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The Information Content of Prices in Derivative Security Markets

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Abstract

Prices in futures markets and option markets reflect expectations about future price movements in spot markets, but these prices can also be influenced by risk premia. Futures and forward prices are sometimes interpreted as market expectations for future spot prices, and option prices are used to calculate the market's expectations for future volatility of spot prices. Do these prices accurately reflect market expectations? The purpose of this paper is to examine the information that is reflected in futures prices and option prices. The issue is examined by reviewing both the relevant analytical models and the empirical evidence.

JEL Classification Numbers:
E44, G12, G14

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Summary

The relationship between expectations and prices in futures and options markets should be interpreted carefully. Futures and forward prices are prices for future delivery, and these markets make it possible for individuals to hedge price risk. When individuals use futures and forward markets to hedge, they really transfer the price risk to someone else, and there should be some form of compensation for those who absorb the risk. As a result, risk premia are built into the prices so that the price is a combination of the expected future price and a risk premium. This paper first demonstrates by arbitrage methods that futures and forward prices should be functions of current spot prices and interest rates. Any direct connection between these prices and expected future spot prices is purely coincidental. Futures and forward prices are, however, affected by expectations through the current spot price, which is determined by expectations. Several examples of actual prices for stock index futures, Treasury bond futures, and forward foreign exchange are examined, and in all cases the prices are very close to the prices predicted by the arbitrage models. Prices in futures and forward markets reveal no additional information on market expectations that is not already revealed in spot prices. In foreign exchange markets, forward rates are very poor predictors of future changes in the exchange rates.

It is possible that implied volatilities computed from option prices may reflect market expectations of future volatility in the spot market. The popular model for computing implied volatilities is the Black-Scholes model, and the paper demonstrates that this model with expected volatility can be interpreted as a first-order approximation for a more complex model that allows the volatility to change randomly. Risk premia may also influence the implied volatilities computed from option prices: the correct first-order approximation is the Black-Scholes model with expected volatility computed form the risk-adjusted volatility process.

Previous empirical studies of implied volatilities are also reviewed, and some new evidence for foreign exchange rates is presented. The results of the empirical studies suggest that implied volatilities are useful for forecasting future volatility, but implied volatilities alone are not optimal predictors. A combination of implied volatilities, past volatilities, and the market factor in volatility appear to be useful in forecasting future volatility. The empirical analysis supports the notion of a volatility-risk premium, but not one that is large enough to break completely the linkage between volatilities in option prices and expectations of future volatility in the spot market.

I. Introduction

Derivative security markets have experienced phenomenal growth in recent years. A wide variety of options and futures contracts are traded on stock indexes, bonds, interest rates, foreign currencies, gold, oil, and numerous commodities. An important issue for financial economists and market analysts is the information content of these prices. Prices on traditional assets, like stocks and bonds, are determined by discounting expected future cashflows and the prices reflect expectations about future events that may effect the underlying cashflows. What kind of information is reflected in the prices of derivative securities. 1/ Futures and forward prices are prices for future delivery of some specified asset. Do these prices reflect expectations of future prices on the asset? Option prices depend on future prices and the potential variability of those prices. Do the option prices reflect expectations of the price and its potential volatility?

In this paper I examine the information content of prices in these derivative security markets. Section II of the paper is devoted to futures and forward prices, and section III is devoted to option prices. In both sections, I examine how arbitrage, expectations, and risk premia influence these market prices. The theoretical analysis is balanced with a review of the relevant empirical research and a presentation of some new empirical results. Because volatility of the underlying asset price plays an important role in the pricing of options, I examine the behavior of implied volatilities which traders and market analysts compute from option prices.

In section II, I argue that the arbitrage relationship is so strong that futures and forward prices are determined primarily as functions of spot prices and interest rates. From this perspective, the expectations reflected in futures or forward prices are the same expectations that are reflected in the spot asset prices. One cannot infer expected future spot prices from futures and forward prices without measuring the relevant risk premia. In section III, I show that there is a relationship between implied volatilities from the options market and expectations of future volatility, but this linkage is weakened by the presence of risk premia associated with volatility. There is some empirical evidence that implied volatilities are useful for forecasting future volatility. By contrast, futures and forward prices, particularly forward foreign exchange rates, do not seem to be useful as predictors of future spot prices.

1/ These futures and options are sometimes called derivative instruments because their payoffs are derived from asset prices or economic variables.

II. Futures and Forward Markets

Prices in futures and forward markets are prices for the future delivery of an asset or a commodity. There are active markets in futures contracts on foreign currencies, stock indexes, long term bonds, interest rates, gold, oil, and numerous commodities. There is a large forward market in foreign currencies, and interest rate swaps and currency swaps are essentially long term forward contracts. Futures and forward contracts are similar, but a few differences are worth mentioning. Futures contracts are standardized and trade on organized exchanges. The delivery dates and terms are set by the exchanges. Forward contracts are primarily negotiated through over the counter markets and the dates and terms of delivery can be set to meet the needs of the customer. The principal markets are operated through the trading rooms of large banks and investment houses. From a pricing perspective, the principal difference between futures and forwards is the timing of the cashflows. Futures contracts are settled each day by the exchanges so that short positions and long positions experience daily cashflows. Forward contracts do not experience any cashflows until the delivery date when the asset or commodity is delivered for cash. In both cases, margin or collateral may be required when the contracts are initiated. It is possible for futures and forward prices on otherwise identical contracts to differ because of the difference in the timing of the cashflows. 1/ If there is a price difference, the potential arbitrage is to buy, or go long, at the lower price and sell, or go short, at the higher price. The apparent arbitrage profit is the price difference, but there is some risk due to the timing of the cashflows. If the futures position experiences early losses there is an initial cash outflow for the arbitrageur and this loss is not offset by the profit on the forward position until the delivery date. The arbitrageur must be able to finance the potential losses on the daily settlement of the futures and this risk is sometimes referred to as interest rate risk. For short term contracts, delivery dates less than a year away, these risks are small and the pricing differences are negligible. The pricing differences could be economically significant on long term contracts, but most of the futures and forward contracts in active markets are short term. 2/ For this reason, I will follow the usual practice of equating futures and forward prices.

1. The Determination of Futures and Forward Prices

The most popular pricing model for futures and forward contracts is the cost of carry model: the forward price is equal to the spot price plus the cost of carry or storage. This model is based on simple arbitrage. If the forward price is too high, the arbitrageur buys the asset or commodity on the spot market and sells forward. The forward price must be just high

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1/ For a careful analysis of these differences, see Cox, Ingersoll, and Ross (1981).

2/ The notable exceptions are the longer term Eurodollar futures and swaps.

enough to offset the storage costs which must be incurred while the arbitrageur waits for delivery. If the forward price is too low, then an arbitrageur who holds the asset or commodity in inventory can sell on the spot market, buy forward, and avoid the carrying costs over the life of the forward contract. This approach to pricing futures and forward contracts has been used by traders for many years and was used by Holbrook Working in his analysis of futures markets over 40 years ago. 1/ The cost of carry model is easy to apply in financial markets because the transportation and transactions costs are small and the cost of carry or storage is simply the interest rate or the opportunity cost on cash that is used to purchase the asset on the spot market. An adjustment must be made for potential cashflows, dividends or interest, on the asset. For futures contracts on stock indexes or stock portfolios the model is

$$F = S(1+R) - D = S(1+R-\frac{D}{S}) \quad , \tag{1}$$

where F is the forward price, S is the spot index or the spot value of the portfolio, R is the risk free interest rate, and D is the dividend to be received from holding the stocks in the index or portfolio. For forward contracts on bonds we replace the dividend yield with the coupon yield:

$$F = S(1+R-\frac{C}{S}) \quad . \tag{2}$$

The relationship between forward prices and spot prices for long term bonds is determined by the relationship between short term interest rates, R, and long term yields, C/S. If the yield curve is upward sloping the forward price of the bond is lower. If the yield curve is downward sloping, then the forward price is higher. When the cost of carry model is applied to forward foreign exchange rates, one must account for the foreign interest rate which is paid on short term positions in the foreign currency. The resulting model is the familiar covered interest rate parity condition:

$$F = S\frac{(1+R_d)}{(1+R_f)} \quad , \tag{3}$$

where the subscripts d and f are used to distinguish the domestic and foreign interest rates. Here the forward and spot exchange rates are expressed as the domestic currency price of the foreign currency, the ratio of domestic currency to foreign currency. 2/

This model for futures and forward prices is based on arbitrage. If prices deviate from the model relationship, then arbitrage opportunities become available, at least for traders who transact at low transactions costs. 3/ For this reason, the cost of carry model provides an accurate description of prices in these markets. The price differences that are observed tend to

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1/ See Working (1948, 1949)

2/ This same convention will be followed in the discussion of foreign currency options.

