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The Monetary Approach to the Exchange Rate: Rational Expectations,
Long-Run Equilibrium and Forecasting

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Abstract

We re-examine the monetary approach to the exchange rate from a number of perspectives, using monthly data on the deutschemark-dollar exchange rate. Using the Campbell-Shiller technique for testing present value models, we reject the restrictions imposed upon the data by the forward-looking rational expectations monetary model. We demonstrate, however, that the monetary model is validated as a long-run equilibrium condition. Moreover, imposing the long-run monetary model restrictions in a dynamic error correction framework leads to exchange rate forecasts which are superior to those generated by a random walk forecasting model.

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Summary

This paper re-examines the monetary model of the exchange rate from a number of complementary perspectives. Using data for the deutsche mark-U.S. dollar exchange rate during 1976-90, the paper demonstrates the following: First, the static monetary approach to the exchange rate is valid when it is considered as a long-run equilibrium condition. Second, when the exchange rate fundamentals suggested by the monetary model are assumed, the speculative bubbles hypothesis is rejected. Third, the full set of rational expectations restrictions imposed by the forward-looking monetary model is rejected. Fourth, the long-run estimate of the monetary model can be used to generate a dynamic error-correction exchange rate equation that has robust in-sample and out-of-sample properties and is superior to a random walk (the usual benchmark) in post-sample forecasting.

I. Introduction

In this paper we re-examine the monetary model of the exchange rate from a number of complementary perspectives. We begin by examining the validity of the model in its forward-looking rational expectations formulation, since although standard reduced-form monetary approach equations have received little empirical support for the recent floating experience (see MacDonald and Taylor (1992a)) the forward-looking monetary model has enjoyed relatively more success (see, *inter alia*, Hoffman and Schlagenhauf (1983); Woo (1985); Finn (1986)) ^{1/}. In this paper, however, we argue that previous tests of the forward model may have been incorrectly implemented. Our method of obtaining a forward-looking solution relies on the exploitation of the recently developed multivariate cointegration methodology (Engle and Granger (1987); Johansen (1988)) and its application to present value models (Campbell and Shiller (1987); MacDonald and Speight (1990); Taylor (1991, 1992)). Using data for the German mark--US dollar exchange rate over the period 1976-1990, we demonstrate the following. First, the static monetary approach to the exchange rate has some validity when considered as a long-run equilibrium condition. Second, when the set of exchange rate fundamentals suggested by the monetary model are assumed, the speculative bubbles hypothesis is rejected. Thirdly, the full set of rational expectations restrictions imposed by the forward-looking monetary model are rejected. Finally, however, we demonstrate that the monetary model can be used to generate a dynamic error correction exchange rate equation which has robust in-sample and out-of-sample properties--including beating a random walk (the usual benchmark for exchange rate forecasting) in post-sample forecasting.

II. The Monetary Model: Forward-Looking Restrictions, Bubbles, and Cointegration

The flexible price monetary model is now well known and requires only the briefest of descriptions here (see MacDonald and Taylor (1992a) for a comprehensive discussion). The model relies on a relative money market equilibrium condition as in (1), an expression linking the exchange rate to the home and foreign price level (2), purchasing power parity (PPP), and uncovered interest rate parity, linking home and foreign interest rates and the expected exchange rate change, (3):

$$m'_t - p'_t = \gamma y'_t - \lambda i'_t \quad (1)$$

^{1/} The forward-looking monetary model was used, for example, as the maintained hypothesis in Meese's (1986) study of foreign exchange market bubbles.

$$s_t = p'_t \quad (2)$$

$$i'_t = E(\Delta s_{t+1}^e | I_t) \quad (3)$$

where m denotes the logarithm of the domestic money supply, p the logarithm of the domestic price level, y the logarithm of domestic income, i the domestic interest rate, s the logarithm of the exchange rate (domestic price of foreign currency) and where, if an asterisk denotes a foreign variable, $m' = m - m^*$, etc. Δ is the first difference operator and $E(.|I_t)$ denotes the mathematical conditional expectation operator, conditional on the information set available to agents at time t , I_t . As is standard in the literature on the monetary approach to the exchange rate, we assume common money demand parameters in the home and foreign country.

