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Welfare Costs of Inflation, Seigniorage, and Financial Innovation

Prepared by José De Gregorio ^{1/}

Authorized for Distribution by Peter Wickham

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Abstract

This paper examines the welfare effects of mitigating the costs of inflation. In a simple model where money reduces transaction costs, a fall in the costs of inflation is equivalent to financial innovation. This can be caused by paying interest on deposits, indexing money, or "dollarizing." Results indicate that financial innovation raises welfare in low inflation economies while reducing it in high inflation economies, due to the offsetting indirect effect of higher inflation to finance the budget.

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	<u>Contents</u>	<u>Page</u>
I.	Introduction	1
II.	A Simple Monetary Model	2
III.	Welfare Effects of Financial Innovation	5
IV.	Financial Innovation with a Banking System	12
V.	Endogenous Government Spending and Optimal Taxation	16
VI.	Concluding Remarks	21
	References	24
	Appendix: The Optimal Tax Problem	27

I. Introduction

A reduction in the costs of inflation may induce an increase in the rate of inflation. This increase in inflation in turn may be large enough to reduce welfare. The issue of reducing the social costs of inflation was recently addressed by Fischer and Summers (1989). Their model is based on a monetary policy game, following Barro and Gordon (1983) model. In these models, inflation arises because government cannot commit to the optimal rate of inflation (assumed to be zero) and tries to exploit a Phillips curve relationship. In its simplest form, reducing the costs of inflation will reduce welfare because of the more than offsetting increase in the equilibrium rate of inflation.

Fischer and Summers (1989) have extended the model to consider imperfect control of inflation. They show that mitigating the costs of inflation may be desirable only in high inflation economies, since in low inflation economies reducing the costs of inflation may lower welfare.

In this paper, the problem of reducing the costs of inflation is studied in the context of a simple monetary model. In the model, inflation arises because of the need to finance government spending rather than as a result of trying to exploit any short-run trade off between output and unexpected inflation. This approach is particularly relevant to the analysis of the experience of high inflation, where the evidence shows that the underlying cause is the existence of a large fiscal deficit. ^{1/} Thus, seigniorage considerations play a crucial role in determining the welfare effects of inflation mitigation.

A monetary model is developed in which money is introduced through a transactions technology, and an exogenous constant flow of government spending is assumed. Only one case of reducing the costs of inflation is considered, namely, decreasing the necessity of people to hold money. There are several potential ways of achieving this: financial deepening, paying interest on deposits, lowering the reserve-deposit ratio, indexing money or allowing foreign exchange denominated deposits, among others.

In all of the above cases, the shift in money demand will require an increase in inflation to finance the deficit. The rise in inflation will produce a further reduction in real balances and results opposite to those of Fischer and Summers (1989) will be shown to hold, i.e., at low levels of inflation a reduction in the social costs of inflation is more likely to increase welfare. In the extreme case where seigniorage equals zero, reducing the costs of inflation will allow people to economize on real money balances without affecting the rate of inflation (zero in this case). At the other extreme, when inflation is close to the revenue maximizing rate of inflation, the resulting shift in money demand will reduce welfare. The larger the interest rate elasticity (in absolute value) of money demand, the larger will be the required increase in inflation to offset a fall in real balances. Provided that the rate of inflation is not on the "wrong side of

^{1/} For example, Sargent (1983), Helpman and Leiderman (1988), Liviatan and Kiguel (1988) and Dornbusch and Edwards (1989).

the Laffer curve", the elasticity is largest at the maximum level of seigniorage.

The transactions technology assumed in this paper allows to interpret reductions in the welfare costs of inflation as financial innovation. Thus, the purpose of this paper can be reinterpreted as an exploration of the welfare effects of financial innovation in economies that are subject to seigniorage constraints. Therefore, the main result can be reformulated as follows: financial innovation will increase welfare in low inflation economies, and will reduce welfare in high inflation economies, because of the offsetting indirect effect of financing the budget.

The paper is organized in six sections. Section II presents a simple monetary model where the only asset and medium of exchange is money. People demand money because it facilitates transactions. So, money reduces transaction costs. In Section III the question of a reduction in required money holdings, which is related 1:1 to the welfare costs of inflation, is analyzed. Section III also contains the main results of the paper: given the rate of inflation, a reduction in required money holdings is welfare improving. Once the indirect effect on inflation is added the result is ambiguous. The direction of the welfare change depends, however, on the level of inflation. For low inflation countries the reduction in the social costs of inflation will increase welfare while at high inflation rates, and consequently high seigniorage, welfare will decrease.

Section IV extends the model to the case where deposits are also available for making transactions. Deposits are assumed to pay a fixed interest rate and are imperfect substitutes of money. Analogous to Section III, it is shown that increasing the interest paid on deposits at high rates of inflation reduces welfare, and increases welfare at low rates of inflation. If the interest rate were determined in a competitive banking system and a reduction in reserve requirements would occur, the same results would hold.

The main part of the paper treats government spending as exogenous and inflation as being the only source of revenue. In Section V, however, both assumptions are relaxed. First, it is assumed that government spending consists of providing a public good, which is optimally chosen. It is shown that the results obtained for a the case of an exogenous government spending are still valid. Next, Section V extends the model to an optimal taxation scheme, in which two taxable goods are assumed: money and consumption goods. It is shown that, with some qualifications, the main results from the previous sections still hold. Finally, Section VI provides the conclusions and discusses other possible extensions of the results.

II. A Simple Monetary Model

The economy is populated by a constant number of identical infinitely lived individuals. The representative consumer maximizes the present discounted value of a concave instantaneous utility of consumption $u(c_t)$:

$$\text{Max } U_s = \int_s^{\infty} u(c_t) e^{-\delta(t-s)} dt \quad (1)$$

There is no production and the individual has a constant flow endowment of y . The only asset is money which is used because it facilitates transactions. The budget constraint is (time subscripts are omitted):

$$c + \theta F(m) + \dot{m} + \pi m = y + g \quad (2)$$

where π is inflation, m is real money balances and g are government lump-sum transfers. 1/

Per capita government transfers are denoted by g and are financed exclusively through the inflation tax. Alternatively, it could be assumed that *seigniorage is not returned to consumers*. The model would be basically the same, because the main result concerns with a constant level of g .

$\theta F(m)$ represents the transactions technology. Modeling money as an intermediate input in transactions has been used recently by Fischer (1983), McCallum (1983), Kimbrough (1986a,b), Benhabib and Bull (1987), Faig (1988) and Végh (1989), among others. As shown by Feenstra (1986) there is a close relationship between this approach and the introduction of money into the utility function.

In contrast to the traditional formulation, it is assumed in this paper that transaction costs are independent of the level of consumption. This is a convenient shortcut to take which does not alter the results. 2/

The transactions (or shopping) technology represents real resources that are foregone in transactions. $\theta F(m)$ will depend on institutional considerations, for example the degree of development of credit markets, where θ is a parameter reflecting those aspects of the transactions technology.

The technology assumed has the advantage that θ can be also interpreted as a parameter of the welfare costs of inflation. In the present model inflation is a distortionary tax, whose welfare costs depends positively on

1/ Money is the only asset in this economy, although in general equilibrium an interest bearing bond can be introduced at the margin, paying a real interest rate equal to δ . The case of debt financing is not considered. Since g is constant, the constant rate of inflation can be interpreted as the result of optimal tax smoothing (Mankiw (1987) and Barro (1988) for recent applications).

2/ This is equivalent to assuming that money provides "shopping services" as suggested in Dornbusch and Frenkel (1973). The individual budget constraint would be: $c + \dot{m} + \pi m = y(1 + v(m)) + g$, so $-yv(m) = \theta F(m)$. Hence, the transactions technology would depend on y rather than c . But, since y is constant, it is omitted as an argument of F .

θ .

