IMF WORKING PAPER

WP/92/109

INTERNATIONAL MONETARY FUND

European II Department

The Volatility of Consumption in a Simple General Equilibrium Model

Prepared by Gunnar Tersman *

Authorized for distribution by Adalbert Knöbl

December 1992

Abstract

This paper studies the volatility of consumption relative to output in the context of a simple general equilibrium model of a small open economy subject to exogenous shocks in productivity. With infinite horizons and exogenous relative prices, the model generates variance estimates that are well above what can be observed in empirical data. While finite horizons and endogenous terms of trade reduce the volatility of consumption, the model fails to generate sufficient serial correlation with respect to the consumption growth rate. If the household's decision problem is modified to take into account durability and adjustment costs, the model does well on both dimensions.

JEL CLASSIFICATION NUMBERS: C68, E21

---

* The views expressed in this paper are those of the author and do not necessarily reflect the views of the International Monetary Fund.

Table of Contents

Summary

This paper studies the volatility of consumption relative to output in a simple general equilibrium model of a small open economy subject to exogenous shocks in productivity. With infinite horizons and exogenous terms of trade, the model generates variance estimates that are typically well above those that can be observed in empirical data. The time series process of consumption in empirical data, judged in this perspective, would thus seem "excessively smooth," too stable to be consistent with intertemporal optimization. However, if one allows for finite horizons, broadly interpreted as liquidity constraints, the model is able to come up with more reasonable estimates.

Although finite horizons and endogenous terms of trade help in one dimension, by reducing the volatility of consumption relative to output, the model still fails to produce a plausible degree of serial correlation with respect to the consumption growth rate. The fact that the growth rate of consumption is positively correlated suggests that durability and adjustment costs are important aspects of consumption behavior. When these aspects are incorporated into the household's decision problem, the model does well on both dimensions.

I. Introduction

Conventional economic wisdom implies that innovations in consumption should be smaller than those in output. Yet changes in consumption typically exceed those in output. According to the traditional view, the volatility of consumption relative to output would seem to be "excessive," too great to be consistent with intertemporal optimization.

How one views the relative volatility of consumption depends on how one views the growth process. According to the long prevailing view of economic growth, output fluctuates around a deterministic trend. In this case, the observed behavior of consumption clearly displays an excessive degree of volatility. However, if one abandons the idea of a smooth trend, this is no longer necessarily true. Following work by Nelson and Plosser (1982), Prescott (1986), Campbell and Mankiw (1987) and others, many researchers now question the traditional view, arguing that there is no reason to believe that technical progress is described by a smooth trend. If this more modern view is correct, consumption could indeed be less smooth than output. In fact, as noted by Deaton (1986), the problem is not that consumption is "excessively sensitive" as argued by Flavin (1981). Rather, consumption appears to be "excessively smooth," a phenomenon which has widely been interpreted as evidence of liquidity constraints. 1/ The fact that aggregate investment and saving often seem to be positively correlated would seem to support this view.

If one allows for permanent shocks in output, the fact that consumption is more volatile than output should come as no surprise. On the other hand, failure to detect enough volatility should not by itself lead to the rejection of intertemporal optimization for at least two reasons. The first reason has to do with intertemporal substitution. Intertemporal optimization may still be consistent with the data if output innovations are partly offset by relative price changes. For example, using a small real business cycle model of a closed economy derived explicitly from first principles, Christiano (1987) examined the role of interest rates and found that even small interest movements were sufficient to explain the lack of volatility of consumption in the United States.

While offsetting interest rate movements would seem to be a remote possibility in most small open economies, what about changes in terms of trade? To the extent that innovations in output are important enough to move the market for domestic goods, one would expect the volatility of consumption to be a decreasing function of the volatility of terms of trade. The problem with endogenous terms of trade as an explanation of the observed

---

1/ See for example Flavin (1984). Recent empirical studies of the role of liquidity constraints include Campbell and Mankiw (1989) and Japelli and Pagano (1989). See Hayashi (1985) for a survey of earlier work.

smoothness of consumption is that it generally implies that investment is less volatile as well. As with endogenous interest rates, this suggests that smaller economies are more likely to be volatile than larger ones.

The second reason why one should be careful not to reject intertemporal optimization too quickly has to do with the durability of consumption. Empirical applications typically exclude purchases of durables, focussing instead on consumption of services and nondurables. However, this approach fails to consider the obvious fact that many nondurables (and indeed even some services) are not expected to be used up immediately after they are acquired. Empirical studies by Hayashi (1985) and Bernanke (1985), for example, that try to model household behavior in a more realistic manner, indicate that durability can successfully explain the observed behavior of consumption with only a very minor role for liquidity constraints.

By looking at how consumption behaves, is it possible to recover the underlying growth process? Generally not, since different time series processes for output may be consistent with the same process for household wealth. However, provided that consumption behaves according to standard theory, the volatility of consumption relative to output can still be seen as an indication of the importance of permanent shocks. 1/ Based on simulations of a small real business cycle model, this note provides some simple numerical examples intended to illustrate these rather simple ideas. The note demonstrates how the observed volatility of consumption can be accounted for in a model explicitly derived from intertemporal optimization.

II. The Model

The model is in the small open economy tradition, constructed along the lines of the recent business cycle theory as, for example, in Cardia (1991), Leiderman and Razin (1989), Mendoza (1991) and Zeira (1987). 2/ The representative firm operates under perfect competition and takes all prices as given. However, if the size of the domestic market is sufficiently large relative to the market for domestic goods as a whole, terms of trade may still be endogenously determined. Economic agents can borrow freely from the rest of the world at a given world interest rate. While the focus in this paper is on productivity shocks, the model can be used to consider other types of shocks as well. Expectations about future variables are

---

1/ A number of researchers have argued that consumption may be subject to "taste shocks," movements in consumption which are unrelated to news about interest rates or incomes, in which case this interpretation may no longer be valid.

2/ There is by now a large and growing literature on saving and investment behavior in equilibrium models. Examples of two-country models include Backus, Kehoe and Kydland (1992), Baxter and Crucini (1992), Finn (1989) and Stockman and Tesar (1990). For an overview of the real business cycle research program, see McCallum (1989) and Plosser (1989).

introduced by assuming rational expectations. Approximate decision rules for the model's dynamic behavior are derived from solving the deterministic version of the model in an environment of perfect foresight.

1. <u>Firms</u>

In each period t, the representative firm produces output Y_t with a Cobb-Douglas production function using capital K_t and labor L_t:

$$Y_t=A_tZ_tK_t^{\alpha}L_t^{(1-\alpha)} \quad 0<\alpha<1$$

where productivity is the sum of a deterministic time trend A_t and an AR(2) process Z_t which is subject to identically and independently distributed shocks ϵ_{Zt} with zero mean and standard error $\sigma_{\epsilon Z}$:

$$A_t=(1+\beta)^{(1-\alpha)t}$$

$$\ln Z_t=\gamma_1\ln Z_{t-1}+\gamma_2\ln Z_{t-2}+\epsilon_{Zt} \quad \gamma_1+\gamma_2=1$$

Employment may either be derived from the representative household's leisure-consumption choice or considered to be exogenous subject to some stochastic process. The apparent failure of efforts to explain employment variations solely through unconstrained utility optimization has triggered a search for alternative explanations which basically fall under any of the three headings "implicit contracts," "insiders" or "efficiency wages." The present version of the model is based on the second approach. Employment is an AR(2) process:

$$\ln L_t=\ln\bar{L}+\zeta_1\ln L_{t-1}+\zeta_2\ln L_{t-2}+\epsilon_{Zt} \quad \zeta_1+\zeta_2\leq 1$$

There are no independent shocks in employment. However, because of real wage rigidity, employment responds positively to shocks in productivity after which it gradually returns to its long-run value. In each period, employment is set to equate the marginal product of labor with the real product wage. Wages are partly predetermined by expectations about the marginal product of labor in the previous period. When the real product is completely predetermined, a shock in productivity ϵ_{Zt} causes a shock in employment equal to $(1/\alpha)\epsilon_{Zt}$. The other extreme is when the real product wage moves with the marginal product of labor so that employment remains unchanged. To strike a balance between these two extremes, it is assumed that employment responds to shocks in productivity in a one to one fashion. Consistent with the stylized fact that real wages seem to be only weakly procyclical, this assumption distributes most of the shocks in productivity to changes in employment.

