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Wage and Public Debt Indexation

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Abstract

This paper examines the relationship between the degree of wage indexation chosen by private agents and the degree of indexation of the public debt. It is shown that the government is likely to respond to an increase in the degree of wage indexation by increasing the portion of the public debt that is indexed. By contrast, the effect of an increase in public debt indexation on the degree of wage indexation is ambiguous. In equilibrium, depending on the sources of shocks to the economy, the degree of wage indexation may be positively or negatively related to that of debt indexation. This relationship is analyzed both in situations where the policymakers are able to precommit policies and in those where precommitment is not possible.

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Contents

I. Introduction

After a period of relative neglect in the literature, the issue of public debt management—namely, policies regarding the characteristics of public debt instruments—has recently received increased attention. 1/ This sudden renewal of interest in public debt management reflects both empirical and theoretical developments. On the empirical side, the last decade has witnessed a marked growth of public debt in both industrial and developing countries (see Guidotti and Kumar (1991)). On the theoretical side, government policy is increasingly being characterized by economists as "endogenous" (see Persson and Tabellini (1990)), and more attention is being devoted to the consequences of incomplete markets. Policy endogeneity results from the presence of political constraints and from the recognition that government behavior may be subject to time-inconsistency, particularly where rational private agents take into account policymakers' reaction functions. One of the consequences of policy endogeneity and market incompleteness is that they provide a role for public debt management policies, which are largely irrelevant in the standard neoclassical model. 2/ Indeed, the focus of mainstream economists on the complete-markets neoclassical model may be the main reason for the past neglect of debt management issues.

An important aspect of public debt management which has recently received attention is that of debt indexation. As Calvo (1988) points out, when public debt is contracted in nominal terms, policymakers have an incentive to resort to inflation as a means of reducing the real value of government liabilities. 3/ Thus the perverse interaction between nominal public debt and policymakers' time-inconsistent behavior provides a strong argument in favor of indexing the public debt. Debt indexation breaks the link between public debt and inflation by making the real debt burden independent, ex-post, of the inflation rate. 4/

1/ Early studies dealing with public debt management policies include Bach and Musgrave (1941), Tobin (1971), Fischer (1975), and Levhari and Liviatan (1976). Recent studies include Lucas and Stokey (1987), Bohn (1988), Persson, Persson, and Svensson (1987), Persson and Tabellini (1990), Alesina and Tabellini (1990), Calvo (1988), Calvo and Guidotti (1990a and b), Missale and Blanchard (1991), and Cukierman and Meltzer (1989).

2/ See, for instance, Barro (1974 and 1979), Levhari and Liviatan (1976), and Peled (1985).

3/ In some cases, as shown by Calvo (1988), nominal debt may even lead to multiple equilibria and, hence, to the loss of a nominal anchor.

4/ Although the role of debt indexation as a tool to influence governments' incentives to use the inflation tax has been analyzed formally only recently, it was recognized early by Bach and Musgrave (1941), who stated that "by imposing upon the government a contingent liability dependent on its failure to check price inflation, the flotation of stable purchasing power bonds may exert a wholesome pressure upon Congress to adopt aggressive anti-inflationary polices."

The time-inconsistency argument identifies a significant potential danger associated with the presence of nominal government liabilities. On this count alone, optimal policy would call for 100 percent indexation of the public debt. 1/ However, Calvo and Guidotti (1990a) pursue the argument further and note that too much debt indexation may excessively limit policymakers' ability to respond to unexpected shocks. In the model of Calvo and Guidotti (1990a), the government faces the optimal-taxation problem of financing a stochastic level of government expenditure plus the interest and amortization on the public debt by resorting to conventional distortionary taxes and the inflation tax. Markets are incomplete because the government issues only two types of bonds: an indexed bond and a nominal bond that pays a fixed—i.e., non state-contingent—nominal interest rate. In the absence of precommitment on the part of policymakers, the following trade-off arises. On the one hand, the presence of nominal government liabilities induces policymakers to resort to sub-optimally high inflation. The larger is the stock of nominal public debt, the higher is equilibrium inflation. On the other hand, too little nominal debt forces the government to rely excessively on distortionary conventional taxes to finance fluctuations in government expenditure. Since the stock of nominal debt is part of the unexpected-inflation tax base, it provides the policymaker with added flexibility to respond to unexpected shocks and, hence, serves the purpose of smoothing taxes across states of nature. 2/ Thus, policymakers need to weigh the inflationary cost associated with nominal debt against the flexibility gain derived from it. This trade-off yields an optimal degree of indexation of the public debt.

The arguments for and against debt indexation that have been explored so far focus essentially on the fiscal policy (optimal taxation) problem. However, it is often an important concern of policymakers to assess whether the presence of debt indexation may lead to other forms of indexation, in particular that of wages. Wage indexation is often regarded as a cause of inflation persistence. At a superficial level, it may be argued that a positive relationship between debt and wage indexation may arise simply because, to the extent that bondholders are insulated from the effects of inflation through debt indexation, it is a matter of fairness to provide the same type of protection to wage earners. At a more subtle level, it may be argued that a higher degree of public debt indexation is indicative of the government's concern about higher inflation and, hence, provides a signal that leads workers to demand higher wage indexation. While the

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1/ In Calvo (1988) optimal policy calls for 100 percent indexation, since the demand for money is assumed to be interest inelastic and the model is deterministic. Since the demand for money is interest elastic in Persson, Persson, and Svensson's (1987) perfect-foresight model, the optimal policy calls for issuing non-indexed government assets (see, however, Calvo and Obstfeld (1991)).

2/ The role of unanticipated inflation in smoothing taxes across states of nature as part of optimal monetary policy is developed formally in Calvo and Guidotti (1989).

interconnection between these two forms of indexation is of considerable relevance for policymakers, it has not yet been formally analyzed. The purpose of this paper is to fill this gap by studying the relationship that may exist between indexation of the public debt and indexation of wages.

The factors that govern the degree of wage indexation have long been a central issue in both macroeconomic theory and policy. 1/ The analysis pioneered by Gray (1976) and Fischer (1977) provides a clear and intuitive way of thinking about wage indexation. They focus on an eminently relevant issue: what is the optimal degree of wage indexation in a world where there are both monetary and real shocks and where a complete set of state-contingent wage contracts cannot be implemented? By focusing on incomplete markets--in particular, where wage contracts are signed under imperfect information and, quite realistically, can only be made contingent on the price level--Gray (1976) and Fischer (1977) provide insights that are particularly relevant for macroeconomic policy. Perhaps the most important insight is the relation of optimal wage indexation to the source of the shocks hitting the economy. In particular, as shown by Gray (1976), it is generally optimal to have partial wage indexation unless the only source of shocks is monetary. Moreover, the optimal degree of wage indexation is positively related to the variance of monetary shocks and negatively related to the variance of real shocks.

Policy is exogenous in Gray (1976) and Fischer (1977), as well as in the related literature. Thus, this paper blends the two approaches by incorporating policy endogeneity in the Fischer-Gray framework, and providing a link between public debt management policy--in particular, debt indexation--and the private sector's choice of the degree of wage indexation. By focusing on realistic forms of wage contracts, the Fisher-Gray approach is very similar in spirit to the approach to public debt indexation taken by Calvo and Guidotti (1990a).

The analysis undertaken in the paper is exploratory in nature. The objective is to obtain basic insights while adding the minimum of changes to the models of Fischer-Gray and Calvo-Guidotti. Nevertheless, blending the two approaches results in surprising richness. Since the wage and public debt indexation decisions are closely interdependent, the two frameworks are extended significantly. The Calvo-Guidotti framework is enriched by adding the concern about output and employment fluctuations in the government's objective function, and by introducing imperfect controllability of inflation in the optimal-taxation problem. 2/ The Fischer-Gray approach--

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1/ See Gray (1976), Fischer (1986), Aizenman and Frenkel (1985), Karni (1983), and Canzoneri and Siebert (1990), among others.