The following assumptions with respect to the transactions technology are made:

$$(A1) \theta \in \mathbb{R}_+, F' \leq 0 \text{ and } F'' > 0$$

$$(A2) F' = 0 \quad \forall m \geq \bar{m}$$

$$(A3) FF'' - F'^2 > 0$$

The reason for (A3) will be clear later. It is a natural requirement since it will guarantee that, other things being equal, a reduction of θ will reduce the amount of resources spent in transactions, thus it will increase welfare.

The necessary conditions for optimality of consumer plans are:

$$u'(c) = \lambda$$

$$\dot{\lambda}/\lambda = \delta + \pi + \theta F'(m)$$

$$\lim_{t \rightarrow \infty} \lambda_t e^{-\delta t} m_t = 0$$

where λ_t is the current marginal utility value of an extra unit of money at time t .

Now we can proceed to characterize the general equilibrium. As is well known from Brock (1974) and Calvo (1979), this economy may have multiple equilibrium paths. However, there is only one bubbleless equilibrium in which $m = 0$. In this equilibrium, inflation is equal to the rate of money growth. This unique saddle path stable equilibrium will be examined. 1/

Because there is no capital and prices are fully flexible the model has no inherent dynamics. Hence, the economy is always at the steady state. This equilibrium is characterized by the following equations: 2/

$$c = y - \theta F(m) \tag{3}$$

$$\theta F'(m) = -(\delta + \pi) \tag{4}$$

$$g = m\pi \tag{5}$$

1/ Bubbles can be ruled out assuming that the transactions technology satisfies $\lim_{m \rightarrow 0} -F'(m)m > 0$, Blanchard and Fischer (1989), Chapter 4 and references therein.

2/ To save in notation and given that the economy is always in steady state, superscripts or subscripts to denote equilibrium values are omitted.

A caveat: The optimal quantity of money and maximum seigniorage

Two recurrent issues in the literature on inflation and inflationary taxation can be reproduced with this model. First, the optimal rate of money growth, which equals the rate of inflation, is the Friedman (1969) rule: $\pi = -\delta$, which is a zero nominal interest rate. This is checked after maximizing consumption in the steady state. This rate of inflation is optimal since it equates the marginal productivity of money with the marginal cost of production. Second, the maximum level of seigniorage is found by solving:

$$g^* = \text{Max}_m -m (\theta F' + \delta)$$

which yields the following expression for the revenue maximizing rate of inflation:

$$\pi^* = \theta F''(m^*)m^* \quad (6)$$

which together with (4) determines (π^*, m^*) . 1/ The revenue maximizing rate of inflation is given by the standard rule that the money-demand inflation elasticity has to be equal to one. Also, it can be checked using the envelope theorem that $dg^*/d\theta > 0$.

Following the tenets of public finance it is known that, up to a first approximation, the deadweight loss of distortionary taxation is proportional to the square of the tax rate. In this model the deadweight loss of inflation will be proportional to the square of $\pi + \delta$, which by (4) implies that it will be proportional to the square of θ . Therefore, θ has a direct interpretation in terms of welfare.

III. Welfare Effects of Financial Innovation

This section looks at the effects of a reduction of θ on welfare in two cases. Because the economy is always in steady state and consumption is the only argument in the utility function, it is enough to analyze the effects of θ on consumption.

The fall in θ is interpreted as financial innovation, which allows people to require lower money balances to carry the same amount of transactions. In Section IV a more concrete example will be discussed, where θ is related to deposits and interest rates paid on deposits. The case developed in this section, however, shows the basis of the model and its main

1/ Assumed is the second order condition holds: $2F''(m^*) + F'''(m^*)m^* > 0$. A sufficient condition for a unique interior solution is that $F'(m)$ goes to $-\infty$ when m goes to zero.

implications in the simplest setting. Two issues are addressed. First, the welfare effect of a change in θ given the rate of money growth is examined. The second is the welfare effect of a change in θ with the constraint that the rate of money growth and the equilibrium real balances must satisfy the government flow budget constraint.

The reason for having inflation in the present model is to finance the budget deficit. Therefore, the only purpose of the first exercise is to show that, other things being equal, a fall in θ will increase welfare. 1/ This formally shows that the welfare costs of inflation are positively related to θ .

Proposition 1: Given the rate of inflation, a decrease in θ increases welfare.

Proof: Differentiating (3) and (4) we have:

$$\frac{dc}{d\theta} = -\theta F' \frac{dm}{d\theta} - F$$

and

$$\theta F'' \frac{dm}{d\theta} + F' = 0, \quad \text{hence} \quad \frac{dm}{d\theta} = \frac{-F'}{\theta F''} > 0$$

therefore:

$$\frac{dc}{d\theta} = \frac{F'^2}{F''} - F < 0 \quad \text{because of (A3)} \quad \blacksquare$$

When θ falls, equilibrium money holdings will fall. This effect does not, however, offset the direct effect on welfare. When the government is constrained to raise a given amount of revenue through an inflation tax, the previous result changes. Denoting as $g(\theta)$, the maximum seigniorage for a given level of θ we have:

Proposition 2: For a given g , the welfare effect of a change in θ is ambiguous. For g close and below g^* a decrease in θ will decrease welfare. For g equal to zero a decrease in θ will increase welfare.

Corollary: If in addition we assume that $F''' \geq 0$, there exists $\bar{g}(\theta)$ such that a decrease in θ increases welfare for all $g \in [0, \bar{g})$ and decreases welfare for all $g \in (\bar{g}, g^*)$.

1/ This exercise assumes that the government budget constraint, equation (5), does not hold.

Proof: Differentiating (3), (4) and (5):

$$\frac{dc}{d\theta} = -\theta F' \frac{dm}{d\theta} - F$$

and

$$\theta F'' \frac{dm}{d\theta} + F' = -\frac{d\pi}{d\theta}, \text{ where } \frac{d\pi}{d\theta} = -\frac{\pi}{m} \frac{dm}{d\theta}$$

therefore:

$$\frac{dm}{d\theta} = -\frac{mF'}{m\theta F'' - \pi} \quad (7)$$

$$\frac{dc}{d\theta} = \frac{m\theta F'^2}{m\theta F'' - \pi} - F \quad (8)$$

which is negative for $g = \pi = 0$, and goes to $+\infty$ as π approaches from below to the revenue maximizing rate of inflation.

To have monotonicity in $dc/d\theta$, $d^2c/d\theta^2 < 0$ is required, so that $dc/d\theta$ will start from $+\infty$ for the lowest feasible θ , denoted as $\underline{\theta}$: this is the value that satisfies $g = g^*(\underline{\theta})$. As θ increases, g will become small relative to $g^*(\theta)$ and $dc/d\theta$ will become negative. Therefore, for any g , there will be a unique $\theta, \bar{\theta}$, such that $dc/d\theta = 0$. Define this function as $\theta = \bar{\theta}(g)$. The inverse is $g = \bar{g}(\theta)$. Hence the rest of the proof consists of showing that the function $c = c(\theta)$ is concave.

$$\frac{d^2c}{d\theta^2} = -\theta F'' \left(\frac{dm}{d\theta} \right)^2 - 2F' \frac{dm}{d\theta} - \theta F' \frac{d^2m}{d\theta^2} \quad (9)$$

Since $\frac{dm}{d\theta} = -\frac{mF'}{m\theta F'' - \pi} \geq -\frac{F'}{\theta F''}$, $d^2c/d\theta^2$ can be bounded by:

$$\frac{d^2c}{d\theta^2} \leq -F' \left(\frac{dm}{d\theta} + \theta \frac{d^2m}{d\theta^2} \right) = -F'K$$

Defining $D = m\theta F'' - \pi$, it can be shown that:

$$K = \left(-2mF''\theta - F'\theta + F'm^2\theta^2F''''/D + F'm\theta^2F''/D + F'\theta\pi/D + D \right) \frac{1}{D} \frac{dm}{d\theta}$$

using again the fact that $D \leq m\theta F''$, it can be seen that the two positive terms in the brackets, $-F'\theta$ and D , are less than the absolute value of $F'm\theta^2F''/D$ and $-2mF''\theta$, respectively. Therefore, for $F'''' > 0$, K is strictly negative, and hence $d^2c/d\theta^2$ is negative ■

In contrast to Proposition 1, a decrease in θ may be welfare reducing, specially for economies with large fiscal deficits which are financed through inflation tax. The reason is that as θ falls, real money balances also fall. When inflation remains constant the benefit of the reduction in θ is larger than the cost of holding lower real balances, as was shown in Proposition 1. For a given g , however, the government has to increase inflation because of the drop in the tax base. This increase in inflation reduces even more real money balances. In the end, the tax revenue requirement may reduce real money balances up to a point where the increase in F outweighs the fall in θ . The above corollary provides a sufficient condition to generalize the result for all relevant g .