Using (1) and (3) in (2) we may obtain a familiar reduced form for the spot exchange rate:

$$s_t = (1+\lambda)^{-1} x_t + \lambda(1+\lambda)^{-1} E(s_{t+1} | I_t) \quad (4)$$

where $x_t = [m' - \gamma y']_t$. Solving (4) forward, we obtain:

$$s_t = (1+\lambda)^{-1} \sum_{i=0}^{\infty} [\lambda/(1+\lambda)]^i E(x_{t+i} | I_t) \quad (5)$$

where the transversality condition $\lim_{i \rightarrow \infty} [\lambda/(1+\lambda)]^i E(s_{t+i} | I_t) = 0$ has been

imposed. Equation (5) is the basic equation of the forward-looking monetary approach to the exchange rate (FMAER). Equation (5) makes clear that the monetary model with rational expectations involves solving for the entire expected future path of the forcing variables (i.e. money supplies and income levels). An interesting implication of the present value model of the exchange rate, which has been exploited for stock prices and interest rates by Campbell and Shiller (1987), but has not been previously exploited in the exchange rate literature, is that the exchange rate should be cointegrated with the forcing variables contained in x_t . ^{1/} To see this, subtract x_t from both sides of (5) and rearrange:

^{1/} The seminal paper on cointegration is Engle and Granger (1987). See Cuthbertson, Hall, and Taylor (1992) for an accessible introduction to the literature on non-stationarity and cointegration.

$$s_t - x_t = \sum_{i=1}^{\infty} [\lambda/(1+\lambda)]^i E(\Delta x_{t+i}^e | I_t) \quad (6)$$

Now, if m_t , m_t^* , y_t , and y_t^* are first-difference stationary, $I(1)$ variables, then the right-hand side of (6) must be $I(0)$, subject only to the caveat that forecasting errors are stationary, which is clearly the case for rational expectations (see Taylor (1991)). ^{1/} Thus, the right hand side of (6) must also be stationary and so, if s_t is also an $I(1)$ series, the exchange rate must be cointegrated with m_t , m_t^* , y_t , and y_t^* , with certain restrictions on the cointegrating parameters:

$$L_t = s_t - m_t + m_t^* + \gamma y_t - \gamma y_t^* \sim I(0) \quad (7)$$

Previously when researchers (see, for example, Hoffman and Schlagenhauf (1983); Meese (1986); Finn (1986)) have implemented the present value exchange rate model they have used a first difference transformation of all the variables. However, as Engle and Granger (1987) have demonstrated, if a vector of variables is cointegrated, using a pure first difference transformation will be inappropriate (if some form of error correction term is incorporated into the equation the estimation will be legitimate; however, none of these researchers adopt this approach). Thus, a preliminary step in testing the FMAER is to test for a cointegration relation of the kind (7). Note that cointegration in (7) is not inconsistent with the existence of a cointegration relation corresponding to the basic flex-price monetary approach equation:

$$s_t = m_t' - \gamma y_t' + \lambda i_t' \quad (8)$$

which follows from equations (1) and (2). This is because, from (3)--the UIP condition, the interest rate differential, i_t' , must be $I(0)$ for $s_t \sim I(1)$.

If cointegration in (7) is found, then a more stringent test may be carried out by testing the forward restrictions. Following Campbell and Shiller (1987), this may be done as follows.

If L_t , as defined in (7), is in fact $I(0)$, we can obtain an estimate of it by cointegration methods. If x_t is $I(1)$ then both Δx_t and L_t are each stationary, $I(0)$, series. Thus, by the multivariate form of Wold's decomposition (Hannan, 1970), there exists a Wold representation which may be approximated by a vector autoregression (VAR) of lag depth p , say. Let

^{1/} A time series is said to be cointegrated of order d , denoted $I(d)$, if it must be differenced d times in order to achieve covariance stationarity.

$z_t = [\Delta x_t, \dots, \Delta x_{t-p+1}, L_t, \dots, L_{t-p+1}]'$. This VAR may be represented in companion form as:

$$\begin{bmatrix} \Delta x_t \\ \Delta x_{t-1} \\ \vdots \\ \Delta x_{t-p+1} \\ L_t \\ L_{t-1} \\ \vdots \\ L_{t-p+1} \end{bmatrix} = \begin{bmatrix} \vartheta_1 & \vartheta_2 & \dots & \vartheta_p & | & \kappa_1 & \kappa_2 & \dots & \kappa_p \\ \hline & I_{p-1} & & & | & & 0 & & \\ & & & & | & & & & \\ \hline \lambda_1 & \lambda_2 & \dots & \lambda_p & | & \mu_1 & \mu_2 & \dots & \mu_p \\ \hline & & & & | & & I_{p-1} & & \\ & 0 & & & | & & & & \end{bmatrix} \begin{bmatrix} \Delta x_{t-1} \\ \Delta x_{t-2} \\ \vdots \\ \Delta x_{t-p} \\ L_{t-1} \\ L_{t-2} \\ \vdots \\ L_{t-p} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \vdots \\ \nu_t \\ \vdots \end{bmatrix} \quad (9)$$

Or, in a more compact notation:

$$z_t = A z_{t-1} + v_t \quad (10)$$

Further, define g' and h' as $(1 \times 2p)$ selection vectors with unity in the $(p+1)$ th and first elements respectively, so that

$$L_t = g' z_t \quad (11)$$

and

$$\Delta x_t = h' z_t \quad (12)$$

The standard multi-period forecasting formula may be used to forecast z in any future period, that is

$$E[z_{t+k} | H_t] = A^k z_t \quad (13)$$

where H_t is a restricted information set consisting of current and lagged values of L_t and Δx_t .