As in Hayashi (1982), there are convex installation costs, implying that the firm only gradually adjusts its capital stock to the desired level. While real net investment is simply the change in the capital stock $K_{t+1}-K_t$, real gross investment I_t also includes installation costs $\eta((K_{t+1}-K_t)/K_t) \times (K_{t+1}-K_t)$ and depreciation δK_t:

$$I_t=(1+\eta(\frac{(K_{t+1}/A_{t+1})-(K_t/A_t)}{(K_t/A_t)})(K_{t+1}-K_t)+\delta K_t \quad \eta>0 \quad \delta>0$$

In carrying out its investment plans, the representative firm uses a combination of domestic and foreign goods, denoted below by superscripts y and x. The composite good is a linearly homogenous CES function in domestic and foreign investment goods. In the special case with a unitary elasticity of substitution between the two goods, it is a Cobb-Douglas function:

$$I_t=(\theta_1^{(\frac{1}{\theta_2})} I_t^{y\,(1-\frac{1}{\theta_2})}+(1-\theta_1)^{(\frac{1}{\theta_2})} I_t^{x\,(1-\frac{1}{\theta_2})})^{(\frac{\theta_2}{\theta_2-1})},$$

$$=\theta_1^{-\theta_1}(1-\theta_1)^{(\theta_1-1)} I_t^{y\,\theta_1} I_t^{x\,(1-\theta_1)} \quad \text{for } \theta_2=1$$

I use the timing convention that all transactions, payments of dividends and wages as well as purchases of investment and consumption goods, are made in the end of each period. With r_t denoting the real interest rate, P_t^y the price of domestic goods, W_t the wage rate and P_t^i the price index of the composite investment good, the value of the firm F_t can be written:

$$F_t=E_t[\sum_{s=t}^{\infty}\prod_{v=t}^{s}(\frac{1}{1+r_v})(P_s^y Y_s-W_s L_s-P_s^i I_s)]$$

Given technology and output and input prices, the firm is assumed to choose its production and investment plans to maximize its expected net present value. Maximizing the firm's net present value subject to the difference equation for the capital stock yields the following first order conditions for an optimal investment program:

$$(K_{t+1}/A_{t+1})-(K_t/A_t)=((q_t/P_t^i-1)/2\eta)(K_t/A_t)$$

$$q_t=E_t[(\frac{1}{1+r_{t+1}})(q_{t+1}+\alpha P_{t+1}^y A_{t+1} Z_{t+1} K_{t+1}^{(\alpha-1)} L_{t+1}^{(1-\alpha)}-$$

$$P_{t+1}^i(\delta-\eta(\frac{(K_{t+2}/A_{t+2})-(K_{t+1}/A_t)}{(K_{t+1}/A_{t+1})})^2))]$$

As usual, investment depends on the value of an installed unit of capital relative to its replacement cost q_t (Tobin's average and marginal q). Similarly, maximizing profits with respect to employment involves setting the real product wage equal to the marginal product of labor. Finally, finding the least-cost combination of domestic and imported investment goods yields a solution which defines the firm's demand for each good as a function of investment and relative prices:

$$I^y_t=\theta_1(P^i_t/P^y_t)^{\theta_2}I_t$$

$$I^x_t=(1-\theta_1)(P^i_t/P^x_t)^{\theta_2}I_t$$

$$P^i_t=(\theta_1 P^{y\ (1-\theta_2)}_t+(1-\theta_1)P^{x\ (1-\theta_2)}_t)^{(\frac{1}{1-\theta_2})}$$

$$=P^{y\ \theta_1}_t P^{x\ (1-\theta_1)}_t \quad for\ \theta_2=1$$

2. <u>Households</u>

There are overlapping generations of households with finite expected lifetimes. Following Blanchard (1985), each household faces a given "probability of survival" from the current period to the next λ so that the "expected remaining lifetime" or planning horizon is equal to $1/(1-\lambda)$. The fact that households have finite horizons implies that they pay a risk premium in the capital market which is one reason why Ricardian equivalence may fail to hold in this model. 1/ The representative household has a CRRA instantaneous utility function defined over consumption C_t and maximizes expected discounted utility which can be written:

$$E_t[\sum_{s=t}^{\infty}(\frac{\lambda}{1+\mu})^{s-t}U_s] \quad U_t=(1-1/\pi)^{-1}C_t^{(1-1/\pi)}=\ln C_t\ for\ \pi=1 \quad \lambda\leq 1 \quad \mu>0$$

where μ is the subjective discount rate and π is the intertemporal elasticity of substitution. Adjustment costs and durability can be taken into account if the household's decision problem is rewritten in a way similar to the firm's. This involves adding an adjustment cost function and a difference equation for the stock of consumption goods. For example, a solution is easily found if the adjustment cost function enters multiplicatively into the utility function:

$$U_t=(1-1/\pi)^{-1}(C_t(C_t/C_{t-1})^{\varphi})^{(1-1/\pi)} \quad \varphi<0 \quad \pi\neq 0$$

Similar to investment, consumption is a composite good consisting both of domestic and foreign goods. The consumption composite is a linearly homogenous CES function in domestic and foreign goods:

1/ Finite horizons increase the sensitivity of consumption to changes in current income and tend to tilt the response to productivity shocks into the future. A convenient alternative to this specification is the Campbell and Mankiw (1989) setup with current income consumers who set consumption equal to their disposable income.

$$C_t=(\omega_1^{(\frac{1}{\omega_2})} C_t^{y\,(1-\frac{1}{\omega_2})}+(1-\phi_1)^{(\frac{1}{\omega_2})} C_t^{x\,(1-\frac{1}{\omega_2})})^{(\frac{\omega_2}{\omega_2-1})}$$

$$=\omega_1^{-\omega_1}(1-\omega_1)^{(\omega_1-1)} C_t^{y\,\omega_1} C_t^{x\,(1-\omega_1)} \quad for\ \omega_2=1$$

