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Fiscal Revenue, Inflationary Finance and Growth

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Abstract

This paper analyzes the optimal rate of monetary expansion when government resorts to inflationary finance to generate additional investment for enhancing growth. If there are lags in tax collection, an increase in inflation erodes real fiscal revenue, thereby worsening the current balance while reducing government investment. This impedes capital accumulation as well as increases the welfare cost of inflation. As such, the optimal rate of monetary expansion, equilibrium capital-labor ratio and output are lower while the marginal cost of inflationary finance is higher than they would be without collection lags. Simulations are performed to highlight empirical implications.

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Contents

Summary

This paper analyzes the optimal rate of monetary expansion when government resorts to inflationary finance to support public investment for enhancing growth. When there are collection lags in the tax system, use of the inflation tax erodes real fiscal revenue, leading to a deterioration of the government's current account balance, and reinforces inflationary pressures. This, in turn, reduces the amount available for government investment, thereby impeding the process of capital accumulation, and in addition increases the welfare cost of inflation owing to the reduction in real balances. The analysis demonstrates that the optimal rate of monetary expansion and the corresponding capital-labor ratio, as well as output and consumption are lower while the marginal cost of using inflationary finance is higher than they would be without collection lags.

Simulations using the empirical evidence on collection lags and the demand for money indicate that the scope of inflationary finance is rather limited, and that it is further diminished with higher rates of private saving and the social discount. As indicated by the shadow price of capital, the marginal cost of inflationary finance is substantially higher than it would be without collection lags. In contrast, the use of taxation to finance government investment leads to higher growth with efficiency and price stability.

These results have important implications for fiscal policy and tax reform. A government that chooses the option of inflationary finance to generate additional investment must take into account the possible effect of real fiscal revenue erosion on the budget deficit. In general, the use of inflationary finance can threaten the objectives of growth with efficiency and price stability. These considerations substantially strengthen the case for improving tax administration and increasing reliance on fiscal revenue to finance an expansion in government expenditure rather than relying on resources generated through inflation.

I. Introduction

Recent evidence on collection lags in the tax system highlights the twofold harmful effects of inflation on the economy. 1/ The use of inflationary finance imposes a substantial welfare cost on the holders of real balances and it also reduces the real fiscal revenue. The possible widening of the fiscal deficit may lead to increased use of the inflationary finance, in addition to increasing the welfare cost of inflation. 2/

The harmful effects of inflation raise the question of what is the optimal amount of inflationary financing a government can use in the pursuit of its growth objectives. When there are no collection lags and the tax system is inadequate, it has been argued that a moderate use of the inflation tax to finance government investment may enhance growth. 3/ The basic argument is that the marginal benefit of higher rates of capital formation outweigh the marginal welfare cost of inflation up to a certain point. However, when there are collection lags, the loss of real fiscal revenue increases the marginal cost of using inflationary finance. Therefore, the inference is that use of the inflation tax for enhancing growth could be much more limited than would be indicated by an analysis that does not consider collection lags in tax revenues.

The main objective of this paper is to analyze the effects of growth on the optimal rate of monetary expansion in light of collection lags. This analysis is based on a neoclassical growth model, with an explicit specification of the government budget constraint in which the inflation tax finances government investment in the production process. An important feature of the model is that a collection lag is explicitly incorporated into the budget constraint. By so doing, the causation that runs between inflation and deficits, via the effects on real fiscal revenue, links the fiscal deficit to the process of capital accumulation, and in turn to saving

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1/ The insight on collection lags and its importance for government finance in developing countries was developed by Tanzi (1977). This analysis has been elaborated further by others, including Tanzi (1978), and most recently by Choudhry (1990, 1991); the latter analyzes and provides evidence on collection lags, the erosion of real fiscal revenue by inflation, and the scope of inflationary finance for a large number of developing countries.

2/ Dixit (1990) has argued that the optimal rates of consumption taxes should also be reconsidered in the light of collection lags. When this is done, the erosion of fiscal revenue from inflation can be recouped by changing the rates on these taxes. While indexation of taxes shifts the burden of fiscal erosion away from the inflation tax, Dixit's analysis ignores the social costs and economic distortions stemming from inflation, as evidenced by the experience of many countries.

3/ Aghevli (1977) made a case in favor of moderate rates of inflation when countries resort to inflationary finance in pursuit of their growth objectives.

and consumption. Also, the model takes into account the welfare cost of inflation, from foregone services of real balances, along with the stream of future benefits derived from higher investment made possible by the inflation tax. Thus, the marginal benefits and costs of using inflationary finance in the process of capital accumulation are incorporated into the model.

