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# Sovereign Debt Relief Schemes and Welfare

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## Abstract

This paper shows that concerted debt reduction may be welfare-improving even when the investment disincentive effect of a debt overhang is not large enough to place the debtor country on the wrong side of the debt Laffer curve. Whether the appropriate relief scheme involves debt reduction or new money, however, depends on whether investment disincentives or liquidity constraints dominate. It is shown that, except under very special circumstances, mixed policy packages involving both debt and liquidity relief may not yield the desired results.

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Contents

## Summary

Although investment disincentives associated with an external debt overhang have become a popular explanation for the collapse in investment in heavily indebted countries, the conditions required by previous authors for efficiency-enhancing debt reduction appear not to be satisfied for most of these countries. In addition, market-based debt reduction instruments, such as buybacks, have been criticized on the grounds that they are unlikely to improve the welfare of the debtor, who is forced to pay the average value of outstanding claims for a reduction in the buybacks' marginal value. Despite these considerations, however, several countries have moved ahead with debt reduction and restructuring under the Brady initiative.

This paper models an economy with an external debt overhang. The model captures two types of investment inefficiency--a disincentive effect caused by the existence of the future debt burden and a liquidity effect arising from a shortage of current resources. Two types of relief schemes are considered: debt reduction, which involves reducing current resources in exchange for a reduction in future obligations; and liquidity relief, which increases current liquidity and the stock of debt.

The appropriate relief scheme depends on which of the two effects dominates. It is shown that, except under special circumstances, mixed policy packages involving both debt and liquidity relief are not worthwhile. Furthermore, debt reduction is shown to be Pareto-improving even when the debtor country is not on the "wrong side" of the debt Laffer curve. This result widens the scope for debt reduction on efficiency grounds.

The paper also discusses the price of a concerted debt-reduction operation, finding that its price must be above the marginal value of outstanding claims but may or may not exceed their average value. Hence, even if a concerted debt-reduction operation improves welfare, a market buyback may not.

## I. Introduction

The debt crisis of the 1980s was associated with the collapse in investment in debtor countries. In Argentina the ratio of investment to GDP fell from an average of 25 percent during 1976-81 to 15 percent during 1982-88. In Venezuela the decline was from 33 percent to 18 percent. On the other hand, developing countries that were free of debt problems saw an increase, on average, in their investment ratios over the corresponding period. 1/

Two competing explanations for the fall in investment have been offered, both with very different policy implications. The first argues that the debt crisis is a liquidity, as opposed to a solvency, issue. 2/ According to this view, lucrative investment opportunities are available in debtor countries, but some sort of market failure associated with the debt crisis has prevented creditors from lending any further. The Baker Plan of the mid-1980s reflected this view and called for new lending by commercial banks.

The second explanation, which has gained popularity more recently, has to do with the debt overhang. 3/ This hypothesis argues that when a country is unable to fully service its debt, actual payments tend to depend on the country's economic performance. The existence of a heavy debt burden, then, depresses the return on investment and weakens the incentive to invest, since part of the profits will need to be diverted toward debt servicing and amortization. When such disincentives become important, debt reduction is the appropriate policy action.

In a model which focuses on this disincentive effect, Froot (1989) argues that debt reduction improves the welfare of both the debtor as well as the creditor if and only if the debtor country is on the wrong side of its debt Laffer curve--that the distortion arising from the debt burden is so large that the expected present discounted value of future repayments actually decreases as the level of debt is increased any further. Claessens (1988), however, finds in an empirical investigation that virtually no debtor country is on the wrong side of its debt Laffer curve, apparently weakening the case for debt reduction on efficiency grounds. In spite of this, however, debt buyback schemes have been prevalent in the past as well as the present. In a comprehensive study of the debt crisis of the 1930s, Eichengreen and Portes (1988) find that repurchases of debt obligations by sovereign debtors occurred extensively. In addition, several countries, including Mexico, Venezuela, Costa Rica, the Philippines, Niger, and Nigeria, are currently experimenting with debt reduction schemes.

