

**IMF WORKING PAPER**

© 1991 International Monetary Fund

This is a working paper and the author would welcome any comments on the present text. Citations should refer to an unpublished manuscript, mentioning the author and the date of issuance by the International Monetary Fund. The views expressed are those of the author and do not necessarily represent those of the Fund.

WP/91/21

INTERNATIONAL MONETARY FUND

Fiscal Affairs Department

Estimating the Base of the Value-Added Tax (VAT) in  
Developing Countries: The Problem of Exemptions

Prepared by G.A. Mackenzie\*

Authorized for Distribution by George Kopits

February 1991

Abstract

Developing countries with VATs typically exempt a large number of goods and services. Following a brief discussion of the rationale for exemptions, this paper presents a formula for the base of a VAT with exemptions. Two basic adjustments must be made to the base without exemptions: subtraction of the value of sales to consumers of exempt industries and addition of intermediate sales of taxable inputs to exempt industries. The paper concludes with a derivation of the elasticity of a VAT with exemptions with respect to aggregate consumption and a discussion of the implications of technological change for the VAT base.

JEL Classification Number

323

---

\* I am indebted to Alan Tait, Ved Gandhi, Ke-Young Chu, George Kopits, Richard Hemming, Christopher Towe, and Adrienne Cheasty for comments on an earlier version of this paper. I am responsible for any remaining errors.

<u>Contents</u>	<u>Page</u>
I. Introduction	1
II. Exemptions in the Developing Countries Context	2
III. The Impact of Exemptions on the Base of a VAT	3
IV. Some Implications of a VAT with Exemptions	9
Tables	
1. Examples of VAT Base Calculation	7
2. Elasticities of VAT Base and Taxed Consumption with Respect to Aggregate Consumption	12
References	14

### Summary

The value-added tax (VAT) has already been adopted by many developing countries and is being actively considered by others. These countries typically exempt a large number of goods and services from this tax.

This paper begins with a brief discussion of the rationale for these exemptions, noting that they are most readily justified on the grounds of either administrative expediency or equity. The paper then derives a formula that, with the aid of an input-output table, may be used to determine the impact of a given list of exempt activities on the size of the tax base. Two basic adjustments must be made to the base of a VAT with no exemptions: the value of sales to consumers of exempt goods and services must be subtracted, and the value of the sales of taxable inputs used in the production of exempt goods and services must be added.

The paper concludes by using the formula to obtain an expression for the elasticity of a VAT with exemptions with respect to aggregate consumption and to illustrate the implications of changes in an economy's productive structure for the VAT base.



## I. Introduction

For a variety of reasons, but chiefly for administrative expediency, the value-added tax (VAT) systems of developing countries typically exempt--as distinct from applying a zero rate--a large share of economic activity. These exemptions have to be taken into account by the procedure used to estimate the base of the tax. It is not enough to rely on data on final sales of exempt goods and services, as it would be for a retail sales tax. Additional information on the economy's productive structure is necessary to take account of the use of taxed inputs by exempt sectors. As a result, the procedure to estimate the base of a VAT when there are significant exemptions can be quite complicated.

This paper offers a discussion of the rationale for exemptions from a VAT in the developing country context. It then examines exactly how the presence of exemptions affects the size of the potential base of a VAT, and derives the formula that should be used in the estimation of the potential base.<sup>1/</sup> The paper concludes with a brief examination of some of the economic implications of a VAT with exemptions.

## II. Exemptions in the Developing Countries Context

In a hypothetical world in which the administration of a tax system was costless, there would be no reason to exempt particular activities from a VAT, except possibly in the case of certain activities where value added is hard to define. Abstracting from administrative considerations, the zero-rating of goods and services is a far better instrument than exemption for the achievement of distributive goals. Unlike exemption, zero-rating should completely purge a good of any tax embodied in the materials, supplies and capital equipment used to produce it. This cannot be done merely by exempting the good.

Optimal tax theory argues that differential taxation of commodities is often justified on efficiency grounds, but the use of exemptions--as opposed to zero-rating--can introduce ambiguity into the structure of tax rates by making the effective tax rate on a commodity a function of the structure of production, instead of determining that rate by the rate of tax applicable to the final sale of the commodity. When exempt sectors using inputs of taxable goods sell their output for further processing by producers of taxed goods, the effective rate of tax on the finished product will exceed the statutory rate and a cascade effect

---

<sup>1/</sup> Aguirre and Shome (1988) make detailed calculations of the base of the Mexican VAT, which has multiple rates and some important exemptions. The method they employ to take account of the impact of exemptions differs from the one presented here.

will be created. <sup>1/</sup> Again, administrative considerations aside, zero-rating would be superior to exemption on efficiency grounds as well.