Projecting both sides of (6) onto H_t , applying the law of iterated

mathematical expectations $\underline{1/}$, and using (11)-(13), we then have:

$$\begin{aligned} g' z_t &= \sum_{i=1}^{\infty} \left(\frac{\lambda}{1+\lambda} \right)^i h' A^i z_t \\ &= h' \psi A (I - \psi A)^{-1} z_t \end{aligned} \quad (14)$$

where $\psi = \left(\frac{\lambda}{1+\lambda} \right)$. If (14) is to hold non-trivially, the following 2p parameter restrictions are imposed on the VAR:

$$g' - h' \psi A (I - \psi A)^{-1} = 0 \quad (15)$$

Postmultiplying (15) by $(I - \psi A)$, we may then obtain a set of 2p linear restrictions which the FMAER imposes on the VAR for $(L_t, \Delta x_t)'$:

$$H_0: g' (I - \psi A) - h' \psi A = 0 \quad (16)$$

Now define the 'theoretical spread' as the right hand side of (14):

$$L_t^* = h' \psi A (I - \psi A)^{-1} z_t \quad (17)$$

Testing the restrictions (16) is tantamount to testing $H_0: L_t = L_t^*$, for all t . However, Campbell and Shiller (1987) point out that (16) may be rejected because of economically unimportant deviations from the null hypothesis such as data imperfections, which are nevertheless statistically significant. Thus, a less formal check on the validity of the restrictions is simply to compare the time series of the actual and theoretical spreads, L_t and L_t^* ; manifest differences in their behavior would be indicative of economically important deviations from the null hypothesis.

Now consider the implications of the present analysis for the detection of foreign exchange market bubbles. Add a bubble term b_t to the right hand side of (5):

1/ That is, $E(E(w|I_t)|H_t) = E(w|H_t)$, for $H_t \subseteq I_t$.

$$s_t = (1+\lambda)^{-1} \sum_{i=0}^{\infty} [\lambda/(1+\lambda)]^i E(x_{t+i} | I_t) + b_t \quad (18)$$

where b_t satisfies:

$$b_t = [\lambda/(1+\lambda)] E(b_{t+1} | I_t) \quad (19)$$

Subtracting x_t from both sides of (18), we have:

$$s_t - x_t = \sum_{i=1}^{\infty} [\lambda/(1+\lambda)]^i E(\Delta x_{t+i}^e | I_t) + b_t \quad (20)$$

Since b_t is explosive by construction, (20) implies that s_t and x_t cannot be cointegrated for $x_t \sim I(1)$. Thus, testing for stationarity of the spread, $L_t = s_t - x_t$, is equivalent to testing for the presence of bubbles. ^{1/} In contrast to Meese's (1986) test for foreign exchange market bubbles, however, we are able to distinguish between testing for bubbles (equivalent to testing non-stationarity of the spread) and testing the restrictions implied by the FMAER (ie (16)).

Note, however, that the procedure just outlined for testing the FMAER is non-operational as it stands, since both the income elasticity γ (needed to form x_t) and the interest rate semi-elasticity λ (needed to form ψ) are unknown. With respect to γ , an estimate can be obtained from the cointegration estimate of (7) and, because of the super consistency of cointegration parameter estimates (Stock (1987)), treated as known in testing (16). With respect to λ , we pursued two options. First, an estimate of λ can be obtained from the cointegration parameter in the basic flex-price equation (8). Again, by appeal to the super consistency result, this can then be treated as known in constructing tests of (16). The second option was to use extraneous estimates of λ from the literature. Bilson (1978), in his Bayesian estimation of the monetary model, uses a prior for λ of 0.015 with a 95 percent confidence interval ranging from zero to 0.03. Accordingly, we tried three values of λ within this interval: 0.015, 0.001, and 0.03.

