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An Empirical Exploration of Exchange Rate Target-Zones

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Abstract

In the context of a flexible-price monetary exchange rate model and the assumption of uncovered interest parity, we obtain a measure of the fundamental determinant of exchange rates. Daily data for the European Monetary System are used to explore the importance of nonlinearities in the relationship between the exchange rates and fundamentals. Many implications of existing "target-zone" exchange rate models are tested; little support is found for existing nonlinear models of limited exchange rate flexibility.

Keywords: EMS, exchange rates, nonlinear, and target-zone.

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Summary

This paper evaluates the empirical content of the most popular model in recent literature dealing with target zones for exchange rates. Such a target zone is a preannounced range for a country's exchange rate. For example, in most Exchange Rate Mechanism (ERM) countries in the European Monetary System (EMS) the exchange rate is allowed to fluctuate within a +/- 2.25 percent band of announced central parity. In the event, countries may further restrict the movement of their exchange rate or may realign the declared central parity. The literature shows that a credible commitment to a target zone dampens exchange-market fluctuations. Moreover, a credible commitment to intervene to keep the exchange rate within a band stabilizes the exchange rate even between episodes of intervention.

In theory, fluctuations in the target zone are dampened because a credible target-zone policy gives foreign-exchange-market participants some grounds on which to base expectations of future intervention. According to the theory, the credible target zone induces exchange-market fundamentals to revert to their mean. This, in turn, induces exchange-rate expectations to revert to their mean. Since market participants are confident that intervention will preserve a credible zone, they assume that a movement of exchange market fundamentals, which would have resulted, for example, in a 1 percent currency appreciation in the absence of a target zone will result in an appreciation of less than 1 percent in the presence of the zone.

In this study the target-zone model is examined using data from several fixed-exchange-rate regimes with special concentration on the ERM members of the EMS. The study finds that virtually none of the predictions of the target-zone literature holds up under empirical scrutiny. The model is subjected to testing at three levels. First, it studies the relationship between the exchange rate and fundamentals graphically. Second, it examines various aspects of the relationship econometrically. Third, it studies graphically some implications of the target-zone model that do not depend on the chosen measure of fundamentals. Almost all of this testing leads to disappointing results--the simple target-zone model is of little help in understanding the data. While exchange-rate models have a long history of empirical failure, the failure in this case is particularly incriminating since the model is used to measure exchange-market fundamentals, implying that test failure cannot be ascribed to mismeasured fundamentals and therefore must be attributed to a fallacious model.

I. Introduction

In this paper we attempt to characterize the behavior of nominal exchange rates during managed exchange rate regimes. We are especially interested in nonlinearities that may exist in the relationship linking the exchange rate to its fundamental determinants; that is, nonlinearities in the conditional mean of exchange rates. These nonlinearities are the focus of a theoretical literature concerned with exchange rate "target-zones." We assess the empirical importance of these nonlinearities, focusing on the six long-term participants in the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS).

There are two motives for this paper. First, a comprehensive description of exchange rate behavior during managed floats is potentially of great value in comparing the merits of alternative exchange rate regimes. Second, this paper is a contribution to the sparse empirical literature on exchange rate target-zone models.

By implicitly using a flexible-price monetary exchange rate model and the assumption of uncovered interest parity, we are able to obtain a daily measure of the fundamental exchange rate determinant. With this variable, we search directly for a nonlinear relationship between the exchange rate and fundamentals. We use three different modes of analysis: graphical study; parametric testing for nonlinear terms; and out-of-sample forecast analysis. We also test five implications of target-zone models that do not rely on our measure of fundamentals. Our EMS findings are corroborated by data drawn from three regimes of limited exchange rate flexibility: the post-WWII Bretton Woods era; and the inter-war and pre-WWI gold standards.

Our findings are mixed. Our graphical analysis suggests that the relationship between the exchange rate and its fundamental determinant "looks different" in an exchange rate target zone than it looks in freely floating exchange rates. However, the exchange rate: fundamentals relationship does not resemble that suggested by current theories. Our parametric testing for nonlinear terms usually indicates that a model which fails to account for the effects of the target zone is misspecified; nonlinear terms are statistically significant determinants of the exchange rate, although the sign pattern of the estimated coefficients is usually inconsistent with theoretical predictions. However, these effects are also apparent for floating rates. More importantly, nonlinearities do not help to predict exchange rates out of sample. Finally, when we examine implications of target-zones that do not depend on our measure of fundamentals, we find little evidence of target-zones.

Our mixed findings make us cautious in our conclusions. Our graphical analysis suggests to us that fixed exchange rates behave at least somewhat differently than freely floating exchange rates; this seems unsurprising. However, our more intensive study of the data, reveals little support for existing target zone models. We think our results are not very surprising. Our more intensive statistical work is often quite model dependent. The

auxiliary assumptions required to derive closed-form solutions for models in this literature seem to be poor assumptions that do not much aid our understanding of the data. We conclude that models of limited exchange rate flexibility work about as poorly as do models of flexible exchange rates.

In the next section of the paper, the relevant theory and our empirical strategy are outlined; Section III provides a brief survey of the existing literature, while a description of the data is contained in the following section. Section V provides a discussion of how we determine α , a parameter that is important in our model because it is required to identify exchange rate fundamentals. Our analysis of nonlinearities in conditional means of exchange rates is contained in the next four sections, which constitute the core of the paper. Section VI provides graphical analysis of the relationship between the exchange rate and fundamentals. Parametric tests for target-zone nonlinearities are reported in the following section; the forecasting abilities of linear and nonlinear models are compared in Section VIII. Various auxiliary implications of target-zone models that do not rely on measurements of fundamentals are analyzed in Section IX. A brief summary and some concluding remarks are contained in Section X.

II. Theory

In this section, we present a simple theoretical model of exchange rate target-zones. We then use this model to derive distributional implications for the exchange rate and fundamentals. Finally, we outline our approach to measuring exchange rate fundamentals.

1. The Model

The model we use in our study is standard in the target-zone literature (e.g., Krugman (1990), and Froot and Obstfeld (1989a)). In the model, the natural logarithm of the spot exchange rate, e_t (measured as the domestic currency price of a unit of foreign exchange) is equal to a scalar measure of exchange rate fundamentals, f_t , plus an opportunity cost term proportional to the rate of change of the exchange rate expected at t , $E_t(de)/dt$:

$$(1) \quad e_t = f_t + \alpha E_t(de)/dt.$$

In the typical derivation of equation (1), f_t is a linear function of variables that enter money market equilibrium, while α is the interest rate semi-elasticity of money demand; we follow that interpretation here. 1/

The expectation operator, E_t is based on information through time t . The latter includes values of the only forcing variable, f_t , and the structure of the model, including the nature of the equilibrium condition and any "process switching" relevant to the forcing process. By "process switching" we mean changes in the process governing f ; Flood and Garber (1983). One type of process switch, for example, might involve a policy switch from benign neglect of exchange market fundamentals to specific interventions to alter the course of f in order to protect an exchange rate zone.

As is typical in rational expectations models, we conjecture that the solution for the exchange rate is a function of the relevant state variable, with the additional condition that the function be a twice continuously differentiable function of the state. We consider only policies and forcing processes where the current value of f summarizes the state:

$$(2) \quad e_t = g(f_t)$$

The precise form of the g function depends on the nature of contemplated process switches. Henceforth we will usually drop the notation for the time of observation, t , writing, for example, $e = g(f)$.

In the absence of any process switches, fundamentals follow:

$$(3) \quad df = \eta dt + \sigma dz$$

1/ A typical simple flexible-price monetary model consists of: a domestic money demand equation ($m-p=\phi y-\alpha i+\epsilon$); the definition of the real exchange rate ($q=e+p^*-p$); and uncovered interest parity ($i-i^*=E(de)/dt$); where m is the log of the money supply, p denotes the log of the price level, y denotes the log of real income, i denotes the nominal interest rate, ϵ is a shock to the domestic money demand equation, q denotes the real exchange rate, and an asterisk denotes foreign variables. Elimination of endogenous prices and interest rates leads to (1), where the fundamental are defined as $f_t=m_t+v_t$ (where v denotes velocity, given by $v_t=-\phi y_t+q_t-p^*_t-\epsilon_t$). See Froot and Obstfeld (1989a) or Svensson (1990c). Certain types of risk premia can be added to the uncovered interest parity equation; this is discussed further below. In future work, we plan to extend our analysis to models with sticky prices.

where η is the drift rate, σ is a positive constant and dz is a standard Wiener process. During process switches, the f process changes to another process dictated by the particular policy switch. 1/

Using our trial solution from (2) and invoking Ito's lemma:

$$(4) \quad E(de)/dt = \eta g'(f) + (\sigma^2/2)g''(f)$$

Substituting from equation (4) into equation (1), we obtain:

$$(5) \quad g(f) = f + \alpha\eta g'(f) + (\alpha\sigma^2/2)g''(f)$$

Equation (5) is a second order linear differential equation, which has the general solution: 2/

$$(6) \quad g(f) = f + \alpha\eta + A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f)$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$ are the roots of:

$$(7) \quad \lambda^2 \alpha \sigma^2 / 2 + \lambda \alpha \eta - 1 = 0$$

The integration constants A_1 and A_2 are determined by process switching side conditions. Different side conditions result in different settings for the constants. Indeed, during periods of policy volatility, agents' settings for the A s should shift with policy perceptions.

Three patterns for the setting of the constants have emerged in the literature. Firstly, if agents pay no attention to the policy side conditions, then (ruling out bubbles), $A_1=A_2=0$. 3/ Secondly, if the target-zone is credible, agents must anticipate that the authorities will stop the drift of fundamentals out of the zone when fundamentals and the exchange rate reach the boundaries of the target-zone. Consequently, credible target-zones give rise to "sure thing" bets about fundamentals at

1/ Pesenti (1990) allows the drift rate to vary so as to induce mean reversion in the exchange rate.

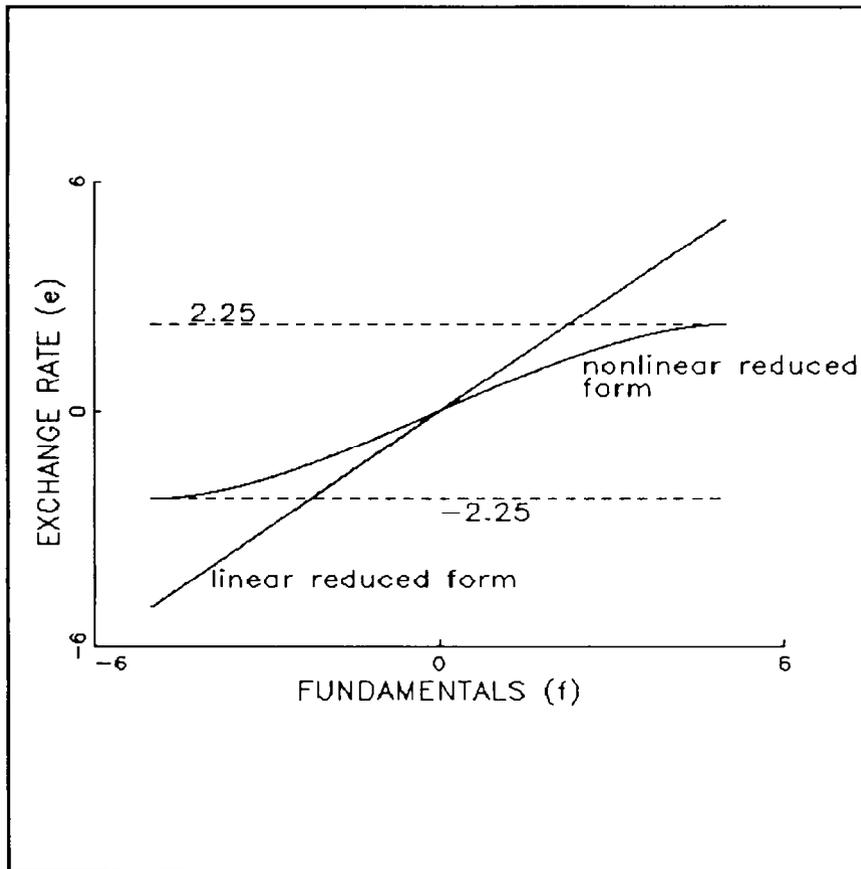
2/ The particular solution is $f + \alpha\eta$, while the solution of the homogenous part is $A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f)$. Lewis (1990) develops a different model with qualitatively similar properties.

3/ Froot and Obstfeld (1989b) provide a discussion of bubbles in the context of the stock market; see also Flood and Hodrick (1989).

the boundaries. In order to keep such bets about fundamentals from being translated into profit opportunities, agents require "smooth pasting" conditions at the boundaries. These smooth pasting conditions ensure that the exchange rate will not change in response to anticipated infinitesimal intervention at the boundaries. Smooth pasting requires $A_1 < 0$ and $A_2 > 0$.

This result is true for all credible zones, with or without intra-marginal interventions. Thirdly, the target-zone may have full credibility. In this case, the constants are unconstrained until alternative policies are specified; see e.g., Bertola and Caballero (1989b). Most of our empirical work does not use an explicit model of interventions, and so allows A_1 and A_2 to be free parameters; thus our empirical work is directed at the general class of target-zone models based on regulated Brownian motion for a single state variable. Figure 1 is a graph of the exchange rate against fundamentals with credible exchange rate limits of ± 2.25 percent. 1/

Figure 1. Credible Target-Zone



1/ In Figure 1 the linear reduced form is $e = \alpha\eta + f$; the nonlinear reduced form is $e = \alpha\eta + f + A_1\exp(\lambda_1 f) + A_2\exp(\lambda_2 f)$ where: $\alpha = 0.1$ yrs., $\sigma = 0.85/\text{day}$, $\eta = -0.06/\text{yr.}$, $A_1 = -0.542$, $A_2 = 0.546$; $\lambda_1 = 0.3317$; $\lambda_2 = -0.3311$.

2. Properties of unconditional distributions

In a credible target-zone, both the distribution of increments to f and the function that transforms f into values of exchange rates and interest rate differentials are known. Hence, a number of properties of the conditional and unconditional joint distribution of the exchange rate (e) and the interest rate differential ($i-i^*$) in the zone can be deduced. These properties were derived by Svensson (1990c); apart from a few comments to aid the reader's intuition, we leave the technical details of the derivation of these properties to the Svensson paper. ^{1/}

If the f increments are normally distributed, and if f and e are bounded by a target-zone, the nature of the distribution of the endogenous target-zone variables can be determined. Since f drives the model, the distribution for f also drives the distributions for e and ($i-i^*$). Harrison (1985, page 90) shows that if the drift rate of fundamentals, η , is zero, the unconditional distribution of f in the target-zone is uniform between the upper and lower f boundaries. If $\eta \neq 0$, f is distributed as truncated exponential.

The exchange rate in a credible target-zone follows the S-shape of Figure 1. Consequently, the unconditional distribution of the exchange rate will be bi-modal with the modes at the e boundaries. This bi-modality follows intuitively from the "flattening" of the S-shape near the zone edges. Because the S-curve is flat, a large range of possible outcomes for f becomes concentrated in a small number of outcomes for e .

A variant of the logic that predicts a bi-modal distribution for the exchange rate also predicts a uni-modal distribution for the interest rate differential. Assuming uncovered interest parity (about which more will be said later), the interest rate differential, from equation (1) is $(de)/dt = (i-i^*) = \delta(f) = (e(f)-f)/\alpha$. Plotted against f , this is a negatively sloping relationship [as $\delta'(f) = ((e'(f)-1)/\alpha)$, and $(0 \leq e'(f) < 1)$], with its steepest slopes at the zone boundaries, since $e'(f)=0$ at the boundaries. It follows that a given number of f -outcomes at the boundaries becomes stretched over a large range of e outcomes, so that little probability is attached to large δ outcomes at the lower zone boundary and little probability is attached to low δ outcomes at the zone's upper bound.

3. Conditional distributions

Conditional distributions correspond to the distributions used for "one-step-ahead" forecasting. Once again, the joint distribution of e and δ will be determined by the distribution of f ; now, however, it is the increments to f that are relevant. In a credible zone, when e is at its

^{1/} Bertola and Caballero (1990b) discuss comparable distributional properties for a model which incorporates realignments.

lower bound δ is at its maximum; when e is at its upper bound, δ is at its minimum. The relationship between e and δ is a nonlinear but monotonic negative relationship.

The target-zone offers a trade-off between exchange rate volatility and interest rate differential volatility. Svensson shows that:

$$(8) \quad \sigma^e(f) + \alpha\sigma^\delta(f) = \sigma$$

That is, in a credible target-zone, conditional exchange rate volatility is negatively related to conditional interest rate volatility in a linear fashion. 1/

4. Empirical strategy

The model presented in equation (1) bears only a limited direct relation to observables. While the exchange rate is observable almost continuously, the model offers little guidance on how to observe the triplet $\{f_t, \alpha, E_t(de)/dt\}$. 2/ We note, however, that if we could observe any two members of the triplet, then, by using equation (1), we would have the third member. Our empirical strategy entails obtaining measures of α and $E_t(de)/dt$, and deducing a measure for exchange rate fundamentals, f_t . This approach obviously precludes tests of equation (1), since the latter is used to construct measured fundamentals. Our strategy does however allow us to construct and compare reduced form equations based on equation (1).

It is relatively easy to observe $E_t(de)/dt$; we defer discussion of α to Section V. Assuming covered interest parity for contracts of length h :

$$(9) \quad 1 + i_{t,h} = (1 + i^*_{t,h})F_{t,h} / ER_t$$

where: $i_{t,h}$ is the interest rate at time t on domestic funds borrowed for a period of length h ; $i^*_{t,h}$ is the corresponding foreign interest rate; $FR_{t,h}$ is the forward exchange rate quoted at time t for delivery at $t+h$; and ER_t is the level of the spot exchange rate at time t . The relationship between the forward rate and the expected future spot rate is given by:

1/ In a cross section, if α is constant across countries and regimes, this becomes an equation for estimating α . This method has the advantage of being not being dependent on measured fundamentals. Actual results are discussed below.

2/ We are unable to use survey data on exchange rate expectations, since this is neither collected at a fine frequency, nor is it collected on bilateral European rates.

$$(10) FR_{t,h} = E_t(ER_{t,h}) + RP_{t,h}$$

where $RP_{t,h}$ is the risk premium at time t for contracts of length h . If agents in the foreign exchange market maximize the expectation of an intertemporally separable utility function, then:

$$(11) RP_{t,h} = [\text{Cov}_t(U'(C_{t+h})/P_{t+h}, ER_{t+h})] / [E_t(U'(C_{t+h})/P_{t+h})]$$

where: $\text{Cov}_t(\dots)$ denotes the covariance operator conditional on information at time t ; $U'(C_{t+h})$ is the marginal utility of consumption at time $t+h$; and P_{t+h} is the price level at time $t+h$.

We intend to ignore risk premia in this study for two reasons. First, Svensson (1990a) has shown that for constant relative risk aversion utility functions, the risk premium in a credible target-zone (with potentially moderate devaluation risk) is small. Second, in the empirical part of this study, we rely on daily observations of two-day interest rates. Regardless of the functional form of the period utility function, the risk premium embedded in such short contracts is likely to be negligible, compared with the expected rate of change of the exchange rate. 1/ 2/

Once risk premia have been assumed away, we combine equations (9) and (10) to yield:

1/ The risk premium in two-day contracts would be due to two-day conditional covariance between $U'(C_{t+h})/P_{t+h}$ and ER_{t+h} where h is two-days. The conditional covariance between two variables is the expected product of surprises in the two magnitudes. We find it hard to believe that consumption and pricing plans can be expected to change much over the course of two-days to match exchange rate surprises over the same two-days. In our view, both prices and consumption are sticky compared with the exchange rate, at least at the two-day horizon. Therefore, while both the risk premium and the expected rate of change of the exchange rate go to zero over short horizons we think that the consumption-based risk premium would go to zero faster than would the expected rate of change of the exchange rate. Over longer contract periods, such as a month, we are much less complacent about assuming away risk premia. Engel (1990) and Hodrick (1987) provide further analysis.

2/ Bertola and Svensson (1990) show that the implied two-day forward rate, $(1+i_{t+h})E_t/(1+i^*_{t+h})$ where $h = \text{two-days}$, should be a biased predictor of E_{t+h} in our data samples (which are between EMS realignments). Standard tests of unbiasedness on our EMS data do in fact reject the null hypothesis of unbiasedness. This is a standard finding for floating rates (Hodrick (1987), Froot and Thaler (1990)).

$$(12) E_t(ER_{t+h})/ER_t = (1+i_{t,h})/(1+i^*_{t,h})$$

Taking natural logarithms of each side of this equation, and applying two approximations, we arrive at: 1/

$$(13) E_t e_{t+h} - e_t = i_{t,h} - i^*_{t,h}$$

We observe interest rates on contracts with a two-day maturity; by equation (12), that is equivalent to observing the two-day expected rate of change of the exchange rate. We treat the two-day expected rate of change of the exchange rate as the instantaneous expected rate of change of the exchange rate.

Succinctly, we measure exchange rate fundamentals as $f_t = e_t - \alpha(i - i^*)_t$. Even assuming that uncovered interest parity holds, this measure will not be literally correct, as long as α is unknown; we use sensitivity analysis to account for uncertainty about α . Also, this measure does not directly link exchange rates to "raw" fundamentals such as money and output. Nevertheless, for reasonable choices of α , all interesting moments of the f distribution will closely match moments of the e distribution *in the sample*. Given the poor performance of exchange rate models that use raw fundamentals, this is a compelling argument for our measure of fundamentals. 2/ 3/

1/ The approximations are: $\ln(1+i) - \ln(1+i^*) \approx i - i^*$ and $\ln(E_t ER_{t+h,t}/ER_t) \approx (E_t e_{t+h,t} - e_t)$. The second approximation is much the more worrisome of the two since the logarithm is a nonlinear operator, which induces Jensen's Inequality problems. Since we are using only two-day forecasts, our error of approximation may be small. We investigated this assertion by simulating the approximation error for a credible target-zone on the exchange rate with the zone boundaries 2.25 percent around central parity and $\alpha = 0.1$. We found that the average absolute approximation error is about 1.1 percent of the average absolute expected rate of change of the exchange rate.

We are also assuming away any measurement error which may be the result of transactions costs. So long as bid-ask spreads are small in relation to interest differentials, this error is likely to be very small.

2/ Alternatively, we could use a McCallum substitution, replacing the expected rate of change of the exchange rate with the exchange rate's actual rate of change, and estimating with IV.

3/ Our methodology can, we think, be extended fruitfully to other environments where fundamentals are difficult to measure, so long as reduced form estimates allow one to answer the question of interest. One example is the existence of bubbles; Froot and Obstfeld (1989b).

III. Previous Findings

Most previous empirical examinations of nonlinearities in exchange rate behavior have focused on nonlinearities that affect even moments of the exchange rate process, often the conditional variance of the exchange rate. For instance, it is known that exchange rates manifest substantial leptokurtosis; conditional forecast variances of exchange rates also exhibit serial dependence (Meese and Rose (1991) provide references). However, relatively little empirical work has been done to link the level of the exchange rate to fundamentals in an intrinsically nonlinear fashion. Until recently, there appeared to be no theoretical reason to pursue such avenues. The papers by Krugman (1990) and Smith and Smith (1990) presented exchange rate models where side conditions imply deviations from the linear exchange rate solution.

There is another, more important, explanation for the dearth of nonlinear empirical work on conditional means of exchange rates. Empirical work on exchange rate determination has been dampened by the negative results of Meese and Rogoff (1983). Meese and Rogoff demonstrated that a forecaster equipped with a variety of linear structural exchange rate models and actual ex-post knowledge of the determinants of such models would not be able to forecast more accurately than a naive random walk model. It should be noted that target-zone models require a structural linear model (that is, a set of fundamentals to which additional nonlinear terms are tacked on in the presence of a target-zone; see equation (6)), so that target-zone models have, at the very least, all the problems of floating exchange rate models.

Only a small amount of relevant empirical research has been conducted to date. Almost without exception, economists have taken heed of the negative results of Meese and Rogoff, and abstained from positing explicit parametric models of fundamentals (in contrast, much of the work presented below is parametric). Meese and Rose (1990) use nonparametric techniques and find little evidence that nonlinear models fit exchange rate data better than linear models during fixed exchange rate periods. Diebold and Nason (1990) and Meese and Rose (1991) find comparable results both in-sample and out-of-sample, during floating exchange rate regimes, using univariate and multivariate data respectively. Spencer (1990) and Smith and Spencer (1990) use the method of simulated moments to avoid positing an explicit empirical model of fundamentals in modeling EMS exchange rates. Bertola and Caballero (1990b) present informal evidence on three aspects of two EMS exchange rates from the early- through mid-1980s. Svensson (1990b, 1990d) uses a variety of techniques with Swedish data to test and corroborate a model of target-zones with realignment risks without relying on a model of fundamentals. Pessach and Razin (1990) is the paper that is closest to ours in spirit; they use Israeli data in a parametric fashion and find some evidence of symmetric nonlinear effects implied by target-zone models in the rate of change of the exchange rate.