The results of Propositions 1 and 2 are shown in Figure 1. For $\theta = \theta_1$, ($\theta_1 > \theta_2$) the demand for money, given g , is at A. In the space (c, m) point A corresponds to A'. A reduction in θ from θ_1 to θ_2 can be decomposed into two parts. The first part is a reduction in m , given the rate of inflation (A to P1 in the upper panel and A' to P1' in the lower panel). Proposition 1 shows that consumption at P1' is larger than consumption at A'. However, since g is fixed, inflation will have to rise. Therefore there is a further reduction in real money balances (from P1 to P2). This effect will offset the increase in welfare, since consumption falls from P1' to P2'.

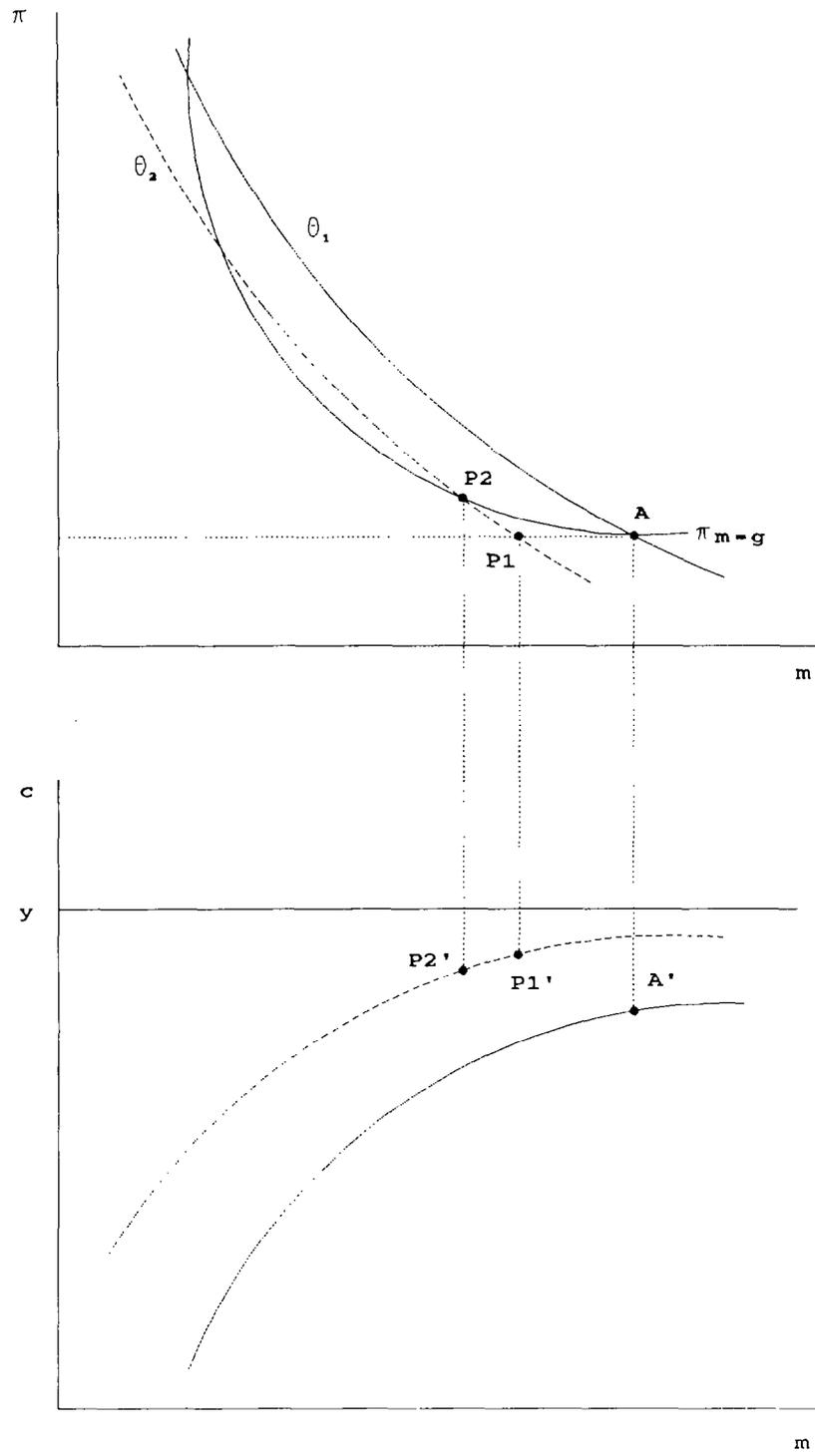
Proposition 2 shows that the total welfare effect of a reduction in θ (A to P2') is ambiguous, and its sign will depend on the size of g . For g close to g^* (the value of g that produces tangency between the hyperbola $\pi m = g$ and money demand for θ_1) consumption decreases. On the other hand, for $g = 0$, there is no need to increase the rate of inflation from P1 to P2.

That welfare increases in the case of $g=0$ is a particular result of Proposition 1. The interesting outcome is the reduction of welfare for g close to g^* . Looking at equation (7), what drives the result is that $dm/d\theta$ goes to infinity as g goes to g^* . The intuition for this result can be provided through analysis of a standard multiplier effect.

Consider a small reduction in θ , which reduces m by Δ percent. Since g is constant, inflation has to rise Δ percent to offset the fall in m . This increase in inflation induces an additional fall of $\epsilon\Delta$ percent in m , where ϵ is the money-inflation elasticity (in absolute value). Then, a new increase in inflation of $\epsilon\Delta$ is required with a consequent $\epsilon^2\Delta$ percent reduction in m . A new increase in inflation will then be required, and so on. Therefore, the total fall in m in response to a small fall in θ is equal to $\Delta(1+\epsilon+\epsilon^2+\dots)$. Hence,

$$\frac{1}{m} \frac{dm}{d\theta} = \frac{\Delta}{1 - \epsilon} \quad (10)$$

Figure 1



The money-demand inflation elasticity varies between 0 and 1 in the "right side" of the inflation-tax-Laffer-curve. For $g=0$, the elasticity of money demand is zero, so that there is no indirect effect from inflation to money demand. As g goes to g^* the indirect effect also increases because ϵ is increasing. In the limit, a second order change in θ causes a first order change in m because of the adjustment in the inflation rate needed to finance the budget. From the welfare (consumption) point of view, the increase in $F(m)$ caused by the reduction in real balances is larger than the direct effect of the fall in θ .

Finally, note that the result is only concerned with inflation rates at the "right side" of the inflation-tax-Laffer-curve. That is, in the increasing portion of the seigniorage-inflation schedule. ^{1/} Since the economy is always in steady state, there is no reason to assume that the government is collecting seigniorage with excessive inflation.

