must equal the average ration. Hence,

$$\Gamma_1 = f - g - y = 0 \quad (6)$$

$$\Gamma_2 = x - y = 0. \quad (7)$$

The equilibrium impact of a small change in exogenous parameters M_0 , p_0 , w , or g can be determined as follows. For V the vector $[x \ h \ m \ \lambda \ y \ p]^T$, where the T indicates transpose, and E the vector $[\mathcal{L}_x \ \mathcal{L}_h \ \mathcal{L}_m \ \mathcal{L}_\lambda \ \Gamma_1 \ \Gamma_2]^T$, let H denote the matrix $\partial E / \partial V$ evaluated at the equilibrium solution, or,

$$H \equiv [H_{ij}]_{i,j=1}^6 = \begin{bmatrix} U_{xx} & U_{xh} & U_{xm} & -p & 0 & -\lambda \\ U_{hx} & U_{hh} & U_{hm} & -w & 0 & 0 \\ U_{mx} & U_{mh} & U_{mm} & -p_0 s & 0 & -s' \lambda \\ -p & -w & -p_0 s & 0 & p-p_0 & -s' m \\ 0 & -f' & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \quad (8)$$

(In equation (8), the element H_{46} has been simplified from $y-x-s'm$ by applying equation (7)). The matrix H will be invertible if and only if the determinant $|H|$ is non-zero, in which case the equilibrium adjustments are given by:

$$dV = -H^{-1}dE = -\frac{[H^{ji}]_{i,j=1}^6}{|H|} \begin{bmatrix} 0 \\ -\lambda dw \\ -\lambda \left(s - s' \frac{p}{p_0} \right) dp_0 \\ dM_0 + (1-h)dw - \left(x + ms - ms' \frac{p}{p_0} \right) dp_0 \\ -dg \\ 0 \end{bmatrix}, \quad (9)$$

where H^{ji} is the cofactor of H_{ji} . ^{1/} The impact on utility is given by:

^{1/} To calculate the cofactor of the (i,j) element of a square matrix, delete the i -th row and j -th column of the matrix, calculate the determinant of the remaining matrix, and multiply by $(-1)^{i+j}$. Note that the inverse of a matrix equals the transposed matrix of cofactors divided by the determinant.

$$\begin{aligned} dU &= U_x dx + U_h dh + U_m dm \\ &= \lambda \left[(p-p_0) dy - ms' dp - \left(y + ms - ms' \frac{p}{p_0} \right) dp_0 + dM_0 + (1-h) dw \right], \end{aligned} \quad (10)$$

where the second equality follows from the envelope theorem.

The next section will try to sign dU and the elements of dV . The remainder of this section reviews the main similarities and differences between the model formulated above and the Barro-Grossman and Bennett models.

As mentioned earlier, households in the Barro-Grossman model maximize a separable function of present consumption, future consumption, and leisure, subject to a rationing constraint on present consumption. This can be viewed as a special case of the model here, with money a proxy for future goods, s constant, and U separable. Also, the component functions for utility of present goods and money should be identical up to a discount factor. (Barro-Grossman's exclusion of black market trade imposes no substantive restrictions, since each household can be viewed as reselling to itself at the shadow price p .) Conversely, the present model can be viewed as the Barro-Grossman model extended to allow a general utility function and partial spillover (via s) of present shortage into the future.

Bennett allows a general utility function but with money entering in nominal rather than in real terms, so that $S = p_0 s$ is constant. In another respect Bennett's model is more general than the present one, since he allows for differences in utility functions and rations across households. After deriving and examining the equilibrium conditions, Bennett proceeds to suggest that trade between households is unlikely to alter the signs of the main comparative statics results. Presumably the present model would behave similarly if household differences were incorporated. However, as the added algebraic complexity entailed by inclusion tends to obscure the core effects, household differences are not pursued further here.

Bennett (1989) also takes a different approach to analyzing the model. He first notes that optimal household consumption, leisure, and money holdings—denoted here by $\hat{\cdot}$ —will depend on the black market price, the wage, and full wealth. He then substitutes into the two market equilibrium constraints and differentiates to obtain (using the terminology of the present model and ignoring differences across households):

$$\begin{bmatrix} -f' \frac{\partial \hat{h}}{\partial p} - f' x \frac{\partial \hat{h}}{\partial Z} & -f' (p-1) \frac{\partial \hat{h}}{\partial Z} - 1 \\ \frac{\partial \hat{x}}{\partial p} + x \frac{\partial \hat{x}}{\partial Z} & (p-1) \frac{\partial \hat{x}}{\partial Z} - 1 \end{bmatrix} \begin{bmatrix} dp \\ dy \end{bmatrix} = \begin{bmatrix} dg + f' \frac{\partial \hat{h}}{\partial Z} (dM_0 - x dp_0) - \frac{\partial \hat{h}}{\partial w} dw \\ -\frac{\partial \hat{x}}{\partial Z} (dM_0 - x dp_0) - \frac{\partial \hat{x}}{\partial w} dw \end{bmatrix}. \quad (11)$$

Denote the left-hand matrix in equation (11) by $K = [K_{ij}]_{i,j=1,2}$. Multiplying both sides by K^{-1} , we see that the supply multiplier $d(g+y)/dg$ equals $1-k_{21}/|K|$, or $[k_{11}k_{22}-k_{21}(k_{12}+1)]/|K|$. Normality implies that k_{21} is less than -1 and, since $p\partial\hat{x}/\partial Z = 1-\partial(\hat{w}h+M)/\partial Z$, that k_{22} is negative. By invoking the Slutsky equations, Bennett rewrites k_{11} and k_{21} as $-f'\partial h^c/\partial p$ and $\partial x^c/\partial p$ respectively, where h^c and x^c refer to the compensated demand functions. From this he concludes that k_{11} and k_{21} are both negative. Provided the determinant $|K|$ is also negative, claims Bennett, the supply multiplier will be positive. His claim is mistaken, however. Without further information, the multiplier cannot be definitively signed. For example, if $k_{11} = k_{21} = k_{22} = -1$, the supply multiplier will be negative for k_{12} between -1 and -2 and positive for k_{12} less than -2.

Moreover, when real balance effects are incorporated into Bennett's model, the Slutsky equation will take a different form. For a small increase dp in the black market price, to repurchase the original utility-maximizing bundle the household will need to spend not $x dp$ but $(x+s'm)dp$, which by the envelope theorem is also the extra income required to maintain utility. It follows that:

$$\frac{\partial x^c}{\partial p} = \frac{\partial \hat{x}}{\partial p} + \frac{\partial \hat{x}}{\partial Z} \frac{dZ}{dp} = \frac{\partial \hat{x}}{\partial p} + \frac{\partial \hat{x}}{\partial Z} (x+s'm), \quad (12)$$

in which case k_{11} and k_{21} can be rewritten as $-f'\partial h^c/\partial p + f's'm\partial h/\partial Z$ and $\partial x^c/\partial p - s'm\partial \hat{x}/\partial Z$, respectively. In the new form k_{21} is readily identified as negative, but k_{11} is not. If k_{11} is positive, so will be the supply multiplier, but a negative k_{11} is compatible with either a positive or a negative supply multiplier. To resolve the uncertainty, or at least to determine the extent to which the uncertainty can be resolved, it is generally simpler to analyze equation (9) directly than to use the nominal shortcut equation (11). ^{1/}

III. Comparative Statics under Rationing

This section tries to evaluate the impact of small parameter changes. The influences of government consumption, initial money holdings, wages, and official prices will be considered separately. The first priority, however, is to sign $|H|$, since, from equation (9), it appears in every element of dV .