2/ It should be noted that because of the assumption that the government's targeted levels of output coincides with that of the private sector, the inflation bias emphasized by Barro and Gordon (1983) is absent in this paper. The inflation bias emphasized here stems from the fiscal problem stressed by Calvo and Guidotti (1990a).

in particular, Gray's (1976) model—is enriched by making the monetary shock endogenous. In particular, the monetary shock reflects the state-contingent response of the rate of monetary expansion to shocks to government expenditure. The endogenous distribution of the rate of monetary expansion is, in turn, a function of the government's choice of debt indexation and the private sector's wage indexation decision.

A number of insights emerge from the analysis. As far as the government's fiscal problem is concerned, wage and debt indexation are positively related: higher wage indexation increases unwanted inflation volatility and induces the government to increase the degree of public debt indexation. Since higher public debt indexation reduces the policymakers' incentive to resort to inflation, it follows that a higher degree of wage indexation induces the government to adopt a more anti-inflationary policy stance. 1/

As far as the private sector is concerned, the degrees of wage and debt indexation may be positively or negatively related. Whether higher debt indexation induces wage setters to choose a higher or a lower degree of wage indexation depends on the effect of debt indexation on the variability of the rate of money growth. If higher debt indexation leads to increased monetary volatility, then wage setters will choose higher wage indexation. If higher debt indexation leads to decreased monetary volatility, the optimal degree of wage indexation falls. The intuition behind these two alternative effects of debt indexation will be discussed in detail.

Equilibrium wage and debt indexation represent a Nash equilibrium. The paper analyzes the response of wage and debt indexation to exogenous changes—such as changes in the distribution of monetary and real shocks and government expenditure, and changes in the level of public debt. Whether wage and debt indexation respond to exogenous changes by moving in the same or opposite directions depends on the initial equilibrium, the nature of the change, and the policy response to it. The analysis shows, however, that the presumption that one form of indexation leads to other forms of indexation does not receive general support. Indeed, there are several cases where the two forms of indexation will move in opposite directions.

Finally, the role of policy precommitment is analyzed. It is shown that policy precommitment increases the role for nominal debt. Unlike Calvo and Guidotti (1990a), where the optimal policy with precommitment called for an unboundedly large swap of nominal debt for indexed government assets, it is shown that the optimal degree of debt indexation is bounded. The reason is that in this paper the government cannot fully control the rate of inflation. Hence, an excessively high stock of nominal debt—although not necessarily a high *net* stock of public debt—will result in excessively high

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1/ This result provides an interesting insight in the discussion about whether wage indexation weakens the resolve of governments to fight inflation (see Fischer and Summers (1989), and De Gregorio 1991).

variability of conventional taxes and inflation. The presence of policy precommitment is shown to alter some of the implications obtained when there is no precommitment. However, many qualitative features of the interaction between wage and public debt indexation remain unchanged.

The remainder of the paper is organized as follows. Section II sets up the basic model. Section III analyzes the factors that determine the equilibrium degree of wage and debt indexation under the assumption that the government cannot credibly precommit its policy. Section IV explores the choice of debt and wage indexation under the assumption of policy precommitment. Section V concludes.

II. The Basic Model

This section sets up the basic framework, by integrating Gray's (1976) analysis of wage indexation with Calvo and Guidotti's (1990a) analysis of debt indexation. Emphasis will be placed on the additional elements that are brought into play by the interaction of the choice of the degree of wage indexation with government fiscal and monetary policy decisions.

1. Wage Contracts

Consider a small open economy in which the production function of the only good is given by:

$$Y = \frac{1}{\alpha} L^{\alpha} e^{u}, \tag{1}$$

where Y denotes real output, L denotes labor—which is assumed to be the only variable input—$\alpha \in (0,1)$ is a constant parameter, and u is a technological stochastic shock, symmetrically distributed with zero mean and constant variance, σ_u^2.

It is assumed that there are two periods. In period 0, a wage contract is decided upon; in period 1 the production decision takes place. It is assumed that in period 1 the representative firm observes the realization of the technological shock, u. Thus, the firm demands labor according to the following first-order condition:

$$L^{\alpha-1} e^{u} = \frac{W}{P}, \tag{2}$$

where W and P denote the nominal wage rate and the price level in period 1. Equation (2) yields the following demand for labor:

$$\ell^d = - \frac{1}{1 - \alpha}(w - p - u),$$ (3)

where ℓ^d, w, and p denote the logarithm of labor demand, the nominal wage, and the price level, respectively.

The supply of labor is assumed to be given by:

$$\ell^s = h(w - p),$$ (4)

where ℓ^s is the logarithm of labor supplied, and h (>0) is the (constant) labor supply elasticity.

Under full information, equilibrium nominal wages, w^*, would be obtained by setting $\ell^d = \ell^s$. This implies, by equations (3) and (4):

$$w^* = p + \frac{u}{1 + h(1 - \alpha)}.$$ (5)

Equation (5) shows that under full information equilibrium, real wages would be directly related to the realization of the technological shock, u. Full information real wages are high (low) in states of high (low) productivity.

The realization of the technological shock, u, however, is assumed to be firms' private information. Thus, wage contracts cannot be made contingent on the realization of u. Following Gray (1976), it is assumed that wage contracts negotiated in period 0 specify the following indexation rule for the nominal wage:

$$w = p^e + \eta(p - p^e),$$ (6)

where a superscript e denotes private expectations, and $\eta \in [0,1]$ is the wage indexation parameter. As will become clear below, the price level is a noisy signal of the technological shock u. Therefore, the wage-indexation decision entails a signal extraction problem.

Once the nominal wage is contracted, employment becomes completely demand determined. Equation (6) can be written as:

$$w - p = (\eta - 1)(\pi - \pi^e),$$ (7)

where π denotes the inflation rate between period 0 and period 1. If there is full wage indexation—i.e., $\eta = 1$—then the real wage is fixed at the expected full-information equilibrium real wage. Note that, by equations (5) and (7), it follows that $w - p = E(w^* - p) = 0$, where $E(\bullet)$ denotes the

expectations operator based on information available at time 0. If there is partial wage indexation—i.e., $\eta < 1$—then the real wage will vary with inflation around its expected full-information level—note that $E(w - p) = E(w^* - p) = 0$.

Following Gray (1976), it is assumed that the degree of wage indexation, η, is chosen so as to minimize the expected deviation of the logarithm of output, y, from its full information level, y^*. Using equations (1)-(7), the following expression for the loss function, LW, that is minimized by wage setters obtains:

$$LW = E(y - y^*)^2 = E\left\{\frac{\alpha}{1 - \alpha}\left[(1 - \eta)(\pi - \pi^e) + \frac{u}{1 + h(1 - \alpha)}\right]\right\}^2. \qquad (8)$$

Note that, at this point, a distinction is being drawn between the mathematical expectation operator, $E(\bullet)$, and the concept of private expectations, denoted by a superscript e. This distinction, which will disappear in equilibrium due to the assumption of rationality of private expectations, exists to highlight the fact that π^e is embodied in the wage contract in period 0 and, thus, is a predetermined variable for decisions taken in period 1. This will become clear below when the government's decision problem is analyzed.

