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Risk-Taking and Optimal Taxation with Nontradable Human Capital

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Abstract

What are the effects of taxation on individual/entrepreneurs' risk-taking behavior? This paper re-examines this old question in a continuous time life-cycle model. We demonstrate that the stream of uncertain income from human capital has systematic effects on demand for the risky physical capital asset. If labor supply is inelastic and real wages are known with certainty, then a labor income tax will reduce holdings of the risky physical asset. However, if there are random fluctuations in labor income, then the effect depends on the nature of interaction between wage risk and investment income risk. A labor income tax may actually raise demand for the risky capital asset if human capital risk and physical capital risk are positively correlated. The idiosyncratic risk and nontradability of human capital also have implications for optimal taxation. When the insurance and disincentive effects are jointly taken into account, a Pareto efficient tax structure implies a strictly positive tax rate.

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Summary

Although governments and the public generally take a favorable view of individual/entrepreneurial risk-taking activity, the question of how public policy such as taxation affects this risk-taking behavior continues to be debated.

This paper re-examines the question in a continuous-time life-cycle model. First, individuals' optimal consumption and portfolio rules in the case of two assets--one risk free, the other risky--are derived. Risk-taking can then be conveniently measured as demand for the risky asset or, alternatively, investment in the real production process. According to common assumptions about investor preference, individuals will hold a constant share of the risky physical asset all the time. When human capital is introduced, however, the optimal portfolio share of the risky asset will be age-dependent insofar as human capital varies over the life cycle. When labor supply is inelastic and real wages are known with certainty, a labor income tax reduces risk taking.

This conclusion will no longer hold true if there are random fluctuations in labor income. The paper demonstrates that the uncertain income from human capital has systematic effects on risk-taking behavior. The exact effects of a labor income tax will generally depend on the covariance of human capital risk and physical capital risk. Surprisingly, when the two are positively correlated, a labor income tax may actually encourage risk-taking, owing to investors' hedging demand.

Finally, the paper examines how the risk and nontradability of human capital can affect the optimal tax structure. If human capital risk is idiosyncratic--that is, if there is no aggregate shock--government taxation of labor income essentially provides insurance for individuals insofar as moral hazard causes a breakdown of private markets. When the insurance role of labor income taxation and its disincentive effects on labor supply (assuming labor supply is elastic, of course) are jointly taken into account, a Pareto efficient tax implies a strictly positive tax rate.

I. Introduction

Economists have long been concerned with the effects of taxation on individual/entrepreneurs' risk-taking behavior. Since the seminal essay by Domar and Musgrave (1944), such issues have been analyzed in a pure portfolio choice framework. See Tobin (1958), Mossin (1968), Feldstein (1969, 1976), Stiglitz (1969), Sandmo (1969) and Dreze and Modigliani (1972), among others. Presumably more risk taking leads to greater demand for risky assets and thus lowers the cost of risky capital. In the long run it helps to increase the economy's capital stock and national income.

While this literature has succeeded in dispelling a popular perception that taxation of return on risky assets will necessarily reduce risk taking, it suffers from some serious limitations. Chief among these is that the earlier work largely ignores some important and closely related decision problems such as individuals' saving-consumption and labor-leisure choices. Moreover, even if labor income, in addition to income from capital assets, is allowed, the pure portfolio choice model usually takes it as exogenously given (i.e., labor supply is fixed) or assumes that individuals receive a certain stream of wage income.

Ignoring labor income fails to capture the crucial role of human capital in individuals' life-cycle consumption and portfolio selection because human capital is often the more important form of wealth for most individuals. An extreme case is that in the absence of bequests the young are endowed with only human capital and no financial wealth.

To assume a certain stream of wage income is also clearly unsatisfactory because in reality individuals cannot foresee the flow of their future labor income with certainty. For example, real wages are uncertain when prices of consumption goods are uncertain. For some people the otherwise smooth, continuous flow of labor income may contain jump components over life cycle if they face a positive probability of temporary layoff or sudden loss of earning ability due to health problems. Thus, like its physical counterpart, human capital is essentially a risky asset. The risk of human capital income likely has important interactions with the risks of financial assets. Moreover, unlike physical capital, human capital cannot be traded due to obvious moral hazard reason. No claims on future wage income are actually traded in the real world financial markets. This feature of human capital will undoubtedly affect society's efficient risk pooling and risk sharing. Consequently, the analysis of effects of taxation on risk taking should explicitly take into account the riskiness and nontradability of human capital.