The expected value intertemporal budget constraint is:

$$E_t[\sum_{s=t}^{\infty}\prod_{v=t}^{s}(\frac{\lambda}{1+r_v})P_s^c C_s]=F_t-B_t+H_t$$

where P_t^c is the price index of the composite consumption good, B_t is the household's net debt obligations and human wealth H_t is the present value of labor income:

$$H_t=E_t[\sum_{s=t}^{\infty}\prod_{v=t}^{s}(\frac{\lambda}{1+r_v})W_s L_s]$$

Maximizing expected future discounted utility subject to the intertemporal budget constraint yields the usual Euler equation:

$$C_t^{\pi}=E_t[(1+r_{t+1})\gamma/(1+\mu)(P_t^c/P_{t+1}^c)C_{t+1}^{\pi}]$$

Under logarithmic utility, which is assumed here, consumption is a constant fraction of household wealth $C_t=\chi(F_t+H_t-B_t)$. I adjust the time preference rate so as to make sure that consumption always grows at a constant rate equal to the steady state growth rate of output. For example, in the infinite horizon case, $\mu=(1+r)/(1+\beta)-1$. With adjustment costs, the solution to the household's optimization problem is generally somewhat more complex. For the specification above, the Euler equation becomes:

$$C_t^{(1+\varphi)(1-1/\pi)-1} C_{t-1}^{-\varphi(1-1/\pi)}$$

$$=E_t[(1+r_{t+1})\lambda/(1+\mu)C_{t+1}^{(1+\varphi)(1-1/\pi)-1} C_t^{-\varphi(1-1/\pi)}]$$

which defines the expected consumption growth rate as an AR(1) process with an autocorrelation coefficient equal to $\varphi(1-1/\pi)/((1+\varphi)(1-1/\pi)-1)=\rho$. As in the case of investment, the share of each good in the consumption composite depends on relative prices:

$$C_t^y=\omega_1(P_t^c/P_t^y)^{\omega_2}C_t$$

$$C_t^x=(1-\omega_1)(P_t^c/P_t^x)^{\omega_2}C_t$$

$$P_t^c=(\omega_1 P_t^{y\,(1-\omega_2)}+(1-\omega_1)P_t^{x\,(1-\omega_2)})^{(\frac{1}{1-\omega_2})}$$

$$=P_t^{y\,\omega_1}P_t^{x\,(1-\omega_1)} \quad for\ \omega_2=1$$

3. Market clearing

Under the standard small open economy assumptions, all prices are given. With endogenous terms of trade, the model is closed by an export demand function which relates exports to demand in the rest of the world and terms of trade. Similarly, endogenizing the real interest rate involves equalizing saving and investment. The present calculations assume that domestic and foreign bonds are perfect substitutes. With a perfectly fixed exchange rate and perfect capital mobility, the economy faces a given world market interest rate.

III. Some Simple Numerical Exercises

Deriving exact solutions under uncertainty for models of this kind is typically only possible if one is willing to make rather restrictive assumptions with respect to the economy's underlying structure. In the case of the household's optimization problem, exact solutions are available only for specific utility functions and specific assumptions about asset returns. Similar restrictions apply to the firm's optimization problem. However, a number of numerical solution methods exist which make it possible to find approximate solutions when exact solutions are not available. 1/

The more advanced methods involves discretizing the random variable (if it is not already discrete) in combination with value function iteration as in Baxter, Crucini and Rouwenhorst (1990). Other options include Fair and Taylor's (1983) extended path method or the linear-quadratic approximation methods described in Christiano (1990). Each method has advantages as well as disadvantages. The more advanced value function-grid methods conform more closely with the spirit of the underlying model and are more exact than methods based on approximate decision rules. However, they are also more complicated and therefore more time consuming. Methods based on linear approximations around a deterministic steady state solution require insignificant amount of computing time as they do not involve any type of iterative procedures but have on the other hand a greater potential for errors.

Although the solutions to the optimization problems in the model are generally different under uncertainty than under certainty, this may be of no great consequence when it comes to the time series behavior of the model.

1/ See Taylor and Uhlig (1990) for a comprehensive survey.

For this reason, and because it is computationally easier, I will work with the deterministic version of the model. However, even in the deterministic case, the model is too complex to allow for an explicit analytical solution and can only be solved numerically. For this purpose, I rely on the deterministic version of the extended path algorithm (see Appendix). While the true model solution requires that expectations are based on the entire probability density of the random variable ϵ_{Zt}, this approach essentially boils down to solving the model under the assumption that future values of ϵ_{Zt} are zero.

In order to study the time series behavior and variance properties of the model, I proceed in two steps. Starting from the initial steady state, I first examine the response of the model to a standardized unanticipated shock in productivity, by definition equivalent to one standard error (i.e. ϵ_{Zt}=0 for t=0,2,3,...,T and equivalent to $\sigma_{\epsilon Z}$ for t=1). The second step is to use the response pattern of the model to derive for each variable a moving average process in log differences with a set of weights normalized to express the variable's dynamic response relative to the underlying productivity process. When this is done, the variance properties of each variable can easily be derived.

Because of the approximate nature of the solution obtained in the first step, the model's response pattern is to some extent particular to each simulation experiment. Although the effects on the time series properties of each variable relative to the underlying productivity process are quite small, the size of the shock considered in the first step is not entirely unimportant for the results. The significance of this problem depends on the degree of nonlinearity of the underlying model. A simple way out is to assume that ϵ_{Zt} has a uniform and discrete distribution $\epsilon_{Zt}=\{\sigma_{\epsilon Z}, -\sigma_{\epsilon Z}\}$ where ϵ_{Zt} is determined so that the response pattern of the model is consistent with the standard deviation of the output growth rate in the empirical data.

In calibrating the model, I take as my point of departure annual data on six small European economies for the period 1971-90 (Figure 1 and Table 1). Rather than to use data from any particular country, I construct a six-country average economy. The model is calibrated so that an initial steady state solution closely reproduces the average shares of private and public consumption, investment, exports and imports in GDP in this six-country average data set (Tables 2 and 3).

I must also make a number of assumptions with respect to asset trade and the composition of household assets. Under uncertainty, the household must make two decisions, to determine the level of consumption and to determine the composition of its wealth portfolio. In the present model, the household can either invest in bonds, earning the risk-free real interest rate, or in equity capital, the return on which is uncertain. Unless there are capital controls, an additional choice is between investing in domestic or foreign capital. A complete treatment of the household's optimization problem thus requires specifying the probability distribution on domestic as well as foreign capital.

Ideally, decision rules for consumption and portfolio selection should both be derived explicitly within the model. I will not attempt to do that in this note. Instead, essentially sidestepping the issue of portfolio selection, I will simply assume that only domestic residents hold claims on the economy's capital stock. In addition, in the initial steady state, net foreign assets are zero, implying that domestic demand equals output. These assumptions are important since portfolio diversification can make household wealth less sensitive to shocks to the domestic economy. One way to justify these assumptions is to think about the capital stock as a composite asset which already incorporates the optimal portfolio-selection decision.

To make the model solution consistent with the data, I have to introduce a government sector. I do this in the simplest possible way, ignoring government production, employment and investment, as well as assuming that government expenditures amount to a constant fraction of output and that the government budget is balanced at all times through lump-sum taxes. 1/ While these assumptions make it possible to work with total-economy data, it is important to realize how they affect the results. Even if consumers would be fully Ricardian and there would be no credit market imperfections or tax distortions, different assumptions with respect to the time series process for government expenditures have different implications for household wealth and thereby also for consumption.

Employing these procedures yields the following results. Consider first the response pattern for a reference case with infinite horizons and exogenous terms of trade (all parameters as in Table 3). If productivity is described by an autocorrelated process in first differences, the shock to the present value of household income is greater than the shock to income itself (Figure 2). As a result, the initial increase in consumption is greater than that in income. If the shock would be temporary, the present value of household income would be less affected, in which case the main effect would be a rise in the household saving rate. Although always likely to increase more than consumption, the same general conclusion also applies to investment. 2/

In the second step, I calculate the volatility of each variable relative to that of output (Table 4). The calculations confirm the conclusions from examining the model's impulse response pattern. Investment is the most volatile variable followed by consumption, employment and the

---

1/ In a more elaborate framework, the time series processes of government taxes and expenditures could be estimated econometrically.