The model and its solution are discussed in Section II. Simulations, using empirical evidence on collection lags and on the demand for money from previous studies, are reported in Section III. The simulations focus on the optimal rate of monetary expansion, the associated steady-state levels of output, consumption, and the shadow price of capital. The latter is an indicator of the marginal cost of using inflationary finance. The empirical parameters, particularly the collection lag and the coefficient of expected inflation, are varied to assess the sensitivity of the steady-state levels to these factors. The results are reported in Section III. Some implications of the theoretical analysis and the simulations are contained in the final section.

II. A Model of Inflationary Finance and Growth

This section first describes a neoclassical model of inflationary finance and growth, highlighting the role of government investment in capital accumulation and the revenue-eroding effects of inflation when there are collection lags in the tax system. The final part of this section analyzes the solution set, its existence and stability, and examines the optimal rate of monetary expansion and the shadow price of capital.

1. The model

Consider an economy in which government is directly involved in the production process so that public and private investments are interchangeable. Further, government supplements private capital accumulation through inflationary financing. In this economy, aggregate output per capita, y, is assumed to be an increasing function of the capital-labor ratio, k, according to the following Cobb-Douglas production function: 1/

$$y = f(k) = \sigma_0 k^{\sigma}, \qquad 0 < \sigma < 1. \tag{1}$$

The parameter σ is the share of capital in total output, whereas σ_0 is a scale variable. Real disposable income, y_d, is total output net of real tax receipts, θ:

1/ The model described below retains the salient features of the model used by Aghevli (1977). Henceforth, all variables are expressed in real per capita terms, unless otherwise specified.

$$y_d = y - \theta . \tag{2}$$

It is assumed that private saving, s_p, is a constant proportion, s, of disposable income and that it is channeled into private investment, i_p. 1/ Thus,

$$s_p = s(y - \theta) = i_p . \tag{3}$$

Government operations involve both current and capital account transactions. The overall budget deficit, g, which is the excess of current consumption, c_g, and government investment, i_g, over tax receipts, θ, is financed by the inflation tax, at the rate μ:

$$g = c_g + i_g - \theta = \mu m , \tag{4}$$

where m is the stock of real balances.

Taxation depends on total income and the collection lag in the tax system, reflecting legal codes, regulations and administrative procedures. Collection lags reduce the real fiscal revenue through inflation, π. Thus, the tax equation can be written as: 2/

$$\theta = \beta_o y^{\beta_1} e^{-\beta\pi}, \qquad \beta, \beta_1 > 0. \tag{5}$$

Government current consumption is a constant fraction of total output:

$$c_g = c_1 y . \tag{6}$$

Given the budget constraint in equation (4), government investment is financed partly by the current account surplus, $(\theta - c_g)$, and partly by the proceeds from deficit financing, μm:

$$i_g = \theta - c_g + \mu m . \tag{7}$$

1/ The assumption of a constant private saving ratio facilitates the analysis to focus on the use of the inflation tax.

2/ A formal derivation of this equation form is given in Choudhry (1991), pp. 2-3.

The demand for real balances, m, can be written as a Cagan-type function, in which inflation is fully anticipated: 1/

$$m = \alpha_0 \, y^{\alpha_1} \, e^{-\alpha\pi}, \qquad \alpha, \alpha_1 > 0. \tag{8}$$

In the steady state, the anticipated inflation is the actual inflation, which is equal to the rate of monetary expansion, μ, net of the rate of population growth, n. Thus,

$$\pi = \mu - n . \tag{9}$$

Capital accumulation, $\dot{k}$, comprises private investment and government investment less the adjustment necessary to maintain the capital stock for the rate of growth of population, n. Using equations (3) and (7), the equation for capital accumulation can be written as:

$$\dot{k} = s(y-\theta) + \mu m + \theta - c_g - nk. \tag{10}$$

For simplicity, assuming $\alpha_1 = \beta_1 = 1$ (i.e., the buoyancy of tax revenue and the income elasticity of real balance holdings are unitary) and using equations (1), (5), (6), (8), and (9), equation (10) can be rewritten as:

$$\dot{k} = s_e(\mu) f(k) - nk, \tag{10a}$$

where,

$$\begin{aligned} s_e(\mu) &= s(1 - \phi(\mu)) + \lambda(\mu) + \phi(\mu) - c_1, \\ \phi(\mu) &= \beta_0 e^{-\beta(\mu-n)}, \\ \lambda(\mu) &= \mu\alpha_0 e^{-\alpha(\mu-n)}. \end{aligned}$$

The component $s_e(\mu)$ can be viewed as the effective saving ratio, which is influenced by both the inflation and fiscal revenue ratios, $\lambda(\mu)$ and $\phi(\mu)$, respectively. As the rate of monetary expansion increases, the effective saving ratio rises upto a certain level because of gains in total saving resulting from forced saving and from a reduction in the tax burden stemming from the erosion of fiscal revenue. The latter, however, causes a