Numerical example

Before explicitly introducing a banking system, a numerical example may help to clarify the nature of the results. The example is illustrative and is not intended to replicate an actual economy. The figures are, however, consistent with empirical evidence on seigniorage and inflation (Fischer, 1982; Dornbusch and Reynoso, 1989; and Giavazzi and Giovannini, 1989).

Table 1 provides the structure of the example. A quadratic transactions technology defined for $m > \beta_1/\beta_2$ is assumed. Because there is a one-to-one mapping from θ to g^* , the simulations can be presented in two ways. The first approach is depicted in Figure 2, where there are two curves. They represent the relationship between consumption and θ under the assumptions of Propositions 1 and 2. The hump-shaped curve considers a constant value of g equal to 0.25; this is seigniorage equal to 2.5 percent of potential GNP (y is normalized to be 10). This value for seigniorage is the maximum revenue for $\theta = 1.1$,

$$0.25 = g^*(\theta=1.1)$$

Therefore, θ has to be restricted to be larger than 1.1. Otherwise it would not be possible to collect $g = 0.25$ through seigniorage. As θ changes, the rate of money growth, and hence inflation, will adjust to keep $g = 0.25$. In this simulation money over GNP is between 5 percent and 10 percent. The hump shape reflects Proposition 2: the curve increases vertically when g corresponds to the maximum revenue, $g^*(\theta=1.1)$. As θ increases the relationship between consumption and θ is negative. In terms of Proposition 2, as g becomes low with respect to g^* welfare decreases monotonically in θ .

^{1/} Eckstein and Leiderman (1989) estimate the relationship of seigniorage and inflation for Israel. In their model they find that the Laffer curve is always increasing, becoming almost flat for rates of inflation above 5 percent per quarter.

For $\theta = 1.1$ the rate of inflation that maximizes revenue is 52 percent. Any inflation rate above that value will be at the "wrong side of the Laffer curve". For the range of θ depicted in figure 2, the rate of inflation changes from 52 percent for $\theta = 1.1$ to 30 percent for $\theta = 2.1$.

The monotonically decreasing curve corresponds to consumption, keeping the rate of money growth, and hence inflation constant. Given this rate of inflation (assumed to be 0) we can find m from the money demand for any value of θ . Therefore this curve illustrates the result of Proposition 1: that the relationship between consumption and θ is negative everywhere when inflation is constant--at zero in this example. Note that the optimal rate of inflation should be -5 percent, therefore in both exercises there is a welfare loss due to inflation tax. This loss varies between 1 percent and 2.5 percent of potential GNP. ^{1/}

Propositions 1 and 2 have been illustrated in terms of the slope of the consumption- θ schedule. This is done in Figure 3. The curves are drawn for a fixed value of θ (1.1), and g is allowed to vary. Proposition 1 says that for any constant rate of inflation the slope is negative. This is the curve at the bottom of Figure 3. For any initial g , a change in θ without adjusting inflation will cause consumption to move in the opposite direction. Once we incorporate the restriction that g has to be constant, and inflation is adjusted accordingly, the welfare effect of a change in θ is ambiguous. This is reflected in the other two curves, drawn for different values of β_2 . As g approaches $g^*(\theta=1.1)=0.25$, the marginal change in welfare goes to infinity. That is $c(\theta)$ becomes vertical as in Figure 2.

The two curves are drawn for different assumptions about β_2 (1 for the curve above and 0.85 for the other) to show that the value of g , at which $dc/d\theta$ changes sign (\bar{g} in prop. 2), is very sensitive to the parameters. For $\beta_2=1$ welfare falls when θ falls for any g larger than 2 percent of potential GNP. Instead, for $\beta_2=0.85$, welfare falls when θ falls for any g larger than 0.8 percent of potential GNP.

The gap between the curve for the inflation constant and the one for g constant can be interpreted as the welfare cost arising from the adjustment of inflation to raise the required revenue. In terms of Figure 15, this is the change in c from $P1'$ to $P2'$.

^{1/} Assuming an inflation rate of 7 percent and $\theta=1.1$, the welfare loss is 1.2 percent of potential GNP. Fischer (1981) computes a welfare loss ("inflation triangle") of 0.3 percent of GNP due to a 12 percent deviation of inflation from its social optimum.

Figure 2

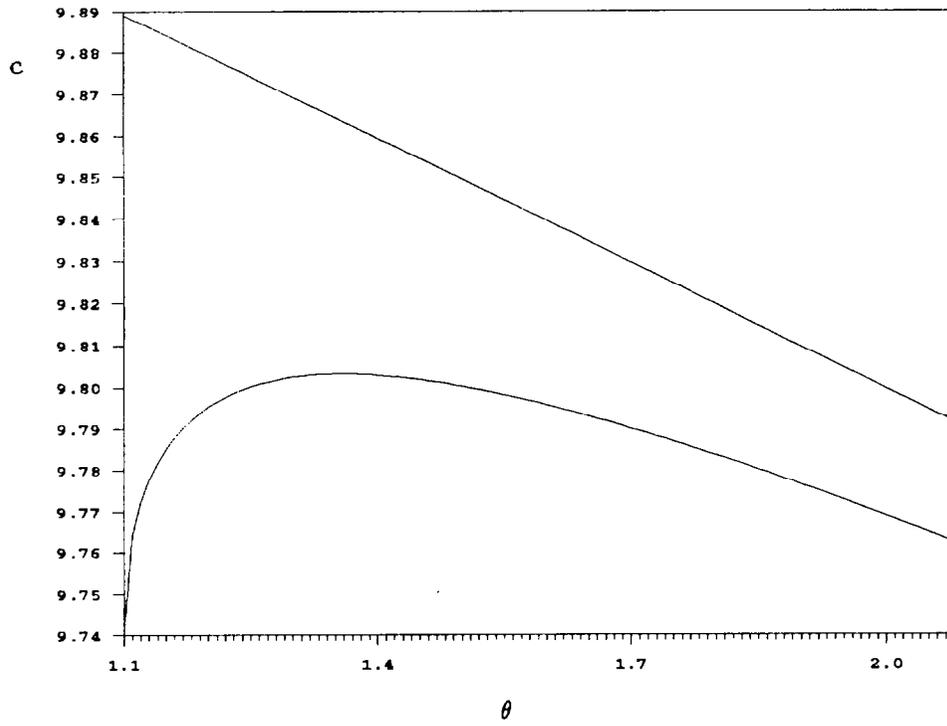


Figure 3
($\theta = 1.1$)

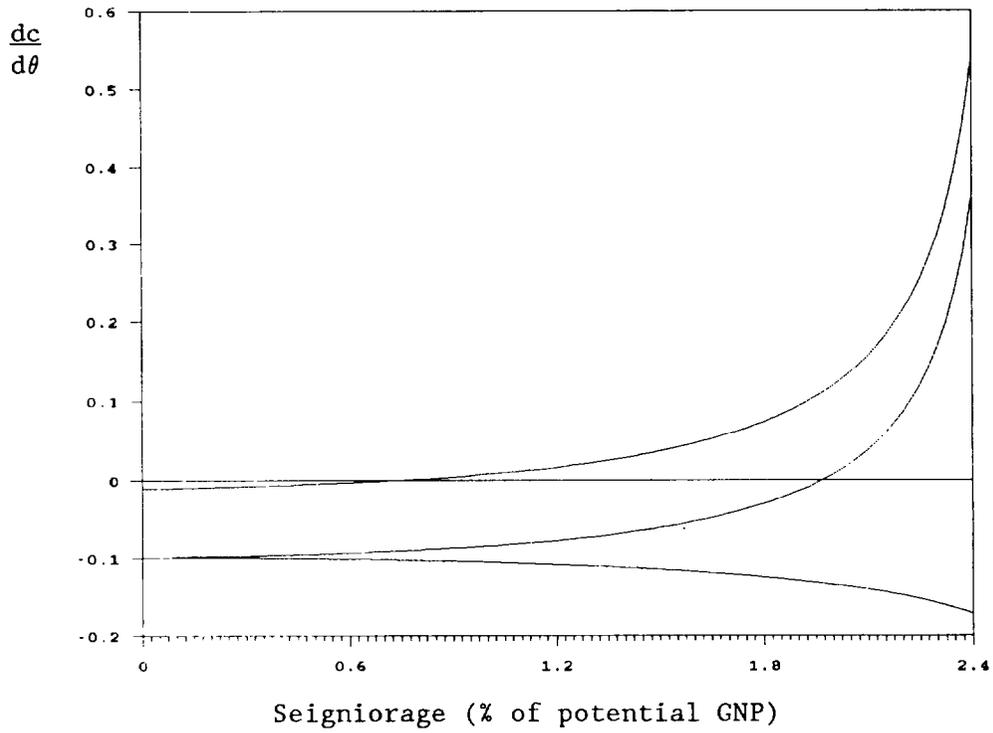


Table 1: Numerical Example

Equations:

Transaction technology:

$$\theta F(m) = \theta \left(\beta_0 - \beta_1 m + \beta_2 m^2 / 2 \right)$$

Consumption:

$$c = y - \theta \left(\beta_0 - \beta_1 m + \beta_2 m^2 / 2 \right)$$

Money demand:

$$m = \left(\beta_1 - (\pi + \delta) / \theta \right) / \beta_2$$

Maximum seigniorage, and money balances at the maximum:

$$g^* = (\theta \beta_1 - \delta)^2 / 4 \theta \beta_2$$

$$m^* = (\theta \beta_1 - \delta) / 2 \beta_2$$

Given g , required money balances (inflation tax curve as function of m):

$$m = m^* + \frac{1}{2 \theta \beta_2} \sqrt{(\delta - \theta \beta_1)^2 - 4 g \theta \beta_2}$$

Parameters:

$$y = 10$$

$$g = 0.25$$

$$\beta_0 = 0.6$$

$$\beta_1 = 1$$

$$\beta_2 = 1$$

$$\delta = 0.05$$

IV. Financial Innovation with a Banking System

In this section the monetary model is extended to incorporate deposits. Deposits are imperfect substitutes of money, but they also provide transaction services. 1/ In this case what is called money is more precisely a monetary base which under the absence of reserves corresponds to currency.

The transactions technology is now $F(m,e)$, where e denotes deposits. Assumptions (A1) to (A3) become:

$$(A1') F_m \text{ and } F_e \leq 0, F_{mm} \text{ and } F_{ee} \geq F_{me} \geq 0$$

$$(A2') \forall e, \text{ there exists } \tilde{m}, \text{ such that } F_m(\tilde{m}, e) = 0, \text{ for all } m > \tilde{m}$$

$$\forall m, \text{ there exists } \tilde{e}, \text{ such that } F_e(m, \tilde{e}) = 0, \text{ for all } e > \tilde{e}$$

$$(A3') F_{mm} F_e - F_m F_{em} < 0 \text{ and } F_m F_{ee} - F_e F_{em} < 0$$

This last assumption simply says that money as well as deposits are normal goods. 2/

Financial intermediation is made through banks. Deposits are the bank's liabilities. On the assets side, because there is no capital in this model, it is assumed that banks have exclusive access to a storage technology with a return equal to δ . 3/ The marginal cost of providing deposits is zero and banks pay an interest rate equal to i . Further, it is assumed that the interest rate paid on deposits is fixed by the government and there are no reserves. The interest rate is kept below its competitive level, representing a financially repressed economy.

Now the consumer's problem is to maximize (1) subject to the following budget constraint:

$$c + F(m,e) + \dot{m} + \dot{e} + \pi(m + e) = y + g + f + ie \quad (11)$$

f is banks profits. The nominal return of banks is $\pi + \delta$. Therefore the

1/ Fischer (1983), and recently Brock (1989), introduce deposits in the transactions technology. Romer (1985) put deposits in the utility function and Walsh (1984) in a cash in advance constraint. In all cases deposits are assumed to be imperfect substitutes of money.

2/ This assumption is equivalent to (A3) noting that an increase in θ is equivalent to a fall in e .

3/ This is equivalent to assume that banks are the only ones allowed to hold foreign assets. The world interest rate is δ . In addition, it should be assumed that PPP holds, and the nominal exchange rate grows at the same rate as money and domestic prices.

competitive interest rate is equal to $\pi + \delta$. 1/ If i is less than $\pi + \delta$, f is equal to $(\pi + \delta - i)e$.

At the competitive interest rate, and for any $i < \delta + \pi$, people do not want to save, although they still maintain deposits since they, as well as money, facilitate transactions.

The steady state is now characterized by:

$$c = y - F(m, e) \quad (12)$$

$$F_m(m, e) = -(\delta + \pi) \quad (13)$$

$$F_e(m, e) = -(\delta + \pi) + i \quad (14)$$

and the government budget constraint (5).

Note that again the optimal rate of inflation (first best) is equal to Friedman's rule. At that optimum, the interest rate is $i=0$. In this case, the marginal productivity of both, money and deposits, are equated to their marginal cost of production. This optimum is attained setting the rate of deflation equal to δ and allowing banks to pay the competitive interest rate.

Note also that by (A1'), the problem can have interior solution only for $i \leq \delta + \pi$. In the rest of this section the focus will be on the interest rate on deposits between 0 and $\delta + \pi$. The question is what are the welfare effects of an increase in the interest rate towards its competitive level. By making the financial system more competitive financial repression is alleviated.

Using equations (13) and (14) it is possible to show that the rate of inflation that maximizes government revenue is given by: 2/

$$\pi^* = m^* \frac{F_{mm} F_{ee} - F_{me}^2}{F_{ee} - F_{me}} \quad (15)$$

Now, the results from Propositions 1 and 2 can be extended to the following proposition:

Proposition 3: (i) For a given inflation rate an increase in the interest rate will increase welfare.

(ii) For a given g , an increase in the interest rate will increase welfare for g around zero and will decrease welfare for g around, but below, g^* .

1/ In the first version of this paper it was assumed zero return on this storage technology, therefore banks were only providers of an alternative asset to make transactions. The competitive interest rate in this case is π , so they provide "indexed money".

2/ It is also assumed that the second order condition holds and the solution is interior.

Proof:

(i) Differentiating (12):

$$\frac{dc}{di} = - F_m \frac{dm}{di} - F_e \frac{de}{di} \quad (16)$$

Now, differentiating (13) and (14):

$$- \begin{bmatrix} F_{mm} & F_{me} \\ F_{em} & F_{ee} \end{bmatrix} \begin{bmatrix} dm \\ de \end{bmatrix} = \begin{bmatrix} d\pi \\ d\pi - di \end{bmatrix} \quad (17)$$

Denoting as H the hessian of $F(\cdot, \cdot)$ and considering $d\pi=0$, we have that:

$$\frac{dc}{di} = \frac{F_m F_{me} - F_e F_{mm}}{\det(H)} > 0 \quad (18)$$

(ii) Using, from (5), $d\pi = -\pi dm/m$ in (17), the system can be written as:

$$- \begin{bmatrix} F_{mm} - \frac{\pi}{m} & F_{me} \\ F_{em} - \frac{\pi}{m} & F_{ee} \end{bmatrix} \begin{bmatrix} dm \\ de \end{bmatrix} = \begin{bmatrix} 0 \\ -di \end{bmatrix} \quad (19)$$

Lets call H' the square matrix on the LHS. Note that for $\pi=0$, $H'=H$, hence the first part of (ii) is true from (18).

