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The Implications of Cross-Border Monetary Aggregation

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Abstract

Some recent studies suggest the possibility of estimating a stable aggregate demand-for-money relationship for the group of countries participating in the European Monetary System. These results are of particular relevance in connection with the task of setting policy targets for a European Central Bank. This paper uses a theoretical error-in-variables framework to identify what is gained and what may be lost through cross-border aggregation of money demand. It provides an analytical basis for such studies, paying particular attention to currency substitution and international portfolio diversification.

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### Summary

Some recent studies suggest the possibility of estimating a stable aggregate demand-for-money relationship for the group of countries participating in the exchange rate mechanism of the European Monetary System (EMS), which would facilitate the setting of policy targets for a European Central Bank.

This paper examines, within a theoretical framework, the implications of using data aggregated across countries to study money demand. Two sources of bias in estimates are considered: aggregation bias occurs to the extent that different countries in the group have different money-demand relationships, while specification bias occurs to the extent that there are omitted variables, errors in measurement of the explanatory variables, or other specification errors.

In the EMS, the possibility of currency substitution--and of international portfolio substitution more generally--may lead to specification bias in single-country money-demand estimates. Currency substitution means that residents of each country hold more than one country's money, so that the demand for each country's money depends on the incomes and interest rates of other countries as well as its own. International portfolio substitution means that each country's residents consider foreign assets among the alternatives to holding money, so that a properly specified money-demand equation would include foreign as well as domestic asset yields as opportunity-cost variables.

The paper demonstrates how these specification errors may bias single-country money-demand estimates, especially by giving the impression of unduly slow adjustment of money balances toward their desired levels. The way in which cross-border aggregation may reduce specification bias at the cost of introducing some aggregation bias is also examined. While single-country money-demand estimates ignore the effects of currency substitution, cross-country aggregate estimates internalize this effect. Moreover, the analysis suggests that such estimates may support the view that currency substitution may be an important consideration in the stages leading up to European Monetary Union.



## I. Introduction

The European Community (EC) has recently begun to move decisively toward Economic and Monetary Union (EMU). The establishment of irrevocably fixed intra-EMU exchange rates, a common currency (the ECU), and the abolition of capital controls within the EC implies that there must also be a common monetary policy. Investigating monetary relationships at a Community-wide level has therefore become increasingly important, as it will be on the basis of these relationships that a European Central Bank (ECB) would eventually set policy. Aggregate monetary relationships are also important during the transition to EMU, when policy is to be implemented by the individual national central banks but coordinated through the European Monetary Institute (EMI).

One issue that has emerged in this connection is therefore the need to identify a stable money-demand relationship for the EMU-area as a whole, and the possibility that under some conditions this relationship might be more stable and predictable than similar relationships at the national level. Under the European Monetary System (EMS), this has not been a particularly pressing issue, given the asymmetrical arrangements that have prevailed: within the EMS, Germany has based its monetary policy on money supply targets relating only to Germany, while the other member countries have sought to stabilize their currencies' values against the deutsche mark (Giavazzi and Giovannini, 1989). Under these conditions, it is mainly the stability of German demand for money that has been important. The transition toward EMU is intended to be a transition toward symmetrical monetary arrangements, however, so the stability of demand for money in other Community countries, or in the EMU-area in the aggregate, figures importantly in determining what targets the EMI and later the ECB, could pursue (Kremers and Lane, 1992(b)).

The stability of the demand for money in the individual EC member countries is in question during the transition to EMU, because of the possibility that the increasing degree of economic and financial integration in the EC will increase the extent of international portfolio substitution in general, and currency substitution in particular. In the extreme case, different EC member countries' moneys would become perfect substitutes, and it would be impossible to predict demand for any individual country's money--only for money in the area as a whole. If this happened, it would be impossible to conduct monetary policy in any single country, but only at an EC-wide level. Short of perfect currency substitutability, if money demand were more predictable for the area as a whole than for individual countries, this would strengthen the case for implementing monetary policy at the Community level (Russo and Tullio, 1988). There is indeed evidence that instability in the demand for money in some of the member countries may be increasing--although in the case of Germany, this effect cannot yet be disentangled from the consequences of German reunification.