One's initial inclination is to try to deduce the sign of $|H|$ from maximization principles. If the last two rows and columns are deleted from H , the remaining 4×4 matrix J is the Jacobian for the household's maximization problem. Requirements of concavity and normality can be expressed as restrictions on the determinants of various submatrices of J .

^{1/} If one does choose to pursue the analysis of equation (11), allowing for real balance effects, the term $x dp_0$ appearing twice on the right-hand side of (11) should be replaced in both places by $[x+m(s-s'p/p_0)]dp_0$.

For example, concavity requires that the Hessian (matrix of cross-partials) of U be negative definite, which in turn implies that $|J|$ is negative. Indeed, even if U were not concave, given that there are four variables and one constraint, a non-positive $|J|$ would be required for a local maximum. Normality of goods implies that J^{41} , the cofactor of J_{14} , is positive, since $dx/dZ = -J^{41}/|J|$. Similarly, normality of leisure and real money balances imply that J^{42} and J^{43} are positive.

Unfortunately, none of these properties indicate anything directly about $|H|$, nor does expansion of the determinant yield an expression with an unambiguous sign. One must appeal instead to local stability of the equilibrium. Fortunately, useful information can be gleaned by examining the local stability of equilibrium, as suggested by Samuelson (1963, pp. 257-269). Local stability says that the system tends to return to equilibrium after small perturbations. Without local stability, comparative statics yields misleading results, since the disruption pursuant to small parameter changes would tend to dominate the shifts in equilibrium values.

To examine local stability, one needs to posit an adjustment process. The process posited here—actually, two subprocesses—is simple and intuitively appealing. First, when production less government consumption exceeds rations, rations increase. Second, when households' demands exceed rations, black market prices rise. In the appendix to this paper, it is shown that local stability under these conditions requires that $|H|$ be negative. ^{1/} A negative $|H|$ is also sufficient for local stability, provided prices do not adjust too fast relative to rations. If H^{55} is positive, as it typically will be, a negative $|H|$ will guarantee local stability regardless of adjustment speeds.

Ideally, one would like to translate the negative $|H|$ condition into more economically intuitive restrictions. The appendix shows that for the special case of Cobb-Douglas utility and linear production, local stability in effect imposes a floor on wages relative to initial money holdings and the official value of marginal product. The extent to which these conclusions apply more generally is unclear.

All of the calculations below assume a locally stable equilibrium. If the equilibrium is unstable the direction of the equilibrium responses will be reversed, to the extent that such responses are relevant at all.

1. Government consumption

From equation (9), the elements of dV/dg will have the same signs as the corresponding elements of $[-H^{51} -H^{52} -H^{53} -H^{54} -H^{55} -H^{56}]^T$. The supply multiplier will have the sign of $-dh/dg$, or,

^{1/} The implication of local stability for the sign of the determinant is noted, without elaboration, in Bennett's sequel article on queuing (1991).

$$H^{52} = - \begin{vmatrix} U_{xx} & U_{xm} & -p & -\lambda \\ U_{hx} & U_{hm} & -w & 0 \\ U_{mx} & U_{mm} & -p_0 s & -s' \lambda \\ -p_0 & -p_0 s & 0 & -s' m \end{vmatrix} \quad (13)$$

and could be positive or negative. If U is separable, equation (13) equals $\lambda p_0 U_h U_{mm} + \lambda s' w U_{xx} (m U_{mm} + U_m)$, which will be negative if, as is typically the case, $m U_m$ is not steeply decreasing in m . Alternatively, since $m U_{mm} + U_m$ equals $(1-r)U_m$, where r is the coefficient of relative risk aversion for money, a negative sign will prevail for the supply multiplier if the agent is not too risk-averse. 1/ A negative sign will also prevail if U is separable and s' is close to or equals zero, as in Barro and Grossman. Hence, contrary to Bennett's claim, the supply multiplier will typically be negative, barring strong cross-effects or a steeply decreasing marginal utility of money.

If supply $x+g$ declines as g rises, then certainly household consumption x will decline. More directly, we have seen that H^{55} , which has the opposite sign as dx/dg , will almost always be positive (and indeed must be positive for some forms of local stability). As for dp/dg , we have

$$-H^{56} = (p-p_0) J^{41} - \begin{vmatrix} U_{xx} & U_{xh} & U_{xm} & -p_0 \\ U_{hx} & U_{hh} & U_{hm} & -w \\ U_{mx} & U_{mh} & U_{mm} & -s \\ -p_0 & -w & -s & 0 \end{vmatrix} \quad (14)$$

For U concave, the last term in equation (14) will be positive, while J^{41} is positive for x normal. So, p can be expected to rise with g .

For an intuitive explanation of these results, consider that the more the government consumes, the higher black market prices will rise, holding supply fixed. Higher black market prices in turn cause non-wage income to rise, while leisure becomes cheaper relative to both goods and real money balances. Hence, supply will typically fall, causing black market prices to rise further.

As for real money balances, these can rise or fall depending on s' . For U separable, $-H^{53}$ equals the sum of $s' U_{xx} (\lambda w^2 - p_0 s m U_{hh})$, which is negative, and $-\lambda U_{hh} (p_0 s - s' p)$, which is positive or zero by equation (1). As s'

1/ The sign of $m U_{mm} + U_m$ also determines whether goods and leisure are gross substitutes for real money; that is, whether an increase in S , leaving w , p , and Z fixed would raise consumption of goods and leisure. Hence, the supply multiplier will be negative unless goods are a strong gross complement for money.

approaches 0, so that the usefulness of money balances ceases to depend on p , the second term will dominate and real money balances will rise. As s' approaches 1, however, the first term dominates and real money balances fall.

From equation (10), dU/dg equals $\lambda(p-p_0)dy/dg - \lambda s' mdp/dg$. Since y falls and p rises with g , households are left unambiguously worse off.

2. Initial money holdings

From equation (9), dV_i/dM_0 has the sign of H^{4i} . As H^{42} equals $-\lambda w U_{mm} + \lambda(p_0 s - s' p) U_{hm} + \lambda s' w U_{xm}$, leisure will increase, provided the cross partials are not too negative, while output and consumption fall. Since H^{46} equals $J^{41} + f' J^{42}$, the black market price will also rise. Part of the extra initial money is spent on leisure, which reduces supply and raises the shadow price. For U separable, H^{43} will equal $-\lambda(p_0 s - s' p) U_{hh} + \lambda w s' f' U_{xx}$, so as before real money balances will rise [fall] depending on how little [much] their value is affected by black market prices.

