IMF WORKING PAPER

WP/91/7

INTERNATIONAL MONETARY FUND

Research Department and Western Hemisphere Department

On Interpreting the Random Walk Behavior of Nominal and Real Exchange Rates

Prepared by Charles Adams and Bankim Chadha 1/

Authorized for Distribution by M.P. Dooley and Y. Horiguchi

January 1991

Abstract

The random walk property of exchange rates is frequently regarded as carrying strong implications for the kinds of shocks that have driven exchange rates and the models appropriate for analyzing their behavior. This paper conducts stochastic simulations of Dornbusch's (1976) sticky-price monetary model, calibrated for representative parameter values for the United States. It shows that the model is capable of generating time series for both real and nominal exchange rates that are statistically indistinguishable from random walks when all shocks are nominal.

JEL Classification Numbers:
210, 431

1/ We would like to thank Tamim Bayoumi, Eduardo Borensztein, Michael Dooley, Guy Meredith, and Mark Stone for helpful comments and discussions. Any errors and all opinions expressed are our own.

Contents

Summary

During the recent floating rate period, the time series for both nominal and real exchange rates have been closely approximated by random walks. The random walk is frequently regarded as carrying strong implications for the shocks that have driven exchange rates and for the models that explain their behavior. If the exchange rate is viewed as being in equilibrium, the random-walk property implies that all shocks driving them must be permanent. Further, for the real exchange rate, this implication suggests that real shocks are more prevalent than nominal shocks, since nominal shocks would be expected to have only transitory effects. Moreover, the random-walk finding argues against the validity of models that ascribe a major role to short-run disequilibrium dynamics, since such dynamics induce systematic movements in exchange rates.

This paper conducts stochastic simulations of Dornbusch's (1976) overshooting model, calibrated for parameter values representative of the United States, where all shocks are nominal. It shows that the model can generate time series for both real and nominal exchange rates that are statistically indistinguishable from random walks. This is the case even though the model generates a "true" process for the nominal exchange rate that has a unit root with systematic components (and hence deviates from a random walk), and the real exchange rate follows a stationary process. The simulations serve, therefore, as a cautionary note against drawing strong inferences about the kinds of shocks that affect exchange rates on the basis of tests that cannot reject a unit root or random walk in exchange rates.

In addition, the data generated by the simulations are employed to test for the existence of cointegration between the nominal exchange rate and the money supply, and between the nominal exchange rate and the (one-period-ahead) forward rate.

I. Introduction

The representation of many non-stationary economic time series by unit root processes has become increasingly common in the empirical literature. (See Stock and Watson, 1988, and the references cited therein.) The time series for (the logarithms of) variables such as real GNP, prices, and employment have been found to be well approximated by unit-root processes (Nelson and Plosser, 1982) implying, in contrast to the assumption of stationarity around a deterministic trend, permanent movements in response to shocks. The extent of the permanent movements--or, alternatively, the size of the random walk component in these series--1/ has been regarded by many analysts as important for identifying the kinds of shocks affecting these variables, and the economic models that are appropriate for analyzing their behavior. (Stock and Watson, 1988; Campbell and Mankiw, 1987; Nelson and Plosser, 1982; Sheffrin, 1989; and Adams, 1990.)

There has also been a growing literature on the representation of the time series for (the logarithms of) nominal and real exchange rates by unit root processes. (See, for example, Meese and Singleton, 1982; Huizinga, 1987; Kaminsky, 1987; Meese and Rogoff, 1985; and Adams and Chadha, 1990.) An important empirical regularity that has emerged over the recent floating rate period is that the time series for both nominal and real exchange rates are closely approximated by random walks. (See Mussa, 1984; Levich, 1979; and Adams and Chadha, 1990.) In the view of several investigators, the random walk carries strong implications for identifying the kinds of shocks that have driven exchange rates. (See Campbell and Clarida, 1987; and Kaminsky, 1987.) If one is to view the exchange rate as being in equilibrium, then the random walk property implies that all shocks driving them must be permanent. Further, the characterization of real exchange rates by random walks has been interpreted as being consistent with a primary role for real rather than nominal shocks, since the latter would be expected on the basis of most models to have only transitory effects on real variables. (See Stockman, 1980; Stockman, 1983; Huizinga, 1987; and Campbell and Clarida, 1987.) Moreover, the random walk finding argues against the applicability of models--such as the sticky-price monetary model of Dornbusch (1976)--that ascribe a major role to short-run disequilibrium dynamics, since they induce systematic movements in exchange rates.

Using simulation techniques, this paper shows that a stochastic version of Dornbusch's (1976) overshooting model in which all shocks are nominal is capable of generating time series for both real and nominal exchange rates that are statistically indistinguishable from random walks. This is the

---

1/ Using the Beveridge-Nelson(1981) decomposition, any unit root process can be decomposed into a random walk and a stationary component. The random-walk component measures the permanent component of the series and the stationary component measures the transitory component. A random walk is thus a special case of a unit-root process in which there is no stationary component.

case even though the model generates a 'true' process for the nominal exchange rate that has a unit root with systematic components (and hence deviates from a random walk), and the time series for the real exchange rate generated by the model follows a stationary stochastic process. The simulations serve, therefore, as a cautionary note against drawing strong inferences about the kinds of shocks which affect exchange rates on the basis of statistical tests that cannot reject a unit root or random walk in exchange rates.

The remainder of the paper is organized as follows. Section II outlines the stochastic version of Dornbusch's overshooting model used in the simulations and major features of the solutions when there are only nominal shocks and the real exchange rate is stationary. Section III describes how the stochastic simulations were undertaken and the values of the parameters used in simulating the model. It then applies a number of statistical tests to the solutions generated by the model with a view to determining their statistical properties. Tests are undertaken for: unit roots in the time series processes for nominal and real exchange rates and other variables generated by the model; the significance of systematic components in exchange rate changes, and hence deviations from a random walk; and for cointegration between the nominal exchange rate and the money supply, and between the nominal exchange rate and the (one-period ahead) forward rate. Concluding comments are contained in section IV.