IV. Description of the Data

The major focus of this paper is the EMS regime of fixed, but adjustable, exchange rates. We concentrate on the EMS both for its intrinsic and current interest, and for easy comparison with the literature. Relevant features of the institutional structure of the EMS are described in Folkerts-Landau and Mathieson (1989) and Giavazzi and Giovannini (1989).

Our EMS data were obtained from the BIS. We also use BIS data for non-EMS countries, and for EMS countries during the period preceding the ERM. 1/ The data are daily; exchange rates are recorded at the daily "official fixing" while interest rates are annualized simple bid rates at 10:00 a.m. Swiss time. 2/ 3/ 4/ We focus on two-day interest rates (which will be taken to be "the interest rate," unless explicitly noted otherwise); we use 1-month and 12-month rates to check our results. Two-day interest rates have been used because they are the shortest available interest rates (they also reflect the yield on a deposit that has the same maturity as the two-day settlement period in foreign exchange markets). 5/ The interest rates are Euro-market rates, and should be relatively free of political, credit, settlement and liquidity risk premia, at least for

1/ We refer to the U.K. as a "non-EMS" country, although the U.K. is actually an EMS member which did not participate in the ERM during our sample period.

2/ The rates are averages across several Euro-markets.

3/ Belgium has a system of dual exchange markets. We use the official rate, which is used for current account transactions. The Belgian central bank is committed to following EMS rules for the official market; the financial rate floats freely. We have also checked our key results with financial rate data, and our conclusions are not affected.

4/ We treat each daily observation identically, and take no special account of e.g., day-of-the-week or holiday effects. By ignoring any "time deformation", we are implicitly assuming that economic time effectively stops on holidays and weekends. As much of our analysis does not depend on the time-series properties of the data, we are not excessively worried about this assumption. Further, the hypothesis that day-of-the-week dummies do not enter significantly into regressions of exchange rate levels and interest rate differentials on a constant, cannot generally be rejected at conventional significance levels. In some of our parametric work below, we have also separated out Friday data from other data; our results are never substantially affected by this division.

5/ The typical two-day settlement period in foreign exchange markets reflects the fact that the ultimate transfer of funds must take place in the domestic payments systems in countries whose currencies are involved in the transaction.

interest rate differentials across different currencies at the same maturity. 1/ Two-day interest rates are unavailable for Denmark and Ireland until February 1982 and November 1981 respectively. 2/ The data have been checked for errors in a number of ways. 3/

Unless otherwise noted, we always use natural logarithms of exchange rates; for interest rates, we almost always use the natural logarithm of one plus the interest rate (in percentage points), divided by 100. 4/ In our EMS work, Germany is treated as the "home" country, so that exchange rates

1/ Political risk reflects the possibility that the bank which issues the Euro-currency deposit may suddenly be confronted by the government of the country in which it is physically located with new restrictions or taxes on the transfer of funds once the deposit matures. As France and Italy have maintained capital controls throughout this period, political risk considerations are important in any study of the EMS. While the extent of the political risk premia might vary with the maturity of the deposit, it should be relatively uniform across different currencies of denomination. Thus the differentials between Euro-currency interest rates on deposits denominated in different currencies should be relatively free of political risk premia. Sampling across several Euro-markets should also help to alleviate this problem. If such capital controls were relatively unchanged during a particular period, they could introduce a wedge between the yields on instruments demonstrated in different currencies, even in the Euro-currency markets, as well as between domestic and offshore instruments denominated in the same currency. However, this wedge may vary over time because capital controls have been progressively eased for countries such as France and Italy. Giavazzi and Giovannini (1989) provide further discussion.

The longer version of this paper contains discussions of credit risk, settlement failure risk, and liquidity premia.

2/ Japanese short-term interest rates are also unavailable until early 1982.

3/ In particular, we checked for outliers from both levels and log-differences of the series by computing descriptive statistics and examining the data graphically. Some 150 apparent outliers were then compared with independent quotations from *The Financial Times*. We have also checked our data against internal IMF data, and provided our data corrections to Hali Edison and Graciela Kaminsky, who are performing independent research with the same data. Our programs, data and documentation are available upon receipt of a box of formatted high-density 3.5" diskettes. Most of the computing was performed in RATS 3.0, Micro-TSP 6.5, STATA 2.0, Gauss 386, and Lotus 1-2-3 2.01; documents are word-processed in WordPerfect 5.1. This offer expires one year after publication.

4/ Thus a typical American interest rate might be $\ln(1+(8/100)) \approx 0.08$.

are always the DM price of one unit of foreign exchange, and interest rate differentials are always German interest rates minus foreign interest rates. 1/

For the purposes of comparison, we also use data for the period of fixed exchange rates that prevailed during the classical Gold Standard. Our exchange rate data are taken from Andrew (1910), who tabulates data on weekly nominal exchange rates of the U.S. vis-à-vis the U.K., France, and Germany for the National Monetary Commission. The rates are the average of weekly highs and lows. Kemmerer (1910) provides weekly data on American interest rates, also gathered for the National Monetary Commission. The rate is a weekly average call loan rate for the NYSE. The National Monetary Commission (1910) tabulates British call money rates and French "market rates of discount." Our German interest rate data were gathered from back issues of The Economist. The classical gold standard data span 1899-1908. We also use data on the inter-war gold standard. These data are monthly, and are taken from Banking and Monetary Statistics 1914-1941; the data span June 1925 through July 1931. The exchange rates are averages of daily rates; interest rates are usually short term "private discount rates." Finally, we use monthly data from the Bretton Woods regime of adjustable pegged exchange rates. This data was obtained from the OECD's Main Economic Indicators. The exchange rates are point-in-time spot rates, while the interest rates are usually quoted for three month domestic treasury-bills. The data are drawn from the longest single period of exchange rate tranquility during the 1960s (e.g., the German data begin after the March 1961 revaluation and end before the October 1969 revaluation). For both the gold standard and Bretton Woods data, the USA is treated as the home country.

Figures 2 and 3 contain plots of the basic daily data for the EMS period. 2/ Figure 2 contains time-series of the nominal exchange rates (measured, as always, as the natural logarithm of the DM price of one unit of foreign exchange); the upper and lower (implied) EMS exchange rate bands are also included in the graph. Tick marks along the bottom of the diagrams delineate calendar years; the ticks along the top mark realignments which affected either of the relevant two currencies (e.g., either the DM, the Belgian Franc or both, in the case of the DM/Bfr rate). Figure 3 contains time-series plots of the two-day interest rate differential (as always, the German rate minus the foreign rate). As is true of most of our graphics,

1/ In doing so, we treat the ERM as a set of bilateral exchange rate pegs, ignoring any multilateral aspects of the EMS. Giavazzi and Giovannini (1989) provide further discussion.

2/ Our presentation has been greatly influenced by Tufte's (1983) superb monograph. Thus we typically present groups of data with greater than twenty observations in graphical format, and we repeatedly use small multiples graphs.

scales are not directly comparable across countries; the Dutch exchange rate has actually been much more stable than the Italian exchange rate even though the relevant exchange rate bands appear wider on the graphs.

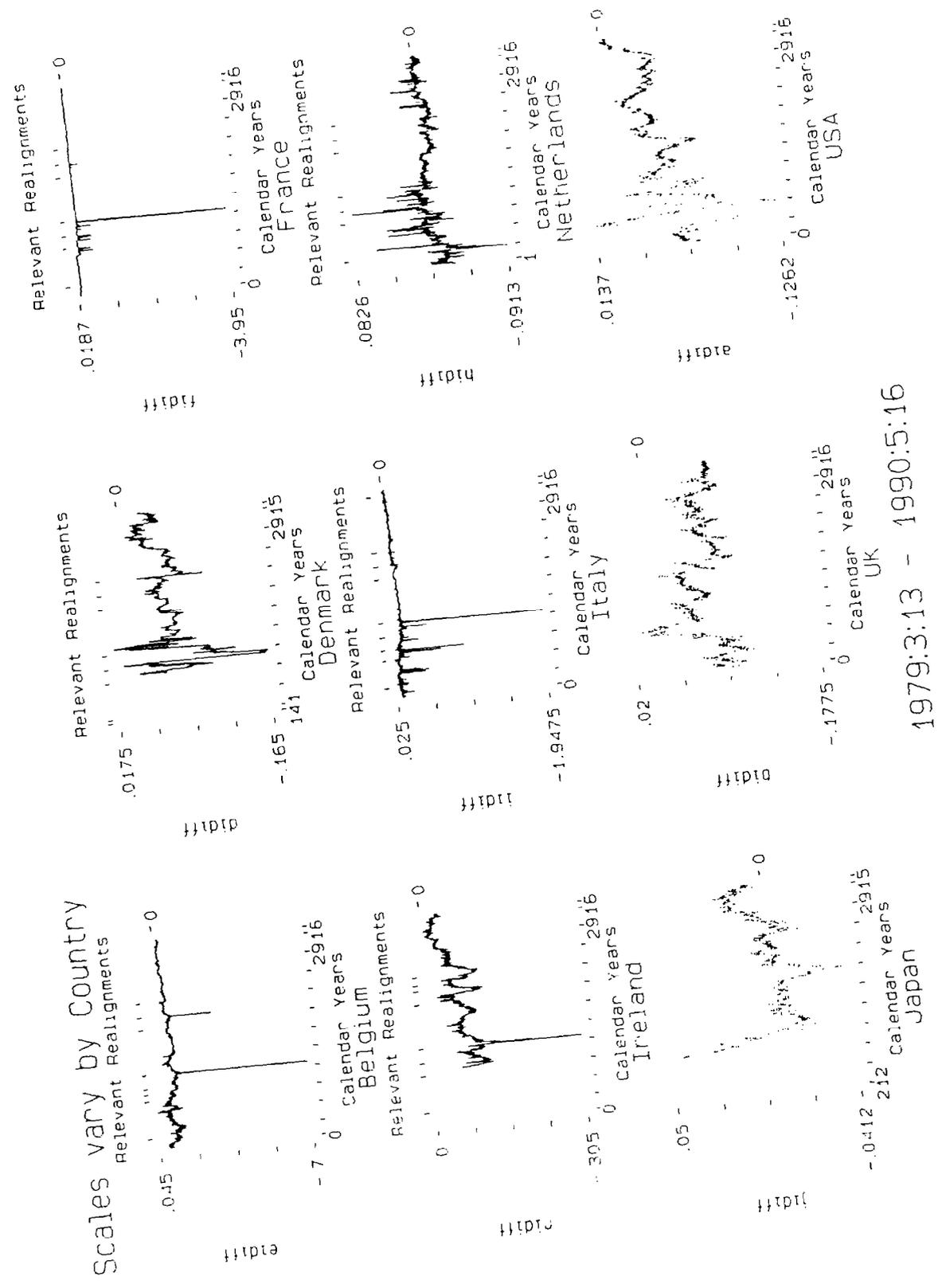
The EMS has experienced a number of realignments. Our use of fine frequency data enables us to split our data into 13 different parts, corresponding to the periods between the twelve different realignments of the EMS. 1/ We divide our data for a number of reasons. A split sample allows us to check the sensitivity of our results. Dividing the sample also allows us to check for policy shifts such as the often-noted increasing credibility of the EMS (which should result in changing types of nonlinearities), and time-varying capital controls. 2/ Bertola and Caballero (1990a) also argue that the nature of the nonlinear relationship is expected to vary over time with the level of reserves. The 13 different samples are tabulated below; it should be noted that the number of potential observations varies dramatically across regimes. In virtually all of regime-specific work below, data for the business weeks immediately before and after realignments are excluded.

1/ The exact ERM realignments were as follows (percentage changes in bilateral central rates are also shown):

Regime	Date	Belgium	Denmark	France	Germany	Ireland	Italy	Neth.
1	13-3-79	EMS Begins						
2	24-9-79		+3		-2			
3	29-11-79		+4.74					
4	22-3-81						+6	
5	4-10-81			+3	-5.5		+3	-5.5
6	21-2-82	+8.5						
7	12-6-82			+5.75	-4.25		+2.75	-4.25
8	21-3-83	-1.5	-2.5	+2.5	-5.5	+3.5	+2.5	-3.5
9	21-7-85	-2	-2	-2	-2	-2	+6	-2
10	6-4-86	-1	-1	+3	-3			-3
11	3-8-86					+8		
12	12-1-87	-2			-3			-3
13	5-1-90						+3.7	

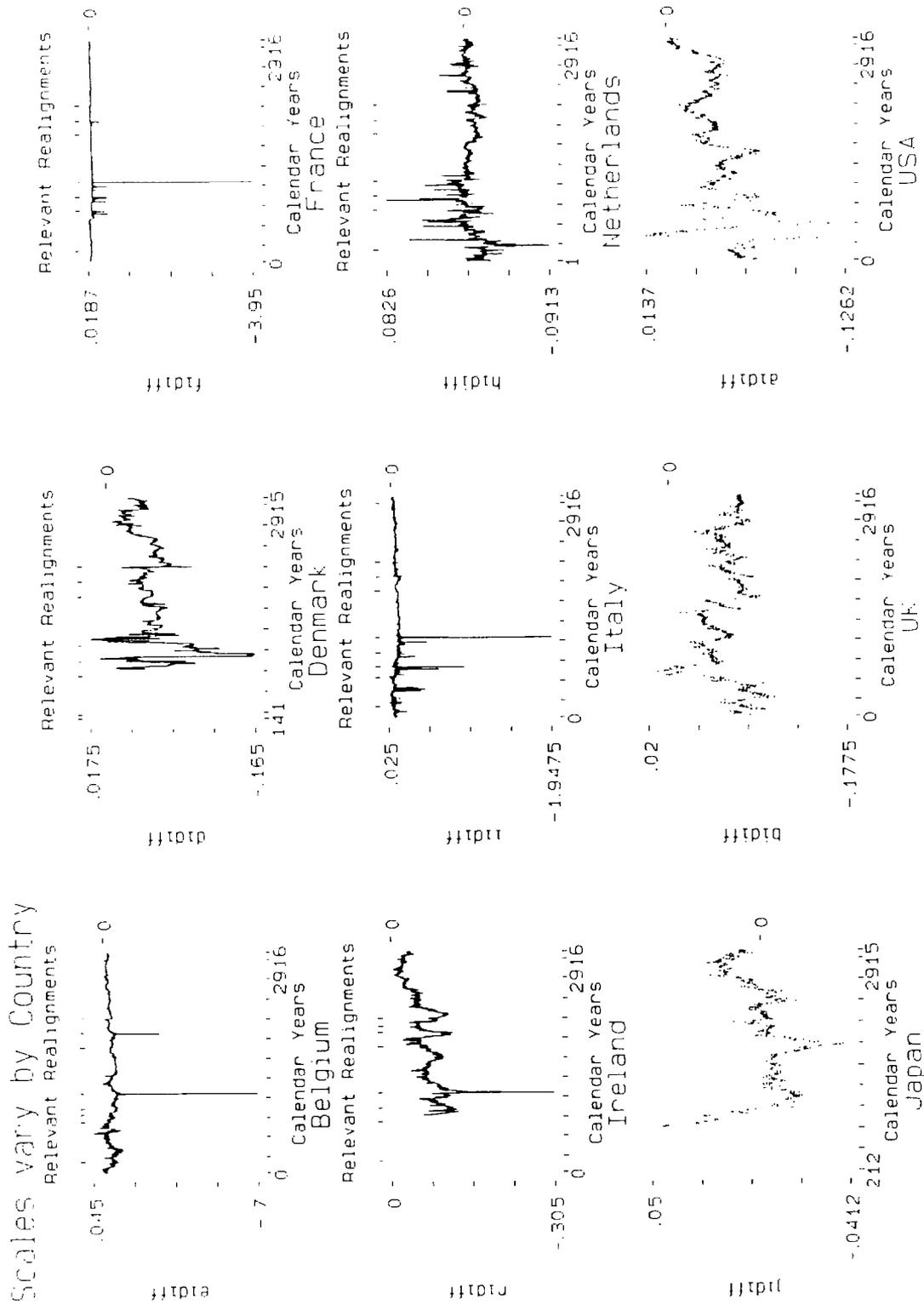
2/ Government authorities may also defend implicit target-zones which change over time and differ from declared target-zones; splitting the sample may alleviate this problem.

Figure 2. Exchange Rates



1979:3:13 - 1990:5:16

Figure 3. Interest Rate Differentials

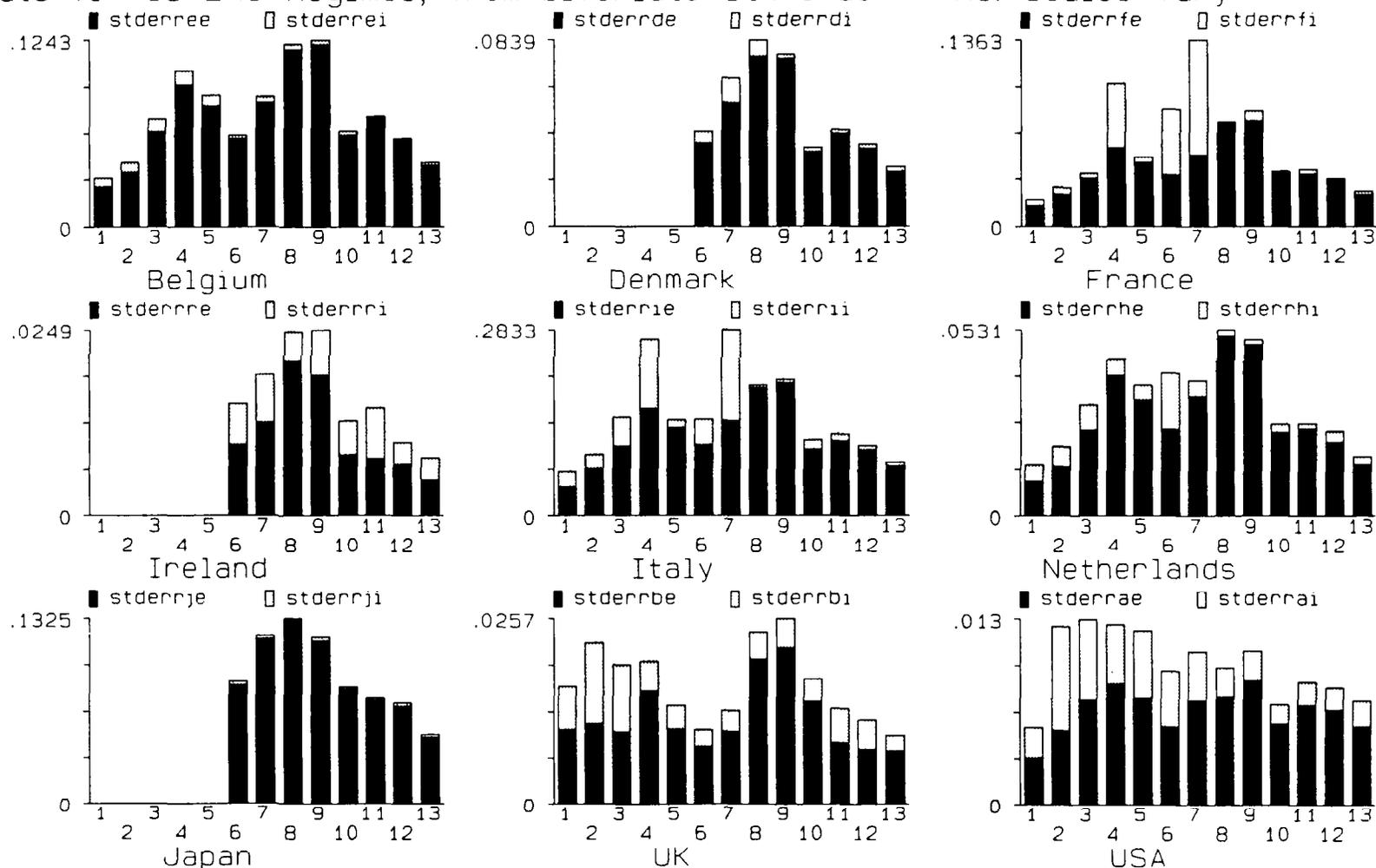


1979:3:13 - 1990:5:16



Figure 4. Conditional Volatility Measures

Residual Standard Errors of Exchange Rate and Interest Differentials
 Data for 13 EMS Regimes, from bivariate 5th order VARs: Scales Vary



Interest Differential and Exchange Rate Standard Errors

1. EMS Regimes used in empirical analysis

EMS Regime	Dates	Potential Number of Observations
Regime 1:	1979:3:30-1979:9:16	134
Regime 2:	1979:9:29-1979:11:25	39
Regime 3:	1979:12:14-1981:3:15	331
Regime 4:	1981:4:4-1981:9:27	130
Regime 5:	1981:10:17-1982:2:14	117
Regime 6:	1982:3:6-1982:6:6	70
Regime 7:	1982:6:26-1983:3:13	190
Regime 8:	1983:4:2-1985:7:14	600
Regime 9:	1985:8:3-1986:3:30	175
Regime 10:	1986:4:19-1986:7:27	75
Regime 11:	1986:8:16-1987:1:4	105
Regime 12:	1987:1:24-1989:12:31	770
Regime 13:	1990:1:20-1990:5:16	87

As is well-known, the EMS has become increasingly credible in the sense that the periods between realignments seem to be growing longer; we intend to test for other manifestations of increasing credibility. In our empirical work we tend to focus on the twelfth regime of the EMS, as it is a long sample of data drawn from a potentially credible target-zone.

2. Volatility in exchange and interest rates

We note that exchange rate volatility varies dramatically over time for each country; this is apparent in Figures 2 and 3, as well as simple descriptive statistics (which are tabulated in the working paper). While more recent regimes are not generally associated with high volatility (measured by historical standards), neither are they associated with exceptionally low volatility. On the other hand, interest rate differentials do seem to be less volatile more recently.

There are large differences across countries in both exchange rate and interest rate volatility. For instance, the Netherlands has much lower exchange rate volatility than the other EMS countries. However, no trade-off between exchange rate and interest differential volatility is apparent in the data. Figure 4 provides stacked bar charts of standard deviations of

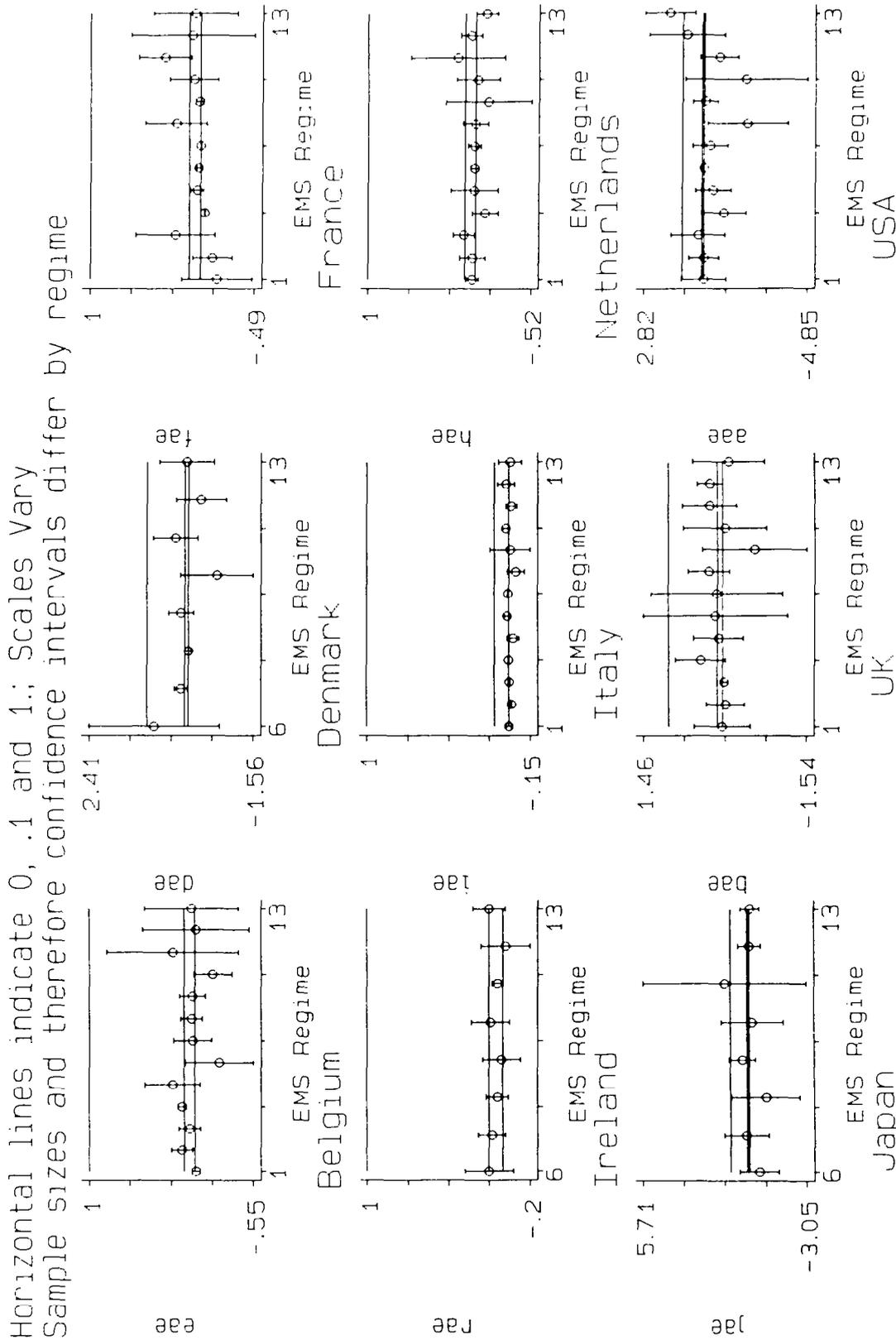
residuals from a bivariate fifth-order VAR of interest rate differentials and exchange rates. 1/ No relationship is apparent between the two measures (unconditional estimates deliver the same result and are contained in the working paper version).