The model presented in this paper captures both the liquidity and the disincentive effects of a debt crisis, and indicates that the appropriate relief scheme depends on which of the two effects dominates. If the disincentive effect is large, efficiency gains can be realized through debt

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1/ International Monetary Fund (1989), pages 65-66.
2/ See, for example, Krugman (1988).
3/ See, for example, Sachs (1988), Corden (1998), and Krugman (1989).

reduction. If, on the other hand, the liquidity effect dominates, liquidity relief is Pareto-improving. It is shown that either debt reduction or liquidity relief must be Pareto-improving; but both should not, except under very special circumstances, be offered simultaneously. The analysis indicates that the conditions for Pareto-improving debt reduction, even on purely efficiency grounds, are considerably weaker than in Froot (1989)--the debtor country need not be on the wrong side of its debt Laffer curve for debt reduction to yield favorable results.

The remainder of this paper is organized as follows: Section II contains the model. Section III discusses the welfare effects of alternate relief schemes and contains an extension of the model. A discussion of the price at which debt reduction is Pareto-improving is contained in Section IV. Some concluding remarks are presented in Section V.

## II. The Model

Consider a two-period, one-good world with a debtor country which has a debt obligation of D to its external creditor. The debt has already been incurred as of period zero, and the repayment is due in period one. The debtor country is populated with a large number of identical citizens. 1/ The representative citizen is endowed with initial resources Q in period zero, and has access to a production technology which transforms period zero investment into period one output.

The debtor country government imposes a marginal tax on period one output of its citizens to make the repayment. This tax is levied after output is realized. If the government fails to make the entire repayment of D, the creditor seizes a fraction $\lambda$ of the country's output. 2/ Since the government does not engage in consumption or investment of its own, it chooses a tax rate just high enough to raise D in revenue, as long as this tax rate is less than $\lambda$. If sufficient revenue to repay the debt cannot be raised with a tax rate less than $\lambda$, the government simply taxes at the rate $\lambda$ and turns over the proceeds to the creditor as partial repayment.

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1/ Although distributional effects of the debt overhang on various types of agents within the debtor country are important, they are outside the scope of this analysis. Instead, the focus here is exclusively on potential efficiency gains and losses associated with alternate debt relief schemes.

2/ The constant $\lambda$ can be interpreted as the result of a bargaining game between debtor and creditor, as in Bulow and Rogoff (1989). In such a game the creditor threatens to impose a trade embargo or to eliminate trade credits--actions which impose a cost on the debtor. The debtor then offers a fraction of its output in order to avoid these penalties. Alternatively, $\lambda$ may be interpreted as a tax ceiling faced by the debtor country government, as in Helpman (1989).

The representative agent in the debtor country chooses an optimal investment level to maximize his expected utility of consumption. 1/ This optimization problem may then be expressed as

$$\max_{I} \; u(Q-I) + \beta E\{v[(1-t)\theta f(I)]\} \quad , \tag{1}$$

where u, v, and f are twice continuously differentiable concave functions which satisfy the Inada conditions, f(0)=0, $\beta$ is the agent's intertemporal discount factor, E is the expectations operator, and t is the tax rate. The shock $\theta$ is a random variable with a probability density function $g(\theta)$, finite supports $\underline{\theta}$ and $\overline{\theta}$, and mean $\hat{\theta}$. $\theta$ is realized in period one, so its value is uncertain in period zero when the investment decision is made. This shock is common across all agents, and may be interpreted as a terms of trade shock.

The tax rate is

$$t = \begin{cases} \dfrac{D}{\theta^* f(I)} & \text{if } D \leq \lambda\theta^* f(I) \\ \lambda & \text{otherwise,} \end{cases} \tag{2}$$

where $\theta^*$ is the realized value of $\theta$. The tax rate, then, depends not only on the level of debt and on the realized shock, but also on the aggregate level of investment. An individual investor, however, cannot influence aggregate investment, and takes the tax rate as given. 2/

The agent's optimal investment choice I satisfies the first order condition

$$u'(Q-\tilde{I}) = \beta E\{v'[(1-t)\theta f(\tilde{I})](1-t)\theta f'(\tilde{I})\} \quad . \tag{3}$$

The impact of a change in the level of debt on I is

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1/ Husain (1990) extends this framework to allow for the possibility of capital flight. Such a modification, however, does not alter the results presented here. Since the country is in a debt crisis, it is assumed that agents are no longer able to borrow from abroad. This has been an important feature of the debt crisis and is well documented. See, for example, International Monetary Fund (1989).