II. Dornbusch's Sticky-Price Model

Dornbusch's (1976) model provides a convenient vehicle for illustrating that near-random walk behavior of nominal and real exchange rates can be generated by a model with sticky prices which is subject only to nominal shocks. The results underscore that the finding that exchange rates are described by random walks could arise as a result of an economic model generating behavior that is sufficiently close to a random walk so as not to be detectable at standard levels of statistical significance. The results illustrate as well that sticky nominal prices interacting with nominal shocks can create enough persistence in real exchange rates to make them statistically indistinguishable from unit root processes, even when long-run purchasing power parity holds in the model generating the data.

The basic version of the original model presented by Dornbusch is used for the simulations; we abstract from the refinements and extensions that have subsequently been made. (See Obstfeld and Stockman, 1985, for a discussion of some of these extensions.) The model is described by equations (1)-(6) where all variables other than interest rates are measured in natural logarithms and foreign variables are distinguished from domestic variables by an asterisk (*).

$$m_t - p_t = \beta \cdot \bar{y}_t - \alpha \cdot R_t \qquad (1)$$

$$R_t = R_t^* + E_t s_{t+1} - s_t \tag{2}$$

$$R_t = R_t^* + f_t^{t+1} - s_t \tag{3}$$

$$p_{t+1} - p_t = \phi \cdot (y_t - \bar{y}_t) \tag{4}$$

$$y_t = \theta \cdot q_t \tag{5}$$

$$q_t = s_t + p_t^* - p_t \tag{6}$$

Equation (1) describes equilibrium in the domestic money market in terms of equality between the supply and demand for money (m); money demand is assumed to depend positively on capacity output ($\bar{y}$) 1/ and negatively on the domestic interest rate (R). Equations (2) and (3) describe uncovered and covered interest rate parity; they imply that the one period forward exchange rate (f_t^{t+1}) is equal to the expected future spot exchange rate in the next period ($E_t s_{t+1}$). Here the foreign interest rate is denoted by R^* and the expected spot exchange rate is assumed to equal the expectation of the future spot rate conditional on information available at time t. 2/

Equation (4) is a simple Phillips curve relating domestic inflation to the gap between actual (y) and capacity output ($\bar{y}$); by construction, prices are assumed to be completely predetermined in the current period. 3/ Finally, equation (5) specifies that the demand for domestic output depends positively on the real exchange rate (q) as defined in equation (6); the real exchange rate is measured as the difference between foreign (p^*) and domestic prices (p) adjusted for the nominal exchange rate.

Equations (1)-(6) can be solved for the six endogenous variables [s, p, f, y, R, q]' in terms of the exogenous variables [R^*, p^*, $\bar{y}$, m]'. Here we consider solutions to the model under the simplifying assumption that all exogenous variables other than the money supply are constant and (their

---

1/ The assumption that money demand depends on capacity rather than actual output is made to simplify the model. Implicitly, Dornbusch (1976) makes this assumption in the version of the model described in the main part of his paper.

2/ The information set is assumed to include the model as well as the current and lagged values of all variables.

3/ Problems with this particular price adjustment rule--which is the rule used by Dornbusch (1976)--are discussed by Obstfeld and Rogoff (1984). For a version of the Dornbusch model with rational staggered contracts see Chadha (1987).

logarithms) equal to zero. The money supply is assumed to follow a random walk which implies that all changes in the money supply are unanticipated and expected to be permanent; it is the stochastic equivalent of the case considered by Dornbusch (1976).

Equations (1)-(6) can be combined to yield a pair of first-order stochastic difference equations in the nominal exchange rate and the price level. For standard parameter values, one of the two roots of this system will lie outside the unit circle and the other inside it. Under these conditions, as in Dornbusch's original model, the long-run equilibrium is a saddle point. The economy is assumed to be on the stable arm of this saddle point, implying that the solutions to equations (1)-(6) are 1/

$$p_t = \lambda \cdot p_{t-1} + (1-\lambda) \cdot m_{t-1} , \tag{7}$$

$$s_t = - \frac{1}{\alpha(1-\lambda)} \cdot p_t + \left[1 + \frac{1}{\alpha(1-\lambda)}\right] \cdot m_{t-1} + \left[1 + \frac{1}{\alpha(1-\lambda)}\right] \cdot u_t , \tag{8}$$

$$q_t = \lambda \cdot q_{t-1} + \left[1 + \frac{1}{\alpha(1-\lambda)}\right] \cdot u_t , \tag{9}$$

$$f_t^{t+1} = - \frac{\lambda}{\alpha(1-\lambda)} \cdot p_t + \left[1 + \frac{\lambda}{\alpha(1-\lambda)}\right] \cdot m_{t-1} + \left[1 + \frac{\lambda}{\alpha(1-\lambda)}\right] \cdot u_t , \tag{10}$$

where $\lambda = 1 - \frac{\phi\theta}{2} - \frac{1}{2}\left[\phi^2\theta^2 + \frac{4\phi\theta}{\alpha}\right]^{1/2}$,

refers to the (assumed) stable eigenvalue of the system and u_t represents the innovation in the money supply in period t.