In practice, zero-rating is a complex administrative procedure that in most VAT systems, even those in the industrialized countries, is confined to exports and in some countries to a number of sensitive goods or services. In most of these countries, exemption is a more common practice than zero-rating for goods and services that are consumed domestically. <sup>2/</sup> It is also more common in those developing countries that have adopted the VAT, mainly because these countries typically exempt basic foodstuffs. These commodities are not exempt in most of the European VATs, although they are zero-rated in some, notably those of Portugal, Ireland, and the United Kingdom. The VATs adopted by developing countries also tend to exempt newspapers and books (Tait (1988)), and services tend to be taxed selectively.

It is often argued that the base of a general sales tax or VAT should be as broad as possible, and that rate differentiation or exemption should not be used to achieve redistributive goals. This argument is weakened in many developing countries by a lack of the necessary capacity to administer a comprehensive income tax and transfer system. In such countries, the indirect tax system, *faute de mieux*, is of necessity an instrument for redistribution, and exempting one or two basic commodities could make a significant difference to the progressivity of the indirect tax system (Ahmad and Ludlow (1989) and Bird (1987)).

For reasons of equity and administrative expediency, a strong case can be made for exempting basic foodstuffs from a VAT in a developing country. The output of the agricultural sector, with the exception in many countries of a few cash or export crops, will typically be very difficult to tax in developing countries, and expenditure on a few basic commodities can represent an important share of the household budget of the poor. As Bird (1987) notes, however, the ways indirect taxes impinge on the lives of poor people are complex and subtle, and hinge very much on the details and structure of the tax in question.

A number of developing countries, including Colombia and Indonesia, have adopted a VAT that is restricted to the manufacturing-import stage. This type of VAT--which is sometimes called a manufacturing-import sales tax with a tax credit feature--is often adopted as a means of reducing

---

<sup>1/</sup> Stern (1987).

<sup>2/</sup> Thus, France exempts medical and educational services, rented housing, sports and entertainment, financial services, secondhand goods, and original art. Germany exempts the same activities exclusive of secondhand goods and original art, but also exempts museums. In neither country, however, is any domestic sector zero-rated. Portugal, Ireland, and the United Kingdom are three exceptions to this rule (Tait (1988), Table 3-1).

the number of tax collection points and hence the costs of tax administration and taxpayer compliance. This restriction of the base is sometimes necessary to make the administration of the tax feasible.

The danger with this version of the tax is that the manufacturers can connive with wholesalers or retailers to understate the true value of sales at the manufacturing stage or integrate vertically and practice transfer pricing. An additional difficulty is the problem created by direct sales by the manufacturer to retailers or final consumers. Instead of exempting the distributive sectors, a similar result can be achieved by exempting traders whose sales or value added fall below some stipulated minimum, which is deliberately set at a high level. This type of exemption by size of trader can put pressure on small manufacturers and wholesalers to register as taxpayers so that their larger customers can receive an invoice stating the tax paid on their purchases and credit this tax against their liability to tax on their sales. Policing the exemption limit can prove difficult, however.

Aside from those goods that are produced in hard-to-tax sectors or are prominent in the budgets of the poor, there is little rationale for exempting particular goods. The exemptions complicate tax administration because they necessitate extra record-keeping to separate taxable from exempt sales. Moreover, as Casanegra (1986) points out, the distinction between what is exempt and what is taxed is often tenuous or arbitrary. The proliferation of exemptions of manufactured goods can seriously erode the base of the tax with little saving in administrative costs. Nonetheless, it is easy to find examples of exemptions with little economic or administrative justification in the indirect tax systems of both developing and industrialized countries.