This paper recognizes the existence of risky, nontradable human capital income and attempts to offer an integrated treatment of individuals' intertemporal consumption and labor supply as well as portfolio choices. Within this context, the classic problem of effects of taxation on risk

taking is re-examined. Section II sets up the basic model and considers the simplest case with fixed labor supply and deterministic wage rate. Section III introduces endogenous labor income. Section IV analyzes the case when income from human capital follows geometric Brownian motion. Finally, in Section V, we explore the implications of the risk and nontradability of human capital for efficient taxation.

II. The Basic Model

We assume that the uncertainty in the economy is generated by a two-dimensional stochastic processes (dz, dq), where dz and dq are standard Brownian motions, and dz dq = η_{zq} dt, where η_{zq} is the instantaneous coefficient of correlation between dz and dq.

Individuals can have two investment opportunities in this economy. One is investment in a risk free asset, with an instantaneous rate of return, $r(t)$, and the other is investment in a risky capital asset, which may be viewed as a real production possibility. The consideration of this two-asset case proves convenient because it provides a well-defined measure of the degree of risk-taking, which is given by the portfolio share of the single risky asset. Moreover, under certain conditions (e.g., with constant consumption and investment opportunity set or logarithmic utility functions), the continuous-time version of Sharp-Lintner-Mossin Capital Asset Pricing Model (CAPM) obtains, and all individuals will hold the market portfolio, so that the many asset case can be simplified as if the market were the only risky asset. The price of this risky asset can be modeled as an Ito process:

$$dP = \alpha P dt + \sigma P dz \qquad (1)$$

where α and σ are the instantaneous mean and standard deviation of the rate of return on the risky asset. With constant α and σ, (1) implies that the price of the risky asset is log-normally distributed.

The flow of stochastic real wage income can be written as

$$dY = \mu_y Y dt + \sigma_y Y dq \qquad (2)$$

Consider an individual who lives for T years and whose preference is described by a time-additive, state independent utility function, $U(c(t), L(t))$. We assume that $U(c(t), L(t))$ exhibits the standard properties of monotonicity and concavity in consumption and leisure, that is, $U_c > 0$, $U_L > 0$, $U_{cc} < 0$ and $U_{LL} < 0$, where the subscripts on U denote the first-order

and second-order partial derivatives with respect to c(t) and L(t), respectively.

At each point in time during the life cycle, the risk averse individual chooses the rate of consumption, c(t), the current leisure, L(t), and a portfolio rule via which he can transfer his wealth over time and across the states of nature. 1/ Let $\omega(t)$ denote the proportion of wealth to be invested in the risky asset. The general problem of intertemporal optimization is

$$\max \ E_0 [\int_0^T \ U(C_t, L_t) \ dt \ ] \tag{3}$$

subject to a budget equation; where E_0 denotes the expectation conditional on the information available at date 0. With income derived form human capital, dI, the budget constraint can be written as a stochastic differential equation: 2/

$$d W = \{[\omega \ (\alpha - r) + r]W - c\} dt \ + \ dI \ + \ \omega W \sigma dz \tag{4}$$

We assume that labor income is subject to a proportional tax rate, r, with the government tax revenue being used to finance a public good that enters the utility function separately. If the individual consumes a fixed amount of leisure, $\bar{L}$, and wage flow is deterministic, that is, dY = Y(t) dt, then the stream of after-tax labor income is: dI = θ (1 - $\bar{L}$) Y(t) dt, where θ = (1 - r). Thus the problem of (3) reduces to the one with a single consumption good and exogenous, risk free labor income. Define the derived utility function of wealth J(W, t) = max $E_t \int_t^T$ U(c(t), L(t)) dt, the Bellman equation is:

$$0 = \frac{\max}{c,\omega}(U(c(t),\bar{L}) + J_t + J_W \{[\omega(\alpha-r)+r]W + \theta(1-\bar{L})Y - c\} dt + \frac{1}{2} J_{WW}\omega^2 W^2 \tag{5}$$

1/ We abstract from the individual's retirement decision in this paper so that the individual has an active working life until the termination date T. The retirement problem has been extensively studied in the public finance and labor economics literature, see Diamond and Mirrlees (1978) and many others. We can similarly explore the problem of retirement with uncertainty in our continuous time framework.