There have already been some empirical studies of the demand for money aggregated over the countries participating in the Exchange Rate Mechanism

(ERM) of the EMS. 1/ Bekx and Tullio (1989) presented estimates of demand for money aggregated across EMS countries for the period 1978-III to 1986-IV. They included the possibility of currency substitution vis à vis the US dollar, as reflected in an influence of uncovered interest rate differentials on money demand. Their results were consistent with the hypothesis that the exchange rate of the deutsche mark vis à vis the US dollar is better explained by supply and demand for money in the EMS as a whole than in Germany alone. 2/

In our own paper (Kremers and Lane (1990)), we explored aggregate demand for narrow money (M1) in the EMS over the 1979 to 1987 period, using a cointegration and error-correction framework that takes account of the non-stationarity of the relevant variables. We found that the long-run demand for real narrow money in the ERM could be expressed as a stable function of a limited number of variables: ERM-wide real income, inflation, long-term interest rates, and the exchange rate of the ECU vis-à-vis the U.S. dollar (where the latter variable may reflect currency substitution). The short-run adjustment of real money demand depended on changes in real income, changes in interest rates, and deviations of the real ERM money stock from its long-run desired position. An important feature of the results was the relatively rapid elimination of any such disequilibria; this contrasts with most earlier econometric work on individual countries, which tends to find implausibly slow adjustment. The model satisfies a broad set of diagnostic tests for possible misspecification, and arguably fits the data at least as well as money demand functions estimated for individual EMS countries. 3/

Some subsequent empirical work has tended to support our conclusions. Artis (1991) estimated a similar money demand relationship, using a slightly different method of aggregation, and found results that were broadly similar. 4/ Bomhoff (1991) reexamined the data using Kalman filter techniques, and confirmed our stability results. Monticelli and Strauss-Kahn (1991), in a study for the Committee of Governors of EC Central Banks, used an error-correction approach to examine aggregate demand for broad money (M3), finding a specification that was stable over the EMS period. Clearly, there is a growing body of evidence suggesting that it may be useful to estimate aggregate relationships for the EMS as a whole--and perhaps that these relationships may be an important guide to monetary policy during the transition to EMU.

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1/ There is an earlier literature on aggregate demand for money in the world economy, including papers by Gray, Ward, and Zis (1976), McKinnon (1982), and Spinelli (1983).

2/ It was later shown that the money demand equation estimated by Bekx and Tullio was unstable over the sample period; see Kremers and Lane, 1990.

3/ Compare for example with Buscher (1984), Atkinson et al. (1985), Heri (1985), von Hagen and Neumann (1988).

4/ Some of the issues involved in alternative methods of aggregation in this context are discussed in Kremers and Lane (1992(a)).

In evaluating this growing body of evidence on the aggregate demand for money in the EMS, it is important to have a clear idea of what can be gained and what may be lost in aggregating across a group of countries. In this paper, we use a classic errors-in-variables framework to consider this issue. Section II sets up a theoretical framework in which we can examine the sources of bias in estimates that may arise with either individual-country or cross-border aggregates, associated with currency substitution and cross-border portfolio diversification. Section III discusses the role of these sources of bias in a simple error-correction model of money demand at the national level. Section IV analyzes the implications of cross-country aggregation in this framework. Section V presents some conclusions.

## II. Alternative Sources of Bias

The econometric literature on aggregation suggests that there are some circumstances under which aggregate estimates may actually perform better than estimates for the individual units. Whether or not this is the case depends on a tradeoff between alternative sources of bias.

In aggregating demand for money across different countries, aggregation bias occurs to the extent that different countries in the group have different money demand relationships; as a result, the aggregate estimates may not converge to the money demand of one particular country or even necessarily to an appropriately weighted average of the group. A second source of error is specification bias: to the extent that there are omitted variables, errors in the appropriate measures of the explanatory variables, or other specific errors, these may lead to biased estimates at both the single-country and the aggregate level (see Pesaran, Pierce, and Kumar (1989)). It is possible that this specification bias will be lessened at the aggregate level--for example, if the specification error arises from the omission of aggregate variables from the individual-unit equations. As a result, it is possible, as first shown by Grunfeld and Griliches (1960), that aggregate estimates will actually perform better, when evaluated in terms of within-sample prediction error, than the corresponding individual unit equation estimates.

Another issue that arises, in connection with aggregative estimates, is the potential efficiency gains that may arise if the error terms in different individual-unit equations are correlated. In this case, the ideal procedure would be to estimate a system of seemingly unrelated regression (SUR) equations for the individual units (Zellner (1962)), exploiting the cross-correlations of the disturbance terms. 1/ There are some instances in which aggregative estimates may be a short cut to achieving some of the same benefits.