2/ Strictly speaking, the tax rate is actually a tax schedule $t(\theta)$. Since D is known at the time the investment decision is made and the agent takes aggregate investment as given, the tax rate is only a function of $\theta$.

$$\frac{d\tilde{I}}{dD} = \frac{\beta \int_{\frac{D}{\lambda f(\tilde{I})}}^{\bar{\theta}} \left[ v''(1-t)\theta f' + \frac{v'f'}{f} \right] g(\theta)\, d\theta}{u'' + \beta E\{v''(1-t)^2\theta^2 f'^2 + v'(1-t)\theta f''\}} \quad . \tag{4}$$

This expression is negative (positive) if the Arrow-Pratt measure of relative risk aversion is less than (greater than) unity. 1/ The optimal investment choice, then, depends on the tax rate. An individual investor takes the tax schedule as given and chooses investment optimally. Since all agents are identical, the equilibrium tax schedule depends on the representative agent's investment choice and must be consistent with the tax schedule used by the agent in choosing the optimal level of investment. In equilibrium, then, the utility of the representative agent is

$$U = u(Q-\tilde{I}) + \beta E\{v[(1-t)\theta f(\tilde{I})]\} \quad . \tag{5}$$

Since all tax revenue is turned over to the creditor, the expected repayment is

$$E\{R\} = E\{t\theta f(\tilde{I})\} \quad . \tag{6}$$

1. Concerted debt reduction operations (CDROs)

Suppose that in addition to taxing period one output, the government also has at its disposal a marginal tax on citizens' initial endowment (Q). The government uses period zero revenue to retire some of its future debt obligations. 2/ The optimization problem of the representative agent can then be expressed as

$$\max_{I} \; u[(1-t_0)Q-I] + \beta E\{v[(1-t_1)\theta f(I)]\} \quad , \tag{7}$$

where $t_0$ and $t_1$ are the tax rates in periods zero and one. The optimal investment choice $I^*$ solves the first order condition

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1/ This result is consistent with Helpman (1989).

2/ A discussion of the price at which debt is retired is contained in Section IV. For the moment, the debt exchange may be thought of as a trade between the debtor government and the creditor in which the government pays some amount today in return for a write-off of some of tomorrow's debt obligations.

$$u'[(1-t_0)Q-I^*] = \beta E\{v[(1-t_1)\theta f(I^*)](1-t_1)\theta f'(I^*)\} \quad . \tag{8}$$

$I^*$ is now a function of both $t_0$ and $t_1$, where

$$\frac{dI^*}{dt_0} = \frac{-u''Q}{u'' + \beta E\{v''(1-t_1)^2\theta^2 f'^2 + v'(1-t_1)\theta f''\}} < 0 \tag{9}$$

and

$$\frac{dI^*}{dD} = \frac{\beta \int_{\frac{D}{\theta f}}^{\bar{\theta}} \left[v''(1-t_1)\theta f' + \frac{v'f'}{f}\right] g(\theta)\, d\theta}{u'' + \beta E\{v''(1-t_1)^2\theta^2 f'^2 + v'(1-t_1)\theta f''\}} \quad . \tag{10}$$

Expression (9) is unambiguously negative--an increase in the tax rate on initial wealth will decrease investment--while the sign of (10) depends, as in expression (4), on the representative agent's period one utility function.

The optimization problem of a debtor government seeking to maximize the welfare of its representative citizen through a concerted debt reduction operation can then be expressed as 1/

$$\max_{t_0, D} \; u[(1-t_0)Q-I^*] + \beta E\{v[(1-t_1)\theta f(I^*)]\} \quad . \tag{11}$$

A CDRO, in order to be implemented, must be agreed to by both the debtor as well as the creditor. The debtor government's choice of $t_0$ and D, then, must also leave the creditor at least as well off as before the debt reduction. The creditor is assumed to maximize the expected present discounted value of net capital flows from the debtor country,

$$E\{R\} = t_0 Q + \rho E\{t_1 \theta f(I^*)\} \quad , \tag{12}$$

where $\rho = 1/(1+r)$ and r is the world interest rate. The debtor country government, then, maximizes (12) subject to the constraint

$$t_0 Q + \rho E\{t_1 \theta f(I^*)\} \geq \rho E\{t\theta f(\tilde{I})\} \quad . \tag{13}$$