2/ Needless to say, all variables are naturally excessively correlated with output. This would of course change if one would allow for simultaneous shocks in other variables, for example terms of trade, the real interest rate or employment.

capital stock. The volatility of consumption increases with the degree of autocorrelation of the output growth rate and with the planning horizon. While not much higher than in the empirical data for two of the countries in the sample, the volatility of consumption seems somewhat too high, particularly as the autocorrelation of the output growth rate roughly matches what can be observed in the empirical data. Similarly, the consumption growth rate is serially uncorrelated while the six-country sample indicates that it ought to be positively autocorrelated.

By raising the propensity to consume out of wealth as well as the rate at which labor income is discounted, finite horizons dampen the initial response of consumption relative to the infinite horizon case (Figure 3). 1/ In order to generate the degree of volatility observed in the empirical data, the average horizon ought to be around 20 years. I also considered the Campbell and Mankiw (1989) model where only a fraction of households (permanent income consumers) base their spending decisions on intertemporal optimization with the rest setting consumption equal to their current income (current income consumers). 2/ To generate the observed consumption volatility, the share of permanent income consumers ought to be around 80 percent, higher than Campbell and Mankiw's estimate for the United States and also higher than their estimates for the other major industrial countries except for the United Kingdom.

Although finite horizons help in one dimension, by making consumption less volatile, the model still fails to produce a plausible degree of serial correlation. The fact that the growth rate of consumption expenditures is positively correlated suggests that durability and adjustment costs are important aspects of consumption behavior. 3/ While the traditional route around the problem with durable goods has been to focus on spending on services and non-durables, the theoretically more appealing approach is to allow for different degrees of durability. A strong argument can also be made for considering adjustment costs. Shopping takes time and has opportunity costs in terms of foregone leisure. In addition, households may

---

1/ The rate of time preference is adjusted to make it possible to generate a steady state path according to which consumption grows in line with output.

2/ The income accruing to the household sector (net of investment expenditures, government taxes and interest expenditures) is split into two parts. One part goes to permanent income consumers who determine consumption on the basis of intertemporal optimization. The other part goes to current income consumers. Total consumption is then a weighted sum of each group's consumption.

3/ Serial correlation may of course also simply reflect time averaging. If the basic permanent income-life cycle model holds in continuous time, then measured consumption is the time average of a random walk. In this case, the change in consumption will be serially correlated even if the underlying model is accepted.

have a preference for steady consumption paths in which case the volatility of consumption enters directly into the utility function.

In the simple case with no adjustment costs, consumption basically follows a random walk with drift, implying that its rate of growth is serially uncorrelated. In the general case with durable goods and adjustment costs, the solution to the household's optimization problem involves decision rules which make the expected level of consumption depend not only on its level in the preceding period (as under the case with no adjustment costs) but also on levels in earlier periods which implies that the consumption growth rate exhibits positive serial correlation. With such a setup, the model can match both the volatility and autocorrelation in the empirical data reasonably well.

In addition to finite horizons, liquidity constraints and adjustment costs, the response of consumption to shocks in productivity also depends on to what extent such shocks lead to offsetting movements in interest rates or relative prices. In order to investigate this idea, I let the price of the domestic good be determined by market clearing and simulate the model under different assumptions for the elasticity of substitution between domestic and foreign goods. The parameters of the CES investment and consumption composite good functions are assumed to be the same. I also assume the same substitution elasticity in the rest of the world as in the domestic economy. Further, in steady state, all variables, including demand in the rest of the world, are assumed to grow at the same rate as output.

While changes in relative prices cause firms and households to reallocate spending over time, the most important effect comes through decreasing (or increasing in case of negative productivity shocks) spending in each and every period (Figure 4). Not surprisingly, endogenous terms of trade serve to reduce the volatility of both consumption and investment. The volatility of consumption and investment increases with the elasticity of substitution between domestic and foreign goods. Thus, to the extent that offsetting terms of trade movements would seem a more realistic possibility in large economies, one would expect such economies to be less volatile than the typical small open economy which faces a close to horizontal demand curve for its products. Given that endogenous terms of trade fail to generate sufficient serial correlation in the consumption growth rate and given the implications for the volatility of investment, an explanation based on offsetting movements in relative prices does not appear to be very appealing.

IV. Concluding Remarks

This note examines the volatility of consumption and other key macro aggregates in a simple real business cycle model of a small open economy subject to exogenous shocks in productivity. Investment and consumption decisions are explicitly derived from intertemporal optimization. The representative firm operates under perfect competition and takes all prices as given. Economic agents can borrow freely from the rest of the world at a given world interest.

If productivity is described by a positively autocorrelated process in first differences, an assumption which seems to be supported by recent empirical work, consumption is generally more volatile than output. The degree of volatility of consumption relative to output depends, among other things, on the representative household's planning horizon and the extent to which productivity shocks generate offsetting movements in relative prices. With exogenous terms of trade and infinite planning horizons, the model generates estimates of the relative volatility of consumption that are typically well above what can be observed in empirical data. However, if one allows for finite horizons, broadly to be interpreted as the result of liquidity constraints, the model is able to come up with more reasonable estimates.

Although finite horizons help in one dimension, by making consumption less volatile, the model still fails to produce a plausible degree of serial correlation. The fact that the growth rate of consumption expenditures is positively autocorrelated suggests that durability and adjustment costs are important aspects of consumption behavior. While the traditional route around the problem with durable goods has been to focus on spending on services and non-durables, the theoretically more appealing approach is to allow for different degrees of durability. There is also a strong argument for considering adjustment costs. With adjustment costs, the model performs well on both dimensions.

Endogenous terms of trade serve to reduce the volatility of consumption and investment. The volatility of both aggregates generally increases with the elasticity of substitution between domestic and foreign goods. Thus, to the extent that offsetting terms of trade movements would seem a more realistic possibility in large economies, one would expect such economies to be less volatile than the typical small open economy which faces a close to horizontal demand curve for its products. However, given that endogenous terms of trade fail to generate sufficient serial correlation in the consumption growth rate and given the implications for the volatility of investment, an explanation of the low degree of volatility of consumption relative to output typically observed in empirical data based on offsetting movements in relative prices does not appear to be very convincing.

Table 1. Volatility of Key Macroeconomic Aggregates 1/

	Austria	Belgium	Denmark	Finland	Holland	Sweden	Average
	Standard deviations of log differences relative to GDP						
Total consumption	0.89	0.98	1.02	0.79	0.99	1.02	0.95
Private consumption	1.09	1.05	1.42	1.00	1.15	1.44	1.18
Total investment	2.65	3.21	4.74	2.55	3.04	3.14	3.24
Capital stock	0.87	0.44	0.74	0.45	0.61	0.51	0.60
Total employment	0.56	0.48	0.62	0.65	0.73	0.58	0.60
	First order autocorrelation of changes in logs 2/						
Output	0.25 (0.21)	0.04 (0.22)	0.08 (0.24)	0.20 (0.23)	0.39 (0.20)	0.09 (0.20)	0.18 (0.22)
Private consumption	0.08 (0.24)	0.44 (0.21)	0.15 (0.23)	0.09 (0.23)	0.63 (0.15)	0.31 (0.22)	0.29 (0.21)

Source: Calculations based on the OECD Economic Outlook database.