3/ Examples include banks, investment houses, and futures trading firms.

be quite small. Arbitrage models perform well empirically because they do not rely on restrictive assumptions about preferences of economic agents or the structure of the economy. Only two assumptions are required: avarice and free trading among individuals. Evidence on the empirical accuracy of the cost of carry will be presented below in section III.2. I turn now to the relationship between futures and forward prices and expectations of future spot prices.

Do futures and forward prices serve as unbiased or optimal predictors of future spot prices? Can these prices be used to infer information about market expectations? The general answer is no; forward prices reveal nothing more about expectations of the future than what is already revealed in spot prices and interest rates. 1/ An alternative model for futures and forward prices is the one in which these prices equal the market's expectation for the spot price at delivery:

$$F_t(t+s) = E(S_{t+s}|I_t) = E_t(S_{t+s}) \quad , \tag{4}$$

where E_t is a conditional expectation and I_t is the information set used by the market at time t. If expectations are rational and the market uses all available information to forecast spot prices, then this model implies (1) that futures and forward prices are unbiased predictors of future spot prices and (2) that futures prices are martingales. The martingale property, $F_t = E_t(F_{t+k})$, implies that changes in futures prices should be unpredictable. Both of these implications have been tested extensively in the efficient markets literature and the results are mixed. Two observations are important, First, even if this simple expectations model is correct, the arbitrage relationship must also be satisfied in an efficient market. Second, a variety of assumptions, including risk neutrality, are necessary to derive this expectations result in an equilibrium asset pricing model.

An auxiliary approach is to allow for a risk premium in the forward price:

$$F_t(t+s) = E_t(S_{t+s}) + RP_t \quad , \tag{5}$$

and consider plausible models for the risk premium. One can then infer the market's expectation by removing the risk premium from the forward price. This exercise has limited usefulness because the structure of the risk premium may be quite complicated and it should be whatever is necessary to move from forward prices, which are determined by arbitrage, to the expected future spot price. When a time varying risk premium is present, the forward price is no longer an optimal predictor of future spot prices and other information variables will be useful in predicting future spot prices.

1/ This position and the related arguments are essentially contained in the writings of Holbrook Working (1949a,b).

There is, however, a connection between forward prices and expectations even in models with risk aversion.

This point is demonstrated by using the Cox, Ingersoll, Ross (1981, 1985a) model, hereafter the CIR model, for pricing futures and forwards in a continuous time equilibrium model. CIR price futures and forwards by using arbitrage to convert futures and forward prices into prices of assets and then they apply a continuous time valuation model. 1/ Let $B_t(t+s)$ be the price of a default-free discount bond that pays $1 at time t+s, and r_t be the instantaneous risk free interest rate. They show that the forward price, now $F_t(t+s)$, must equal the value of an asset that will pay at delivery the spot price, S_{t+s}, divided by $B_t(t+s)$. The futures price, now $f_t(t+s)$, should equal the value of an asset that will pay at delivery the following cashflow:

$$S_{t+s} \exp\left\{\int_t^{t+s} r(u)\,du\right\} \quad . \tag{6}$$

In this continuous time model, the value of an asset that has a single cashflow is equal to the risk adjusted expectation of the cashflow discounted as follows:

$$V_t = \hat{E}_t\left(\exp\{-\int_t^{t+s} r(u)\,du\}\, C_{t+s}\right) \quad . \tag{7}$$

The risk adjusted expectation is determined by first performing a risk adjustment on the state variables that determine r and C, and then taking the expectation. Let $Y = \{y_i\}$ represent the relevant state variables, which are assumed to be diffusion processes. The risk adjustment is accomplished by reducing the mean parameter of each state variable by its risk premium:

$$dy_i = [\mu_i(Y) - \lambda_i(Y)]\,dt + \sigma_i(Y)\,dz_i \quad , \tag{8}$$

where dz_i is a Wiener process, and μ_i and σ_i represent the instantaneous mean and variance. The risk premia are determined by the covariability of the state variable with the marginal utility of wealth. The model for the forward price becomes:

1/ No distinction is made here between real prices and nominal prices. CIR (1985b), at the end of their term structure paper, show that the important results of their valuation model also work if one uses nominal cashflows and nominal interest rates to determine nominal asset prices. The results for futures and forward prices follow from propositions 1 and 2 in CIR (1981).

$$F_t(t+s) = \hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du\}\left(\frac{S_{t+s}}{B_t(t+s)}\right)\right) = \frac{\hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du\}\, S_{t+s}\right)}{\hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du\}\right)} \quad . \tag{9}$$

The bond price is known at time t and it is also an asset price:

$$B_t(t+s) = \hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du\}\ \$1\right) \quad . \tag{10}$$

The model for the futures price is

$$f_t(t+s) = \hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du + \int_t^{t+s} r\,du\}\, S_{t+s}\right) = \hat{E}_t(S_{t+s}) \quad . \tag{11}$$

As was previously noted, short term futures and forward contracts have prices that are roughly equal. If the spot price is uncorrelated with interest rates in this model, then we have the result that the forward price equals the futures price.

The important observation here is that the futures price is the risk adjusted expectation of the spot price, not the actual expectation of the spot price. Now consider the spot price of an asset that is traded in financial markets. One investment strategy is to buy the asset with the intention of selling it at the delivery date, t+s. If the asset pays no dividend or interest, then the current price is also determined by the valuation model:

$$S_t = \hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du\}\, S_{t+s}\right) \quad . \tag{12}$$

Now compare this to the forward price. If we divide this expression for the current spot price by the price of the discount bond, we get the forward price, $F_t(t+s) = S_t/B_t(t+s)$. The bond price is just the reciprocal of one plus the interest rate so that $F_t(t+s) = S_t(1+R)$, which is the cost of carry, or arbitrage, model for the forward price with no dividends or interest. The trivial result here is that the asset pricing model is consistent with the results of arbitrage-free pricing. The important observation, however, is that the expectations reflected in the forward price are exactly the same expectations reflected in the current spot price. If the covariability between interest rates and the spot price is ignored, the same statement applies to the futures price. The expectations reflected in forward prices are the same expectations reflected in the spot price and the forward price is not necessarily the market's expectation of the future spot price.

There is one special case worth considering. Suppose that the underlying spot price is uncorrelated with the marginal utility of wealth. The risk premium for the spot price is zero and the futures price is equal to the expectation of the spot price. If the covariability between the spot price and interest rates is zero, then the forward price is also equal to the expectation of the spot price. The spot price is equal to the expected value of the future spot price discounted at the risk-free rate:

$$S_t = \hat{E}_t\left(\exp\{-\int_t^{t+s} r\,du\}\ S_{t+s}\right) = \hat{E}_t\left(\exp\{-\int_t^{t+s}\}\right)\ \hat{E}_t(S_{t+s}) \tag{13}$$

$$= B_t(t+s)\ \hat{E}_t(S_{t+s}) = \frac{\hat{E}_t(S_{t+s})}{(1+R)} \quad . \tag{14}$$

In this special case, market expectations can be inferred from futures and forward prices, but they can also be inferred directly from the spot price. Even in this special case, the futures and forward prices do not provide any additional information on market expectations beyond what is available in the spot price.

What are futures and forward prices, if they are not market expectations of future spot prices? Futures and forward prices are prices for future delivery that permit individuals to transfer price risk. It is natural that these prices contain risk premium, so that those individuals who are willing to bear the price risk are appropriately compensated. In financial markets these prices can be easily determined by arbitrage relationships which are based on current spot prices and interest rates. The connection between these prices and actual market expectations of future spot prices is purely coincidental. To infer actual market expectations from these prices, one would need a careful analysis of the risk premium which is based on the behavior of the spot price.

2. Some Empirical Evidence on Pricing in Futures and Forward Markets

In this section II present a few empirical observations on the pricing of futures and forward contracts. The empirical literature on futures and forward pricing is quite voluminous, and no attempt will be made to either survey this literature or to present a complete empirical study. I begin with some applications of the arbitrage models for stock index futures, bond futures, and forward foreign exchange. In Table 1, I present calculations for the futures contract on the Major Market Index (MMI); the MMI is a stock index of twenty large U.S. companies that is designed to mimic the movements of the Dow Jones Industrial Average. This contract has been chosen because

Table 1. Cost of Carry Model, MMI Stock Index Futures
July 22, 1991

MMI Spot Price = 633.71

Delivery Date	Interest Rate	Annual Interest Rate	Dividend	Theoretical Futures Price	Actual Futures Price
	(In percent)				
August	.3762%	5.49%	2.360	633.73	633.75
September	.9264	5.64	4.367	635.21	635.15

NOTES: The short term interest rates are computed from T-Bills with maturities closest to the delivery date. The dividends on the MMI are from CBOT Financial Update, June 21, 1991.

it is easier to construct the dividends needed to calculate theoretical futures prices for the arbitrage model. 1/ For the day chosen, the theoretical futures prices are extremely close to the actual futures prices. Investment firms perform these calculations on a daily basis and trade whenever the price differences are large enough to generate profits over the transactions costs. Because the arbitrage is simple, and many investment firms stand ready to take positions, it should be no surprise that the arbitrage model is very accurate.