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1/ Lane and Poloz (1992) use a SUR approach to examine demand for money in a multi-country setting using data for the G7 countries, paying particular attention to the possibility of currency substitution.

In examining the demand for money in a relatively integrated financial area such as that corresponding to the EMS, we would like to allow for two possibilities. First, there may be currency substitution in the demand for money, meaning that individuals and/or firms resident in any country may hold transactions balances in more than one country, and/or denominated in more than one currency. 1/ Currency substitution implies that total money holdings in a currency include holdings by domestic and foreign residents, and thus conceivably depend not only on incomes and interest rates in that country but also on incomes and interest rates in some other countries. Moreover, with currency substitution, a shock to money balances in one country may be correlated with shocks to money balances in other countries; to the extent that such shocks are related to irregularities in payments and receipts, or to unobserved changes in the anticipated relative return on balances held in different currencies, they may be negatively correlated, while to the extent that they correspond to common changes in transactions technology or preferences, they may be positively correlated.

A second issue is portfolio diversification more generally (of which currency substitution is a special case): residents of each country may hold foreign as well as domestic assets, so interest rates in other countries (adjusted for any anticipated changes in exchange rates), especially in other EC member countries, would affect the demand for money in any one country.

These issues can be incorporated into the error correction model analyzed by Engle and Granger (1987), which forms the framework of our earlier paper, as well as the work of Artis (1991) and of Monticelli and Strauss-Kahn (1991) cited above. 2/ The simplest error correction money demand model for a single country  $j$  consists of the long run or static equation

$$m_{jt} - p_{jt} = \beta_{j0} + \beta_{j1} \bar{y}_{jt} + \beta_{j2} \bar{i}_{jt} + \epsilon_{jt}, \quad (1)$$

$$\beta_{j1} > 0, \beta_{j2} < 0; \epsilon_{jt} \sim I(0)$$

$$t = 1, 2, \dots, n$$

Here  $m_{j,t}$  and  $p_{j,t}$  are the country's money stock and price level.  $\bar{y}_{j,t}$  and  $\bar{i}_{j,t}$  are, respectively, weighted averages of the log of income and the level of interest rates in different countries:

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1/ Angeloni, Cottarelli, and Levy (1991) distinguish between substitution of currency and location. They construct new aggregates to incorporate cross-border deposits, and show that these aggregates perform better than the national definitions.

2/ For further discussions of this approach, see also Hendry (1985, 1986), and Granger (1986).

$$\bar{y}_{jt} = \sum_{i=1}^m \alpha_{ij} y_{it} \quad (2a)$$

$$\bar{i}_{jt} = \sum_{i=1}^m \omega_{ij} (i_{it} + s_{ij,t+1}^e - s_{ij,t}) \quad (2b)$$

where the weights  $\alpha_{ij}$  and  $\omega_{ij}$  reflect the influence of different countries' incomes and interest rates on demand for money in country  $j$ , and  $s_{ij,t}$  is the spot exchange rate between currencies  $i$  and  $j$ . The foreign interest rate  $i_{it}$  plus the expected change in the exchange rate,  $(s_{ij,t+1}^e - s_{ij,t})$  is the total expected domestic-currency return on foreign-currency-denominated assets. The formulation in equations (1), (2a), and (2b) is equivalent to assuming that demand for country  $j$  money depends upon  $y_{1t}, y_{2t}, \dots$  with coefficients  $\beta_{ji1} = \beta_{j1} \alpha_{ij}$ ; we use the weighted average approach for expositional convenience. In the absence of currency substitution and international portfolio diversification,  $\alpha_{ij} = 0$  and  $\omega_{ij} = 0$  for all  $i \neq j$ ; the possibility of currency substitution and portfolio diversification implies that  $\alpha_{ij} > 0$  and/or  $\omega_{ij} > 0$  for some  $i \neq j$ .

There is also a short-run adjustment equation

$$\Delta(m_{jt} - p_{jt}) = \gamma_{j0} + \gamma_{j1} \Delta y_{jt} + \gamma_{j2} \Delta i_{jt} + \gamma_{j3} e_{j,t-1} + \eta_{jt} \quad (3)$$

$$\gamma_{j1} > 0, \gamma_{j2} < 0, \gamma_{j3} < 0, \eta_{jt} \sim \text{i.i.d.}$$

$$t = 2, 3, \dots, n$$

where  $e_{j,t-1}$  is the residual from least-squares estimation of equation (1); the role of this variable is typically interpreted as corresponding to disequilibrium from long-term money holdings. 1/ Disequilibrium money holdings are frequently viewed as reflecting costs that individuals may face in adjusting their portfolios, although, as pointed out by Laidler (1982) and Lane (1990), such costs at the individual level will not generally lead to disequilibrium money holdings in an economy as a whole if prices, incomes or interest rates can adjust freely to equate money supply and money demand.