2/ See Merton (1971) for a detailed derivation.

we can write the first order conditions for this problem:

$$U_c = J_W \tag{6}$$

$$0 = J_W(\alpha - r) + J_{WW} \, \omega \, W \, \sigma^2 \tag{7}$$

where subscripts on U and J denote partial derivatives with respect to c, W and t, respectively.

From (7) we have the share of the portfolio in the risky asset:

$$\omega^* = -\frac{J_W}{W J_{WW}} \frac{\alpha - r}{\sigma^2} \tag{8}$$

In general, the optimal proportion in the risky asset (the market portfolio in our case) depends on the investor's risk preference, wealth endowment and age as well as the parameters of the returns on traded assets, α, r and σ^2. The individual's investment in the risky asset is determined by his desire to attain a preferred risk-return tradeoff in wealth. The more risk averse he is (i.e., the smaller the term $- J_W / (W J_{WW})$ is), the smaller his portfolio share in the risky asset will be. On the other hand, the larger the premium on the market, $\alpha - r$, or the smaller the market risk, σ^2, the larger the proportion of his wealth he will hold in the risky asset.

Note that a labor income tax affects the optimal portfolio demand by changing individuals' risk aversion behavior and total wealth. To see this, let us consider the case of utility function with constant relative risk aversion (CRRA) of consumption. 1/ Merton (1971) first derived explicit consumption and portfolio rules for the CRRA utility function:

$$\omega^* W = \frac{\alpha - r}{\sigma^2 \delta} (W + \frac{\theta (1 - \bar{L}) \, Y \, \{1 - \exp[r(t-T)]\}}{r}) \tag{9}$$

where δ is the reciprocal of the relative risk aversion of consumption let W_H denote the value of human capital, then

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1/ The constant relative risk aversion (CRRA) or isoelastic utility function belongs to the family of hyperbolic absolute risk-aversion utility functions.

$$W_H = \int_t^T Y_t \exp[-r(s - t)]ds = \frac{Y\{1 - \exp[r(t - T)]\}}{r} \tag{10}$$

that is, the value of human capital is the present value of the lifetime flow of maximal labor income discounted at the risk free market rate of interest. The crucial point here is that although human capital is not directly traded the individual can in effect sell his risk free future labor income for present consumption by trading (shortselling) financial assets. Then, we can write (9) as

$$\omega^* W = \frac{\alpha - r}{\sigma^2 \delta} (W + \theta(1 - \bar{L})W_H) \tag{11}$$

If there is only investment income (i.e., $W_H = 0$), the individual will wish to maintain a constant share of the risky asset, ω^*, in his portfolio over the life cycle. It implies that the demand function for the risky asset is linear in the level of wealth. We thus obtain the usual separation theorem which states that ω^* should be independent of the investor's wealth level and age for the CRRA utility function. If the individual also receives labor income, he will treat his net worth of human capital as an addition to the current stock of wealth and his portfolio share of the risky asset will be age-dependent. He is willing to take more risk by investing more in the risky asset when he is young since the value of his human capital is the greatest in the early stage of his life. When he gets older, his ratio of W_H to W will decline and he will thus take smaller position in the risky asset.

A labor income tax reduces the value of his human capital and, therefore, decreases his holding of the risky asset. Fiscal policy is not neutral with respect to the individual's risk-taking behavior. A current tax cut matched by a future tax increase tends to encourage investment in the risky asset. Since the Government does not share the investment risk faced by private agents through a labor income tax, total (social) risk taking must also be reduced.