Unit-root tests (allowing for serial dependence through the method suggested by Perron (1988)) indicate, unsurprisingly, that unit-roots are pervasive throughout the data (the statistics are tabulated in the working paper). More precisely, the null hypothesis that a unit-root exists cannot usually be rejected at conventional significance levels in each of: the exchange rate; fundamentals (using $\alpha=0.1$); and the interest differential. 2/ While this may be the result of low power (Froot and Obstfeld (1990b)), it is extremely disturbing that the interest differential appears to be nonstationary. Ignoring drift, the difference between the exchange rate and fundamentals is the expected rate change of the exchange rate; uncovered interest parity implies that the latter is the same as the interest differential. A nonstationary interest differential is inconsistent with credible target-zones; the persistence in this series which cannot be accounted for by fundamentals will return to haunt our hypothesis tests later.

1/ Svensson (1990c) asserts that there should be a trade-off between the conditional variances of interest rates and the width of the fundamentals band. Indeed, the slope of the $\text{stderr}(e):\text{stderr}(i-i^*)$ relationship should provide an estimate of $-\alpha$. However, regression techniques that pool data across regimes and countries, lead to a positive relationship between conditional interest rate differential volatility and exchange rate volatility; this result is insensitive to inclusion of regime-specific effects. If the data are first-differenced (taking into account any country-specific "fixed effect"), this effect is wiped out. Statistics are fully tabulated in the working paper version of this paper. Also, the estimated unconditional standard deviation of the exchange rate is essentially uncorrelated with the estimated standard deviation of the interest rate differential; this result is also robust to inclusion of year or country fixed effects. There is also little evidence of any nonlinearity in the latter relationship, although Svensson (1990c) derives a nonlinear relationship between the width of a target-zone and unconditional interest rate variability.

2/ It should be noted that a Wiener process that is reflected between two barriers still has all moments and is not integrated of order one.

Figure 5. Estimates of Alpha



2 standard error interval also shown

The hypothesis that fundamentals have a unit-root cannot typically be rejected at conventional significance levels. In fact, the assumption that fundamentals follow a driftless random walk, while not literally true, seems to be a good approximation. 1/

V. Determination of Alpha

Our strategy will be to find an appropriate range for α ; we then conduct our analysis for reasonable values of α spanning this range. We estimate α by two methods. First, we use our data to estimate α . Second, we use estimates from the literature.

1. Estimating α from daily data

If the increments to f are generated by equation (3), then integrating df over one day results in:

$$(3') f_t - f_{t-1} = \eta + \epsilon_t$$

where the discrete-time period is one day, η is the daily growth rate of fundamentals and ϵ_t , which is the integral over one day of σdz , is the daily disturbance to the f process. Substituting from equation (3') into equation (1), we obtain:

$$(14) e_t - e_{t-1} = \eta + \alpha \{ [E_t(de)/dt] - [E_{t-1}(de)/dt] \} + \epsilon_t$$

For estimation we replace $E_{t-j}(de)/dt$ with $(i_{t-j} - i^*_{t-j})$. Equation (14) can then be used to estimate η and α . The structural parameters α and η are identified because fundamentals are exogenous almost everywhere.

Our estimates of alpha are presented in Figure 5. This figure graphs the point estimate of alpha with a two standard-error band. 2/ The

1/ Judged by conventional Box-Ljung Q-statistics, the residuals from a regression of the first-difference of fundamentals on a constant look like white-noise for most EMS regimes and countries, while the intercepts are usually close to zero both statistically and economically. However, even in this linear framework, there are some clear violations; lagged first-differences of fundamentals sometimes have explanatory power for first-differenced fundamentals, and some constants are significant. Of course, in a target-zone set-up, reflection terms (at the bands) should also contribute explanatory power.

2/ As sample size varies by regime, the two standard error bands correspond to intervals of varying confidence levels.

estimates are almost uniformly small, although they vary considerably across country and EMS regime. With the exception of a few imprecise estimates for Denmark and France, there is little statistical evidence that alpha exceeds 0.25 for EMS countries; lack of interest rate volatility makes it much more difficult to pin down α for non-EMS countries. Indeed, there are numerous negative estimates of alpha; the hypothesis that alpha is zero does not seem unreasonable from a purely statistical point of view (although we exclude that possibility, and hence many of the point estimates, a priori). ^{1/}

2. Estimates of α in the literature

We have interpreted α as the negative of the interest rate semi-elasticity of money demand, a parameter that plays a widespread role both in theoretical and empirical macroeconomics. This parameter has previously been estimated in the literature; Goldfeld and Sichel (1990) provide a survey. The short-run semi-elasticities reported are quite similar to one

^{1/} We have also tried to estimate α with a technique which relies on McCallum's substitution of actual exchange rate changes in place of anticipated movements; this technique typically delivers estimates of α near -1. As discussed above, we have also tried to estimate α by regressing regime-specific conditional volatilities of exchange rates on conditional volatilities of interest rate differentials; this method typically delivers an estimate of α near zero. The latter technique could be extended within regimes by employing an ARCH-like specification for conditional volatilities (this would deliver more observations for estimation purposes). One could also measure f by regressing $(i-i^*)$ on e and defining the residual plus the constant to be f . This approach has the advantage of not depending on additional assumptions about f ; it is potentially important with data sampled less finely than is our data, since the target-zone reflections of fundamentals can bias coefficient estimates for the f process.

another and average -0.4. 1/ 2/ These estimates are converted to long-run elasticities by dividing by the average quarterly speed of adjustment, 0.32/quarter, giving a long run semi-elasticity estimate of -1.25, which we take to be representative of semi-elasticity estimates for industrial countries during normal times.

There are two ways to apply these estimates to daily data. First, in the spirit of the models upon which equation (1) is based, one can think of a model of continuous long-run money market equilibrium such that an appropriate choice of α is 1.25. More realistically, one can think of equation (1) as resulting from a Goldfeld-style partial-adjustment model of the money market. 3/ In this view, it is the short-run interest rate semi-elasticity which is relevant to the problem; this is obtained by dividing -0.4 by 90 days per quarter, giving a daily short-run semi-elasticity of -0.0044, so that $\alpha=0.0044$ seems appropriate.

Our various methods of uncovering α have led us to a range for this parameter. We think of $\alpha=0.1$ as being a reasonable value; $\alpha=1$ is certainly representative of the high end of the range, especially given our point estimates. In most of our work below, we report results based on $\alpha=0.1$ and $\alpha=1$. Several different manifestations of the data indicate that $\alpha=0.1$ is a good choice for this key parameter; see also Diebold (1986).

1/ The average number Goldfeld and Sichel report is -0.004, but they chose interest rate units so that 10 percent per year, for example, was entered as 10. We choose units so that 10 percent per year is entered as 0.10. Under our convention, the Goldfeld and Sichel estimates need to be multiplied by 100.

The estimates Goldfeld and Sichel report are the product of a speed of adjustment, which has units percent per quarter, and the semi-elasticity of money demand, having time units which are the inverse of the time units of the interest or expected rates of change of asset prices. Throughout this study we use annualized interest rates so our interest rate semi-elasticities have units years.

Quarterly domestic interest rates rather than two-day Euro-rates are usually used as interest rates in money demand equations. Also, increased currency substitution may mean that historical estimates of α are too low; Canzoneri and Diba (1990) discuss the effects of currency substitution further.

2/ The estimates Goldfeld and Sichel report involve the following countries and data periods; Canada 1962:1-1985:4, Japan 1966:1-1985:4, France 1964:1-1985:4, Germany 1969:1-1985:4, Italy 1971:1-1985:4, and U.K. 1958:1-1986:1. The results for these countries match quite closely with the results for the U.S. in terms of the magnitude of the short-run interest rate semi-elasticity.

3/ However, it is important to recall that the assumption that fundamentals can be summarized in a single model-determined state variable is maintained throughout the analysis.

Given an α value, the fundamentals can be measured at the monthly frequency and compared with the traditional reduced-form determinants of flexible-price exchange rate models, money and output. 1/ We obtained monthly IFS measures of M1 and industrial production, 2/ computed logarithmic differentials between German and foreign variables, and regressed our measure of fundamentals on actual money-supply and output differentials. The regressions are computed from 1979 through 1990 on a country-by-country basis. Our measures of fundamentals are typically highly correlated in levels with actual money and output differentials; for instance, the R^2 s for our six countries have an average of 0.63. On the other hand, the coefficients on actual fundamentals are not signed consistently, and there is substantial residual autocorrelation. In first-differences, our fundamental measures are essentially uncorrelated with money and output.

VI. Graphical Analysis of Nonlinearities

1. A direct examination of the exchange rate: fundamentals relationship

In this section of the paper, we analyze the relationship between exchange rates and fundamentals, using graphical techniques. Our conclusions will be corroborated below with more rigorous econometric techniques. We begin with the assumption $\alpha=0.1$.

Figures 6 through 11 contain a wealth of descriptive graphical information about the relationship between the exchange rate (e) and fundamentals (f). Each figure (except those for Denmark and Ireland) contains 14 "small multiple" $e:f$ scatter-plots; one for each of the 13 EMS regimes, and another covering the whole sample from 1979 through 1990. The use of small multiple graphs allows the data to be compared easily across regimes and countries.

In any given scatter-plot, each of the individual points represents a single daily observation. To guide the eye in connecting the dots of the joint distribution, a nonparametric "data smoother" is drawn as a solid line. 3/ We use the shapes of these smoothers extensively in our search for nonlinear relationships between e and f . The smoother can easily handle

1/ We temporally average fundamentals (instead of selectively sampling fundamentals), to correspond to the way that industrial production is measured.

2/ Quarterly in the cases of Belgium and France.

3/ The smoother divides the horizontal axis into a number of bands (we generally use five), and calculates the cross-median of e and f within each band. The cross-medians are then connected with cubic splines. Meese and Rose (1991) use a different nonparametric smoothing technique (locally-weighted regression) and arrive at results consistent with ours. See also Diebold and Nason (1990) and Meese and Rose (1990).

