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The Determinants of U.S. Real Interest Rates in the Long Run

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Abstract

This paper examines the factors which influence the behavior of real interest rates in the United States over the long run. Data on real and nominal returns to bonds and equities are tested for unit root non-stationarity. The results indicate that real and nominal interest rates and inflation are integrated of order one while the evidence on returns to equities is mixed. Short- and long-term real rates were found to be cointegrated with government deficits, government debt relative to GNP, private wealth, real balances relative to GNP, demographic factors and the marginal productivity of capital; demographic, fiscal, and monetary policy variables appear to be particularly significant.

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Summary

Following two decades of relative stability after World War II, real interest rates in the United States have swung widely from the 1970s to the early 1990s, challenging the commonly held notion that, apart from some short-run deviations, real interest rates tend to revert to a fairly constant level over the long run. The stationarity of real interest rates is also predicted by several theoretical models of neoclassical growth and asset pricing. This paper explores the behavior of real interest rates by analyzing their time-series properties and by examining whether a long-term relationship can be found among these rates and other similarly non-stationary economic variables.

The paper estimates real yields on bonds and stocks over different holding periods on the basis of three hypotheses of expectations formation regarding inflation: ex post, a forward forecast, and static expectations. An examination of the data since the mid-nineteenth century reveals large and sustained swings in ex post real interest rates even before the Great Depression. Although real rates on bonds reached a post-World War II peak in the early 1980s, their levels were not particularly high relative to the levels in the late nineteenth century and the 1920s.

Tests for stationarity indicate that real and nominal interest rates and inflation contain a unit root and hence are nonstationary, although, in some cases, this conclusion depends on the processes containing moving average as well as autoregressive components. The evidence on real and nominal returns to equity is mixed, according to the choice of data set and sample period and to the existence of moving average components in the time-series process.

In light of the evidence that real interest rates are nonstationary, cointegrating regressions were run for the post-World War II period. These long-run regressions, which exclude all short-run dynamics, appear to capture the major movements in real interest rates, particularly for ex post returns. Irrespective of the expectations formation process specified, short- and long-term real interest rates were found to be cointegrated with government deficits, government debt relative to GNP, private sector wealth, real balances relative to GNP, demographic factors and the marginal productivity of capital. Fiscal and monetary policy variables were found to be important determinants of real interest rates, although fiscal deficits appeared to have a greater effect on short-term than on long-term real rates. The strong presence of the fiscal variables is significant because this result has usually been difficult to establish in the literature.

I. Introduction

The marked swings in real interest rates in the period from the 1970s to the beginning of the 1990s have focused attention on the determinants of long-run changes in real interest rates. In particular, the question arises of the role played by policy variables in bringing about sustained shifts in real interest rates. This paper approaches these issues by first examining the possible stationarity of rates of return in the United States over different historical periods. It then attempts to identify the major determinants of the long-run level of short- and long-term real interest rates in the post-World War II period.

The time series processes followed by real interest rates is in and of itself an important issue in some areas of theoretical and empirical work. Brock and Mirman (1972), Donaldson and Mehra (1983) for instance, have shown that under appropriate assumptions about preference and technology, the real interest rate would be stationary in neoclassical growth models with uncertainty even if technological progress were an integrated stochastic process of order one. The consumption based capital-asset pricing model as presented by Lucas (1978) and Hansen and Singleton (1983) indicates that the growth rate of real consumption and the real interest rate would be integrated of the same order. Real consumption in the United States appears to contain a unit root (Schwert (1988), Neusser (1991)) implying that the growth rate of consumption and the real interest rate would be stationary.

Section II of this paper outlines the methodology used to estimate real returns on a number of assets--short and long, risky and riskless--and reviews the broad movements in these rates since the late nineteenth century. Section III examines whether the time series processes followed by these real rates are unit root nonstationary implying, in contrast to stationary series, the absence of a tendency to revert to a constant mean over time. In light of these results, Section IV explores whether a long-term relationship can be found among real interest rates and other similarly nonstationary economic variables that theory suggests determine real interest rates. These variables include demographic factors, fiscal and monetary variables, private wealth, and the marginal productivity of capital. The concluding section briefly discusses the implications of the empirical evidence for the long-run behavior of real interest rates.

II. Movements in Real Interest Rates

1. Definition of the real interest rate

The real interest rate on an asset may be approximated by its nominal interest rate minus the inflation rate expected over the holding period of the asset. This section describes the methodology used in this paper to estimate real yields on two basic types of assets--bonds and stocks--over three holding periods--one quarter, one year, and ten years.

Real interest rates on bonds are estimated on the basis of three different hypotheses of expectations formation:

(1) Ex post: on the assumption that expected inflation in every period equals actual inflation. Although this corresponds to a perfect

foresight real interest rate, the ex post definition is used in Section III to test a more general ex-ante real interest rate process for stationarity. Since expected inflation may be expressed as the sum of realized inflation π and a forecast error ϵ_t, the real interest rate r may be written as $r = i - (\pi + \epsilon)$ where i is the nominal interest rate. Under the not very restrictive assumption that the forecast error is I(0), if the ex post real rate $(i - \pi)$ is integrated of order k, or I(k), then the ex ante expected real interest rates must be I(k) as well.

(2) An explicit forecast on inflation: on the assumption that expected inflation rates are forecast on the basis of a rolling ARIMA (4,1,0) process for the CPI over 24 quarters. The forecasting equation was estimated in first difference form; forecasts for the level of the CPI were made for the duration of the relevant holding period (e.g., one quarter, one year, ten years), and the geometric average of the inflation rate during the period was taken as the expected inflation rate. Chart 1 plots the annual averages of three-month and one-year expected inflation forecasted by this methodology along with actual inflation, and expected inflation based on survey data. For the most part, expected rates based on regression forecasts can be observed to follow actual rates except at certain turning points in the early 1970s and in the early and late 1980s.

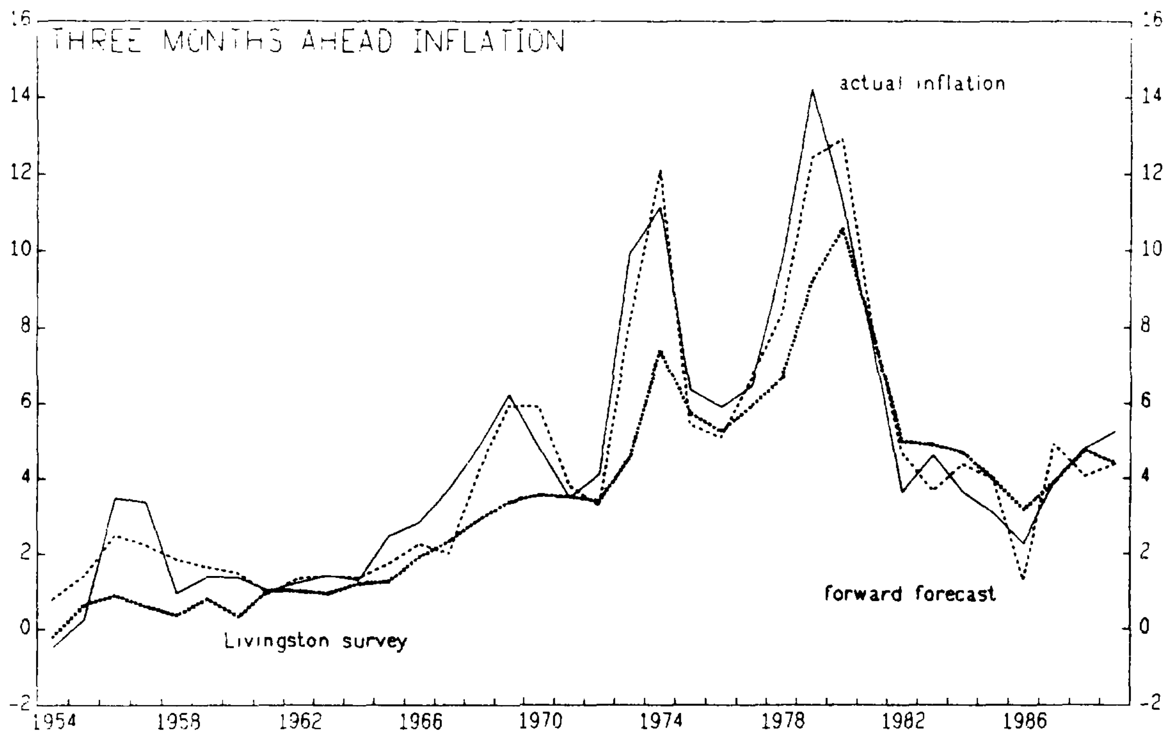
(3) Static expectations: on the assumption that expected inflation is equal to the past inflation rate. For the one-quarter and one-year holding periods, inflation rates over the past quarter or one year, respectively, were used. For the ten-year holding period, the geometric average of the inflation rate over the past seven years was used in order to limit the number of observations lost.

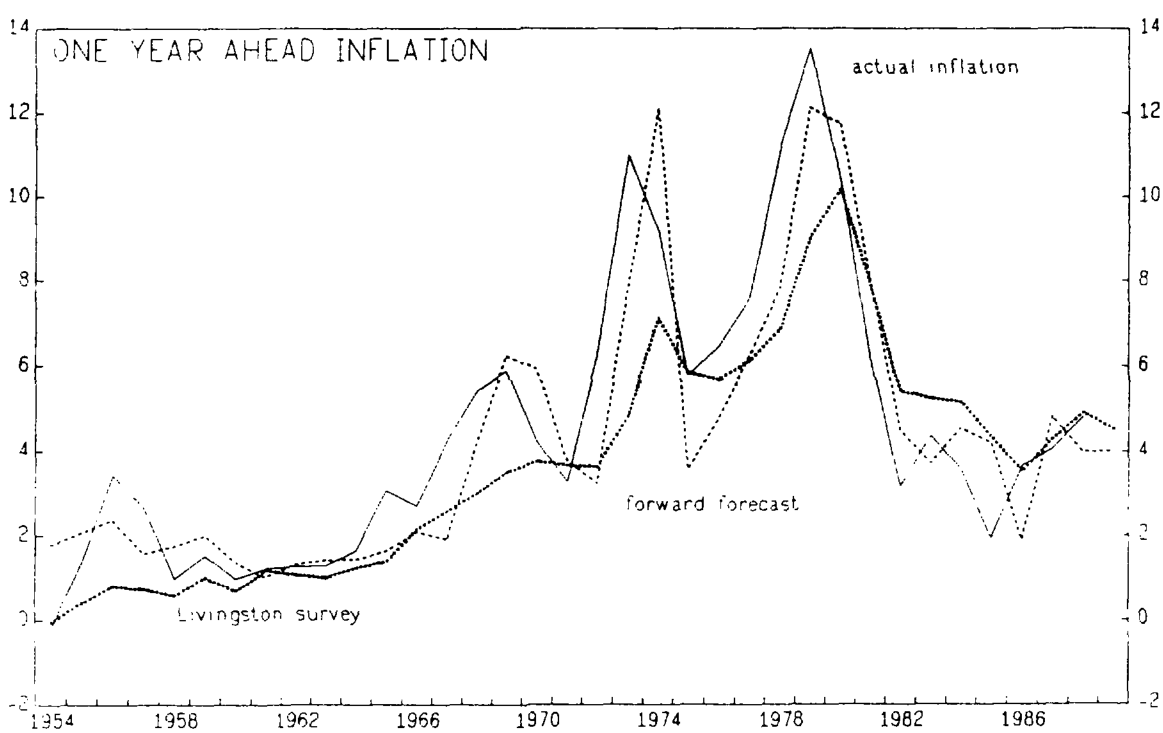
The calculation of returns on equity requires a forecast of the stream of dividends expected over the holding period and the expected end of period price. Following Blanchard and Summers (1984), the real expected rate of return on equity is defined implicitly by the relation,

$$P_t = \sum_{i=1}^{T} \left[\frac{D_{t,\ t+i}}{(1+e_t)^i} \right] + \left[\frac{P_{t,\ T}}{(1+e_t)^T} \right]$$

CHART 1

EXPECTED AND ACTUAL INFLATION RATES





where

$P_{t,T}$ = real stock price at the end of time T, expected at time t

$D_{t,t+i}$ = real dividends at time t+i, expected at time t

P_t = actual real stock price at time t

e = the real expected return on equity at time t (i.e., the internal rate of return)

Real dividends and stock prices were obtained by deflating the corresponding nominal series by the CPI. Forecasts of real dividends and stock prices were obtained by two methods:

(1) estimate a rolling autoregression over 24 quarters for P and D separately and, on the basis of this estimate, generate forecasts of P and D over the relevant holding period (i.e., an univariate method).