III. Interaction Between Labor Supply and Portfolio Choice

In this section, we introduce labor-leisure choice to the individual's decision problem but continue to assume that income from human capital is nonstochastic.

The individual's problem now is

$$\max \; E_0 \int_0^T U(c_t, Lsubt) \; dt \tag{12}$$

$$s.t. \qquad dW = \{[\omega(\alpha - r) + r] + \theta(1 - L)Y - c\} \, dt + \sigma \omega W \, dz$$

From the Bellman equation:

$$0 = \max \; (U(c, L) + J_t + J_W\{[\omega(\alpha - r) + r]W + \theta(1 - L)Y - c\} + \frac{1}{2}J_{WW}\sigma^2\omega^2W^2) \tag{13}$$

we have the first order conditions:

$$U_c = J_W \tag{14}$$

$$U_L = \theta \, Y \, J_W \tag{15}$$

Combining (14) and (15) gives

$$U_L = \theta \, Y \, U_c \tag{16}$$

Differentiate (16) with respect to τ and assume that U is separable in c and L, we have:

$$\frac{dL}{d\tau} = - \frac{Y U_c}{U_{LL} - \theta Y U_{cc} \frac{\partial c}{\partial L}} > 0 \tag{17}$$

Thus an uncompensated increase in the tax on riskless labor income unambiguously discourages the individual's labor supply when his preference is separable in the consumption good and leisure.

The optimal demand for the risky asset given by the first order conditions takes the same form as (8). To derive a closed form solution, we use a two-stage procedure. First, we solve the following static

optimization problem at each t:

$$\max \quad V(c(t), L(t)) \, dt \tag{18}$$

$$s.t. \quad C = c + \theta Y L$$

Define $U(C, t) = V(c^*(C,t), L^*(C,t))$. Because real wages are nonstochastic, there is no intertemporal uncertainty in the price of leisure relative to the consumption good. Therefore $U(C, t)$ remains state-independent. We then proceed to solve the following dynamic optimization problem:

$$\max \quad E_0 \int_0^T U(C, t) \, dt \tag{19}$$

$$s.t. \quad dW = \{[\omega(\alpha - r) + r]W + \theta Y - C\} \, dt + \omega W \sigma \, dz$$

Note that the leisure variable, $L(t)$, no longer enters the budget constraint in (19). The problem now is formally identical to the one with a single consumption good. For CRRA utility function, the optimal portfolio rule has the familiar form:

$$\omega^{**} W = \frac{\alpha - r}{\sigma^2 \delta} (W + \theta W_H) \tag{20}$$

where W_H is given by (10).

Note that $\omega^{**} > \omega^*$. For the same ratio of human wealth to current wealth, the individual with elastic labor supply is more risk taking than when his labor supply is fixed at $(1 - \bar{L})$. Intuitively, the ability to adjust labor supply conditional on his investment performance provides insurance for his investment risk. The effect of a proportional labor income tax, however, is to reduce private (and therefore social) risk-taking, as in the case with fixed labor supply.

IV. Uncertain Income from Human Capital

The presence of risky income from human capital complicates our problem in two ways. First, it introduces uncertainty in the relative price of leisure (denominated in the consumption good). As a result, the utility function becomes state dependent, with the state variable represented by the wage variable, i.e., $U = U(C, Y, t)$, where C is as in (18) redefined as the consumer's aggregate expenditure, inclusive of the value of leisure measured in terms of the consumption good. Second, the presence of risky labor income has a wealth effect. It affects the dynamics of wealth accumulation,

so that both the drift and the diffusion terms of the investor's budget constraint are modified.