Several features of the solutions in equations (7)-(10) are noteworthy. (a) Even though the money supply follows a random walk and all changes in money are expected to be permanent, the endogenous nominal variables--spot and forward exchange rates, and nominal prices--contain systematic and predictable components. These components arise as a result of the assumption that commodity price adjustment is sluggish and are reflected in movements along the stable arm of the saddle point. (See Adams and Boyer, 1986.) (b) In response to innovations in the money supply, the nominal exchange rate will overshoot its long-run value on impact. The degree of overshooting is determined by all the parameters in the model but depends importantly on the speed of commodity price adjustment (ϕ). (c) Even though the real exchange rate is influenced in

---

1/ It is straightforward to derive the solutions for the interest rate and output from the following equations.

the short run by nominal shocks, it tends to return over time to a fixed mean; that is, purchasing power parity holds in the long run. 1/

The solutions to the Dornbusch model have strong implications for the stochastic processes describing the endogenous variables. Given that the money supply follows a random walk and hence has a unit root, all of the nominal variables in the model (with the exception of the nominal interest rate) will also contain unit roots. Compared with the money supply, however, these variables will contain systematic components as a result of the intrinsic dynamics of the model. They will therefore have both permanent (random walk) and transitory (stationary) components, and while they may drift apart from the money supply in the short run they will move with it in the long run. In short, these variables will be cointegrated with the money supply. 2/ By contrast, the assumptions of long-run purchasing power parity and monetary neutrality imply that the real exchange rate and all other real variables in the model (output, the forward premium, and real money balances) are stationary and integrated of order zero. The real exchange rate will be closely correlated with the nominal exchange rate in the short run given stickiness in commodity prices, as will other real variables, but they will tend to return over time to fixed long-run values.

III. Stochastic Simulations

The Dornbusch model given by equations (1)-(6) was simulated using numerical values for the parameter vector $[\alpha, \theta, \phi]$ and the assumption that the (logarithm of the) money supply follows a random walk with innovations having a 1 percent standard deviation. 3/ The simulations were carried out using the random number generator on PC-TSP. Rather than perform the simulations on wide ranges of alternative parameter values, for purposes of empirical relevance representative values for the United States were chosen. The parameters were selected to be consistent with data at a quarterly frequency.

For the (absolute) value of the semi-interest elasticity of money demand (α) a value of 4 was used. At an (average) annual interest rate of 8 percent (the average short-term interest rate in the United States during the first half of 1990) this parameter gives a relatively small (quarterly) interest

---

1/ This feature of the solution reflects the assumptions of monetary neutrality and an absence of real shocks.

2/ Two unit root processes y_t and x_t are said to be cointegrated if there exists a constant A such that $z_t = y_t - A \cdot x_t$ is stationary and integrated of order zero. The constant A is said to be the cointegrating parameter. (See Engle and Granger, 1987.) In the Dornbusch model the cointegrating parameter for the nominal variables (other than the nominal interest rate) is unity, given that money is neutral in the long run.

3/ A drift term is not included since it is assumed that there is no output growth in the model. It is straightforward to allow for such a drift.

elasticity of money demand of -0.08, which is at the mid-point of Friedman's (1978) range for the elasticity of M1 demand with respect to short-term interest rates. This value is also consistent with the money demand elasticity referred to by Mussa (1984) in his survey of models of exchange rate determination. The use of a relatively small elasticity (in absolute value) of money demand with respect to interest rates tends to increase the amount of systematic movement in exchange rates; if anything it biases the results towards finding significant deviations from a random walk.

The parameter θ, which measures the (reduced-form) elasticity of aggregate demand with respect to the real exchange rate, was assumed to equal 0.25. Unlike the interest elasticity of money demand, there are few direct estimates of this parameter available in the literature. A value of 0.25, however, appears fairly neutral and is not grossly at odds with comparable parameters in several large econometric models.

The price stickiness parameter, ϕ, which plays a key role in the simulations, was assumed to equal 0.05 on quarterly data. This value is consistent with estimates of price stickiness made for the United States by Taylor (1980) 1/ and Rotemberg (1982); 2/ it implies that around 20 percent of the gap between actual prices and their flexible equilibrium solution is made up in one year. 3/ While this is obviously low when judged against flexible price models, it is clearly not at odds with the view that there is considerable price inertia in the United States.

Based on these representative parameter values, the solutions to the Dornbusch model are characterized by one root equal to 0.9375 and the other equal to 1.05, implying that the long-run equilibrium is a saddle point. The parameter values imply that a 1 percent increase in the money supply leads on impact to a 5 percent depreciation of the nominal and real exchange rate. Over time the nominal and real exchange rate are then expected, in the absence of shocks, to appreciate as in the original Dornbusch (1976) model. The adjustment of the real exchange rate over time is described by an AR(1) process

---

1/ Taylor (1980) estimates the response of "new" wage contracts to excess demand as 0.087 on a quarterly basis. Since in his framework only a subset of wage/price setters revise their prices in each period, our value of 0.05 for the responsiveness of the aggregate price level seems appropriate.

2/ Rotemberg (1982) jointly estimates a system of price, output, and money equations with a reduced-form equation for the general price level very similar to the solution for the general price level obtained in equation (7). His preferred estimates for the coefficient on the lagged aggregate price level lie between 0.92 and 0.946; this coefficient corresponds to the stable characteristic root, λ, of our system which is equal to 0.9375 (see below). For further details see Rotemberg (1982) and Chadha (1989).

3/ Krugman's (1990) discussion of real exchange rate dynamics employs a similar value.

with a coefficient equal to the stable eigenvalue of the system (i.e., 0.9375). 1/ The change in the nominal exchange rate can be described by an infinite order moving-average process. 2/

Simulations were carried out for 100 runs of the model for three different sample sizes. The runs covered: a small sample size of 75 observations (approximately the number of quarterly observations since the beginning of the recent floating rate experience); a medium sample size of 150 observations (almost 40 years); and a large sample size of 400 observations (100 years). Summary statistics for the simulation runs are shown in Table 1 and provide an indication of the amount of variability in the simulated data and the average changes in the variables over simulation runs. Reflecting the overshooting feature of the Dornbusch model, the (sample) standard deviation of changes in the nominal exchange rate is about five times as large as the standard deviation of money supply innovations. Given price stickiness, the standard deviation of changes in the real exchange rate is dominated by that of changes in the nominal exchange rate, a characteristic not unlike that observed in the actual data. (See Adams and Chadha, 1990.) Given that the innovations in the money supply process are drawn from a distribution with a zero mean, the average first differences of all series are close to zero in all the observation sets. This feature of the data allows us to ignore constant terms in the statistical tests.