A more complicated example is the pricing of the Treasury bond futures contract traded at the Chicago Board of Trade. There are typically more than twenty long term Treasury bonds available for delivery on this con-

1/ The dividends are reported in the CBOT Financial Update. To calculate the arbitrage model for the S&P 500 futures contracts, one must collect the dividends on the 500 stocks in the index.

tract, and the short position has the option to deliver his or her choice of bonds during the delivery period. To price this futures contract, one must apply the arbitrage model to all of the deliverable bonds and determine the cheapest bond to deliver. To adjust for different coupon rates, the exchange assigns a conversion factor to each bond: the price that a short receives on a delivered bond is the futures settlement price for the day multiplied by the conversion factor for the bond. 1/ We divide the spot price by the conversion factors to restate all of the prices in terms of the futures price. In Table 2, I present the calculations for the September Treasury bond futures contract as of July 22, 1991. The bond that produces the lowest theoretical futures price is the cheapest bond to deliver; in this case it is the November 2016 bond. All of the theoretical bond prices are higher than the futures price, a pattern which is typical. The lower futures price reflects the value of several options available to the short during the delivery period, but the arbitrage model does produce a theoretical futures price that is close to the actual futures price if we use the cheapest bond to deliver.

For the last example on the accuracy of arbitrage models, I use a data set of foreign exchange rates and interest rates that includes spot exchange rates, 3 month forward rates, and 3 month Eurocurrency interest rates for the two countries. The exchange rates are the U.S. dollar with the British pound, the German mark, the Japanese yen, and the Swiss franc. The time period covered is roughly 1983 to 1986 and the observations are weekly, every Thursday. The difference between the theoretical forward rate and the quoted forward rate was greater than 0.1 percent for only 8 of the 706 weekly observations. Most of the differences were less than 0.05 percent. Another way to measure this difference is to compute the interest rate that domestic investors could earn by engaging in the covered interest rate arbitrage:

$$R_d^* = \frac{F}{S}(1+R_f) \quad , \tag{15}$$

which follows by simply rearranging the covered interest rate parity equation. I calculated the difference between the annualized rates for R_d and R_d^*, and I found that the absolute deviation averaged 13 basis points for the British pound, 10 basis points for the German mark, 10 basis points for the Japanese yen, and 8 basis points for the Swiss franc. 2/ These differences are quite small, and the results imply that the arbitrage model provides an accurate description of the determination of forward rates. Whenever there is sufficient trading activity, sufficient liquidity in the market, the arbitrage model will provide an accurate description of futures and forward price determination. In the active foreign exchange markets, the forward rate is essentially a function of the spot rate and the two interest rates. The difference between the forward rate and the spot rate

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1/ The buyer must also pay accrued interest at the time of delivery, but this extra charge can be ignored in the calculation of the arbitrage model.
2/ One basis point is equal to .01%.

Table 2. Cost of Carry Model, Treasury Bond Futures
July 22, 1991

September Futures Price = 93.625
Interest Rate = 0.6577% (5.58% per annum)

Maturity	Coupon Rate	Conversion Factor	Ask Price	Theoretical Futures Price	Price Difference
Nov 2006	14.000	1.5188	145.7500	95.5116	-1.8866
Nov 2007	10.375	1.2122	115.2500	94.6883	-1.0633
Aug 2008	12.000	1.3653	129.9375	94.7604	-1.1354
May 2009	13.250	1.4899	141.8438	94.7790	-1.1540
Aug 2009	12.500	1.4224	135.3125	94.7190	-1.0940
Nov 2009	11.750	1.3545	128.8438	94.7242	-1.0992
Feb 2015	11.250	1.3404	126.8438	94.2650	-0.6400
Aug 2015	10.625	1.2769	120.7188	94.1793	-0.5543
Nov 2015	9.875	1.1987	113.1875	94.0753	-0.4503
Feb 2016	9.250	1.1327	106.8750	94.0123	-0.3873
May 2016	7.250	0.9200	86.6563	93.8858	-0.2608
Nov 2016	7.500	0.9463	89.0625	93.8024	-0.1774*
May 2017	8.750	1.0811	101.9688	93.9892	-0.3642
Aug 2017	8.875	1.0946	103.1875	93.9304	-0.3054
May 2018	9.125	1.1230	105.9688	94.0227	-0.3977
Nov 2018	9.000	1.1100	104.7188	94.0094	-0.3844
Feb 2019	8.875	1.0963	103.4688	94.0506	-0.4256
Aug 2019	8.125	1.0137	95.6563	94.0441	-0.4191
Feb 2020	8.500	1.0555	99.7188	94.1501	-0.5251
May 2020	8.750	1.0837	102.5630	94.3101	-0.6851
Aug 2020	8.750	1.0837	102.5310	94.2827	-0.6577
Feb 2021	7.875	0.9858	93.3130	94.3421	-0.7171
May 2021	8.125	1.0141	96.2500	94.5939	-0.9689

* - cheapest to deliver bond

Theoretical future price = (S/CF)(1+R-C/S)

NOTE: The short may deliver any bond, and a different bond may become cheaper to deliver. During the delivery period the futures settlement price is set when the futures market closes, but the spot market remains open for several more hours. If the spot prices drop after the futures market closes, the short can buy the cheapest to deliver bond, send a delivery notice to the exchange, and sell at the futures settlement price.

is determined by the interest rate differential, and there is no special role for expectations of futures spot rates. Expectations of future spot rates are, of course, important in the determination of current spot rates.

Numerous empirical studies have examined tests of forward rates as predictors of future spot exchange rates. Numerous tests of the predictability of futures price changes have also been executed in the efficient markets literature. The hypothesis that forward rates are optimal predictors of future spot rates in foreign exchange markets is frequently rejected in these studies. For a discussion of these results, see the papers by Hansen and Hodrick (1980) and Fama (1984) and the review of the literature contained in Hodrick (1987). Because the forward rates are determined by the interest rate differentials, the results imply that interest rate differentials are poor predictors of future changes in spot exchange rates, at least over the short time horizons used in the empirical studies. These results are evidence of the importance of risk premia in the forward rates. To reconfirm the results of these previous studies, I repeated the tests on a data set of foreign exchange rates for a recent period, 1983-89. The time series are weekly observations on the spot rate and the 90 day forward rate, and I ran the following regression:

$$\ln\left(\frac{S_{t+k}}{S_t}\right) = a + b \ln\left(\frac{F_t(t+k)}{S_t}\right) + e_t \quad , \tag{16}$$

where k is 13 weeks, or roughly 90 days.

Because the time intervals for the forecast errors overlap, there is serial correlation in the error term. I estimate the regression with ordinary least squares, which is consistent, and I use the techniques described in Hansen (1982) and Hansen and Hodrick (1980) to account for the serial correlation in the forecast errors. 1/ The results, similar to those obtained in previous studies, are summarized in Table 3. The coefficient on the forward rate should equal one if the forward rate is an unbiased predictor, but the coefficients are negative and statistically significant.

1/ I use a spectral estimator for the variance of the parameter estimates. Let X be the T×2 matrix of observations on the two right hand side variables and let $\mathbf{x}_t$ be the vector of observations at time t. The variance matrix for the parameter estimates is $T(X'X)^{-1}f(X'X)^{-1}$, where f is 2π times the spectral density matrix of $(\mathbf{x}_t e_t)$ evaluated at the zero frequency. To estimate the spectral density, I prewhiten the series first, and then I use a smoothed periodogram estimator with a flat window. The last step is to recolor the estimate by the appropriate filter. For a description of this estimator, see Nerlove, Grether, and Carvalho (1979). Generalized least squares is not used because it is not consistent in this application.

The hypothesis that forward rates are unbiased predictors is easily rejected by the data. The regression results suggest an alternative view: an increase in the domestic interest rate relative to the foreign interest rate predicts that the exchange rate will drop and the domestic currency will increase in value over the next 3 months. Results of this kind do not imply systematic expectation errors in the forward markets, but they do suggest a serious specification error in models that assert that expected changes in the exchange rate are a simple function of the interest rate differential.

Another example of the errors that arise when simple expectation models are applied to futures and forward prices can be found in the case of stock index futures. It is generally accepted that a risk premium can be earned by holding a large portfolio of common stocks. Here the risk premium is the difference in the expected return on the portfolio and the risk-free interest rate. Estimates for the risk premium on the S&P 500 portfolio have varied from 5 percent to 9 percent on an annual basis. Now assume for the moment that the S&P 500 futures price is equal to the expected spot price at delivery. If this were true, then an investor could buy the S&P 500 portfolio, sell the futures contract, and capture the risk premium on the S&P 500 without incurring the risk associated with holding the risky portfolio. An inconsistency exists. We have allowed a risk premium for holding the stock portfolio, but no risk premium in the futures price. The arbitrage portfolio should earn the risk-free rate; for this to occur, the futures price must be less than the expected spot price. The resulting risk premium in the futures price is just a mirror image of the risk premium for holding the risky stock portfolio.