^{1/} Note that bubbles were implicitly ruled out in the earlier discussion of the present value model by the imposition of the transversality condition $\lim_{i \rightarrow \infty} [\lambda/(1+\lambda)]^i E(s_{t+i} | I_t) = 0$

III. Econometric Methods

As MacDonald and Taylor (1991) have indicated, previous tests of the long-run relationship between the exchange rate and the monetary variables, which rely on the Engle-Granger (1987) two-step methodology, suffer from a number of deficiencies. In order to test for cointegration we use the multivariate cointegration technique proposed by Johansen (1988,1989) and Johansen and Juselius (1990). This technique is superior to the simpler regression-based technique because it fully captures the underlying time series properties of the data, provides estimates of *all* of the cointegrating vectors that exist within a vector of variables (i.e. using ordinary least squares to estimate a cointegration relationship for an N dimensioned vector does not clarify whether one is dealing with a unique cointegrating vector or simply a linear combination of the potential N-1 distinct cointegrating vectors which may exist within the system) and offers a test statistic for the number of cointegrating vectors (again, this contrasts with the regression-based methodology). This test may therefore be viewed as more discerning in its ability to reject a false null hypothesis. We now present a brief discussion of the Johansen technique. 1/.

Let X_t be an $N \times 1$ vector of $I(1)$ variables and assume that this vector has a k -th order vector autoregressive (VAR) representation with Gaussian errors ϵ_t : 2/

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \dots + \Pi_k X_{t-k} + \epsilon_t, \quad t=1,2,\dots,T. \quad (21)$$

where, for the purposes of exposition, we have excluded a constant. 3/

The long-run static equilibrium corresponding to (21) is: 4/

1/ For a more complete exposition, see Cuthbertson, Hall, and Taylor (1992).

2/ Phillips (1987) suggests that the Johansen technique may also be applicable in the presence of heterogeneously distributed error processes. In our empirical analysis, an intercept term was included in the VARs, as in Johansen and Juselius (1990).

3/ A constant term was included in our empirical work.

4/ Dynamic steady state equilibrium simply involves the addition of a term in the constant vector of steady-state growth rates to (7), which we omit here for expositional purposes; this does not affect the subsequent discussion.

$$\Pi X = 0, \quad (22)$$

where the long-run coefficient matrix Π is defined:

$$I - \Pi_1 - \Pi_2 - \dots - \Pi_K = \Pi. \quad (23)$$

Π is an $N \times N$ matrix whose rank determines the number of distinct cointegrating vectors which exist between the variables in X . Define two $N \times r$ matrices, α and β , such that

$$\Pi = \alpha\beta' \quad (24)$$

The rows of β' form the r distinct cointegrating vectors such that, if β'_i is the i th row of β' :

$$\beta'_i X_t \sim I(0) \quad (25)$$

Johansen demonstrates that the likelihood function for this problem is proportional to :

$$\mathcal{L} = \left(\prod_{i=1}^N (1 - \hat{\lambda}_i) \right)^{-T/2} \quad (26)$$

where $\hat{\lambda}_1, \dots, \hat{\lambda}_N$ the N squared canonical correlations between the X_{t-k} and ΔX_t series (arranged in descending order, so that $\hat{\lambda}_i > \hat{\lambda}_j$ for $i > j$), corrected for the effect of the lagged differences of the X process (for details of how to extract the $\hat{\lambda}_i$'s see Johansen 1988, 1989; Cuthbertson, Hall, and Taylor, 1992). Further, the number of distinct cointegrating vectors is shown to be equal to the number of non-zero $\hat{\lambda}_i$'s. Thus, the likelihood ratio statistic for the null hypothesis of at most r cointegrating vectors, the TRACE statistic, is seen to be:

$$\text{TRACE} = T \sum_{i=r+1}^N \ln(1 - \hat{\lambda}_i) \quad (27)$$

Additionally, the likelihood ratio statistic for testing at most r cointegrating vectors against the alternative of $r+1$ cointegrating

vectors--the maximum eigenvalue statistic--is given by:

$$\lambda_{MAX} = T \ln(1 - \lambda_{r+1}) \quad (28)$$

TRACE and λ_{MAX} will have non-standard distributions under the null hypothesis, although approximate critical values have been generated by Monte Carlo methods and tabulated by Johansen (1988), Johansen and Juselius (1990), and Osterwald-Lenum (1990). ^{1/}

An additional advantage of using the Johansen methodology is that it allows direct hypothesis tests on the coefficients entering the cointegrating vectors. We may therefore test the hypothesized values of the coefficients noted above and, additionally, the implicit restrictions that the coefficients on the money and income terms are equal and opposite. Such tests have in fact been conducted previously in the exchange rate literature (see *inter alia* Haynes and Stone, 1981, and Rasulo and Wilford, 1980). The novel feature of the present tests is that they are robust to the non-stationarity of the data; previous tests, which use the levels of the variables and standard t-tests or F-ratios, are not (see MacDonald and Taylor, 1991, for a further discussion).