1/ For 1971-90. Data on investment refers to gross fixed investment. Data on capital stock refers to the business sector only.
2/ Standard deviations in parenthesis.

Table 2. Data on Six European Countries--Averages for 1971-90

	Austria	Belgium	Denmark	Finland	Holland	Sweden	Average	Model 1/
				In percent of GDP				
Private consumption	57.6	66.2	55.8	55.7	59.2	55.1	58.1	57.5
Public consumption	18.1	17.1	24.9	19.5	17.8	26.7	20.7	20.0
Gross fixed investment	25.4	18.5	20.1	25.9	21.6	19.7	21.9	22.5
Private fixed investment	21.1	15.5	17.0	22.6	18.3	16.7	18.6	..
Public fixed investment	4.3	3.0	3.1	3.2	3.3	3.0	3.3	..
Exports of goods and services	35.7	67.0	34.1	27.5	55.5	32.2	42.0	40.0
Imports of goods and services	36.7	69.2	35.2	29.2	54.1	33.6	43.0	40.0
Business sector capital stock	214.5	195.4	223.4	251.8	183.0	212.0	213.3	..
				In percent				
GDP growth rate	2.9	2.6	2.1	3.3	2.3	1.9	2.5	2.5
Standard deviation of GDP growth rate	1.9	2.0	2.1	2.1	1.7	1.4	1.9	1.9
Return to capital in business sector	11.5	12.5	10.0	10.0	14.7	9.7	11.4	..
Share of capital in business sector value added	32.5	32.5	32.9	33.5	33.8	30.3	32.6	..

Source: Calculations based on OECD Economic Outlook database.

1/ Initial steady state values.

Table 3. Key Parameters for the Reference Case

α	0.3	Share of capital in output
β	0.025	Average growth rate
γ_1	1.5	First order autocorrelation of productivity
γ_2	-0.5	Second order autocorrelation of productivity
ς_1	0.7	First order autocorrelation of employment
ς_2	0.0	Second order autocorrelation of employment
δ	0.05	Depreciation rate
η	1.5	Adjustment cost factor
θ_1	0.4	Investment composite share parameter
θ_2	1.0	Investment composite substitution parameter
λ	1.0	Household time horizon factor
χ	0.025	Propensity to consume out of wealth
π	1.0	Intertemporal substitution elasticity
ω_1	0.4	Consumption composite share parameter
ω_2	1.0	Consumption composite substitution parameter
σ_y	0.0185	Standard deviation of output growth rate
r	0.05	Real interest rate

Table 4. Time Series Properties and Volatility of Simulated Data 1/

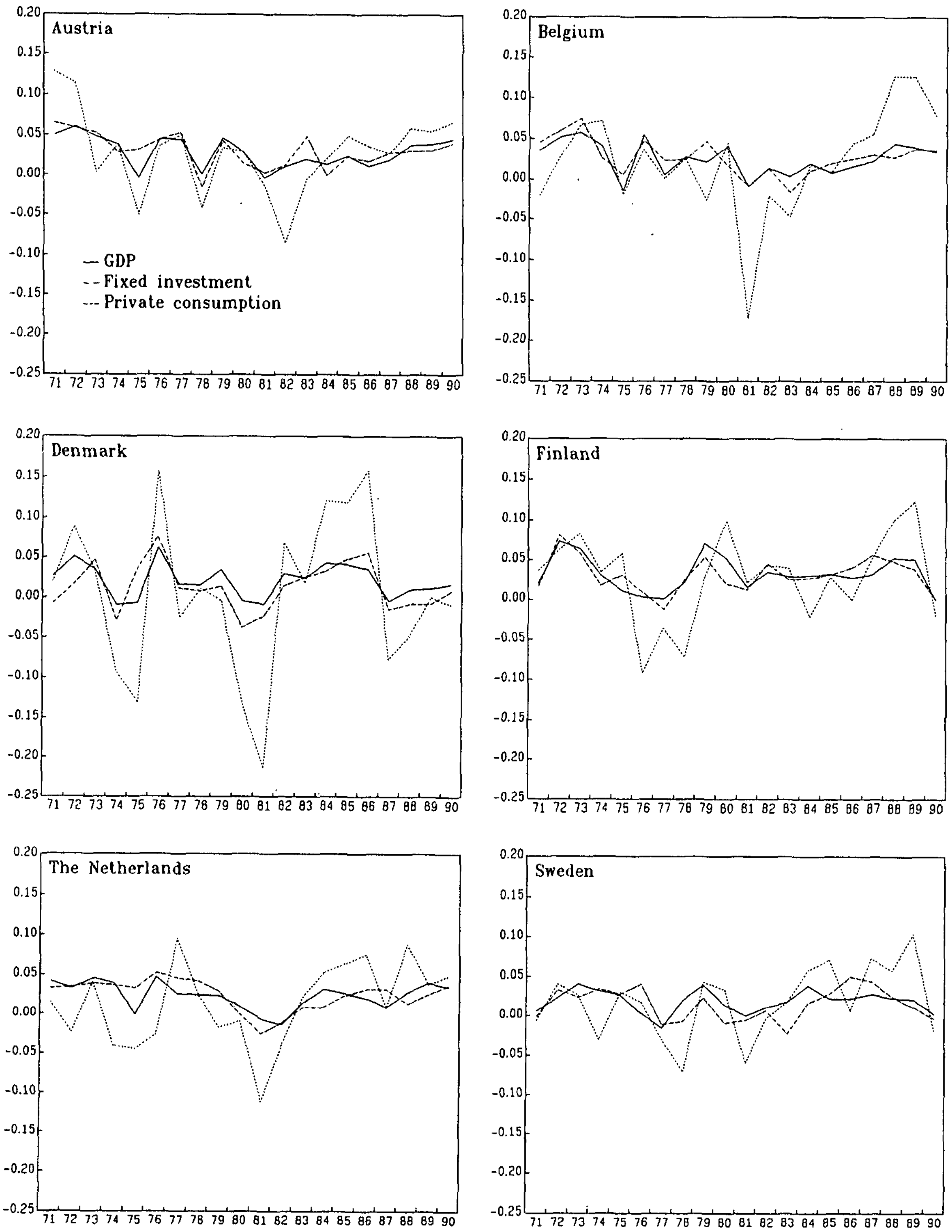
	γ_1=1.25	γ_1=1.50	γ_1=1.75
	Standard deviations of log differences relative to output		
Consumption			
ρ=0.00			
λ=1.00	1.02	1.44	2.30
λ=0.95	..	1.22	..
λ=0.90	..	1.09	..
λ=1.00			
ρ=0.10	..	1.30	..
ρ=0.20	..	1.18	..
ρ=0.30	..	1.06	..
20 percent current income consumers λ=1.00 ρ=0.00	..	1.18	..
Investment	2.39	3.15	4.21
Capital stock	0.31	0.43	0.67
Employment	0.62	0.60	0.50
	First order autocorrelation of changes in logs 2/		
Output	0.07 (0.01)	0.25 (0.01)	0.57 (0.01)
Consumption λ=1.00 ρ=0.00	-0.01 (0.01)	-0.00 (0.01)	0.00 (0.01)

1/ Rate of time preference adjusted to make model solution consistent with steady state path according to which consumption grows at the same rate as output. All other parameters as in reference case except as indicated.
2/ Least squares regressions based on 5000 observations. Standard deviations in parentheses.