(2) estimate a rolling VAR of both P and D over 36 quarters and, on the basis of this estimate, generate forecasts for P and D over the relevant holding period (i.e., a bivariate method).

The second method, similar to that used by Blanchard and Summers, allows for the possibility that stock prices contain information not contained in current and past dividends and vice versa. An estimate of the nominal expected return on holding equities was obtained by substituting nominal, rather than real variables, for P and D above.

2. Movements in real interest rates in the late nineteenth and twentieth centuries

Chart 2 shows ex post real rates of return from the mid-nineteenth century to the 1980s on short- and long-term bonds and corresponding holding period yields on stocks. 1/ These returns underwent large and sustained swings even in the period before the Great Depression and World War II. In fact, the movements in real rates in the 1970s and 1980s appear relatively mild in comparison to the swings that took place before 1930. The top panel shows that, although one-year real interest rates did rise sharply in the 1980s, their levels in the early and mid-1980s were not particularly high in comparison to those prevailing during various periods from the late nineteenth century to World War II. The bottom panel shows that ten-year real rates stood at about 10 percent in the 1860s and drifted down until 1910 before rising again to about 8 percent in the 1920s. Although these rates began to climb to a post-World War II peak in the 1970s, their levels

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1/ Data are from the U.S. Department of Commerce, Bureau of Statistics (1975) and Homer (1963). Stocks returns are holding period yields calculated from dividend and price data as explained in the previous section.

in the early 1980s do not appear to be high relative to the levels observed in the late 19th century and the 1920s.

While real interest rates on short- and long-term bonds reached postwar highs in the 1980s, real returns on equity, which also rose strongly in the 1980s, do not appear to be unusually high in comparison to levels observed in the 1950s and 1960s (Chart 3). Both the univariate and bivariate methods of generating expected returns indicate that the real long-term returns on equities were higher in the 1960s and lower in the late 1970s than ex post returns. Ex post real returns were high in the 1970s because stock prices were low at the time but rose sharply in the mid-1980s, which may not have been anticipated given the economic conditions and expectations in the 1970s.

3. The equity premium puzzle

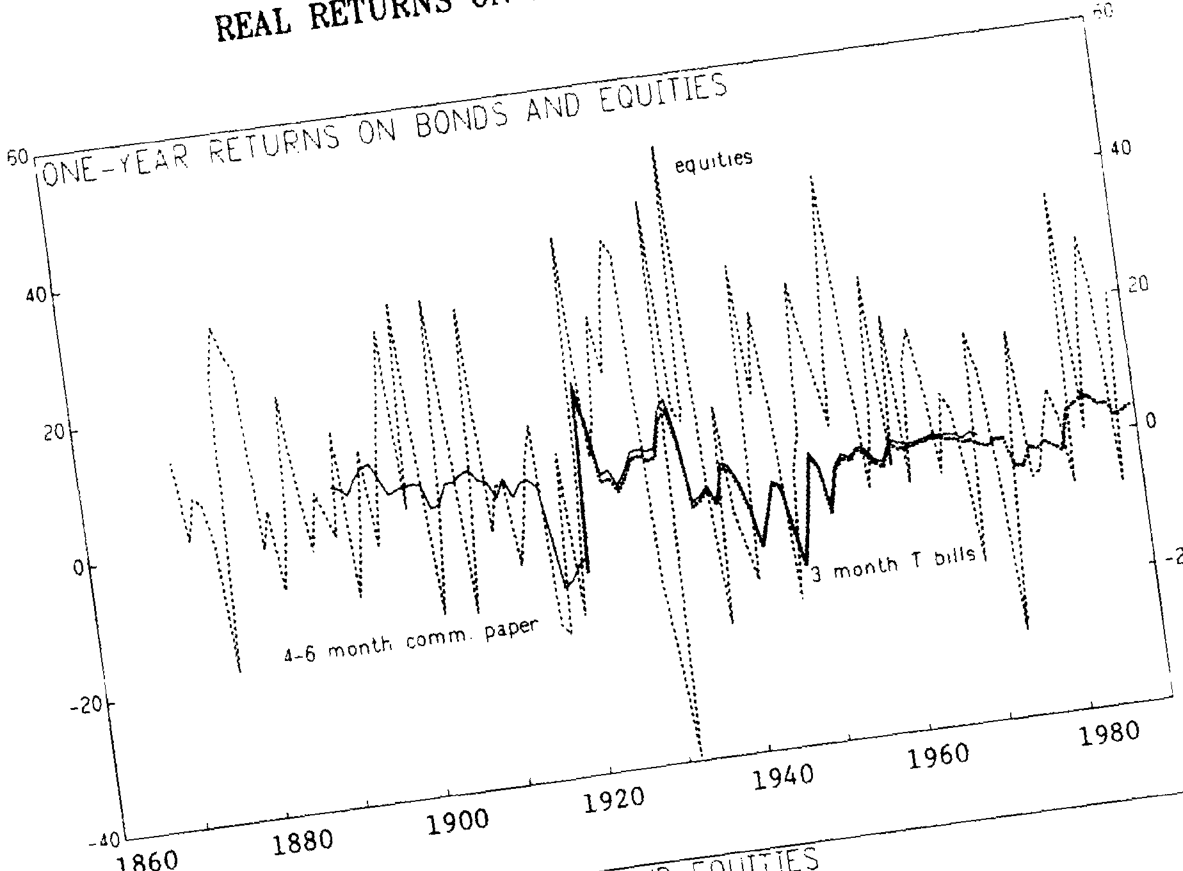
Looking at the relationship between rates on different types of assets, Chart 2 demonstrates that returns on equities display a far greater variance than returns on bonds, particularly over shorter holding periods. In addition, over the 1870-1989 period, average ex post real equity returns turn out to be approximately 7-8 percentage points higher than average ex post real returns on Treasury bills for a one-year holding period.

In their influential paper, Mehra and Prescott (1985) posed this sizable difference in average returns between equities and relatively risk-free government bonds in terms of an equity premium puzzle. They demonstrated that the size of the differential cannot be accounted for by a class of competitive pure exchange models that abstract from transactions costs, liquidity constraints, and other frictions. Even if the parameter measuring the curvature of the utility function (the risk aversion parameter) is allowed to vary from zero to ten (most estimates being in the range of one to two) and the subjective time discount factor is permitted its full range from zero to one (corresponding to a discount rate from zero to infinity), the largest premium obtainable is 0.35 percent compared to the observed value of 6.18 percent in their data set which covers the period 1889-1978. Mehra and Prescott also note that the results are robust to measurement error in the growth rate of real consumption and the risk-free real rate, to model misspecification, to allowance for capital accumulation and production.

One potential problem with the equity premium puzzle lies in the possible nonstationarity of asset returns since Mehra and Prescott's analysis is restricted to a class of models where the equilibrium growth rate processes on consumption and equilibrium asset returns are stationary. The question of the possible nonstationarity of asset returns is dealt with in greater detail in Section III. In addition, the bottom panel of Chart 3 suggests that the equity premium may have narrowed substantially since the late 1960s and early 1970s (Mehra and Prescott's sample ended in 1978). Estimates of expected real rates based on forward forecasts indicate that the equity premium over government bonds for long-term (ten-year) holding periods disappeared in the 1970s and early 1980s. A small premium re-