After solving the system, we have that:

$$\frac{dc}{di} = \left[F_m F_{me} - F_{mm} F_e + F_e \frac{\pi}{m} \right] \frac{1}{\det(H')} \equiv K \frac{1}{\det(H')} \quad (20)$$

For $\pi < \pi^*$: $\det(H') > 0$, and for $\pi = \pi^*$: $\det(H')=0$. So provided that the expression in square brackets is non-zero, dc/di will diverge to $\pm \infty$ depending on whether K is larger or less than zero, respectively.

Replacing in K the value of π from (15), we have that:

$$K = F_{me} \left[F_m + F_e \left\{ \frac{F_{mm} - F_{me}}{F_{ee} - F_{me}} \right\} \right] < 0 \quad (21)$$

Therefore close to g^* , dc/di goes to $-\infty$, and welfare falls ■

The intuition for part (i) is simply that an increase in the interest rate will increase the productivity of holding deposits. The negative effects of economizing in money holdings is smaller given the assumption that money and deposits are normal goods.

Part (ii) follows from Proposition 2 in the previous section, which says that an increase in the interest rate paid on deposits will decrease welfare when there is a high budget deficit, and hence a high rate of inflation. In

contrast, increasing interest on deposits will increase welfare at low rates of inflation.

As in the previous section, the increase in the interest rate paid on deposits requires a further increase in inflation to collect the required revenue. This effect reduces money balances even more, leading to an unambiguous welfare loss at high rates of inflation. As was already shown, the welfare effect depends on the elasticity of the money demand, which determines the change in inflation and its implied further reduction in real balances required to finance the budget.

Reserve-deposit ratio in a competitive banking system

The critical element for the results of this paper is the unitary elasticity of the money demand at the revenue maximizing rate of inflation. The results may be not robust under some specifications that do not satisfy the unitary elasticity rule, which may be the case with positive reserve-deposit ratio.

Brock (1989) shows that for a positive reserve-deposit ratio the elasticity of the money demand (currency) that maximizes revenue is not one. The reason for this is that the tax base is high powered money. If ρ is the actual reserve-deposit ratio, seigniorage, which by assumption is equal to g , will be given by:

$$g = (m + \rho e)\pi \equiv h\pi$$

Therefore, the rate of inflation that maximizes revenue is that at which the elasticity of h (in absolute value) with respect to π (ϵ_h) is equal to one. Hence, the elasticity of currency (m) will, in general, be different than one. In what follows it will be shown that the result of Proposition 3 can be extended to this case, and reinterpreted in the context of a competitive banking system where the reserve-deposit ratio falls.

The increase in the interest rate paid on deposits can be envisioned as a reduction in reserve requirements in a competitive banking system (Calvo and Fernández, 1983). If banks were allowed to compete and required to hold a fraction ρ of their deposits as non-interest bearing reserves, the competitive interest rate would be $(\delta + \pi)(1 - \rho)$.

In this case (14) would become:

$$F_e(m, e) = - (\delta + \pi)\rho \quad (22)$$

The effect of a change in ρ on consumption is:

$$\frac{dc}{d\rho} = - F_m \frac{dm}{d\rho} - F_e \frac{de}{d\rho} \quad (23)$$

and the effect on high powered money is:

$$\frac{dh}{d\rho} = \frac{dm}{d\rho} + \rho \frac{de}{d\rho} + e \quad (24)$$

Substituting (24), (22) and (13) in (23), the following expression for the welfare effects of a change in ρ is obtained:

$$\frac{dc}{d\rho} = (\delta + \pi) \left[\frac{dh}{d\rho} - e \right] \quad (25)$$

The same intuitive argument explaining Proposition 2 can be used to show that at high levels of seigniorage welfare falls when ρ falls. It is enough to note that $dh/d\rho$ goes to infinity as the elasticity of high powered money (ϵ_h) goes to one:

A change in ρ that causes a Δ percent change in h , will require an increase of Δ percent in inflation to offset it. The increase in inflation will reduce high powered money by an additional $\epsilon_h \Delta$. Inflation will then be required to increase further by $\epsilon_h \Delta$ percent, where h will consequently fall by $\epsilon_h^2 \Delta$, and so on. Therefore the total fall in h will be $\Delta(1 + \epsilon_h + \epsilon_h^2 + \epsilon_h^3 + \dots)$, which goes to ∞ when ϵ_h goes to one. Then, according to (25) welfare will unambiguously fall "close" to the maximum inflation-tax revenue because of the excessive increase in the inflation rate and the fall in real balances.

V. Endogenous Government Spending and Optimal Taxation

In previous sections the government has been assumed to have a passive role by imposing an exogenous tax structure to finance a given budget. This section examines government behavior by analyzing two separate cases. First, government spending is assumed to be a public good which is optimally provided, and hence one that will be modified when financial innovation occurs. Second, an optimal taxation approach is followed by assuming that, in addition to the inflation tax, the government can resort to a consumption tax in order to finance a given level of government spending.

Endogenous Government Spending

This section considers g to be a public good which enters into the individual utility function. For simplicity the utility function is assumed to be separable in consumption good and public good. In this case there will also be an optimal response of g to a reduction in θ (financial innovation). The response will be a reduction in the provision of the public good as the welfare cost of providing it increases.

It is shown below that the main results of previous sections hold: in high inflation economies financial innovation will reduce welfare while in low inflation economies welfare increases.

Consider the same model as that of Section II, but now the instantaneous utility function is:

$$U(c, g) = u(c) + v(g)$$

where both, $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave functions. The government will maximize this utility function subject to consumer behavior and its budget constraint. That is, the government problem is to maximize $U(c, g)$ subject to (3), (4) and (5). This problem can be conveniently written as:

$$W = \text{Max}_m u(y - \theta F(m)) + v(m[-\theta F'(m) - \delta]) \quad (26)$$

Thus, the government implicitly chooses m , by setting accordingly the rate of inflation. The first order condition to this problem is:

$$-u'(c)\theta F'(m) = v'(g)(m\theta F''(m) - \pi) \quad (27)$$

The effects of financial innovation can be determined using the envelope theorem (arguments of the functions are omitted):

$$\frac{dW}{d\theta} = -u'F - v'F'm \quad (28)$$

The first term at the RHS represents the standard direct effect of a reduction in transactions costs, which increases welfare when θ falls. The second term is the welfare-reducing effect of financial innovation, by which the provision of the public good will be reduced because the increasing costs of raising revenue. Although, in this case $dW/d\theta$ will not necessarily diverge to $+\infty$ as inflation approaches the level that maximizes government revenue, thus the same results from Propositions 1 and 2 hold. To see this, we can replace (27) in (28) to obtain:

$$\frac{dW}{d\theta} = -u' \left[F - \frac{m\theta F'^2}{m\theta F'' - \pi} \right] \quad (29)$$

$$= -\frac{v'}{\theta F'} \left[(m\theta F'' - \pi)F - m\theta F'^2 \right] \quad (30)$$

It is clear from (29) and (30) that the direct effect (first term within square brackets) is of a second order when compared to the indirect effect of rates of inflation close to the revenue maximizing rate of inflation (equation (6)). When inflation is low, because g is low, the direct effect will dominate. Therefore, the result from Propositions 1 and 2 apply to the case of endogenous government spending.

Optimal Taxation

In addition to the inflation tax, governments use several instruments to raise revenue. Throughout this paper g has been financed via inflation only. Therefore, g can be interpreted as the fraction of government spending that cannot be financed through nondistortionary taxes. In high inflation countries, the heavy reliance on seigniorage may be due to the large size of the underground economy or the inefficiency of the tax system. It may also be caused by political factors as has been discussed recently in Cukierman, Edwards and Tabellini (1989). They argue that political instability and the degree of polarization are key determinants of seigniorage.

Under a non-single tax system, when the base of one tax falls, the government will, in general, adjust several other tax rates. Hence, a permanent shift in money demand will be accommodated not only by an increase in inflation, but also through an increase in other taxes.