From equation (10), dU/dM_0 equals $\lambda + \lambda(p-p_0)dy/dM_0 - \lambda s' mdp/dM_0$. The last two terms have a negative impact but the first term is positive, leaving the aggregate sign indeterminate. However, if p is close to p_0 and s' is close to 0, dU/dM_0 will be positive. By making use of equilibrium conditions (5)-(7), dU/dM_0 can also be rewritten as $\lambda(w - pf')dh/dM_0 + \lambda p_0 sdm/dM_0$. When p is high, the first term is typically negative, while the second term tends to be negative when s' is close to 1. Hence, when money is mainly saved for the indefinite future or used to purchase goods currently in deficit, a "helicopter drop" of money serves to reduce average household welfare. However, when shortage is mild and is expected to ease soon, or when money is used mainly to purchase currently surplus goods, utility will typically rise with M_0 .

3. Wages

From equation (9), dV_i/dw will have the sign of $-\lambda H^{2i} + (1-h)H^{4i}$, where (ignoring the factor $-1/|H|$) H^{4i} is the wealth effect explored above and $-H^{2i}$ is a substitution effect. For leisure, $-H^{22}$ equals $s' m(p U_{mm} - p_0 s U_{xm}) + \lambda p_0 s(s' p - p_0 s)$. If demand for m is normal holding x fixed, the first term will be negative, while the second term is negative from equation (1). Hence the substitution effect of higher wages on leisure is negative, but since the wealth effect H^{42} is positive, leisure can rise or fall.

The complexities can be illustrated by considering extreme cases for s . With $s' = 0$, dh/dw is a positive multiple of $-\lambda(1-h)(w U_{mm} - p_0 U_{mh})$ and remains difficult to sign. The conventional wealth effect encourages leisure, while the conventional substitution effect discourages it. With $s' = 1$, dh/dw has the sign of $U_{mm} - U_{xm}$ times $p_m - w(1-h)$. Substituting from equations (5)-(7), the latter expression can be rewritten as $M_0 - p_0 x$. Hence, when s' is large, the labor supply curve (which is inverse to leisure demand) will be

backward-bending for low values of M_0 and upward-sloping for high values, assuming that demand for m , holding x fixed, is normal.

To appreciate the above effects, bear in mind that the initial surge in demand for goods and real money serves to drive up the shadow prices, which in turn entails additional income and substitution effects. The second-round substitution effects encourage consumption of leisure and thereby neutralize in part the first-round substitution. The total income effects are positive for rations (the cost of which falls relative to market value), negative for initial money holdings (the real value of which is eroded through inflation), and zero for black market transactions (since net purchases are zero). When s equals p , the second-round effects are so strong that the sign of $M_0 - p_0x$ determines the overall impact. In contrast, when s is fixed at 1, the second-round effects are weaker than the first-round effects and the net impact is difficult to sign.

The multiplicity of substitution effects also makes it more difficult to sign dp/dw . For U separable, the net substitution effect $-H^{26}$ equals $p_0^2 s^2 f' U_{xx} + (p_0 f' - w) p U_{mm}$, of which the first term is negative and the second term is either positive or negative depending on whether wages are greater or less than the marginal product valued at the official price. 1/ If the marginal utility of money is relatively flat ($U_{mm} \approx 0$), dp/dw could be negative. In this case, marginal household wealth would be channeled disproportionately to real money balances rather than to goods, while labor supply would increase. 2/ If individual labor supply is relatively inelastic, the black market price seems unlikely to fall with a higher wage.

Without further information, it is difficult to sign the impact of higher wages on real money balances and household welfare.

4. Official prices

Since household demand is homogeneous of degree 0 in (M_0, w, p_0, p) , a 1 percent increase in official prices has the same impact on real consumption and the price ratio p/p_0 as a 1 percent decrease in both wages and initial money holdings. However, since the impact of lower wages is generally indeterminate, this approach to determining the impact of a higher official price for goods is not very useful here.

From equation (9), dV_i/dp_0 will have the sign of $-xH^{4i} - (s - s'p/p_0)(\lambda H^{3i} + mH^{4i})$. If s' equals 1, the second term will vanish, so that

1/ This is not the same as whether the employing firm is profit-maximizing, since the wholesale prices received by the firm for goods may differ substantially from the retail prices. Moreover, employment may be taxed or subsidized directly.

2/ For U separable, analysis of the household's utility maximization problem shows that labor supply will be backward-bending if, and only if, $U_m U_{xx} + U_x U_{mm} + (1-h) U_{xx} U_{mm}$ is positive.

higher p_0 will have exactly the opposite qualitative impact as higher M_0 . Hence, output, consumption, and household welfare will typically rise with the official price while leisure, black market prices, and real money holdings fall.

With the possible exception of household welfare, the preceding conclusions also apply whenever utility is separable and logarithmic in money, since in that case household choices will be unaffected by s' . Otherwise, it is difficult to sign the equilibrium responses.

The typical responses are summarized in Table 1.

Table 1. Typical Response in Rationing Equilibrium
to a Small Increase in Parameter

Parameter	Variable						U
	h	f	x	($s^m=0$)	($s^m=1$)	p	
g	+	-	-	+	-	+	-
M_0	+	-	-	+	-	+	?
w	?	?	?	?	?	?	?
p_0 ($s'=0$)	?	?	?	?		?	?
p_0 ($s'=1$)	-	+	+		+	-	?

IV. Extension to Queuing

Rationing is not the only conceivable method for distributing deficit goods. An alternative approach is to sell arriving goods to shoppers on a first-come, first-served basis. With perfect black markets, shoppers who are able to purchase at the official price in effect receive a prize or "shortage rent" per good of $p-p_0$, which is the profit from resale. Competition for these prizes will cause "shopping jams", and households as a whole will spend extra time queuing. If queuing time and labor time are perfect substitutes, and if variances in shopping outcomes are small, shoppers will queue until the marginal expected return from queuing—the expected goods purchased times the shortage rent—equals the wage.

As in Tullock (1967), suppose that every minute of shopping time is equally likely to result in a purchase, and that households take total public queuing time Q as fixed. With a total of Y goods offered for sale, the average number of goods sold per shopping minute is Y/Q . The average return from queuing, which is also the expected marginal return, equals

$(p-p_0)Y/Q$. Hence, the value wQ of shopping time in equilibrium must equal $(p-p_0)Y$, so that the entire shortage rent is dissipated in queuing. 1/

Dissipation occurs because of a congestion externality. Shoppers weigh the marginal costs and benefits of shopping without taking into account the lower shopping success rates they impose on others. While from an individual perspective the extra queuing is rational, for society as a whole it is waste.