CHART 2

REAL RETURNS ON BONDS AND EQUITIES



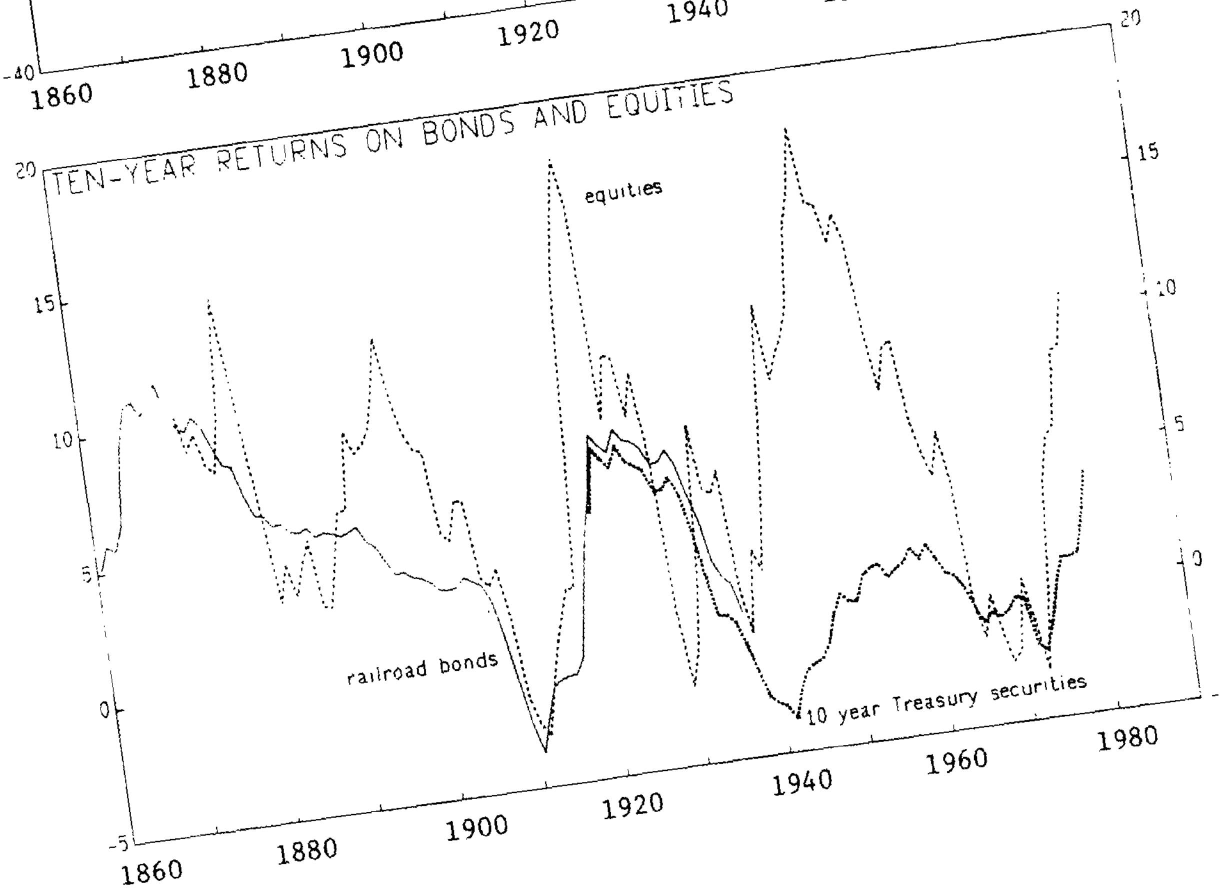
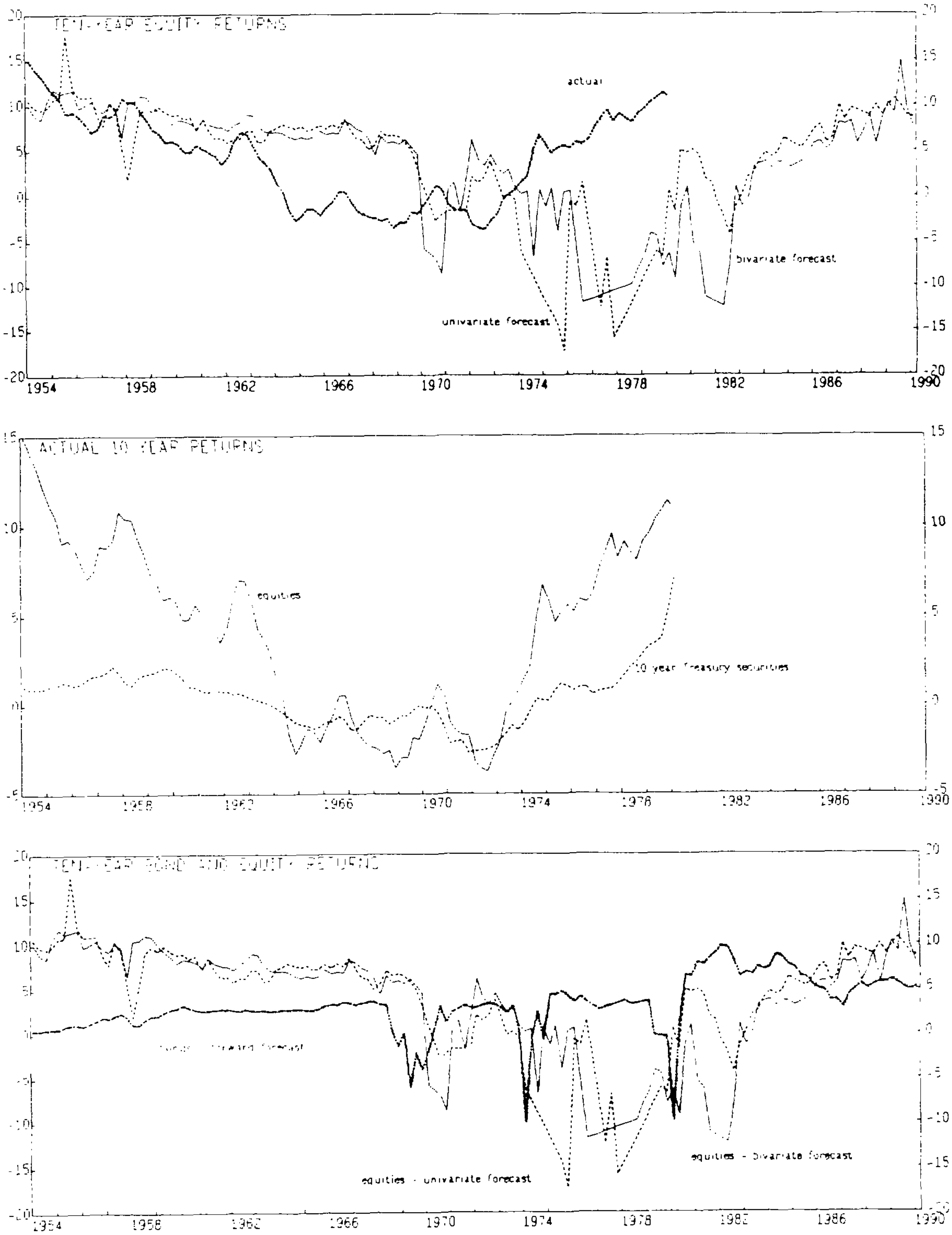


CHART 3

REAL TEN-YEAR RETURNS ON BONDS AND EQUITIES



emerged from 1986 to the first quarter of 1990, albeit generally narrower than the premia observed in the late 1950s and early 1960s.

Indeed, the equity premium puzzle may relate not so much as to why the average return on equity is so high as to why the average risk free rate is so low. According to asset return data computed by Ibbotson and Sinquefield (1979), the sample average of the real interest rate on Treasury bills over the 1920-79 period was only 0.8 percent with a standard deviation of the average of 0.6. Mehra and Prescott find that if the curvature parameter were 2 (at the upper end of the range of empirical estimates) their model's average risk free rate would be at least 3.7 percent a year.

III. Time Series Properties of Real Interest Rates

This section reports the results of tests on real interest rates for unit root nonstationarity. These tests are carried out on three data sets, to allow data over long periods to be analyzed and to facilitate comparisons with results in the literature. Ex post real interest rates are tested for the period before World War II; real interest rates calculated under the other hypotheses of expectations formation described in Section II are tested only for the post-World War II period. Since postwar data are on a quarterly basis, only this data set allows a meaningful distinction to be made between three-month and one-year returns.

The series are tested for unit root nonstationarity using the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests. 1/ It should be noted that the conclusions of these tests depend critically on the assumption that the underlying process followed by the variable in question is a pure autoregressive model. There is, however, much evidence that many economic time series contain moving average components. It has been shown that the distributions of the DF test statistic can be quite different from the distributions originally reported by Fuller when the underlying process contains a moving average component. 2/ Although tests have been proposed by Said and Dickey, Phillips, and Phillips and Perron to allow for mixed ARIMA processes as well as pure AR processes, Schwert (1988) has demonstrated through Monte Carlo simulations that even for very large finite samples these tests can give misleading results, particularly when the moving average parameter is large. Applying these tests to several impor-

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1/ Suppose a variable x can be modeled as an autoregressive process of order (p), the estimate of β, from an OLS regression of

$$\Delta x_t = \alpha+\beta x_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta x_{t-i} + \nu_t$$

is tested to be significantly different from zero, under the null hypothesis that X is I(1). In the DF test all γ_i are set equal to zero while in the ADF test p-1 lags are included to ensure that the residual is empirical white noise.

2/ See for example, Pagan and Wickens (1989).

tant U.S. macroeconomic time series, Schwert (1987) also found that many of the tests would falsely reject the null hypothesis of a unit root.

The Monte Carlo simulations carried out by Schwert indicate that the least misleading test for all values of the moving average parameter is an ADF test with a larger number of autoregressive parameters than would be suggested by a pure AR process. (The intuition behind this result is the fact that a first order moving average process has infinite order autoregressive representation). The order of this autoregressive process can be quite large for high values of the moving order parameter, however, and can give misleading results in small sample sizes (of less than 50 observations). A rule of thumb to select the order of the autoregressive process follows the suggestion of Said and Dickey to use a high order autoregressive process to approximate an unknown ARIMA process where the order of the autoregression grows with the sample size. 1/

The unit root tests on interest rates were performed using these finite sample results as a guide and the results are reported in Tables 1 to 4. Table 1 shows results for one- and ten-year real interest rates based on annual historical data going back to the second half of the nineteenth century and early twentieth century. 2/ Table 2 reports test results on asset returns computed by Ibbotson and Sinquefield (1979). Their data set is of interest not only because it is widely used in the literature but also because it appears to have wide coverage and to have been compiled with considerable care. 3/ Tables 3 and 4 show results for three months, one year and ten year rates for the postwar period under the different expectations formation hypotheses noted in Section II. The issues discussed above are of special relevance to the interpretation of these test results because

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1/ The rule of thumb is:

$$\ell_4 \simeq 4 \times (T/100)^{1/4}$$

and

$$\ell_{12} \simeq 12 \times (T/100)^{1/4}$$

where T is equal to the sample size and ℓ is the number of lags in the ADF regression. Thus 4 and 12 lags would be used for a sample of 100.

2/ The sources for these data are U.S. Department of Commerce, Bureau of Census (1975) and Homer (1963).

3/ In this data set, Treasury bill returns are based on an index of the shortest-term bills not less than a month in maturity. The long-term government and corporate bonds are constructed from a 20-year term bond portfolio. Equity returns are based on the Standard and Poor's Composite Index with holding period yields calculated as in Section II above. Monthly returns are computed for each asset and geometric means are then calculated for different holding periods.

Table 1. Unit Root Tests for One-Year and Ten-Year Rates of Return: Annual Data

	DF	ADF(4)	ADF(8)
One Year			
Real rates of return			
Three-month treasury bills (1920-1989)	-4.97	-1.82	-1.97
Four- to six-month commercial paper (1890-1970)	-3.90	-2.60	-2.35
Equities (1871-1988)	-9.22	-5.42	-3.64
Nominal rates of return			
Three-month treasury bills	-1.23	-0.78	-0.39
Four- to six-month commercial paper	-1.88	-0.90	-1.05
Equities	-9.11	-6.11	-3.12 1/
Memorandum item			
One-year inflation rate (1851-1989)	-5.82	-3.61	-2.58 2/
Dividend-price ratio (1871-1989)	-3.67	-3.77	-2.12
Ten-year			
Real rates of return			
Long-term government securities 3/ (1919-79)	-1.23	-2.76	-3.04 4/
AAA corporate bonds (1919-79)	-1.22	-3.02	-3.28 5/
Railroad bonds (1857-1936)	-0.72	-1.88	-2.51
Equities (1871-1979)	-2.45	-2.98	-4.54
Nominal rates of return			
Long-term government securities (1919-1989)	-0.75	-0.47	0.75
AAA corporate bonds (1919-1989)	-0.54	-0.52	0.45
Railroad bonds (1857-1936)	-3.16	-2.41	-1.85
Equities (1871-1979)	-2.12	-2.47	-3.53 6/
Memorandum item			
Ten-year inflation rate (1850-1979)	-1.14	-3.08	-3.13 7/
Dickey Fuller critical values: 8/		5 percent	1 percent3
50 observations		-2.93	-3.58
100 observations		-2.89	-3.51
250 observations		-2.88	-3.46

1/ With 12 lags, the ADF statistic = -2.93.
2/ With 12 lags, the ADF statistic = -3.11, above the critical value.
3/ In the early years of the sample, bonds maturing in 8-12 years; after 1947, ten-year treasury securities.
4/ With 12 lags, the ADF statistic = -2.16.
5/ With 12 lags, the ADF statistic = -2.41.
6/ With 12 lags, the ADF statistic = -2.71.
7/ With 12 lags, the ADF statistic = -1.52.
8/ For the "t test" for a unit root against the alternative hypothesis that the process is stationary around a constant mean.