III. Option Markets

Option contracts in financial markets are options to buy or sell securities at fixed prices. There are actively traded option contracts on stocks, stock indexes, bonds, foreign currencies, financial futures, and specific interest rates. In addition to the option markets in Chicago, which once dominated option trading, there are active markets located in New York, London, Paris, Frankfurt, Tokyo, and Singapore. Options are different from futures and forward contracts because the holder of an option has the right to buy or sell an asset at a fixed price, but the holder may elect not to carry out the transaction. If underlying asset prices, spot prices, move against the holder of the option, he or she can allow the option to expire and the loss is simply the original premium paid for the option. Options, like futures and forward contracts, can be used to hedge price or interest rate risk, but the hedge with options is like purchasing insurance. In this section, I examine the information content of option prices. Specifically, do option prices provide additional information about future volatility in financial markets?

1. Option Pricing and Implied Volatilities

Option prices are determined by several important factors: the spot price or the price of the asset on which the option is written, the exercise or strike price, the time to maturity of the option, and the potential volatility of the spot price. In many cases, interest rates also effect option prices, but the impact of interest rate changes tends to be small. Some options can be exercised prior to the expiration date, and these are called American options. Other options can be exercised only on the expiration date, and these are called European options. This distinction can have an effect on the option value. All of these elements that influence option values are easily observable, except for the volatility of the spot price. Because option prices typically move up and down with spot price volatility, the option prices reflect the market's expectations for future volatility.

Option traders and market analysts use mathematical models to value options and one of the important parameters in these models is volatility. The most popular model is the Black-Scholes (1973) model for valuing call options on stocks:

$$C(S,t) = S\,N(d_1) - e^{-r(T-t)}K\,N(d_2) \tag{17}$$

$$N(d) = \int_{-\infty}^{d} \frac{\exp(-\tfrac{1}{2}x^2)}{\sqrt{2\pi}}\,dx \tag{18}$$

$$d_1 = \frac{\ln(S/K) + (r + \tfrac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2 = d_1 - \sigma\sqrt{T-t} \tag{19}$$

N(d) is the standard normal distribution function. S is the current stock price, K is the exercise price, r is the instantaneous interest rate, (T-t) is the time to maturity, and σ is the standard deviation, or volatility, of the stock price. 1/ Option models are frequently derived by using arbitrage methods, but the models rely on dynamic trading strategies in continuous time, and all of the models are dependent on the assumptions made for changes in the stock price. The Black-Scholes model is based on the following diffusion process for stock price changes:

1/ European put options can be valued by using the following relationship known as put-call parity: Call - Put = S - $e^{-r(T-t)}K$.

Table 3. Forward Rates and Future Spot Rates in the Foreign Exchange Market

$$\ln(S_{t+k}/S_t) = a + b \ln(F_t(t+k)/S_t) + e_t$$

	a	b	$\chi^2(2)$
British Pound - $	-.03600	-7.7050	19.47
	(.01105)	(2.0333)	
German Mark - $	.1235	-12.9289	33.39
	(.0304)	(2.6885)	
Japanese Yen - $	.1064	-10.9962	17.37
	(.0264)	(2.8800)	
Swiss Franc - $	.1148	-9.9866	36.40
	(.0273)	(2.0116)	

Sample Sizes = 316   Sample Period: March 1983 to June 1989

NOTES: The time series are weekly. The standard errors, shown in parentheses, have been calculated to allow for serial correlation and conditional heteroskedasticity in the error term. The $\chi^2(2)$ statistic is the test statistic for the joint test that a=0 and b=1. k = 13 weeks (90 days).

$$dS = \mu S dt + \sigma S dz \quad . \tag{20}$$

In this model stock prices and stock returns have lognormal distributions. If one changes the distribution for the stock price, then a new option pricing model must be derived. Other models for option prices have been developed, but the Black-Scholes model remains popular because it is easy to use.

The model can be extended to value other types of options. To value stock index options, one simply replaces the stock price with the index. When pricing options on indexes and options on stocks that pay dividends, one should make an adjustment for dividend payments. For stock index options, the typical adjustment is to replace the stock price, S, with $e^{-\delta(T-t)}S$, where δ is the continuous dividend yield. For a stock with discrete dividend payments, the adjustment is made by subtracting from the stock price the present value of the dividends that will be paid before the expiration of the option. 1/ To value foreign currency options the model is extended as follows:

$$C(S,t) = e^{-r_f(T-t)} S\, N(d_1) - e^{-r_d(T-t)} K\, N(d_2) \tag{21}$$

$$d_1 = \frac{\ln(S/K) + (r_d - r_f + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad , \qquad d_2 = d_1 - \sigma\sqrt{T-t} \tag{22}$$

S now represents the exchange rate, and the two interest rates, r_d and r_f, are assumed to be fixed. 2/ The model applied to futures options is frequently called Black's model:

$$C(f,t) = e^{-r(T-t)} \left(f\, N(d_1) - K\, N(d_2) \right) \tag{23}$$

$$d_1 = \frac{\ln(f/K) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \quad , \qquad d_2 = d_1 - \sigma\sqrt{T-t} \tag{24}$$

1/ For American call options, one needs to also consider the effects of early exercise.

2/ A similar model can be derived if one allows interest rates to vary, but assumes that the interest rate differential remains fixed.

Most of the bond and interest rate options are futures options and these options are typically valued by using Black's model even though the model assumes that interest rates are fixed. 1/

The option traders use these models by imputing their forecasts for future volatility. From this perspective the option prices should reflect the market's expectation of future volatility in the spot price. Market analysts use these models to infer implied volatilities from option prices: the σ for volatility is adjusted so that the model price matches the option price quoted in the market. The standard practice now is to use several at-the-money options, options with exercise prices closest to the current price, to calculate implied volatilities, and to allow for different volatilities across different maturities. Some analysts have referred to the differences in volatilities across maturities as the term structure of volatility. If market volatility is currently low and traders expect it to rise in the future, then one should observe an upward sloping term structure of volatility. If market volatility is unusually high and traders expect it to drop in the future, then the term structure will be downward sloping.

2. Random Variance Option Pricing and the Behavior of Implied Volatilities

The common practice of using the Black-Scholes option pricing models to infer values of the volatility parameter and then allowing it to vary from one day to the next would appear to be logically inconsistent. The option pricing models discussed in the previous section are based on the assumption that volatility is fixed. If volatility changes randomly, then one must derive a new option pricing model. Random variance option pricing models have been developed by Scott (1987), Hull and White (1987), and Wiggins (1987). The models do not produce closed form solutions for option prices, but the analysis in Scott and Hull and White can be used to examine the potential behavior of implied volatilities from the Black-Scholes model.

The random variance models consider a second diffusion process for volatility so that it becomes a random variable. The diffusion equations are now:

$$dS = \mu_1 S dt + \sigma S dz_1 \tag{25}$$

$$d\sigma^2 = \mu_2(\sigma^2)\, dt + \gamma(\sigma^2)\, dz_2 \quad . \tag{26}$$

One cannot use arbitrage methods alone to derive unique option pricing functions in this revised model. It is necessary to appeal to an equilibrium asset pricing model like the CIR (1985a) model. The solution for a European call option on a stock that pays no dividends has the following form:

1/ The models work by allowing for variability in the bond futures price or the futures interest rate.

$$C(S_t, \sigma_t^2, t) = \hat{E}_t\{e^{-r(T-t)}\max[0, S_T - K]\} \quad , \tag{27}$$

where the risk adjusted expectation is taken with respect to the following system of diffusion equations:

$$dS = rSdt + \sigma S dz_1 \tag{28}$$

$$d\sigma^2 = [\mu_2(\sigma^2) - \lambda(\sigma^2)]dt + \gamma(\sigma^2)dz_2 \quad . \tag{29}$$

Numerical techniques, like Monte Carlo simulation or the finite difference method, can be used to compute prices in this model.