The first set of tests are for unit roots in the stochastic processes for the variables in the model. By construction, all the nominal variables in the model (except the nominal interest rate) contain unit roots so one would expect to be unable to reject the null hypothesis of a unit root in these variables in a 'large' number of cases. The real exchange rate, however, while highly correlated with the nominal exchange rate in the short run, is a stationary variable and the null hypothesis of a unit root in its stochastic process should be rejected. Given that the other real variables in the model, such as output, follow an AR(1) process that is a linear transformation of the AR process followed by the real exchange rate, nothing is gained by applying tests to these variables.

In order to reduce the reliance on any one single test, three different tests for unit roots were employed: Sargan-Bhargava, Dickey-Fuller, and Augmented Dickey-Fuller tests. As discussed in the literature, the power

---

1/ The time series processes for all the other real variables in the model can be expressed as linear transformation of the process followed by the real exchange rate.

2/ Equation (7) can be manipulated to yield

$$s_t - s_{t-1} = (1-c)\cdot Z_t + c\cdot u_t \ ,$$

where $Z_t = \lambda 1\cdot Z_{t-1} + (1-\lambda 1)\cdot u_{t-1}$, so that the change in the exchange rate is a weighted sum of all past innovations in the money supply.

Table 1. Summary Statistics for Data Generated in the Simulations

First Difference of Variable	100 Runs With 75 observations		100 Runs With 150 observations		100 Runs With 400 observations	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Nominal Exchange Rate	-0.0019	0.0558	-0.0001	0.0524	-0.0001	0.0499
Real Exchange Rate	-0.0011	0.0561	0.0000	0.0527	0.0000	0.0501
Forward Exchange Rate	-0.0019	0.0530	-0.0001	0.0497	-0.0001	0.0473
Price Level	-0.0008	0.0016	-0.0001	0.0016	0.0000	0.0017
Money Supply	-0.0010	0.0110	-0.0001	0.0104	0.0000	0.0100

Note: Statistics are based on 100 runs of the model with the indicated number of observations. The mean of the first difference of a variable refers to the average value of the changes in that series over these runs. The standard deviation refers to the average standard deviation of changes in these variables obtained over the runs. All variables are expressed in logarithms.

of these tests depends on the form of the stochastic process describing the variables, with the possibility that in the case of variables with large moving average errors the tests may tend incorrectly to accept a unit root even when a series is stationary (see Schwert, 1988). While this possibility may cause difficulties in the case of the nominal exchange rate, the forward rate, and the price level which follow MA processes, there is no simple solution available. 1/ In any event, the real exchange rate follows a pure AR(1) process in this model, and the Dickey-Fuller and Sargan-Bhargava tests were set up for this case.

The tests conducted can be described as follows. The Sargan-Bhargava test examines whether the Durbin-Watson statistic for each series is significantly above zero, its value under the null of a unit root. The null hypothesis of a unit root is rejected if the Durbin-Watson statistic is significantly above zero. The Dickey-Fuller test is based on regressing the first difference of each series on its one-period lagged level, and testing whether the estimated coefficient on this lagged value is significantly less than zero. 2/ When the coefficient is significantly <u>less</u> than zero, the null hypothesis that a series has a unit root is rejected. The Augmented Dickey-Fuller test adds lagged changes in the series to the Dickey-Fuller regression, and tests whether the coefficient on the lagged level of the series is significantly less than zero. 3/ All the test statistics have non-standard distributions under the null hypothesis of a unit root; the critical values for the tests at a 5 percent significance level are shown in the accompanying tables.

The results from the unit root tests are summarized in Table 2. 4/ The Dickey-Fuller and Augmented Dickey-Fuller tests do not reject the null hypothesis of a unit root in all the nominal variables in the model in a large number of cases with a sample size of 75 or 150 observations. (The Sargan-Bhargava test, on the other hand, shows a tendency to over-reject a unit root with 75 observations). These tests, however, tend to over-reject the null

---

1/ One possibility would have been to apply the tests proposed by Phillips and Perron (1988). Given that these tests have not been widely used in the literature, we conserve space by focusing on the Sargan-Bhargava, Dickey-Fuller and Augmented Dickey-Fuller tests.

2/ With the times series process for a variable x described by

$$x_t = \rho \cdot x_{t-1} + v_t,$$

the regressions are of the form

$$Dx_t = (\rho - 1) \cdot x_{t-1} + v_t.$$

When x_t follows a unit root process (ρ=1) it is apparent that the coefficient on the lagged level of x in the regressions will be 'close' to zero.

3/ Lagged changes in the variable are added to soak up any autocorrelation in the residuals. For further discussion see Schwert(1988).

4/ Given that the changes in all variables have a zero mean, constant are not included in the Dickey-Fuller or Augmented Dickey-Fuller regressions.