Table 2. Unit Root Tests for One-Year and Ten-Year Rates of Return: Annual Data from Ibbotson and Sinquefield

	DF	ADF(4)	ADF(8)
One Year (1926-78)			
Real rates of return			
Treasury bills	-3.51	-2.24	-2.53
Long-term government bonds	-6.00	-2.11	-2.73
Long-term corporate bonds	-4.97	-1.84	-2.97
Equities	-6.99	-3.23	-1.24
Nominal rates of return			
Treasury bills	-0.89	0.14	--
Long-term government bonds	-7.61	-2.81	-2.11
Long-term corporate bonds	-6.23	-2.35	-1.98
Equities	-6.90	-3.77	-1.77
Memorandum item			
One-year inflation rate	-3.43	-1.75	-2.43
Ten-year (1926-69)			
Real rates of return			
Treasury bills	-2.02	-3.64	-3.13
Long-term government bonds	-2.47	-3.38	-3.14
Long-term corporate bonds	-2.63	-4.00	-3.69
Equities	-1.25	-0.92	-1.10
Nominal rates of return			
Treasury bills	-3.11	-3.40	-0.10
Long-term government bonds	-1.59	-1.31	-1.43
Long-term corporate bonds	-1.65	-1.96	-1.52
Equities	-1.59	-1.77	-1.13
Memorandum item			
Ten-year inflation rate	-1.44	-3.77	-1.51

Table 3. Unit Root Tests for Three-Month Rates of Return: Quarterly Data, Post-World War II Period

	DF	ADF(4)	ADF(8)
Three-month			
Real rate of return			
Three-month Treasury bills			
Ex post	-5.29	-4.23	-2.65
Forward forecast	-4.88	-2.38	-2.00
Static expectations	-4.49	-2.34	-2.33
Equities			
Ex post	-9.24	-5.89	-4.25
Forward forecast			
Univariate AR	-10.56	-4.48	-3.54 1/
Bivariate VAR	-10.87	-4.15	-3.52 2/
Nominal rates of return			
Three-month Treasury bills	-1.83	-1.74	-1.63
Equities			
Ex post	-9.62	-6.32	-4.70
Univariate AR	-11.65	-5.48	-4.22
Bivariate VAR	-12.12	-4.29	-3.53 3/
Memorandum item			
Three-month inflation rate	-4.51	-3.31	-2.39
Dividend price ratio	-1.61	-2.14	-1.98

1/ With 12 lags, the ADF statistic = -2.33.
2/ With 12 lags, the ADF statistic = -2.29.
3/ With 12 lags, the ADF statistic = -2.47.

Table 4. Unit Root Tests for One-Year and Ten-Year Rates of Return: Quarterly Data, Post-World War II Period

	DF	ADF(4)	ADF(8)
One Year			
Real rate of return			
One-year Treasury securities			
Ex post	-2.06	-2.61	-2.44
Forward forecast On inflation	-4.16	-3.29	-2.26
Static expectation On inflation	-2.54	-2.55	-1.75
Equities			
Ex post	-3.91	-4.12	-3.03 1/
Forward forecast			
Univariate AR	-6.25	-3.21	-2.50
Bivariate VAR	-4.13	-3.74	-2.75
Nominal rates of return			
One-year Treasury securities	-1.78	-2.06	-1.72
Equities			
Ex post	-4.11	-4.59	-3.47 2/
Forward forecast			
Univariate AR	-7.96	-4.30	-3.45 3/
Bivariate VAR	-3.58	-3.88	-2.79
Memorandum item			
One-year inflation rate	-2.11	-2.35	-1.46
Ten-year			
Real rates of return			
Ten-year Treasury securities			
Ex post (1953:3-1980:1)	0.36	0.49	0.27
(1953:3-1979:1)	-0.25	-1.20	-1.04
Forward forecast On inflation	-4.51	-3.39	-2.56
Static expectation On inflation	-2.77	-3.20	-1.92
Long-term corporate bonds			
Ex post (1947:2-1980:1)	3.28	0.68	0.63
(1947:2-1979:1)	0.03	-1.38	-0.96
Forward forecast On inflation	-4.37	-3.24	-2.58
Static expectation On inflation	-2.30	-2.56	-1.74
Equities			
Ex post	-1.06	-1.56	-1.71
Forward forecast			
Univariate AR	-2.28	-2.14	-1.48
Bivariate VAR	-2.37	-1.95	-1.52
Nominal rates of return			
Ten-year Treasury securities	-1.31	-1.74	-1.38
Long-term corporate bonds	-1.06	-1.30	-1.16
Equities			
Ex post	-1.05	-1.54	-1.55
Forward forecast			
Univariate AR	-3.26	-2.29	-2.03
Bivariate VAR	-5.26	-2.65	-1.99
Memorandum item			
Ten-year inflation rate	-0.73	-1.50	-1.25

1/ With 12 lags, the ADF statistic = -2.19.
2/ With 12 lags, the ADF statistic = -2.46.
3/ With 12 lags, the ADF statistic = -1.97.

the results on some series, particularly short-term real interest rates, depend on the number of lags included in the ADF regression. The ADF regressions were run with lags of 4, 8 and 12 since the sample size is for the most part moderate--about 50 to 150 observations. 1/ If the ADF test statistic fell below the critical value at 12 lags this is noted in the tables. The DF test results are also reported.

The higher order autoregression ADF (8 to 12 lags) tests do not reject the null hypothesis of a unit root for all one-year real and nominal interest rates for the different data sets and expectations formation processes considered. 2/ With the exception of long-term real interest rates in the Ibbotson and Sinquefield data set, the tests also indicate a unit root in nominal and real ten-year interest rates. 3/ Real ex post one-year and ten-year equity returns appear to be I(0) in the historical data in Table 1 even with a large number of lags (up to 12 were tried), but in the Ibbotson and Sinquefield annual data set and in the quarterly postwar data (Tables 2 and 4), the tests do not reject the unit root hypothesis when a high order autoregression (of 8 or 12 lags) is considered.

The ADF tests indicate a unit root on three-month Treasury bills for the postwar period, although this hypothesis cannot be rejected for ex post real returns only with a higher order autoregression (Table 3). An auto-regression on the level data for ex post real returns on three-month Treasury bills indicated that the coefficients of the lag level terms summed to unity over a period of about ten lags which is consistent with the presence of a large moving average parameter. Real three-month holding period yields on equity appear to be I(0) under an ex post definition; they also appear to be I(0) under the other hypotheses of expectations formation with 8 lags in the ADF test but not with 12 lags.

1/ Dickey and Fuller have shown that asymptotically the unit root "t test" reported in these tables is not affected by estimation of higher order autoregressive parameters. Dickey and Fuller also proposed a test, $T * \beta$, which is more powerful against the alternative that $(\beta-1) < 1$ than the t test. This statistic would have to be scaled by a constant c where c is a function of the moving average parameter when the process is the general ARIMA model. Schwert's simulation results suggest, however, that the t test is less sensitive to model misspecification and hence only the t results of the test are reported in the tables.

2/ Note that the one-year returns on Treasury bills and commercial paper in Tables 1 and 2 are comparable to returns on one-year securities, rather than to returns on securities of shorter maturity.

3/ The ten-year rate in the Ibbotson and Sinquefield data covers 46 observations and hence does not justify the use of 12 lags according to the rule of thumb noted above. Moreover Schwert's simulation results suggest that an ADF test performs better with a ℓ_4 rule than with a ℓ_{12} rule when the sample size is below or at about 50. If 12 lags are included, however, the test statistics for ten-year real rates in the Ibbotson and Sinquefield data also fall below the critical value and the unit root hypothesis cannot be rejected for all long-term interest rates.

Inflation rates over different periods were also tested for unit roots. As discussed in Section II, since expected inflation may be viewed as actual inflation minus a stationary forecast error, actual and expected inflation rates are likely to be integrated of the same order. The results reported in Tables 1 to 4 indicate that higher order autoregressive ADF tests do not reject the unit root hypothesis for inflation rate over different periods and different samples, although ADF tests with four lags would reject the null hypothesis in some instances. 1/ Although Nelson and Plosser (1982) and Rose (1988) find inflation as measured by the CPI to be I(0), Schwert (1987) argues that the time series process followed by CPI inflation appears to have a large moving average coefficient, so that low order ADF and DF tests tend to falsely reject the unit root hypothesis. Rose uses lags of 2 for annual data, 4 for quarterly, and 12 for monthly data. He does find that inflation appears to have a unit root in the monthly data for the subsample 1959 to 1979, but that the result is sensitive to the choice of both lags and sample period.

Overall, the empirical evidence analyzed in this paper appears to support the hypothesis that real and nominal interest rates and inflation are I(1), although in some instances this conclusion depends on the processes containing moving average as well as autoregressive components. 2/ While the evidence on real and nominal returns on equity is mixed, it would be difficult to explain a priori, why returns on bonds and equities are not integrated of the same order.

IV. Long-Run Determinants of Real Interest Rates

In light of the evidence that real interest rate processes are nonstationary, it is legitimate to look for a long-term relationship among real rates and other variables in the economy. As there already exists an extensive literature on the determination of real interest rates, a structural model is set out below which attempts to capture some of the key variables identified in this literature within a consistent analytical framework.

1. A structural model

In a fully employed economy with a Cobb Douglas technology, the marginal product of capital can be expressed as a function of total factor

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1/ One-year and ten-year inflation rates are calculated as the geometric average of the increase in the CPI over one year and ten years, respectively.

2/ If the processes were assumed to be purely autoregressive and lags to the ADF test added only until the Durbin Watson statistic indicated no autocorrelation, the ADF test would reject the unit root hypothesis for many more real interest rates and for the three-month and one-year inflation rate.

productivity and the capital-labor ratio. Over the long run, the real return on equity may be expected to be a function of the marginal product of capital so that

$$r_s = r_s(\overset{+}{TFP}, \overset{-}{K/L})$$

where

r_s = real rate of return on equity
TFP = total factor productivity
K/L = capital labor ratio.

The bond rate is determined by the savings-investment balance and all variables are defined in terms of a single good. In a Metzler type savings function agents have a desired stock of wealth and save less when current wealth rises above desired wealth, and vice versa. With higher wealth, private savings fall and the real interest rate rises, other things equal. In addition, the life cycle theory of savings suggests that agents' marginal propensities to save differ depending on the stage of the life cycle they happen to be in at a given point in time. This implies that aggregate savings is a function, among other things, of the age structure of the population. Private savings would decline as the population ages because a greater proportion of people would be past their peak earning years. Hence, assuming a Metzler type savings function and including demographic variables to represent life cycle considerations,

$$(1) \quad S_p = \dot{W} = S_p(\overset{+}{D}, \overset{-}{W})$$

where

S_p = private savings
W = private wealth; (a dot above a variable denotes a rate of change).
D = demographic variables; the proportion of population in their earning years.

Private wealth is accumulated in the form of claims on capital (K), real balances (M), government bonds (B) and foreign bonds valued in terms of the domestic good (B*). Foreign bonds are imperfect substitutes for domestic bonds. Furthermore, it is assumed that in the long run expected exchange rate changes are equal to actual changes and that the demand for real balances is homogeneous of degree one with respect to real output. Using Walras law, the three independent asset demand functions may be expressed in terms of the real rates of return, the level of real private wealth and real output:

$$(2) \quad K^d = K(\overset{-}{r}_b, \overset{+}{r}_s, \overset{-}{r}^*, \overset{+}{W})$$

$$(3) \quad B^{*d} = B^*(\overset{-}{r}_b, \overset{-}{r}_s, \overset{+}{r}^*, \overset{+}{W})$$

$$(4) \quad M^d = M(\overset{-}{r}_b, \overset{-}{r}_s, \overset{-}{r}^*, \overset{+}{W})Q$$

$$(5) \quad W = K + B + B^* + M$$

where

r_b = real interest rate on bonds
r^* = real interest rate on foreign bonds in terms of the domestic good
Q = full employment output.

and

$$r^* = r^{**} + \dot{\pi}$$

where

r^{**} = foreign interest rate which is fixed in terms of the foreign good
π = real exchange rate between domestic and foreign goods.

The demand for physical capital is thus a positive function of the expected profitability of an additional unit of installed capital, given rates of return on other assets. The return to holding a foreign bond is an exogenous foreign interest rate and an endogenously determined change in the real exchange rate. Asset market equilibrium is given by

$$(6) \quad B^d = B_o + \dot{B}$$

$$(7) \quad M^d = M_o + \dot{M}$$

$$(8) \quad W = W_o + \dot{W}$$

Monetary policy can affect real interest rates in the short run to the extent that price adjustment is slow and the policy change is unanticipated. In some models money is not neutral because sticky prices and wages allow a higher rate of monetary expansion to raise transitory income and thereby increase the desired saving rate. 1/ In this model, a fall in real balances relative to GNP instantaneously creates an excess demand for real balances and, to the extent that money and bonds are close substitutes, an excess supply of bonds. Hence, bond prices fall and interest rates rise to restore equilibrium to asset markets.