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1/ In practice, more lagged values of the differenced variables may be included in the dynamic equation; some of the variables in the static equation may also be excluded from the dynamic equation, as they already have their influence through the error-correction term. The simplified formulation shown here is used for illustrative purposes.

### III. Money Demand at the National Level

Using the framework that has been laid out in the previous section, let us examine the implications of estimating money demand at the national level.

#### 1. The static equation

First, let us examine the implications of an estimate of the static money demand equation (1) using national data. Equation (1) can be written in the form  $Y_j = X_j\beta_j + \epsilon_j$ , where

$$(4a) Y_j = \begin{bmatrix} m_{jt} - p_{jt} \\ m_{j2} - p_{j2} \\ : \\ : \end{bmatrix}$$

$$(4b) X_j = \begin{bmatrix} 1 & \bar{y}_{j1} & \bar{i}_{j1} \\ 1 & \bar{y}_{j2} & \bar{i}_{j2} \\ : & : & : \\ : & : & : \end{bmatrix}$$

$\beta_j$  is a vector of coefficients and  $\epsilon_j$  a vector of disturbances whose expectation is zero.

If demand for money is estimated using a single country's income and interest rate, there may be measurement error, represented by the matrix  $Z_j$ :

$$Z_j = \begin{bmatrix} 0 & z_{1j,1} & z_{2j,1} \\ 0 & z_{1j,2} & z_{2j,2} \\ : & : & : \\ : & : & : \end{bmatrix}$$

where

$$(5a) z_{1j,t} = y_{jt} - \bar{y}_{jt}$$

$$(5b) z_{2j,t} = i_{jt} - \bar{i}_{jt}$$

Thus, error potentially results from estimating money demand as a function of one rather than all countries' income and interest rate.

It is important to consider the time-series structure of the measurement errors. For simplicity, we assume that these errors follow a random walk so that  $Ez_{qj,t} = z_{qj,t-1}$  and thus  $E\Delta z_{qj,t} = 0$  ( $q = 1, 2$ ).

The least-squares estimator for the coefficient vector in equation (1) is (omitting the  $j$  subscript for convenience)

$$\hat{\beta} = (\tilde{X}'\tilde{X})^{-1} \tilde{X}'Y \quad (6)$$

where  $\tilde{X} \equiv X + Z$ .

Substituting from (1) and (5) into (6), the value of this estimator  $\hat{\beta}$  will be

$$\hat{\beta} = \beta - (\tilde{X}'\tilde{X})^{-1} \tilde{X}'Z\beta + (\tilde{X}'\tilde{X})^{-1} \tilde{X}'\epsilon \quad (7)$$

Now, from equation (7), the bias in the OLS estimate is:

$$E(\hat{\beta} - \beta) = -E(\tilde{X}'\tilde{X})^{-1} \tilde{X}'Z\beta + E(\tilde{X}'\tilde{X})^{-1} \tilde{X}'\epsilon. \quad (8)$$

The second term on the right-hand side is

$$E(\tilde{X}'\tilde{X})^{-1} \tilde{X}'\epsilon = E(\tilde{X}'\tilde{X})^{-1} (X'\epsilon + Z'\epsilon)$$

The unconditional expectation  $EX'\epsilon$  is zero if the explanatory variables are exogenous, i.e. orthogonal to the disturbance term  $\epsilon$ . The conditions required for  $\epsilon$  to be orthogonal to the measurement error matrix  $Z'$ , so that  $E(Z'\epsilon) = 0$ , are less fundamental, but there is no obvious reason that disturbances to money demand should be correlated with fluctuations in relative income and interest rates. If these conditions are satisfied, the second term in equation (8) vanishes, simplifying the expression to yield

$$E(\hat{\beta} - \beta) = -E(\tilde{X}'\tilde{X})^{-1} \tilde{X}'Z\beta \quad (8')$$

As we can see, this bias is zero if there is no currency substitution or portfolio diversification (or measurement error from some other unspecified source), since then  $Z$  is identically zero. Otherwise, the expression for the bias in equation (8) becomes

$$E(\hat{\beta} - \beta) = -E(\tilde{X}'\tilde{X})^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \text{Var } z_1 & \text{Cov}(z_1, z_2) \\ 0 & \text{Cov}(z_1, z_2) & \text{Var } z_2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} \quad (8'')$$