Consider next the determination of domestic inflation. It is assumed that purchasing power parity (PPP) holds and that, for simplicity, there is no foreign inflation; this implies that the domestic rate of inflation is identical to the rate of nominal devaluation of the domestic currency. Furthermore, it is assumed that the small open economy operates under a flexible exchange rate, and that the demand for money is interest-inelastic. Hence, money-market equilibrium implies that inflation satisfies:

$$\pi = \mu + \epsilon - \frac{\alpha}{1 - \alpha}(1 - \eta)(\pi - \pi^e) - \frac{u}{1 - \alpha}, \qquad (9)$$

where μ denotes the rate of monetary expansion and ϵ can be interpreted either as an unobservable shock to the demand for money, or as a shock to the money supply process which is not under the control of policymakers. It is assumed that ϵ is symmetrically distributed with zero mean and constant variance, σ_ϵ^2. The last two terms on the R.H.S. of equation (9) sum up to the change in output. (For simplicity, but without loss of generality, it is assumed that the economic equilibrium in period 0 corresponds to a realization where both shocks ϵ and u equal zero.) Equation (9) can be solved to yield:

$$\pi = \frac{\mu + \epsilon + \alpha\beta(1 - \eta)\pi^e - \beta u}{1 + \alpha\beta(1 - \eta)}, \tag{10}$$

where $\beta \equiv 1/(1 - \alpha)$. Under rational expectations, $\pi^e = E\pi$ in equilibrium. Thus, by equation (10):

$$\pi^e = E\pi = E\mu. \tag{11}$$

Note that in equation (11) the rate of monetary expansion, μ, is stochastic. The rate of monetary growth, μ, is endogenous and its distribution will be derived in the next section from the government's decision problem. The government problem, to which I now turn, also determines the optimal degree of debt indexation.

2. The government's decision problem

The government in period 0 inherits a stock of public debt with a real value, denoted by b, which is assumed to be given exogenously. To focus sharply on the issue of debt indexation, it is assumed that the only decision taken by government 0 is how much of the total public debt should be indexed to the price level; denote this proportion by $1 - \theta$. Thus most of the action occurs in period 1, where the government decides how to finance the interest and amortization of the inherited public debt, as well as government expenditure, g.

In period 1, the government budget constraint is given by:

$$x = g + (1 - \theta)b(1 + i^*) + \theta b\frac{(1 + i)}{1 + \pi} - \frac{M}{P}\frac{\pi}{1 + \pi}, \tag{12}$$

where i^* is the (constant) international interest rate, x denotes conventional (distortionary) taxes, i is the domestic nominal interest rate, and M/P is the real quantity of money. Equation (12) states that conventional taxes equal government expenditure plus the service and amortization of the public debt, minus the inflation tax on cash balances. The interest rate on indexed bonds equals the foreign interest rate.

It is assumed that individuals are risk neutral and that government bonds are pure assets. Hence, under perfect capital mobility:

$$E\left[\frac{1 + i}{1 + \pi}\right] = 1 + i^*. \tag{13}$$

Government expenditure is stochastic; g is symmetrically distributed with mean equal to $\bar{g}$ and constant variance σ_g^2. As in Calvo and Guidotti (1990a), it is assumed that the nominal interest rate on nominal government

bonds is not state contingent. Hence, shocks to government expenditure cannot be financed by state-contingent variations in the interest paid on the public debt. This implies that markets are incomplete. 1/ The presence of this form of market incompleteness focuses the analysis on a realistic menu of government bonds and, hence, on meaningful debt management policies.

The presence of nominal debt implies that the government is subject to time-inconsistency. This occurs because, from the perspective of period 1, the amount of nominal debt, θb, as well as the nominal interest rate, i, are predetermined variables. Thus, government 1 has an incentive to use the inflation tax to reduce the real value of government liabilities beyond what would be optimal from the perspective of period 0.

Government 1 chooses taxes, x, and the rate of monetary expansion, μ, to minimize the following loss function:

$$LG_1 = E_1[Ax^2 + \pi^2 + B(y - y^*)^2],$$ (14)

where A and B are constants that denote the relative weights of taxes and output deviations in the government loss function, and $E_1(\bullet)$ denotes the expectations operator based on information available to government in period 1. 2/ It is assumed that government 1 knows the realization of g, but cannot observe the realizations of ϵ and u. Equation (14) assumes that taxes, inflation, and output deviations are costly.

Government 0 chooses the degree of debt indexation, θ, to minimize:

$$LG = E[Ax^2 + \pi^2 + B(y - y^*)^2].$$ (15)

Government 0 does not know g, but knows its distribution function. Government 0's optimization takes into account the reaction function of government 1 with respect to government 0's actions.

Two differences between the present analysis and that of Calvo and Guidotti (1990a) are worth mentioning at this time. First, government 1 is subject to uncertainty, since it does not observe shocks ϵ and u. Thus, it does not have full control over the inflation rate. Second, policymakers

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1/ See Calvo and Guidotti (1990a) for more discussion.
2/ Note that the target level of employment for the government is the same as that of wage setters. Therefore, the time-inconsistency considerations explored by Barro and Gordon (1983) are not present in this analysis. As will become clear in the next section, inflation occurs in this model because of a fiscal problem as in Calvo and Guidotti (1990a), and not because of the government's attempt to push output above its equilibrium level, as in Barro and Gordon (1983).

take into account in their objective function the effect that inflation has on output, in addition to taxes and inflation. (The government loss function is the same as in Calvo and Guidotti (1990a) if $B = 0$.) In particular, the effect of inflation on output depends crucially on the degree of wage indexation chosen by the private sector. The degree of wage indexation, in turn, is chosen in period 0 by the private sector, taking account of the behavior of the government. These are the issues to which I now turn.

Before proceeding to the next section, it is worth noting that wage contracts in this model make the nominal wage contingent only on the price level. In particular, it may be argued that the nominal wage could also be made contingent on government expenditure. For the sake of realism, the analysis focuses only on price indexation, arguing that in practice it may be too costly to make wage contracts contingent on the realization of fiscal variables, which are usually known with substantial lags and are subject of frequent controversy and revisions. Furthermore, this is consistent with the assumption that the interest rate on the public debt is not state-contingent.

III. Wage and Debt Indexation

In order to maintain analytical tractability, I linearize government budget constraint (12) and the interest parity condition (13). (This, in addition, maintains the analysis closely comparable to Calvo and Guidotti (1990a).) Assuming—without loss of generality—that hereafter the international interest rate, i^*, is equal to zero, and taking the first-order terms of the Taylor series expansion of equations (12) and (13) around the point $i=\pi=i^*=0$, it follows that:

$$x = g + b + \theta b(i - \pi) - k\pi, \tag{16}$$

where $k\pi$ is seigniorage on cash balances, k is a constant, and: 1/

$$i = \mathrm{E}\pi. \tag{17}$$

1/ Implicitly in equation (16) it is assumed that the variation in seigniorage which results from fluctuations in the real money demand (because of output fluctuations) is absorbed through lump-sum taxes. Given that the focus of the paper is on the role of nominal debt in fiscal policy, adding those terms would make the algebra unnecessarily cumbersome. In addition, this assumption allows us to maintain closer comparability with the analysis on debt indexation contained in Calvo and Guidotti (1990a), where the demand for money is non-stochastic.

Let us now solve government 1's optimization problem. Government 1 chooses the pair of functions {$E_1 x(g)$, $\mu(g)$} to minimize:

$$ LG_1 = E_1 \left\{ Ax^2 + \pi^2 + B(\alpha\beta)^2 \left[(1 - \eta)(\pi - \pi^e) + \frac{u}{1 + h(1 - \alpha)} \right]^2 \right\}, \quad (18) $$

subject to equation (10) and budget constraint (16), taking π^e and i as predetermined—recall that π^e is part of the wage contract. Note that since the government cannot fully control inflation—because of the unobservable shocks ϵ and u—it cannot fully control what taxes need to be once inflation is realized. Government 1, however, can specify a function $E_1 x(g)$ which indicates what taxes are expected to be—given the information available to government 1—as a function of government spending.