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1/ This simplification avoids the complications associated with the adjustment of price expectations. The divergence between actual and anticipated inflation, as has been frequently observed, leads not only to the transfer of real resources to the government but to other costs associated with the distribution of income and wealth. For the dynamic aspects of the welfare costs of inflation, when expectations adjust with a lag to the actual price developments, see Cathcart (1974) and Frenkel (1975, 1976). Also, for a survey of the recent theoretical and empirical work on the costs of inflation, see Driffill, et al (1989).

deterioration in the government current account balance, which dampens the rise in effective savings because part of the proceeds from the inflation tax is diverted to meet government's current obligations. The rate of monetary expansion at which the effective saving ratio reaches its maximum is μ_{se}. 1/ If inflationary financing continues beyond this rate, the resulting increase in the erosion of fiscal revenue more than offsets the gain in total saving, thereby causing the effective saving ratio to decline. As shown in the Appendix, the rate of μ_{se} is greater than the rate of monetary expansion, μ_r, which maximizes total revenue from taxation and inflation (i.e., total revenue, $R = \theta + \mu m$). Since the maximum total revenue is an upper limit on total government expenditure, it follows that even though the effective saving can be increased up to the rate μ_{se}, total government expenditure, hence government investment, which is constrained by the maximum total revenue, will be less than what it could have been at the rate of monetary expansion, μ_r. In equilibrium, the rate of μ_r, instead of μ_{se}, therefore becomes the binding upper limit on the effective saving ratio.

Capital accumulation, which depends on the effective saving ratio, is importantly affected by the fiscal erosion from inflation. In the process of augmenting capital accumulation through inflationary finance, the erosion of fiscal revenue widens the fiscal deficit by generating an "unintended" gap in the government current account. This may lead to increased use of the inflation tax or a shortfall in planned government investment as resources are diverted to meet government current obligations. Either way, the fiscal erosion dampens effective saving and therefore impedes the pace of capital accumulation, slowing growth. 2/

Private consumption can now be expressed as a function of the capital-labor ratio and the rate of monetary expansion. Using equations (3) and (4), and substituting $\lambda(\mu)f(k)$ for μm into equation (1), private consumption, c_p, can be expressed as:

$$c_p = f(k)\{(1-s)(1-\phi(\mu)) - \lambda(\mu)\}. \qquad (11)$$

1/ The rate μ_{se} is derived from the properties of $\phi(\mu)$, and $\lambda(\mu)$ in the Appendix, section 1, which also formally demonstrates other statements made later in the text.

2/ This is also seen by the steady state capital-labor ratio. By setting $\dot{k} = 0$ and substituting equation (1) into equation (10a), this ratio can be expressed as a function of the rate of monetary expansion:

i.e., $k = [\frac{\sigma_o s_e(\mu)}{n}]^{\sigma/1-\sigma}$. As the effective saving ratio is impeded by the fiscal erosion, the steady state capital-labor ratio would be less than if there were no erosion of fiscal revenue.

The bracketed expression is the consumption ratio, which is equal to $1 - c_1 - s_e(\mu)$. As such, it is influenced by inflationary finance and fiscal erosion in a manner that is opposite to that of $s_e(\mu)$. The consumption ratio thus steadily declines up to the rate of monetary expansion μ_{se} and then begins to rise as monetary expansion is increased beyond this rate.

The steady-state private consumption can be derived as a function only of the rate of monetary expansion. By setting $\dot{k} = 0$ in equation (10) and substituting the steady state capital-labor ratio into equation (11), the steady-state level of private consumption is given by:

$$c_p(\mu) = \sigma_o^{1/1-\sigma} \left[\frac{s_e(\mu)}{n}\right]^{\sigma/1-\sigma}(1 - c_1 - s_e(\mu)) \tag{12}$$

By creating forced savings through the inflation tax (up to a certain point) and spending the proceeds on capital formation, the steady-state consumption can be increased. This increase is not without cost since the marginal benefit of higher consumption of goods is also accompanied by an increase in the marginal welfare cost of inflation on real balances.

The welfare cost of inflation, w_c, of a reduction in real balances from $m(0)$ to $m(\pi)$, corresponding to the inflation rates of zero and π, respectively, can be expressed by substituting equations (8) and (9) into the standard expression: 1/

$$w_c = \int_{m(\pi)}^{m(o)} \pi \, dm = f(k)w(\mu), \tag{13}$$

where $w(\mu) = \alpha_0/\alpha - (\mu - n + 1/\alpha)\,\alpha_0 e^{-\alpha(\mu-n)}$ is the welfare cost ratio.