Figure 6. Belgium

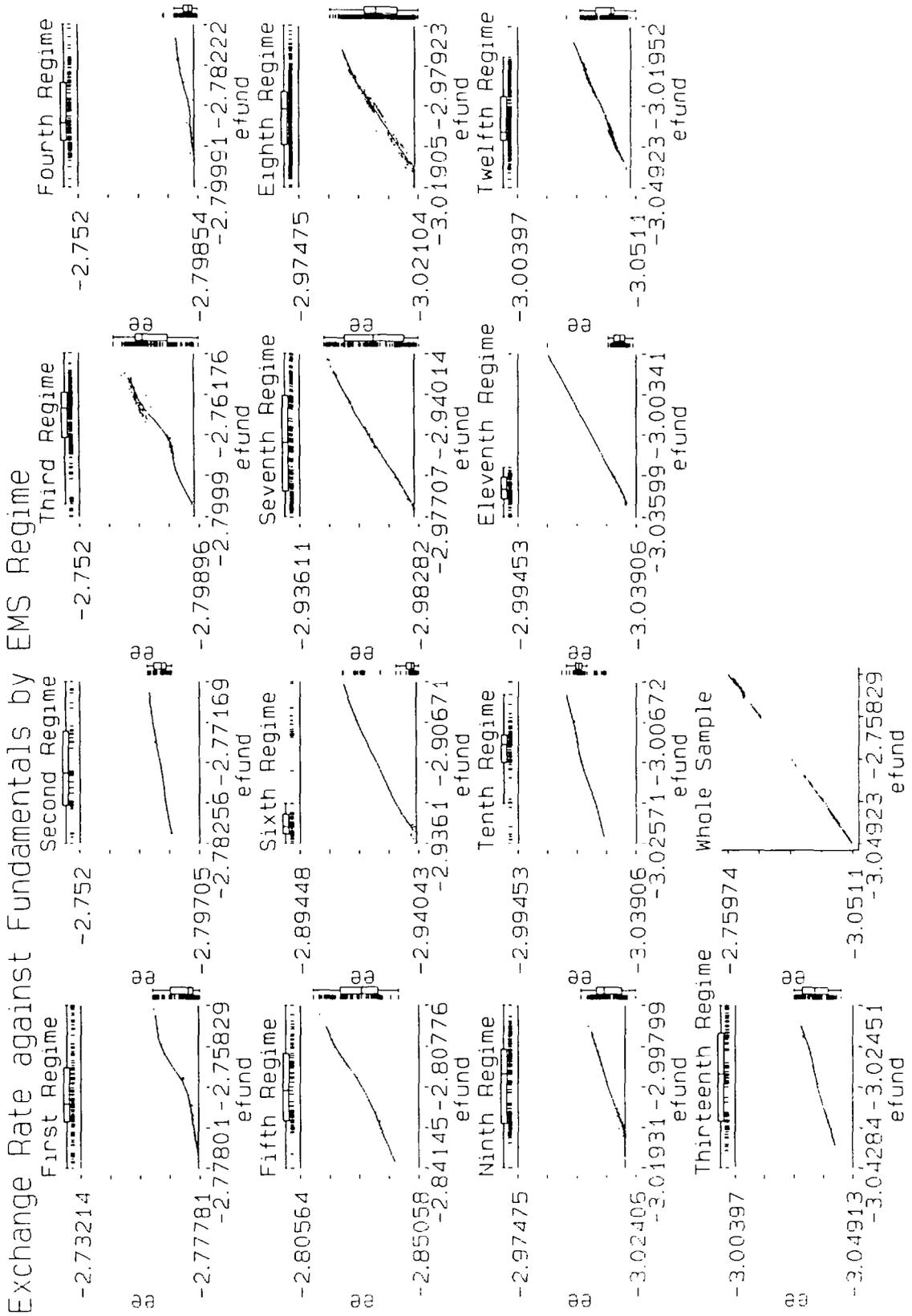


Figure 7. Denmark

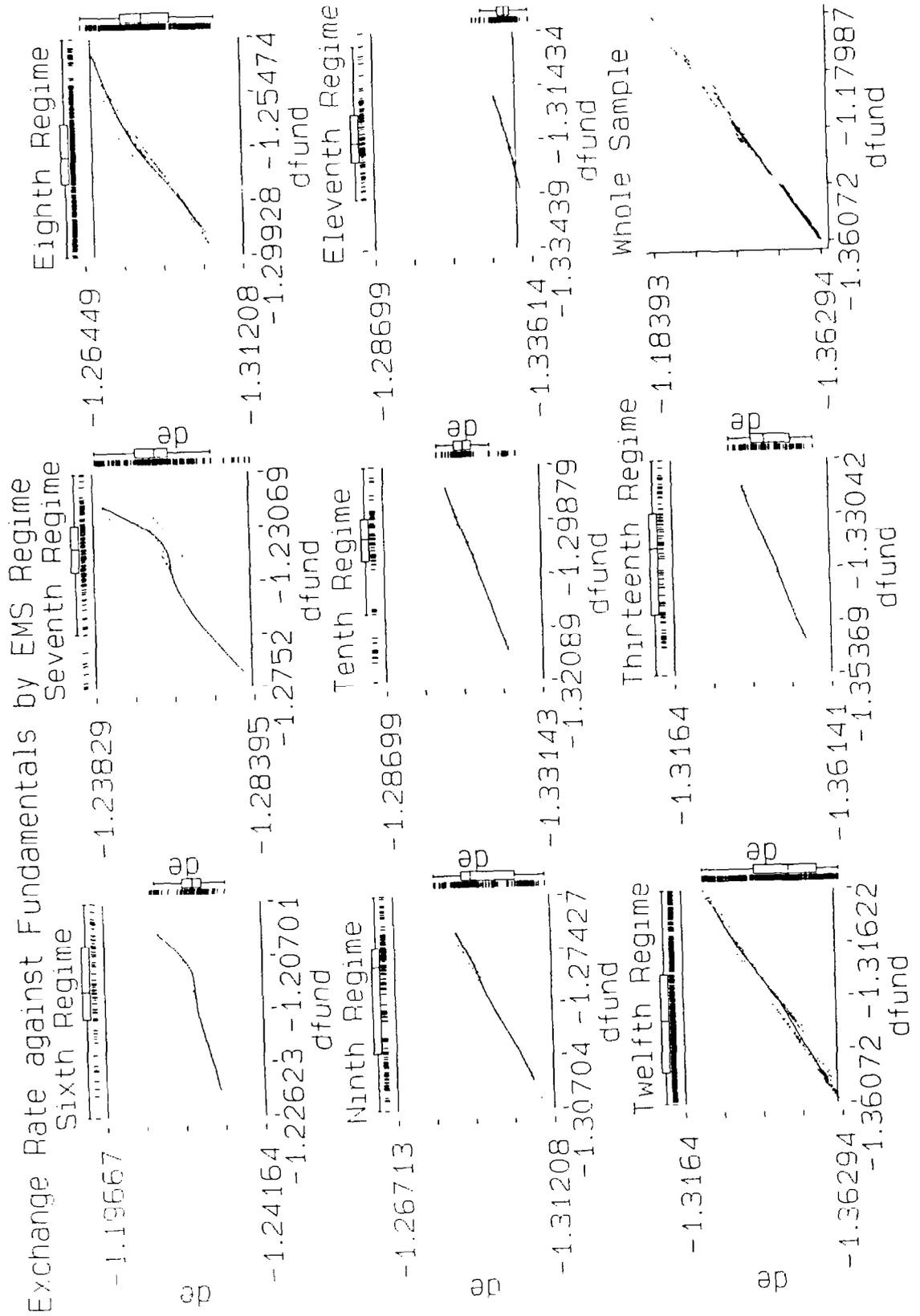


Figure 8. France

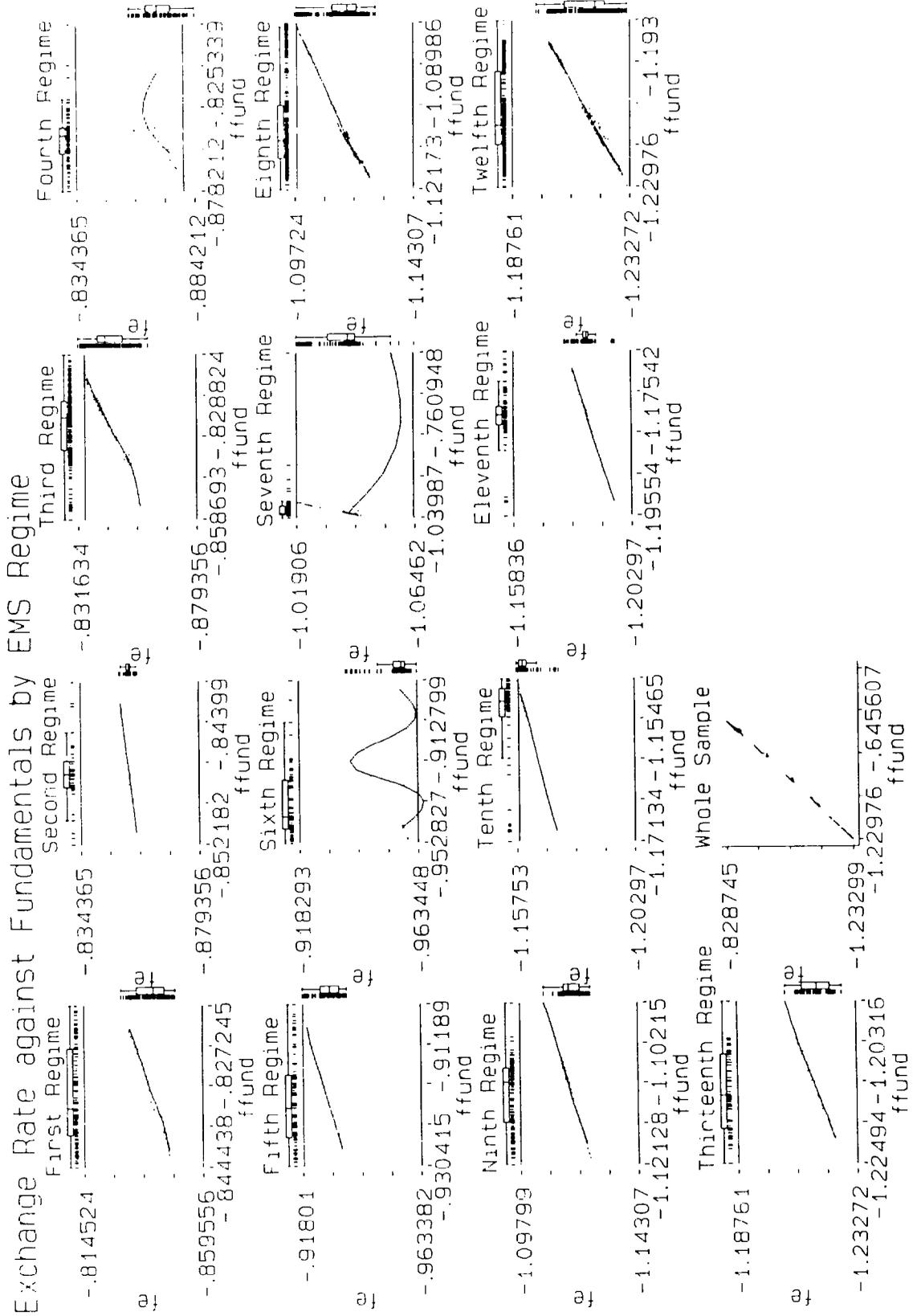


Figure 9. Ireland

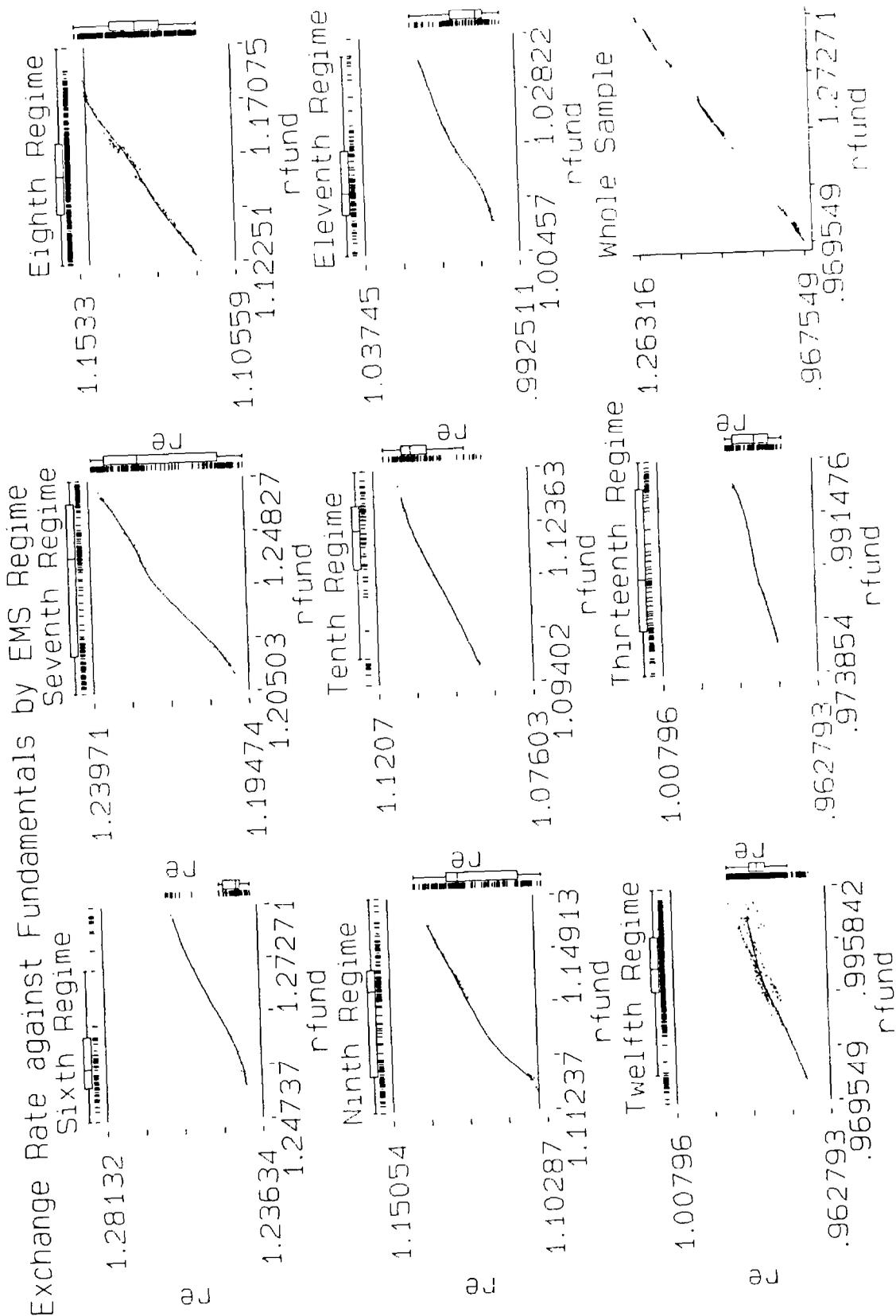


Figure 10. Italy

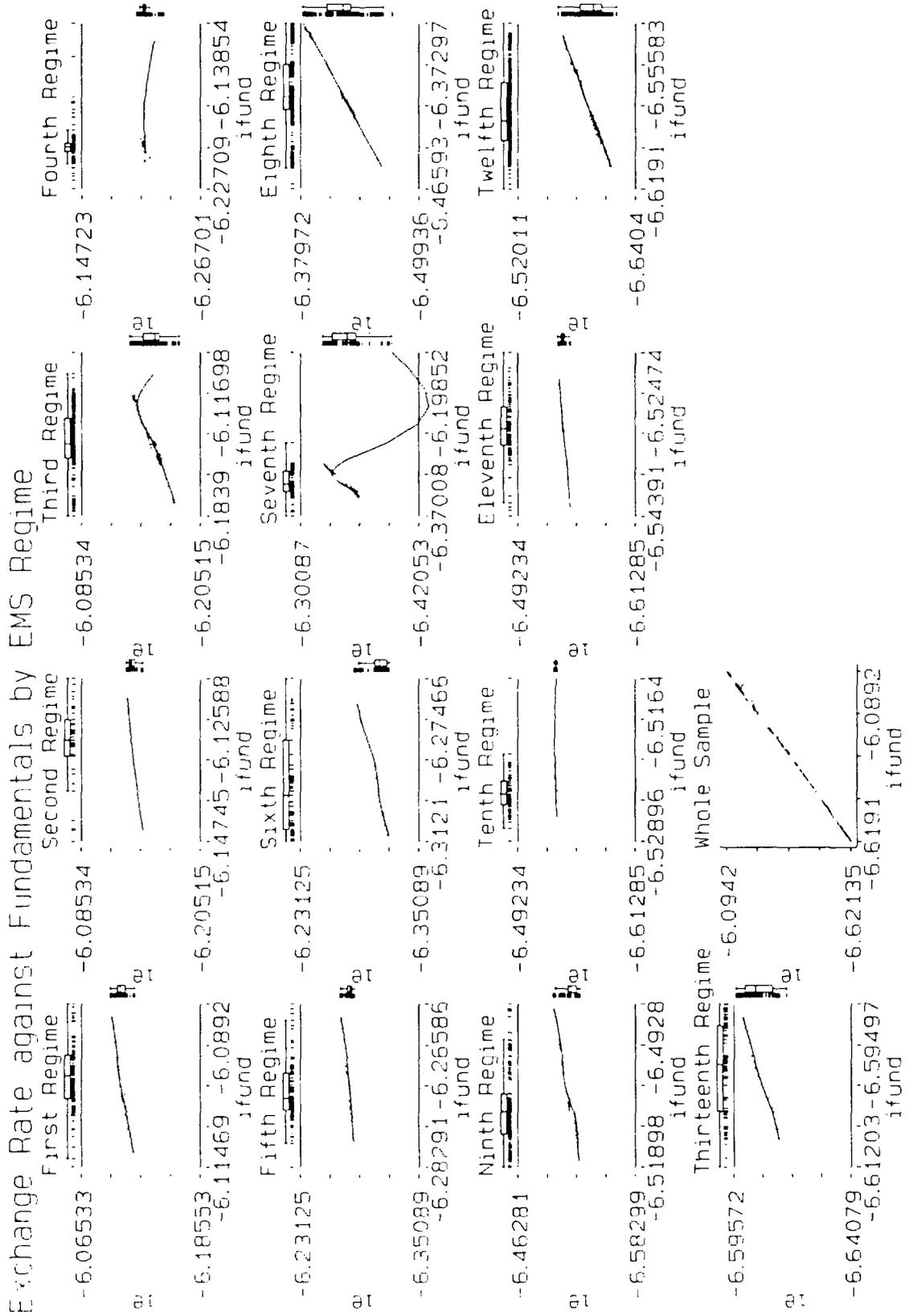
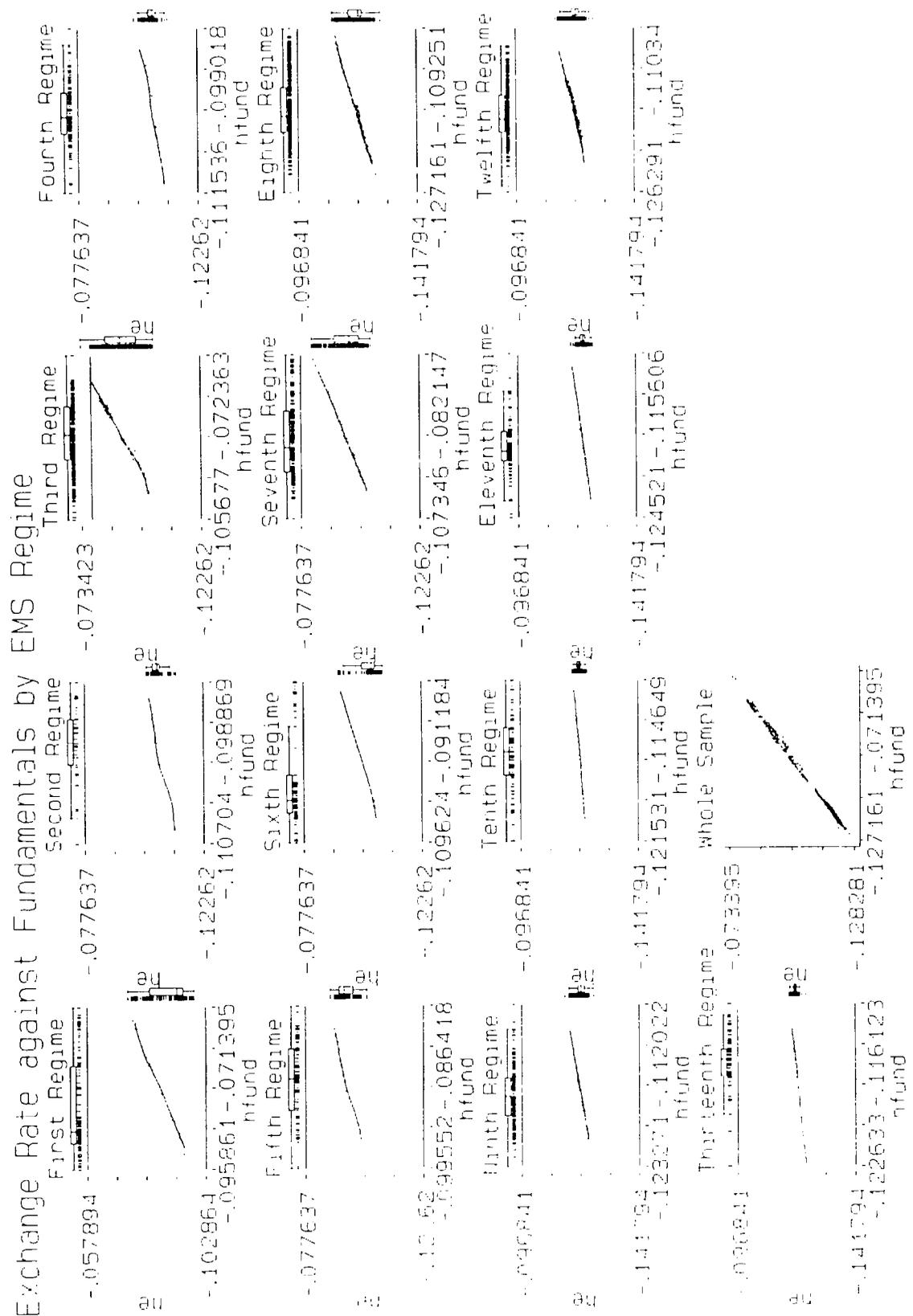


Figure 11. Netherlands



the nonlinear patterns implied by the target-zone theories above; conversely, the absence of sensible nonlinear smoother patterns suggests (though it does not prove) that the theories work poorly.

The marginal density for e is displayed to the right of the scatter-plot; each observation is represented with a single tick mark. Immediately to the right of the marginal density, a box-and-whiskers plot of the marginal density is also displayed. The line in the middle of the box marks the median of the marginal distribution; the box covers the interquartile range (i.e., from the 25th percentile range to the 75th percentile range). The whiskers extend to upper- and lower-"adjacent values"; points beyond adjacent values are usually considered outliers. ^{1/} A comparable marginal density and box plot for f is graphed above the diagram. This combination of graphs allows one to evaluate the marginal and joint distributions simultaneously.

Target-zone theories place a number of restrictions on the marginal distributions of e and f , as discussed above. For instance, the simple model of Krugman (1990) implies that, (with perfect credibility and infinitesimal interventions on the bands); the exchange rate is expected asymptotically to have a bimodal symmetric density which would be directly apparent in the marginal distribution, and manifest in the box plot as a relatively wide symmetric interquartile range with small whiskers. The model of Bertola and Caballero (1990b) delivers a very different set of restrictions. In addition, some theories (e.g., Bertola and Caballero (1990a)) present restrictions on the relationship between e and f across regimes; hence the scatters for the entire sample.

The (implied) EMS exchange rate bands are drawn as horizontal lines in the figures, so that the vertical dimension of almost all the EMS graphs represents +/-2.25 percent.

Consider the top left graph in Figure 6, describing the relationship between e and f for Belgium during the first EMS regime, which prevailed from March 13, 1979 through September 23, 1979. The data are grouped in the lower portion of the graph, indicating that the Belgian Franc was relatively weak during this period; the box plot for e indicates that the median value of the exchange rate is quite low in the band, and there are no positive outliers. This is true despite the fact that fundamentals are approximately symmetrically distributed in an apparently normal distribution. The relationship between e and f appears to be monotonic, positive and slightly nonlinear in a manner reminiscent of Krugman's S-shape, though it is very close to the lower boundary.

No simple general characterization can be made about the $e:f$ relationships. However a number of features do seem apparent. First, and

^{1/} Adjacent values are defined as 150 percent of the interquartile range rolled back to the nearest data point.

most importantly, remarkably few nonlinearities are apparent. Second, currencies that are typically viewed as being more committed to the ERM have fewer (not more) manifestations of nonlinearities. For instance, nonlinearities are not readily apparent in the Dutch data compared with the other five countries, although the Netherlands is generally considered to be a country that maintains a credible exchange rate band (Holland has only experienced two realignments vis-à-vis Germany).

Third, nonlinearities appear to be growing less important over time, rather than more important; the absence of nonlinearities in the twelfth regime is noticeable. However, increased credibility should be manifest in an relationship between the exchange rate and fundamentals that comes increasingly to resemble Krugman's S-shape, as realignments become more unlikely. 1/

Fourth, while some nonlinearities are apparent, they tend not to have shapes that are even vaguely similar to those implied by extant theories. Countries that have experienced frequent realignments (such as Italy) do not appear to have inverted S-shapes, as implied by the Bertola and Caballero (1990b) model; credible countries (such as the Netherlands) do not have Krugman's S-shape. That is, the nonlinearities that are apparent do not seem to have sensible identifiable patterns across either time or country.

Fifth, much of the data is clustered in the middle of the declared exchange rate bands, especially for later regimes. Assuming that the actual exchange rate bands coincide with the declared bands, nonlinearities are difficult to detect visually if the exchange rate stay in the middle of the zone. 2/ This may indicate that the authorities defended implicit bands well within the declared bands; in this case our theoretical analysis applies for the actual implicit bands, so long as the market recognized this fact. 3/ The fact that exchange rates spend much of their time in the interior of the band may instead be a small sample problem. Given the sample sizes involved and the nature of the forcing process under the null hypothesis, we are skeptical of this view; however, nonlinearities would be much more difficult to detect if exchange rates happened to have avoided the periphery of the bands.

1/ The analysis of Bertola and Caballero (1990a,b) implies that the shape of the nonlinearities should be changing over time from an inverted S-shape to Krugman's S-shape.

2/ On the other hand, the problem is explicitly a small sample problem. In a credible target-zone, the exchange rate should spend most of its time near the bands.

3/ This is true so long as the implicit bands are constant (as the declared bands are). Hali Edison and Graciela Kaminsky are currently testing the hypothesis of constant implicit bands.

Finally, the e:f relationship appears to be approximately linear over the entire sample, consistent with the model of Bertola and Caballero (1990a).

Figures 6 through 11 rely on our assumption $\alpha=0.1$. Clearly as α falls, the scatter-plots in these figures move closer towards an exact affine relationship between e and f; if $\alpha=0$, $e=f$ exactly. Figures A1 and A2 are the analogues to Figures 10 and 11, but computed with $\alpha=1$, a value that is implausibly large in our view. These figures indicate nonlinear effects of substantively greater importance, although it is again difficult to detect patterns over time or country. Again, the smoother shapes bear little resemblance to those implied by extant exchange rate models. 1/

2. Comparison with other exchange rate regimes

While the scatter-plots of Figures 6 through 11 do not seem consistent with the implications of known nonlinear exchange rate theories, we hasten to add that countries participating in the EMS do not look similar to countries in (relatively) free floats. Figures 12 through 14 are graphs (comparable in every way to Figures 6-11) for three exchange rates which are floating (relatively) freely against the DM: the Japanese yen; the British pound; and the American dollar (all rates are again bilateral DM rates). Again, each figure has 14 small graphs, one for each of the 13 regimes, as well as one for the whole sample. While actions such as the Plaza Accord and the Louvre Agreement clearly lead one to doubt the assumption of perfectly free floating, the e:f scatters look much more linear for non-EMS countries than they do for EMS countries.

Another natural comparison can be made between the EMS countries during the EMS 1980s and the pre-EMS 1970s. Figure 15 contains e:f scatters for four of the six EMS currencies (Danish and Irish data are unavailable) during the period which preceded the EMS from 1977:9:1 through 1978:10:10. During this period, Belgium and the Netherlands participated in the European common margins arrangement, commonly known as the "Snake," the precursor to the EMS. The graphs appear to be conspicuously linear.

Finally, the EMS can be compared with other regimes of fixed exchange rates. Figure 16 provides graphs for the post-WWII Bretton Woods regime of pegged but adjustable rates (the data is drawn from the 1960s); Figure 17

1/ The working paper version contains analogues to Figure 6 through 11 with Italy as the base country.

provides comparable data for the pre-WWI and inter-war gold standards. Both figures use $\alpha=0.1$. 1/

The relationship between the exchange rate and fundamentals seems to be decidedly more nonlinear for the gold standard than for the EMS; the dollar/yen rate also appears to be nonlinearly related to fundamentals during the Bretton Woods era. 2/ However, most of the Bretton Woods data appear consistent with linear e:f relationships, while the smoother shapes in the gold standard data are not implied by existing target-zone models. 3/

3. Is there a "honeymoon" effect?

As discussed above, the thrust of the original target-zone proposal was to make the exchange rate less responsive to fluctuations in exchange rate fundamentals, the celebrated "honeymoon effect" of Krugman (1990). The theoretical framework of Section II implies that the e:f slope should be unity in a floating rate regime, lower in a credible target-zone. If the diminished impact of fundamentals on the exchange rate in a credible target zone is the "honeymoon effect," then the possibility that the impact might be magnified in an incredible target zone (Bertola and Caballero (1990a,b)) might be the "divorce effect." It should be remembered, however, that we are studying government policies, not interpersonal relationships; the start of a target-zone is more likely, we think, to be characterized by low policy credibility than high policy credibility.

Estimates of the slope thus provide a specification test of the target-zone model. Actual estimates of the slopes for all countries and EMS regimes are presented in Figure 18; we simply regress e_t on f_t and an intercept; Newey-West covariance estimators are used.

1/ Using a higher value of α (say 1) changes the Bretton Woods graphs considerably; the smoothers do not tend to be positively sloped, and are extremely wiggly. Below, we show that much higher values of alpha (e.g., 1.) appear unreasonable in a number of different dimensions. Higher alpha values (say 0.5) for the gold standard data do not greatly change the graphs.

2/ The smoother shapes are vaguely reminiscent of Krugman's S-shape for parts of the lower tails; however, upper tails appear to be essentially linear.

3/ This may be, in part, the result of movement in the gold points. These are the exchange rates at which arbitrage gains from physical transportation of gold exceed transportation costs; the gold points were market forces which limited fluctuations in exchange rates during the gold standard. Myers (1931), Officer (1986), and Spiller and Woods (1988) provide further analysis. Movements in the gold points are conceptually similar to movements in implicit EMS exchange rate bands (when the authorities defend bands which differ from declared bands); however, the smoother patterns are very different.