The above minimization problem yields the following first-order condition:

$$ E_1 \pi + B(\alpha\beta)^2 (1 - \eta)(E_1 \pi - \pi^e) = A(\theta b + k) E_1 x. \quad (19) $$

The L.H.S. of equation (19) is the expected (as of period 1) cost of an increase in inflation triggered by a higher rate of monetary expansion, μ. The expected cost of inflation is composed of a direct cost—the first term on the L.H.S. of (19)—and a cost in terms of the expected (as of period 1) deviation of output from its optimal level—the second term on the L.H.S. of (19). 1/ The expected output deviation depends directly on the degree of wage indexation chosen by the private sector. With full wage indexation—i.e., when $\eta=1$—the output deviation is expected at time 1 to be zero, independently of the inflation rate. The R.H.S. of (19) equals the marginal cost reduction associated with government 1's expected conventional-tax saving obtained by higher inflation. Ex post, higher inflation reduces the real value of government debt in addition to raising seigniorage on cash balances. This is why the stock of nominal debt, θb, appears as part of the inflation tax base in equation (19).

Calvo and Guidotti (1990a) have shown that in equilibrium inflation produces no revenue *on average* from the stock of nominal debt. However, the inflation tax on nominal debt generates revenue on a state-by-state basis, so that it can be used to smooth conventional taxes across states of nature. This property is at the heart of Calvo and Guidotti's (1989 and 1990a) concept of flexibility, which provides a role for nominal government debt.

1/ Note that actual output deviations also depend on the technological shock, u.

In equilibrium, equations (11) and (17) hold. Using these, it can be shown (see the Appendix for details) that the policy reaction functions of government 1 are given by:

$$\mu = E\mu + \frac{A(\theta b + k)[1 + \alpha\beta(1 - \eta)]}{1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1 - \eta)^2} \, (g - \bar{g}), \tag{20}$$

$$E_1 x = Ex + \frac{1 + B(\alpha\beta)^2(1 - \eta)^2}{1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1 - \eta)^2} \, (g - \bar{g}), \tag{21}$$

$$E\mu = E\pi = \frac{A(\theta b + k)}{1 + Ak(\theta b + k)} \, (\bar{g} + b), \tag{22}$$

and

$$Ex = \bar{g} + b - kE\pi. \tag{23}$$

It is shown in the Appendix that, using equations (10), (12), (20), and (21), the following expressions for taxes and inflation obtain:

$$\pi = E\pi + \frac{1}{1 + \alpha\beta(1 - \eta)}\left(\mu - E\mu + \epsilon - \beta u\right), \tag{24}$$

$$x = E_1 x - \frac{\theta b + k}{1 + \alpha\beta(1 - \eta)}\left(\epsilon - \beta u\right). \tag{25}$$

In period 0, government 0 chooses the optimal degree of debt indexation, θ, while the private sector chooses the optimal degree of wage indexation, η. Both take into account the behavior of government 1, as described by equations (20)-(25). Formally, the resulting pair (θ, η) is a Nash equilibrium.

Using equations (20)-(25), and assuming that ϵ, u, and g are uncorrelated, government 0's loss function is given by:

$$LG = \frac{A\left[1 + B(\alpha\beta)^2(1 - \eta)^2\right]\sigma_g^2}{1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1 - \eta)^2} +$$

$$\frac{\left[1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1 - \eta)^2\right]\sigma_\epsilon^2}{\left[1 + \alpha\beta(1 - \eta)\right]^2} +$$

(26)

$$\left\{\frac{1 + A(\theta b + k)^2}{\left[1 + \alpha\beta(1 - \eta)\right]^2} + B\left[\frac{\eta}{1 + \alpha\beta(1 - \eta)} - \frac{h}{\beta + h}\right]^2 \alpha^2\right\}\beta^2\sigma_u^2 +$$

$$\frac{A\left[1 + A(\theta b + k)^2\right]}{\left[1 + Ak(\theta b + k)\right]^2} (\bar{g} + b)^2.$$

Similarly, the loss function of wage setters is given by:

$$LW = (\alpha\beta)^2\left\{\left[\frac{1 - \eta}{1 + \alpha\beta(1 - \eta)}\right]^2\sigma_M^2 + \left[\frac{\eta}{1 + \alpha\beta(1 - \eta)} - \frac{h}{\beta + h}\right]^2\sigma_u^2\right\},$$

(27)

where

$$\sigma_M^2 \equiv \sigma_\epsilon^2 + \sigma_\mu^2 = \sigma_\epsilon^2 + \left\{\frac{A(\theta b + k)\left[1 + \alpha\beta(1 - \eta)\right]}{1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1 - \eta)^2}\right\}^2\sigma_g^2.$$

(28)

The loss function given in equation (27) looks exactly the same as that given in equation (14) in Gray's (1976) paper. The difference between equation (27) and Gray's (1976) loss function is that, in the present framework, the variance of monetary shocks, σ_M^2, is endogenous since it is obtained from the government's optimization problem. In particular, the variance of monetary shocks equals the sum of the variance of the exogenous (money-demand) shock ϵ, σ_ϵ^2, and the variance of the rate of monetary expansion, σ_μ^2. The variance of the rate of monetary expansion, in turn, is a function of both θ and η, as well as of the variance of government expenditure, σ_g^2.

I now turn to the determination of equilibrium wage and debt indexation. To facilitate intuition, I will focus on special cases that

provide the essential elements driving the choice of θ and η. During the ensuing discussion, θ will be restricted to the interval $[0, 1]$. 1/

Case I: $A = 0$. If $A = 0$, then conventional taxes are not distortionary. This implies that there is no time-inconsistency problem in government behavior. It is easy to verify that in this case the government will finance government expenditure only by raising conventional taxes. Hence, the rate of monetary expansion, μ, will be set equal to zero, and the degree of debt indexation, θ, is irrelevant since it does not affect inflation (θ, however, affects the variability of conventional taxes). Since $\mu = 0$, the variance of monetary shocks, σ_M^2, equals σ_ϵ^2.

The choice of the degree of wage indexation, η, follows from minimizing loss function (17). This is exactly the problem studied by Gray (1976). It is well known that in the presence of both monetary and real shocks it is optimal to have partial wage indexation; i.e., $0 < \eta^o < 1$, where superscript o stands for "optimal". Moreover, the optimal degree of wage indexation depends positively on the variance of monetary shocks, σ_M^2, and negatively on the variance of the real shock, σ_u^2. It can also be shown that η^o is positively related to the labor supply elasticity, h, and negatively to the labor demand elasticity, $1/(1 - \alpha)$.

The intuition behind the choice of the degree of wage indexation follows from Gray (1976). In the presence of shocks to productivity it is optimal to have real wage flexibility. Indeed, if the variance of monetary shocks, σ_M^2, equals zero, then Gray (1976) shows that the optimal degree of wage indexation, η^o, equals $h/(1 + h)$, which is non-negative and less than one. This degree of wage indexation achieves the optimal response of the real wage to real shocks. At the other extreme, if the only shocks are monetary—i.e., if $\sigma_u^2 = 0$—then it is optimal to index wages fully—i.e., $\eta^o = 0$. This reflects the fact that in the absence of shocks to productivity it is not optimal to have real wage variability.

Case II: $B = 0$. In this case the government carries out its monetary and fiscal policy without taking output deviations into account in its objective function.

Government 0 chooses θ to minimize:

1/ In theory θ could take values outside the $[0, 1]$ range, as discussed in Calvo and Guidotti (1990a). When $\theta > 1$, the government swaps nominal debt for indexed assets. Conversely, when $\theta < 0$ the government lends in nominal terms against indexed debt. Nothing essential is lost by assuming that $\theta \in [0, 1]$, since parameters can always be chosen to make optimal θ lie in the unit interval.

$$LG = \frac{A\sigma_g^2}{1 + A(\theta b + k)^2} + \frac{1 + A(\theta b + k)^2}{[1 + \alpha\beta(1 - \eta)]^2} (\sigma_\epsilon^2 + \beta^2\sigma_u^2) +$$

$$\frac{A[1 + A(\theta b + k)^2]}{[1 + Ak(\theta b + k)]^2} (\bar{g} + b)^2.$$

(29)

Loss function (29) is comparable to the government loss function given in equation (20) in Calvo and Guidotti (1990a). In fact, both are equal in the case that $\sigma_\epsilon^2 = \sigma_u^2 = 0$. A new term appears in equation (29), as compared to the government loss function studied by Calvo and Guidotti (1990a): the second term on the R.H.S. of the equation, which contains the variance of shocks ϵ and u, and the expression $[1 + \alpha\beta(1 - \eta)]$. The latter, in particular, makes the government's choice of θ dependent on the degree of wage indexation set by the private sector.