IV. Data and Results

The data for this study relate to the German mark - US dollar exchange rate, are taken from the International Monetary Fund's International Financial Statistics data tape, and run from January 1976 through to December 1990. In particular, the exchange rate used (s) is line ag (expressed as home currency per unit of foreign currency), the monetary aggregates (m) are M1, line 34, the income measure (y) is industrial production, line 66c, and the short-term interest rate (r) is line 60c. The money supply and income measures are seasonally adjusted. In the remainder of the paper, an asterisk denotes a series corresponding to the United States; those without asterisks correspond to Germany. All series except interest rates were put into logarithmic form.

We conducted two cointegration exercises. The first involved testing for cointegration within the vector $(s_t, m_t^*, m_t, y_t^*, y_t, i_t^*, i_t)'$. This was done in order to obtain a preliminary estimate of λ . In the second exercise, we tested for cointegration among $(s_t, m_t^*, m_t, y_t^*, y_t)'$.

^{1/} The critical values recorded in Johansen's 1988 paper are for a VAR without an intercept term. Johansen (1989) reports critical values for VAR systems with a constant for systems of up to 5 variables. These critical values have been extended by Osterwald-Lenum (1990) for systems of up to 11 variables. We utilize these latter critical values in the present study.

In order to implement the Johansen procedure a lag length must be chosen for the VAR and the orders of integration of the series entering the VAR must be determined. Our procedure for choosing the optimal lag length was to test down from a general thirteen-lag system until reducing the order of the VAR by one lag could be rejected using a likelihood ratio statistic. The residuals from the chosen VAR were then checked for whiteness. If the residuals in any equation proved to be non-white, we sequentially chose a higher lag structure until they were whitened. For the system involving interest rates, we found that an eighth-order lag satisfied these criteria. For the system excluding interest rates, a twelfth-order system was necessary. The orders of integration of the series were determined using the standard Dickey-Fuller and Phillips-Perron statistics . ^{1/}

In Table 1 our tests for a unit root indicate that all series are I(1) processes. ^{2/} In Table 2 we report the trace and maximum eigenvalue statistics derived from the full system including interest rates. These indicate the presence of three statistically significant cointegrating vectors among the series. Imposing homogeneity on both relative money and income produces, moreover, an insignificant test statistic. With these restrictions imposed, the cointegration coefficients on German and US interest rates (taking the eigenvector corresponding to the largest eigenvalue of the system) are opposite in sign and almost equal in absolute magnitude: 0.049 and 0.05 respectively. However, imposing the additional restriction of equal and opposite interest rate coefficients on all three significant cointegrating vectors (as is necessary with the Johansen procedure--Johansen, 1988, p 236) led to a rejection of the null hypothesis at the 5 percent level (test statistic not reported). Since, however, this restriction is close to being satisfied on the most significant cointegrating vector (as reported in Table 2b), we inferred a value of λ of 0.05 as the cointegration estimate of this parameter. Note that these results provide evidence supportive of the flex-price monetary model, interpreted as a long-run equilibrium model. ^{3/} This in contrast to

^{1/} We report the former in addition to the latter since as Schwert (1987) has noted the latter statistics may reject the null of a unit root too often in the presence of a first order moving average process).

^{2/} For US industrial production, there is some sign that the series may be stationary about a trend in mean. However, given the power of these tests, and the values of the statistics obtained, we would argue that this evidence is slight, and treat this series as a unit root process. This view was confirmed by including a time trend in the VARs used to generate the Johansen results: the trend term was insignificant and the Johansen results were qualitatively unaffected.

^{3/} This confirms the findings of MacDonald and Taylor (1991) for the mark-dollar exchange rate. The results reported in that study differ slightly from those reported here since long-term as opposed to short-term interest rates are used in the former.

Table 1. Unit Root Tests

	τ_{μ}	τ_{τ}	$Z(\tau_{\mu})$	$Z(\tau_{\tau})$
s_t	2.86	0.05	-1.96	-1.83
Δs_t	-9.37	-15.68	-10.47	-10.40
m_t	-1.28	-1.48	1.28	-1.85
Δm_t	-14.66	-14.64	-14.75	-14.71
y_t	-1.90	-1.81	-1.83	-1.67
Δy_t	-9.29	-9.29	-9.79	-9.26
i_t	-0.11	-1.12	-0.52	-2.02
Δi_t	-4.19	-4.20	-5.13	-5.08
m_t^*	-0.56	-2.62	-0.70	-1.75
Δm_t^*	-9.34	-8.32	-11.53	-11.51
y_t^*	-0.77	-3.03	-0.66	-2.57
Δy_t^*	-2.71	-8.74	-8.77	-8.74
i_t^*	0.69	-2.36	-2.12	-2.21
Δi_t^*	-3.45	-3.85	-10.93	-10.89