### III. The Impact of Exemptions on the Base of a VAT

A VAT without exemptions, with zero-rating limited to exports and purchasers of capital goods allowed a credit on their tax liability, is a tax on consumption, so that the base of the tax is given by the value of final sales of consumer goods and services. <sup>1/</sup> In analyzing the impact of exemptions on the base of a consumption-type VAT, however, it is useful to begin with an expression for the total value of sales in an

---

<sup>1/</sup> The theoretical base of the VAT in this case may be derived with the following formula:

$$\text{GDP}_{\text{market prices}} + \text{imports} - \text{exports} - \text{gross capital formation}.$$

GDP at market prices would include indirect taxes net of subsidies but not the VAT itself, if the base is to be measured on a tax exclusive basis. The value of imports would also include customs duties because these are typically included in the base of the tax.

economy, both intermediate and final. This is given by the following expression:

$$C_d + I_d + X + IS_d + IS_m + I_m + C_m \quad (1)$$

where C stands for the value of final sales of consumer goods and services, I for final sales of capital goods, IS for intermediate sales, and the subscripts d and m for domestically produced and imported.

A VAT without exemptions is levied on all sales, both intermediate and final; however, when there are no exemptions, the tax liability generated by an intermediate sale is offset by the credit received by the purchaser. When purchases of capital goods are treated, as they are in a consumption-type VAT, as intermediate purchases, and exports are zero-rated, the only sales that generate a net tax liability are sales to consumers--just as in the case of a broad-based retail sales tax. Using the terminology introduced above, the formula for the base would be:

$$C_d + C_m = C. \quad (2)$$

Once some industries are exempt, however, it is no longer the case that intermediate sales cannot generate a net tax liability. To explore the consequences of exemptions further, it is useful to restate formula (1) as follows:

$$C_d + I_d + X + \sum_{i=1}^n \sum_{j=1}^n a_{ij} GO_j + I_m + C_m \quad (3)$$

where  $GO_j$  is the value of gross output of industry j, which produces good j, and  $a_{ij}$  represents the amount of product i necessary to produce one unit of product j, which is fixed for all combinations of products. Formula (3) is derived from the identity of the sum of intermediate sales of each industry and imports of intermediate goods with the value of goods used up in the production process. The vector of intermediate demands in the economy, which gives the value of each good used up in the production process may be expressed as

$$A \cdot GO$$

where A is the matrix of technological coefficients of which  $a_{ij}$  is the ijth element, and GO is a vector of gross outputs of the goods and services produced in the economy.

The intermediate sales represented by the fourth term in formula (3) can be divided into one of four groups: sales of exempt industries to other exempt industries; sales of exempt industries to taxed industries; sales of taxed industries to other taxed industries; and sales of taxed industries to exempt industries.



Reflecting this categorization, the matrix A can be partitioned into four sub-matrices:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (4)$$

where the subscripts 1 and 2 stand for the exempt and taxed sectors. Thus, on the assumption that there are n products produced by n industries of which k are exempt,  $A_{11}$  represents the k by k matrix of the values of each exempt product used in the production of one unit of each exempt product;  $A_{12}$  represents the k by n - k matrix of values of each exempt product used in the production of one unit of each taxed product;  $A_{21}$  represents the n - k by k matrix of values of taxed products used in the production of one unit of each exempt product; and  $A_{22}$  represents the n - k by n - k matrix of values of each taxed products used in the production of one unit of each taxed product.

By partitioning the vector of gross output in a similar way between exempt and taxed products, and by distinguishing between final sales of exempt and taxed products, (3) may be restated as follows:

$$\begin{aligned} & C_1 + C_2 + I_1 + I_2 + X \\ & + \sum_{i=1}^k \sum_{j=1}^k a_{ij} GO_j + \sum_{i=1}^k \sum_{j=k+1}^n a_{ij} GO_j \\ & + \sum_{i=k+1}^n \sum_{j=1}^k a_{ij} GO_j + \sum_{i=k+1}^n \sum_{j=k+1}^n a_{ij} GO_j \end{aligned} \quad (5)$$

The last four terms of this expression represent the four categories of intermediate sales described above. If there are three exempt products, the first of these terms could be written as:

$$\begin{aligned} & (a_{11} + a_{21} + a_{31}) GO_1 + (a_{12} + a_{22} + a_{32}) GO_2 \\ & + (a_{13} + a_{23} + a_{33}) GO_3 \end{aligned} \quad (6)$$

where the first term in this expression gives the value of the three exempt products used in the production of one unit of the first exempt product; the second term gives the value of the three exempt products used in the production of the second exempt product; and the third term gives the value of three exempt products used in the production of the third exempt product.