We assume that the stochastic behavior of wage income is as specified in (2), that is, the wage income follows the geometric Brownian motion. 1/ Formally, we have the stochastic dynamic programming problem:

$$\max \quad E_0 \int_0^T U(C(t), Y(t), t) \, dt \tag{21}$$

$$s.t. \quad dW = \{[\omega(\alpha - r) + r] W + \theta Y \mu_y - C\} \, dt + \sigma \omega W dz + \theta Y \sigma_y dq$$

from the Bellman equation:

$$0 = \max(U(C,Y,t) + J_t + J_W\{[\omega(\alpha - r) + r] W + \theta Y \mu_y - C\} + J_Y \theta Y \mu_y$$

$$+ \frac{1}{2} J_{WW}(\omega^2 \sigma^2 W^2 + 2\omega W \theta Y \sigma_{zq} + \theta^2 Y^2 \sigma_y^2) + J_{YW} \omega W \theta Y \sigma_{zq} + \frac{1}{2} J_{YY} \theta^2 Y^2 \sigma_y^2) \tag{22}$$

where $\sigma_{zq} = \sigma \sigma_y \eta_{zq}$ is the covariance between the return on the risky asset and income from human capital, we have the first order conditions:

$$U(C^*, Y, t) = J_W(W, Y, t) \tag{23}$$

$$J_W(\alpha - r) + J_{WW} \theta Y \sigma_{zq} + J_{WW} \omega^* \sigma^2 W + J_{YW} \theta Y \sigma_{zq} = 0 \tag{24}$$

We can now derive the optimal demand for the risky asset:

$$\omega^* W = (-\frac{J_W}{J_{WW}}) \frac{\alpha - r}{\sigma^2} - \theta Y \frac{\sigma_{zq}}{\sigma^2} - \frac{J_{YW}}{J_{WW}} \theta Y \frac{\sigma_{zq}}{\sigma^2} \tag{25}$$

The optimal demand for the risky asset consists of three terms. The first term , $(-J_W/J_{WW})(\alpha-r)/\sigma^2$, is the usual speculative demand for the

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1/ Merton (1971) solved a problem in which the wage income is given exogenously and is a Poisson process.

mean-variance maximizer. 1/ The second term, $-\Theta Y \sigma_{zq}/\sigma^2$, is a labor income hedging demand. The last term, $-\theta Y (J_{YW}/J_{WW})(\sigma_{zq}/\sigma^2)$, is the state variable hedging demand. These hedging terms arise from the nontradability of risky human capital. When the individual has human capital income as well as financial investment income, the risk of human capital income will play a role in determining his optimal portfolio behavior. In other words, the interaction between the human capital risk and the financial risk will affect the individual's optimal holding of the risky asset. Comparing to the case with riskless labor income in the previous sections, we can see that now in addition to holding the risky market portfolio to attain the desired risk-return tradeoff in wealth, the individual also uses the risky market portfolio to hedge against the unanticipated and possibly unfavorable changes in his labor income.

From the first-order condition (23), we have:

$$U_{CC} \, \partial C/\partial W = J_{WW} \tag{26}$$

$$U_{CC} \, \partial C/\partial Y + U_{CY} = J_{WY} \tag{27}$$

If $U_{CY} = 0$, that is, the direct utility function is state independent, then $J_{WY}/J_{WW} = (\partial C/\partial Y)/(\partial C/\partial W) > 0$, because the propensities to consume out of income and wealth are positive. Then the signs of both hedging terms depend on the sign of the covariances between the return on the risky asset and wage income, σ_{zq}. If the return on the risky asset and wage income are negatively correlated, that is, $\eta_{zq} < 0$, the existence of the risky human capital income produces a positive hedging effect on the demand for the risky asset. The same observation was made by Fischer (1975) in his exposition on demand for indexed bonds with state-independent utility function.

Since

$$\frac{\partial(\omega^* W)}{\partial \tau} = \frac{Y}{\sigma^2} \left(1 + \frac{J_{YW}}{J_{WW}} \right) \sigma_{zq} \tag{28}$$

1/ With continuous trading, the risk premium on the traded asset, $\alpha - r$, is determined by Breeden's (1979) consumption β model even with the existence of nontraded human capital, as long as both the asset prices and the consumption rate follow the Ito processes, as shown by Grossman and Shiller (1982).

the effects of labor income taxation on risk taking will in general depend on the covariance of human capital risk and physical capital risk. The standard result with riskless real wage income no longer carries over to the more general case when income stream from human capital is uncertain.