Figure 12. Japan

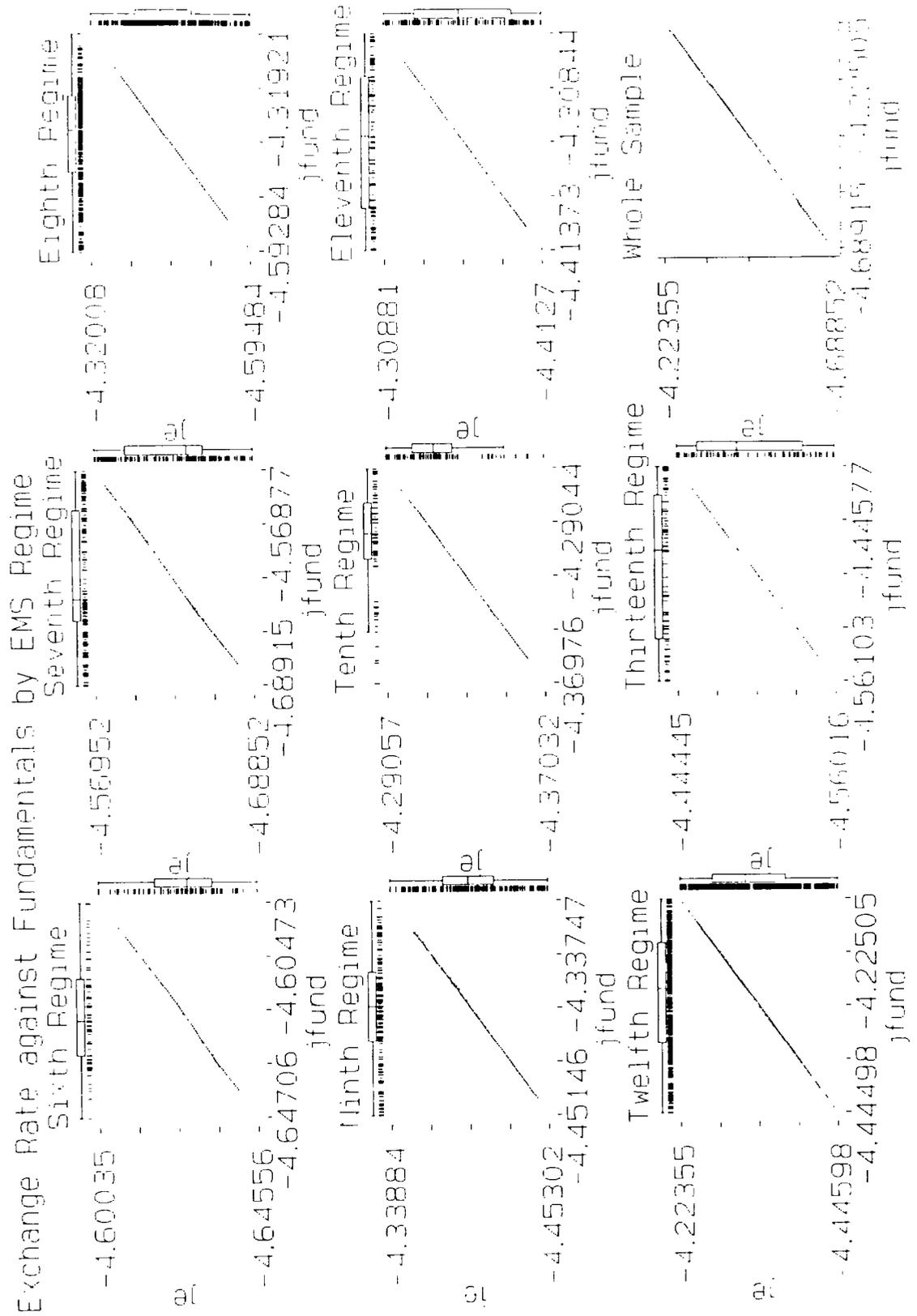




Figure 13. United Kingdom

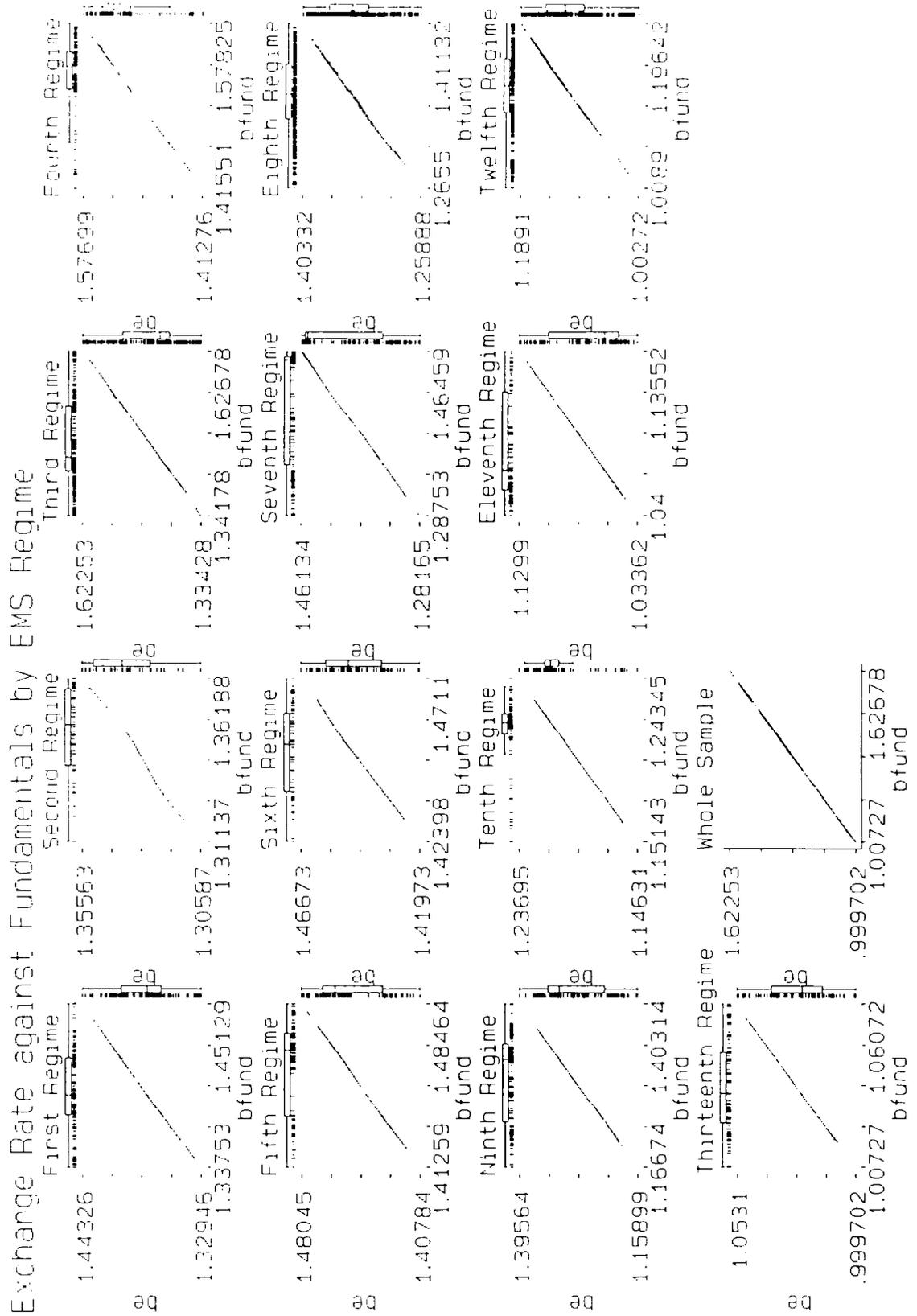


Figure 14. United States

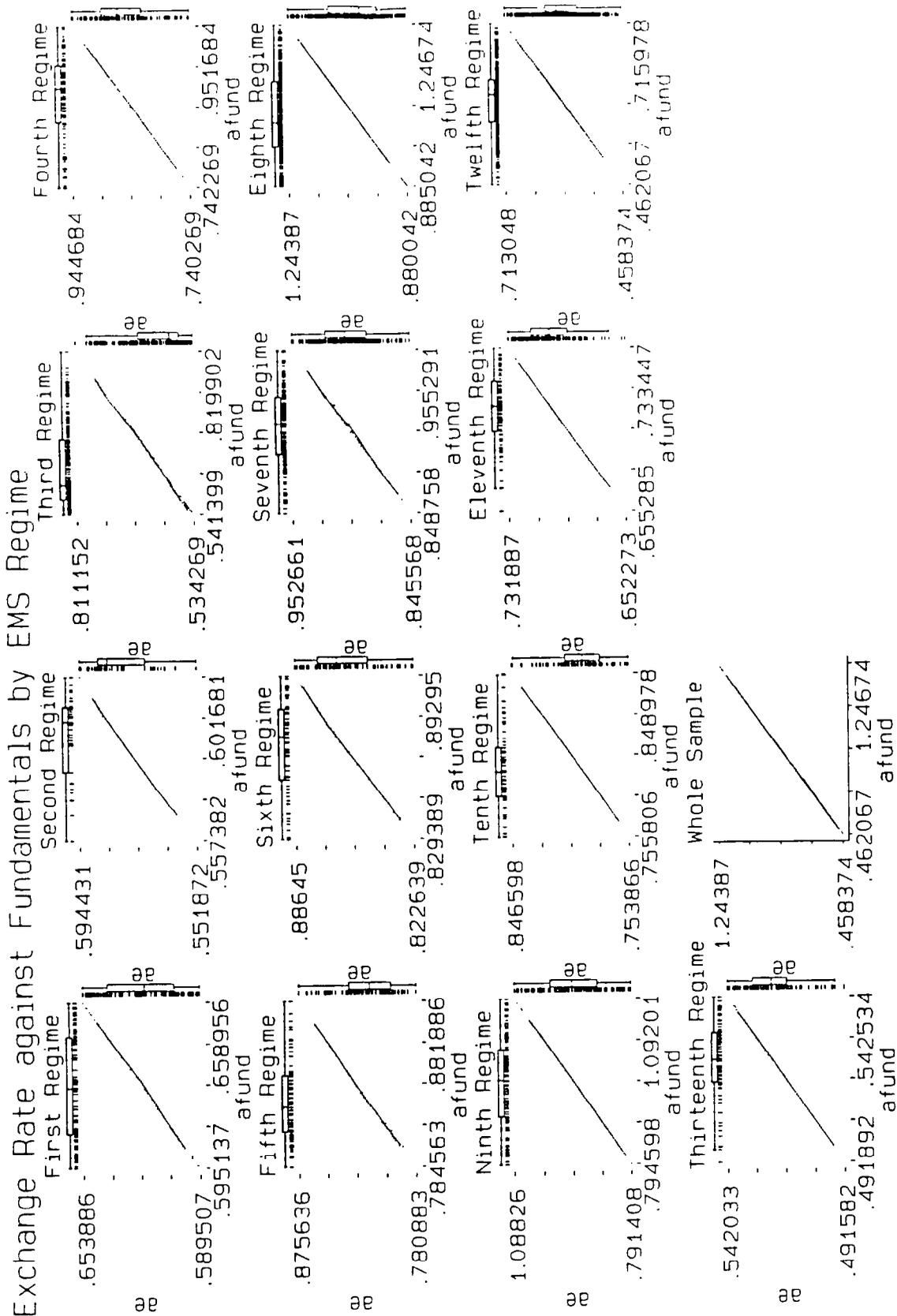


Figure 15. EMS Currencies in Non-ERM Regime

Exchange Rate against Fundamentals
9-1-1977 through 10-10-1978

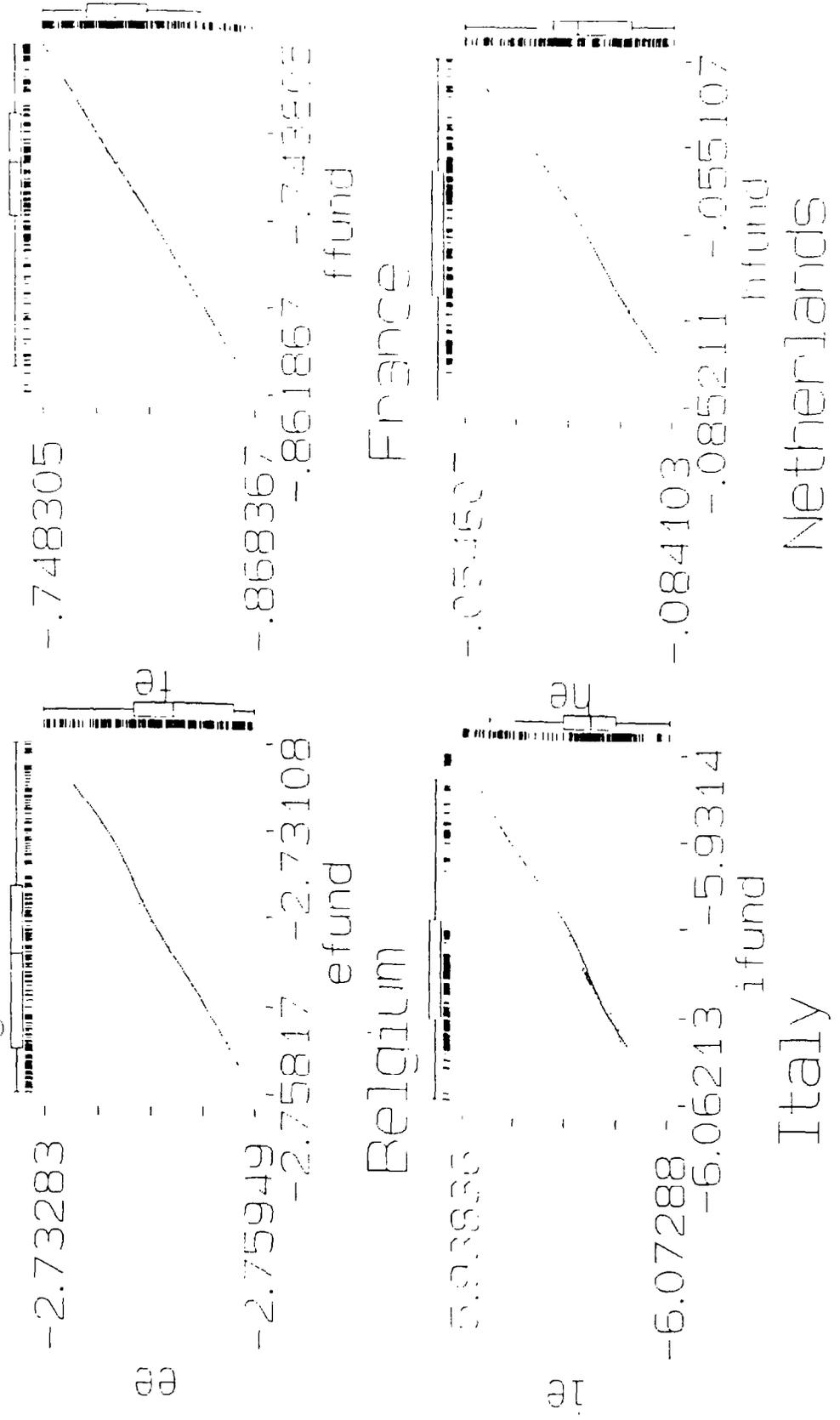


Figure 16. Bretton-Woods Regime: 1960s

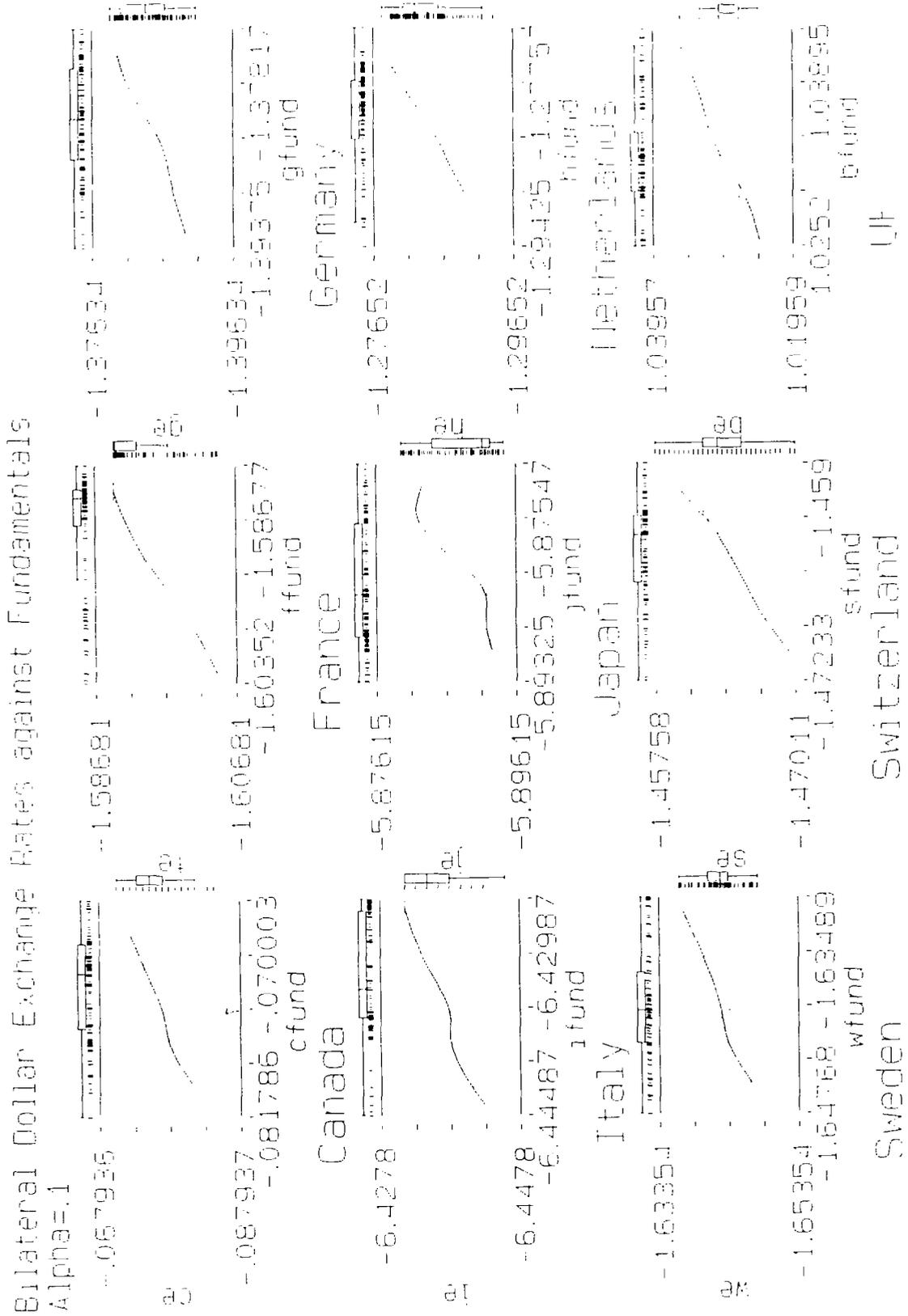


Figure 17. Gold Standards

Bilateral Dollar Exchange Rate Against Fundamentals
Alpha=1

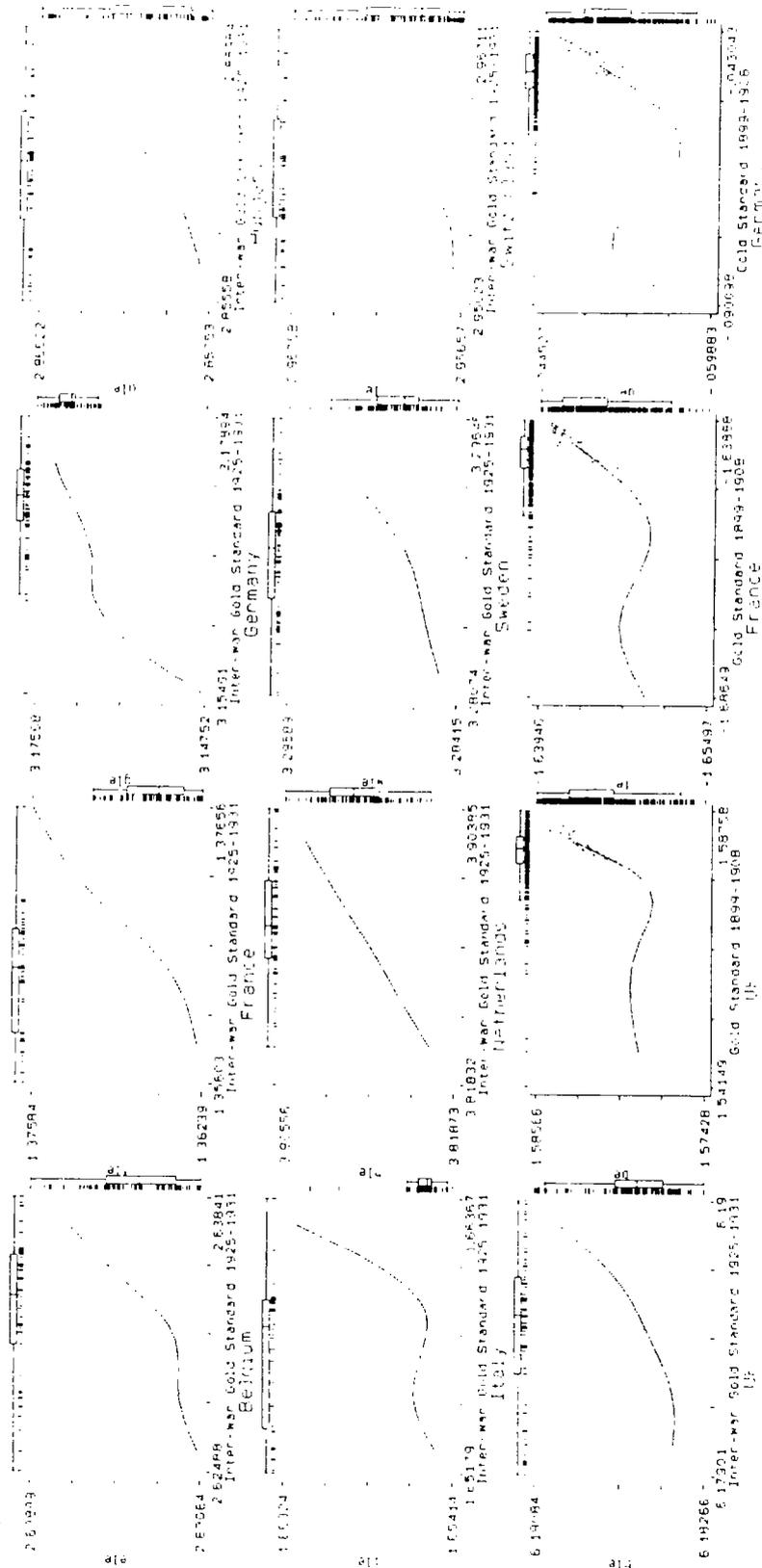
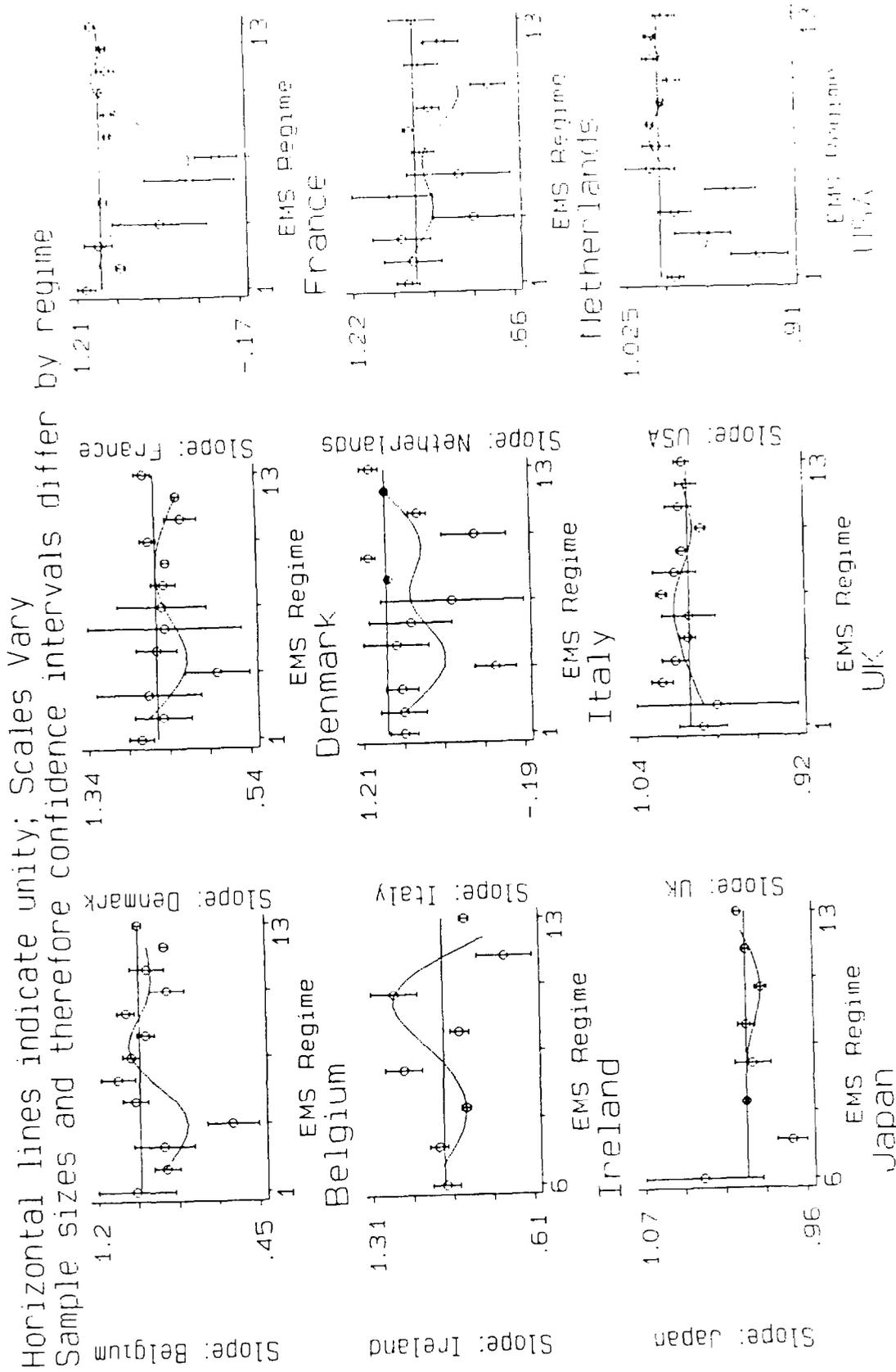


Figure 18. E:F Slopes, Alpha = 0.1



2 standard error interval also shown

Consistent with the honeymoon effect (and inconsistent with the work of Bertola and Caballero (1990b)), for $\alpha=0.1$, the e:f slope is often less than unity for EMS countries, though rarely by a large margin. However, for any given country, our point estimates of the slope vary considerably over time, being greater than unity for around a third of the regimes considered; point estimates of small slopes also tend to be imprecise. Further, there are few identifiable patterns in the slope estimates. For instance, the unstable regimes of the early 1980s are associated with small slopes, while the credible regimes of the late 1980s seem to have higher slopes. Also, slope estimates for countries as different as Italy and the Netherlands do not appear to be very different. It will be shown below that the nonlinear effects which give rise to the honeymoon effect in target-zone models, are not usually found in the data. 1/ Unsurprisingly, non-EMS countries have e:f slopes very close to unity. 2/

An errors-in-variables argument leads to the conclusion that a choice of α which is too high will lead to an e:f slope which is too low. Given our uncertainty about α , we conduct sensitivity analysis. Figure 19 is comparable to Figure 18, but uses $\alpha=1$ (the graphs with $\alpha=0.05$, for which there is essentially no evidence that the e:f slope strongly differs from unity, are in the working paper version). For $\alpha=1$, all point estimates (across six exchange rates and thirteen EMS regimes) are less than unity, virtually always by statistically significant margins. Indeed, the e:f slopes are clustered closer to zero than to unity. We view this as another manifestation of our hypothesis that unity is an excessively high choice for α .

4. Summary

Some nonlinearities are apparent in the scatter-plots between the exchange rate and fundamentals; the e:f relationship tends to look much more linear for floating exchange rates than it does for fixed exchange rates. However, in a number of different dimensions, the nonlinearities do not seem to conform to the patterns implied by target-zone models. The few nonlinearities that do exist do not appear as one might expect in more credible exchange rates (such as the Dutch Guilder), more recently (e.g., since 1987), or in the S-shapes implied by existing theories. Similarly, although there is modest evidence of a "honeymoon effect," the size of this

1/ Slopes are also unrelated to the spread between maximal and minimal values of e.

2/ Potentially important statistical problems afflict the standard errors for non-EMS countries if exchange rates and fundamentals are nonstationary. We suggest that if bubbles in the flexible-price model were important for explaining exchange rates then it might be expected that the honeymoon slopes would be different than unity. Of course, to take this suggestion seriously one would need to confront possible nonstationarities.

effect does not vary in a sensible way across regimes; in any case, the existence of the effect depends strongly on α , and reasonable values of α are consistent with no honeymoon effect.

Our relatively naive graphical approach has yielded, at best, weak support for target-zone nonlinearities. We now attempt to clarify the issue by applying more econometric firepower.

VII. Parametric Tests for Nonlinear Effects

In this section, we estimate target-zone models directly, and test the significance of nonlinear terms. We find that the nonlinear terms often add significant explanatory power in sample. However, the finding of statistically significant nonlinearities in-sample is too robust; it occurs for both fixed and floating exchange rates. Also, coefficient signs are not those predicted by target-zone models, and a number of other aspects of the model are rejected.

The structural model which we wish to estimate is:

$$(3') f_t = \eta + f_{t-1} + \epsilon_t$$

$$(14) e_t = \alpha\eta + f_t + A_1\exp(\lambda_1 f_t) + A_2\exp(\lambda_2 f_t)$$

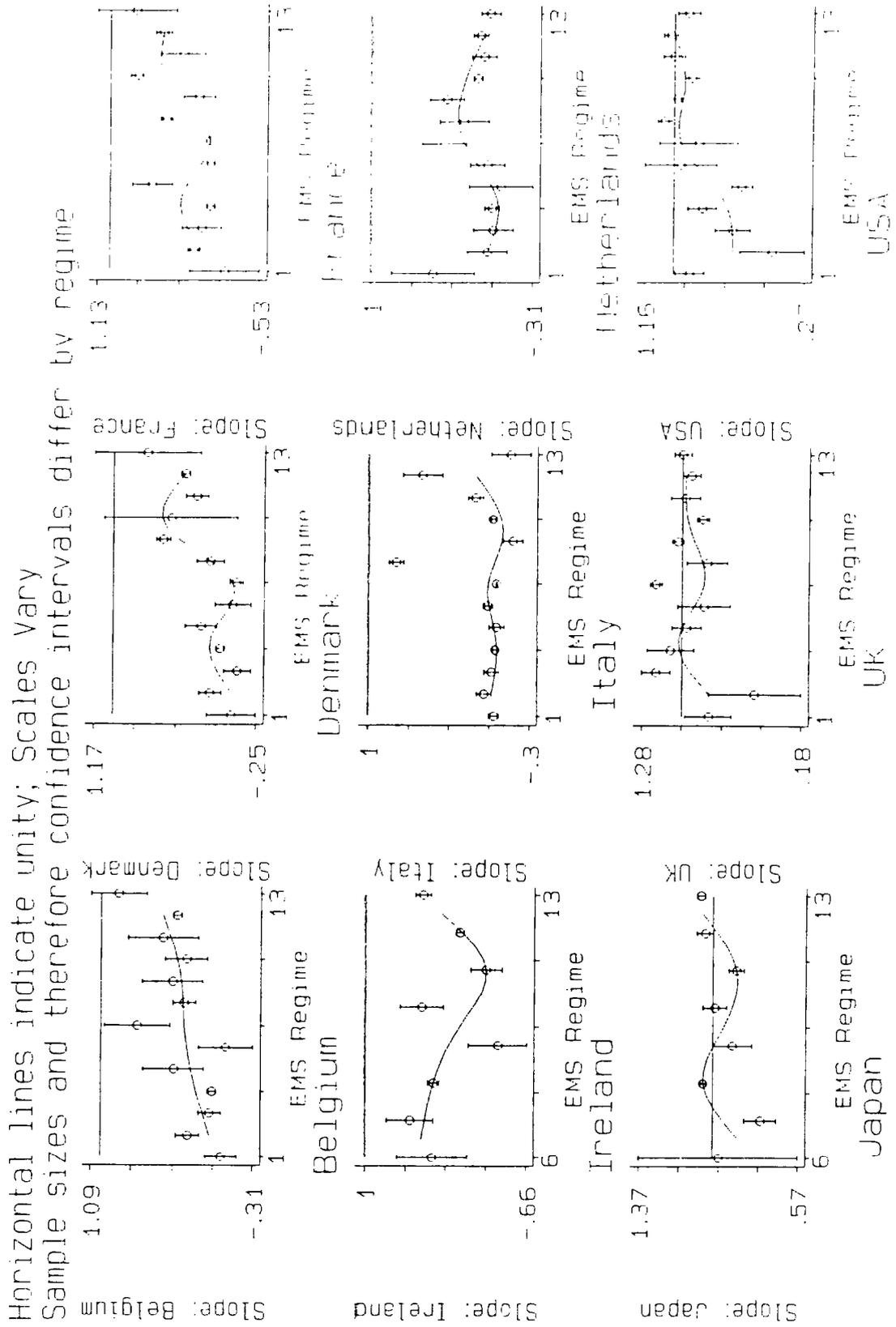
where we selectively sample e_t and f_t daily. In our empirical work, we work with a slight extension of (14):

$$(15) e_t - \hat{\alpha}\hat{\eta} - f_t = \theta_0 + \theta_1\exp(\hat{\lambda}_1 f_t) + \theta_2\exp(\hat{\lambda}_2 f_t) + \theta_3 f_t + w_t$$

where: $\hat{\eta}$ is the estimate of η from equation (3') (adjusted to an annual rate); $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are the roots to equation (7) with estimates of σ and η used in place of true σ and η ; and $\hat{\sigma}$ is the estimated standard of the residual of equation (3') (adjusted to annual rates). We estimate (15) with OLS, ignoring any biases in η and σ which may result from e.g., small-sample bias, generated regressors, or misspecifications of (3'). We maintain $\alpha=0.1$ for most of the analysis which follows.