In posing the CDRO in this manner, we are letting all efficiency gains accrue to the debtor. A more general formulation would allow both debtor

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1/ The function $t_1(\theta)$ is the period one tax schedule, as expressed in (2), when the level of debt is D. Choosing D, then, amounts to choosing $t_1(\theta)$.

and creditor to bargain over these gains and involve a derivation of the outcome of a complex bargaining game. 1/ The purpose of this exercise, however, is to see whether a welfare-improving CDRO exists. Thus, as long as $t_0 > 0$ and $D^* < D$, there exists a Pareto-improving CDRO in which the debtor government pays the creditor $t_0^*Q$ now in exchange for a reduction in tomorrow's debt obligations by $D-D^*$. Rather than solve for the optimal CDRO, the following analysis derives conditions under which a marginal reduction in debt improves the welfare of both agents. As long as these conditions are satisfied, there exists a Pareto-improving CDRO.

Recall that the creditor's welfare E{R} is expressed in (12), and

$$dE\{R\} = Bdt_0 + \rho C dD \quad , \tag{14}$$

where

$$B = \left\{ Q + \left[ \rho \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda \theta f'(I^*) g(\theta) d\theta \right] \frac{dI^*}{dt_0} \right\} \tag{15}$$

and

$$C = \left\{ \int_{\frac{D}{\lambda f}}^{\bar{\theta}} g(\theta) d\theta + \left[ \rho \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda \theta f'(I^*) g(\theta) d\theta \right] \frac{dI^*}{dt_0} \right\} \quad . \tag{16}$$

As the tax rate on initial wealth is raised, the creditor's welfare is affected in two ways. The creditor gains revenue today, and this makes it better off. At the same time, however, debtor country citizens are poorer and invest less (expression (10)), which translates into lower expected repayment to the creditor in the future. These two effects together are captured in expression (15). An increase in the level of indebtedness of the debtor also affects the creditor's welfare in two ways. The number of states in which full repayment occurs is reduced. Additionally, the increased debt reduces investment, which results in lower repayment in states of default. These two effects are captured in (16).

The representative debtor country citizen's utility, for given $t_0$ and D, is

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1/ Fernandez and Rosenthal (1990) assess which agent, debtor or creditor, gets the efficiency gains under alternate bargaining formulations of debt forgiveness.

$$U = u[(1-t_0)Q-I^*] + \beta E\{v[(1-t_1(D))\theta f(I^*)]\} \quad , \tag{17}$$

and the change in her welfare resulting from small changes in $t_0$ and D, then, is

$$dU = -u'Qdt_0 - \left[\beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v'g(\theta)\,d\theta\right] dD \quad . \tag{18}$$

A marginal CDRO must satisfy the condition

$$dt_0 = \frac{-\rho C}{B}\, dD \quad , \tag{19}$$

in order to leave the creditor as well off after the CDRO as before. Thus,

$$dU = \left[\frac{\rho u'QC}{B} - \beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v'g(\theta)\,d\theta\right] dD \quad , \tag{20}$$

and in order for a marginal CDRO ($dD < 0$) to be Pareto-improving ($dU > 0$), the following must hold:

$$C < \frac{B\beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v'g(\theta)\,d\theta}{\rho u'Q} \quad . \tag{21}$$

Note that the debtor is on the wrong side of its debt Laffer curve if and only if $C < 0$. The debtor need not be on the wrong side, then, for a buyback to be Pareto-improving. This result contrasts with Froot (1989), who finds that Pareto-improving debt reduction and forgiveness are equivalent in the sense that they both require the debtor country to be on the wrong side of its debt Laffer curve. Expression (20) indicates that a Pareto-improving CDRO only requires that the creditor's marginal valuation of debt be less than the debtor's. 1/ On the other hand, Pareto-improving forgiveness, which involves a reduction in debt without any payment out of current resources ($dD < 0$, $dt_0 = 0$), requires that the creditor's marginal valuation be negative.