Total utility can now be defined as the utility of consumption goods less the utility loss associated with the reduction in real balances. 2/ Assuming that the marginal utility of consumption goods is unity, total utility, u, is the difference between private consumption and the welfare cost, w_c. Thus, using equations (11) and (13), total utility can be

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1/ This measure of w_c, which is the area under the demand curve for real balances, is based on the assumption that the marginal utility of consumption goods is constant. For a detailed analysis of the welfare cost of inflation and the derivation of the standard expression, see Bailey (1956) and Marty (1967, 1973). Also for the associated adjustment costs pertaining to price expectations, see Cathcart (1974) and Frenkel (1975 and 1976).

2/ This definition, which is also used by Aghevli (1975), is based on the assumption that real balances are a substitute for consumption goods.

expressed as a function of the capital-labor ratio and the rate of monetary expansion:

$$u = c_p - w_c = f(k)\psi(\mu), \tag{14}$$

where $\psi(\mu) = 1 - c_1 - s_e(\mu) - w(\mu)$

$$= (1-s)(1-\phi(\mu)) - \{\alpha_0/\alpha - (1/\alpha - n)\alpha_{oe}^{-\alpha(\mu-n)}\}.$$

The component $\psi(\mu)$ may be considered as the effective consumption ratio, the behavior of which is like the ratio discussed earlier, although, as the rate of monetary expansion increases, it falls more rapidly because of the increase in the marginal welfare cost of the reduction in real balances. The effective consumption ratio bottoms out at the rate of monetary expansion, μ_ψ, which is higher than the effective saving maximizing rate μ_{se} (see the Appendix, section 1). Beyond this rate, the marginal increase in tax savings more than outweighs the marginal increase in the welfare cost of inflation, thereby causing effective consumption to rise.

In order to derive the optimal rate of monetary expansion, the utility function is maximized over time. The maximization process depends on the time path of the capital-labor ratio, given by equation (10). It is assumed that the future stream of effective consumption, u, is discounted at the rate, δ. Given the social discount rate, the problem is to choose a rate of monetary expansion that maximizes

$$\int_0^\infty u(k,\mu)e^{-\delta t}dt, \tag{15}$$

subject to: $\dot{k} = s_e(\mu)f(k) - nk$.

2. Analysis of solution

The solution to the above maximization process determines the optimal rate of monetary expansion, μ^*, which satisfies the relationship: 1/

$$\mu^* = n + \frac{s_e'(\mu^*)}{\alpha\, \nu(\mu^*)} \left[\frac{\sigma\psi(\mu^*)}{(1+\delta/n-\sigma)s_e(\mu^*)} - 1 \right], \qquad (16)$$

where $\nu(\mu^*) = \alpha_0 e^{-\alpha\pi^*}$ is the real balance ratio at the equilibrium inflation, $\pi^* = \mu^* - n$. Given this rate, the steady-state capital-labor ratio, k^*, and the shadow price of capital, p^*, can be obtained from the solution of the maximum conditions of the maximization process set out in equation (15). 2/

The conditions for the existence of an optimal rate of monetary expansion, which yields a non-negative equilibrium inflation rate, are discussed in the Appendix, Section 2. The initial level of the fiscal deficit is critically important. If the level of the fiscal deficit requires the creation of more inflation revenue than would be forthcoming at the total revenue-maximizing rate, μ_r, which is less than the effective saving maximizing rate, μ_{se}, the deficit cannot be closed because the erosion of fiscal revenue more than offsets the gain in inflation revenue. Moreover, the deficit increases continuously as monetary expansion is stepped up to close the fiscal gap. In this situation, there is no equilibrium rate of monetary expansion, that can guarantee a non-negative equilibrium inflation rate and the return to equilibrium requires a fiscal shock. Without such a shock, continual deficit financing leads to an ever-widening fiscal gap and the economy moves to hyperinflation, with the

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1/ The solution of the control problem of equation (15) is derived in the Appendix, section 2. If the buoyancy of fiscal revenue is greater than unity ($\beta_1 > 1$), the relationship determining the optimal rate of monetary expansion is given by:

$$\mu^* = n + \frac{s_e'(\mu^*)}{\rho(\mu^*)\alpha\nu(\mu^*)} \left[\frac{\sigma\psi(\mu^*)}{(1+\delta/n-\sigma)s_e(\mu^*)} - 1 \right],$$

where,

$$\rho(\mu^*) = 1 - \frac{\sigma(\beta_1 - 1)(1-s)\tau(\mu^*)}{(1+\delta/n-\sigma)s_e(\mu^*)}$$

$$\tau(\mu^*) = \beta_0 \sigma_0^{\frac{\beta_1 - 1}{1-\sigma}} \left[\frac{s_e(\mu^*)}{n} \right]^{\frac{(\beta_1 - 1)\sigma}{1-\sigma}} \phi(\mu^*)$$ is the average fiscal revenue ratio

at the optimal rate of monetary expansion. As $\rho(\mu^*)$ is expected to be less than unity, the optimal rate of monetary expansion is higher when $\beta_1 > 1$ than when $\beta_1 = 1$.