Then, for simplicity, let us consider the further assumption that  $Ez_2z_1 = 0$ , that is that the error in income and interest rates are uncorrelated. In this case, it can readily be seen from (8'') that estimating the static equation using a single country's income and interest rate biases both the coefficients on income and on interest rates. The bias in the coefficient on income is then  $E(\hat{\beta}_1 - \beta_1) = -\Omega_{22}\beta_1\text{Var } z_1 - \Omega_{23}\beta_2\text{Var } z_2$ , while the bias in the coefficient on interest rates is  $E(\hat{\beta}_2 - \beta_2) = -\Omega_{32}\beta_1\text{Var } z_1 - \Omega_{33}\beta_2\text{Var } z_2$  where the  $\Omega_{ij}$  are corresponding entries in the  $E(\tilde{X}'\tilde{X})^{-1}$  matrix. Thus, recalling that  $\Omega$  is positive definite and that  $\beta_1 > 0 > \beta_2$ , the direction of the bias depends upon the relative variances of errors in interest rates and income. If the variance of the difference between the national interest rate and the appropriately-weighted international average rate,  $\text{Var } z_2$ , is small relative to the variance of the corresponding income variable,  $\text{Var } z_1$ , both coefficients are biased downward. The latter assumption is likely to be valid in a single financial area like the EC, in which expected returns on financial assets denominated in different currencies are highly correlated.

## 2. The dynamic equation

In the error correction model, the residual from the static equation is used as a measure of the "disequilibrium" in demand for money, which is then used as an explanatory variable in the dynamic equation (3). The residuals from the static equation are

$$\begin{aligned} \hat{e} &= Y - \tilde{X}\hat{\beta} \\ &= (I - \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}')\epsilon + (I - \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}')Z\beta \end{aligned} \quad (9)$$

The dynamic equation (3) can be rewritten as

$$\Delta Y = \gamma X^D + \eta \quad (10)$$

where

$$X^D = \begin{bmatrix} 1 & \bar{\Delta y}_{j2} & \bar{\Delta i}_{j2} & e_{j0} & \epsilon_{j2} \\ 1 & \bar{\Delta y}_{j3} & \bar{\Delta i}_{j3} & e_{j1} & \epsilon_{j3} \\ : & : & : & : & : \\ : & : & : & : & : \end{bmatrix}$$

The least-squares estimate of the parameter vector  $\gamma$ , using the lagged residuals from the static equation as characterized in equation (9), can be expressed as

$$\hat{\gamma} = (\tilde{X}^D \tilde{X}^D)^{-1} \tilde{X}^D \Delta Y \quad (11)$$

where  $\tilde{X}^D = X^D + Z^D$

and

$$Z^D = \begin{bmatrix} 0 & \Delta z_{12} & \Delta z_{22} & e_1 - \epsilon_1 \\ 0 & : & : & : \\ 0 & : & : & : \end{bmatrix}$$

This estimator will thus be

$$\hat{\gamma} = \gamma - (\tilde{X}^D \tilde{X}^D)^{-1} \tilde{X}^D Z^D \gamma + (\tilde{X}^D \tilde{X}^D)^{-1} \tilde{X}^D \eta \quad (12)$$

Then, assuming again that the explanatory variables are exogenous, so they are uncorrelated with the dynamic equation disturbance  $\eta$ , the bias in the estimate is

$$E(\hat{\gamma} - \gamma) = -E(\tilde{X}^D \tilde{X}^D)^{-1} \tilde{X}^D Z^D \gamma \quad (13)$$

or equivalently to

$$E(\hat{\gamma} - \gamma) = -E(\tilde{X}^D \tilde{X}^D)^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \text{Var}(\Delta z_1) & \text{Cov}(\Delta z_1, \Delta z_2) & \text{Cov}(\Delta z_1, e - \epsilon) \\ 0 & \text{Cov}(\Delta z_1, \Delta z_2) & \text{Var}(\Delta z_2) & \text{Cov}(\Delta z_2, e - \epsilon) \\ 0 & \text{Cov}(\Delta z_1, e - \epsilon) & \text{Cov}(\Delta z_2, e - \epsilon) & \text{Var}(e - \epsilon) \end{bmatrix} \gamma$$