We allow for two potential misspecifications of the model by including θ_0 and θ_3 ; a finding either of $\theta_0 \neq 0$ or $\theta_3 \neq 0$ is an indication of model misspecification (multi-collinearity considerations often preclude free estimation of θ_0). An error term has also been added to the equation; Froot and Obstfeld (1989b) suggest that this can be interpreted in a domestic

Figure 19. E:F Slopes, Alpha - 1



2 standard error interval also shown

context as the result of time-varying income tax rates that are conditionally independent of f_t . We also examine the serial correlation properties of this disturbance below.

Since there are cross-equation restrictions, estimation of these equations should be conducted jointly; for convenience, we pursue two-step estimation below. 1/ 2/ Thus, we estimate (3') with OLS; consistent estimates of η and σ are obtained from the intercept and standard error of the residual respectively. These estimates are then used to estimate λ_1 and λ_2 ; (15) can then be estimated directly with OLS. A_1 and A_2 can be consistently estimated by θ_1 and θ_2 ; from the latter, the exchange rate bands, e^L and e^U can be estimated. In practice, we test the hypothesis $\theta_1 - \theta_2 = 0$ ($\leftarrow A_1 - A_2 = 0$).

Two problems affect this work in practice. First, our regressors are exponential functions, which can lead to computational complexities. Such problems can be avoided by appropriate rescaling of the data. Second, there is often severe multicollinearity between the regressors of (15), making tests of individual coefficients unreliable. For this reason, tests of the joint hypothesis $\theta_1 - \theta_2 = 0$ are tabulated in Table 1. Table 1 also presents the estimated signs of the θ coefficients. As shown in the theoretical section, A_1 and A_2 are of opposite sign in most theoretical target-zone models. 3/

Table 1 also presents two specification tests (the restriction $\theta_0 = 0$ was imposed for the analysis reported in Table 1). First, the marginal significance level from a standard Q-test to examine the serial correlation properties of the residual from (15) is tabulated; a high number indicates statistically significant autocorrelation. Second, the marginal significance level of a t-test of the hypothesis $\theta_3 = 0$ is also presented. Rejection of this hypothesis is also another indication of model failure.

The results of Table 1 indicate that the joint hypothesis $\theta_1 = \theta_2 = 0$ is almost always rejected at conventional significance levels. This result is

1/ Simultaneous estimation is complicated by two facts: (1) the well-known leptokurtosis in exchange rates is manifest in gross violations of normality of the shocks to the fundamentals equation (3'); and (2) choice, rather than estimation, of α precludes serious statistical work, unless one is willing to guess the covariances of α with other parameters. IV estimation using Durbin's ranked instrumental variables does not change results; Bartlett's variant of Wald's indicator instrumental variables leads to enormously higher standard errors.

2/ Equations for individual bilateral exchange rates can also be estimated jointly with Zellner's seemingly unrelated technique for greater efficiency.

3/ Froot and Obstfeld (1989b) show that θ_1 and θ_2 are well-behaved with the additional assumptions of normality of w_t and independence of ϵ_t . Froot and Obstfeld (1989b) provide further analysis.

Table 1. Hypothesis Tests for Nonlinear Terms, $\alpha=0.1$

(Joint hypothesis tests for nonlinear terms)

Regime	Belgium	Denmark	France	Ireland	Italy	Nether-lands	Japan	United States	United Kingdom
1	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
2	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
3	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
4	0.00	n/a	0.00	n/a	0.00	0.76	n/a	0.00	0.00
5	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
6	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Entries are marginal significance level for joint test $\theta_1-\theta_2=0$ in regression $e_t-f_t-\alpha\eta = \theta_1\exp(\lambda_1f_t)+\theta_2\exp(\lambda_2f_t)+\theta_3f_t+w_t$. Throughout, $\alpha=0.1$; σ_2 and η (and therefore λ_1 and λ_2) are country- and regime-specific. Newey-West covariance estimators are used, with six lags.

Signs of θ_1 and θ_2

Regime	Belgium	Denmark	France	Ireland	Italy	Nether-lands	Japan	United States	United Kingdom
1	++	n/a	+-	n/a	+-	-+	n/a	+-	++
2	+-	n/a	-+	n/a	-+	-+	n/a	+-	-+
3	-+	n/a	-/-	n/a	-+	-+	n/a	--	+-
4	+-	n/a	-+	n/a	-+	+-	n/a	-+	-+
5	++	n/a	--	n/a	+-	--	n/a	-+	-+
6	-+	++	-+	++	-+	++	-+	-+	-+
7	--	+-	--	--	-+	-+	+-	++	-+
8	+-	--	+-	+-	++	--	--	--	--
9	+-	+-	+-	--	+-	+-	-+	++	-+
10	-+	-+	-+	-+	--	++	-+	-+	-+
11	+-	+-	+-	--	-+	-+	-+	-+	-+
12	+-	+-	+-	--	+-	+-	++	--	--
13	-+	+-	-+	-+	-+	--	-+	-+	-+

Entries are signs of estimates of θ_1 and θ_2 in regression $e_t-f_t-\alpha\eta = \theta_1\exp(\lambda_1f_t)+\theta_2\exp(\lambda_2f_t)+\theta_3f_t+w_t$; $\lambda_1>0>\lambda_2$.

Q-tests for residual serial correlation

Regime	Belgium	Denmark	France	Ireland	Italy	Nether-lands	Japan	United States	United Kingdom
1	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
2	0.27	n/a	0.00	n/a	0.99	0.02	n/a	0.01	0.00
3	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
4	0.00	n/a	0.00	n/a	0.00	0.00	n/a	0.00	0.00
5	0.00	n/a	0.23	n/a	0.00	0.00	n/a	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Entries are marginal significance levels for serial correlation of w_t from regression $e_t - f_t - \alpha\eta = \theta_1 \exp(\lambda_1 f_t) + \theta_2 \exp(\lambda_2 f_t) + \theta_3 f_t + w_t$, $\alpha=0.1$.

T-Tests of $\theta_3=0$

Regime	Belgium	Denmark	France	Ireland	Italy	Nether-lands	Japan	United States	United Kingdom
1	0.97	n/a	0.28	n/a	0.54	0.00	n/a	0.00	0.00
2	0.00	n/a	0.33	n/a	0.11	0.06	n/a	0.31	0.00
3	0.01	n/a	0.04	n/a	0.00	0.04	n/a	0.48	0.00
4	0.01	n/a	0.13	n/a	0.29	0.52	n/a	0.00	0.00
5	0.60	n/a	0.42	n/a	0.00	0.22	n/a	0.00	0.00
6	0.00	0.05	0.01	0.05	0.00	0.98	0.00	0.00	0.00
7	0.00	0.25	0.00	0.00	0.00	0.17	0.03	0.25	0.09
8	0.00	0.00	0.00	0.00	0.00	0.83	0.24	0.08	0.64
9	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.03	0.00
10	0.00	0.25	0.00	0.00	0.00	0.20	0.00	0.00	0.00
11	0.00	0.00	0.00	0.35	0.93	0.45	0.00	0.00	0.00
12	0.00	0.00	0.00	0.01	0.00	0.00	0.57	0.97	0.96
13	0.00	0.36	0.29	0.00	0.00	0.93	0.00	0.53	0.00

Entries are marginal significance level of t-statistics of hypothesis $\theta_3=0$ in regression $e_t - f_t - \alpha\eta = \theta_1 \exp(\lambda_1 f_t) + \theta_2 \exp(\lambda_2 f_t) + \theta_3 f_t + w_t$.

quite strong; rejections occur for most countries and most EMS regimes. The existence of nonlinearities of the type implied by target-zone models seems, at first blush, to be overwhelmingly supported. We have also examined a number of perturbations of the basic regression framework including: setting $\alpha=1$; and a first-differenced version of the test. Neither perturbation changes the basic results of Table 1. The rejection of $\theta_1=\theta_2=0$ is also insensitive to: use of $\alpha = 0.05$; choice of 30-day (as opposed to two-day interest rates); the exact sample period (we tried excluding both (1) only the day; and (2) the whole month before and after realignments); and day-of-the-week effects (we estimated (15) for both Fridays and non-Fridays separately). This rejection also characterizes all the currencies in the Bretton Woods and gold standard regimes of fixed rates. The hypothesis $\theta_1=\theta_2=0$ is usually strongly rejected; we conclude that the finding of statistically significant in-sample nonlinearities in the conditional means of exchange rates is quite robust.

The signs of $\hat{\theta}_1$ and $\hat{\theta}_2$ are also tabulated in Table 1. As demonstrated in the theoretical section, these are expected to be of opposite sign in most target-zone models (both credible and incredible). Around a third of the time, the signs of $\hat{\theta}_1$ and $\hat{\theta}_2$ correspond to those implied by target-zone models.

However, the statistical model does not withstand further scrutiny. There is strong evidence of severe residual autocorrelation (Newey-West covariance estimators have been used, both because of this autocorrelation, as well as the censoring induced by target-zones; residual ARCH is also apparent). Only in a few cases can one reject the null hypothesis of no autocorrelation. Furthermore, the model seems to be misspecified, in that θ_3 is often significantly different from zero. Again, these results are relatively robust. Most importantly, the hypothesis $\theta_1=\theta_2=0$ is usually rejected for floating exchange rates as well as fixed exchange rates, as is apparent from Table 1. This indicates that our nonlinear terms may be picking up some generic misspecification in our model that is not particular to target-zone regimes.

Summary

Parametric tests for nonlinearities leave us with a mixed verdict. On the one hand, nonlinearities of the type implied by target-zone models seem to be statistically significant in-sample. The hypothesis that nonlinearities do not exist in conditional means of exchange rates can easily be rejected in a robust fashion. However, these nonlinearities arise in a model which is usually rejected on other statistical criteria. In any case, the economic meaning of these terms is far from clear. Although the signs of the coefficients correspond to target-zone nonlinearities, the fact that these nonlinear terms are often significant during regimes of floating rates seems to bolster the notion that the nonlinear terms do not represent target-zone effects. To study this issue further, we now turn to a forecasting methodology.

VIII. Forecasting with Linear and Nonlinear Models

In this section of the paper, we compare the forecasting ability of linear exchange rate models with models that have additional nonlinear terms implied by the target-zone literature. We find that the presence of additional nonlinear terms does not produce better "ex-post" forecasts than those of linear models. This result, combined with the in-sample analysis of the previous section mirrors the results of Diebold and Nason (1990).

Our baseline forecasting experiment proceeds as follows. Consider a given country (say Belgium) and a given EMS regime (say the period before the first realignment, from March 1979 through September 1979). Using the first thirty observations, we estimate the drift term for fundamentals by regressing the first-difference of exchange rate fundamentals on a constant. This provides us with estimates of σ^2 and η . Given these estimates and our choice of α , we can solve for $\hat{\lambda}_1$ and $\hat{\lambda}_2$; hence we can generate the two nonlinear terms, $\exp(\hat{\lambda}_1 f_t)$ and $\exp(\hat{\lambda}_2 f_t)$. We then run two regressions: (1) (the linear model) $e_t = \pi_0 + \pi_1 f_t + v_t^L$; and (2) (the nonlinear model) $e_t = \phi_0 + \phi_1 f_t + \phi_2 \exp(\hat{\lambda}_1 f_t) + \phi_3 \exp(\hat{\lambda}_2 f_t) + v_t^{NL}$. We then generate forecast errors by substituting in the actual future values of the regressors to generate a forecast; thus, the one-step nonlinear forecast error is given by $\hat{v}_t^{NL} = e_{t+1} - [\hat{\phi}_0 + \hat{\phi}_1 f_{t+1} + \hat{\phi}_2 \exp(\hat{\lambda}_1 f_{t+1}) + \hat{\phi}_3 \exp(\hat{\lambda}_2 f_{t+1})]$. We then add an observation to the initial set of (30) observations, and repeat the procedure until we arrive at the week before the next EMS realignment.

The square roots of the mean squared forecast errors (RMSEs) from linear and nonlinear models (computed with $\alpha=0.1$) are presented in a graphical format in Figure 20; this portrays the ratio of the linear to nonlinear RMSE for the six different countries and thirteen different EMS regimes. There is little evidence that nonlinear models provide superior forecasts, for either EMS or floating currencies. In particular, the ratios of linear to nonlinear RMSEs are typically around one; there is no evidence that they tend to vary systematically over time, or that they tend to be larger for countries with credible reputations like the Netherlands.

We have checked the sensitivity of these results extensively. Figures 21 and 22 are comparisons of a number of different perturbations of linear and nonlinear forecast errors. Figure 21 presents ratios of linear to nonlinear mean absolute errors (MAEs); Figure 22 uses $\alpha=1$. ^{1/} A number of other perturbations are contained in the working paper version, including: rolling regression techniques; the imposition of $\pi_1 = \phi_1 = 1$; 20-step ahead

^{1/} Choosing α to maximize the forecast error ratios represents yet another way to estimate α .

forecasts. The finding that linear models seem to forecast EMS exchange rates as well as nonlinear models appears to be robust to our sensitivity checks. 1/ 2/

Summary

It is well known that sophisticated exchange rate models that appear to be satisfactory on the basis of in-sample criteria, often do not forecast out-of-sample data better than extremely naive alternatives. 3/ In this section, we have shown that nonlinear models do not forecast better than simpler, linear, models; this finding appears to be robust.

We have used three different techniques to examine the nature of the relationship between exchange rates and fundamentals; none has yielded compelling evidence of nonlinearity, at least of the sort implied by target-zone models. There are three potential reasons for this finding: (1) mismeasurement of α ; (2) violations of uncovered interest parity; or (3) an invalid theoretical model. Two arguments discredit the first explanation: low point estimates of α (lower α estimates lead to more linear relationships); and the fact that many of our results are insensitive to choice of α . The short time horizon leads us to believe that any risk premium (which would violate uncovered interest parity) would be too small to account for our results. We are therefore attracted to the conclusion that the theory is not useful in modelling the data. Nevertheless, to confirm our doubts we now use techniques that do not rely on our measure of exchange rate fundamentals.

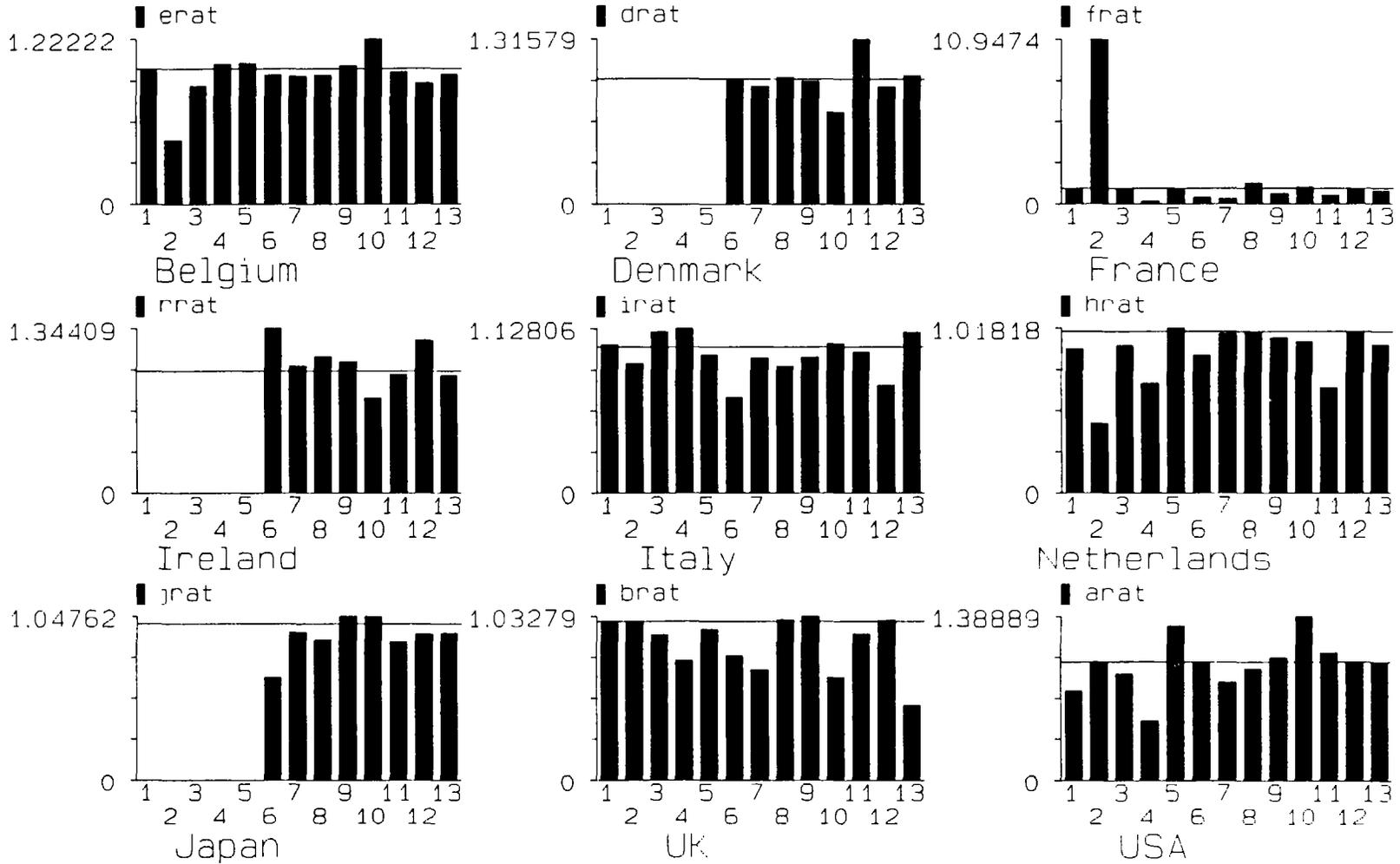
1/ Linear and nonlinear models produce approximately equal RMSEs for the Bretton Woods data. For the gold-standard data, nonlinear models produce RMSEs which are around 20 percent smaller than linear models.

2/ One can rigorously test the hypothesis of equality of forecast error variances. Denote the estimated linear and nonlinear forecast errors u_t^L and u_t^{NL} , and define $v_{1,t} = u_t^L - u_t^{NL}$, $v_{2,t} = u_t^L + u_t^{NL}$. Assuming that $E(v_1, v_2) = 0$ and that the vector (u_t^L, u_t^{NL}) is iid $N(0, W)$, a test of the null hypothesis $w_{11} = w_{22}$ can be computed from $t(T-2) = \psi(T-2)^{0.5} / (1-\psi^2)^{0.5}$ where T is the number of errors and ψ is the estimated sample correlation between v_1 and v_2 . Under the null hypothesis, this test statistics is distributed as Student's t with $T-2$ degrees of freedom. Such standard tests often do not reject the null hypothesis of equal variances. There are also many rejections, as might be expected from the RMSE bar-charts.

3/ Meese and Rogoff (1983) showed that linear structural exchange rate models do not forecast better than a random walk; Diebold and Nason (1990) and Meese and Rose (1991) extend this finding to nonparametric techniques.

Figure 20. Forecast Comparison of Target Zone Models

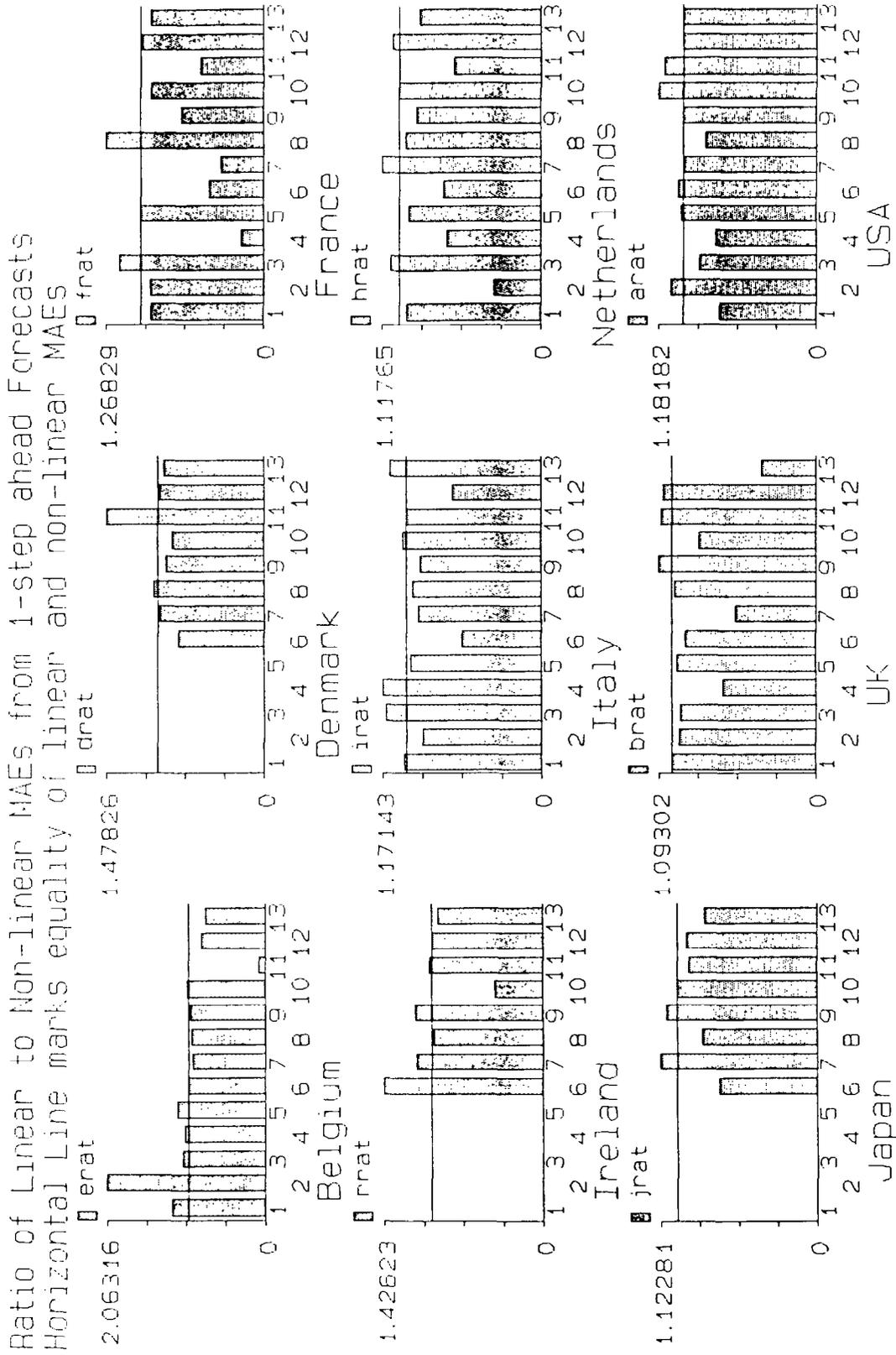
Ratio of Linear to Non-linear RMSEs from 1-step ahead Forecasts
 Horizontal Line marks equality of linear and non-linear RMSEs



RMSE Comparisons by EMS Regime: Alpha=.1



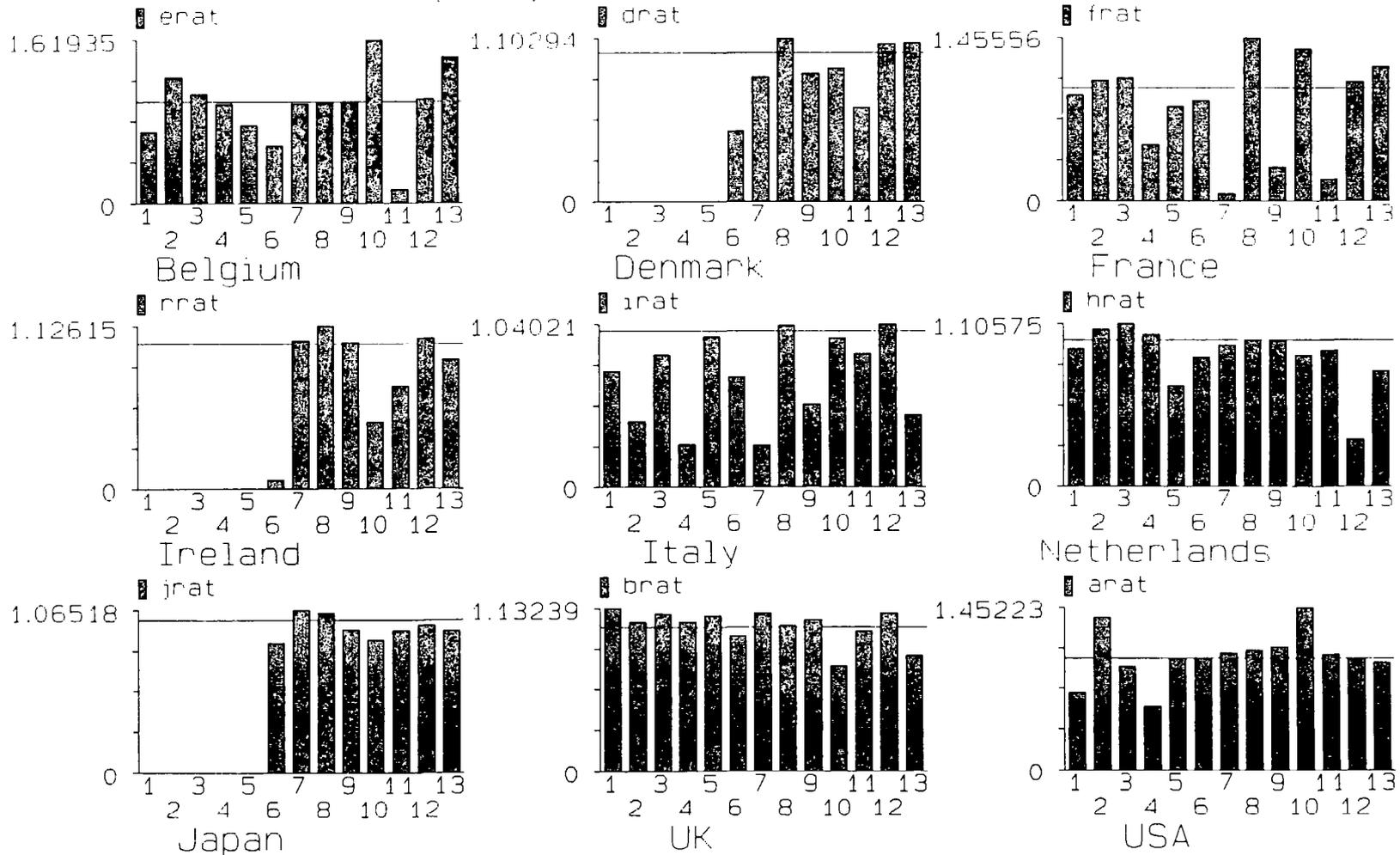
Figure 21. Forecast Comparison of Target Zone Models



MAE Comparisons by EMS Regime: Alpha=.1

Figure 22. Forecast Comparison of Target Zone Models

Ratio of Linear to Non-linear RMSEs from 1-step ahead Forecasts
 Horizontal Line marks equality of linear and non-linear RMSEs



RMSE Comparisons by EMS Regime: Alpha=1.