2/ See Appendix, equation (27).

capital-labor ratio declining as the erosion of fiscal revenue eventually causes a decline in the effective saving. Stabilization requires a sharp reduction in the fiscal deficit to regain control of monetary expansion.

The solution point (k^*, μ^*) is locally stable. 1/ This implies that, for the optimal rate of monetary expansion, $\mu^*(t) = \mu^*$ at time t, whenever the capital-labor ratio k is lower than its steady-state level k^*, then the capital-labor ratio increases, i.e., $\dot{k} > 0$. If k is at a higher level than k^*, then $\dot{k} < 0$ and the capital-labor ratio declines to k^*. Notice that whenever the rate of monetary expansion is below μ^*, the effective saving can be increased by raising the rate of monetary expansion to μ^*, thereby increasing the capital-labor ratio to k^*. Any further monetary expansion that is below the total revenue maximizing rate of μ_r, yields additional inflation revenue, which is greater than the erosion in fiscal revenue, thus reducing the fiscal deficit. This leads to a lower rate of monetary expansion and this process continues until the rate of monetary expansion reverts to μ^*. Monetary expansion beyond the rate μ_r moves the economy to high inflation, a declining capital-labor ratio and, hence, negative growth.

Several interesting points emerge from the above analysis. 2/ First, the smaller the collection lag (β), the higher will be the optimal rate of inflation because a moderation in the erosion of fiscal revenue allows the creation of more effective saving. This increases the steady-state capital stock, as well as output and consumption, even after taking into account the increase in the marginal welfare cost of inflation. Second, the higher the value of the coefficient of expected inflation (α), the lower will be the value of μ^* and the lower is the sustainable level of fiscal deficit. The latter is determined by the optimal level of inflation revenue, $\lambda(\mu^*)$. 3/ It thus follows that the sustainable level of fiscal deficit depends importantly on inflationary expectations. Third, the greater the initial fiscal revenue ratio or the initial level of fiscal revenue relative to real

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1/ The local stability requires the conditions:

(i) $s_e'(\mu^*) > 0$

(ii) $\psi'(\mu^*)\, s_e''(\mu^*) - \psi''(\mu^*) s_e'(\mu^*) > 0$.

Both are satisfied. A formal analysis of local stability of solutions (equations (31)-(33)) is provided in the Appendix, section 2.

2/ These observations are based on the sign of the derivatives of μ^* with respect to the structural parameters in equation (16). See equation (34) in the Appendix, section 2.

3/ A high value of α implies that the elasticity of inflation of real balances is greater for a given expected rate of inflation. Thus, an increase in the expected rate of inflation would lower the desired real balance holdings more than if the value of α were small. Hence, with a smaller value of α, inflation revenue would be higher at a relatively low rate of inflation.

balance holdings, the smaller will be the optimal rate of monetary expansion. Finally, the lower the government current expenditure ratio, the lower is the optimal rate of monetary expansion.

In addition, the smaller the private saving ratio or the social discount rate, the higher will be the optimal rate of monetary expansion. A smaller private saving rate would require the creation of more forced saving to generate the same increase in the effective saving rate. The same result can be seen intuitively for the discount rate. A smaller discount rate gives less weight to present consumption, thereby permitting the extraction of more resources from current consumption to finance a larger government investment, which contributes to the future stream of consumption.

The cost of using inflationary finance can be analyzed through the shadow price of capital, p, which is given by 1/

$$p = - \frac{\psi'(\mu)}{s_e'(\mu)} . \tag{17}$$

The shadow price of capital equals the marginal loss of current effective consumption (in utility terms) per unit of additional effective saving created by the inflation tax. It thus reflects the marginal cost of the additional resources acquired through the inflation tax and is, therefore, an indicator of the marginal cost of using inflationary finance. The shadow price of capital can be expressed in terms of the marginal welfare cost of inflation and the marginal effective saving ratio by substituting the derivative, $\psi'(\mu) = -s_e'(\mu) - w'(\mu)$ into equation (17). Thus,

$$p = 1 + \frac{w'(\mu)}{s_e'(\mu)} . \tag{17a}$$

The shadow price of capital is above unity by the amount $\frac{w'(\mu)}{s_e'(\mu)}$, which is the marginal welfare cost of inflation per unit of additional effective forced saving. When there is no forced saving, the marginal welfare cost of inflation is zero, and the shadow price of capital is unity, implying that an additional unit of capital is generated by an equivalent unit of additional saving, thereby indicating that the utilization of resources is efficient. As additional resources acquired through forced saving increase the welfare cost of inflation, the shadow price of capital increases rapidly

1/ See Appendix, equation (28).

with inflation as well as with the degree of fiscal erosion. 1/ Thus, the higher the rate of monetary expansion or the extent of fiscal erosion, the higher is the marginal cost of using inflationary finance and the less efficient is the utilization of resources.