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1/ Note that this is not equivalent to requiring heterogeneous beliefs between debtor and creditor about the probability of repayment. Rather, the difference in the marginal valuations of the two agents stems from the difference in the agents' attitudes toward risk--the debtor is risk-averse while the creditor is risk-neutral--and the inability of the debtor to borrow in order to smooth consumption.

2. Grants and new lending

If the debtor country has severely underutilized investment opportunities, an injection of resources would result in higher investment and repayment. Such an increase in repayment could potentially be greater than the injection of liquidity. Negativity of expression (15) is required for this to be true, and serves to define a severely liquidity constrained debtor. If $B < 0$, the debtor is severely liquidity constrained and a grant from the creditor improves the welfare of both agents. 1/ Such a grant involves an injection of $Qdt_0$ resources today, without any increase in tomorrow's debt claims.

Even when the debtor is not severely liquidity constrained, new lending can be Pareto-improving. In order for the creditor to voluntarily engage in a new lending scheme, the expected present discounted value of the increase in future claims must equal the injection of liquidity. This constraint is contained in expression (19). Thus, as long as

$$C > \frac{B\beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v' g(\theta)\, d\theta}{\rho u' Q}, \tag{22}$$

new lending of $Qdt_0$ today, accompanied by an appropriate increase in debt claims, improves the welfare of the debtor without hurting the creditor. Thus, if the creditor's marginal valuation of debt claims is greater than that of the debtor, new lending is Pareto-improving. If the creditor's valuation is greater than its cost of funds, a grant is welfare-improving.

3. Debt versus liquidity relief

The model captures two types of investment inefficiency--liquidity constraints and intemporal distortions--which serve to reduce debt repayment capacity. The appropriate relief scheme differs according to which of these two inefficiencies dominates. If investment disincentives arising from the debt burden are so great that the country is on the wrong side of its debt Laffer curve ($C < 0$), debt forgiveness is welfare-improving. If, on the other hand, the country is severely liquidity constrained ($B < 0$), a grant to the debtor is Pareto-improving. If both $B < 0$ and $C < 0$, then the appropriate relief scheme involves both a grant and forgiveness. 2/

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1/ Note that an injection of liquidity implies $dt_0 < 0$. Since $t_0 = 0$ before such a scheme is implemented, $t_0$ is actually reduced to a negative level.

2/ An alternate scheme in which the debtor government finances a CDRO with a tax on period zero consumption is analyzed in the appendix. Due to the additional proincentive effect of the consumption tax scheme, both liquidity relief and a CDRO financed with a consumption tax can improve welfare under some circumstances.

Even if neither liquidity nor debt overhang inefficieny is sufficiently large to merit unilateral action by the creditor, there are other Pareto-improving schemes which affect the mix between current and future flows among the two agents. If liquidity effects are relatively more important than debt overhang effects (so that (22) holds), a decrease in current payments in exchange for an increase in future payments is appropriate. If debt overhang effects dominate (so that (21) holds), a scheme involving more current payments in exchange for fewer future payments is Pareto-improving. Note that the ratio of B to C is important in determining which scheme is appropriate. The smaller is this ratio, the more important are liquidity considerations. Note also that either (21) or (22) must hold, except in the very special case that both hold as equalities.

Finally, unless $B < 0$ and $C < 0$, both new money and debt reduction cannot be appropriate at the same time. Either the debtor's marginal valuation of future claims exceeds the creditor's, or vice versa. This result also contrasts with Froot (1989), who finds that when debt reduction is efficiency enhancing, an injection of liquidity may also be desirable. In Froot's model, however, debt reduction is useful only if the country is on the wrong side of the debt Laffer curve ($C < 0$). In addition, Froot finds that providing liquidity in addition to debt relief is profitable only if the debtor is sufficiently liquidity constrained--if an additional dollar of liquidity raises investment by so much that the resulting increase in expected repayment compensates the creditor for its cost of funds. This is equivalent to $B < 0$ in our model. Hence, Froot's result is valid only in the case where both B and C are negative. Our model indicates that even when these extreme conditions are not satisfied, there still exists an efficiency enhancing relief scheme. But when Froot's conditions are not satisfied, the optimal relief package does not involve both debt and liquidity relief.