Allocation of deficit goods by queuing rather than rationing changes the household wealth constraint. It remains true that labor income plus shortage rent equals the value of goods consumed plus nominal savings. However, labor income now equals $w(1-h-q)$, where q denotes household queuing time, instead of $w(1-h)$. Hence, $w(1-h-q)+wq$ equals $px+M-M_0$, or, alternatively,

$$M_0 + w - px - wh - Sm = 0. \quad (15)$$

Comparing equation (15) with equation (5), the wealth constraint under rationing, the only difference is that the full income Z no longer includes the shortage rent. Note in particular that q is absent from equation (15). This reflects the complete indifference of individual households, under the preceding assumptions, between queuing and working. To simplify the exposition, however, it will be assumed that households behave identically, so that wq equals $(p-p_0)y$.

All of the other equilibrium conditions take exactly the same form as under rationing. 2/ However, it must be remembered that, under queuing, the supply f appearing in equation (6) is a function of $1-h-q$, or equivalently of $1-h-(p-p_0)y/w$. Denoting values in a queuing equilibrium by the subscript q , the fundamental comparative statics equation can be written as $dV_q = -H_q^{-1}dE_q$, where,

1/ If each shopper takes all others' shopping time as fixed but not the total, and if there are N identical households, then with the linear shopping technology specified above, all but $1/N$ of the total shortage rent will be dissipated. For large N , dissipation is virtually complete.

2/ Because shortage rent nets out of full income, y does not appear anywhere in the first-order conditions and so the two market equilibrium equations (6) and (7) could be merged. Nevertheless, the six-equation format will be retained here in order to make the comparisons between queuing and rationing equilibria easier to follow.

$$H_q \equiv \begin{bmatrix} U_{xx} & U_{xh} & U_{xm} & -p & 0 & -\lambda \\ U_{hx} & U_{hh} & U_{hm} & -w & 0 & 0 \\ U_{mx} & U_{mh} & U_{mm} & -p_0 s & 0 & -s' \lambda \\ -p & -w & -p_0 s & 0 & 0 & -x - s' m \\ 0 & -f' & 0 & 0 & -1 - \frac{f' (p - p_0)}{w} & -\frac{f' x}{w} \\ 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \quad (16)$$

$$dE_q = \begin{bmatrix} 0 \\ -\lambda dw \\ -\lambda \left\{ s - s' \frac{p}{p_0} \right\} dp_0 \\ dM_0 + (1-h) dw - m \left\{ s - s' \frac{p}{p_0} \right\} dp_0 \\ -dg + (p - p_0) \frac{f' x}{w^2} dw + \frac{f' x}{w} dp_0 \\ 0 \end{bmatrix}. \quad (17)$$

Comparing equation (16) to equation (8), it is seen that H and H_q differ only in the fifth and sixth elements of the fourth and fifth rows. Comparison of equations (17) and (9) shows that E_q keeps the same dg and dM_0 terms as dE , leaves out x in the fourth position, and in the fifth position adds terms in $(p - p_0)f'xdw/w^2$ and $f'xdp_0/w$. The impact dU_q on utility is given by:

$$U_x dx + U_h dh + U_m dm = \lambda \left[-s' m dp - \left\{ s - s' \frac{p}{p_0} \right\} m dp_0 + dM_0 + (1-h) dw \right]. \quad (18)$$

To sign the determinant $|H_q|$, we once again appeal to stability conditions. As with rationing, local stability of the equilibrium requires that the determinant of the system be negative. Since H_q^{66} is clearly positive (it is a negative multiple of $|J|$), a negative $|H_q|$ is also sufficient for at least some forms of local stability, while H_q^{55} must be positive to assure local stability regardless of relative adjustment speeds (see appendix for a comparison of local stability requirements under queuing and rationing).

The model described above is easily extended to incorporate a mixture of queuing and rationing. Let σ denote the fraction—assumed constant—of goods sold randomly to shoppers, with the remainder supplied initially via rations. Then in the relevant comparative statics equation, H_q should be replaced by $\sigma H_q + (1-\sigma)H$, and E_q by $\sigma E_q + (1-\sigma)E$. Since inversion is not a

linear operator, the equilibrium response to a small shift in parameters will not be exactly $\sigma dV_q + (1-\sigma)dV$. Nevertheless, in general one should expect a response intermediate to the responses under pure queuing and pure rationing.

Before pursuing the comparative statics analysis further, it is instructive to compare the preceding formulation with the queuing model found in Bennett (1991). As in his analysis of rationing, Bennett allows for differences between households while neglecting real money balance effects. However, he focuses on a difference not considered in his earlier paper—disparities between households in the wages they can earn in the state sector—and restricts the number of types of households to two. In equilibrium, the queuing wage must lie between the low state wage and the high state wage. If it lies at the high-wage [low-wage] boundary, high-wage [low-wage] households may both work and queue. If the queuing wage lies strictly between the high and low wage, then high-wage households will work without queuing and low-wage households queue without working. After noting these possibilities, Bennett proceeds to analyze only the case of complete specialization.

In contrast, the queuing model of this paper can be viewed as addressing a particular boundary case; namely, the high-wage boundary when the two types have identical utility functions and identical initial money holdings. The low wage will not be observed, as the low-wage type will queue without working. Each household will consume the same amount of leisure and retain the same amount of money. Average queuing time per high-wage household can be calculated from price and consumption data and the share of high-wage households in the total population. Given leisure and average queuing time, the average productive effort per high-wage household follows immediately. Alternatively, it can be calculated from data on the production function, quantity produced, and the population share of high-wage households.

V. Impact of Parameter Changes under Queuing

This section analyzes the equilibrium impact under queuing of small parameter changes assuming local stability requirements are met. Unfortunately, the seemingly small changes from H to H_q considerably complicate the elements of the inverse matrix. To simplify the calculations, and to strengthen the reader's intuition for what should normally be the determining factors, it will also be assumed that U is separable.

1. Government consumption

If government consumption rises, the black market price will also rise, as dp/dg takes the sign of $-H_q$ ⁵⁶ or $-|J|$. From equation (18), it follows immediately that utility falls. Household consumption also falls, since

$-H_q^{55}$ equals $w(p_0s-s'p)U_mU_{hh}+w^2U_hU_{mm}-wp(x+s'm)U_{hh}U_{mm}$, all of whose elements contribute negatively.

The derivative of leisure with respect to g has the opposite sign to $U_{mm}(xU_{xx}+U_x)+s'U_{xx}(mU_{mm}+U_m)$. For the Cobb-Douglas (logarithmic) utility, leisure will be constant. Otherwise, leisure will rise or fall depending on whether utility is less or more concave than Cobb-Douglas.

Not surprisingly, the impact of a higher g on real money balances varies with s' . When $s' = 0$, $-H_q^{53}$ equals $-wp_0U_{hh}(xU_{xx}+U_x)$, so nominal money balances rise or fall with xU_x . When $s' = 1$, real money balances are certain to decline as $-H_q^{53}$ equals $-wU_{xx}$ times $p(x+m)U_{hh}-\lambda w^2$.