IX. Other Implications of Target-Zone Models

The empirical work that we have pursued so far has depended on our measure of exchange rate fundamentals. If this measure is flawed, our empirical work will also be faulty. For this reason, we now turn to tests of target-zones that do not depend on fundamentals.

Target-zone models have a variety of implications that can be examined without a measured exchange rate fundamental (Bertola and Caballero (1990b), Svensson (1990a,b,c,d) and Smith and Spencer (1990)), given a specific process for interventions. For instance, as noted in Section II, the interest differential in a credible target-zone is expected to be declining in the deviation of the exchange rate from its central parity; the exchange rate should spend most of its time near the boundaries; and exchange rate volatility should be greatest in the middle of the band. In this section, we examine some of these other aspects of the data.

1. Exchange rate volatility by band position

Figures 23 and 24 are scatter-plots of the absolute value of the daily change in the exchange rate against the deviation of the exchange rate from its central parity (in percentage points). For brevity, we present results for Italy and the Netherlands only. The upper and lower exchange rate bands are marked by vertical lines (at +/- 2.25 percent); a nonparametric smoother is also provided. The graphs are intended to convey a sense of the relationship between the volatility of the exchange rate and its position inside the band. It is not easy to find a clear pattern in the smoothers, either by country or by EMS regime (credible or not). The relationship is occasionally U-shaped (as suggested by Bertola and Caballero (1990b)), but the smoother is just as likely to have an inverted U-shape (as implied by Krugman's (1990) model). Monotonic or flat smoothers are also apparent throughout the figures. ^{1/}

The evidence from other regimes of fixed exchange rates is similar to that of the EMS; results are in the working paper version.

2. Interest rate differentials by band position

Figures 25 and 26 provide comparable scatter-plots of two-day interest rate differentials against the deviation of the exchange rate from its central parity. As noted in Section II, models of credible target-zones imply that the interest rate differential should be a nonlinear deterministic declining function (e.g., Krugman (1990)), graphed against the exchange rate: the model of Bertola and Caballero (1990b) implies the opposite slope. However, there are again no clear patterns (and much

^{1/} The negative results imply that there is little point to testing the parametric model of conditional heteroskedasticity presented in Section II.

evidence of randomness) in the data. 1/ The Bretton Woods and gold standard analogues to the interest rate differential: exchange rate position graphs are again in the working paper.

3. Exchange rate distributions by band position

Figures 27 and 28 provide histograms of exchange rates. Single peaks appears to be the norm (though results for other EMS exchange rates indicate bi-modality). Despite the widespread perception of increasing EMS credibility, we also see no clear indications of a change in the pattern of the histograms over time. 2/ Figures for the Bretton Woods and gold standards are in the working paper. Again, the data do not seem particularly close to the patterns predicted by existing exchange rate theories.

4. Svensson's "simplest test"

Another (nonstatistical) "test" of target-zone credibility has been proposed by Svensson (1990b). Svensson uses uncovered interest parity (which should hold closely in a credible target-zone as shown in Svensson (1990a)) to derive expected future exchange rates. 3/ Svensson's test is simply to graph the time-series of expected future exchange rates and see whether they lie within the exchange rate bands.

Figure 29 provides time-series plots of the exchange rates expected as of time t to prevail one year in the future. Exchange rate bands are also presented. With the exception of the Dutch exchange rate, exchange rates expected to prevail in a year are often outside the bands for prolonged periods of time, even for the more recent, credible, 12th EMS regime. This is a further inconsistency between the predictions of credible target-zone models and the EMS data.

Summary

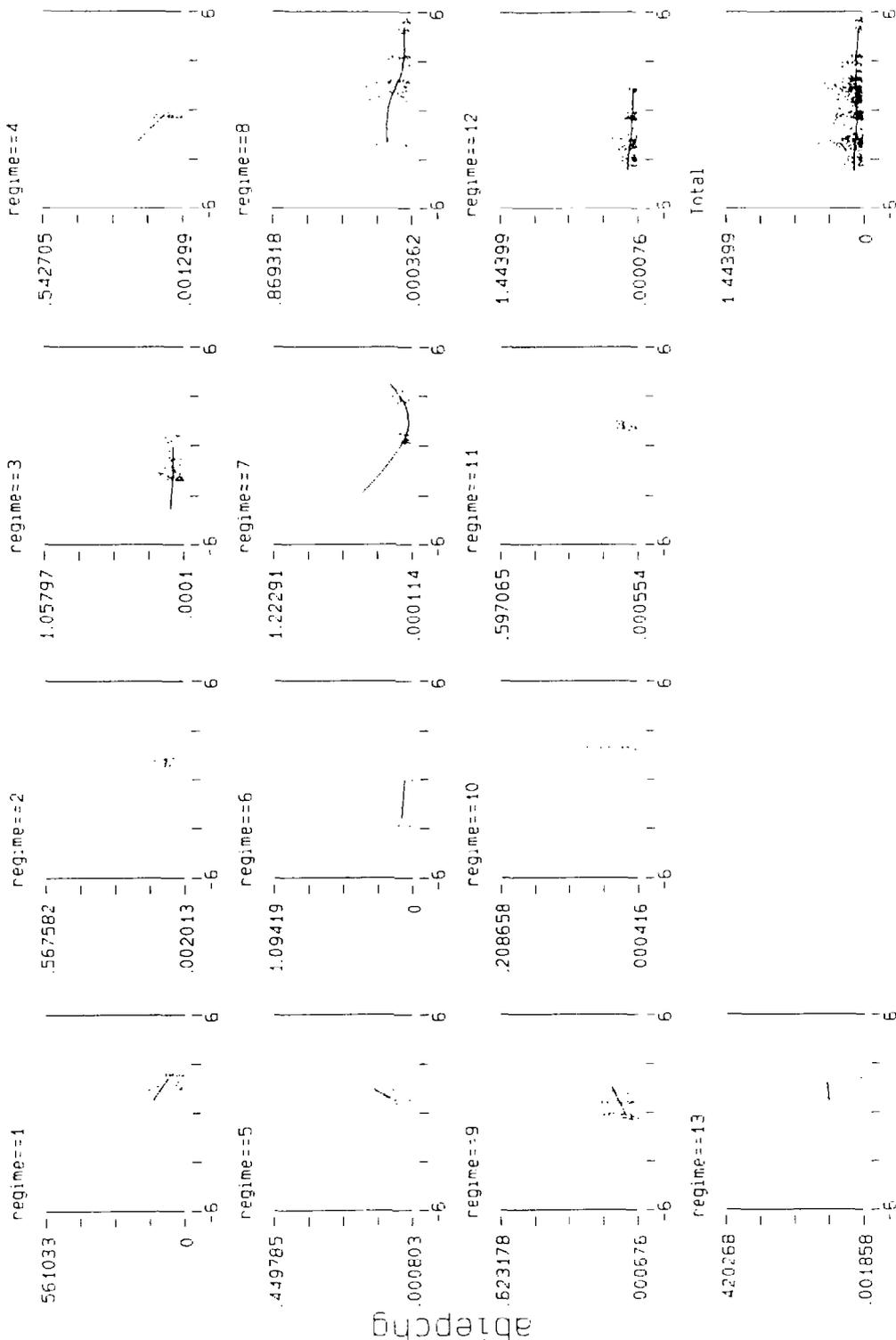
Target-zone models have a number of implications that can be examined empirically without relying on a measure of exchange rate fundamentals. In

1/ Svensson (1990d) also derives implications for the entire term structure of interest rate differentials for a credible target-zone. When we use two-day, 30-day interest rate data, we find no clear pattern of differences between the slopes of various maturities of interest rate differential/exchange rate position smoothers.

2/ There is also widespread evidence of excess leptokurtosis, although the model presented in Section II implies the opposite.

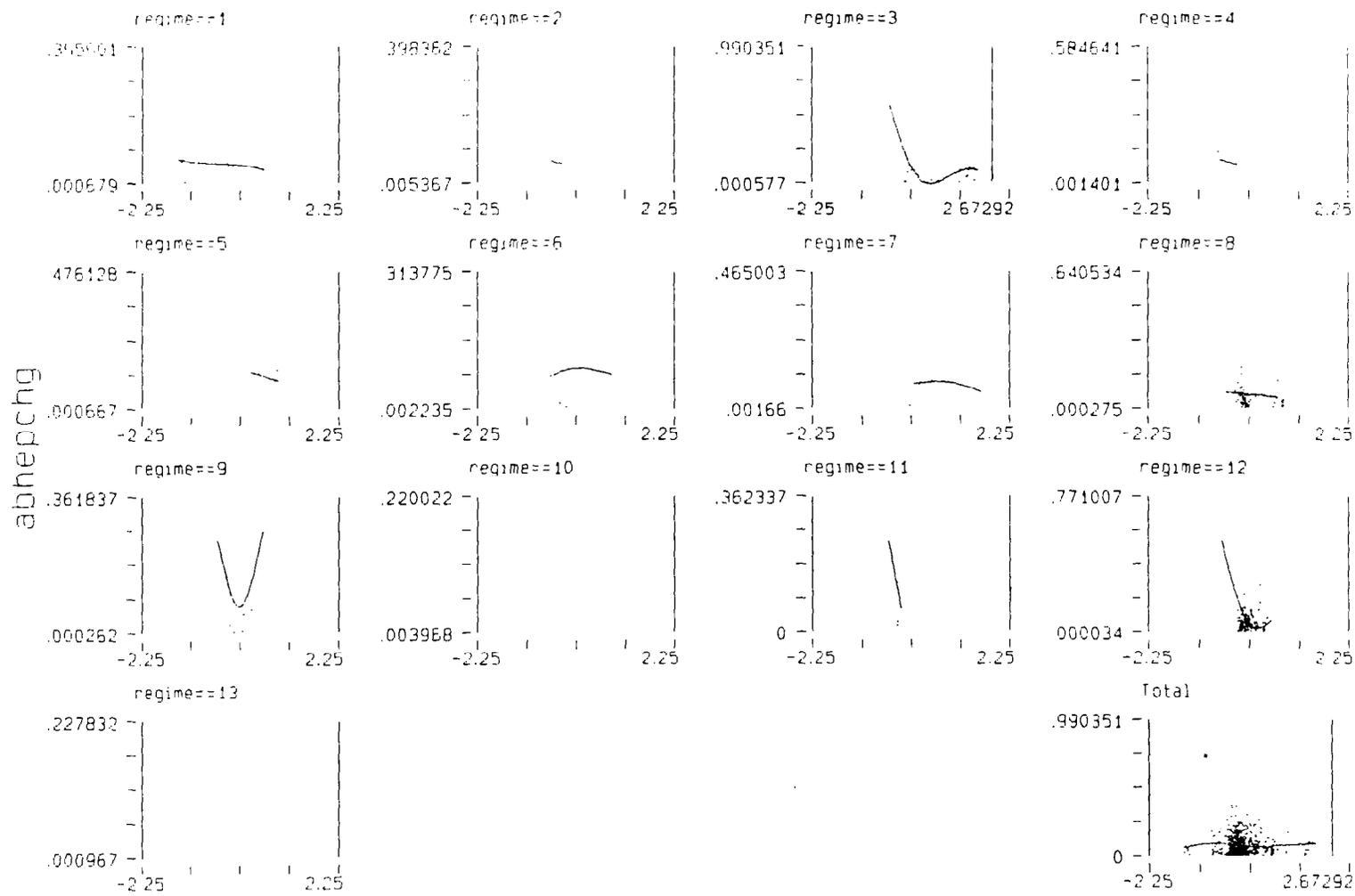
3/ Algebraically, uncovered interest parity implies $E_t e_{t+k} = e_t [(1+i_t)/(1+i_t^*)]^{(\tau/360)}$ where: $E_t e_{t+k}$ is the exchange rate which is expected at time t to prevail at time $t+k$; and i_t (i_t^*) is the return on a domestic (foreign) bond with τ days to maturity. This assumes away any risk premium, possibly a dubious claim at this horizon.

Figure 23. Volatility: Band-Position for Italian Exchange Rate



Absolute Daily Changes against Deviation from Central Parity, in %

Figure 24. Volatility: Band-Position for Dutch Exchange Rate



- 34b -

Absolute Daily Changes against Deviation from Central Parity, in %

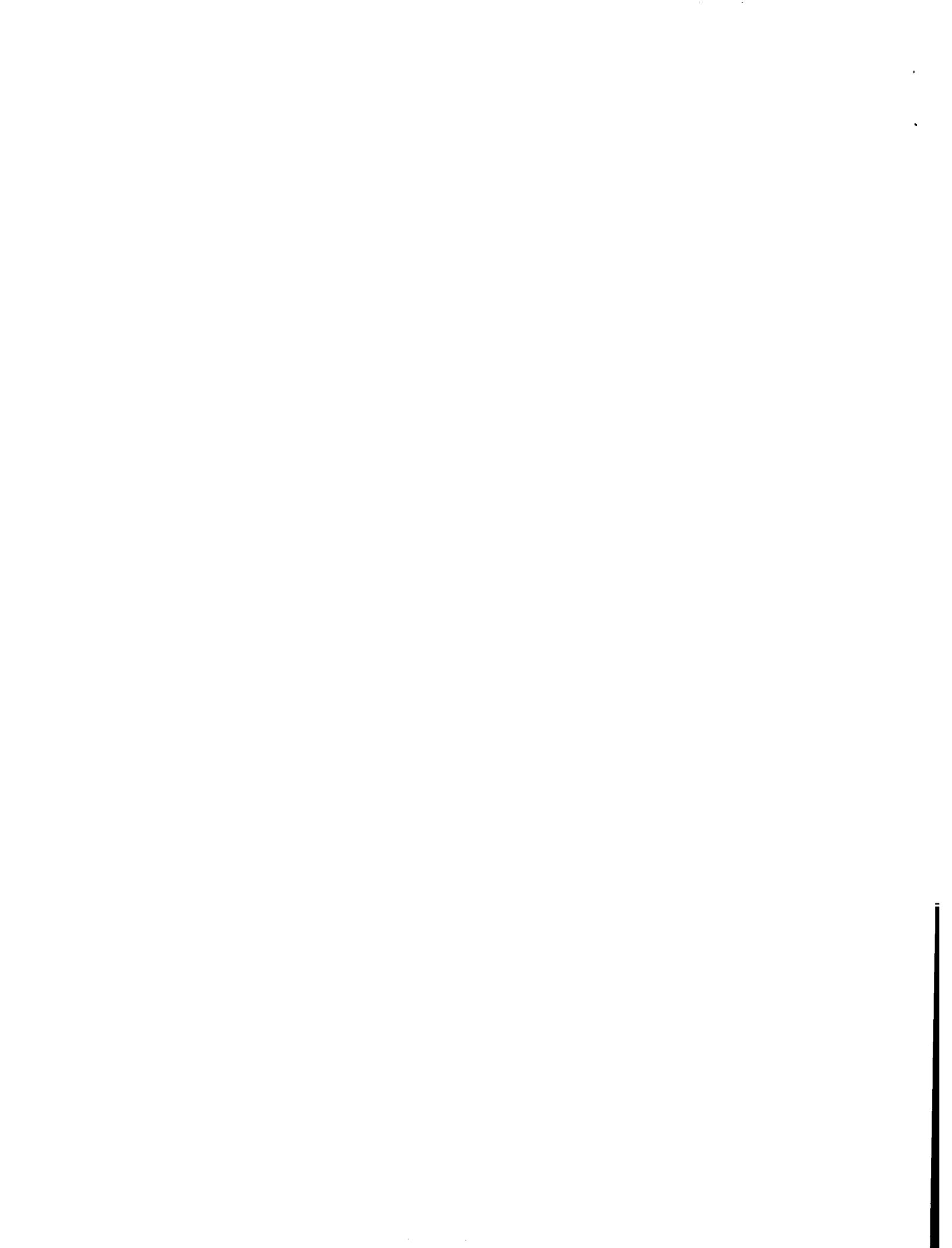
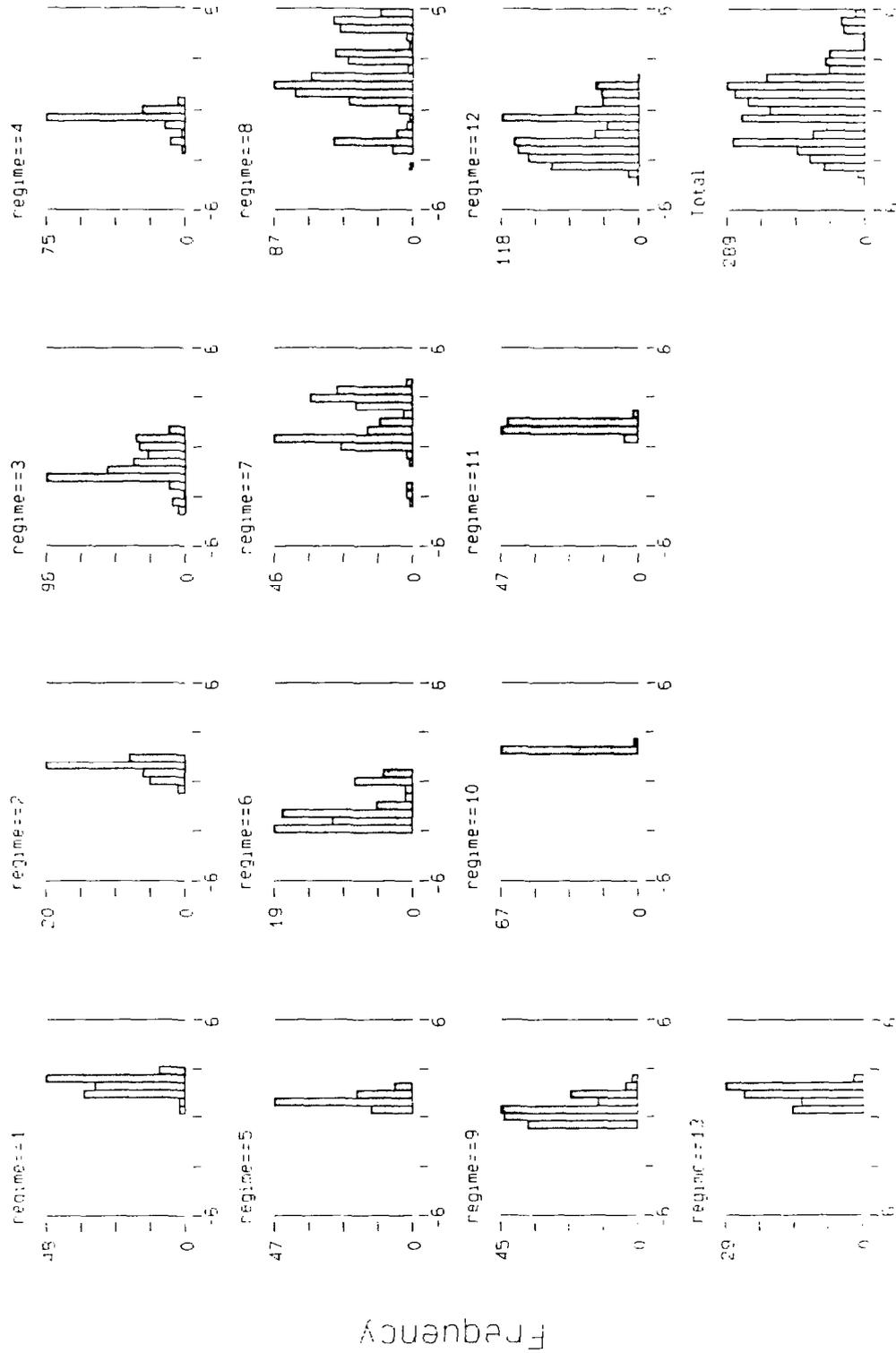
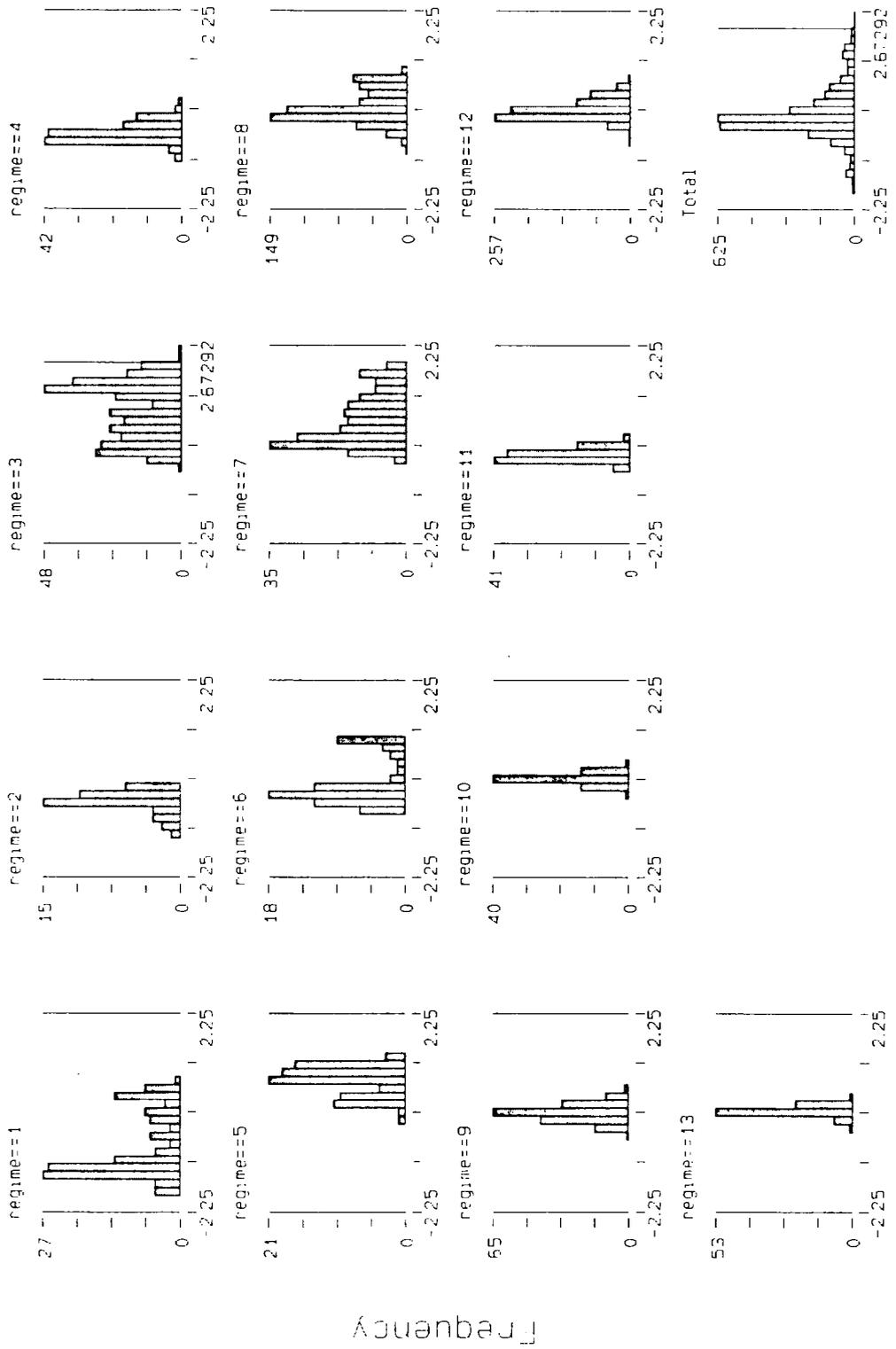