Scott and Hull and White have shown that the option pricing problem can be simplified if dz_1 and dz_2 are uncorrelated. The result is

$$C(S_t, \sigma_t^2, t) + \int_0^\infty \{S_t N(d_1) - e^{-r(T-t)} K N(d_2)\} dF(V; \sigma_t^2, t) \quad , \tag{30}$$

where

$$d_1 = \frac{\ln(S_t/K) + r(T-t) + \frac{1}{2}V}{\sqrt{V}} \tag{31}$$

$$d_2 = d_1 - \sqrt{V} \quad \text{and} \quad V = \int_t^T \sigma^2(u)\,du \quad . \tag{32}$$

$F(V;\sigma_t^2,t)$ is the distribution function for V, which is the volatility of the stock price over the life of the option. The integral is the expectation of the Black-Scholes solution with the random variable V in place of $\sigma^2(T-t)$. One can value European calls in this model by simulating V in a Monte Carlo simulation; it is not necessary to simulate the stock price process and a substantial reduction in computing time is achieved. One can also develop analytic approximations for this model. Let

$$C^*(S_t, V, t) = S_t N(d_1) - e^{-r(T-t)} K N(d_2) \quad . \tag{33}$$

Now do a Taylor series expansion about the point $V = \hat{E}_t(V)$.

$$\hat{E}_t[C^*(S_t,V,t)] \approx C^*(S_t,\hat{E}_t(V),t) + \tfrac{1}{2}\frac{\partial^2 C^*(S_t,\hat{E}_t(V),t)}{\partial V^2}\hat{E}_t(V-\hat{E}_t(V))^2 + \cdots \quad (34)$$

$\hat{E}_t(V-\hat{E}_t(V))^2$ is the variance of the volatility over the life of the option and the omitted terms in the expansion involve higher moments and derivatives. The first term of the approximation, $C^*(S_t,\hat{E}_t(V),t)$, is the Black-Scholes model with expected volatility in place of $\sigma^2(T-t)$, the form of the Black-Scholes model that is typically used. In other words, the Black-Scholes model is a first order approximation for a random variance option pricing model.

The approximation model can be used to examine several issues. First, is the Black-Scholes model with expected volatility a good approximation? Second, what is the relationship between expected volatility in the model and actual volatility? The first question can be answered by simulating the random variance model and comparing the option values with the Black-Scholes approximation. In Tables IV and V, I present simulation results for two cases: a low volatility stock and a high volatility stock. The following diffusion process is used for volatility:

$$d\sigma^2 = \kappa(\theta - \sigma^2)\,dt + \gamma\sigma\,dz_2 \quad , \quad (35)$$

and for now I assume that there is no volatility risk premium. The parameters for the low volatility stock have been set to approximate sample second and fourth moments for the S&P 500; the mean reversion parameter, κ, has been set so that the mean half life for volatility shocks is six months, which is close to the values estimated by Poterba and Summers (1986). The values are $\theta = .0324 = (.18)^2$, $\kappa = 1.3863$, and $\gamma = .22$, and r is set at 8 percent. The parameters for the high volatility stock have been set to approximate sample moments for a volatile stock, National Semiconductor. The parameters values are $\theta = 0.2976 = (0.5455)^2$, $\kappa = 1.94$, and $\gamma = 0.8956$. The stock price is set at \$50, the strike prices range from \$45 to \$60, and the maturities are three months, six months, and nine months. The two tables include the Black-Scholes approximation, the second order random variance approximation, and the Monte Carlo solution. The random variance approximation is very close to the Monte Carlo solution: the largest pricing errors are \$0.01 in Table 4 and \$0.05 in Table 5. The Black-Scholes approximation is reasonably accurate, but the pricing errors are larger: the largest pricing error in Table 4 is \$0.06, and the largest pricing error in Table 5 is \$0.29. The approximation errors for the Black-Scholes are small percentages of the correct random variance price, and the implied volatilities computed from the Black-Scholes model should provide reasonably accurate approximations for the expected volatility under the risk adjusted volatility process.

In the random variance model, the expectations are risk adjusted expectations. The model uses the risk free interest rate in place of the expected return on the stock and there should be a risk adjustment on the volatility process. Consider the following risk adjusted process for volatility:

$$d\sigma^2 = (\kappa\theta - \kappa\sigma^2 - \lambda\sigma^2)\, dt + \gamma\sigma\, dz_2 \quad . \tag{36}$$

When the first order, Black-Scholes, approximation is reasonably accurate, the implied volatility computed from the option prices is the risk adjusted expectation of volatility over the life of the option. The implied σ^2 is approximately equal to $\hat{E}_t(V)/(T-t)$, the average expected volatility. If the risk premium is zero, then the implied volatility should be an unbiased predictor of future volatility, or at least a close approximation. If the volatility risk premium is significant, then the implied volatility will not be an unbiased predictor of future volatility.

The approximation model can be used to explain the term structure of volatility that is sometimes used by market analysts. If volatility is currently high relative to the long run average, $\sigma_t^2 > \theta$, then volatility is expected to decline and we should observe a downward sloping term structure for volatility. Implied volatilities on longer term options should be lower than implied volatilities on shorter term options. If volatility is low, $\sigma_t^2 > \theta$, then the results are reversed and we should observe an upward sloping in this term structure. The conventional wisdom suggests that volatility risk premia should be negative for stocks . Increases in volatility tend to be associated with decreases in the stock market. The negative correlation between volatility and returns on aggregate stock portfolios suggest a negative risk premium. If λ in the model above is negative, there is a slower rate of mean reversion under the risk adjusted process. If $\lambda = -\kappa$, the volatility under the risk adjusted process behaves like a random walk with growth. In this case, one would observe large differences between implied volatilities and actual expected volatilities. Even if the risk premia are significant, implied volatilities should move with actual volatilities because the risk adjusted expectation takes current volatility as its starting point. Implied volatilities may not be unbiased or optimal predictors of future volatility, but they should reflect some information that is useful for forecasting future volatility. The relationship between implied volatilities and future volatilities in actual markets is discussed in the next two sections.

Table 4. Option Prices for a Low Volatility Stock

Strike Price	Time to Maturity	Black-Scholes Model	Random Variance Approx.	Monte Carlo Solution	Initial Volatility	
45	0.25	6.59	6.58	6.58	Sigma =	30.00%
	0.50	7.92	7.91	7.91		
	0.75	9.03	9.01	9.01		
50	0.25	3.33	3.32	3.32		
	0.50	4.82	4.78	4.79		
	0.75	5.98	5.93	5.93		
55	0.25	1.40	1.39	1.39		
	0.50	2.68	2.65	2.65		
	0.75	3.74	3.69	3.69		
60	0.25	0.49	0.49	0.49		
	0.50	1.37	1.36	1.36		
	0.75	2.22	2.19	2.19		
45	0.25	6.05	6.06	6.05	Sigma =	18.00%
	0.50	7.14	7.14	7.14		
	0.75	8.15	8.16	8.16		
50	0.25	2.32	2.30	2.30		
	0.50	3.59	3.55	3.55		
	0.75	4.69	4.64	4.64		
55	0.25	0.53	0.52	0.52		
	0.50	1.43	1.39	1.40		
	0.75	2.34	2.28	2.28		
60	0.25	0.07	0.08	0.08		
	0.50	0.45	0.46	0.46		
	0.75	1.02	1.00	1.00		
45	0.25	5.92	5.94	5.93	Sigma =	12.00%
	0.50	6.91	6.93	6.92		
	0.75	7.88	7.90	7.90		
50	0.25	1.85	1.83	1.83		
	0.50	3.06	3.02	3.02		
	0.75	4.17	4.11	4.12		
55	0.25	0.21	0.22	0.22		
	0.50	0.91	0.87	0.88		
	0.75	1.76	1.69	1.70		
60	0.25	0.01	0.01	0.01		
	0.50	0.18	0.20	0.20		
	0.75	0.59	0.59	0.59		

Table 5. Option Prices for a High Volatility Stock

Strike Price	Time to Maturity	Black-Scholes Model	Random Variance Approx.	Monte Carlo Solution	Initial Volatility
45	0.25	8.58	8.50	8.51	Sigma = 54.55%
	0.50	11.00	10.84	10.85	
	0.75	12.95	12.71	12.73	
50	0.25	5.88	5.77	5.78	
	0.50	8.52	8.30	8.31	
	0.75	10.59	10.30	10.32	
55	0.25	3.89	3.79	3.80	
	0.50	6.52	6.30	6.31	
	0.75	8.63	8.32	8.34	
60	0.25	2.50	2.44	2.44	
	0.50	4.95	4.76	4.78	
	0.75	7.02	6.71	6.75	
45	0.25	10.07	10.00	10.01	Sigma = 76.00%
	0.50	12.76	12.60	12.62	
	0.75	14.73	14.49	14.51	
50	0.25	7.58	7.50	7.51	
	0.50	10.47	10.28	10.29	
	0.75	12.55	12.28	12.30	
55	0.25	5.63	5.55	5.56	
	0.50	8.56	8.36	8.38	
	0.75	10.69	10.41	10.43	
60	0.25	4.14	4.07	4.08	
	0.50	6.99	6.80	6.82	
	0.75	9.12	8.83	8.86	
45	0.25	7.24	7.18	7.18	Sigma = 32.00%
	0.50	9.47	9.32	9.34	
	0.75	11.45	11.22	11.26	
50	0.25	4.24	4.11	4.11	
	0.50	6.76	6.52	6.55	
	0.75	8.90	8.60	8.64	
55	0.25	2.26	2.15	2.16	
	0.50	4.69	4.45	4.47	
	0.75	6.85	6.52	6.56	
60	0.25	1.11	1.08	1.09	
	0.50	3.18	3.00	3.02	
	0.75	5.22	4.92	4.97	