These differences notwithstanding, it is shown in the Appendix that the choice of the optimal degree of debt indexation maintains the qualitative properties discussed by Calvo and Guidotti (1990a). In particular, it is shown that for appropriate parameter values optimal θ, θ^o, is greater than zero and less than one. Moreover, the optimal degree of debt indexation, $1 - \theta^o$, decreases with the variance of government expenditure, σ_g^2, and increases with the expected value of government expenditure, g, and the level of public debt, b.

The intuition for these results follows from considering the trade-off faced by government 0. On the one hand, a higher level of nominal debt— i.e., less debt indexation—creates incentives for government 1 to generate a sub-optimally high inflation-rate because of the time-inconsistency problem. On the other hand, a higher level of nominal debt enlarges the base of the *unanticipated* portion of the inflation tax. (Recall that, since the nominal interest rate reflects expected inflation point-for-point, no inflation tax can be collected *on average* on nominal debt.) Since inflation is costly, a higher inflation tax base makes the inflation tax more efficient for smoothing out conventional taxes—i.e., it raises the amount of government revenue that can be generated with a given level of inflation distortion. Moreover, since nominal bonds are part of the inflation tax base corresponding to unanticipated inflation, the gains from nominal debt accrue only to the extent that there is budget-constraint variability. Thus, a higher variance of g increases the scope for nominal debt. Conversely, because of the time-inconsistency problem a higher g as well as higher b increase the incentives to raise average inflation, thus inducing a reduction in θ^o.

It is apparent from equation (29) that the level of wage indexation, as well as the variances of shocks ϵ and u, affect the optimal degree of debt indexation. In particular, the Appendix shows that optimal debt indexation,

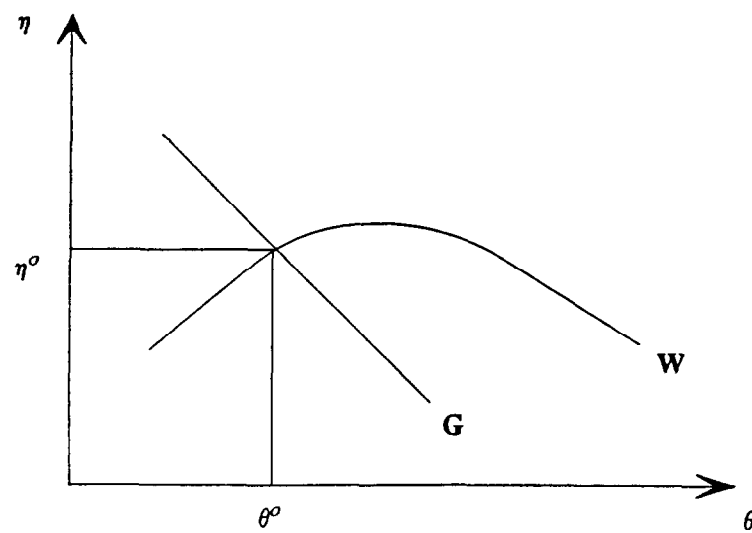$1 - \theta^o$, is positively related to η, as well as to the variances of shocks ϵ and u.

The relation between θ^o and σ_ϵ^2 and σ_u^2 derives from the following considerations. The variability of unobservable shocks ϵ and u induces output variability which, in turn, generates—through money market equilibrium—inflation variability that is not under the control of the policymaker. This element of inflation variability is costly and plays no useful role in smoothing conventional taxes. On the contrary, since unanticipated inflation generates revenue fluctuations, it results in unwanted tax variability, which is uncorrelated with government expenditure. Since this effect is magnified by the presence of nominal debt, it follows that the higher is the uncontrollable portion of inflation variability the larger is the share of the public debt that should be indexed.

The presence of partial wage indexation implies that output variability reflects the variance of real wages, in addition to the variance of the technological shock u. By equation (24) it can be observed that the higher is the degree of wage indexation the higher, *ceteris paribus*, is the variability of inflation. This reflects the fact that unexpected inflation is associated with an expansion of output—and hence of money demand. By equation (20), however, it can be observed that the variance of the rate of monetary expansion, μ, takes into account the role of wage indexation—i.e., $[1 + \alpha\beta(1 - \eta)]$ appears in the numerator of the second term on the R.H.S. of (20). Hence the relationship between the variance of government expenditure and the variance of inflation is independent of the degree of wage indexation.

The magnification role played by wage indexation, however, applies to the uncontrollable—and, hence, unwanted—portion of inflation variability. The higher is the degree of wage indexation, the higher is the unwanted variability of inflation, for given σ_ϵ^2 and σ_u^2. This explains why there is a positive relationship between the degrees of wage and debt indexation in the government's optimization problem. More wage indexation generates higher unwanted inflation variability, thereby inducing the government to index more the public debt. Thus, the government's reaction function, which provides θ^o as a function of η, is downward sloping in the (η, θ) plane, as depicted by schedule G in Figure 1. Interestingly, the fact that higher wage indexation induces the government to choose higher debt indexation implies that higher wage indexation leads the government to choose a more anti-inflationary policy stance. This follows from the fact that higher debt indexation reduces government 1's incentive to inflate.

Consider now the private sector's wage indexation choice. Wage setters choose η to minimize loss function (27), where σ_M^2 is now given by:

Figure 1: Wage and Public Debt Indexation

$$\sigma_M^2 = \sigma_\epsilon^2 + \sigma_\mu^2 = \sigma_\epsilon^2 + \left\{ \frac{A(\theta b + k)\left[1 + \alpha\beta(1 - \eta)\right]}{1 + A(\theta b + k)^2} \right\}^2 \sigma_g^2. \tag{30}$$

The full characterization of wage indexation is provided in the Appendix; in particular, the basic insights of Gray (1976) are shown to remain valid in this case. Equation (30) illustrates that the variance of the rate of monetary expansion is a non-monotonic function of θ. In particular, it is easy to show that it is increasing for low values of θ, and decreasing for high values of θ. This non-monotonicity reflects two factors. At low values of θ, an increase in θ makes the inflation tax more efficient; thus it induces a substitution away from conventional taxes to the inflation tax. For sufficiently high levels of θ, additional nominal debt decreases the variability of *both* conventional taxes and the inflation tax by enlarging the unanticipated-inflation tax base. Thus, a *substitution effect* dominates at low levels of θ, while a *scale effect* dominates at high levels of θ.

The non-monotonicity of σ_M^2 as a function of θ implies that the reaction function of wage setters is non-monotonic as well, as illustrated by schedule W in Figure 1. At low values of θ, an increase in θ—i.e., less debt indexation—increases the variance of monetary shocks and, hence, increases the optimal degree of wage indexation, η. At high levels of θ, less debt indexation decreases the variance of monetary shocks and, hence, decreases the optimal degree of wage indexation.

The intersection of schedules G and W provides the Nash equilibrium (η^o, θ^o). Depending on parameter values, schedule G can intersect schedule W to the left of the point of maximum η (shown in Figure 1) or to the right. 1/ The implications of these two alternative configurations will be explored below. 2/

I now turn to study the effects of changes in various exogenous parameters on the equilibrium levels of wage and debt indexation. (The corresponding analytical derivations can be found in the Appendix.) Consider first the effects of an increase in the variance of government expenditure, σ_g^2. A rise in σ_g^2 shifts schedule G to the right, while it shifts schedule W upwards. Schedule G shifts to the right because, by previous considerations, a rise in σ_g^2 increases the tax-smoothing role of nominal debt. Schedule W shifts up because an increase in σ_g^2 increases the variance of monetary shocks, thus inducing the private sector to choose a higher degree of wage indexation. In the case where schedule G intersects schedule W on its upward-sloping segment, a rise in σ_g^2 results in an increase in η^o—i.e., more wage indexation—but has an ambiguous effect on

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1/ Existence of these two possible configurations was established by means of numerical simulations.