III. Simulations

The simulation provided in this section are used to draw some empirical implications from the preceding analysis. The simulations of the optimal rate of monetary expansion and the corresponding levels of output, consumption of goods, and the shadow price of capital are all based on the recent evidence on collection lags and the demand for money in developing countries. The remainder of the section provides a list of parameter values that were selected for the simulations and then discusses the results.

1. Parameter values

The values of the parameters selected for the simulations are shown in Table 1. The typical value for the collection lag, $\beta = 0.35$, and for the coefficient of expected inflation, $\alpha = 2.0$, are taken from the results of studies on collection lags (Choudhry (1990)) and the demand for money (Khan (1980)), respectively. The unit of the parameter β is in years. The value of the coefficient α, reflects the extent to which individuals adjust the demand for real balances in accordance with their revised expectations of inflation. 2/ The value of both of these parameters are varied to assess the sensitivity of the simulations. The fiscal revenue and real balance ratios are typical of tax and nontax revenue and of the monetary base in developing countries. The government current expenditure ratio, c_1, is chosen to keep the initial current account in balance. The output scale variable, σ_0, is any arbitrary number, whereas the share of capital, σ, is generally considered to be about 0.25.

The private saving and social discount rates should be broadly consistent. From the modified Golden Rule, a low private saving rate is

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1/ This is seen by the sign of the derivative, $\frac{dp}{d\mu}$ from equations (17) and (17a):

$$\frac{dp}{d\mu} = \frac{\psi'(\mu)s_e''(\mu) - \psi''(\mu)s_e'(\mu)}{[s_e'(\mu)]^2} = \frac{s_e'(\mu)w''(\mu) - s_e''(\mu)w'(\mu)}{[s_e'(\mu)]^2} > 0$$

for all $\mu < 1/\alpha + n$.

2/ This implies that, for a given expected inflation elasticity of real balances, the higher the value of this coefficient, the lower will be the optimal rate of monetary expansion whether or not there is fiscal erosion.

Table 1. Parameter Values for Simulation Experiment

Parameters		Values
Real fiscal revenue	β_o	0.5
Collection lag (in years)	β	(0, 0.10, 0.35, 0.50, 0.75) 1/
Government consumption	c_1	0.15
Real balance	α_o	0.15
Coefficient of expected inflation	α	(1.0, 2.0, 5.0, 20.0) 1/
Output scale (arbitrary number)	σ_o	25
Capital's share	σ	0.25
Private saving rate	s	0.05
Social discount rate	δ	0.05
Population growth rate	n	0.03

1/ Typical empirical value of β is 0.35 and α = 2.0. See Choudry (1990) and Khan (1980).

associated with a high discount rate, and vice versa. 1/ Thus, with no forced saving and a population growth of 3 percent, a 5 percent private saving rate implies a social discount rate of about 13 percent. This value appears to be high enough to indicate a strong preference for present consumption. Since large fiscal deficits divert substantial resources away from present consumption, the choice of a relatively high social discount rate provides little incentive for additional capital formation through forced saving. However, the effective saving ratio is expected to be increased by the creation of forced saving; hence, the selection of a discount rate of lower value is permissible. Thus, a value of 5 percent for the social discount rate may be considered to provide sufficient incentive for the authorities to augment capital formation through inflationary finance. 2/

1/ The Golden rule prescribes that in the steady state:

$$f'(k) = n + \delta.$$

Using the steady-state expression for $\dot{k}$ in equation (10) and substituting it into the derivative above, we have $f'(k) = \frac{n\sigma}{s'_e(n)}$, where $\mu = n$ implies no forced saving. Equating the two expressions results in the following:

$$\frac{n\sigma}{s'_e(n)} = n + \delta.$$

Therefore, given n and σ, and since $s_e(n) = s(1-\beta_0) + n\alpha_0$, the lower the rate of private saving, the higher is the social discount rate.