3. A Review of Empirical Research on Implied Volatilities

Only a few papers in the finance literature have addressed the predictability of implied volatilities, but there are some recent working papers. Most of the research has focused on the use of implied volatilities or historical volatilities in option pricing models. One of the first empirical applications of the Black-Scholes model was a paper by Black and Scholes (1972). They found that actual option prices were closer to model prices when they used future volatility instead of past volatility. Past volatility tended to produce larger pricing errors in the model: the model overestimated option prices on high volatility stocks and underestimated option prices on low volatility stocks. The paper by Latané and Rendleman (1976) was one of the first to use implied volatilities. For each stock on a given day, they computed a weighted implied standard deviation, WISD, from all of the options traded; the weights were determined by the sensitivity of the option price to volatility. Most financial economists now calculate implied standard deviations, ISD's, by using at-the-money options, options with strike prices closest to the current price, and setting the ISD to minimize the sum of squared errors. The main point of the Latané-Rendleman paper was a comparison of correlations across the WISD's, past standard deviations, current standard deviations, and future standard deviations. The data were primarily cross-sectional and they found a high correlation between WISD's and future volatility. They also found evidence of a common market factor in the WISD's over time. In a subsequent paper, Schmalensee and Trippi (1978) ran regressions to explain the variation of ISD's, but they did not examine the predictability of ISD's. They found some evidence of mean reversion in ISD's and they found that changes in ISD's are negatively correlated with changes in stock prices, but they concluded that ISD's do not seem to be related to current measures of volatility, like the price range or the square of the stock price change. These measures, however, represent very noisy estimates of current instantaneous volatility, σ_t, in the random variance model.

The papers by Chiras and Manaster (1978) and Beckers (1981) were the first to directly examine the predictability of implied volatilities. Both of these papers used cross sectional regressions of future volatility on implied volatility and past volatility. Given that there is a common factor in volatility, there is correlation across the error terms in a cross sectional regression; the result is that standard errors are understated, t statistics are overstated, and statistical inference is unreliable. Neither of these papers account for the correlation of volatility shocks across securities, which can be significant. 1/ Chiras and Manaster used cross-section time series data, but all of the regressions were cross sectional regressions. They found that WISD's were better than past standard deviations as predictors of future volatility. Beckers used at-the-money ISD's,

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1/ In the next section, I present some regressions for volatility in foreign exchange rates and the correlation across exchange rates is substantial.

WISD's, and Black's volatility estimates, and he found that Black's estimates were the best predictors of future volatility. Black had an investment service through which he sold volatility estimates. His method for predicting future volatility included implied volatilities, past volatility, and a market factor for volatility. 1/ In a recent paper, Stein (1989) found some evidence of overreaction in the option market. He studied a series of implied volatilities computed from at-the-money options on the S&P 100 index and he found that changes in implied volatilities were greater for longer term options. If there is mean reversion in volatility, then implied volatilities for longer term options should be less sensitive to current volatility shocks. The longer term options should allow for the longer time period over which volatility can revert back to the long run average. He concluded that his results were evidence of overreactions in the option market, but he did concede that the results could be generated by having a risk premium in the volatility process. In the model of the previous section, if $\lambda < -\kappa$, the risk adjusted volatility process is nonstationary and the reaction to volatility shocks is actually amplified over longer time horizons.

In a recent working paper, Lamoureux and LaStrapes (1991) present a detailed analysis of implied volatilities and future volatility within the framework of a GARCH model for stock returns: 2/

$$R_t = \mu + e_t \tag{37}$$

$$h_t^2 = c + \alpha e_t^2 + \beta h_{t-1}^2 + \gamma IV_t \quad , \tag{38}$$

and e_t is normally distributed with mean zero and variance h_t^2. Their data set consists of two years of daily observations on ten stocks, and they find that the coefficients on implied volatility in the GARCH equations are all positive, but only a few are statistically significant if α and β are not set equal to zero. Past information in stock returns is useful for forecasting future volatility, and implied volatilities contribute only marginally. Their optimal predictor of future volatility, however, is one that uses both the information from current stock returns, the GARCH structure, and the implied volatilities. It should be noted that their GARCH model for volatility is a model of volatility over the time interval for each observation, which is one day. The implied volatilities pertain to volatilities over the remaining life of the option used. In the next section, I present some additional empirical analysis of the relationship between implied volatilities and actual volatilities.

1/ For a description, see Black (1976).

2/ GARCH stands for generalized autoregressive conditional heteroskedasticity.

4. Empirical Analysis of Implied Volatilities

The results of the previous studies suggest that implied volatilities from option prices contain information that is useful for forecasting future volatility, but implied volatilities alone are not unbiased or optimal predictors of future volatility. Information from stock returns, such as past volatility or GARCH models that use past variation, is also useful for forecasting future volatility. This evidence suggests the presence of a volatility risk premium, but one that is not too large in magnitude. Most of the research on implied volatilities has focused on stock markets. In this section I present some analysis of implied volatilities in foreign currency markets. At the end of the section I present some analysis of the term structure of volatility for stock index options and interest rate options.

My data set for foreign currencies consists of actual volatilities and implied volatilities for exchange rates of the U.S. dollar with four currencies: the British pound, German mark, the Japanese yen, and the Swiss franc. Options on these exchange rates have been traded at the Philadelphia exchange since 1983. The implied volatilities have been calculated from at-the-money options that have three months to expiration. The volatilities are from the Black-Scholes model, modified for foreign currency options. Call options have been used for the German mark, the Japanese yen, and the Swiss franc because the interest rates on these currencies were lower than the U.S. interest rates during the sample period, and it would not have been optimal to exercise these calls early. Call options on the British pound were used when the British interest rates were lower than the U.S. rates, and where possible, put options were used when the British rates were higher. 1/ The observations are quarterly and are taken from the third week of March, June, September, and December of each year from 1983 to 1989; The options expire during the third week of the expiration month. Each implied volatility, IV_t, is matched with the actual volatility, V_{t+1}, over the subsequent three months. The actual volatility is calculated as the sample variance of the daily changes in the log of the exchange rate, and the numbers are annualized. The implied volatility for this analysis is the implied variance, instead of the ISD. Past volatilities, V_t, are also included in the prediction equation:

$$E_t(V_{i,t+1}) = a_i + b_i V_{it} + c_i IV_{it} \quad . \tag{39}$$

If the implied volatilities are optimal predictors of future volatility, then $a_i = b_i = 0$ and $c_i = 1$. I also run a test of $c_i = 0$ to test whether implied volatilities are useful for forecasting future volatility. The regression equations have the following form:

1/ The implied volatilities have been taken from joint research with Marc Chesney. Implied volatilities from a model that incorporates an analytic approximation for the American premium have also been used, and the results are virtually the same.

$$\Delta V_{i,t+1} = a_i + b_i V_{it} + c_i (IV_{it} - V_{it}) + e_{it} \quad . \tag{40}$$

The regressions are specified in this manner to allow for a possible root on the unit circle in the volatility process, and to allow us to interpret the R^2 of the regression as the percentage of the variation in volatility changes that is predictable from past information.

The results of the regression analysis for the four exchange rates are presented in Tables 6 and 7. Graphs of the data are presented in Figures 1-4. The solid lines in the graphs represent the ISD's and the boxes represent the actual standard deviations. An inspection of the graphs suggests that, excluding the Japanese yen, the ISD's do vary with the actual standard deviations. The first set of regression results in Table 6 is for single equation ordinary least squares. There is substantial correlation across the error terms of the equations. In the second half of that table, I present the results for the estimation of the entire system with seemingly unrelated regressions, SUR. There is one subtle econometric issue that requires discussion. The data for actual volatility are sample variances which contain sampling error: the sample variance is the actual variance over the three month period plus a measurement error. Under the null hypothesis that the implied volatility is the optimal predictor, the error term is a combination of the forecast error and a measurement error. Both of these should be uncorrelated with the implied volatility which is calculated at the beginning of the period. Under the hypothesis that past volatility is also useful for forecasting, $b_i \neq 0$, then the measurement error from V_{it} is included in the error term of the regression. There will be correlation between the error term and the right hand side variables and the error term will be a first order moving average. To handle this econometric problem one should use implied volatilities, IV_{it}, and past volatilities lagged one period, $V_{i,t-1}$, as instrumental variables and apply Hansen's generalized method of moments, GMM. The results of the GMM estimation are presented in Table 7. Each equation is estimated separately, but all of the instruments are used for each equation to take advantage of the correlation across exchange rate volatility. I use a total of thirteen instrumental variables: a constant plus $V_{i,t-1}$, IV_{it}, and $IV_{i,t-1}$ for each currency.