## IV. The Price of a CDRO

In studying the welfare impact of market-based debt buyback schemes, Bulow and Rogoff (1988) argue that such schemes are unlikely to improve the welfare of the debtor. A buyback in a competitive market must take place at the market price, or the average value of debt. Debt is reduced, however, by only its marginal value.

A CDRO in this model is not a competitive market transaction. Rather, it is a trade between debtor and creditor which involves a payment by the debtor of some resources today in exchange for some of tomorrow's debt obligations. In the analysis presented above, all efficiency gains arising from such a transaction accrue to the debtor. The implicit price at which this trade occurs is 1/

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1/ Recall that this trade involves a reduction of claims by dD in exchange for a payment of $Qdt_0$.

$$P = \frac{\rho Q C}{B} \quad . \tag{23}$$

If, on the other hand, all efficiency gains were to accrue to the creditor, the price of this exchange (setting (18) equal to zero) would be

$$P^* = \frac{\beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v' g(\theta)\, d\theta}{u'} \quad . \tag{24}$$

Since CDROs are Pareto-improving only when inequality (21) holds, P is less than $P^*$. The implicit price of the CDRO, when it is worthwhile on efficiency grounds, must be lower if efficiency gains are captured by the debtor than if all such gains are captured by the creditor. In principle, then, if (21) holds a CDRO at any price between P and $P^*$ is Pareto-improving.

From expressions (9) and (15), Q is greater than B. Thus, P is greater than the marginal value of debt ($P^M$), where

$$P^M = \rho \int_{\frac{D}{\lambda f}}^{\bar{\theta}} g(\theta)\, d\theta + \rho \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda \theta f'(I^*) g(\theta)\, d\theta = \rho C \quad . \tag{25}$$

A CDRO at a price equal to the marginal value of debt is a losing proposition for the creditor. At this price the creditor is forced to give up some claims at their marginal value and also suffer the additional loss resulting from lower investment by the debtor. 1/ On the other hand, the average value of debt ($P^A$), where

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1/ A reduction in debt affects $P^A$ in two ways: first, through a reduction in the stock of debt, and second, through the investment incentive effect which increases expected future repayment. The increase in $P^A$ affects the welfare of both agents, then, to the extent that it reflects a change in expected repayment. Thus, it is the change in expected repayment, rather than the change in $P^A$, that is important. As long as (21) holds and debt is exchanged at a price between P and $P^*$, the change in expected repayment is such that both agents are better off, and the ensuing change in $P^A$ can then be obtained by differentiating (35) with respect to D.

$$P^A = \rho \int_{\frac{D}{\lambda f}}^{\bar{\theta}} g(\theta)\,d\theta + \rho \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \frac{\lambda\theta f(I^*)}{D}\, g(\theta)\,d\theta = \frac{\rho E\{t_1 \theta f(I^*)\}}{D} \quad , \qquad (26)$$

may well be greater than $P^*$. 1/ In this case, a buyback at the competitive market price would reduce the welfare of the debtor.

The analysis indicates that the menu of policy actions to alleviate the debt crisis via debt reduction should not be limited to competitive market debt buybacks. Even when market buybacks are not Pareto-improving, there exists a range of prices at which "non-market" buybacks improve the welfare of both debtor and creditor.

## V. Conclusions

The model presented above indicates that whenever there is a debt crisis in which citizens of a debtor country can no longer borrow from abroad, there must exist a Pareto-improving relief scheme. Two sources of investment inefficiency--a wealth effect and a distortionary effect--are identified. The wealth effect stems from the inability of agents to borrow, even when there exist lucrative investment opportunities. If this effect is sufficiently large, liquidity relief is Pareto-improving. The distortionary effect, on the other hand, has to do with the tax distortion associated with future debt obligations. If the distortionary effect is large, debt reduction is Pareto-improving.