The physical capital stock adjusts slowly over time due to adjustment costs. Thus

$$(9)\quad \dot{K} = \overset{+}{\lambda}(q) = \overset{(+)}{\lambda}(r_s/r_b)$$

$$\lambda(1) = 0$$

where

$\dot{K}$ = investment in physical capital
q = the present discounted value of future marginal products of installed capital

Government dissaving is reflected in increases in the supply of bonds and money, hence

$$-S_g = \dot{B} + \dot{M}$$

The savings-investment balance requires net increases in holdings of all assets to equal net savings in the economy. Hence,

$$S_g + S_p = \dot{K} + \dot{B}^*$$

$$\therefore S_p = \dot{K} + \dot{B}^* + \dot{B} + \dot{M} = \dot{W}$$

$$(10)\quad \dot{B}^* = \dot{W} - (\dot{K} + \dot{B} + \dot{M})$$

where

$\dot{B}^*$ = increase in net foreign assets, or the current account.

1/ See for example Mundell (1971); Barro and Sala i Martin, (1990).

The model has ten equations and ten endogenous variables: r_b, $\dot{\pi}$, K^d, M^d, B^{*d}, B^d, W, $\dot{K}$, $\dot{B}^*$, $\dot{W}$. Hence a reduced form expression for the real interest rate r_b at a given point in time may be obtained as

$$r_b = r_b\,(\overset{-}{D},\ \overset{+}{W},\ \overset{+}{B},\ \overset{-}{M/Q},\ \overset{-}{S_g},\ \overset{+}{r_s},\ \overset{+}{r^{**}})$$

or

$$r_b = r_b\,(\overset{-}{D},\ \overset{+}{W},\ \overset{+}{B},\ \overset{-}{M/Q},\ \overset{-}{Sg},\ \overset{+}{TFP},\ \overset{-}{K/L},\ \overset{+}{r^{**}})$$

Note that the aging of the population can affect the real interest rate in opposite directions. On one hand, D, the proportion of the population in their earning years would decline, reduce private saving and increase the real interest rate; on the other hand, K/L, the capital-labor ratio would rise, reduce the marginal product of capital and the demand for investment in physical capital and lower the real interest rate. The model assumes that larger government deficits raise real interest rates because private agents do not completely offset these deficits with increases in private savings as suggested by Ricardian equivalence in its extreme form. In addition, while in this model the impact of fiscal policy works through S_g, B and W, in other models it could also work through r_s and r^{**}. Blanchard (1981), for instance, demonstrates the effect of fiscal policy on output, the stock market and real interest rates while two country models (Dornbusch (1980)) show that fiscal policy in a large country affects foreign interest rates.

2. Estimation

The cointegrating regressions were carried out on the basis of the two reduced form expressions for the real interest rate derived in the previous section. The real interest rate would be expected to be a negative function of a demographic variable, the ratio of real money to GNP, the fiscal balance, and the capital-labor ratio and a positive function of private wealth, total factor productivity, and the rate of return on foreign assets. The real return on equity which would be positively related to the real interest rate could be substituted for total factor productivity and the capital-labor ratio. Several variables which theory has shown to be important for the determination of real rates had to be excluded out of necessity. For example, parameters of consumer preferences such as the rate of time preference and the degree of risk aversion are inherently unobservable, while concepts such as the timing and uncertainty associated with endowment income and the tax aspects of the structure of asset returns are difficult to capture empirically.

The demographic variable was initially measured as the percentage of population over the age of 65. However, this variable appeared to be highly collinear with the private wealth term so the proportion of the total population in the labor force was used instead. A lower labor force participation rate suggests a smaller number of people in their earning years, and all other things equal, lower private saving and a higher real rate of interest. It must be recognized, of course, that labor force participation is influenced by other factors, such as the increasing entry of women into the workforce, which may have ambiguous effects on private saving and the real interest rate.

The private wealth variable was calculated as the sum of the value of physical capital, private net foreign assets and base money at constant prices. Since the Ricardian equivalence hypothesis would suggest that private agents do not view government bonds as a part of private wealth, government debt was entered as a separate variable. On account of the different time series property of this variable, real government debt was expressed as a ratio to real GNP. 1/ Government debt was calculated as federal debt outstanding (excluding holdings by Federal Reserve banks) at constant prices. 2/ For purposes of estimation, the government savings variable was expressed as the primary balance of the federal government in real terms. 3/ Real balances were defined as M2 at constant prices and were divided by real GNP.

Measures of total factor productivity and capital labor ratios in the private nonfarm business sector were used directly in some regressions as indicators of the marginal product of capital. 4/ The regressions were also run substituting measures of stock market returns in place of these two

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1/ The stock of real federal government debt appeared to be I(2). Its ratio relative to real GNP appeared to be I(1), however, and was used instead. Correspondingly, private wealth including government debt appeared to be I(2), but was I(1) when government debt was excluded.

2/ A continuous series was not available for the debt of state and local governments for the entire sample period.

3/ This variable appeared to be borderline in terms of being I (1) in that the DF Statistic was -3.64 and the ADF Statistic at 8 lags was -2.66. The total balance was clearly I(1). However, the primary balance appeared to be more appropriate for use as an exogenous variable since there would be some simultaneity between the total balance and the interest rate. In any event, the results were not significantly different when the total federal government balance was used instead. Some of the regression results reported in Tables 5-8 use the total rather than the primary balance.

4/ The index of total factor productivity from 1960-88 was based on previous staff work; see Appendix XIII, SM/89/176.

variables. 1/ As Hayashi (1982) has noted, stock market data give average rates of return while investment demand depends on the marginal return on installed capital.

An additional variable, the price of oil relative to the non-oil producer price index was also considered on the basis of arguments in the literature that this was an important determinant of real interest rates in the 1970s and 1980s. (See for example, Fried and Burgess (1989), Barro and Sala i Martin (1990)). The price of oil could, a priori, have ambiguous effects on real rates. On one hand, it would lower the marginal product of capital and reduce investment demand and hence, real interest rates. The size of this effect would depend in part on the substitutability of oil for other factors of production. On the other hand, a rise in oil prices may be perceived as a temporary fall in income and thus lead to lower savings and higher interest rates in the short run.

Given the openness of the U.S. economy, real interest rates could also be influenced by returns on foreign assets. In practice, because U.S. interest rates clearly affect world interest rates, a problem of simultaneity arises in using a world interest rate as a measure of the return from foreign assets. Hence, the rate of return on world stocks, which are less substitutable for U.S. bonds, was used instead. Following Barro and Sala i Martin (1990), this was approximated by a GNP weighted index of industrial share prices in nine OECD countries converted into U.S. dollars and deflated by the CPI. 2/

All variables were tested to be I(1) before being used in the cointegrating regressions. The regression results are reported in Table 5-8 for three-month, one-year, and ten-year bonds under different hypotheses about expectations formation. Cointegration tests in the form of DF, ADF, and Sargan-Bhargava (SB) tests are presented with each regression. 3/ Plots of actual and estimated long-run values for some of these ex post real rates are shown in Charts 4 and 5. The charts demonstrate that the estimated equations capture the major swings in short- and long-term rates

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1/ These holding period yields were calculated as described in Section II. Unless otherwise noted, the cointegrating regression in Tables 5-8, use the univariate method to forecast dividends and stock prices. Unit root tests reported in Tables 3 and 4 indicate that three month holding period yields calculated under univariate and bivariate methods and ex post one year real holding period yields may be borderline in terms of being I(1).

2/ Barro and Sala i Martin use the change in the index; however, the index itself appeared to be I(1) and the change in the index to be I(0), and thus not appropriate for use in a cointegrating regression.

3/ The critical values of these tests at a 5 percent significance level are approximately DF = 3.37; ADF = 3.13; and SB = 0.37. The SB test is simply the Durbin-Watson statistic under the null hypothesis that the autocorrelation parameter equals one rather than zero.

Table 5. Cointegrating Regressions on Real Three-Month Treasury Bill Rates

Ex post real rates 1/

(1) r3M = 294.98 - 4.41D + 0.02W + 0.42(B/GNP) - 0.89(M/GNP) - 0.02SG - 5.21ln(K/L)

R^2 = 0.58 DF = -6.8 ADF(4) = -3.6 SB = 1.1
F(6,117) = 27.1 ADF(8) = -3.7

(2) 2/ r3M = 243.73 - 3.71D + 0.008W + 0.38(B/GNP) - 0.84(M/GNP) - 0.02SG + 0.10PO + 20.85ln(TFP)

R^2 = 0.64 DF = -7.1 ADF(4) = -3.7 SB = 1.2
F(7,112) = 27.9 ADF(8) = -3.7

(3) 3/ r3M = 188.77 - 3.0D + 0.008W + 0.22(B/GNP) - 0.58(M/GNP) - 0.02SG + 0.07PO + 0.16RSL

R^2 = 0.66 DF = -7.8 ADF(4) = -4.6 SB = 1.4
F(7,116) = 32.0 ADF(8) = -4.7

(4) r3M = 258.20 - 4.07D + 0.01W + 0.37(B/GNP) - 0.88(M/GNP) - 0.02SG + 0.05RSM

R^2 = 0.60 DF = -7.0 ADF(4) = -3.7 SB = 1.2
F(6,117) = 28.8 ADF(8) = -3.9

Forward looking forecast 4/

(5) r3M = 280.25 - 3.69D + 0.02W + 0.34(B/GNP) - 0.56(M/GNP) - 0.02SG - 16.20ln(K/L)

R^2 = 0.48 DF = -7.3 ADF(4) = -3.4 SB = 1.2
F(6,118) = 18.1 ADF(8) = -3.0

(6) r3M = 203.35 - 3.04D + 0.01W + 0.36(B/GNP) - 0.79(M/GNP) - 0.03SG + 19.45ln(TFP)

R^2 = 0.47 DF = -7.1 ADF(4) = -3.4 SB = 1.2
F(6,113) = 16.7 ADF(8) = -3.2

(7) r3M = 115.30 - 1.92D + 0.005W + 0.14(B/GNP) - 0.28(M/GNP) - 0.02SG + 0.06PO + 0.18RSL

R^2 = 0.57 DF = -8.4 ADF(4) = -4.4 SB = 1.5
F(7,117) = 22.5 ADF(8) = -3.8

(8) r3M = 173.65 - 2.9D + 0.008W + 0.28(B/GNP) - 0.42(M/GNP) - 0.01SG + 0.07PO + 0.06RSM

R^2 = 0.52 DF = -7.9 ADF(4) = -3.5 SB = 1.4
F(7,117) = 18.3 ADF(6) = -4.0
ADF(8) = -2.9

(9) r3M = 203.90 - -3.36D + 0.01W + 0.35(B/GNP) - 0.62(M/GNP) - 0.02SG + 0.01RSS
R^2 = 0.46 DF = -7.2 ADF(4) = -3.2 SB = 1.2
F(6,118) = 16.9 ADF(6) = -3.9
ADF(8) = -3.0

Static expectations 4/

(10) r3M = 296.80 - 3.79D + 0.02W + 0.36(B/GNP) - 0.55(M/GNP) - 0.02SG - 19.23ln(K/L)

R^2 = 0.50 DF = -6.3 ADF(4) = -3.3 SB = 1.0
F(6,118) = 19.9 ADF(8) = -3.0

(11) 2/ r3M = 173.31 - 2.67D + 0.003W + 0.32(B/GNP) - 0.54(M/GNP) - 0.02SG + 0.13PO + 26.78ln(TFP)

R^2 = 0.58 DF = -6.6 ADF(4) = -3.4 SB = 1.1
F(7,112) = 22.3 ADF(6) = -3.2
ADF(8) = -3.0

(12) r3M = 114.95 - 1.78D + 0.006W + 0.12(B/GNP) - 0.40(M/GNP) - 0.03SG + 0.22RSL

R^2 = 0.59 DF = -6.8 ADF(4) = -4.4 SB = 1.1
F(6,118) = 28.3 ADF(8) = -3.9

(13) r3M = 190.38 - 3.11D + 0.01W + 0.31(B/GNP) - 0.59(M/GNP) - 0.02SG + 0.06RSM

R^2 = 0.50 DF = -6.3 ADF(4) = -3.4 SB = 1.0
F(6,118) = 20.0 ADF(6) = -3.4
ADF(8) = -3.0

(14) r3M = 204.55 - 3.38D + 0.01W + 0.36(B/GNP) - 0.62(M/GNP) - 0.02SG + 0.02RSS

R^2 = 0.49 DF = -6.2 ADF(4) = -3.2 SB = 1.0
F(6,118) = 18.9 ADF(6) = -3.3
ADF(8) = -2.9

1/ Sample period: 1959(1)-1989(4).
2/ Sample period: 1959(1)-1988(4)
3/ Fitted values from this regression are plotted in Chart 4.
4/ Sample period: 1959(1)-1990(1).