Thus, the bias in the coefficient vector can be expressed as the product of a positive definite matrix and the covariance matrix of the three measurement errors--the errors in measuring national income, the interest rate, and the disequilibrium term. Recall that the error in the change in income,  $\Delta z_{1,t}$ , is the change in the ratio of national income to a suitably-weighted average of different countries' national incomes. The error in interest rates,  $\Delta z_{2,t}$ , is the change in the deviation between domestic interest rates and an appropriate average of the expected returns

on assets in different countries. The error in the estimated disequilibrium term, as expressed in (9), depends on the disturbance term in the static equation and on the measurement errors in income and interest rates. Then equation (13) can be used to find the bias in all of the estimated coefficients.

Let us consider, for example, the bias in the coefficient of adjustment in response to disequilibrium:

$$\begin{aligned} E(\hat{\gamma}_3 - \gamma_3) = & -\Omega_{42}[\gamma_1 \text{Var}\Delta z_1 + \gamma_2 \text{Cov}(\Delta z_1, \Delta z_2) + \gamma_3 \text{Cov}(\Delta z_1, e-\epsilon)] \quad (14) \\ & -\Omega_{43} [\gamma_2 \text{Cov}(\Delta z_2, \Delta z_2) + \gamma_3 \text{Var}\Delta z_2 + \gamma_4 \text{Cov}(\Delta z_2, e-\epsilon)] \\ & -\Omega_{44} [\gamma_1 \text{Cov}(\Delta z_1, e-\epsilon) + \gamma_2 \text{Cov}(\Delta z_2, e-\epsilon) + \gamma_3 \text{Var}(e-\epsilon)] \end{aligned}$$

Again, recalling that the matrix  $\Omega$  is positive definite, and using our simplifying assumption that the errors  $z_1$  and  $z_2$  follow random walks in which the innovations are mutually uncorrelated,

$$\text{Cov}(\Delta z_1, \Delta z_2) = \text{Cov}(\Delta z_1, e-\epsilon) = \text{Cov}(\Delta z_2, e-\epsilon) = 0$$

so that

$$E(\hat{\gamma}_3 - \gamma_3) = -\Omega_{42} \gamma_1 \text{Var}\Delta z_1 - \Omega_{43} \gamma_2 \text{Var}\Delta z_2 - \Omega_{44} \gamma_3 \text{Var}(e-\epsilon) \quad (15)$$

If we make the following further assumptions, some of the terms in expression (15) can be signed: assume (as is empirically plausible) that innovations in national income are positively serially correlated, but that innovations in interest rates are serially uncorrelated and have no discernible trend. These assumptions are sufficient to yield  $\Omega_{42} \geq 0$ , and also imply that the sign of  $\Omega_{43}$  is the same as that of the covariance of innovations to income and interest rates. The fact that  $\Omega$  is a positive definite matrix implies that  $\Omega_{44} > 0$ . If we again assume that  $\text{Var}\Delta z_2$  is relatively small, as would be implied by a high degree of financial market integration, the error correction coefficient  $\gamma_3$  will be biased downward, giving the appearance of implausibly slow adjustment. This is a variant of Goodfriend's (1985) result that misspecification of a money demand equation would give rise to an implausibly slow estimated speed of adjustment of money balances to their long-run equilibrium level.

#### IV. Cross-Border Aggregation

Now let us consider the consequences of aggregating across several countries. Aggregation across ERM countries replaces one set of restrictions with another: it replaces the exclusion of foreign income from each country's demand for money with the restriction that all countries' money demand has roughly the same structure, and that the weight of any country's income in influencing another's demand for money is roughly proportional to the weight of that country's money in aggregate money.

Suppose that the static equation for each of  $m$  countries is as given by equation (1). Then consider the consequences of estimating the aggregate demand for money, as represented in the following equation; we use an  $A$  superscript to represent cross-country aggregates.

$$(16) \ Y^A = \bar{X}^A \beta^A + \epsilon^A$$

where

$$Y^A = \sum_{i=1}^m \psi_i Y_i$$

$$\bar{X}^A = \sum_{i=1}^m \psi_i \bar{X}_i$$

where, as above,  $\bar{X}$  is the matrix of constant terms, and measured national incomes, and interest rates, as defined below (6), and where the weights  $\psi_i$  reflect the sizes of the different countries. 1/ Then, of course, if different countries' parameter vectors  $\beta_i$  differ, an estimate of the aggregate demand for money will not necessarily correspond to the demand for money in any of the countries, but to a weighted average of the countries' parameters. In addition, there is still potentially measurement error in income and interest rates, since in this context we would ideally like the weights on income and interest rates to correspond not directly to the size of different countries' share of the aggregate national income but also to the importance of different countries' incomes