The results from the first part of Table 6 appear to support the hypothesis that the implied volatilities are optimal predictors for three of the four currencies: the British pound, the German mark, and the Swiss franc. The coefficients on implied volatility for these currencies are all close to one. The coefficient for implied volatility in the Japanese equation is negative and not significantly different from zero. The F test for (a_i=0, b_i=0, c_i=1) indicates rejection for the Japanese yen, but not for the other three currencies. There is, however, substantial correlation across the error terms of these equations and the results do change when the equations are estimated as a system. The results for the system estimation are contained in the second half of Table 6. The coefficients on implied

Table 6. Implied Volatility Versus Actual Volatility

$$\Delta V_{i,t+1} = a_i + b_i V_{it} + c_i (IV_{it} - V_{it}) + e_{it}$$

Sample Period: 1983:III to 1989:III T = 25 quarters

A. Single Equation OLS

	a	b	c	F Test	R^2	DW
British Pound	.001781	-.02782	.9499	.46	.30	1.86
	(.006565)	(.05137)	(.4567)			
German Mark	.007550	-.08046	1.0858	2.79	.70	2.36
	(.005426)	(.04410)	(.2407)			
Japanese Yen	.01745	-.1755	-.2209	4.68	.34	2.35
	(.00656)	(.0647)	(.3290)			
Swiss Franc	.007285	-.05724	.9971	.61	.49	2.42
	(.007175)	(.05838)	(.3706)			

B. Seemingly Unrelated Least Squares

	a	b	c	R^2	DW
British Pound	.003807	-.04179	.7013	.29	1.91
	(.004587)	(.03550)	(.2736)		
German Mark	.01185	-.1117	.7012	.66	2.48
	(.00380)	(.0305)	(.1513)		
Japanese Yen	.01564	-.1574	-.1424	.34	2.40
	(.00512)	(.04999)	(.2606)		
Swiss Franc	.01052	-.08392	.6787	.48	2.47
	(.00468)	(.03740)	(.2087)		

Correlation Matrix

British Pound	1	.73	-.13	.74
German Mark		1	.39	.85
Japanese Yen			1	.46
Swiss Franc				1

Test of (a_i=0, b_i=0, c_i=1) $\chi^2(12)$ = 49.85
Test of (c_i=0) $\chi^2(4)$ = 26.06
Test of (a_i=0, b_i=0, c_i=1) $\chi^2(9)$ = 25.83 excluding Japan
Test of (c_i=0) $\chi^2(3)$ = 24.15 excluding Japan

NOTE: The numbers in parentheses are standard errors.

Table 7. Implied Volatility Versus Actual Volatility

$$\Delta V_{i,t+1} = a_i + b_i V_{it} + c_i (IV_{it} - V_{it}) + e_{it}$$

GMM - Instrumentals Variables Estimation

Sample Period: 1983:III to 1989:III T = 25 quarters

	a	b	c	$\chi(3)$ Test	DW
British Pound	.001768 (.001477)	-.2198 (.09151)	.5759 (.3123)	22.43	1.82
German Mark	.002338 (.000834)	-.2651 (.05175)	.9980 (.1403)	87.44	2.64
Japanese Yen	.009619 (.001070)	-.8540 (.06983)	-.5333 (.08734)	463.47	2.81
Swiss Franc	.004495 (.001364)	-.2871 (.09977)	.6873 (.1979)	16.65	2.55

Instruments: Constant, $V_{i,t-1}$, IV_{it}, and $IV_{i,t-1}$, $i=1,\ldots,4$

volatility for the three European currencies are no longer close to one. In the system estimation, the tests for implied volatilities as optimal predictors are rejected at low marginal significance levels, but the hypothesis that implied volatilities provide no information, $c_i=0$, is also rejected by the data. When the correlation across volatility shocks is considered, the implied volatilities alone are not optimal predictors, but a combination of implied volatilities and past volatilities are useful for forecasting future volatility. The results from Table 7 for the GMM estimation are similar to the results for the system estimation. The hypothesis that implied volatilities are optimal predictors is rejected, but the implied volatilities are useful for forecasting future volatility. It is interesting to note that Black's method for forecasting volatility included implied volatility, past volatility, and consideration of the market effect. The regression analysis suggests that a similar approach would be useful in foreign currency markets.

In Table 8, I present some calculations of ISD's across different maturities for the S&P 500 index options, the S&P 100 index options, Treasury bond futures options, and Eurodollar futures options. The number of observations is limited, but an attempt has been made to find examples of upward and downward sloping term structures for volatility. During the month of July 1991, the term structure of volatility was upward sloping in the stock index options market and in the interest rates options market. For both of these markets, volatilities were below long run averages. Downward sloping term structures are less common. One week after the stock market crash of October 1987, volatility in the U.S. stock market, as implied in the S&P 100 options, was approximately three times greater than the level from earlier in the month. By historical standards, volatility was incredibly high and it did decrease gradually over the subsequent six months, but the term structure of volatility on October 27, 1987, was upward sloping: 61.9 percent for November, 65.6 percent for December, and 77.5 percent for January of 1988. There are several possible explanations. One is that the market overreacted to the dramatic volatility shock. or that the options market did not have any confidence that security markets would stabilize over the next three months. Alternatively, a significant negative risk premium on volatility would have generated these numbers even if the market had anticipated a gradual decline in volatility. A downward sloping term structure of volatility was observed in this market in January of 1988. The bond markets also experienced a volatility shock during October of 1987: volatility increased to high levels by historical standards, but the shock was not as dramatic as in the stock market. On October 27, 1987, the term structure of volatility was downward sloping for options on Treasury bond futures and options on Eurodollar futures. Many of the volatility term structures are relatively flat which suggests that the risk adjusted volatility process used in pricing options is close to a random walk.