Table 6. Cointegrating Regressions on Real One-Year Treasury Security Rates

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Ex post real rates 1/

(1) 2/ rlY = 311.89 - 4.77D + 0.01W + 0.43(B/GNP) - 1.13(M/GNP) - 0.02SG + 0.06PO + 6.01 ln(TFP)

R^2 = 0.76 DF = -4.8 ADF(4) = -4.0 SB = 0.7
F(7, 112) = 50.6 ADF(6) = -3.6
ADF(8) = -2.7

(2) 3/ rlY = 273.28 - 4.17D + 0.01W + 0.31(B/GNP) - 0.99(M/GNP) - 0.02SG + 0.05PO + 0.10RSL

R^2 = 0.78 DF = -5.1 ADF(4) = -4.5 SB = 0.7
F(7, 113) = 55.9 ADF(8) = -3.1

(3) rlY = 317.43 - 4.89D + 0.02W + 0.43(B/GNP) - 1.17(M/GNP) - 0.02SG + 0.03RSMA 4/

R^2 = 0.76 DF = -4.6 ADF(4) = -4.0 SB = 0.6
F(6, 114) = 59.5 ADF(6) = -3.6
ADF(8) = -2.8

(3)' rlY = 303.71 - 4.76D + 0.01W + 0.41(B/GNP) - 1.03(M/GNP) - 0.01SG + 0.05PO + 0.03 RSMA 4/

R^2 = 0.77 DF = -4.7 ADF(4) = -3.9 SB = 0.6
F(6, 114) = 54.3 ADF(6) = -3.4
ADF(8) = -2.7

Forward looking forecast 5/

(4) rlY = 263.21 - 3.37D + 0.01W + 0.29(B/GNP) - 0.60(M/GNP) - 0.03SG - 14.65 ln(K/L)

R^2 = 0.51 DF = -6.3 ADF(4) = -4.5 SB = 1.0
F(6,118) = 20.7 ADF(8) = -3.7

(5) 2/ rlY = 192.34 - 2.76D + 0.01W + 0.01(B/GNP) - .80(M/GNP) - 0.04SG + 17.67 ln(TFP)

R^2 = 0.50 DF = -6.1 ADF(4) = -4.4 SB = 1.0
F(6, 113) = 19.2 ADF(8) = -4.0

(6) rlY = 124.70 - 1.94D + 0.01W + 0.13(B/GNP) - 0.40(M/GNP) - 0.03SG + 0.04PO + 0.14RSL

R^2 = 0.56 DF = -6.5 ADF(4) = -5.4 SB = 1.0
F(7,117) = 21.3 ADF(8) = -4.4

(7) rlY = 166.93 - 2.68D + 0.01W + 0.23(B/GNP) - 0.50(M/GNP) - 0.02SG + 0.05PO + 0.07RSM

R^2 = 0.54 DF = -6.7 ADF(4) = -4.7 SB = 1.1
F(7,117) = 19.8 ADF(8) = -3.8

Static expectations 5/

(8) rlY = 303.75 - 3.20D + 0.02W + 0.30(B/GNP) - 0.30(M/GNP) - 0.01SG - 32.83 ln(K/L)

R^2 = 0.53 DF = -3.8 ADF(4) = -3.6 SB = 0.42
F(6,118) = 21.7 ADF(6) = -3.7
ADF(8) = -2.4

(9) rlY = 70.86 - 1.16D + 0.004W + 0.11(B/GNP) - 0.22(M/GNP) - 0.01SG + 0.19RSL

R^2 = 0.53 DF = -3.9 ADF(4) = -4.1 SB = 0.5
F(6,118) = 21.9 ADF(8) = -3.1

(10) rlY = 138.07 - 2.35D + 0.01W + 0.29(B/GNP) - 0.39(M/GNP) - 0.01SG + 0.04RSM

R^2 = 0.44 DF = -3.5 ADF(4) = -3.5 SB = 0.4
F(6,118) = 15.3 ADF(6) = -3.4
ADF(8) = -2.6

1/ Sample period: 1959(1)-1989(1).
2/ Sample period: 1959(1)-1988(4).
3/ Fitted values from this regression are plotted in Chart 4.
4/ Ex post one year real returns on equities.
5/ Sample period: 1959(1)-1990(1).

Table 7. Cointegrating Regressions on Real Ten-Year Treasury Securities

Ex post real rates 1/

(1) r10Y = 82.97 - 0.74D + 0.01W + 0.24(B/GNP) - 0.51(M/GNP) - 0.005SG + 0.05PO - 7.36 ln(K/L)

R^2 = 0.93 DF = -3.9 ADF(4) = -3.4 SB = 0.7
F(7,77) = 148.7 ADF(6) = -2.9
ADF(8) = -2.8

(2) 2/ r10Y = 44.30 - 0.46D + 0.004W + 0.30(B/GNP) - 0.59(M/GNP) - 0.005SG + 0.06PO + 10.52ln(TFP)

R^2 = 0.94 DF = -4.3 ADF(4) = -3.6 SB = 0.8
F(7,77) = 158.8 ADF(8) = -3.2

(3) r10Y = 45.44 - 0.54D + 0.003W + 0.19(B/GNP) - 0.49(M/GNP) - 0.003SG + 0.09PO + 0.03RSL

R^2 = 0.93 DF = -4.1 ADF(4) = -3.2 SB = 0.7
F(7,77) = 146.8 ADF(6) = -2.8
ADF(8) = -2.9

Forward looking forecast 3/

(4) r10Y = 271.96 - 3.20D + 0.02W + 0.26(B/GNP) - 0.44(M/GNP) - 0.03SG - 21.80 ln(K/L)

R^2 = 0.48 DF = -6.8 ADF(4) = -5.2 SB = 1.1
F(6,118) = 18.1 ADF(8) = -5.0

(5) r10Y = 150.45 - 2.61D + 0.007W + 0.27(B/GNP) - 0.33(M/GNP) - 0.02SG + 0.08PO

R^2 = 0.49 DF = -6.6 ADF(4) = -5.2 SB = 1.0
F(6,118) = 18.6 ADF(8) = -4.9

Static expectations 3/

(6) 4/ r10Y = 102.10 - 1.33D + 0.003W + 0.15(B/GNP) - 0.48(M/GNP) + 17.32ln(TFP)

R^2 = 0.50 DF = -3.9 ADF(4) = -3.7 SB = 0.5
F(5,114) = 23.0 ADF(6) = -2.9
ADF(8) = -2.7

(7) r10Y = 143.94 - 1.80D + 0.01W + 0.15(B/GNP) - 0.43(M/GNP) - 0.002SGT 5/ - 0.05PO - 6.78 ln(K/L)

R^2 = 0.50 DF = -3.8 ADF(4) = -3.9 SB = 0.4
F(7,117) = 16.9 ADF(6) = -3.2
ADF(8) = -2.5

(8) r10Y = 47.91 - 0.55D + 0.003W - 0.28(M/GNP) - 0.003SGT 5/ - 0.03PO + 0.11RSL

R^2 = 0.57 DF = -4.2 ADF(4) = -5.4 SB = 0.5
F(6,118) = 26.2 ADF(8) = -3.3

(9) r10Y = 75.24 - 1.05D + 0.005W + 0.06(B/GNP) - 0.33(M/GNP) - 0.03PO + 0.09RSL

R^2 = 0.58 DF = -4.2 ADF(4) = -5.1 SB = 0.5
F(6,118) = 27.6 ADF(8) = -3.2

1/ Sample period: 1959(1)-1980(1).
2/ Fitted values from this regression are plotted in Chart 5.
3/ Sample period: 1959(1)-1990(1).
4/ Sample period: 1959(1)-1988(4).
5/ The real total federal budget deficit is used as the fiscal indicator.

Table 8. Cointegrating Regressions on AAA Corporate Bonds

Ex post real rates 1/

(1) $raaa = 54.79 - 0.60D + 0.007W + 0.31(B/GNP) - 0.52(M/GNP) - 0.007SG - 3.06 \ln(K/L)$

$R^2 = 0.95$	DF = -4.0	ADF(4) = -3.3	SB = 0.8
F(6,78) = 246.7		ADF(6) = -2.9	
		ADF(8) = -2.6	

(2) $raaa = 38.20 - 0.50D + 0.006W + 0.34(B/GNP) - 0.54(M/GNP) - 0.006SG + 2.88 \ln(TFP)$

$R^2 = 0.95$	DF = -4.1	ADF(4) = -3.3	SB = 0.8
F(6,78) = 242.9		ADF(6) = -2.9	
		ADF(8) = -2.6	

(3) 2/ $raaa = 42.12 - 0.52D + 0.004W + 0.23(B/GNP) - 0.49(M/GNP) - 0.006SG + 0.05PO + 0.02RSL$

$R^2 = 0.96$	DF = -4.3	ADF(4) = -3.3	SB = 0.8
F(7,77) = 234.3		ADF(8) = -3.2	

(4) $raaa = 40.14 - 0.47D + 0.004W + 0.20(B/GNP) -0.46(M/GNP) - 0.004SG + 0.04PO + 0.04RSLA$ 3/

$R^2 = 0.95$	DF = -3.7	ADF(4) = -3.2	SB = 0.8
F(7,77) = 231.7		ADF(8) = -3.3	

Forward looking forecast 4/

(5) $raaa = 232.29 - 3.02D + 0.01W + 0.27(B/GNP) - 0.40(M/GNP) - 0.04SG - 14.6 \ln(K/L)$

$R^2 = 0.50$	DF = -6.8	ADF(4) = -5.0	SB = 1.1
F(6,118) = 20.3		ADF(8) = -5.0	

(6) $raaa = 141.02 - 2.43D + 0.006W + 0.24(B/GNP) - 0.28(M/GNP) - 0.02SG + 0.09PO + 5.43\ln(TFP)$

$R^2 = 0.52$	DF = -6.6	ADF(4) = -5.0	SB = 1.1
F(7,112) = 17.1		ADF(8) = -5.0	

Static expectations 4/

(7) 5/ $raaa = 103.05 - 1.43D + 0.005W + 0.15(B/GNP) - 0.46(M/GNP) - 0.006SGT$ 6/ $- 0.04PO + 9.09\ln(TFP)$