2/ When schedule W is decreasing it is less steep than schedule G.

the degree of debt indexation, $1 - \theta^o$. The ambiguous effect on θ^o responds to two opposing elements. On the one hand, for a given degree of wage indexation the government will reduce debt indexation when government expenditure becomes more volatile. On the other hand, higher volatility of government expenditure increases the variance of monetary shocks and induces more wage indexation. Higher wage indexation, by previous arguments, induces more debt indexation.

When schedule G intersects schedule W on its downward-sloping part, an increase in σ_g^2 has an ambiguous effect on both wage and debt indexation. The ambiguity with respect to η^o comes from two opposing effects. On the one hand, *ceteris paribus* an increase in σ_g^2 increases the variance of monetary shocks. On the other hand, the reduction in the equilibrium degree of debt indexation tends to reduce the variance of monetary shocks. This provides an example in which government policy regarding debt indexation dampens the private sector's incentive to increase the degree of wage indexation. The rise in σ_g^2 increases the variability of inflation and, as one would expect, tends to reinforce the incentive to increase wage indexation. The government response to the rise in σ_g^2—by indexing more the public debt—reduces inflation volatility and, hence, dampens the private sector's incentive to raise wage indexation. This example shows that more debt indexation may indeed reduce wage indexation.

Consider the effects of an increase in expected government expenditure, $\bar{g}$. An increase in $\bar{g}$ has no effect on schedule W and shifts schedule G to the left. Schedule G shifts to the left because an increase in expected government expenditure tends to raise expected inflation. Hence, it is optimal for government 0 to reduce the amount of nominal debt outstanding to reduce government 1's incentives to generate inflation. Thus an increase in $\bar{g}$ induces a fall in both η^o and θ^o if the initial equilibrium is on the upward-sloping portion of schedule W, and induces an increase in η^o and a fall in θ^o if the initial equilibrium is on the downward-sloping portion of schedule W. The fall or increase in η^o reflects the different effects that less debt indexation may have on the variance of monetary shocks affecting wage contracts.

Consider the effects of an increase in σ_ζ^2. From the government's point of view, an increase in σ_ζ^2 raises the optimal degree of debt indexation for any given wage indexation level. More debt indexation is optimal because a higher σ_ζ^2 implies that the policymaker has less control over inflation variability. This implies that an increase in σ_ζ^2 shifts schedule G to the left. Similarly, *ceteris paribus* a higher σ_ζ^2 increases the variance of monetary shocks and, hence, shifts schedule W up. As a result, a higher σ_ζ^2 reduces θ^o—i.e., more debt indexation—but has an ambiguous effect on η^o if the initial equilibrium is on the upward-sloping portion of schedule W, and induces an increase in η^o and a fall in θ^o if the initial equilibrium is on the downward-sloping portion of schedule W.

Comparing the effects of an increase in σ_g with those of an increase in σ_ϵ illustrates the importance of the policy response in determining the equilibrium relation between wage and public debt indexation. From the point of view of wage setters an increase in both variances would, *ceteris paribus*, increase monetary volatility, thereby resulting in higher wage indexation. This is why schedule W is affected in the same way in both cases. From the point of view of the government, however, increases in σ_g and σ_ϵ yield opposite policy responses--causing schedule G to shift in opposite directions. While higher volatility of government expenditure induces it to reduce debt indexation, higher volatility in the demand for money induces the government to increase debt indexation. This illustrates a case in which exogenous changes that would have similar effects in the absence of a government response may end up having opposite equilibrium effects because of the different policy responses they induce.

Consider the effects of an increase in the variance of the technological shock, σ_u^2. A rise in σ_u^2 shifts schedule G to the left, and shifts schedule W down. The shift of schedule G reflects the fact that higher σ_u^2 increases the portion of inflation variability that is not controlled by the government. Schedule W shifts down because, *ceteris paribus*, an increase in the variance of real shocks makes it optimal for wage setters to reduce the degree of wage indexation. Hence, when schedule G intersects schedule W on its upward-sloping portion, an increase in σ_u^2 results in a fall of η^o--i.e., less wage indexation--but has an ambiguous effect on the degree of debt indexation. When schedule G intersects schedule W in its downward-sloping section, an increase in σ_u^2 has ambiguous effects on both η^o and θ^o.

Consider the effects of an increase in the debt level, b. A higher public debt level shifts schedule G to the left since, as discussed earlier, it induces the government to increase the degree of debt indexation. Schedule W shifts to the left; in particular, for given levels of θ, wage setters choose a higher η for low values of θ and a lower η for high levels of θ. This reflects the different effects that an increase in b exerts on the variance of monetary shocks, depending on whether one is situated in the upward or on the downward-sloping part of schedule W. As a result, an increase in b reduces θ^o but has an ambiguous effect on η^o if the initial equilibrium was on the upward-sloping part of schedule W, and has an ambiguous effect on both θ^o and η^o if the initial equilibrium was on the downward-sloping part of schedule W.

Finally, consider the effects of a change in the elasticity of labor supply. An increase in the labor supply elasticity, h, has no effect on schedule G and shifts schedule W up. This implies that an increase in h results in higher indexation of both wages and debt.

The above analysis shows that, depending on the type of exogenous disturbance, the degrees of wage and debt indexation may move in the same or in opposite directions. While in Gray (1976) the effects on wage indexation

can be cleanly classified in terms of the source of shocks (i.e., on whether they are monetary or real), in the present framework the relationship between the sources of shocks and the degrees of wage and debt indexation is less clear cut. There are two reasons for this. First, government policy responses are introduced into the analysis. Second, it is shown that the effect of changes in the degree of debt indexation on the variance of the rate of monetary expansion is non-monotonic.

<u>Case III: The general case</u>. In addition to the effects considered in Case II, this case incorporates the deviations of output from its full information level in the government objective function.

Consider first the effect of introducing output deviations into the government's loss function—i.e., letting B be greater than zero. In particular, I will henceforth focus on the effects of an increase in B at an initial equilibrium where $B = 0$. As indicated from equations (20)-(25), inflation and tax variability reflect the variability of shocks ϵ and u, and the government's choice of the degree to which the rate of monetary expansion responds to shocks in government expenditure. In particular, since the rate of monetary expansion is uncorrelated with the technological shock, u, its variance adds to output variability. Thus, introducing output deviations in the government's loss function induces the policymakers to reduce the optimal variance of the rate of monetary expansion. By equations (20) and (21), it can easily be observed that an increase in B reduces the variance of the rate of monetary expansion but increases the variance of conventional taxes, reflecting a substitution from unanticipated inflation into conventional taxes as means of financing shocks to government expenditure.

What is the effect on the optimal degree of debt indexation? Increasing B may be thought of as making inflation variability more costly to the policymaker. Thus, one would expect that the optimal share of the public debt would move in the direction of reducing inflation variability. Indeed, the Appendix shows that, for given η and for *low* initial values of θ^o, optimal debt indexation increases as B increases. For *high* values of θ^o and given η, optimal debt indexation declines following an increase in B. This implies that an increase in B shifts schedule G to the left for low initial values of θ^o, and shifts schedule G to the right for high initial values of θ^o. The Appendix shows that a *low* initial value of θ^o in the previous sentences refers to values of θ at which $\partial \sigma_M^2/\partial \theta > 0$, implying that schedule W is upward-sloping. Similarly, a *high* initial value of θ^o refers to values of θ at which $\partial \sigma_M^2/\partial \theta < 0$, for which schedule W is downward-sloping.