Figure 25. Histograms of Italian Exchange Rates by EMS Regime



Percentage Deviation of Exchange Rate from Central Parity

Figure 26. Histograms of Dutch Exchange Rates by EMS Regime



Percentage Deviation of Exchange Rate from Central Parity

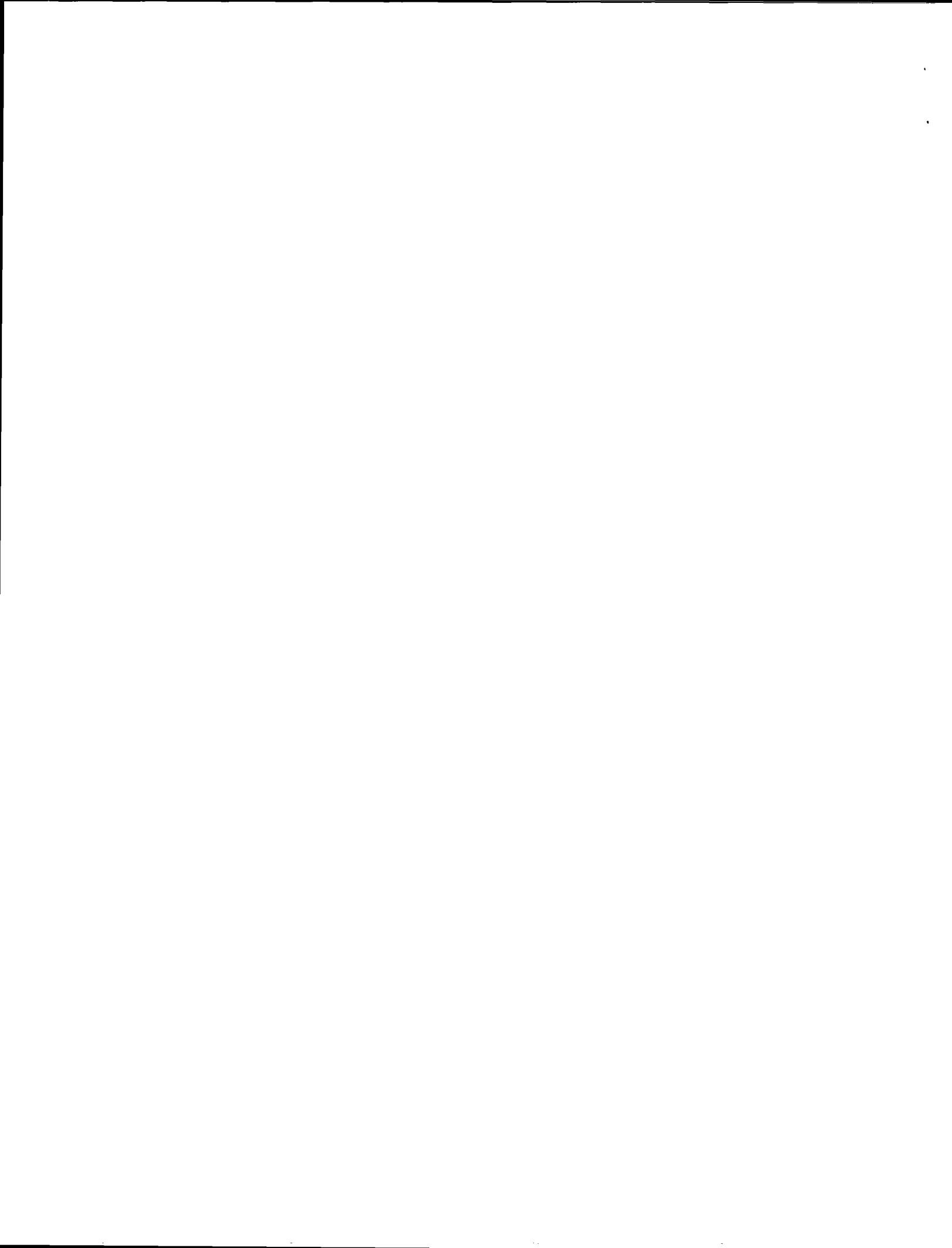
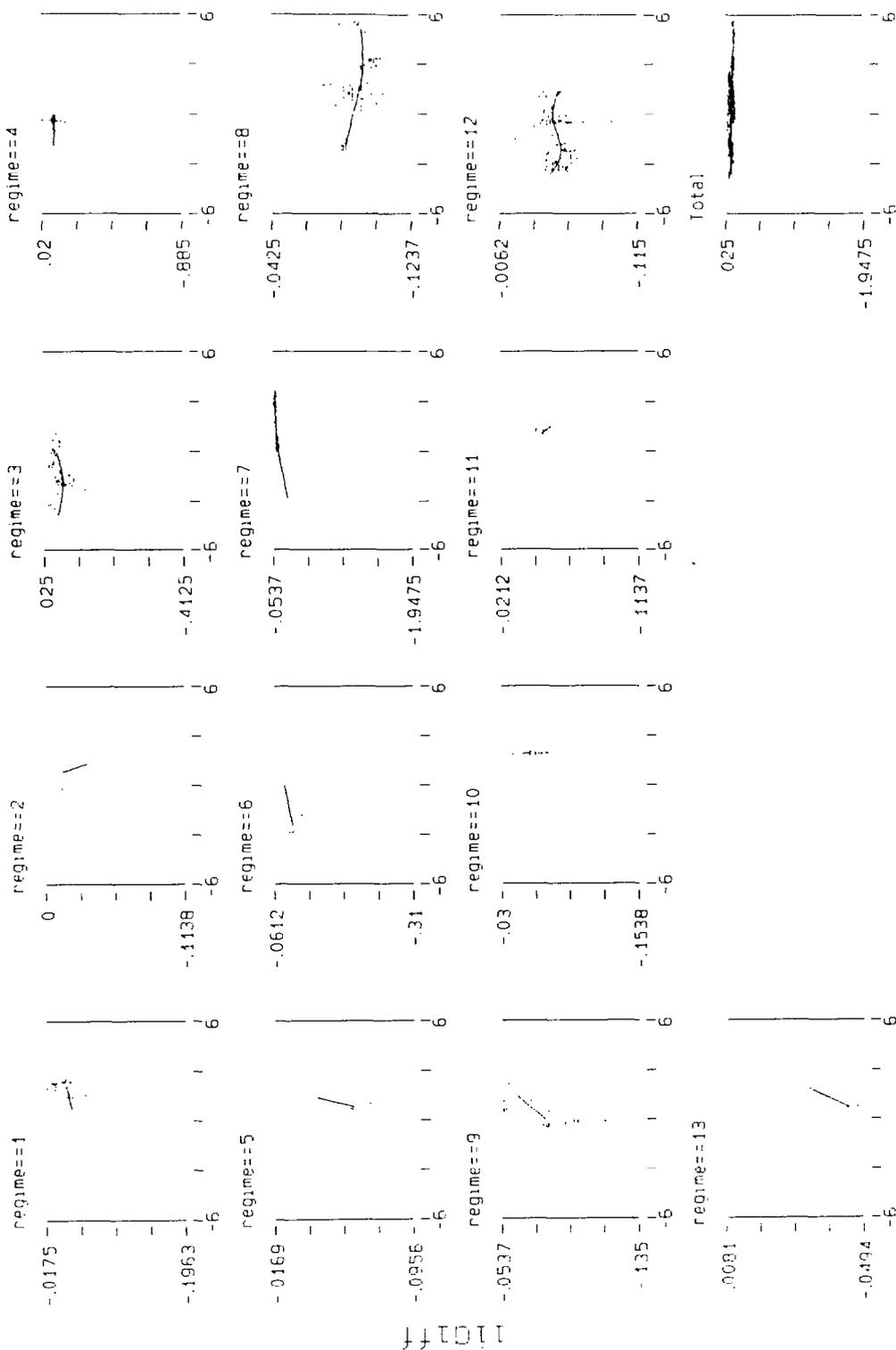
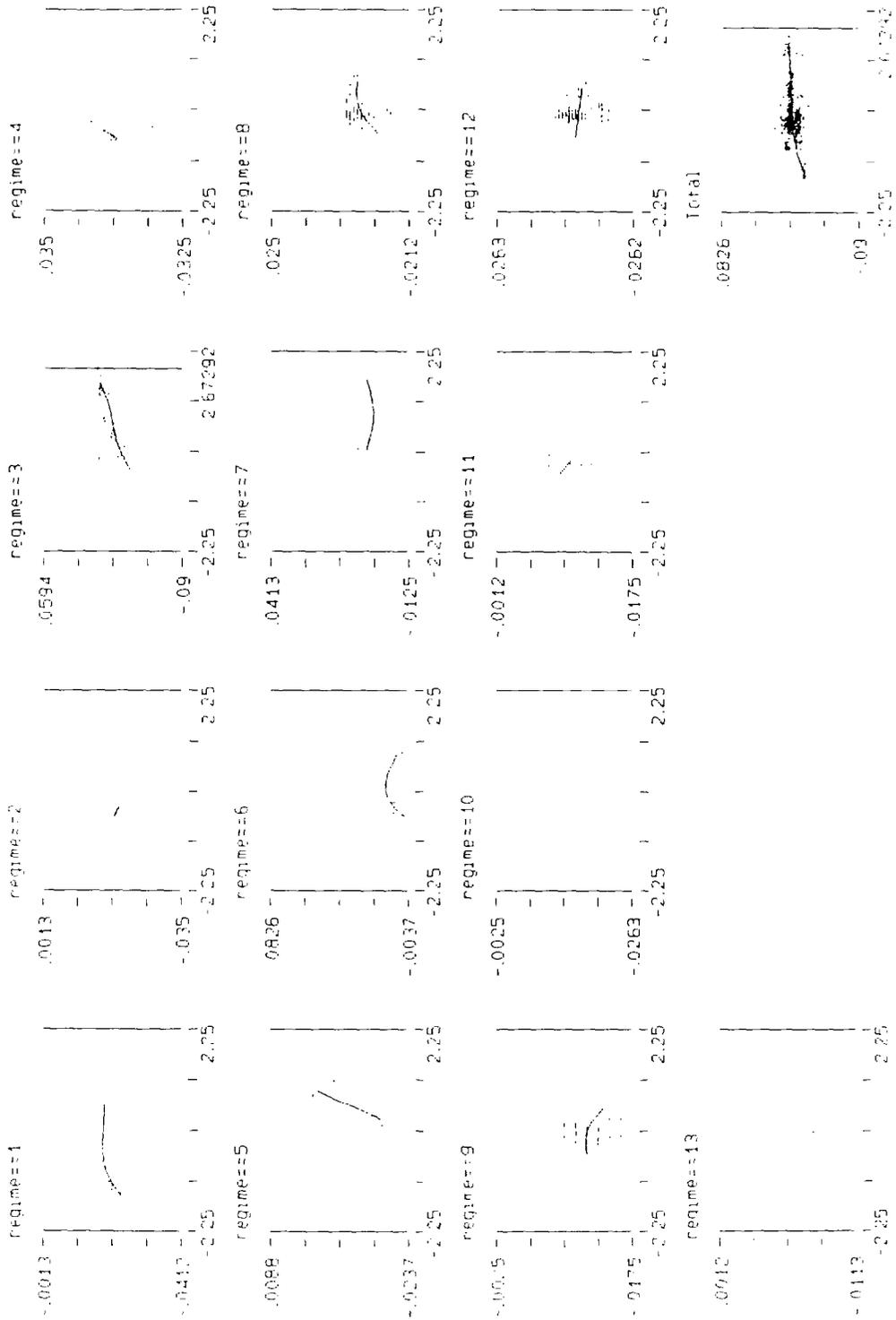


Figure 27. Interest Differential: Band-Position for Italy



Differentials against Deviation from Central Parity, in %

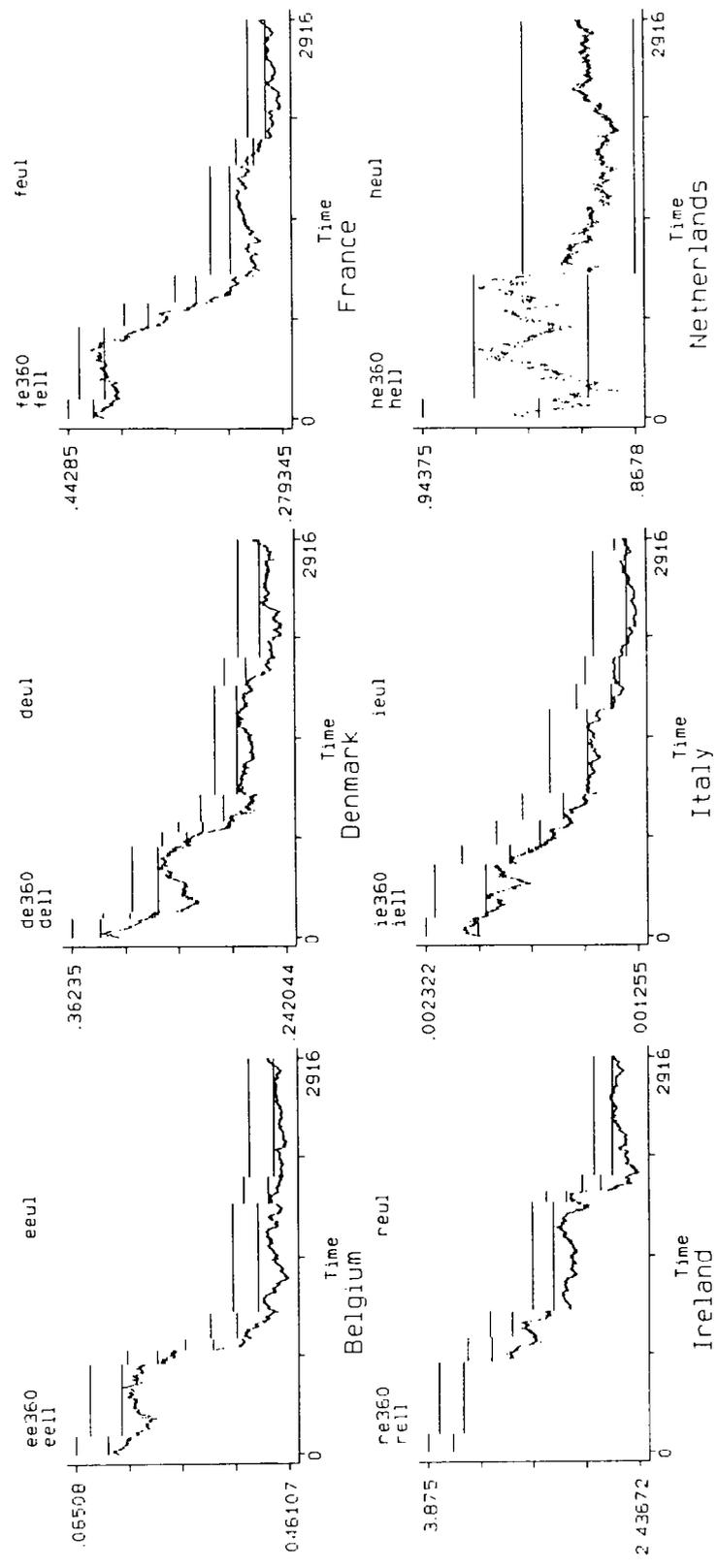
Figure 28. Interest Differential: Band-Position for Holland



Differentials against Deviation from Central Parity, in %



Figure 29. Expected Exchange Rates



Rates Expected in One Year



this section, we examined: interest rate differentials; exchange rate volatility; exchange rate distributions; and expected future exchange rates. These auxiliary (albeit informal) tests provide no support for models of credible target-zones, and only weak support for models with realignments such as Bertola and Caballero (1990b). 1/

X. Summary and Conclusion

Using uncovered interest parity in a framework that implicitly depends on a flexible-price exchange rate model, we derived a measure of exchange rate fundamentals. With the aid of this measure of fundamentals, we tested target-zone models of exchange rate behavior in a number of ways. Graphical examination of the relationship between exchange rate levels and

1/ We have conducted simulation experiments to check our results. Using actual daily data we found that the ratio of the range of possible fundamental values to σ was around 12 (using data across EMS regimes and α values between 0.1 and 1). We therefore set the corresponding ratio in the simulations to 12, and generated f data using the reflection principle in a credible model without drift; exchange rate data was then generated from f . In a typical replication, the data set is 200 observations long and begins at a random starting point. Our simulation results were generated for two values of α : 0.1 and 1. For both of these settings, we investigated a grid of investigator beliefs, which range from 0.1 to 1. These simulation were carried out with and without noise added to the true exchange rate. Regardless of the match between the true α and the investigator's α , we always found that instrumental variable estimation of α , as proposed in the text using lagged interest differentials and lagged exchange rates as instruments, resulted in numerically small estimates of α which were well within two standard errors of zero. We also found that the honeymoon regressions (the linear regressions of e on f), deliver an $e:f$ slope "significantly" less than unity. Also, we always found that the estimation of the constants of integration in the expression for the exchange rate (equation (15)) had coefficients that were "significant" and of the appropriate opposite signs. However, adding noise to the exchange rate makes the significance of these coefficients disappear (the noise was set equal in standard deviation to the noise generating f). These in-sample results are based on 500 replications per simulation. By "significant" we mean that the absolute value of the ratio of the mean value of an estimated coefficient to the standard deviation of such coefficients across 500 replications is greater than 2.

Without added noise, the forecasting exercise always indicated a huge forecasting advantage to the nonlinear model. Ratios such as those in Table 20 were never less than 1000. Ratios remained greater than one until the volatility in the noise was that of the fundamental innovations.

Our simulations indicate that the sample distribution of the exchange rate resembles its unconditional counterpart if 200 observations are available.

fundamentals did not yield strong evidence of economically meaningful and important nonlinearities, certainly not those implied by existing target-zone models. There is little clear evidence of an important "honeymoon effect." Explicit in-sample parametric tests of the nonlinear terms implied by target-zone models yield the conclusion that nonlinearities are usually statistically significant; however, a number of aspects of these models work poorly in-sample, on both economic and statistical grounds. More importantly, linear models forecast out-of-sample data just as well as models with additional nonlinear terms. Finally, a number of additional implications of target-zone models that do not depend on our measure of fundamentals, have been tested and found not to be in accord with the data. For instance, there does not appear to be any particular relationship between exchange rate and interest rate volatility, and expected future exchange rates often fall outside the EMS bands. Moreover, few of the relationships between the exchange rate and (1) interest rate differentials; (2) exchange rate volatility; and (3) exchange rate distributions seem to be in accord with existing theories. Succinctly, we have been unable to provide a characterization of exchange rate behavior during managed exchange rate regimes.

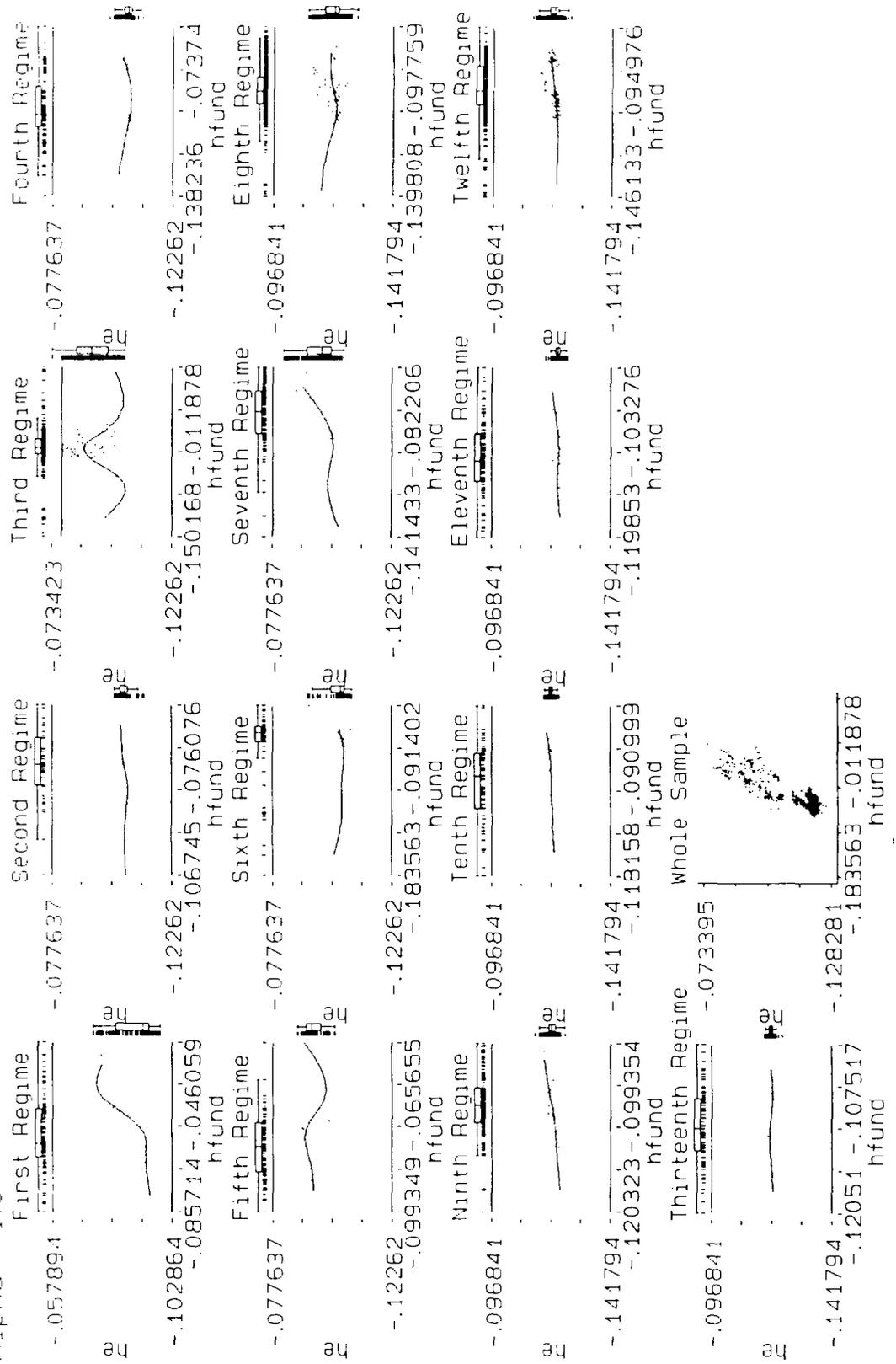
We conclude that, at an empirical level, there is little advantage apparent in working with nonlinear, rather than linear, models of exchange rate conditional means. This result is exactly analogous to the conclusions of Meese and Rose (1991) for flexible exchange rate regimes. Our results also imply that there is little empirical support for existing target-zone models of exchange rates.

The models that we have dealt with in this paper have a number of restrictive features. For instance, policy rules were usually modelled as explicit and time-invariant, without intra-marginal or mean-reverting interventions. More importantly, our model incorporates only a single state variable (thereby ignoring sticky prices and certain types of devaluation risk). 1/ We expect future research to identify the importance of these factors in explaining our negative results.

1/ There is little reason to believe that either sticky prices or devaluation risk can easily reconcile target-zone models with the data. The lack of interest rate differential variability for floating rate countries implies that a model with sticky prices must rely heavily on shocks from goods markets; however, it is difficult to reconcile this feature with the data. Further, the Dutch guilder has been firmly fixed to the deutsche mark since early 1983, and we find it hard to believe that devaluation risk could account for our negative results.

Figure A2. Netherlands

Exchange Rate against Fundamentals by EMS Regime
Alpha = 1.0



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