$R^2 = 0.49$	DF = -3.5	ADF(4) = -3.5	SB = 0.4
F(7,112) = 15.1		ADF(6) = -3.0	
		ADF(8) = -2.5	

(8) $raaa = 71.05 - 1.0D + 0.005W + 0.06(B/GNP) - 0.30(M/GNP) - 0.005SGT$ 6/ $- 0.05PO + 0.09RSL$

$R^2 = 0.61$	DF = -3.8	ADF(4) = -4.8	SB = 0.4
F(7,117) = 26.5		ADF(6) = -4.1	
		ADF(8) = -2.8	

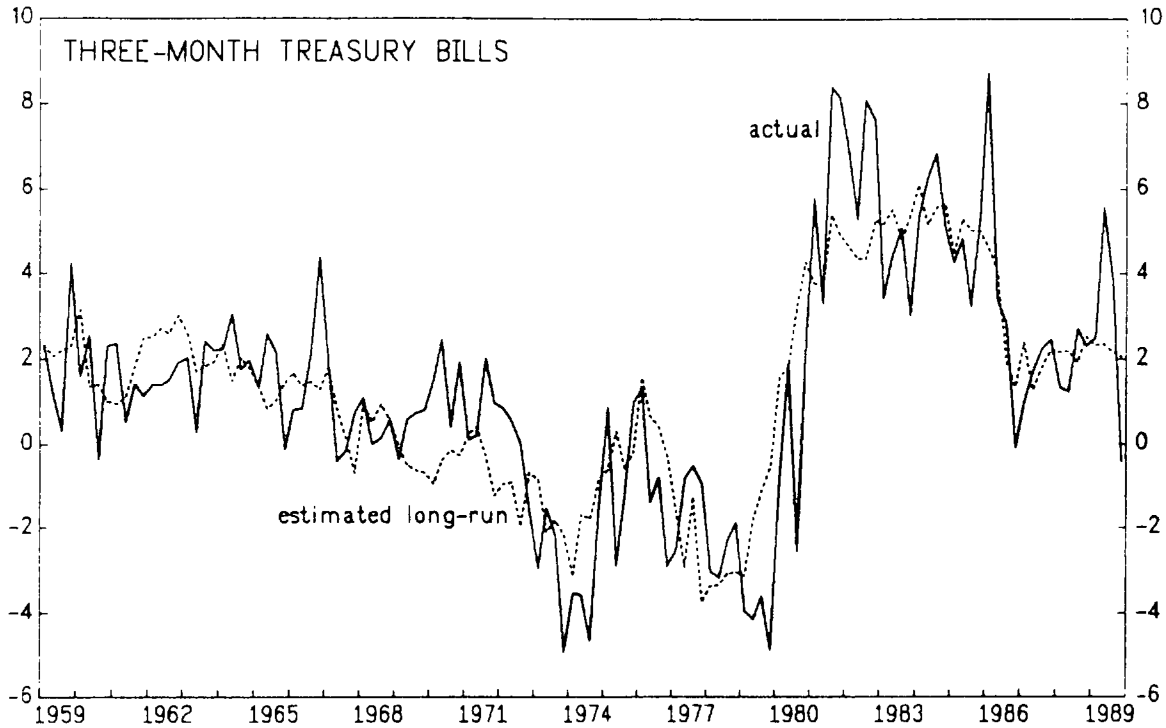
(9) $raaa = 99.0 - 1.49D + 0.006W + 0.13(B/GNP) - 0.38(M/GNP) - 0.003SGT$ 6/ $- 0.05PO + 0.03RSL$

$R^2 = 0.53$	DF = -3.4	ADF(4) = -3.7	SB = 0.4
F(7,117) = 18.7		ADF(6) = -3.2	
		ADF(8) = -2.5	

1/ Sample period: 1959(1)-1980(1).
2/ Fitted values from this regression are plotted in Chart 5.
3/ Ex post ten year real returns on equities.
4/ Sample period: 1959(1)-1990(1).
5/ Sample period: 1959(1)-1988(4).
6/ The real total federal budget deficit is used as the fiscal indicator.

CHART 4

ACTUAL AND ESTIMATED REAL INTEREST RATES



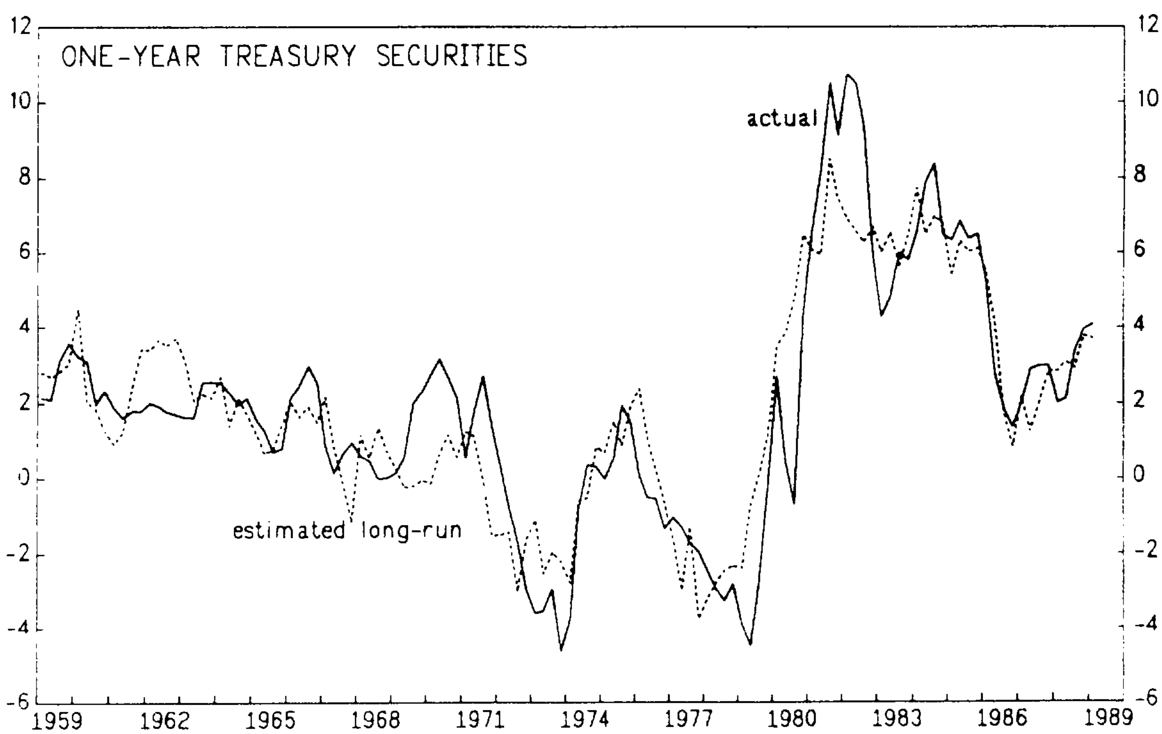
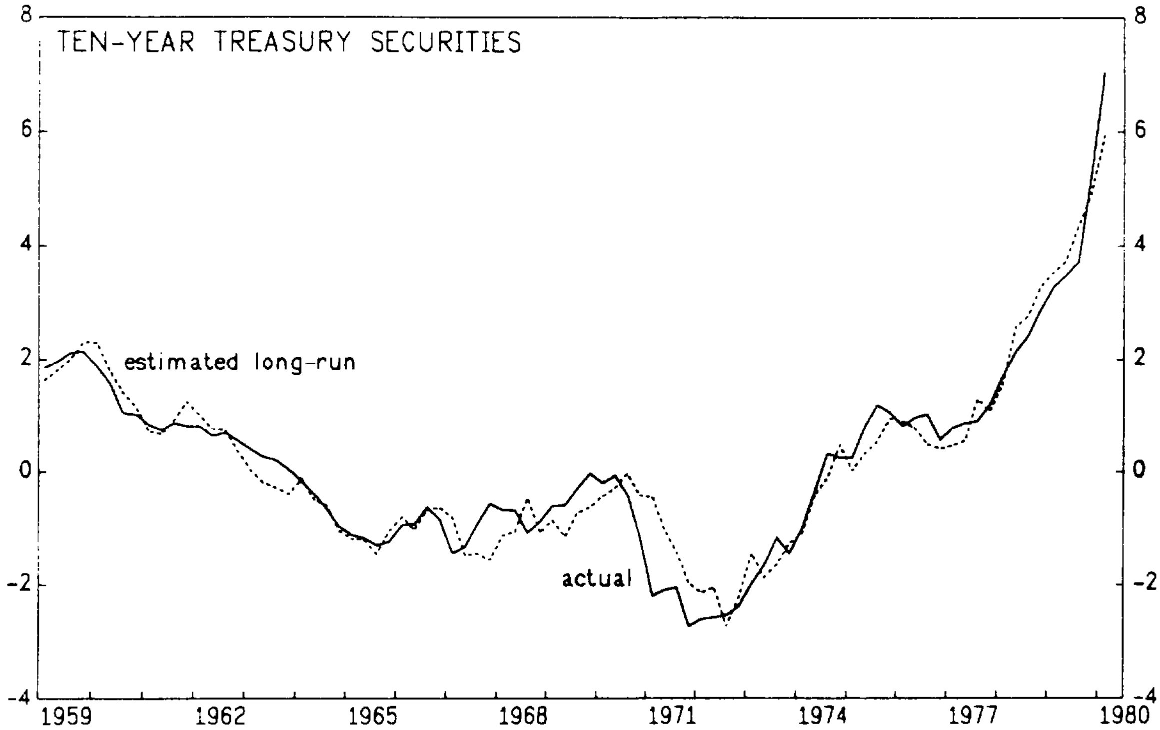
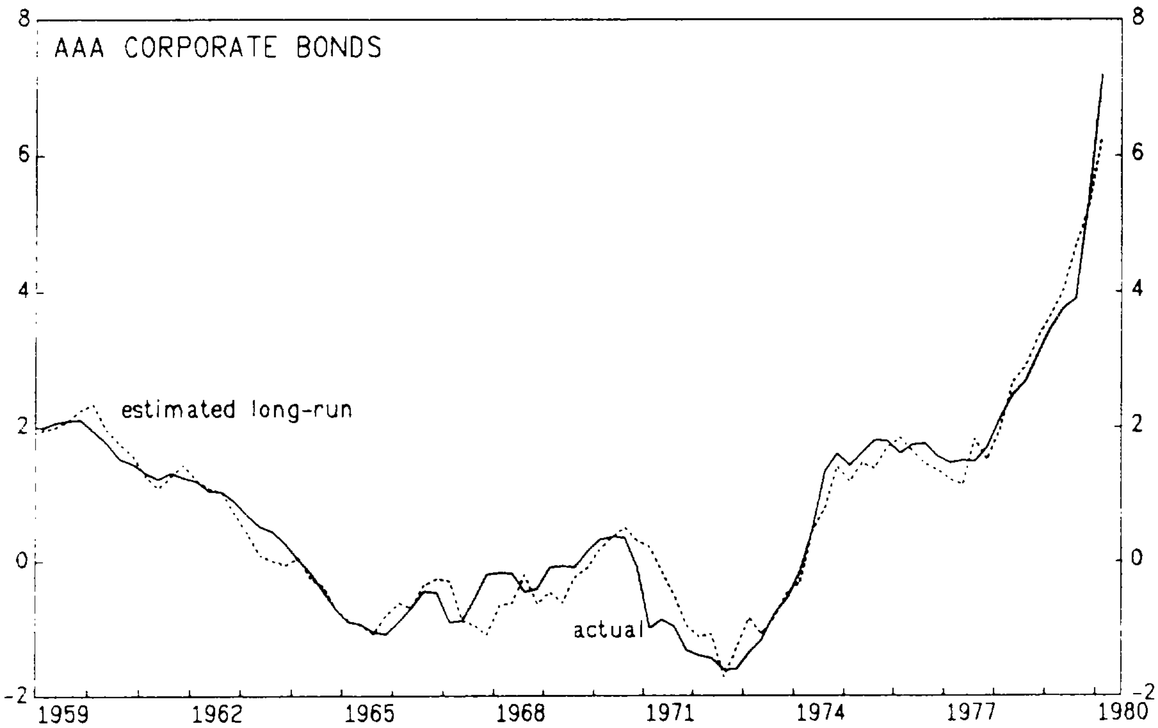


CHART 5

ACTUAL AND ESTIMATED REAL INTEREST RATES





very closely, particularly the surge in the early 1980s and the decline in 1985-86. It is interesting that these long-run regressions which only include level terms and hence exclude all short-run dynamics are able to capture so much of the movement in real interest rates.