Consider next the effects on the choice of η of letting B be greater than zero. Since, by previous considerations, an increase in B results in lower inflation variability, it is to be expected that it will reduce the optimal degree of wage indexation for given θ^o. The Appendix shows that, starting from $B = 0$, an increase in B indeed induces wage setters to choose,

ceteris paribus, lower wage indexation. This implies that an increase in B shifts schedule W down.

The above considerations imply that if the initial equilibrium is on the upward-sloping portion of schedule W, then a higher B induces a fall in η^o—i.e., less wage indexation—but has an ambiguous effect on the degree of debt indexation. If the initial equilibrium is on the downward-sloping portion of schedule W, then a higher B induces a fall in η^o—i.e., less wage indexation—and an increase in θ^o—less debt indexation.

The effects of comparative statics exercises in the general case have also been explored by means of numerical simulations. These suggest that the qualitative response to shocks analyzed in Case II remains robust.

IV. Wage and Debt Indexation under Policy Precommitment

In this section, the model is modified by allowing government 0 to credibly precommit the policies of government 1. Calvo and Guidotti (1990a) showed that under full policy precommitment the flexibility role of nominal debt becomes the dominant element in deciding the optimal degree of debt indexation. In their model, the absence of costs from time-inconsistency implies that the optimal policy calls for expanding the amount of nominal public debt indefinitely with the objective, in the limit, of completely smoothing out both inflation and conventional taxes. In fact, in Calvo and Guidotti (1990a), swapping an unboundedly large nominal stock of debt for and unboundedly large stock indexed assets—note that *net* public debt is unaffected by such swap—attains, in the limit, the complete-markets solution.

It turns out that this remarkable result does not necessarily apply to the present framework. The reason is that in the present model there are unobservable shocks that impart unwanted volatility to inflation and conventional taxes. As long as this uncontrollability problem exists, a swap of nominal debt for indexed assets of the type described above would magnify the unwanted variability of inflation and taxes by enlarging without bound the base on the unexpected-inflation tax. Hence, in the present framework, the optimal share of nominal debt, θ^o, will remain bounded as long as either σ_ϵ^2 or σ_u^2 is positive.

As in Calvo and Guidotti (1990a), it can be shown that optimal government policies under precommitment differ from those without precommitment only insofar as the expected levels of monetary expansion and taxes are concerned. The state contingent part of the policy is the same in the presence or absence of precommitment. In particular, the optimal policy regarding taxes and the rate of monetary expansion under precommitment is given by equations (20), (21), (22), (24), and (25), where $E_1(\bullet)$ is simply interpreted as the expectation over ϵ and u, and $E\mu$ is given by:

$$E\mu = E\pi = \frac{Ak}{1 + Ak^2} (\bar{g} + b). \tag{31}$$

Equation (31) shows that if government 0 can precommit policies, then the optimal policy calls for expected inflation to be independent of the stock of nominal debt. This reflects the fact that, as mentioned earlier, the inflation tax collects no revenue on average on nominal debt because the nominal interest rate incorporates any expected inflation, point-for-point.

In order to fix ideas, I concentrate on Case II, where $B = 0$. Thus, the government loss function under precommitment is given by:

$$LG = \frac{A\sigma_g^2}{1 + A(\theta b + k)^2} + \frac{1 + A(\theta b + k)^2}{[1 + \alpha\beta(1 - \eta)]^2} (\sigma_\epsilon^2 + \beta\sigma_u^2) +$$
$$\frac{A}{(1 + Ak^2)} (\bar{g} + b)^2. \tag{32}$$

Equation (32) is the precommitment analog of equation (29). As indicated earlier, the only difference between the two loss functions comes from the term involving expected inflation and taxes. 1/ If σ_ϵ^2 and σ_u^2 equal zero, then the minimization of equation (32) with respect to θ reproduces the result of Calvo and Guidotti (1990a): θ should be set unboundedly large. If either σ_ϵ^2 or σ_u^2 is positive, then optimal $\theta < \infty$. Moreover, if $(\sigma_\epsilon^2 + \beta\sigma_u^2)/[1 + \alpha\beta(1 - \eta)]^2 < \sigma_g^2$, then optimal $\theta > 0$; it is henceforth assumed that this condition holds. 2/

It is easy to verify that the optimal degree of debt indexation is obtained from the following first-order condition:

$$[1 + A(\theta b + k)^2]^2 = \frac{\sigma_g^2[1 + \alpha\beta(1 - \eta)]^2}{\sigma_\epsilon^2 + \beta\sigma_u^2} > 1. \tag{33}$$

In accord with intuition it can be shown that, *ceteris paribus*, optimal θ is higher—i.e., optimal debt indexation is lower—in the presence of precommitment than in its absence. 3/ Furthermore, equation (33) shows that, as in the no-precommitment case, the government reaction function is

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1/ The derivation of equation (32) follows the same methodology set out in the Appendix to Calvo and Guidotti (1990a).

2/ A stricter condition is assumed for the case of no precommitment in order to make $\theta^o > 0$, as the Appendix shows.

3/ The proof follows directly from comparing equation (33) to its analog—equation (A7) in the Appendix—in the absence of precommitment.

negatively sloped in the (η, θ) plane. The Nash equilibrium with no policy precommitment (θ^o, η^o) is determined by equation (33) and the solution to the wage indexation problem of the previous section—i.e., the minimization over η of equation (27) where σ_M^2 is given by equation (30). The fact that the optimal degree of debt indexation is smaller in the presence of precommitment implies that, for given parameter values, it is more likely that the precommitment equilibrium (θ^o, η^o) be situated on the downward-sloping portion of schedule W than the no-precommitment equilibrium.

It can be shown that several comparative statics exercises under policy precommitment yield the same qualitative (although not quantitative) results as in the absence of precommitment. This is, for instance, the case for the effects of an increase in σ_g^2, σ_μ^2 on (θ^o, η^o).

Other comparative static exercises, however, yield different qualitative results depending on whether government 0 can precommit its policies. Consider, for instance, the effects of an increase in expected government expenditure, g. Without precommitment, an increase in g shifts schedule G to the left. This reflects the fact that expected inflation is a function of the degree of debt indexation because of the time-inconsistency problem. Under precommitment, however, expected inflation is independent of the degree of debt indexation because policymakers realize that the inflation tax collects no revenue on average on the stock of nominal debt. Thus, under precommitment, schedule G is independent of θ and changes in g have no effect on the equilibrium (θ^o, η^o).

V. Conclusions

This paper has explored the interconnections that may exist between government policy regarding the degree of indexation of the public debt, and the private sector's choice of wage indexation. The analysis has been carried out in a framework where incomplete markets and incentives in government policy play central roles.

The analysis suggests that the relationship between wage and public debt indexation may be quite complex. As far as the government's decision problem is concerned, it was shown that the optimal degree of debt indexation is positively related to the degree of wage indexation chosen by the private sector. As far as wage setters are concerned, it was shown that the relationship between the optimal degree of wage indexation and the level of debt indexation is non-monotonic because of the different effects that debt indexation may have on inflation variability.

Wage and public debt indexation are determined as a Nash equilibrium. Whether equilibrium wage and debt indexation are positively or negatively related in response to changes in exogenous parameters depends on the nature of the changes being analyzed, as well as on some characteristics of the initial equilibrium. It also may depend on whether or not the government

has the ability to precommit future policies. Hence, the analysis does not support the presumption that, in general, equilibrium wage and debt indexation move in the same direction. Moreover, the analysis provides examples in which more wage indexation may induce the government to adopt a more anti-inflationary policy stance. Therefore, the results of the analysis weaken the presumption that indexation encourages policymakers to tolerate inflation.