Notes. See text for data definitions. An asterisk denotes a US variable, those without asterisks are German variables. τ_{μ} and τ_{τ} are standard augmented Dickey-Fuller test statistics with allowance for a constant mean and for a trend in mean respectively (Fuller, 1976). $Z(\tau_{\mu})$ and $Z(\tau_{\tau})$ are the Phillips-Perron transforms of these statistics (Phillips, 1987). The null hypothesis is that the variable in question is first-difference stationary, $I(1)$. Approximate 5 percent critical value for τ_{μ} and $Z(\tau_{\mu})$ is -2.89, with rejection region $\{\phi | \phi < -2.89\}$; the 5 percent rejection region for τ_{τ} and $Z(\tau_{\tau})$ is $\{\phi | \phi < -3.43\}$ (Fuller (1976)).

Table 2. Cointegration Results

a) System involving $s, m, m^*, y, y^*, i, i^*$

(i) Johansen statistics

Null Hypothesis	TRACE Statistic	5% Critical Value	Null Hypothesis	λ MAX Statistic	5% Critical Value
$r \leq 6$	1.43	9.09	$r=6 r=7$	1.43	9.09
$r \leq 5$	6.33	19.96	$r=5 r=6$	4.90	15.67
$r \leq 4$	18.17	34.91	$r=4 r=5$	11.84	22.00
$r \leq 3$	39.19	53.12	$r=3 r=4$	21.02	28.14
$r \leq 2$	77.10	76.07	$r=2 r=3$	35.81	34.40
$r \leq 1$	115.87	102.14	$r=1 r=2$	40.87	40.30
$r=0$	159.87	131.70	$r=0 r=1$	43.99	46.46

(ii) Testing homogeneity on relative money and relative income

Likelihood ratio statistic = $18.77 \sim \chi^2(12)$

Restricted cointegrating relationship (largest eigenvalue only):

$$s_t = (m_t - m_t^*) - (y_t - y_t^*) + 0.049i_t - 0.050i_t^*$$

b) System involving s, m, m^*, y, y^*

(i) Johansen statistics

Null Hypothesis	TRACE Statistic	5% Critical Value	Null Hypothesis	λ MAX Statistic	5% Critical Value
$r \leq 4$	0.14	9.09	$r=4 r=5$	0.14	9.09
$r \leq 3$	5.58	19.96	$r=3 r=4$	5.44	15.67
$r \leq 2$	19.95	34.91	$r=2 r=3$	14.38	22.00
$r \leq 1$	44.40	53.12	$r=1 r=2$	24.45	28.14
$r=0$	79.08	76.07	$r=0 r=1$	34.67	34.40

(ii) Testing homogeneity on relative money and relative income

Likelihood ratio statistic = $6.13 \sim \chi^2(4)$

Notes: r denotes the number of distinct cointegrating vectors. Critical values for the TRACE and λ MAX statistics are taken from Osterwald-Lenum (1990).

Table 3. Tests of Forward-Looking Restrictions

a) Assuming $\lambda = 0.05$

$$\text{Linear Wald statistic} = 0.29\text{E}+07 \sim \chi^2(12)$$

$$\text{Var}(L_t)/\text{Var}(L_t^*) = 105.65$$

b) Assuming $\lambda = 0.015$

$$\text{Linear Wald statistic} = 0.33\text{E}+08 \sim \chi^2(12)$$

$$\text{Var}(L_t)/\text{Var}(L_t^*) = 0.12\text{E}+04$$

c) Assuming $\lambda = 0.001$

$$\text{Linear Wald statistic} = 0.73\text{E}+10 \sim \chi^2(12)$$

$$\text{Var}(L_t)/\text{Var}(L_t^*) = 0.29\text{E}+06$$

d) Assuming $\lambda = 0.03$

$$\text{Linear Wald statistic} = 0.81\text{E}+07 \sim \chi^2(12)$$

$$\text{Var}(L_t)/\text{Var}(L_t^*) = 302.71$$

much previous work on the monetary model which has utilized the inferior two-step cointegration methodology (see *inter alia* Boothe and Glassman (1987); Meese (1987)), but is consistent with the results of MacDonald and Taylor (1991).