Figure 1

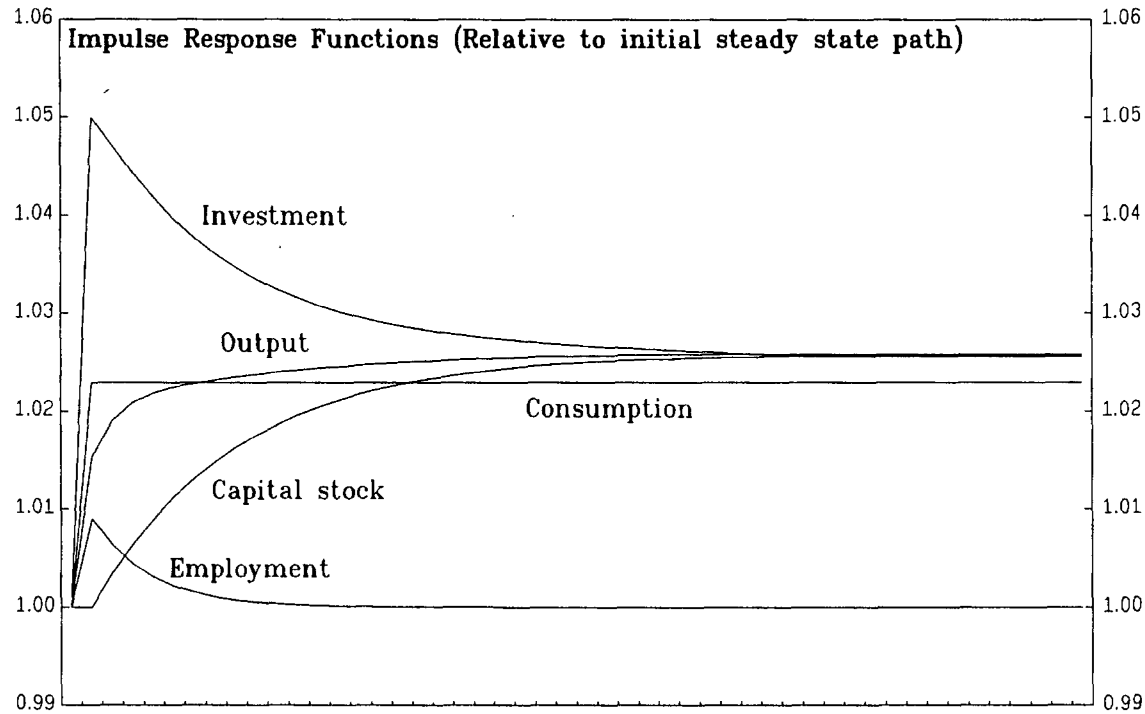
Output and Demand in Six Small Economies

(Changes in logs)



Source: OECD Economic Outlook database.

Figure 2
Response to Productivity Shocks



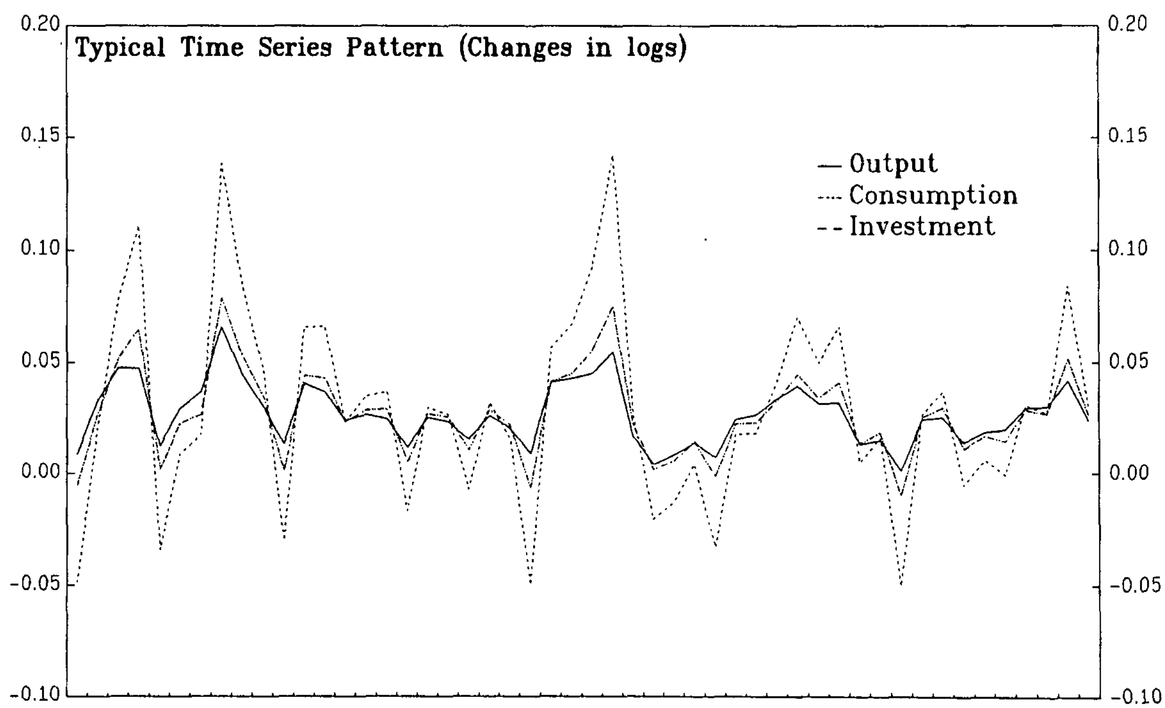


Figure 3
Consumption with Finite Horizons
(Percent differences relative to reference case consumption and output)