Table 2. Tests for Null Hypothesis of a Unit Root

Variable	Sargan-Bhargava Statistic				Dickey-Fuller Statistic				Augmented Dickey-Fuller Statistic			
	Mean Median	Standard Deviation	Extreme Values Min., Max.	Percent Rejection of Null	Mean Median	Standard Deviation	Extreme Values Min., Max.	Percent Rejection of Null	Mean Median	Standard Deviation	Extreme Values Min., Max.	Percent Rejection of Null
					100 Runs with 75 observations							
Nominal Exchange Rate	0.222 0.199	0.129	0.035 , 0.757	15	-1.201 -1.187	0.767	-3.924 , 0.327	1	-1.050 -1.092	0.849	-3.223 , 1.168	1
Real Exchange Rate	0.239 0.218	0.126	0.047 , 0.761	20	-1.502 -1.527	0.679	-3.998 , 0.052	2	-1.347 -1.381	0.768	-3.344 , 0.598	2
Forward Rate	0.221 0.199	0.129	0.034 , 0.756	15	-1.184 -1.158	0.772	-3.918 , 0.353	1	-1.034 -1.078	0.853	-3.214 , 1.212	1
Price Level	0.009 0.006	0.007	0.002 , 0.039	0	4.905 3.589	4.562	-0.422 , 18.901	0	-0.195 -0.277	1.180	-2.805 , 2.729	0
Money Supply	0.135 0.098	0.115	0.012 , 0.658	7	-0.098 -0.252	1.137	-3.279 , 2.657	1	-0.127 -0.265	1.072	-2.391 , 2.979	0
					100 Runs with 150 observations							
Nominal Exchange Rate	0.161 0.155	0.075	0.040 , 0.462	4	-1.708 -1.682	0.716	-3.734 , -0.111	4	-1.549 -1.488	0.739	-3.661 , 0.636	4
Real Exchange Rate	0.184 0.166	0.084	0.054 , 0.532	4	-2.200 -2.108	0.606	-3.805 , -0.669	13	-2.044 -2.035	0.631	-3.760 , -0.204	7
Forward Rate	0.160 0.154	0.074	0.039 , 0.456	3	-1.682 -1.648	0.722	-3.727 , -0.064	3	-1.524 -1.469	0.745	-3.651 , 0.681	4
Price Level	0.003 0.002	0.002	0.001 , 0.010	0	4.469 4.275	3.862	-0.459 , 16.353	0	-0.127 -0.112	1.186	-3.209 , 2.708	1
Money Supply	0.063 0.049	0.049	0.006 , 0.271	0	-0.228 -0.299	1.032	-2.703 , 2.211	0	-0.235 -0.185	1.026	-2.474 , 2.825	0
					100 Runs with 400 observations							
Nominal Exchange Rate	0.116 0.109	0.039	0.053 , 0.272	0	-2.469 -2.676	0.845	-4.683 , -0.568	36	-2.274 -2.444	0.868	-4.750 , -0.458	23
Real Exchange Rate	0.146 0.140	0.040	0.079 , 0.281	0	-3.598 -3.605	0.518	-5.361 , -2.390	93	-3.392 -3.363	0.476	-5.144 , -2.510	88
Forward Rate	0.114 0.107	0.040	0.050 , 0.270	0	-2.414 -2.622	0.854	-4.655 , -0.513	34	-2.221 -2.379	0.877	-4.715 , -0.390	20
Price Level	0.001 0.001	0.001	0.000 , 0.011	0	3.766 2.907	3.603	-0.430 , 15.105	0	-0.313 -0.303	1.061	-3.660 , 2.192	1
Money Supply	0.028 0.020	0.027	0.004 , 0.196	0	-0.307 -0.464	0.980	-2.498 , 2.344	0	-0.319 -0.490	1.019	-2.605 , 2.244	0

Notes: (1) The test statistics were computed for each of 100 runs of the model with the indicated number of observations. The mean, median, standard deviation, and extreme values describe the values of the respective statistics obtained over the 100 runs.

(2) The Sargan-Bhargava statistic (SB) was computed by regressing the variable on a constant and computing the Durbin-Watson for the residual.

(3) The Dickey-Fuller statistic (DF) was computed as the t-ratio of the coefficient on the lagged level, in a regression of the first difference of the variable on the lagged level.

(4) The Augmented Dickey-Fuller statistic (ADF) was computed as the t-ratio of the coefficient on the lagged level, in a regression of the first difference of the variable on the lagged level and 4 lags of the first difference of the variable.

(5) All three test statistics have non-standard distributions and in practice various critical values have been employed. We impose a uniform significance level of 5 percent, and as critical values: 0.334 for SB from Hall and Henry (1988); -2.89 for both the DF and the ADF from Schwert (1988).

hypothesis of a unit root in many of the nominal variables with 400 observations. Most worrisome is that with 75 and 150 observations all of the test statistics are consistently unable to reject a unit root in the real exchange rate even though its 'true' process is stationary. 1/ The failure to reject a unit root in the real exchange rate reflects the fact that the deviation from a unit root implied by the model are 'small.' As the number of observations is increased to 400, the Dickey-Fuller and Augmented Dickey-Fuller correctly reject the null hypothesis of a unit root in the real exchange rate around 90 percent of the time. This result is tempered by the fact that with 400 observations these tests also incorrectly reject the null of a unit root in the spot and forward exchange rate about 20 and 35 percent of the time.

Given the results of these tests, working with a small number of observations one would correctly conclude that all of the nominal variables tested have unit roots. Most of the time, however, one would incorrectly conclude that the real exchange rate is a nonstationary unit-root process. 2/ Since one is unlikely to expect that nominal shocks could permanently influence real exchange rates one would turn naturally to real factors to explain their nonstationarity.

The next set of tests are for the null hypothesis that each variable is described by a random walk under the maintained (and incorrect in the case of the real exchange rate) assumption that each series has a unit root in its stochastic representation. (For a recent application of these and other tests to exchange rates, see Adams and Chadha, 1990.) The tests can be regarded as determining whether there are significant transitory components in the time series for each variable. If there are no transitory components, the series are judged to follow random walks. As noted earlier, the only series that is described by a random walk is the money supply. Exchange rates--spot and forward--contain systematic components as a result of the intrinsic dynamics of the model, but these may not, of course, be large enough to be detected by the

---

1/ Given that the AR(1) representation for real output contains the same root as that for the real exchange rate (0.9375), the results suggest that we would also be unable to reject a unit root in output. West (1988) has argued that under certain kinds of monetary policy, and inertia in wage/price adjustment, real GNP in the United States may have a root sufficiently close to one as to make it indistinguishable from a unit root process.