If wages and return from investment in physical capital are negatively correlated, then a proportional labor income tax will reduce the hedging demand for the risky asset. Because part of the labor income risk is now shifted to the government 1/, the individual then holds less the risky asset to hedge the risk of nontradable human capital.

If the instantaneous rate of return on the risky financial asset is positively correlated with the unanticipated changes in wage income, i.e., $\eta_{zq} > 0$, then the individual's hedging demands for the risky asset are negative, that is, reverse hedging occurs. A labor income tax actually will encourage risk taking in this case.

If the return on the risky asset and wage income are uncorrelated, $\eta_{zq} = 0$, then both the hedging terms in (25) will vanish. In this case, the individual's labor income risk is idiosyncratic. Holding the risky financial asset cannot provide insurance for the unanticipated changes in his labor income. Although the idiosyncratic labor income risk will affect the individual's risk aversion and consumption behavior, it does not affect his demand for the risky asset 2/. Therefore the hedging demand for the risky asset will be zero. In this case the only mechanism for sharing and pooling individuals' idiosyncratic labor income risk is the government's taxation on labor income.

Which of these possibilities is empirically more relevant remains a subject for further research. Fama and Schwert (1977) studied the role of human capital in the classical CAPM model. They found that the relationships between the return on human capital and the returns on various portfolios of traded assets are weak, so that the presence of human capital do not significantly affect the measurement of risk for traded capital assets. However as the authors themselves acknowledged their results cannot be viewed as definitive due to a number of measurement and estimation problems.

In addition to these hedging effects, the existence of labor income, of course, also produces a wealth effect. The risky stream of labor income, when appropriately capitalized, is treated as part of wealth in the consumption and portfolio demand functions.

1/ Implicitly we assume that the tax code contains full loss offset provisions.

2/ Although the idiosyncratic labor income risk per se does not affect the demand for the risky asset, the presence of human capital and labor income taxation will still produce a wealth effect and the financial risk-taking will be affected.

V. Optimal Taxation with Nontradable Risky Labor Income

Our analysis above points to the importance of the interaction between the risks of financial assets and of the income from human capital. The individual will invest more of his wealth in the risky capital asset if the return on the asset is negatively correlated with his labor income. In effect, the financial market can provide insurance for the labor income risk. This interaction between the individual's nonhuman capital risk and human capital risk will, in addition to its effects on portfolio choice and risk-taking behavior, have implications for efficient income taxation, to which we now turn our discussion.

Because human capital is nontradable, it is clearly welfare improving to have some kind of social schemes to pool and share the human capital risk, when the private market fails to provide insurance instruments to hedge against the uncertain stream of labor income. The government, through its ability to tax, can precisely play such a role in diversifying the idiosyncratic human capital risk. Indeed, it is tempting to suggest imposing a 100 percent wage tax combined with a lump-sum transfer. In reality this type of public insurance program is unlikely to work because of its disincentive effect on labor supply. Therefore we need jointly consider the insurance and efficiency aspects of labor income taxation.

We assume that there are a large number of individuals in the economy who are identical in preferences, beliefs, initial endowments and abilities, so that variations in labor income arise only from differences in "luck" that different individuals may have. 1/ In other words, the random fluctuations in labor income are caused by "idiosyncratic" risks that are uncorrelated between individuals. Formally, for each individual i, the uncertain stream of wage income is represented by (2), with $dq_i \, dz = \eta_{qz} \, dt$, $\forall$ i and $dq_i \, dq_j = 0$, $\forall$ i $\neq$ j.

Consider a representative consumer who chooses his optimal consumption, labor supply and portfolio share at each point in time, taking the tax rate on his labor income, r, as given. The individual essentially faces the

following problem:

We can write the first-order conditions with respect to c, L and ω:

1/ Stiglitz (1982) examined the problem of Pareto efficient taxation when individuals differ in their productivities and the government has imperfect information about the "true" distribution of abilities across people. Our assumption here is similar to that employed by Barsky, Mankiw, and Zeldes (1986) who examined the interaction between individual income uncertainty and income taxation in the face of a debt-financed tax cut.