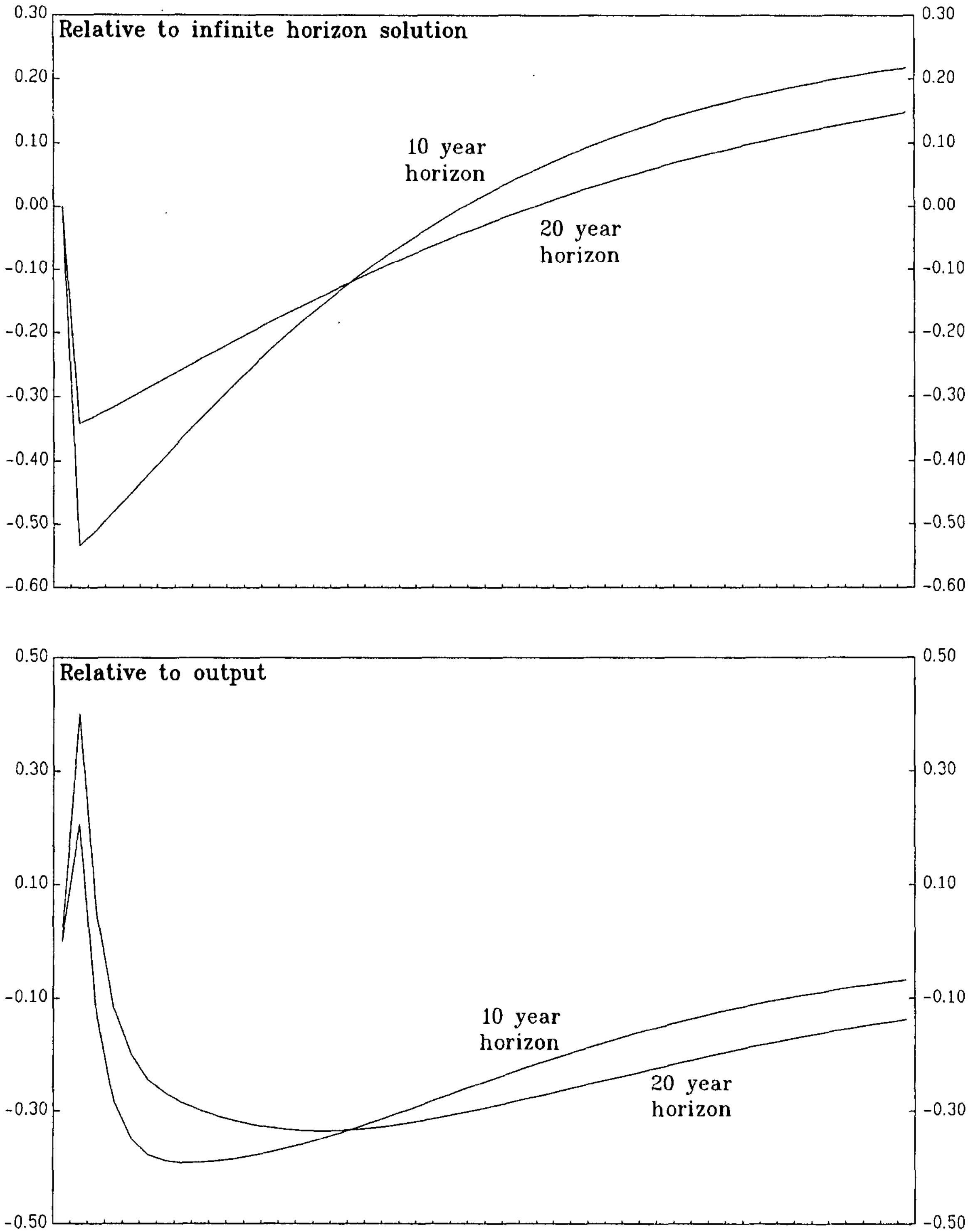
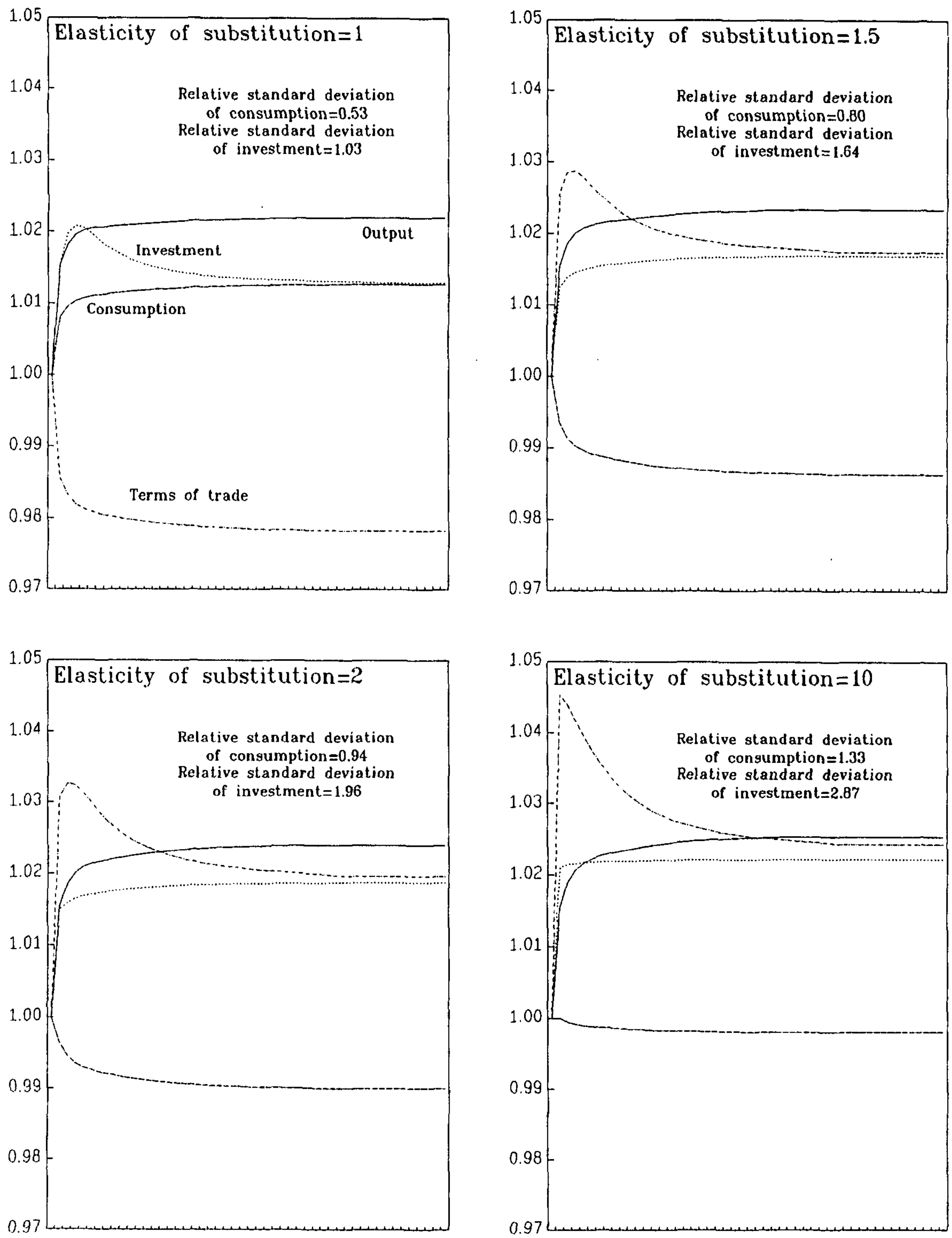


Figure 4
Productivity Shocks with Endogenous Terms of Trade
(Relative to initial steady state path)



Appendix on Solution Procedures and Derivation of Variances

Models based on intertemporal optimization can generally not be solved analytically. The basic problem is that some endogenous variables, like for example the shadow price of capital q, are conditioned on the entire future path of all variables in the model. There is also a number of predetermined variables like the capital stock K which depend on the past history of the forward-looking variables. While the initial values of the backward-looking variables are known, all that can be done initially with respect to the forward-looking variables is to impose certain terminal conditions. This poses a problem usually referred to as a two-point boundary value problem.

In the present model, the main problem is to find the exact path for the capital stock and the shadow price of capital consistent with optimizing the firm's objective function. Once this problem has been solved, the rest of the model can be solved recursively. The solution to the firm's decision problem is characterized by saddle-point instability. Initially small deviations from the optimal path result in a sequence that eventually diverges away from steady state. It can be shown, however, that a certain path for the shadow price of capital results in an optimal path for the capital stock which gradually approaches steady state. This path can only be approximated through iteration. The extended path method by Fair and Taylor (1983) has proved to be very useful in this regard. 1/ The basic steps involved in solving the model can be described as follows.

1. The first step is to calculate a steady state growth path solution which can be used to define terminal conditions for the forward-looking variables. Such a solution requires that all real variables grow at the same rate as the exogenous growth in productivity and that all relative prices are constant. Imposing these conditions yields a steady state value for the shadow price of capital q_S. With endogenous terms of trade, the steady state price of capital must be determined simultaneously with the price of output. It is difficult to do this analytically. However, the model can be iterated to yield a combination of the two prices consistent with a steady state growth path. 2/

1/ While the expanded path method remains popular, the increased demand for numerical solution methods for non-linear rational expectations models has stimulated rapid growth in alternative solution techniques. For a recent survey, see Taylor and Uhlig (1990).