The supply multiplier $1+dx/dg$ has the sign of $-|H|-H_q^{51}$. With $s' = 0$, the latter equals p_0f' times $wU_hU_{mm}-x(p_0U_{xx}+pU_{mm})U_{hh}-(p-p_0)U_mU_{hh}$, of which the first two terms are negative and the third is positive. However, if the black market price is not too much higher than the official price, the third term will be close to zero and the negative terms will dominate. At the other extreme where $s' = 1$, the supply multiplier will have the sign of $mp(p-p_0)U_{hh}U_{mm}+w^2U_{xx}(U_m+mU_{mm})+p_0wU_hU_{mm}-pxU_{hh}(p_0U_{mm}+pU_{xx})$. In this expression, the term in $p-p_0$ is positive, but it will be small if shortage is not too severe. The term in U_m+mU_{mm} could be positive or negative, but it will not be very positive unless households are extremely risk-averse in money. The remaining terms are strictly negative. The supply multipliers for other values of s' tend to be intermediate to the supply multipliers for the extreme cases. Hence, the supply multiplier is likely to be negative under queuing regardless of s' , unless shortage is severe and households are strongly risk-averse in money.

Intuitively, higher government consumption under queuing serves to raise the black market price without providing any extra full household income in compensation (since potential shortage rents are exhausted in queuing). The immediate income and the substitution effects discourage consumption of goods. As for leisure, substitution effects encourage and income effects discourage consumption, leaving the net response indeterminate.

The higher black market price also raises the shortage rent per item, and thereby tends to divert effort from productive labor to queuing. However, the contraction in supply exerts a countervailing influence, decreasing the incentive to queue. If black market prices are not too much higher than official prices, higher government spending will increase total shortage rent, and with it the amount of queuing. However, when price differentials per item are large, additional government expenditure may serve to reduce queuing.

2. Initial money holdings

From equation (17), dV_i/dM_0 takes the same sign as H_q^{4i} . Since H_q^{46} equals $[1+f'(p-p_0)/w]J^{41}+iJ^{42}$, which is positive for goods and leisure

normal, and H_q^{41} equals $f'U_hU_{mm}-f'p_xU_{hh}U_{mm}/w$, the black market price rises and output falls with M_0 . Leisure, however, could move either way, since H_q^{42} equals $-U_{mm}$ times $\lambda(w-f'p_0)+f'(U_x+xU_{xx})$. As before, the shift in real money balances depends on s' . When $s' = 0$, H_q^{43} is a positive multiple of H_q^{42} , so real money balances move in the same direction as leisure. When $s' = 1$, H_q^{43} is a positive multiple of H_q^{41} , so that m falls in step with output. The movement in utility is indeterminate, although a decline is relatively more likely the higher s' is.

The results are easily explained. The initial increase in wealth serves to increase demand for leisure. It also increases the demand for goods, so that the black market price rises, which in turn induces more queuing. Hence, output and consumption of goods must contract, which reduces the incentive to queue. While the income effects of a higher p and s discourage the consumption of leisure, they are unlikely to overcome both the countervailing substitution effects and the income effect of higher initial wealth. With consumption falling for goods and typically rising for labor, utility not surprisingly can either rise or fall.

3. Wages

In the previous discussion of equilibrium under rationing, it was noted that multiple, competing income and substitution effects make the net responses difficult to sign. With queuing, wages exert yet a third influence. Higher wages increase the opportunity cost of queuing, so that a given shortage rent is exhausted more quickly. Algebraically, this influence is captured in the sixth element of dE_q/dw . Whereas this element is zero under rationing, under queuing it equals $f'x(p-p_0)/w$. Working in the opposite way as government purchases, this factor serves to increase output and consumption, all else being equal, and to reduce black market prices. However, the relative strength of these effects remains unclear, and the net responses are indeterminate.

4. Official prices

In a pure queuing equilibrium, unlike a rationing equilibrium, higher official prices do not reduce full household income. However, they do reduce incentives to queue and thereby improve the productive labor supply. They also influence the real value of money, unless $s' = 1$. Formally, from equation (29), dV_i/dp_0 has the sign of $-(s-s'p/p_0)(\lambda H^{3i}+mH^{4i})+(f'x/w)H^{6i}$. For $s' = 1$, the first term in parentheses vanishes, so that a higher p_0 has the qualitative impact of an exogenous boost in supply. For comparison, when $s' = 1$ under rationing, a higher p_0 has the same qualitative impact as lower M_0 . The net responses, while not identical, are similar. The black market price falls, while real money balances, output, and consumption of goods rise. Leisure may rise or fall.

Again, with the possible exception of household welfare, the preceding conclusions also apply whenever utility is logarithmic in money, since

household choices will be unaffected by s' . For a general utility function the response are indeterminate.

Table 2 summarizes the typical responses in a queuing equilibrium.

Table 2. Typical Response in Queuing Equilibrium to
a Small Increase in Parameter

Parameter	Variable						
	h	f	x	$(s'^m=0)$	$(s'^m=1)$	p	U
g	?	-	-	?	-	+	-
m_0	?	-	-	?	-	+	?
w	?	?	?	?	?	?	?
$p_0 (s'=0)$?	?	?	?		?	?
$p_0 (s'=1)$?	+	+		+	-	+

VI. Discussion

Tables 1 and 2 show that, even when normality, local stability, and other likely or desirable economic properties are invoked, many equilibrium responses to parameter shifts cannot be signed. This is not surprising, since many cross-substitution and income effects are possible in a general three-good utility function. Indeed, it is perhaps surprising that so many typical responses could be signed.

Of the various responses, the sign of df/dg , the supply multiplier, has received most attention in the literature. A positive supply multiplier indicates that extra government spending—say, for investment or social services—can be financed in part out of additional current production stimulated by such spending. A negative supply multiplier indicates, on the contrary, that the immediate burden of government spending exceeds its direct cost. This paper's finding that the supply multiplier is likely to be negative extends the original Barro-Grossman result and refutes a contrary claim voiced by Nuti and Bennett. Simply put, shortage makes it harder to tap labor reserves.

Infusions of money not tied to labor also tend to reduce output. By raising black market prices, they serve to divert productive labor to leisure or queuing. In response to higher wages, however, output may either expand or contract, as there are several competing income and substitution effects.

In any case, the length of queues may be a misleading indicator of shortage. When excess demand at official prices is modest, worsening imbalances do tend to be associated with more queuing. However, when excess

demand is extreme, further deterioration will result in less queuing, as the contraction in supply dominates the rise in black market prices to reduce the aggregate shortage rent. Higher wages unbacked by higher productivity may also reduce queuing insofar as the opportunity cost of queuing rises. At the time of writing (spring 1991), Albania appears to provide an example of an economy with extreme shortages but relatively little queuing.