$$\max \quad E_0 \int_0^T U(c(t),L(t)) \, dt \tag{29}$$

$$S.t. \ dW = \{[\omega(\alpha - r) + r]W + \theta(1 - L)Y\mu_y - c\}dt + \omega W\sigma dz + \theta(1 - L)Y\sigma_y dq$$

$$U_c = J_W \tag{30}$$

$$U_L = \theta Y\mu_y J_W + [\omega W\theta Y\sigma_{zq} + \theta^2 Y^2(1 - L)\sigma_y^2] J_{WW} \tag{31}$$

$$\omega W = -\frac{\alpha - r}{\sigma^2} \frac{J_W}{J_{WW}} - \theta Y(1 - L) \frac{\sigma_{zq}}{\sigma^2} \tag{32}$$

where $\Theta = 1 - r$, $J = J(W, t)$ is the derived utility function of wealth, and $\sigma_{zq} = \sigma \sigma_y \eta_{zq}$ is the covariance between the individual's financial risk and human capital risk.

How might the uncertainty about future wage income affect the individual's labor supply decision? To see this let us consider some simple cases. Substituting (32) into the first-order condition (31) we have:

$$U_L = \theta Y[\mu_y - \frac{\sigma_{zq}}{\sigma^2}(\alpha - r)] J_W + \theta^2 Y^2(1 - L^*)\sigma_y^2(1 - \eta_{zq}^2) J_{WW} \tag{33}$$

Suppose that the individual's utility function is additively separable in the consumption good and leisure. If the individual's human capital risk and financial risk are perfectly correlated, that is, $|\eta_{zq}| = 1$, then differentiating (33) with respect to r will give us:

$$U_{LL} \frac{\partial L^*}{\partial r} = -YJ_W[\mu_y - \frac{\sigma_{zq}}{\sigma^2}(\alpha - r)] \tag{34}$$

Note that the term in the square bracket is positive. To see why, imagine that we have a traded asset that replicates the income from a unit of human capital. At equilibrium this traded asset is priced by the continuous-time CAPM:

$$\mu_y - r = \frac{\sigma_{zq}}{\sigma^2} \, (\, \alpha - r \,) \qquad\qquad (35)$$

where μ_y is the instantaneous rate of return on the traded asset. It is then easy to see that the square-bracketed term in (34) is positive when the rate of return on the riskless asset $r > 0$. According to our standard assumptions about the utility function $U(c(t), L(t))$, we establish from (34) that $\partial L^* / \partial r > 0$. Therefore just as in the case of risk free labor income in Section III, an increase in proportional wage tax will unambiguously reduce labor supply when the labor income risk and financial investment income risk are perfectly correlated.

If, as a polar case, the human capital risk and the financial risk are uncorrelated, that is, $\eta_{zq} = 0$, then differentiating (33) with respect to r ill give us:

$$(U_{LL} + \theta^2 Y^2 \sigma_y^2 J_{WW}) \, \frac{\partial L^*}{\partial r} = - Y J_W [\mu_y - 2\theta \, \sigma_y^2 \, \frac{Y(1 - L^*)}{W} \, (\, - \frac{W J_{WW}}{J_W})] \qquad (36)$$

Equation (36) clearly shows that there exists the possibility of a positive response of labor supply to a proportional wage tax increase. The greater the nondiversifiable labor income risk, the larger the share of current labor income in wealth and more risk-averse the individual is, then more likely the right-hand side of (36) will turn into positive and the individual will increase his labor supply when he faces higher wage tax. We have this surprising result because the government income taxation provides insurance for the individual's labor income risk.

Suppose that the government chooses a linear income tax schedule. At each point in time, the government taxes each individual's labor income at the proportional rate r and makes a lump-sum income transfer $dS(t)$ to each individual. The government faces the following intertemporal budget constraint

$$dS = r \, (1 - L) E_t (dY) - dG = r (1 - L) Y \mu_y dt - dG \qquad\qquad (37)$$

where dG is the government revenue requirement. The government wishes to maximize the representative consumer's life-time utility given its budget constraint (37) so as to determine the optimal tax rate r^*. That is:

From the Bellman equation we have the first-order condition with respect to r:

$$\max \quad E_0 \int_0^T U(C(t), L(t)) \, dt \tag{38}$$

$$S.t. \, dW = \{[\omega(\alpha-r)+r]W+\theta(1-L)Y\mu_y-c\} \, dt+dS \, +\omega W\sigma dz+\theta(1-L)Y\sigma_y dq$$

$$U_L \frac{\partial L^*}{\partial \tau} = \{Y\mu_y J_W + \theta Y[\omega W\sigma_{zq} + \theta Y(1-L^*)\sigma_y^2]J_{WW}\} \frac{\partial L^*}{\partial \tau} \tag{39}$$

$$+ Y(1-L^*)[\omega W\sigma_{zq} + \theta Y(1-L^*)\sigma_y^2] J_{WW}$$

The other two first-order conditions with respect to c and ω are not presented here because they have the same forms as (30) and (32).

Using these first-order conditions from the individual's maximization problem, we can, after some manipulations, obtain

$$\tau Y\mu_y J_W \frac{\partial L^*}{\partial \tau} + \theta Y^2(1-L^*)^2\sigma_y^2(1-\eta_{zq}^2) \, J_{WW} - \frac{\alpha-r}{\sigma^2}\sigma_{zq}Y(1-L^*)J_W = 0 \tag{40}$$

Let $F(\tau)$ denote the left-hand side of (40). Obviously F depends on the interaction between the labor income risk and financial risk (η_{zq}) as well as the labor income risk itself (σ_y^2).

Suppose that individuals' labor income is risk free, i.e., $\sigma_y^2 = 0$, then $F(0) = 0$. That is to say, the optimal taxation structure implies a zero wage tax rate if labor supply is elastic and if individuals do not face human capital risk. This is just due to the usual reason that, under certainty, we can have lump-sum taxation when all individuals are identical.

In general, a zero wage tax cannot be optimal. Consider the case when the human capital risk and financial risk are uncorrelated, i.e., $\eta_{zq} = 0$. The expression of F becomes

$$F(\tau^*) = \tau^* Y\mu_y J_W \frac{\partial L^*}{\partial \tau} + \theta Y^2(1-L^*)^2\sigma_y^2 J_{WW} \tag{41}$$

Because evaluating F at $\tau = 0$ gives

$$F(0) = Y^2(1 - L^*)^2 \sigma_y^2 J_{WW} < 0 \tag{42}$$

the first-order condition cannot be satisfied at zero tax rate. In fact, if we set $F(\tau) = 0$, then (40) yields

$$\tau^* = \frac{(-Y^2)(1 - L^*)^2 \sigma_y^2 J_{WW}}{Y \mu_y J_W \frac{\partial L^*}{\partial \tau} - Y^2(1 - L^*)^2 \sigma_y^2 J_{WW}} \tag{43}$$

Therefore, if labor supply responds negatively to wage tax changes, i.e., $^*/\partial \tau > 0$, the optimal marginal tax rate should lie strictly between 0 and 1.

For the case of perfect correlation, it is straightforward to show that $0 < \tau^* < 1$ if $\eta_{zq} = +1$, and it may be optimal to have a wage subsidy, that is, $\tau^* < 0$ if $\eta_{zq} = -1$. Therefore, even when individuals can diversify the risk of human capital by holding risky financial capital asset, the optimal tax on their labor income is usually not zero.

These results are closely parallel to those obtained by Eaton and Rosen (1980), who used a static model of uncertainty with endogenous labor supply. 1/ They did not consider the interaction between investment risk and wage risk because the non-labor income in their paper is assumed to be sure.

VI. Conclusion

We have re-examined the effects of taxation on risk-taking and labor supply in a continuous time life-cycle model. We show that the existence of income from human capital has systematic effects on individuals' optimal portfolio choice, and that the risk and nontradability of human capital has implications for efficient income tax structure. Further work, especially empirical study, is needed in order to better understand how taxation affects individuals' lifetime saving and portfolio decisions.

1/ Varian (1980) also examined the social insurance aspect of redistributive taxation in a static model of uncertainty. He did not explicitly consider the incentive effect of taxation on labor supply.

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