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1/ In Kremers and Lane (1990), the aggregate variables are constructed by first adding up the levels of the variables in real terms (using PPP exchange rates) and then taking logarithms of the total. The approach in equation (16), a first-order approximation of this procedure, is used for expository convenience.

in each country's demand for money. That is, we would like the measure of aggregate income to be

$$\bar{y}_t^A = \sum_{j=1}^m \psi_j \sum_{i=1}^m \alpha_{ij} y_{it} \quad (17)$$

and the aggregate interest rate variable to be

$$\bar{i}_t^A = \sum_{i=1}^m \psi_j \sum_{j=1}^m \psi_i \omega_{ij} (i_{it} + s_{ij,t+1}^e - s_{ij,t}) \quad (18)$$

The measurement errors arising from the discrepancy between the variables measured as in equation (16) and as in (17) and (18), are:

$$z_{1,t}^A = \bar{y}_t^A - y_t^A \quad (19)$$

$$z_{2,t}^A = \bar{i}_t^A - i_t^A, \quad (20)$$

and these become the components of an error matrix  $Z^A$ .

Although the measurement error in the single-country equation estimates carry over to the aggregate model, errors in the aggregate model may be less than those in the single-country model, to the extent that the errors in the relevant measure of income for different countries may cancel out; for instance, the effects of excluding German income from demand for French money may partly offset those of excluding French income from demand for German money. More formally, the variance in the error in aggregate national income, in relation to an appropriate weighted average of the measured countries' incomes taking proper account of the influence of each country's income on demand for money in each other country, is

$$\sigma_{z_{1A}}^2 = \sum_i \sum_j \psi_i \psi_j \sigma_{z_{1j}} \sigma_{z_{1j}} \rho_{z_{1ij}} \quad (21)$$

where  $\sigma_{z_{1i}}$ ,  $\sigma_{z_{1j}}$  are the variances of the measurement error in each country, and  $\rho_{z_{1ij}}$  is the correlation coefficient between the income

measurement errors in the two countries. In general, the  $\rho_{z_{1ij}}$ 's are less than unity, and tend to be negative. As a result, we would expect that in

general  $\sigma_{Z1A}^2$  will be less than a similarly weighted average of  $\sigma_{Z1i}^2$  for the member countries.

The ordinary-least-squares estimate of the parameter vector  $\beta^A$  in the aggregate model differs from an average of the parameter vectors of the individual countries  $\bar{\beta}^A$  by

$$E(\hat{\beta}^A - \beta^A) = E(\bar{X}^{A'} \bar{X}^A)^{-1} \bar{X}^{A'} \left[ \sum_{i=1}^m \psi_i X_i (\beta_i - \bar{\beta}) + Z^A \right] \quad (22)$$

Thus, equation (22) shows that there may be bias in the parameter estimates (even as estimates of the average parameters of the group of countries) for two reasons. First, there may be divergences between the parameters in different countries, and these may be correlated with the countries' incomes and interest rates. Second, there may still be measurement error at the aggregate level, as a result of currency substitution and portfolio diversification.

Moreover, the disturbance term in the aggregate money demand equation  $\epsilon^A$  may be smaller than that for equations estimated for individual countries. The variance of the aggregate disturbance is related to the variances of the single countries' disturbances according to

$$\sigma_{\epsilon^A}^2 = \sum_{i=1}^m \sum_{j=1}^m \psi_i \psi_j \sigma_i \sigma_j \rho_{ij} \quad (23)$$

where  $\rho_{ij}$  is the correlation coefficient between the disturbances to money demand in countries  $i$  and  $j$ . The variance of the aggregate disturbance may be less than a similarly weighted sum of the variances of the individual money demand disturbances, to the extent that different countries' money demand disturbances may be at least partly offsetting: with currency substitution, a shock to money balances in one country may be negatively correlated with shocks to money balances in other countries, to the extent that such shocks are related to transactions noise, or to unobserved changes in the anticipated relative return on balances held in different currencies. It is also possible that shocks in different countries may be positively correlated, to the extent that they correspond to changes in transactions technology or preferences. In any event the correlation among money demand shocks may well be substantially less than unity, which tends to reduce the variance of the aggregate money demand disturbance and therefore enhances the efficiency of the aggregated money demand estimates.