The model indicates that, except in the very special circumstances that the debtor country is on the wrong side of the debt Laffer curve and and severely liquidity constrained, new money and debt reduction packages should not be offered simultaneously. Although this result appears to contrast with very recent experience under the Brady Plan, in which creditors were given the option of providing new loans or selling part of their debt at a discount, the net effect of these packages has generally been a reduction in the face value of obligations in exchange for a small amount of current liquidity. 2/ Mexico, for example, paid $ 7.1 billion to reduce its debt by $ 15 billion, implying an exchange price of about 47 cents per dollar of face value. The secondary market price of Mexican debt around the time of the debt restructuring rose from below 40 cents to over 50 cents, suggesting that Mexico paid close to post-restructuring prices for the operation. However, $ 5.7 billion of the repurchase payment was obtained through new

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1/ In order for $P^A$ to be less than $P^*$, the expected after-tax return on investment must be sufficiently low and/or the covariance of the marginal utility of consumption tomorrow with the after-tax return must be sufficiently negative.

2/ Details of recent debt restructurings are contained in International Monetary Fund (1991), pages 74-83. El-Erian (1991) describes Mexico's external debt negotiations.

borrowing from official creditors, and another $ 1.1 billion was obtained through new money from commercial lenders. Thus, the various components of the Brady deal, when put together, involved a net debt reduction of $ 8.2 billion for a payment of $ 0.3 billion out of current resources. The simultaneous options of new lending and debt reduction, then, should be interpreted as a means for subsidizing the debtor's repurchase of debt at market prices. The existence of the subsidy is consistent with the model, which indicates that even when a CDRO can improve the welfare of both debtor and creditor, a buyback at market prices may not.

Finally, the model indicates that the wealth effect (distortionary effect) needs to be very large for a grant (forgiveness) to be Pareto-improving. Even when these effects are not large enough to merit forgiveness or a grant on efficiency grounds, however, concerted debt reduction or new lending may still be welfare-improving. Thus, even though Claessens (1988) finds that few of the heavily indebted countries are on the wrong side of the debt Laffer curve, there may still be scope for efficiency-enhancing debt reduction.

APPENDIX

## Consumption Tax-Financed CDROs

A scheme which has been ignored in the analysis above, and indeed in the literature, is a CDRO which the debtor government finances by levying a tax on period zero consumption. Under such a scheme, the representative citizen's optimization problem becomes

$$\max_{I} \; u[(1-t_0)(Q-I)] + \beta E\{v[(1-t_1)\theta f(I)]\} \quad , \tag{27}$$

with the associated first order condition

$$u'[(1-t_0)(Q-\hat{I})](1-t_0) = \beta E\{v'[(1-t_1)\theta f(\hat{I})](1-t_1)\theta f'(\hat{I})\} \quad , \tag{28}$$

where $t_0$ now denotes the consumption tax. The response of investment to changes in $t_0$ and D is

$$\frac{d\hat{I}}{dt_0} = \frac{-u''(Q-\hat{I})(1-t_0) - u'}{(1-t_0)^2 u'' + \beta E\{v''(1-t_1)^2\theta^2 f'^2 + v'(1-t_1)\theta f''\}} \tag{29}$$

and

$$\frac{d\hat{I}}{dD} = \frac{\beta \int_{\underline{\theta}}^{\bar{\theta}} \left[v''(1-t_1)\theta f' + \frac{v'f'}{f}\right] g(\theta)\, d\theta}{(1-t_0)^2 u'' + \beta E\{v''(1-t_1)^2\theta^2 f'^2 + v'(1-t_1)\theta f''\}} \quad . \tag{30}$$

An increase in $t_0$ affects investment in two ways. The tax increase makes the investor poorer. This wealth effect causes her to invest less. The increase in $t_0$, however, makes consumption more costly than before. This is a substitution effect, and induces the agent to invest more. If the substitution effect dominates, investment actually rises in response to an increase in $t_0$.

The creditor's welfare is now

$$E\{R\} = t_0(Q-\hat{I}) + \rho E\{t_1\theta f(\hat{I})\} \quad , \tag{31}$$

and

$$dE\{R\} = \left\{ (Q-\hat{I}) - t_0 \frac{d\hat{I}}{dt_0} + \left[ \rho \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda\theta f'(\hat{I})\, g(\theta)\, d\theta \right] \frac{d\hat{I}}{dt_0} \right\} dt_0$$
$$+ \left\{ -t_0 \frac{d\hat{I}}{dD} + \left[ \rho \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda\theta f'(\hat{I})\, g(\theta)\, d\theta \right] \frac{d\hat{I}}{dD} + \left[ \rho \int_{\frac{D}{\lambda f}}^{\bar{\theta}} g(\theta)\, d\theta \right] \right\} dD \qquad (32)$$

measures the change in expected repayment as $t_0$ and D are changed. Before the CDRO is implemented, $t_0=0$ and a marginal debt forgiveness increases repayment if and only if the country is on the wrong side of its debt Laffer curve.