2/ The choice of $\delta = 0.05$ may be considered an admissible value. This can be seen by rewriting the $\dot{p} = 0$ equation in (27) as:

$$f'(k) = \frac{(\delta + n)}{\frac{\psi(\mu)}{p} + s_e(\mu)}.$$

Also, using the $\dot{k} = 0$ equation in (10) and substituting the steady state value for k, in terms of μ, into the derivative of f(k) above, we have:

$$f'(k) = \frac{n\sigma}{s'_e(\mu)}$$

Equating these two relationships, we have: $\frac{n\sigma}{s'_e(\mu)} = \frac{(\delta + n)}{\frac{\psi(\mu)}{p} + s_e(\mu)}$, and

rearranging, we can write: $\delta = n\sigma(\frac{\psi(\mu)}{ps_e(\mu)} + 1) - n.$

Given the values for the other parameters, as shown in Table 1, and for the values of: $\mu = 0.012$; $\alpha = 2.0$ and $\beta = 0.35$ and $p = 1.41$, which are obtained from Table 2, it is seen that for $s = 0.05$, the value of δ is 0.056.

2. Analysis of simulations

The simulations are reported in Table 2. The optimal rate of monetary expansion was obtained first, by solving equation (16). Given this rate, the steady-state capital-labor ratio, which was calculated by setting $\dot{k} = 0$ in equation (10), was used to compute the level of output and the consumption of goods from equations (1) and (14). Finally, the shadow price of capital is calculated from equation (17a).

Simulations of optimal rate of monetary expansion indicate that the scope of inflationary finance, which guarantees stability, is substantially limited. For the empirically relevant values of collection lags, from zero to six months ($\beta = 0.5$) and the values of the coefficient of expected inflation, between 2 and 5, the optimal rate of monetary expansion is in the 6-15 percent band, implying an inflation band of 3-12 percent. This rate falls sharply with higher collection lags. What is noteworthy is that the optimal rate of monetary expansion falls even more sharply with higher values of α. This indicates that if inflationary expectations are high, the scope of inflationary finance is progressively limited, whether or not there is fiscal erosion. This is an important result, with serious implications for the sustainable level of fiscal deficit that is aimed at promoting growth with price stability. The sustainable fiscal deficit corresponding to the optimal rate of monetary expansion appears to be significantly smaller (about 0.6-2 percent of real income or less with a higher rate of private saving) than implied by the total revenue maximizing rate of μ_r, which was found to be about 25 percent for the value of $\alpha = 2.0$ and $\beta = 0.35$. Although not reported in Table 2, the sustainable level of fiscal deficit is significantly narrowed with higher rates of private saving or of the social discount. Thus, if the rate of private saving is, say 7.5 percent or more, the authorities should either lower the deficit through fiscal adjustment or use a lower than 5 percent discount rate in order to make a limited use of inflationary finance for promoting growth with stability. 1/

The corresponding levels of steady-state output and consumption are also influenced by the harmful effects of inflation. Surprisingly, for the given values of the parameters for collection lags and the coefficient of expected inflation, the equilibrium steady-state levels of output and consumption were found to be a few percentage points higher, compared with the corresponding levels implied by a zero inflation rate. 2/ In contrast, the steady-state levels of output and consumption were found to be higher with a private serving rate of 7.5 percent; these levels would be even higher with a lower than 5 percent discount rate.

1/ While there is still debate in the literature on the choice of an appropriate rate of discount, a rate of 3 percent provides some room for use of inflationary finance.

2/ The steady-state values of output and consumption of goods were 84.9 and 68.2, respectively, when $\mu = n$.

Table 2. Simulations of the Optimal Rate of Monetary Expansion, Output and Consumption and the Shadow Price of Capital 1/

β/α	20.0	5.0	2.0	1.0
		Optimal rates of monetary expansion 2/ (in percent)		
0.00	3.86	9.15	15.19	19.37
0.10	3.61	8.48	14.75	19.76
0.35	3.12	6.71	12.65	19.43
0.50	2.86	5.65	10.79	17.89
		Output 3/		
0.00	85.13	88.14	92.28	95.63
0.10	85.03	87.47	91.23	94.69
0.35	84.91	86.10	88.60	91.72
0.50	84.91	85.53	87.19	89.68
		Consumption of goods 3/		
0.00	68.32	70.28	72.87	74.86
0.10	68.26	69.85	72.23	74.31
0.35	68.18	68.97	70.58	72.53
0.50	68.18	68.59	69.67	71.26
		Shadow price of capital		
0.00	1.75	1.57	1.35	1.20
0.10	1.76	1.61	1.40	1.24
0.35	1.77	1.69	1.54	1.38
0.50	1.77	1.73	1.62	1.48

1/ Other parameter values used in simulations are given in Table 1.

2/ The optimal rate of inflation is obtained by substracting the 3 percent rate of population growth.

3/ These values can be regarded as index numbers since the value of $\sigma_o = 25$ is an arbitrary number. Percentage changes can be directly derived from these values.