Next, we tested for cointegration among the vector of variables excluding interest rates, as suggested by our analysis in Section II. The results are given in Table 2b. These demonstrate the existence of a unique cointegrating vector among the series, in which the restrictions of homogeneity of relative money and of relative incomes cannot be rejected at the 5 percent level. The finding of cointegration of these series--and hence the stationarity of the spread, L_t --is thus tantamount to a rejection of the speculative bubbles hypothesis, independently of whether or not the full set of FMAER restrictions can be imposed.

In Table 3 we report tests of the FMAER restrictions on the VAR for $(\Delta x_t, L_t)'$, for $L_t = s_t - m_t' + y_t'$. For each of the values of λ discussed in the previous section, namely $\lambda=0.05$, $\lambda=0.03$, $\lambda=0.015$, and $\lambda=0.001$, we constructed heteroskedastic-robust linear Wald statistics of for the null hypothesis (16). In each case, the restrictions are hideously rejected.

Moreover, the deviations from the null hypothesis appear to be economically as well as statistically significant: The ratio of the variance of the actual spread to the theoretical spread (computed according to (17)) is in each case massively different from unity. Moreover, plots of the time series behavior of L_t and L_t^* (for all of the chosen values of λ) also revealed important differences and evidence of "excess volatility" in L_t . ^{1/} Figure 1 graphs L_t and L_t^* for the case $\lambda=0.05$.

V. Forecasting with the Monetary Model

We demonstrated in the previous section that, for the dollar-mark during the period of investigation, a cointegration relation exists which corresponds to the static monetary approach exchange rate equation. We suggested, therefore, that the monetary model can be interpreted as having at least long-run validity. According to the Granger Representation Theorem, however, if a cointegrating relationship exists among a set of I(1) series, then there also exists a dynamic error correction representation of the data. That is to say, there should exist a stable vector autoregression

^{1/} Our results thus echo the excess volatility finding of Huang (1981), although in implementing the Campbell-Shiller technique and thereby allowing for non-stationarity and long-run constraints, our analysis is technically superior to Huang's.