The impact of changes in official prices and/or initial money balances tends to depend on the magnitude of s' . For several reasons, it would seem that s' in most shortage economies is high. First, insofar as money is saved for future purchase of goods currently in deficit, a low s' would require that households be hopelessly optimistic, since there is no realistic indication that shortages at low official prices will end soon. 1/ Second, a rich institutional literature (e.g., Grossman (1977, 1979) on the Soviet economy) suggests that black markets are widespread in centrally planned economies and generate substantial transaction demands for money. Third, insofar as difficulties in obtaining supplies at official prices lead households to increase their precautionary money balances, higher black market prices will be associated with higher money demands even for trade through official channels. Fourth, empirical work on demand in centrally planned economies has tended to find strong spillover effects in present markets, either directly, as in Podkaminer (1988), or indirectly in the form of "discrete switching" effects, i.e., that at any given time, either the great majority of goods markets will be in excess supply, or the great majority will be in excess demand (e.g., Burkett (1988)). 2/

When s' is high, an increase in official prices will serve to reduce black market prices, while raising output and goods consumption. Leisure will fall in a pure rationing equilibrium, so that household welfare could rise or fall. In a queuing equilibrium, however, the reduction in shopping leads to an unambiguous improvement in welfare, and leisure could rise.

This finding should be reassuring news for policymakers in shortage economies contemplating large-scale price reform. Often price reform in shortage economies is postponed out of concern for consumer living standards, or is enacted only with extensive compensatory payments that hinder fiscal and monetary stabilization. The analysis in this paper suggests that the concern may be misplaced, as price reform may directly

1/ One might draw a different conclusion for households in Western countries facing the end of wartime rationing after World War II. One of the weaknesses of the so-called "disequilibrium" literature is the slighting of distinctions between temporary imbalances and chronic ones.

2/ "Discrete switching" refers to the tendency for the product of present excess demands and

benefit households. To the extent that compensation is provided, it should be modest in scale and targeted to disadvantaged groups. 1/

When s' is high, both an increase in government purchases and an increase in initial money holdings are likely to reduce real money balances, while an increase in official prices has the opposite effect. Nominal money balances, it can be shown, may move in either direction. These results suggest that conventional notions of "forced savings" are extremely misleading. Not only are savings voluntary, to the extent that households are allowed to select their labor and leisure, 2/ but also savings (identified here with money balances) tend to decline in real terms as shortages grow more severe. Neither should increases in nominal savings, to the extent they occur, be interpreted to mean necessarily that the public expects to buy more in the future at the low official prices.

The distinction between real and nominal saving trends may help to explain the high initial rates of inflation experienced by shortage economies undergoing price decontrol. Estimates of future price levels implicitly assume a certain level of real money holdings by households. To the extent that policymakers overestimate the latter, they would tend to underpredict the post-decontrol surge in the price level.

The results above also may help to reconcile claims that shortages are endemic in centrally planned economies with empirical work finding forced money savings to be relatively minor (e.g., Portes and Winter (1980), Portes, Quandt, and Yeo (1988), Burkett (1988)). When s' is high, the savings impact of discouragement in present markets may be substantially offset by discouragement in future markets. However, whether and to what extent the model presented here is amenable to direct empirical testing remains to be determined. 3/

1/ Even if this analysis is accepted by policymakers, the public at large may continue to perceive price reform as an assault on living standards. To be politically acceptable, price reform may have to be garnished with income indexation schemes, even though indexation may not only hurt the public at large but ultimately deepen its distrust of reform. The long-term solution is better public education to help the public appreciate the connection between low official prices, shortage, and productivity.

2/ The extent to which households' labor choices are constrained in centrally planned economies is a matter of some debate. Historically, the leaderships of these economies have employed a variety of legal and administrative measures to reduce slacking and increase labor force participation. As Brada and King (1986) note, however, one should be careful not to confuse hours worked with real labor effort, or to neglect household choice over human capital formation and second jobs.

3/ For arguments—perhaps not applicable here—that macroeconometric models of excess demand are not identifiable, see Podkaminer (1989).

VII. Remarks on Methodology

In this paper, the Barro-Grossman model of general equilibrium under shortage has been generalized to incorporate both money demand and spillovers in deficit goods markets. The approach appears to be more consistent with the available empirical evidence, and offers new insights into the nature of savings in shortage economies. To avoid exaggerating the power of the present model, however, two methodological shortcomings should be noted.

The first shortcoming is common to all models with money in the utility function. It is doubtful that money provides much direct utility simply by being held in one's pocket. Rather, money is presumably useful as a vehicle for something else: purchasing goods, reducing shopping time, insuring against risk, etc. While a model with money in the utility function may be a "reduced form" embracing these processes, by beginning with the reduced form one sacrifices potentially useful information. It may be fruitful to instead derive money demand under shortage starting from the specification of money's functions (e.g., as a medium for purchase or saving).

A second shortcoming, not unrelated to the first, is the condensation of a dynamic process to a one-period decision. As a result, one cannot distinguish very well between intertemporal and intratemporal substitution, nor can one study very well the repercussions of continual growth in money holdings relative to income (the so-called "monetary overhang"). In this respect the present model takes a step backward from the Barro-Grossman model, although the dynamics of the latter are very primitive (essentially it is a two-period model). The task of addressing these shortcomings is left for future research.

Implications of Local Stability

As mentioned in the text, a system is said to be locally stable, for a given adjustment process, if it returns to equilibrium after small perturbations. The adjustment process posited here has two aspects. First, when production less government consumption exceeds rations, rations increase. Second, when households' demands exceed rations, the black market price rises. Mathematically, adjustment can be represented by two differentiable functions ϕ and ξ and a time parameter t , such that

$$\frac{dy}{dt} = \phi(f-g-y) = \phi(\Gamma_1); \quad \phi(0) = 0; \quad \phi'(0) > 0; \quad (A1)$$

$$\frac{dp}{dt} = \xi(x-y) = \xi(\Gamma_2); \quad \xi(0) = 0; \quad \xi'(0) > 0. \quad (A2)$$

For small deviations from the equilibrium values y^* and p^* , the system can be linearized as:

$$\begin{bmatrix} \frac{dy}{dt} \\ \frac{dp}{dt} \end{bmatrix} = \begin{bmatrix} \phi' \frac{d\Gamma_1}{dy} \big|_{p=p^*} & \phi' \frac{d\Gamma_1}{dp} \big|_{y=y^*} \\ \xi' \frac{d\Gamma_2}{dy} \big|_{p=p^*} & \xi' \frac{d\Gamma_2}{dp} \big|_{y=y^*} \end{bmatrix} \begin{bmatrix} y-y^* \\ p-p^* \end{bmatrix} \quad (A3)$$

where all variables in the Γ 's are treated as functions of y and p , and the derivatives of ϕ and ξ are evaluated at 0. Denote the middle 2×2 matrix by D , with elements D_{ij} . Necessary and sufficient conditions for stability are that the trace $D_{11}+D_{22}$ be negative and the determinant $|D| = D_{11}D_{22}-D_{21}D_{12}$ be positive. ^{1/} If it is further required that the system be locally stable for any relative adjustment speed ϕ'/ξ' , then D_{11} and D_{22} must each be negative.