No tax is payable on sales of either exempt consumer or capital goods. The exemption of capital goods does not affect the base, but the base of a consumption-type VAT must be reduced by the value of final sales of exempt consumer goods. But the existence of exemptions also means that intermediate sales no longer vanish, with the exemption of sales of taxed industries to other taxed industries (given by

$\sum_{i=k+1}^n \sum_{j=k+1}^n a_{ij} GO_j$ ). Sales of exempt industries to other exempt industries (given by  $\sum_{i=1}^k \sum_{j=1}^k a_{ij} GO_j$ ) do not generate either a credit or a payment and neither do sales of exempt industries to taxed industries (given by  $\sum_{i=1}^k \sum_{j=k+1}^n a_{ij} GO_j$ ). Intermediate sales of taxed industries to exempt industries do, however, generate a tax payment that is not offset by a credit. It is, therefore, necessary to add the estimated value of these sales ( $\sum_{i=k+1}^n \sum_{j=1}^k a_{ij} GO_j$ ) to the base of the tax.

To sum up, two adjustments should be made to aggregate consumption to take account of exemptions: (1) the value of exempt consumer goods must be subtracted; and (2) the value of intermediate sales of the taxed to the exempt sector should be added. 1/

With these adjustments, the formula for the base of a consumption type VAT that zero-rates exports and exempts certain industries is: 2/

$$C - C_1 + \sum_{i=k+1}^n \sum_{j=1}^k a_{ij} GO_j. \quad (7)$$

The calculation of the base of the VAT with exemptions may be illustrated with a simple example of a closed economy with two sectors, one exempt and one taxed where all final sales are to consumers (Table 1 upper panel).

The table gives the values of intermediate sales from each sector to itself and to the other sector, as well as value added, final demand--equal in this case to consumption--and gross output. Reading

---

1/ An additional adjustment to be made is the addition of purchases of taxed capital goods by the exempt sector. These purchases should be treated as intermediate sales because the purchaser, being exempt, does not receive credit for the tax content of the capital goods.

2/ A qualification is necessary. If the products of an exempt industry contain taxed inputs and if these products are sold to another taxed sector, then the component of the sales value accounted for by the tax on the inputs to the exempt sector should be included in the base of the tax as well.

Table 1. Examples of VAT Base Calculation

		Exempt Sector	Taxed Sector	Final Demand	Gross Output
<hr/>					
1. General case					
VAT base = 480					
		Purchase of inter- mediate inputs			
Exempt sector	Sales of intermediate inputs	60	150	170	380
Taxed sector		90	100	390	580
	Value added	230	330		
	Gross output	380	580		
2. Special case assuming no intermediate sales from exempt to taxed sector.					
VAT base = 380					
		Purchase of inter- mediate inputs			
Exempt sector	Sales of intermediate inputs	60	0	270	330
Taxed sector		90	100	290	480
	Value added	180	380		
	Gross output	330	480		

Source: Staff calculations.

the first column down, the exempt sector is seen to purchase 60 units of intermediate inputs from itself, and 90 from the taxed sector, and to generate value added of 230, and gross output of 380. Reading the table horizontally gives the use of gross output: the second line shows that the taxed sector sells 90 units to the exempt sector, 100 to the taxed sector, makes final sales of 390, and thus has gross output of 580.

The two sector economy has total value added of 560, which equals total final demand. The base of a VAT without exemptions would therefore be 560. With exemptions, this base is reduced by the value of exempt consumption, 170, then increased by the value of sales of the taxed to the exempt sector, 90 and equals 480.

In general, the base of a VAT with exemptions will be greater than the value added of the taxed sector, and the reduction to the base that results from exempting part of the economy is less than the value added of the exempt sector. <sup>1/</sup> There is a special case, however, in which the reduction to the base that results from exempting part of the economy will equal the value added of the exempt sectors illustrated by the lower panel of Table 1. This is the case where the exempt sector does not make any intermediate sales to the taxed sector, when the sub-matrix  $A_{12}$  has only zero elements. It may be seen that subtracting the value of exempt consumption, 270, and adding the value of sales of the taxed to the exempt sector, 90, to the base of a VAT without exemptions, which is 560, gives 380, which is the value added of the taxed sector.

In this special case, none of the value added of the exempt sector gets taxed by being embodied in the output of the taxed sector. If the exemptions of a real-world economy did lead to this result, the calculation of the adjustment to be made to the base of a VAT with no exemptions to derive the effect of exemptions would be greatly simplified.