2/ Krugman (1990) argues that based on an AR(1) representation of the (annual) real exchange rate with an AR parameter of 0.8, approximately 40 years of data would be required for the standard error on the parameter to decline sufficiently to reject a unit root at a given significance level. His argument relies on knowing the true value of the parameter. As our simulations have demonstrated, across finite samples estimates of this parameter will vary, introducing an additional source of variation in the test statistic. Numerical simulation, based on representative parameter values thus provides a way of characterizing the behavior of the test statistics at different sample sizes.

statistical tests. The price level contains large systematic and predictable components given the assumption of price stickiness. In addition, the 'true' process for the real exchange rate is characterized by systematic movements around a fixed mean so the random walk should be rejected for this series.

The results for the random walk tests are summarized in Table 3. The tests are based on the autocorrelations of the first difference of each series at successive lags and testing whether these sample autocorrelations are significantly different from zero using the Box-Ljung Q-statistic. (See Granger and Newbold, 1977.) The Q-statistic is computed at a lag length of 20 quarters and a longer lag length of 40 quarters so as to allow for long-run systematic movements in the series. When the autocorrelations are significantly different from zero at these lag lengths, a series is judged to have systematic components and the null hypothesis of a random walk is rejected. *It should be noted that because the tests are based on the first* differences of the series (rather than their levels), standard distributional assumptions can be applied to the tests. Critical values at the 5 percent significance level for the null hypothesis that the autocorrelations for the first difference of each series at 20 and 40 lags are zero are shown in Table 3.

Under the maintained assumption that each series has a unit root, the null hypothesis of a random walk cannot be rejected for either the nominal or real exchange rate from 90 to 95 percent of the time (Table 3). Furthermore, there is no tendency for the null hypothesis to be rejected more often when a larger numbers of observations is used. It is only in the case of nominal prices which exhibit considerable inertia that the tests consistently (and correctly) reject the random walk. The null hypothesis that the money supply follows a random walk is rejected about 5 percent of the time.

Based on the Box-Ljung tests, one would conclude that both nominal and real exchange rates are subject only to permanent shocks. As Stockman (1980) has argued such findings are consistent with a dominant role for shocks that *affect* nominal and real exchange rates in the *same* way. One would be correct in assuming that the data are consistent with a primary role for permanent shocks (the money supply follows a random walk) but wrong in identifying these shocks as real.

The last set of tests are for cointegration between the nominal exchange rate and the money supply, and between the nominal exchange rate and the (one-period) forward exchange rate. As noted above, the unit root nonstationarity in the 'true' processes for spot and forward rates derives from a single source in the model (the unit root in the money supply). It is interesting to determine whether standard tests for cointegration are able to detect this common unit root. Following Granger and Engle (1985), we test for cointegration by estimating a cointegrating regression of the form $y_t = A \cdot x_t + v_t$ and testing to see whether the residual v_t contains a unit root. Under the null hypothesis

Table 3. Test for the Null Hypothesis of No Autocorrelation in First Differences

First Difference of Variable	Box-Ljung Q-statistic at 20 Lags: Mean	Median	Standard Deviation	Extreme Values Minimum, Maximum	Percent Rejection of Null	Box-Ljung Q-statistic at 40 Lags: Mean	Median	Standard Deviation	Extreme Values Minimum, Maximum	Percent Rejection of Null
100 Runs with 75 observations										
Nominal Exchange Rate	20.534	19.914	6.891	6.286 , 38.667	10	41.219	40.251	10.906	20.060 , 71.970	11
Real Exchange Rate	20.577	20.009	6.859	6.372 , 38.648	9	41.262	40.372	10.877	19.673 , 71.883	11
Forward Rate	20.531	19.910	6.891	6.282 , 38.669	10	41.216	40.241	10.907	20.084 , 71.950	11
Price Level	270.500	227.493	114.620	64.622 , 588.870	100	424.610	397.574	203.140	92.542 , 1017.100	100
Money Supply	20.159	19.071	6.910	6.446 , 41.344	8	40.893	39.272	11.104	21.761 , 78.539	11
100 Runs with 150 observations										
Nominal Exchange Rate	20.408	19.566	6.468	6.099 , 42.609	7	39.611	38.730	9.663	20.313 , 68.646	4
Real Exchange Rate	20.519	19.706	6.477	6.122 , 42.656	7	39.739	38.738	9.705	20.316 , 68.644	4
Forward Rate	20.398	19.559	6.466	6.095 , 42.599	7	39.599	38.726	9.660	20.314 , 68.638	4
Price Level	699.490	644.849	322.200	181.430 , 1765.700	100	851.790	784.092	396.930	267.020 , 2134.700	100
Money Supply	19.586	18.862	5.963	5.521 , 40.915	5	38.610	37.541	9.194	20.361 , 66.815	4
100 Runs with 400 observations										
Nominal Exchange Rate	21.419	21.527	6.732	10.310 , 52.206	6	42.673	41.116	10.649	18.770 , 86.993	6
Real Exchange Rate	21.657	21.701	6.723	10.563 , 52.374	6	42.950	41.247	10.656	19.030 , 87.234	6
Forward Rate	21.395	21.506	6.733	10.286 , 52.187	6	42.646	41.086	10.649	18.745 , 86.966	6
Price Level	2301.200	2202.190	593.850	1090.900 , 4087.900	100	2541.100	2407.920	746.270	1187.900 , 5130.200	100
Money Supply	18.803	18.772	6.546	7.209 , 48.792	3	39.536	37.982	10.474	15.408 , 82.352	5

Notes: (1) The Box-Ljung Q statistics were computed for each of 100 runs of the model with the indicated number of observations. The mean, median, standard deviation, and extreme values describe the values of the Q statistics obtained over the 100 runs.