Since Phelps (1973), a large body of literature has focused on inflation as part of an optimal tax system. In this case the optimal structure consists of equating the social marginal costs of different distortions. 1/

It is assumed for the model in Section II, that money is the only asset and is used in transactions. Government optimally sets taxes on money holdings and consumption to finance g of government spending, which is then returned to consumers as a lump sum transfer.

The consumer problem is the same as before, but now consumption is taxed at a rate τ :

$$\text{Max } U_s = \int_s^{\infty} u(c_t) e^{-\delta(t-s)} dt$$

subject to:

$$c(1+\tau) + \theta F(m) + \dot{m} + \pi m = y + g$$

Note that despite having taxes on the only two goods, consumption and real balances, lump sum taxation cannot be reproduced. The reason is that $\theta F(m)$ plays the role of a third good which is not taxed.

Individual behavior is characterized by the following equations:

$$c(1+\tau) = y + g - \theta F(m) - \pi m \quad (31)$$

1/ Poterba and Rotemberg (1990) and Grilli (1989) cast serious doubts on the empirical validity of the optimal seigniorage theory, when tested in industrialized countries.

$$\theta F'(m) = - (\delta + \pi) \quad (32)$$

Although taxes and g are related through the government budget constraint, the individual takes g as given. These equations describe consumption and real balances as functions of the two tax rates, which have to be considered in solving the optimal tax problem. Note that m depends only on the inflation tax.

As will be shown later, in the present model, an optimal tax structure involves no tax on money, where all the tax burden should fall on the consumption tax. 1/ Therefore, to make an inflation tax positive, the model has to be amended. For this purpose real costs in collecting consumption tax are, realistically, assumed, whereas the cost of collecting an inflation tax is zero (Aizenman, 1983 and Végh, 1989). Collection costs are increasing in the amount of revenue collected. They are described by a function $\phi(\tau c)$, where $0 < \phi'(\cdot) < 1$ and $\phi''(\cdot) > 0$. The case $\phi'(\cdot)=0$ is equivalent to the no collection cost case. Since utility depends only on consumption, which is constant in equilibrium, the optimal tax problem consists of:

$$\begin{aligned} & \text{Max } c(\tau, \pi; \theta) \\ & \pi, \tau \end{aligned}$$

subject to:

$$g = \tau c(\tau, \pi; \theta) + \pi m(\pi; \theta) - \phi(\tau c)$$

The lagrangian of this problem is:

$$\mathcal{L} = c(\tau, \pi; \theta) - \mu (g - \tau c(\tau, \pi; \theta) - \pi m(\pi; \theta) + \phi(\tau c)) \quad (33)$$

It can be shown that the first order conditions from the government problem are: 2/

$$\mu = 1/(1-\phi') \quad (34)$$

$$-(\delta - \mu\tau\theta F' - \mu\tau\phi'\delta + \pi\mu) m_{\pi} = [\mu(1 + \phi\tau) - 1] m \quad (35)$$

Since $\phi' \in (0,1)$, μ is greater than 1. In the case where $\phi'=0$, i.e. there are no collection costs, $\mu=1$. Solving (35) for the case of no collection costs, the following equation is obtained for the optimal tax problem:

1/ The specification of the problem in this section is consistent with the result of Kimbrough (1986b), who extended the inflation tax problem in Diamond and Mirrlees' (1971) principle that intermediate inputs should not be taxed. As pointed out in Guidotti and Végh (1990), however, these results hold true only under particular characteristics of the transactions technology, which in the above case applies.

2/ For a full derivation of the remaining equations, see appendix.

$$(1 + \tau)(\delta + \pi)m_{\pi} = 0 \quad (36)$$

Then, optimal taxation calls for $\delta + \pi = 0$, which recovers Friedman's optimal money rule as in Kimbrough (1986b). In the presence of collection costs, however, the optimal inflation tax departs from the zero nominal interest rate rule. In this case combining (35) and (36) and substituting $\partial m / \partial \pi$ by $-1 / \theta F''(m)$, the optimal tax problem is reduced to:

$$m = \frac{(1 - \phi')\delta + \pi}{\phi' \theta F''} \quad (37)$$

In this case, both money and consumption goods have to be taxed.

The question now is what is the welfare effect of a change in θ , considering that all taxes will be optimally adjusted. The total effect can be written as:

$$\frac{dc}{d\theta} = \frac{\partial c}{\partial \theta} + \frac{\partial c}{\partial \pi} \frac{d\pi}{d\theta} + \frac{\partial c}{\partial \tau} \frac{d\tau}{d\theta} \quad (38)$$

The direct effect ($\partial c / \partial \theta$) will be the same as the total effect computed in Proposition 1, which by the assumptions made on $F(\cdot)$ guarantee this to be negative (financial innovation increases welfare). 1/ The other effects will have the opposite sign since taxes will have to be risen to offset the fall in the inflation-tax base. After some algebra it can be shown that:

$$\frac{dc}{d\theta} = \frac{F'^2}{F''} - F - \frac{G}{1 + \tau} \left[(1 - \alpha)\tau + \frac{\delta + \pi}{(1 - \phi')\delta + \pi} \alpha \pi \right] \quad (39)$$

where α is the share of inflation tax revenue on total spending ($G = g + \phi$), and x denotes the percentage increase in the tax rate x ($x = \pi, \tau$) on account of the fall in θ .

When the tax rates are not adjusted, allowing a fall in revenue, a fall in θ will increase welfare. Therefore, only the direct effect matters. This result is merely a generalization of Proposition 1; thus, given the tax rates, a fall in θ is welfare improving.

Nevertheless, once we consider the government budget constraint, there is a negative effect because of the increase in the tax rates. A reduction

1/ One could suspect by the envelope theorem that only the direct effect of θ matters. Equations (28) and (33) show why this intuition is wrong. The government does not set $\partial c / \partial (\text{tax rate})$ equal to zero, but rather $\partial \mathcal{L} / \partial (\text{tax rate})$ equal to zero. In fact, both taxes are distortionary, so a decrease in θ will cause both tax rates to increase such that at the margin the distortions are equated, but the indirect effects through the tax rates cannot be eliminated.

in the inflation tax base requires an adjustment in tax rates. Then, a fall in θ implies that the term within square brackets is positive, offsetting the beneficial direct effect.

It is not possible to generalize Proposition 2 without making additional assumptions, although it can be argued that under general conditions the results still hold. Looking at equation (39)) two remarks that support this presumption can be made. First, the negative effect of a fall in θ is positively related to g , although the change in tax rates and their share in revenue will also depend on g . Second, the increase in the rate of inflation required to raise a given amount of seigniorage is increasing in the elasticity of money demand. Therefore, the larger the elasticity of money the larger the increase in the rate of inflation and the more distortionary commodity taxation will become to offset the distortion in money holdings.

It is possible, however, that a fall in θ will always increase welfare. The fact that the indirect effect of tax rate changes is of a first order when compared to the direct effect of financial innovation--which resolved the ambiguity--is not a general proposition under the assumptions of this section. The reason is that when the elasticity of money demand is close to one, the required revenue will be raised mainly through commodity taxation. Thus, the welfare effect will also depend on the elasticity of the demand for consumption goods. Nevertheless, the direction of the result is the same: the larger the seigniorage, the larger will be the distortion introduced in consumption because of the increase in the tax rate.

VI. Concluding Remarks

Only two interpretations have been fully developed for the change in the marginal productivity of money, and consequently for the reduction in the welfare costs of inflation. The results of Section IV can, however, be easily extended to include other sources of reduction in the welfare costs of inflation.