The argument of this section has confirmed the well-known view that exempt products remain in the base of a VAT to the extent that they are sold as intermediate inputs to the taxed sector. This proposition is sometimes used as an excuse to limit the base of the VAT, and it is argued that the output of a particular industry takes the form of intermediate sales to another industry. Transportation services or business services are sometimes given as examples. In practice, however, it is very difficult to find industries that make no final sales. It also needs to be remembered that the existence of exempt industries creates incentives for misclassification of taxed sales as exempt.

---

<sup>1/</sup> This follows from the relationship that the value added of the exempt sector minus intermediate sales to the taxed sector will equal final sales of the exempt sector minus intermediate purchases from the taxed sector. As long as intermediate sales to the taxed sector are positive, the reduction to the base entailed by exemptions will be less than the value added of the exempt sector.

In applying the method of base estimation described in this paper, it will inevitably be necessary to make some additional modifications to take into account the special features of the tax or the limitations of the data base available. For example, it is unlikely that the definition of industries used to derive the input-output table will conform exactly to the scheme used to classify industries as exempt or taxed; specifically, the food preparation industry in the input-output table may contain subindustries that are taxed and others that are exempt. One way of circumventing this problem would be first to estimate the base on the assumption that the entire industry was exempt, then to estimate it assuming the entire industry was taxable, and then to take a weighted average of the two estimates using whatever additional information on the sales of the food processing industry was available.

Another problem that will be faced is the impact on the base of an exemption of small traders that can include firms from all industries. In this case, once the base of the tax has been estimated on the assumption that there is no small trader exemption, some additional information will have to be used to derive an estimate of the further adjustment that must be made to the initial estimate. One possibility is to use industry surveys, where they exist, that give the distribution of sales by size of firm for each industry.

#### IV. Some Implications of a VAT with Exemptions

The potential base of a consumption-type VAT with no exemptions will display an elasticity of one with respect to the value of aggregate consumption expenditure. <sup>1/</sup> When there are exemptions to the VAT, the elasticity of the potential base with respect to aggregate consumption may differ from one. Its value will be determined by the elasticity of the value of exempt consumption and that of the value of taxed consumption with respect to total consumption and by their shares in total consumption--that is, by the evolution of final demand for exempt and taxed goods--and by the extent to which the exempt sector relies on taxed inputs and hence on the economy's productive structure.

The relationship between the elasticity of the potential base and the evolution of consumption for taxed and exempt goods can be derived from equation (7) by making use of the relationship between gross output and final sales given by:

$$A \cdot GO + FS = GO \quad (8)$$

---

<sup>1/</sup> The elasticity of the effectively exploited base with respect to consumption will differ from one if the rate of tax evasion is nonzero, provided that this rate varies from one sector to another, and also that the share in aggregate consumption of various products varies as aggregate consumption grows.

$$\text{or } FS = [I-A] GO \quad (9)$$

$$\text{or } GO = [I-A]^{-1} FS. \quad (10)$$

Letting the inverse of  $I-A$  be represented by  $B$ , the formula for the base of the VAT given by equation (7) can be restated as:

$$C - C_1 + \sum_{i=k+1}^n \sum_{j=1}^k a_{ij} \sum_{p=1}^n b_{jp} FS_p \quad (11)$$

where  $b_{jp}$  is the  $j$ th element of the  $n$  by  $n$  matrix  $B$ . (Recall that there are  $k$  exempt products and  $n - k$  taxed products.)

By expressing the final sales of the  $i$ th exempt product in terms of the final sales of the group of exempt products ( $FS_1$ ) as in  $FS_i = \lambda_i FS_1$ , and by making a similar transformation for taxed products, formula (11) can be restated as:

$$C - C_1 + \sum_{i=k+1}^n \sum_{j=1}^k a_{ij} [\hat{b}_{j1} FS_1 + \hat{b}_{j2} FS_2] \quad (12)$$

where  $\hat{b}_{j1} = b_{j1} \lambda_1 + b_{j2} \lambda_2 + b_{j3} \lambda_3 + \dots + b_{jk} \lambda_k$

and  $\hat{b}_{j2} = b_{jk+1} \lambda_{k+1} + \dots + b_{jn} \lambda_n$ .