Finally, this paper has shown that uncontrollability of inflation as a tax instrument is an additional element that favors indexing the public debt. On the other hand, introducing output fluctuations in the government objective function does not have a clear-cut effect on optimal public debt indexation.

The analysis has relied on existing theories of wage and public debt indexation. The insights obtained should be viewed as a first step towards understanding the macroeconomic interactions of different forms of indexation. Further analysis could be directed at exploring in greater depth the microfoundations of the problem, as well as the nature of wage contracts.

The government's decision problem. Consider first the government problem of section III. In period 1, the government chooses the pair of functions $\{E_1x(g),\ \mu(g)\}$ to minimize loss function (18), subject to equations (10) and (16). The corresponding first-order condition for a minimum is given by equation (19). Taking expectations of constraint (16) based on information available in period 1, it follows that:

$$E_1x = g + b + \theta b E\pi - (\theta b + k)E_1\pi, \qquad (A1)$$

where equilibrium condition (17) has been taken into account.

Equations (22) and (23) are obtained by taking expectations of equations (16) and (19), based on information available at time 0. Adding $A(\theta b + k)(g + b)$ and subtracting $[1 + Ak(\theta b + k)]E\pi$—note that these expressions are equal, by equation (22)—from (19), after replacing E_1x by the R.H.S. of (A1), the following equation obtains:

$$- A(\theta b + k)(g - \bar{g}) + \left[1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1 - \eta)^2\right](E_1\pi - E\pi) = 0. \quad (A2)$$

From equations (10) and (11), it follows that:

$$E_1\pi - E\pi = \frac{\mu - E\mu}{1 + \alpha\beta(1 - \eta)}. \qquad (A3)$$

Hence, equation (20) follows from combining equations (A2) and (A3). Equation (24) follows from equations (10) and (11).

By taking expectations of equation (19), it follows that:

$$Ex = \frac{E\pi}{A(\theta b + k)}. \qquad (A4)$$

Equation (21) follows from combining (19), (A3), and (A4). Equation (25) is obtained by combining equations (16), (A1), and (A3). Finally, loss function (26) is obtained by using equations (20)-(25).

Assume that $B = 0$. Then loss function (26) may be written as:

$$f(z) = \frac{\Omega}{z} + \phi z + \Gamma \frac{z}{\left(1 + k[A(z - 1)]^{\frac{1}{2}}\right)^2}, \tag{A5}$$

where $z \equiv 1 + A(\theta b + k)^2$, $\Omega \equiv A\sigma_g^2$, $\Phi \equiv [A(\sigma_\epsilon^2 + \beta^2\sigma_u^2]/[1 + \alpha\beta(1 - \eta)]^2$, and $\Gamma \equiv A(\bar{g} + b)^2$. Of course, $z \geq 1$. It can be verified that $f(1) = \Omega + \Phi + \Gamma < f(\infty) = \infty$. Moreover,

$$f'(z) = -\frac{\Omega}{z^2} + \Phi + \Gamma \frac{\omega - kA}{(1 + k\omega)^2\omega}, \tag{A6}$$

where $\omega \equiv A(z - 1)^{\frac{1}{2}}$. By (A6), $\lim (z \to 1) f'(z) = -\infty$. Hence, argmin $f(z) > 1$, and by previous considerations $< \infty$. In particular, optimal z satisfies the following first-order condition:

$$\frac{\Omega}{z^2} = \Phi + \Gamma \frac{\omega - kA}{(1 + k\omega)^2\omega}. \tag{A7}$$

It is assumed that

$$\frac{\Omega}{z^2} > \Phi \quad \Rightarrow \quad \frac{\sigma_g^2}{\left[1 + A(\theta b + k)^2\right]^2} > \frac{\sigma_\epsilon^2 + \beta^2\sigma_u^2}{\left[1 + \alpha\beta(1-\eta)\right]^2}. \tag{A8}$$

Hence, (A6) and (A7) imply that $\omega > kA$; this implies that optimal $\theta > 0$.

Using the fact that, at a minimum, $f''(z) > 0$, one can use (A5) to compute comparative statics exercises. It can be easily shown that optimal θ decreases with η, $\bar{g}$, b, σ_ϵ^2, σ_u^2, and increases with σ_g^2. The comparative statics with respect to η, in particular, indicates that the government's reaction function G is downward-sloping in the (η, θ) plane.

Consider next the case in which $B > 0$. In the general case, the first-order condition for the choice of θ is given by:

$$f'(z) = -\frac{(1 + \gamma)\Omega}{(z + \gamma)^2} + \Phi + \Gamma \frac{\omega - kA}{(1 + k\omega)^2\omega} = 0, \tag{A9}$$

where $\gamma \equiv 1 + B(\alpha\beta)^2(1 - \eta)^2$. By (A7), optimal $\theta > 0$. Differentiating equation (A9) one obtains:

$$f''(z)dz - \Omega \frac{z + \gamma - 2(1 + \gamma)}{(z + \gamma)^2} d\gamma = 0. \tag{A10}$$

Recalling that $f''(z) > 0$, equation (A10) implies that, at $B = 0$, $d\theta^o/dB > 0$ for $z < 2$ and $d\theta^o/dB < 0$ for $z > 2$. In particular, it is easy to check that the condition which signs $d\theta^o/dB$, at $B = 0$ is the same as the one that determines whether σ_M^2 increases or decreases with θ. This is the same condition that determines whether schedule W is upward-sloping or downward-sloping. From equation (28), it can be verified that, at $B = 0$, σ_M^2 increases with θ as long as $z < 2$, and decreases with θ when $z > 2$.

The wage setters' decision problem. The wage setters' loss function (27) is obtained by equations (8), (20) and (24). Equation (27) may be written as

$$LW = (\alpha\beta)^2 \left\{ \left[\frac{1-\eta}{1+\alpha\beta(1-\eta)}\right]^2 \sigma_\epsilon^2 + (1-\eta)^2 \Psi(\theta,\eta,B)\sigma_g^2 + \left[\frac{\eta}{1+\alpha\beta(1-\eta)} - \frac{h}{\beta+h}\right]^2 \sigma_u^2 \right\}, \tag{A11}$$

where

$$\Psi(\theta,\eta,B) \equiv \left[\frac{A(\theta b + k)}{1 + A(\theta b + k)^2 + B(\alpha\beta)^2(1-\eta)^2}\right]^2.$$

The first-order condition for the choice of η is given by:

$$\frac{\partial LW}{\partial \eta} = \frac{2(\alpha\beta)^2}{[1+\alpha\beta(1-\eta)]^3} \left\{ -(1-\eta)r\left[\sigma_\epsilon^2 + \frac{1+h}{h+\beta}\beta^2\sigma_u^2\right] + \frac{\beta^2\sigma_u^2}{h+\beta} \right\} +$$

$$- (1-\eta)\Psi(\theta,\eta,B)\sigma_g^2 + \frac{1}{2}(1-\eta)^2\sigma_g^2\frac{\partial\Psi(\theta,\eta,B)}{\partial\eta} = 0. \tag{A12}$$

Consider first the case where $B = 0$. This implies that $\partial\Psi/\partial\eta = 0$ in equation (A12). It can be shown that, as in Gray (1976), $\eta^o = h/(1 + h)$ when $\sigma_\epsilon^2 = \sigma_g^2 = 0$, and $\eta^o = 1$ when $\sigma_u^2 = 0$. When there are both monetary and real shocks, η^o lies in the interval $[h/(1 + h), 1]$. Moreover, it can be shown that η^o is positively related to σ_ϵ^2, σ_g^2 and h, and it is negatively related to σ_u^2. From equation (A12), it can also be shown that η^o is positively related to θ when $\partial\sigma_M^2/\partial\theta > 0$, while it is negatively related to θ when $\partial\sigma_M^2/\partial\theta < 0$. This explains the form of schedule W. Finally, equation (A12) can be used to show that, at $B = 0$, $d\eta^o/dB < 0$.

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