A grant from the creditor to the debtor has precisely the same effect on expected repayment here as it did in section II above. Such a grant involves a reduction in the period zero tax rate to a negative level. The tax rate considered in this section, however, is a tax/subsidy on consumption. Hence the condition for a grant to be welfare-improving is the same as in section II--that the country be severely liquidity constrained.

If the debtor country is on the wrong side of its tax Laffer curve, or if it is severely liquidity constrained, or both, the relief scheme which improves welfare is the same as before. The only case left to consider is if the country is neither severely liquidity constrained nor on the wrong side of its tax Laffer curve. Proceeding as before, the change in the representative citizen's welfare may be expressed as

$$dU = -u'(Q-\hat{I})\, dt_0 - \left[ \beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v' g(\theta)\, d\theta \right] dD \quad . \qquad (33)$$

Holding dE(R)=0 and assuming $t_0=0$,

$$dU = \left[ \frac{\rho u'(Q-\hat{I})\, C'}{B'} - \beta \int_{\frac{D}{\lambda f}}^{\bar{\theta}} v' g(\theta)\, d\theta \right] dD \quad , \qquad (34)$$

where

$$\frac{C'}{B'} = \frac{-\rho \left\{ \left[ \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda\theta f'(\hat{I})\, g(\theta)\, d\theta \right] \frac{d\hat{I}}{dD} + \left[ \int_{\frac{D}{\lambda f}}^{\bar{\theta}} g(\theta)\, d\theta \right] \right\}}{(Q-\hat{I}) + \rho \left[ \int_{\underline{\theta}}^{\frac{D}{\lambda f}} \lambda\theta f'(\hat{I})\, g(\theta)\, d\theta \right] \frac{d\hat{I}}{dt_0}} \, dD \quad . \tag{35}$$

If the term inside the brackets in (34) is negative, a CDRO financed by a consumption tax is Pareto-improving. New lending, on the other hand, is Pareto-improving if condition (22) is satisfied.

A marginal CDRO financed by a consumption tax is welfare-improving whenever a CDRO financed by a tax on initial wealth is, but a wealth tax-financed CDRO may not be Pareto-improving even when a consumption tax-financed CDRO is. To see this, let $t_0$ and D denote the tax rate and the level of debt associated with the consumption tax scheme, and let $t_0^*$ and $D^*$ be associated with the wealth tax scheme. Initially, before any scheme is implemented, $t_0^* = t_0 = 0$ and $D^* = D$. Thus, $\hat{I} = I^*$, and we need to show

$$\frac{\rho u' Q C}{B} > \frac{\rho u' (Q-\hat{I})\, C'}{B'} \quad . \tag{36}$$

This is true if and only if 1/

$$Q \frac{d\hat{I}}{dt_0} > (Q-\hat{I}) \frac{dI^*}{dt_0^*} \quad . \tag{37}$$

From expressions (9) and (29), and using the fact that $t_0 = 0$,

$$\frac{d\hat{I}}{dt_0} = \frac{[u''(Q-\hat{I}) + u']}{u''Q} \frac{dI^*}{dt_0^*} \quad . \tag{38}$$

Thus, expression (37) is true if and only if

$$\{u''(Q-\hat{I}) + u'\} \frac{dI^*}{dt_0^*} < u''(Q-\hat{I}) \frac{dI^*}{dt_0^*} \quad . \tag{39}$$

This is equivalent to $u' > 0$, which is always true. Due to the additional investment proincentive effect of the consumption tax scheme, then, both liquidity relief and a CDRO financed with a consumption tax can improve welfare under some circumstances.

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1/ Note also that $d\hat{I}/dD = dI^*/dD^*$, since $D = D^*$ and $\hat{I} = I^*$.

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