When all final sales are to consumers, the formula for the base can be shown to take the following form:

$$C - C_1 + k_1 C_1 + k_2 C_2 \quad (13)$$

$$\text{or } (1 + k_2) C_2 + k_1 C_1 \quad (14)$$

where  $k_1 = \sum_{i=k+1}^n \sum_{j=1}^k a_{ij} \hat{b}_{j1}$  and  $k_2 = \sum_{i=k+1}^n \sum_{j=1}^k a_{ij} \hat{b}_{j2}$ .

The value of the parameter  $k_1$  reflects the demand for inputs from the taxed sector for use in the exempt sector that are generated by final sales of exempt products; similarly, the value of  $k_2$  reflects the demand for taxed inputs for use in the exempt sector that are generated by final sales of taxed products. These parameters depend not only on the extent to which production of exempt goods requires taxed inputs (in other words, the value of the elements of the sub-matrix  $A_{21}$ ) but also on the extent to which an increment of final demand generates increased production in the exempt sector.

The expression for the elasticity of the tax base with respect to total consumption ( $\epsilon_{tb}$ ) is readily derived from this last expression, and is given by

$$\epsilon_{tb} = \frac{k_1 C_1}{(1+k_2)C_2 + k_1 C_1} \epsilon_{c1} + \frac{(1+k_2) C_2}{(1+k_2)C_2 + k_1 C_1} \epsilon_{c2}. \quad (15)$$

Because  $k_1$  and  $k_2$  will be greater than zero but less than 1, for given values of  $k_1$  and  $k_2$  the elasticity of the base will vary positively with the share of consumption of taxed goods in total consumption, provided  $\epsilon_{c2}$  exceeds  $\epsilon_{c1}$ . Note that expression (15) is simply a weighted average of the elasticities of consumption of exempt ( $\epsilon_{c1}$ ) and taxed ( $\epsilon_{c2}$ ) goods with respect to aggregate consumption. On the plausible assumption that the share of taxed products in total consumption will generally exceed 50 percent, and noting that the value of  $k_1$  will not exceed  $(1+k_2)$ , the weight of the elasticity of taxed goods will exceed that of exempt goods.

Supposing that the data--specifically, a technological coefficients matrix--that would allow the application of the approach presented in the paper were not obtainable, the elasticity of the tax base with respect to aggregate consumption could be approximated by an estimate of  $\epsilon_{c2}$ . It is instructive to examine the degree to which the latter is an overestimate of the elasticity of the tax base derived from (15).

In the example shown in the upper panel of Table 1, the share of exempt products is 30 percent of aggregate consumption. Because the matrix is only 2 by 2, the  $\hat{b}_{i1}$  vector collapses to  $b_{11}$  and the  $\hat{b}_{j2}$  vector to  $b_{12}$ . The values for  $k_1$  and  $k_2$  are 0.31 and 0.10, respectively. <sup>1/</sup>

The elasticity calculated with the formula derived above does not differ substantially from the approximation obtained by using the elasticity of taxed goods with respect to aggregate consumption (Table 2) in the case of the hypothetical economy of Table 1's upper panel. In particular, when the consumption of exempt goods is completely inelastic with respect to aggregate consumption, the elasticity of consumption of taxed goods with respect to aggregate consumption will be 1.44 (Table 2). The elasticity of the tax base with respect to aggregate consumption will be 1.28, so that the error resulting from using the approximation is not especially large. This error declines as the elasticity of taxed goods with respect to aggregate consumption falls, and that of exempt goods rises. Thus, with an elasticity of taxed goods with respect to aggregate consumption of 1.22 and an elasticity of exempt goods of 0.5, the error falls to 0.08.

This conclusion is reasonably robust to changes in the values of  $k_1$  and  $k_2$  when the share of exempt consumption in total consumption is held at 30 percent. For example, with  $k_1$  increased to 0.51, the elasticity

---

<sup>1/</sup> The elements of the A matrix are:  $a_{11} = 0.158$ ;  $a_{12} = 0.259$ ;  $a_{21} = 0.237$ ; and  $a_{22} = 0.172$ . The elements of the B matrix are:  $b_{11} = 1.302$ ;  $b_{12} = 0.407$ ;  $b_{21} = 0.373$ ; and  $b_{22} = 1.325$ .