Applying the chain rule, $\partial\Gamma_1/\partial y$ equals $-f'\partial h/\partial y - 1$ equals $-f(p-1)\partial\hat{h}/\partial Z - 1$, which is Bennett's k_{12} . In fact, it is readily checked (bearing in mind that $\partial\hat{h}/\partial p$ equals $\partial h/\partial p + x\partial h/\partial Z$ and $\partial\hat{x}/\partial p$ equals $\partial x/\partial p + x\partial x/\partial Z$) that:

$$D = \begin{bmatrix} \phi' k_{12} & \phi' k_{11} \\ \xi' k_{22} & \xi' k_{21} \end{bmatrix}. \quad (A4)$$

Hence $|K|$ equals $-|D|/\phi'\xi'$, and must be negative if the equilibrium is to be locally stable. Also, both k_{12} and k_{21} must be negative if equilibrium is stable for any relative adjustment speeds.

Terms in $\partial x/\partial y$, $\partial x/\partial p$, $\partial h/\partial y$, and $\partial h/\partial p$ can be further reduced by differentiating the first-order conditions of equations (2)-(5) for

^{1/} See, for example, Samuelson (1963, pp. 430-431).

household utility maximization and solving. The results can be expressed more succinctly using a matrix notation. Let A denote the vector $[x \ h \ m \ \lambda]^T$ of household choice variables, and B the vector $[y \ p]^T$. Partition E into $\partial \mathcal{L}/\partial A$ and $\Gamma \equiv [\Gamma_1 \ \Gamma_2]^T$. Finally, let C denote a 2×2 diagonal matrix with upper-left-hand element ϕ' and lower-right-hand element ξ' . It follows that D can be rewritten as C times $\partial \Gamma(A(B), B)/\partial B$, J as $\partial^2 \mathcal{L}/\partial A^2$, and H as:

$$J = \begin{bmatrix} \frac{\partial^2 \mathcal{L}}{\partial A^2} & \frac{\partial^2 \mathcal{L}}{\partial A \partial B} \\ \frac{\partial \Gamma}{\partial A} & \frac{\partial \Gamma}{\partial B} \end{bmatrix}. \quad (A5)$$

The next few steps relate D to H. Applying the chain rule,

$$\frac{\partial \Gamma(A(B), B)}{\partial B} = \frac{\partial \Gamma}{\partial A} \frac{\partial A}{\partial B} + \frac{\partial \Gamma}{\partial B}, \quad (A6)$$

while differentiation of the first-order conditions $\partial \mathcal{L}/\partial A = 0$ shows that:

$$\frac{\partial^2 \mathcal{L}}{\partial A^2} \frac{\partial A}{\partial B} + \frac{\partial^2 \mathcal{L}}{\partial A \partial B} = 0. \quad (A7)$$

Solve for $\partial A/\partial B$ in (A7) and substitute into (A6) to obtain:

$$\frac{\partial \Gamma(A(B), B)}{\partial B} = \frac{\partial \Gamma}{\partial B} - \frac{\partial \Gamma}{\partial A} \left[\frac{\partial^2 \mathcal{L}}{\partial A^2} \right]^{-1} \frac{\partial^2 \mathcal{L}}{\partial A \partial B}. \quad (A8)$$

Note that the elements of the right-hand side of (A8) are also the partition elements of H in (A5). Application of the formula for inverting a partitioned matrix 1/ yields the following relationship:

$$C^{-1}D = \frac{\partial \Gamma(A(B), B)}{\partial B} = \begin{bmatrix} \frac{H^{55}}{|\Gamma|} & \frac{H^{65}}{|\Gamma|} \\ \frac{H^{56}}{|\Gamma|} & \frac{H^{66}}{|\Gamma|} \end{bmatrix}^{-1} = \frac{|H|}{H^{55}H^{66} - H^{65}H^{56}} \begin{bmatrix} H^{66} & -H^{65} \\ -H^{56} & H^{55} \end{bmatrix}. \quad (A9)$$

Meanwhile, direct evaluation of (A8) establishes that:

$$\frac{\partial \Gamma(A(B), B)}{\partial B} = \frac{1}{|J|} \begin{bmatrix} H^{66} & -H^{65} \\ -H^{56} & H^{55} \end{bmatrix}. \quad (A10)$$

Equating (A9) and (A10),

1/ See, for example, Johnston (1972, p. 93).

$$|J| = \frac{H^{55}H^{66} - H^{56}H^{65}}{|H|}, \quad (A11)$$

which implies:

$$|D| = |C| \frac{|H|^2}{H^{55}H^{66} - H^{56}H^{65}} = \delta' \sigma' \frac{|H|}{|J|}. \quad (A12)$$

Since a positive $|J|$ is ruled out by utility maximization, local stability of equilibrium requires that $|H|$ be negative. Furthermore, either H^{55} or H^{66} must be positive, and both must be positive if stability is assured regardless of relative adjustment speeds. In fact, H^{66} works out to $-|J| + f'(p-1)J^{42}$, and this will be positive for leisure normal. As for H^{55} , it equals $\lambda J^{11} + s'mJ^{41} + s'\lambda J^{31}$, where J^{11} is positive for U quasi-concave, J^{41} is positive for goods normal, and J^{31} cannot be unambiguously signed but typically is positive too. Hence, a negative $|H|$ is necessary and sufficient for local stability provided prices do not adjust too fast relative to rations, and typically will be sufficient for stability regardless of adjustment speeds.

In an effort to translate the determinant condition into more economically intuitive restrictions, the remainder of this appendix explores the special case of Cobb-Douglas (logarithmic) utility and a linear production function. Let U equal $\alpha \ln x + \beta \ln h + \gamma \ln m$, where the weights α , β , and γ are strictly positive and sum to one, and let production equal f' times labor effort for some constant f' . Note that in this case the shadow price s of money is irrelevant to equilibrium demands and the black market price, since $\ln m$ equals $\ln M - \ln S$.

Since Cobb-Douglas utility weights equal full-income budget shares, black market price, household goods consumption, and leisure they are defined by the equations:

$$px = \alpha[M_0 + w + (p-p_0)x] \quad (A13)$$

$$wh = \beta[M_0 + w + (p-p_0)x] \quad (A14)$$

$$x + g = f'(1 - h), \quad (A15)$$

with solutions for x and p of:

$$x = \frac{[\gamma f' - (\beta + \gamma)g]w - \beta f' M_0}{(\beta + \gamma)w - \beta f' p_0} \equiv \frac{N_1}{N_2}, \quad (A16)$$

$$p = \frac{bw[M_0 + w - (f' - g)p_0]}{[\gamma f' - (\beta + \gamma)g]w - \beta f' M_0} \equiv \frac{N_3}{N_1}, \quad (A17)$$

where N_1 is the numerator in (A16) and the denominator in (A17), N_2 is the denominator in (A16), and N_3 is the numerator in (A17). The conditions of the problem require that x be positive and p be greater than p_0 (in which case h and m will also be positive, so we need not examine them separately). It follows that the indices N_1 , N_2 , and $N_3 - p_0 N_1$ must either all be positive or all be negative. Let us call the range for w yielding positive indices the "upper branch" and the range for w yielding negative indices the "lower branch". Depending on the parameters, either or both of the branches may be empty. For example, the positive branch will be empty if $(\beta + \gamma)g$ equals or exceeds $\gamma f'$, while the negative branch will be empty if p_0 equals zero.