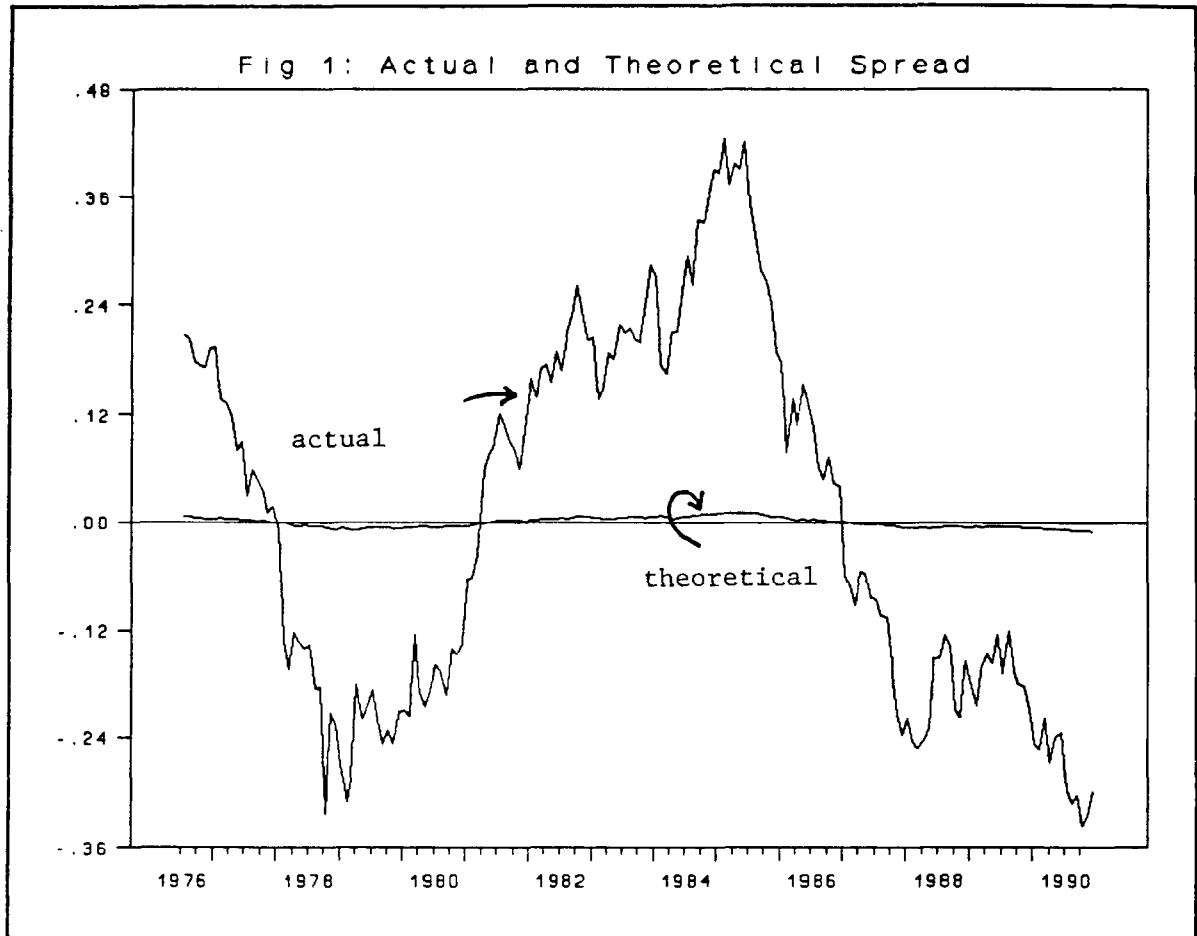


Table 4. Dynamic Forecast Statistics

Horizon (months)	RMSE from dynamic error correction monetary model	RMSE from random walk model
12	0.131	0.148
9	0.103	0.112
6	0.084	0.088
3	0.051	0.053
2	0.038	0.040
1	0.029	0.300

Note. Figures are logarithmic differences and are therefore approximately equal to percentage differences divided by 100.

with degrees of freedom as indicated in parentheses. Subscripted figures in parentheses are estimated (heteroskedastic-consistent) standard errors; those in square brackets are marginal significance levels. Chow and PF denote, respectively, Chow's (1960) and Hendry's (1979) tests for post-sample predictive failure, when the model is used to forecast over the period 1989(1)-1990(12); ^{1/} the Chow statistic is distributed as central F if the model is stable, while the PF statistic is χ^2 .

The model's performance is impressive. Not only does it pass a wide range of in-sample diagnostic tests, it also appears to forecast well out of sample. Note, however, that the CHOW and PF statistics are based on *static* forecasts. That is to say, the forecasts of the exchange rate generated by the model are not fed back in, as in dynamic forecasting. Accordingly, we also carried out a dynamic forecasting exercise over the same remaining 24 data points which followed a similar procedure to that of Meese and Rogoff (1983). That is to say, we sequentially re-estimated the model for every data point from 1989(1) onward, computing dynamic forecasts for a number of forecasting horizons. When this had been done for the whole sample, we then computed the root mean squared error (RMSE) of the forecast at each horizon. As a point of comparison, we also computed the RMSEs generated by a naive random walk model.

The results of this exercise are reported in Table 4, and are very interesting indeed: the dynamic error correction model outperforms the random walk forecast at every forecast horizon. The results of this section thus suggest that, treated as a long-run equilibrium condition, the monetary model of the exchange rate may still be useful in forecasting the exchange rate.

VI. Conclusion

In this paper we have re-examined the monetary model of the exchange rate for the German mark - US dollar exchange rate over the period January 1976 to December 1990. We generated a number of new results.

Our results indicated rejection of the speculative bubbles hypothesis for the dollar-sterling exchange rate, 1976-1990. However, tests of the full set of restrictions which the forward-looking monetary model imposes on the relevant time series processes resulted in an overwhelming rejection of the model. The deviations of the data from the forward model were shown, moreover, to be economically as well as statistically important. This finding differs substantially from the results of other researchers who empirically test the forward-looking monetary model. We attribute the difference to the (more appropriate) way in which we have implemented the model.

^{1/} A guide to all of the diagnostics discussed in this section can be found in Cuthbertson, Hall, and Taylor (1992).

However, some support for the flex-price monetary model--interpreted as a *long-run* model--could, nevertheless, be adduced from our results since we found evidence of cointegration between the exchange rate, relative money supply, relative income, and relative interest rates. This result contrasts with many other cointegration tests of the monetary model, but confirms the findings of MacDonald and Taylor (1991).

Finally, and perhaps most significantly, we showed that imposing the monetary model as a long-run equilibrium condition on a dynamic, error correction model, led to dynamic exchange rate forecasts which were better than the random walk forecast at every horizon considered.

Given that the monetary model appears to hold as a long-run equilibrium condition, it may be that our rejection of the forward model is due to the presence of mean-reverting deviations from the fundamentals which are generated by the presence of speculators who do not conform to the rational expectations hypothesis--such as technical analysts or chartists (Frankel and Froot (1986); MacDonald and Young (1986); Allen and Taylor (1990); Taylor and Allen (1992)). This would explain why allowing the data to determine the form of the short-run dynamics, while imposing theoretically consistent long-run constraints--ie estimating an error correction form--proved fruitful.

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