Next, let us consider the properties of the residuals from estimating the multi-country aggregate demand for money. Using a procedure analogous to the derivation of (9), we find

$$\begin{aligned}
 (24) \quad e^A &= y - \bar{X}^A \hat{\beta} \\
 &= \epsilon^A + Z^A \beta^A + \bar{X}^A (\beta^A - \hat{\beta}^A) \\
 &= \epsilon^A + Z^A \beta^A + \bar{X}^A (\bar{X}^A{}' \bar{X}^A)^{-1} \left[ \sum_{i=1}^m \psi_i X_i (\beta_i - \beta^A) + Z^A \right] \\
 &\quad + \bar{X}^A (\bar{X}^A{}' \bar{X}^A)^{-1} \bar{X}^A{}' \epsilon^A
 \end{aligned}$$

As a result, there will be error in measuring the "disequilibrium" variable, which is used as an explanatory variable in the dynamic equation, due to the measurement error in aggregating income and interest rates, as well as the divergence between each country's parameter vector and the EMS-wide average. The error in measuring the disequilibrium variable, together with the errors in measuring the relevant aggregates of income and interest rates, then feed into the dynamic equation (similar to equation (3)), potentially biasing the coefficient estimates in that dynamic equation as shown in equation (13). The bias resulting from these aggregate estimates may then be compared with that in the single-country estimates: if the bias in the aggregate demand for money is less, it may give rise to more plausible estimates of the disequilibrium adjustment parameter  $\gamma_3$ --as has indeed been found in some cross-border aggregative studies of money demand.

## V. Conclusion

We are now ready to summarize our argument. As we have seen, in the presence of currency substitution and portfolio diversification, there may potentially be measurement error in the relevant concepts of both income and interest rates. Currency substitution implies that demand for each country's money depends not only on its own national income but on a suitably weighted average of its own income and those of other countries. As a result, there is likely to be measurement error in the use of a single country's income in a demand-for-money equation. Second, to the extent that individuals in a given country can diversify their portfolios internationally, their demand for money may depend not only on interest rates in their own country, but also those in other countries; this is another potential source of measurement error, which may either increase or diminish in importance with increasing international financial market integration, as market integration increases the scope for portfolio diversification while on the other hand tending to equalize the expected returns obtainable by holding assets issued in different countries. Both of these sources of measurement error seem likely to diminish, although not to vanish, as we aggregate across a group of countries whose economies

are closely linked. Furthermore, to the extent that disturbances in any one country's money-demand relationship result from unexplained shifts out of one country's money into another's, so that these disturbances may be negatively correlated (or at least less than perfectly positively correlated) the variance of the disturbance in the aggregate relationship will be less than the correspondingly weighted sum of the variances of the disturbance in the single-country relationships. This is another consideration that may tend to make the aggregate demand for money more predictable than that of a typical single country.

Working in the other direction, going from single-country to aggregate demand for money may introduce additional error as a result of possible differences in the money-demand parameters in different countries. To begin with, the coefficient estimates in the aggregate demand for money will be estimates of a multi-country average of parameters, and will thus tend to be inaccurate estimates of money demand parameters of any single country. Second, aggregate money demand estimates will have additional estimation error as a result of the divergences between different countries' money demand parameters, in addition to the remaining measurement error referred to earlier.

Aggregation entails some additional sources of bias while reducing the magnitude of other sources of bias which occur at the single-country level. In this note, the bias that currency substitution and international portfolio diversification may introduce into single-country money demand estimates has been explored. It has been shown that one particularly important place the bias is likely to appear is in the error correction term,  $\gamma_3$ . The implausibly slow adjustment of aggregate money balances implied by many single-country estimates of the error-correction term suggests misspecification in these estimates. Correspondingly, the relatively rapid adjustment found in empirical studies of aggregate demand for money in the EMS weighs in favor of a multi-country approach.

In principle, it may be possible to identify an appropriate level of aggregation arising from the tradeoff between aggregation bias and specification bias--although money demand estimates at various different levels of aggregation will no doubt continue to be useful. In particular, our analysis suggests that it may be desirable to aggregate over a group of countries within which there is a significant degree of currency substitution and portfolio diversification, but might be undesirable to aggregate over a broader group of countries. The appropriate level of aggregation is an empirical question, on which the studies cited in the introduction have already shed some light. This issue will no doubt be studied more intensively as the movement toward EMU gathers further momentum.

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