2/ The model can be solved both with exogenous and endogenous terms of trade. The steps taken to solve the model are basically the same. However, with endogenous terms of trade, the price of the domestic good is determined by market-clearing. Generally, the market-clearing price can only be calculated through iteration. This makes the solution procedure somewhat more complicated and therefore more time-consuming than in the simple case with exogenous terms of trade.

2. The second step is to choose a terminal period T. The terminal period must be chosen so as to allow the model to be solved a sufficient number of periods beyond the actual simulation period. Guess an initial path $i=q_t, q_{t+1}, \ldots, q_T$ and assume that $q_T=q_S$. This yields a starting point for further iterations.

3. The third step is to solve the model in each period using the initial path and the starting values of all predetermined variables. With endogenous terms of trade, this includes finding the price which clears the market for the domestic good in each period. As already noted, this is by itself not an entirely straightforward exercise as it involves finding the market-clearing price through iteration.

4. Using the initial path to represent the expected next-period value of the shadow price of capital in each period, the fourth step is to calculate a new path $i+1=q_t, q_{t+1}, \ldots, q_T$ which replaces the old path and is used to represent expectations in the next iteration. Go back to the previous step and iterate until the difference between any two estimates for the same period of two consecutive iterations is sufficiently small to meet a convergence criterion.

5. The fifth step is to choose a new terminal period $T'>T$ and repeat the third and fourth steps until convergence is reached for the new extended simulation period. The solution period is extended until the difference between any two estimates for the same period of two consecutive solutions for the original solution period 0-T is small enough to meet a convergence criterion.

After a solution has been found, deriving the variance properties of each variable is simple. This involves calculating a set of moving average weights which describes each variable's dynamic response pattern in log differences relative to the underlying productivity process. For example, the variance of the change in productivity $\Delta Z_t = \ln Z_t - \ln Z_{t-1}$ is given by:

$$\sigma^2_{\Delta Z} = (1 + MA^2_{Z1} + MA^2_{Z2} + \ldots + MA^2_{ZT})\sigma^2_{\epsilon \Delta Z}$$

where $MA_{Z1}, MA_{Z2}, \ldots, MA_{ZT}$ are the moving average coefficients and $\sigma^2_{\epsilon \Delta Z} = \sigma^2_{\epsilon Z}$ is the variance error of ΔZ. Since the model can only be solved approximately, the weights are to some extent dependent on the size of the original shock. The size of the original shock $\varepsilon_{Zt} = \sigma_{\varepsilon Z}$ is therefore determined so that the transformed system of moving average weights reproduces the standard deviation of the output growth rate in the empirical data.

References

Backus, David K., Kehoe, Patrick J. and Finn E. Kydland, "International Real Business Cycles," Working Paper 426R (New York: New York University, Department of Economics, February 1991).

Baxter, Marianne and Mario J. Crucini, "Business Cycles and the Asset Structure of Foreign Trade," Discussion Paper No. 59 (Minneapolis: Federal Reserve Bank of Minneapolis, Institute for Empirical Macroeconomics, March 1992).

_____, and K. Geert Rouwenhorst, "Solving the Stochastic Growth Model by a Discrete-State-Space, Euler-Equation Approach," Journal of Business & Economic Statistics, Vol. 8, No. 1 (January 1990).

Bernanke, Ben S., "Adjustment Costs, Durables, and Aggregate Consumption," Journal of Monetary Economics, Vol. 15, No. 1 (January 1985).

Blanchard, Olivier J., "Debts, Deficits, and Finite Horizons," Journal of Political Economy, Vol. 93, No. 2 (March 1985).

Campbell, John Y. and N. Gregory Mankiw, "Consumption, Income and Interest Rates: Reinterpreting the Times Series Evidence," NBER Macroeconomics Annual, 1989 (Cambridge: National Bureau of Economic Research, 1989).

_____, "Permanent and Transitory Components in Macroeconomic Fluctuations," American Economic Review, Vol. 77, No. 2 (May 1987).

Cardia, Emmanuela, "The Dynamics of a Small Open Economy in Response to Monetary, Fiscal, and Productivity Shocks," Journal of Monetary Economics, Vol. 28, No. 3 (December 1991).

Christiano, Lawrence J., "Is Consumption Insufficiently Sensitive to Innovations in Income?" American Economic Review Papers and Proceedings, Vol. 77, No. 2 (May 1987).

_____, "Solving the Stochastic Growth Model by Linear-Quadratic Approximation and by Value Function Iteration", Journal of Business & Economic Statistics, Vol. 8, No. 1 (January 1990).

Deaton, Angus, "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?" NBER Working Paper No. 1910 (April 1986).

Fair, Ray C. and John B. Taylor, "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models," Econometrica, Vol. 51, No. 4 (July 1983).

Finn, Mary G., "On Savings and Investment in a Small Open Economy," Journal of International Economics, Vol. 29, No. 1-2 (August 1990).

Flavin, Marjorie, "The Adjustment of Consumption to Changing Expectations about Future Income," Journal of Political Economy, Vol. 89, No. 5 (October 1981).

_____, "Excess Sensitivity of Consumption to Current Income: Liquidity Constraints or Myopia?" NBER Working Paper No. 1341 (May 1984).

Hayashi, Fumio, "Tobin's Marginal q and Average q: A Neoclassical Interpretation," Econometrica, Vol. 50, No. 1 (January 1982).

_____, "Tests for Liquidity Constraints: A Critical Survey," NBER Working Paper No. 1720 (October 1985).

_____, "The Permanent Income Hypothesis and Consumption Durability: Analysis Based on Japanese Panel Data," Quarterly Journal of Economics, Vol. 100, No. 4 (November 1985).

Japelli, Tullio and Marco Pagano, "Consumption and Capital Market Imperfections: An International Comparison," American Economic Review, Vol. 79, No. 5 (December 1989).

Leiderman, Leonardo and Assaf Razin, "Current Account Dynamics: The Role of Real Shocks," Working Paper No. 89/80 (Washington: International Monetary Fund, September 1989).

McCallum, Bennett T., "Real Business Cycle Models," in Robert Barro ed., Modern Business Cycle Theory, (Cambridge MA: Harvard University Press, 1989).

Mendoza, Enrique G., "Real Business Cycles in a Small Open Economy," American Economic Review, Vol. 81, No. 4 (September 1991).

Nelson, Charles and Charles Plosser, "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications," Journal of Monetary Economics, Vol. 10, No. 2 (September 1982).

Plosser, Charles, "Understanding Real Business Cycles," Journal of Economic Perspectives, Vol. 3, No. 3 (Summer 1989).

Prescott, Edward C., "Theory Ahead of Business Cycle Measurement," Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 10, No. 4 (Fall 1986).

Stockman, Alan and Linda Tesar, "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," NBER Working Paper No. 3566 (December 1990).

Taylor, John B., and Harald Uhlig, "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods," Journal of Business & Economic Statistics, Vol. 8, No. 1 (January 1990).

Zeira, Joseph, "Risk and Capital Accumulation in a Small Open Economy," Quarterly Journal of Economics, Vol. 102, No. 2, (May 1987).