(2) The Box-Ljung Q statistic is distributed approximately as a chi-squared variable, with the degrees of freedom equal to the number of lags employed in its construction. We impose a significance level of 5 percent, and the critical values are: at 20 lags 31.41; at 40 lags 55.75.

that y_t and x_t are not cointegrated, v_t will have a unit root. The Sargan-Bhargava, Dickey-Fuller, and Augmented Dickey-Fuller test are used to test for a unit root in the residuals. If the null hypothesis of a unit root can be rejected, the two variables y and x are cointegrated. 1/

The cointegrating regression also provides estimates of the cointegrating parameter A in the equation $y_t = A \cdot x_t + v_t$. Given the use of logarithms, this parameter represents the long-run elasticity of y_t with respect to x_t. As argued by Stock (1987), when series are cointegrated, estimates of the cointegrating parameter should be highly efficient and converge rapidly to their true values (super consistency). The cointegrating regression between the exchange rate and the money supply should therefore deliver a super consistent estimate of the long-run elasticity of the exchange rate with respect to the money supply. The cointegrating regression between the spot and forward exchange rates should also converge rapidly to its true long-run value of unity. The test statistics for the cointegrating parameters have non-standard distributions, implying that standard tests cannot be applied to test whether coefficients are significantly different from their 'long-run' values (see Stock and Watson, 1988).

Table 4 summarizes the results from the cointegration tests. Two features of the results stand out. First, the null hypotheses of no cointegration between the nominal exchange rate and money supply, and between the nominal exchange rate and (one-period) forward rate, are only rejected in a small number of cases with sample sizes of 75 and 150 observations. In short, the tests do not find much evidence for cointegration between these variables. It is only with 400 observations in the case of the nominal exchange rate and the money supply that the null hypothesis of no cointegration is rejected at least 50 percent of the time. Second, only in the case of spot and forward exchange rates--for which the null hypothesis of no cointegration cannot be rejected--do the cointegrating regressions give a "good" estimate of the true cointegrating parameter of unity. The estimate of the cointegrating parameter for the nominal exchange rate and money supply is heavily influenced by overshooting effects; it deviates substantially from its true value of unity, particularly with sample sizes of 75 and 150 observations. As the number of observations increases, however, on average it becomes closer to its 'true' value. An unwary investigator using a small sample set would incorrectly conclude on average that there is little evidence consistent with cointegration between exchange rates and money, and that long-run homogeneity was not supported. He would no doubt be puzzled by the lack of cointegration between the nominal exchange rate and forward rate since both series appear to have large permanent components and they move closely together--even in the short run.

1/ An alternative would be to use the Johansen(1988), and Johansen and Juselius(1988) maximum likelihood procedure for estimating the cointegrating vector as in Adams and Chadha(1990). The Granger-Engle procedure was adopted simply because it was computationally more feasible given the large number of simulations carried out.

Table 4. Granger-Engle Tests for the Null Hypothesis of No Cointegration

	100 Runs with 75 observations				100 Runs with 150 observations				100 Runs with 400 observations			
	Mean Median	Standard Deviation	Extreme Values Min., Max.	Percent Rejection	Mean Median	Standard Deviation	Extreme Values Min., Max.	Percent Rejection	Mean Median	Standard Deviation	Extreme Values Min., Max.	Percent Rejection
Statistic	Nominal Exchange Rate and Money Supply											
Sargan-Bhargava	0.172 0.137	0.111	0.031 , 0.627	10	0.149 0.131	0.081	0.041 , 0.516	3	0.130 0.130	0.038	0.054 , 0.251	0
Dickey-Fuller	-1.431 -1.391	0.689	-3.598 , 0.243	1	-2.121 -2.006	0.688	-4.420 , -0.437	3	-3.519 -3.529	0.553	-5.139 , -2.314	57
Augmented Dickey-Fuller	-1.412 -1.373	0.626	-3.459 , 0.388	3	-2.020 -2.027	0.659	-3.665 , -0.031	4	-3.330 -3.304	0.435	-4.523 , -2.507	60
Cointegrating coefficient	2.721 2.484	0.662	1.892 , 4.409	---	2.118 1.952	0.526	1.412 , 3.677	---	1.495 1.366	0.368	1.114 , 2.923	---
	Nominal Exchange Rate and Forward Exchange Rate											
Sargan-Bhargava	0.102 0.071	0.095	0.005 , 0.431	3	0.078 0.064	0.067	0.007 , 0.371	1	0.071 0.069	0.040	0.008 , 0.177	0
Dickey-Fuller	-0.718 -0.502	0.718	-2.970 , 0.361	0	-1.174 -1.092	0.801	-3.445 , 0.550	2	-2.387 -2.378	0.833	-4.290 , -0.443	12
Augmented Dickey-Fuller	-0.921 -0.868	0.631	-2.739 , 0.623	0	-1.225 -1.253	0.655	-2.941 , 0.444	0	-2.261 -2.263	0.615	-3.768 , -0.980	9
Cointegrating coefficient	1.044 1.044	0.005	1.036 , 1.052	---	1.040 1.041	0.007	1.025 , 1.052	---	1.031 1.032	0.010	1.012 , 1.050	---

Notes: (1) The Granger-Engle tests for cointegration were carried out on the data from each of 100 runs of the model with the indicated number of observations. The mean, median, standard deviation, and extreme values describe the values of the respective statistics obtained over the 100 runs.