Now let us see what additional restrictions must be imposed for local stability. Straightforward calculation shows that:

$$|H| = \frac{wp_0^2 s^2 [\beta f' p_0 - (\beta + \gamma)w]}{\beta \gamma Z^3}, \quad (A18)$$

where Z as before denotes full income $M_0 + w + (p - p_0)x$. Local stability requires that the term in brackets, which is $-N_2$, be negative, thereby ruling out the lower branch. Conversely, any wage from the upper branch will yield a locally stable equilibrium. In economic terms, local stability requires that the wage exceed a fraction $\beta/(\beta + \gamma)$ of the official value of the marginal production.

For the queuing equilibrium, black market price, output, and goods consumption are defined by the equations:

$$px = \alpha(M_0 + w) \quad (A19)$$

$$wh = \beta(M_0 + w) \quad (A20)$$

$$x + g = f'[1 - h - (p - p_0)x/w], \quad (A21)$$

which has a solution for p of:

$$p = \frac{\alpha(w + M_0)(w - f'p_0)}{(\gamma f' - g)w - (\alpha + \beta)f'M_0} \equiv \frac{N_4}{N_5}. \quad (A22)$$

The conditions of the problem require that p exceed p_0 , in which case all household consumption variables are positive and need not be checked further. Feasible ranges for w again divide into two branches, where the upper branch values yield a positive N_5 and $N_4 - p_0 N_5$ and the lower branch values yield a negative N_5 and $N_4 - p_0 N_5$.

Local stability requires that:

$$|H_q| = \frac{wp_0^2 s^2 (f' p_0 - w)}{\beta \gamma (M_0 + w)^3} \quad (A23)$$

be negative, so the wage must exceed the official value of the marginal product. Again, none of the lower branch wages yield locally stable equilibria, but all of the upper branch wages do.

For this example, the local stability conditions are more restrictive for the queuing equilibrium than for the rationing equilibrium. The greater stringency appears to reflect the danger that the extra queuing induced by a temporarily higher black market price will impinge excessively on labor effort (output), thereby increasing excess demand.

The extent to which these conclusions generalize to a broader class of production and utility functions is not clear.

Bibliography

- Barro, Robert J., and Herschel I. Grossman, "A General Disequilibrium Model of Income and Employment", American Economic Review, Vol. 61, No. 1, March 1971, pp. 82-93.
- Barro, Robert J., and Herschel I. Grossman, "Suppressed Inflation and the Supply Multiplier", Review of Economic Studies, Vol. 41, No. 1, January 1974, pp. 97-104.
- Bennett, John, "Repressed Inflation, Queuing, and the Resale of Goods in a Centrally Planned Economy", European Economic Review, Vol. 35, No. 1, January 1991, pp. 49-60.
- Bennett, John, "Resale of Goods under Repressed Inflation: Implications for Supply Multipliers", Journal of Comparative Economics, Vol. 14, No. 1, March 1990, pp. 1-14.
- Bennett, John, and Michael Phelps, "The Supply Multiplier with a Self-Employed Private Sector", Economics of Planning, Vol. 22, No. 3, 1988, pp. 101-108.
- Brada, Josef C., and Arthur E. King, "Taut Plans, Repressed Inflation, and the Supply of Effort in Centrally Planned Economies", Economics of Planning, Vol. 20, No. 3, 1986, pp. 162-178.
- Burkett, John P., "Slack, Shortage, and Discouraged Consumers in Eastern Europe: Estimates Based on Smoothing by Aggregation", Review of Economic Studies, Vol. 55, 1988, pp. 493-505.
- Charlesworth, Harold K., The Economics of Repressed Inflation, London: Allen and Unwin, 1956.
- Grossman, Gregory, "The Second Economy in the USSR", Problems of Communism, Vol. 26, No. 5, September/October 1977, pp. 25-40.
- Grossman, Gregory, "Notes on the Illegal Private Economy and Corruption," in Joint Economic Committee, U.S. Congress, ed., Soviet Economy in a Time of Change, Washington, D.C.: Government Printing Office, 1979, pp. 834-854.
- Hansen, Bent, A Study in the Theory of Inflation, London: Allen and Unwin, 1951.
- Hare, Paul, "Supply Multipliers in a Centrally Planned Economy with a Private Sector", Economics of Planning, Vol. 21, No. 3, 1987, pp. 53-61.

- Johnston, J., Econometric Methods, Second Edition, New York: McGraw-Hill, 1972.
- Kemme, David M., "The Chronic Shortage Model of Centrally Planned Economies", Soviet Studies Vol. 41, No. 3, July 1989, pp. 345-364.
- Kornai, Janos, Economics of Shortage, Amsterdam: North-Holland, 1980.
- Nuti, Mario, "Hidden and Repressed Inflation in Soviet-Type Economies: Definitions, Measurement, and Stabilization", Contributions to Political Economy, Vol. 5, March 1986, pp. 37-82.
- Osband, Kent, "Economic Crisis in a Shortage Economy", IMF Working Paper WP/91/38, April 1991.
- Podkaminer, Leon, "Disequilibrium in Poland's Consumer Markets: Further Evidence on Intermarket Spillovers", Journal of Comparative Economics, Vol. 12, 1988, pp. 43-60.
- Podkaminer, Leon, "Macroeconomic Disequilibria in Centrally Planned Economies: Identifiability of Econometric Models Based on the Theory of Household Behavior under Quantity Constraints", Journal of Comparative Economics, Vol. 13, 1989, pp. 47-60.
- Portes, Richard, "The Theory and Measurement of Macroeconomic Disequilibrium in Centrally Planned Economies", in Davis, Christopher and Wojciech Charemza, ed., Models of Disequilibrium and Shortage in Centrally Planned Economies, London: Chapman and Hall, 1989, pp. 27-44.
- Portes, Richard, Richard E. Quandt, and Stephen Yeo, "Tests of the Chronic Shortage Hypothesis: The Case of Poland", Review of Economics and Statistics, Vol. 70, No. 2, 1988, pp. 288-295.
- Portes, Richard, and David Winter, "Disequilibrium Estimates for Consumption Goods Markets in Centrally Planned Economies", Review of Economic Studies, Vol. 47, 1980, pp. 137-159.
- Samuelson, Paul A., Foundations of Economic Analysis, Cambridge, Massachusetts: Harvard University Press, 1963.
- Shu-ki, Tsang, "A Note on the Aggregation of Slack and Shortage in Centrally Planned Economies", Economics of Planning, Vol. 23, No. 3, 1990, pp. 193-207.
- Tullock, Gordon, "Efficient Rent Seeking", Western Economic Journal, 1967; reprinted in Buchanan, James M., Robert D. Tollison, and Gordon Tullock, ed., Toward a Theory of the Rent-Seeking Society, College Station, Texas: Texas A&M Press, 1980, pp. 97-112.