Deposits can be interpreted as indexed money which is an imperfect substitute for non-indexed money. More importantly, it represents an alternative asset to money. This asset can be used in transactions and it yields higher interest than money. Policies that make the use of this asset more attractive will increase the velocity of money. Consequently the inflation tax base will fall. This is the case of "dollarization", where there is a shift from domestic to foreign money. Dollarizations are usually

observed when a country's inflation rate increases (Fischer, 1982). 1/2/

In the model presented here, for a constant rate of money growth an increase in velocity is welfare improving. When a given revenue has to be raised through inflation tax, however, the result can be reversed. The main conclusion of this paper is that negative effects on welfare will occur in large seigniorage economies. In contrast, the lower the inflation tax, the more beneficial is the reduction in inflation costs.

Faced with high inflation, people seek institutional changes that will protect them from inflation. For example, wage indexation and short-term financial instruments become very important. Usually there are demands for government to introduce changes that reduce the costs associated with transactions, such as a reduction in the reserve-deposit ratios. The perverse effect that some of these changes have, when the deficit is financed mainly by seigniorage, may, however, explain why governments are reluctant to accept changes that may look, *other things being equal*, positive.

Note that the results of this paper could easily be extended to any form of taxation and its relationship to technical progress. The parameter θ is equivalent to a Hicks neutral parameter of technical progress and m is equivalent to an input. Therefore technical progress can be immiserizing when it produces a fall in the demand for taxed inputs. The required increase in the tax rate, and hence in the degree of distortion, may end up reducing welfare. For example, imagine an economy where the only available tax is a tax on gasoline. Technical progress that saves on inputs will reduce the use of gasoline. Therefore, the required increase in the gasoline tax rate may end up reducing welfare.

Inflation and capital flight are frequently observed in inflationary economies. In this paper, Propositions 1 and 2 assume that θ is exogenous. Making this variable endogenous may explain inflation and capital flight as a coordination failure. If θ is interpreted as a parameter of the structure of credit markets, it can be related to the extent of capital flight. Proposition 1 shows the private incentive to reduce θ because it takes inflation as given: individual decisions have no effect on inflation. Under the assumptions of the model, the private incentive to reduce θ is always positive. Instead, Proposition 2 shows that the total effect is uncertain and depends on the current level of inflation (through seigniorage). Therefore, a welfare reducing increase in inflation may be the result of spillover effects from capital flight to real balances, and hence to inflation.

Finally, this paper connects the degree of inflationary finance with

1/ For the case of Mexico, see Ortiz (1983). In November 1989 in the middle of an unsuccessful stabilization program in Argentina the government allowed deposits in foreign currency, which were to be guaranteed.

2/ Arrau and De Gregorio (1990), in a framework similar to the one in this paper, estimated empirically the role of financial innovation in money demand equations for Chile and Mexico. The results showed that an important component of money demand fluctuations corresponds to financial innovation.

developments in financial markets, as was recently addressed by Dornbusch and Reynoso (1989). Financial deepening may lead to a deterioration in welfare through an increase in the rate of inflation. Therefore, a requisite for the removal of financial repression is fiscal discipline. Relying heavily on seigniorage to finance the budget may outweigh the advantage of a more developed financial market.

In summarizing the results of the original question, what are the welfare effects of a reduction in the costs of inflation, this paper concludes that they are positive (negative) in low (high) seigniorage economies. In a broader interpretation, concerning the welfare effects of financial innovation, it can be concluded that the benefits of improved financial intermediation may be offset by the negative effects of a higher rate of inflation.

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The Optimal Tax Problem

Consumer behavior is given by the following equations:

$$c(1+\tau) = y + g - \theta F(m) - \pi m \quad (A1)$$

$$\theta F'(m) = -(\delta + \pi) \quad (A2)$$

Note that at the optimum c will be a function of both taxes, τ and π , while m is only a function of π . Although in equilibrium g equals $\pi m + c\tau$ the individual takes g as a given transfer. The following partial derivatives can be obtained from (A1) and (A2):

$$\partial m / \partial \pi = -1 / \theta F'' \quad (A3)$$

$$\partial c / \partial \pi = -\delta / (1+\tau) \theta F'' - m / (1+\tau) \quad (A4)$$

$$\partial c / \partial \tau = -c / (1+\tau) \quad (A5)$$

The effect on consumption, then welfare, of a change in θ is given by:

$$\frac{dc}{d\theta} = \frac{\partial c}{\partial \theta} + \frac{\partial c}{\partial \pi} \frac{d\pi}{d\theta} + \frac{\partial c}{\partial \tau} \frac{d\tau}{d\theta} \quad (A6)$$

The direct effect, is the one total derivative from Proposition 1, that is assuming all tax rates as constants:

$$\frac{\partial c}{\partial \theta} = \frac{F'^2}{F''} - F \quad (A7)$$

The government optimal tax problem consists of:

$$\begin{aligned} &\text{Max } c(\tau, \pi; \theta) \\ &\pi, \tau \end{aligned}$$

subject to:

$$g = \tau c(\tau, \pi; \theta) + \pi m(\pi; \theta) - \phi(\tau c)$$

The lagrangian of this problem is:

$$\mathcal{L} = c(\tau, \pi; \theta) - \mu (g - \tau c(\tau, \pi; \theta) - \pi m(\pi; \theta) + \phi(\tau c)) \quad (A8)$$

where m is a lagrange multiplier. The first order conditions of this problem are (subscripts denote partial derivatives):

$$-c_{\tau} [1 + \mu\tau(1 - \phi')] = \mu c(1 - \phi') \quad (A9)$$

$$(1 + \mu\tau)c_{\pi} + \mu m + \mu\pi m_{\pi} - \mu\phi' \tau c_{\pi} = 0 \quad (A10)$$

Substituting (A5) in (A8), this first order condition becomes

$$\mu = 1/(1-\phi') \quad (A11)$$

Since $\phi' \in (0,1)$, μ is greater than 1. In the case that $\phi'=0$, i.e. there is no collection cost, $\mu=1$.

Using (A4) and (A5) in (A10), it can be written as:

$$-(\delta - \mu\tau\theta F' - \mu\tau\phi'\delta + \pi\mu) m_{\pi} = [\mu(1 + \phi\tau) - 1] m \quad (A12)$$

From this equation it can be seen that when $\phi'=0$ and hence $\mu=1$, (A12) collapses to:

$$(1 + \tau)(\delta + \pi)m_{\pi} = 0 \quad (A13)$$

So optimal taxation calls for $\delta+\pi=0$ which recovers Friedman's optimal money rule as in Kimbrough (1986). But in the presence of collection costs the optimal inflation tax departs from the zero nominal interest rate rule. Replacing (A11) and (A3) in (A12) we obtain the following expression characterizing the optimal tax scheme, 1/

$$m = \frac{(1-\phi')\delta + \pi}{\phi'\theta F''} \quad (A14)$$

Substituting (A3), (A4) and (A5) in (A6) (consumer behavior in equation for the total effect of θ in consumption) we obtain

$$\frac{dc}{d\theta} = \frac{\partial c}{\partial \theta} - \left[\frac{\delta}{1+\tau} \frac{1}{\theta F''} + \frac{m}{1+\tau} \right] \frac{d\pi}{d\theta} - \frac{c}{1+\tau} \frac{d\tau}{d\theta} \quad (A15)$$

which after using the optimal tax rule (A14) becomes equation (39) in the text:

$$\frac{dc}{d\theta} = \frac{F'^2}{F''} - F - \frac{g}{1+\tau} \left[(1-\alpha)\hat{\tau} + \frac{\delta+\pi}{(1-\phi')\delta+\pi} \hat{\alpha}\pi \right] \quad (A16)$$

Note that this equation can be further reduced to find expression for the percentage change of the tax rate by totally differentiating the government budget constraint.

1/ The final solution could be obtained replacing m as a function of π from (A2) in (A13) to have a single equation for π . Then, substituting c from (A1), as a function of π and τ , in the government budget constraint the solution for τ would be obtained.