(2) The Sargan-Bhargava statistic (SB), The Dickey-Fuller statistic (DF), and the Augmented Dickey-Fuller statistic (ADF) were computed as in Table 2 on the residuals of a regression of the nominal exchange rate on the money supply, and the nominal exchange rate on the forward exchange rate , respectively. Since this is a two-step procedure the critical values used were different from those in Table 2; at the 5 percent significance level they were: 0.334 for the SB from Hall and Henry (1988) , -3.37 for the DF, and -3.17 for the ADF from Hall and Henry (1988).

Engle, R.F., and C.W.J. Granger, "Cointegration and Error-Correction: Representation, Estimation and Testing," Econometrica, 55 (1987), pp. 251-276.

Friedman, B.M., "Crowding Out or Crowding In? Economic Consequences of Financing Government Deficits," Brookings Papers on Economic Activity, (1978), pp. 593-641.

Granger, C.W.J., and P. Newbold, Forecasting Economic Time Series, (New York: Academic Press, 1977).

Hall, S.G., and Henry, S.G.B., Macroeconomic Modelling, (North-Holland, 1988).

Hodrick, R., The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets, (Harwood Academic Publishers, 1987).

Huizinga, J., "An Empirical Investigation of the Long-run Behavior of Real Exchange Rates," in K.Brunner and A. Meltzer (eds), Empirical Studies of Velocity, Real Exchange Rates, Unemployment and Productivity, Carnegie-Rochester series on Public Policy, (Amsterdam: North-Holland, 1987).

Johansen, S., "Statistical Analysis of Cointegration Vectors," Journal of Economic Dynamics and Control, 12 (1988), pp. 231-54.

Johansen, S. and K. Juselius, "Maximum Likelihood Estimation and Inference on Cointegration--with Applications to the Demand for Money," (mimeographed, University of Copenhagen, 1989).

Kaminsky, G., "The Real Exchange Rate in the Short and in the Long Run," (mimeographed, University of California, 1987).

Krugman, P., "Equilibrium Exchange Rates," in W. Branson, Jacob Frenkel and Morris Goldstein, eds., International Policy Coordination and Exchange Rate Fluctuations, National Bureau of Economic Research, (University of Chicago Press, Chicago, 1990).

Levich, R.M, "On the Efficiency of Markets for Foreign Exchange," in R. Dornbusch and J. Frenkel (eds), International Economic Policy: Theory and Evidence, (Baltimore, MD, John Hopkins, 1979).

Meese, R.A, and K. Rogoff (1983a), "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?," Journal of International Economics, 14 (1983), 3-24.

Meese, R.A., and K. Rogoff (1983b), "The Out of Sample Behavior of Empirical Exchange Rate Models: Sampling Error or Misspecification?," in J.A. Frenkel (ed.), Exchange Rates and International Macroeconomics, NBER, (Chicago, 1983).

Meese, R.A., and K. Rogoff, "Was it Real? The Exchange Rate-Interest Differential Relation, 1973-84," Journal of Finance, 37 (1985), pp. 1029-35.

Meese, M., and K. Singleton, "On Unit-Roots and the Empirical Modeling of Exchange Rates," Journal of Finance, 37 (1982), pp. 1029-35.

Mussa, M., "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," in K. Brunner and A. Meltzer (eds), Policies for Employment, Prices and Exchange Rates, Carnegie-Rochester Conference series, 11, (Amsterdam: North Holland, 1979).

Mussa, M., "The Theory of Exchange Rate Determination," in Exchange Rate Theory and Practice, eds. J.F.O. Bilson and R.C. Marston, (University of Chicago Press, Chicago, Illinois, 1984), pp. 13-78.

Mussa, M., "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications," in K. Brunner and A. Meltzer (eds), Real Business Cycles and Actual Policies, Carnegie-Rochester Conference Series on Public Policy, (North-Holland, 1986).

Nelson, C.R., and Plosser, C.I., "Trends and Random Walks in Macroeconomic Time Series," Journal of Monetary Economics, 10 (1982), pp. 129-62.

Obstfeld, M., and K. Rogoff, "Exchange Rate Dynamics with Sluggish Prices under Alternative Price-Adjustment Rules," International Economic Review, 25 (1984), pp. 159-174.

Obstfeld, M., and A. C. Stockman, "Exchange-Rate Dynamics," in R.W. Jones and P.B. Kenen (eds.), Handbook of International Economics, Vol. 2, (North-Holland, 1985).

Phillips, P.C.B., and Perron, P., "Testing for a Unit Root in Time Series Regressions," (mimeographed, 1988).

Roll, R., "Violations of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets," in M. Sarnat and G.P. Szego (eds.), International Finance and Trade, Vol. 1, (Cambridge MA, Ballinger Publishing Company, 1979).

Rotemberg, J.J., "Sticky Prices in the United States," Journal of Political Economy, 90 (1982), pp. 1187-1211.

Sheffrin, S., The Making of Economic Policy, (Basil Blackwell, Cambridge, 1989).

Stock, J.H., "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors," Econometrica, 55 (1987), pp. 1035-1056.

Stock, J.H. and M.W. Watson, "Variable Trends in Economic Time Series," Journal of Economic Perspectives, Vol. 2, No. 3 (1988), pp. 147-174.

Stockman, A.C., "Real Exchange Rates under Alternative Nominal Exchange Rate Systems," Journal of International Money and Finance, 2 (1983), pp. 147-66.

Stockman, A.C., "A Theory of Exchange Rate Determination", in Journal of Political Economy, 88 (1980), pp. 673-98.

Schwert, W.G., "Tests for Unit Roots: A Monte Carlo Investigation," National Bureau of Economic Research, Technical Working Paper No. 73, (1988).

Taylor, J.B., "Output and Price Stability: An International Comparison", Journal of Economic Dynamics and Control, 2 (1980), pp. 109-32.

West, K.D., "On the Interpretation of Near Random-Walk Behavior in GNP," American Economic Review, 78, 1 (1988), pp. 202-209.