The demographic variable, private wealth, government debt relative to GNP, and real balances relative to real GNP were found to affect real interest rates on bonds of all maturities, under the three different hypotheses of expectations formation considered (Tables 5-8). Demographic factors (D) appear to have a strong effect on interest rates. For example, the regressions indicate that a 1 percentage point increase in the proportion of population in the labor force would decrease three-month and one-year rates by approximately 2 to 4 percentage points and decrease ten-year rates by about 1/2 to 3 percentage points over the long run. The estimated coefficient on private wealth net of government debt (W) is fairly stable over the different specifications and definitions of the real rate. A $100 billion increase in real wealth (in 1982 prices) is estimated to increase both short- and long-real interest rates by about 1/2-1 percentage point.

Monetary "tightness" as measured by the ratio of real M2 to real GNP (M/GNP) has a stronger effect on short-term rates than on long-term rates, although this parameter estimate is sensitive to the specification of expectations formation. Short rates may be expected to be more responsive than long rates if money and short-term bonds were closer substitutes than money and long-term bonds. For three-month rates, a permanent 1 percentage point increase in the ratio of real balances to GNP reduces real rates by about 80-90 basis points under an ex post definition and by about 30-60 basis points under forward forecasts or static expectations of inflation. For one-year rates, there appears to be a very strong one to one relationship with M/GNP under an ex post definition, although this estimate is markedly lower under other hypotheses of expectations formation. For long-term rates, a 1 percentage point increase in M/GNP lowers real interest rates by about 50-60 basis points under an ex post definition and by about 30-40 basis points under forward forecasts or static expectations of inflation. Given the actual movements in the ratio of real balances to GNP, this suggests (in partial equilibrium) that monetary tightness contributed significantly to the rise in real rates in the early 1980s. This effect was entirely reversed by the mid-1980s but monetary tightening appears to have raised real interest rates again in the late 1980s.

The regressions also suggest a strong impact of fiscal policy on real interest rates through current deficits and the accumulation of government debt. 1/ The coefficient estimates for the fiscal balance (SG) are quite

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1/ As noted above, fiscal policy could also affect real interest rates through its impact on stock market returns and world interest rates. In the case of regressions where stock returns are included as an explanatory variable, the coefficients on SG and B/GNP should be interpreted as the effect of deficits and debt on real interest rates given the real return on equity.

stable for three-month real rates under different hypotheses of expectations formation. They suggest that a $10 billion real increase in the primary fiscal deficit (in 1982 prices) would raise short-term interest rates by 20-30 basis points. A similar increase in the primary deficit would raise one year rates by 10-20 basis points under ex post and static definitions and by 20-40 basis points under a forward forecast. The evidence on the contribution of deficits to ten-year interest rates is mixed and the effect on the whole appears to be weaker. The variable has a very small coefficient under an ex post definition of long-term real rates (although this sample period necessarily excludes the 1980s) and does not, in some instances, appear to be statistically significant under static expectations of inflation. However, with a forward forecast of inflation, the fiscal balance variable has a coefficient of a size comparable to that found for three-month and one-year rates. While the errors associated with forecasting inflation over a ten-year period and the consequent difficulty of obtaining accurate measures of ten-year rates should be noted, deficits may be expected to have a greater impact on short-term rather than long-term rates when short- and long-term bonds are not perfect substitutes.

Fiscal deficits also have a cumulative effect on real interest rates through the growth of government debt. The regressions indicate that a 1 percentage point increase in the federal debt to GNP ratio (B/GNP) generally raises three-month interest rates by about 30-40 basis points. One-year rates rise by a similar magnitude under an ex post definition and by about 10-30 basis points under a forward forecast or static expectations. Long time rates increase by 20-30 basis points except under static expectations of inflation where they increase by about 5-15 basis points. Looking at the actual movements in the data, fiscal policy appears to have contributed significantly to the rise in interest rates in the 1980s both through increases in the size of real deficits and through the accompanying increase in government debt.

Shifts in investment demand were captured through the capital-labor ratio and total factor productivity, or by real returns on holding equities, and/or by the price of oil relative to the non-oil PPI. Not all these factors could be present in a given equation because of collinearity in the data, hence separate estimates are reported in Tables 5-8. The evidence on the effect of the capital-labor ratio is varied, with a 1 percent increase in the capital-labor ratio (K/L) estimated to lower real rates of all maturities by 5-10 basis points (particularly under an ex post definition) or by about 20 basis points. 1/ A 1 percent increase in total factor productivity (TFP) is estimated to raise three-month and one-year rates by 15-25 basis points and long term rate by 5-10 basis points, although some exceptions were observed. When estimates of the real return on equity were used, a relatively small response of interest rates to changes in three-month, one-year and the ten-year holding period yields on equities (RSS, RSM

1/ As discussed above, for a given capital stock, the rise in real rates due to a decrease in labor force participation may be offset to some extent by the rise in the capital-labor ratio.

and RSL, respectively) was found. Three-month real rates rise by 1-2 basis points for a 1 percentage point increase in the three-month holding period yields; three-month and one-year real rates rise by about 5 basis points for similar increase in one-year holding period yields and by about 10-20 basis point for a similar increase in ten-year holding period yields. Ten-year interest rates rise by about 2-10 basis point in response to a 1 percentage point increase in ten-year holding period yields on equities.

When the relative price of oil (PO) is included in the cointegrating regression, it appears with a positive sign for interest rates of all maturities except ten-year rates under static expectations. A 10 index point increase in the relative price of oil increases all real interest rates by about 1/2-1 percentage point, except the ten-year rates under static expectations which decline by 30 to 50 basis points. The negative effect on long-term rates is consistent with a scenario where a rise in oil prices reduces the marginal product of capital, lowers returns on equity, and hence long-term interest rates. This effect may be compounded by an increase in private savings due to the reduction in private sector wealth. The positive relationship between real rates and the relative price of oil over the long run is more difficult to explain, although the result is similar to those found by Barro and Sala i Martin with respect to both U.S. and world interest rates. An argument could be made that a rise in the price of oil temporarily lowers income, reduces private saving, and increases real rates in the short run. With frequent shocks to the price of oil, the data show a positive relationship with short rates over long periods of time even though each shock is viewed as a temporary change in income.

The index of world stock market prices did not appear to be significant in the determination of U.S. interest rates. This result should be viewed with some caution since the stock market variable had to be defined in level rather than change terms because the first difference in the index was clearly I(0). While the implication is that changes in share prices in the rest of the world may have a temporary but not a permanent effect on real rates, the index may not be an adequate measure of the return from claims on foreign capital.

A significant feature of these results is the strong presence of fiscal variables as determinants of the real rate and hence, an implied rejection of the strict Ricardian debt equivalence view. This result needs to be qualified to the extent that world interest rates, and hence fiscal deficits in the rest of the world, are not fully captured in the estimates. Blanchard and Summers (1984), for example, argue against a strong fiscal effect on U.S. interest rates by showing that the U.S. fiscal expansion in the 1980s was offset to a significant extent by fiscal contractions in other nations. (This assumes that U.S. and foreign government bonds are perfect substitutes.) However, when the regressions in Tables 5-8 were run for the sample period excluding the 1980s, fiscal deficits still appeared to be significant.

Even in a closed economy setting, the link between fiscal variables and real rates has usually been difficult to establish in the literature and there is no consensus as to whether current or expected future government deficits affect real interest rates. 1/ Results in the literature tend not to be robust in that they are sensitive to changes in specification, sample periods, and/or choice of fiscal measures. In general, there appears to be some support for the effects of fiscal deficits on long-term rather than short-term interest rates and for a link between interest rates and government debt, rather than deficits. 2/ However, in these cointegrating regressions, the deficit and debt variables appeared to be significant under different definitions of these variables and specifications of the model (this is partly evident in Tables 5-8). Cointegrating regressions were run with deficits defined as the total federal government deficit, the total government deficit (including state and local government), the change in the real value of government debt and with debt defined to include total federal government debt issued and federal government debt held by the public (excluding government holdings). 3/ The deficit variable was not cyclically adjusted because this often leads to problems of interpretation; estimation results may, for example, be sensitive to the type of adjustment made. Moreover, the review of the literature by the CBO noted above indicates that studies that use unadjusted deficits are less likely to find a statistically significant relationship with interest rates. It is difficult to compare the results in this paper directly with other results in the literature because of differences in model specification. For example, many models appear to have been specified with a mixture of I(1) and I(0) variables and many do not include demographic factors and shifts in the productivity of capital. However, some of the empirical work on private savings suggests that deficit increases are only partially matched by an increase in private saving, implying that government deficits and debt have effects on real interest rates. 4/

The finding that real balances, oil prices, and stock market variables are cointegrated with real interest rates is consistent with the literature. Blanchard and Summers, find some evidence that monetary tightening is partly responsible for the high real interest rates observed in the early 1980s as do Barro and Sala i Martin. Both sets of authors also find evidence to support the hypothesis that expected profitability increased in the early 1980s beyond what might have been expected from cyclical factors and contributed to the rise in real interest rates. They argue that such a shift can explain both high real interest rates and strong real stock prices in the 1980s. In addition, Barro and Sala i Martin find a significant role for the relative price of oil in explaining real interest rate movements in the

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1/ This literature is extensive. For a survey see Congressional Budget Office (1989).

2/ See also Seater (1985).

3/ Since these debt variables appeared to be I(2), a debt to GDP ratio which was I(1) was always used.

4/ This literature is also extensive; for a survey see Bernheim (1987); Leiderman and Blejer (1988).

1970s and 1980s. Fried and Burgess (1989) also argue that a heightened level of uncertainty associated with the 1973-74 and the 1979-80 oil shocks increased the demand for safe assets and lowered the rate of return on the risk-free assets; conversely, the fall in oil prices reduced uncertainty and raised expectations of future income after 1982.

V. Conclusion

The presence of a unit root in real interest rates indicates that real rates do not have a tendency to revert to a constant mean over time. The nonstationarity of real interest rates raises the question of which economic variables influence movements in real rates over the long run. Economic theory suggests that real interest rates are determined by the balance of saving and investment in the economy, and hence, by the variables that influence these two aggregates. The empirical evidence indicates that short- and long-real rates are cointegrated with government deficits, government debt relative to GNP, private sector wealth, real balances relative to real GNP, demographic factors, and the marginal productivity of capital. The long-run impact of demographic, fiscal, and monetary policy variables on interest rates appears to be particularly significant. The strong influence of the fiscal variable is noteworthy since this result usually has been difficult to establish in the empirical literature.

One implication of the empirical results is that the upward pressure on real interest rates due to present demographic trends in the form of the aging of the population may be offset by increases in the capital-labor ratio. The extent of the offset is difficult to judge since capital-labor ratios are measured in terms of effective labor units which may increase despite a decline in the number of people in the work force (due to increases in skill levels, for instance). In addition, while reductions in current fiscal deficits would have an appreciable negative impact on real interest rates, the extent of the fall in these real rates would be limited by the stock of government debt relative to GNP. Thus if fiscal factors are to contribute significantly to a decline in real interest rates in the long run, substantial fiscal action needs to be taken not just to reduce deficits but also to bring down the level of government debt relative to GNP.

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