Figure 1

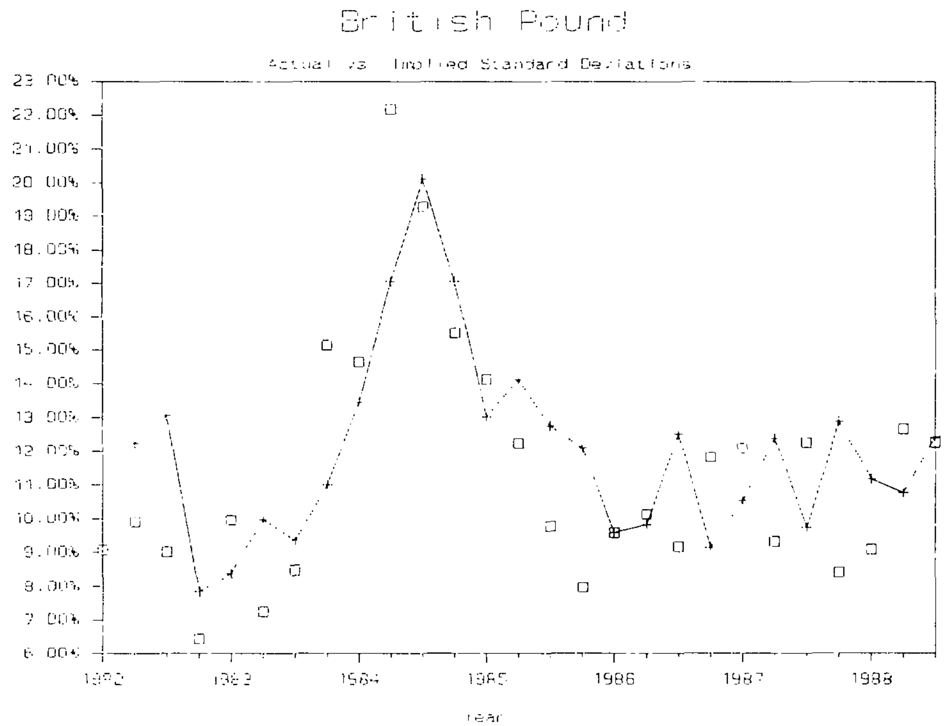

Figure 2

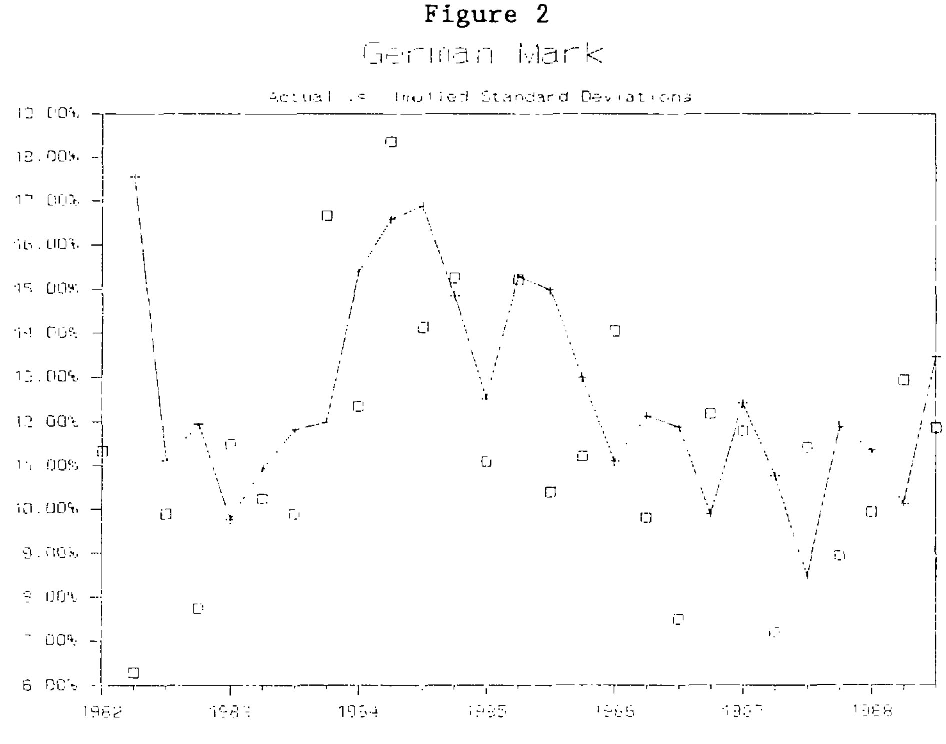

Figure 3

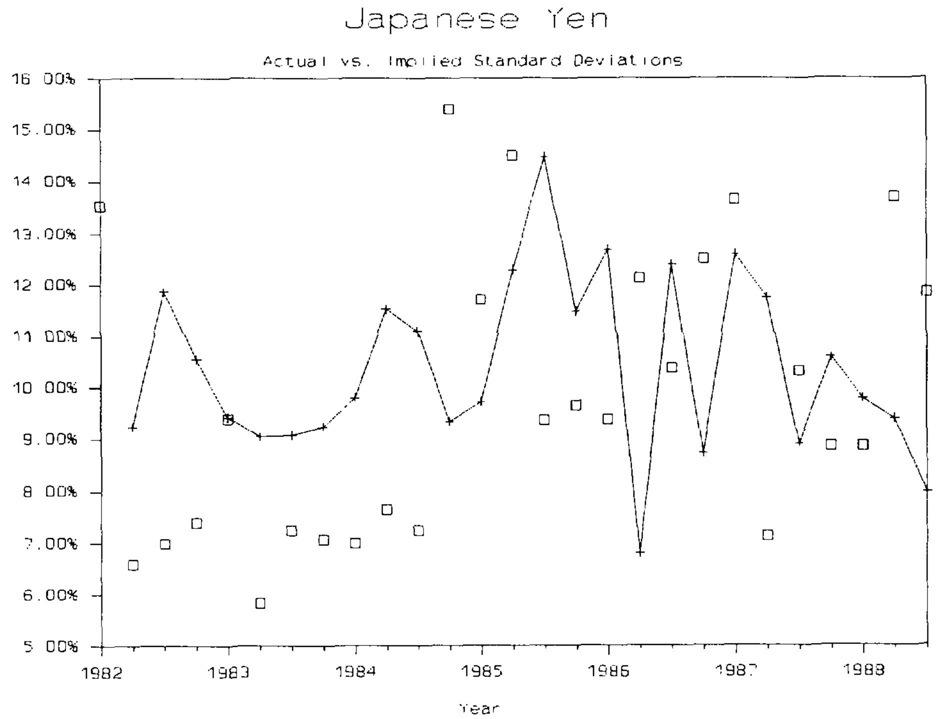

Figure 4

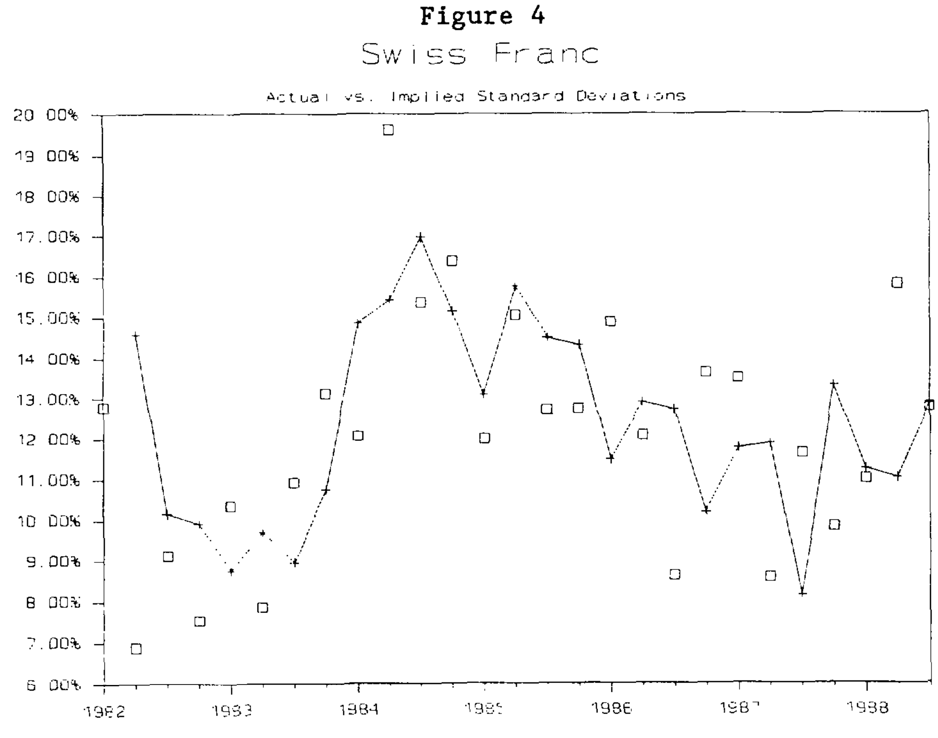

Table 8. Summary of Implied Volatilities
Selected Days

S&P 500 Index Options (European)

Jan. 22, 1991:	Feb.	Mar.	June	
	20.9%	21.4%	21.8%	
July 25, 1991:	Aug.	Sept	Dec.	Dec 93
	14.3	15.7	17.1	19.8

S&P 100 Index Options

	Nov.	Dec.	Jan 88
Oct. 8, 1987:	20.2	20.4	
Oct. 27, 1987:	61.9	65.6	77.5
Jan. 19, 1988:	Mar.	June	
	39.4	35.1	
Sept. 22, 1988:	Nov.	Dec.	
	17.5	17.7	
Jan. 22, 1991:	Mar.	April	Dec 92
	21.2	23.7	20.9

Treasury Bond Futures Options

Oct. 27, 1987:	Dec.	Mar.	June
	22.2	20.2	17.4
June 8, 1989:	Sept	Dec.	Mar.
	10.4	10.2	10.2
July 15, 1991:	Sept.	Dec.	
	7.8	8.3	

Eurodollar Futures Options (ISD for Futures Rate)

Oct. 27, 1987:	Dec.	Mar.	June
	36.4	37.6	31.9
June 8, 1989:	Sept	Dec.	
	20.7	20.8	
July 15, 1991:	Set.	Dec.	Mar 92
	11.4	14.6	14.7

IV. Summary and Conclusions

The relationship between expectations and prices in futures and options markets should be interpreted carefully. Futures and forward prices are prices for future delivery, and these markets make it possible for individuals to hedge price risk. When individuals use futures and forward markets to hedge, they really transfer the price risk to someone else, and there should be some form of compensation for those who absorb the risk. As a result, risk premia are built into the prices so that the price is a combination of the expected future price and a risk premium. In section I, it was demonstrated by arbitrage methods that futures and forward prices should be functions of current spot prices and interest rates. Any direct connection between these prices and expected future spot prices is purely coincidental. Futures and forward prices are, however, affected by expectations through the current spot price which is determined by expectations. Several examples of actual prices for stock index futures, Treasury bond futures, and forward foreign exchange were examined, and in all cases the prices were very close to the prices predicted by the arbitrage models. Prices in futures and forward markets do not reveal any additional information on market expectations that is not already revealed in spot prices. In foreign exchange markets, forward rates are very poor predictors of future changes in the exchange rates.

It is possible that implied volatilities computed from option prices may reflect market expectations of future volatility in the spot market. The popular model for computing implied volatilities is the Black-Scholes model, and it was demonstrated in section III that this model with expected volatility can be interpreted as a first order approximation for a more complex model that allows the volatility to change randomly. Risk premia may also influence the implied volatilities computed from option prices: the correct first order approximation is the Black-Scholes model with expected volatility computed from the risk adjusted volatility process. Previous empirical studies of implied volatilities were reviewed in Section III and some new evidence for foreign exchange rates was presented. The results of the empirical studies suggest that implied volatilities are useful for forecasting future volatility, but implied volatilities alone are not optimal predictors. A combination of implied volatilities, past volatilities, and the market factor in volatility appear to be useful in forecasting future volatility. The empirical analysis supports the notion of a volatility risk premium, but not one that is large enough to completely break the linkage between implied volatilities in option prices and expectations of future volatility in the spot market.

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