Table 2. Elasticities of VAT Base and Taxed Consumption  
with Respect to Aggregate Consumption 1/

Elasticity of Exempt Consumption with Respect to Aggregate Consumption (1)	Elasticity of Taxed Consumption with Respect to Aggregate Consumption (2)	Elasticity of Tax Base with Respect to Aggregate Consumption		Difference	
		(3a)	(3b)	(4a)	(4b)
		$k_1 = 0.31$ $k_2 = 0.10$	$k_1 = 0.51$ $k_2 = 0.10$	(2)-(3a)	(2)-(3b)
--	1.44	1.28	1.19	0.16	0.24
0.10	1.39	1.25	1.18	0.14	0.22
0.20	1.35	1.22	1.16	0.13	0.19
0.30	1.31	1.20	1.14	0.11	0.17
0.40	1.26	1.17	1.12	0.09	0.14
0.50	1.22	1.14	1.10	0.08	0.12
0.60	1.17	1.11	1.08	0.06	0.10
0.70	1.13	1.08	1.06	0.05	0.07
0.80	1.09	1.06	1.04	0.03	0.05
0.90	1.04	1.03	1.02	0.02	0.02
1.00	1.00	1.00	1.00	--	--

Source: Staff calculations.

1/ Derived from general case in Table 1, assuming a 30 percent share of exempt products in aggregate consumption.



of the tax base with respect to aggregate consumption in the case when the elasticity of exempt goods is zero will fall to 1.19, so that the error that results from using the elasticity of taxed goods with respect to aggregate consumption increases to 0.24. As in the first case, the error falls as the elasticity of taxed goods declines (Table 2). The error that results from the use of this approximation will increase as either the share of exempt consumption in total consumption or the value of  $k_1$  increases.

The elasticity with respect to aggregate consumption of the potential base of a consumption-type VAT with significant exemptions can differ from one even if the shares in consumption of exempt and taxed goods are constant: that is, even if  $\epsilon_{c1}$  and  $\epsilon_{c2}$  are both equal to one, provided the productive structure of the economy is evolving. It will not be possible to describe systematically the way in which a haphazard change in technological coefficients will affect the observed elasticity of the base, although to the extent that this change is sluggish its effect on the observed elasticity will not be great. Technological change that tends either to increase or to decrease the values of the  $a_{ij}$ 's will, however, have a systematic effect on the observed elasticity of the potential tax base. Consider technological change that tends to reduce the amount of input  $i$  necessary for the production of a unit of product  $j$ , whereby over time the value of  $a_{ij}$  tends to fall. This change will lower the value of both the  $b_{jp}$  and the  $a_{ij}$  terms in formula (11); the more rapidly this occurs, the greater the effect on the potential base.

The paper has derived a formula for the base of a VAT with exemptions, and has discussed some of its implications. The applicability and practical significance of the formula will depend on the quality of the relevant economic statistics and the relative size of the exempt industries. In practice, it may be much easier to estimate the final sales of exempt products than it is to estimate the sales of taxed industries to exempt industries. Nonetheless, the formula illustrates the need to take account of the impact of these sales on the size of the tax base.

References

- Aguirre, Carlos A., and Parthasarathi Shome, "The Mexican Value-Added Tax (VAT): Methodology for Calculating the Base," National Tax Journal, Vol. XLI, No. 4 (1988), pp. 543-54.
- Ahmad, Ehtisham, and Nicholas Stern, "The Reform of Indirect Taxes," Chapter 7 in The Theory and Practice of Tax Reform in Developing Countries (Cambridge, United Kingdom, forthcoming).
- Bird, Richard M., "A New Look at Indirect Taxation in Developing Countries," World Development, Vol. 15, No. 9 (1987), pp. 1151-61.
- Casanegra de Jantscher, Milka, "Problems of Administering a Value-Added Tax in Developing Countries," IMF Working Paper 86/15 (Washington: International Monetary Fund, 1986).
- Newbery, David, and Nicholas Stern, The Theory of Taxation for Developing Countries (International Bank for Reconstruction and Development, 1987).
- Stern, Nicholas, "Aspects of the General Theory of Tax Reform," Chapter 3 in The Theory of Taxation for Developing Countries, ed. by David Newbery and Nicholas Stern (International Bank for Reconstruction and Development, 1987).
- Tait, Alan A., Value-Added Tax--International Practice and Problems (International Monetary Fund, 1988).