The shadow price of capital is found to be substantially greater than unity, which corresponds to the price when there is no forced saving. As expected, the shadow price of capital is higher, the longer the collection lag or the larger the coefficient of expected inflation. What is striking about the shadow price simulations is that the marginal cost of using inflationary finance increases rapidly with the extent of fiscal erosion or the extent of adjustment of real balance to inflationary expectations. 1/

The above results suggest that enhancing growth by using inflationary finance is rather costly. This raises the question of whether it is more desirable to share some of the burden of inflationary finance with taxation. 2/ Simulations with a fiscal revenue ratio of 20 percent, with $\beta = 0.35$ and $\alpha = 2.0$ while maintaining the values of other parameters as stated, indicate that the level of steady state output is significantly higher (about 23 percent) compared with the level corresponding to the fiscal revenue ratio of 15 percent. Interestingly, at the fiscal revenue ratio of 20 percent, the optimal rate of monetary expansion is less than 3 percent, indicating that the erosion of fiscal revenue is too high to permit a greater use of the inflation tax. At this fiscal revenue ratio, the corresponding shadow price of capital was found to be slightly below unity, indicating using taxation is more efficient than using inflationary finance to achieve growth objectives.

IV. Conclusions

The paper has analyzed the optimal rate of monetary expansion when the government resorts to inflationary finance to supplement private capital formation. When there are collection lags in the tax system, the process of creating forced saving and spending of the proceeds on capital accumulation is substantially impeded by the consequent erosion of fiscal revenue. This causation, running from inflation to budget deficits via the erosion of real fiscal revenue, leads to a deterioration of the government's current account balance and reinforces inflationary pressures, in addition to increasing the welfare cost of inflation. At the same time, it also reduces the amount of

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1/ Whether or not there is fiscal erosion, the conventional measure of the welfare cost of inflation, which is based on the assumption that output is fixed, does not reflect the "true" cost of resources. The measure, $\frac{w'(\mu)}{s_e'(\mu)}$ can be expressed as $\frac{w'(\mu)}{(1-s)\phi'(\mu)+\lambda'(\mu)}$ when there is fiscal erosion, or as $\frac{w'(\mu)}{\lambda'(\mu)}$ when there is no fiscal erosion. Also since $\frac{d\lambda'(\mu)}{d\alpha}<0$, the marginal cost of resources using inflationary finance increases with the coefficient of expected inflation.

2/ A paper on the optimal mix of income taxation and inflationary finance by Abbas Mirakhor and this author will be forthcoming shortly.

available forced saving for government investment, thus slowing the rate of capital accumulation and growth. As such, the steady-state levels of the capital-labor ratio, output and consumption are lower, while the marginal cost of using inflationary finance is higher than they would be without collection lags. Going beyond the optimal rate of monetary expansion can increase the risk of moving the economy to hyperinflation and low or even negative growth.

Simulations provide valuable insight into the cost of using inflationary finance to promote growth. The results indicate that the use of inflationary finance to enhance growth is rather limited and quite costly. Also, its use beyond the optimal rate of monetary expansion can have adverse effects for growth and price stability. These effects are further accentuated with tax collection lags. Moreover if inflation is chronic, the public may anticipate that a fresh spurt in prices is not temporary but signals further inflation to come and is, therefore, likely to quickly alter its behavior to minimize these effects. If this occurred, it would force the authorities to step up monetary expansion to acquire the same amount of resources, thereby increasing the risk of losing monetary control while endangering the economy to move to high inflation unless appropriate economic adjustments are made to reduce the fiscal deficit to a sustainable level. Without such adjustment, inflationary finance becomes increasingly destabilizing and costly.

The simulation results should also be interpreted in light of the omission of several additional factors. First, the omission of adjustment path of prices, which has been observed to lag behind growth in the monetary base or money supply, understates the amount of real resources acquired through inflationary finance because inflation expectations need not be formed instantaneously. This omission also understates the welfare cost of inflation in the dynamic adjustment process. Second, neglect of the adverse allocative and distributive effects of inflation, while difficult to analyze, has important socio-economic implications. These effects are what ultimately must be corrected through structural adjustments in order to constrain the economy from hyperinflation. Third, the assumption that government consumption is a fixed proportion of output is not supported by the experience of developing countries, particularly because of the increased burden of interest payments on the budget. 1/ Fourth, the use of inflationary financing need not solely be for productive purposes. Finally, inflation from external factors can exacerbate the impact of deficit financing. Most of these factors would tend to further limit the scope of using inflationary finance.

Empirical implications of the simulations appear to be consistent with the experience of several debt-laden and high inflation countries. The

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1/ Analytically, the increased burden of interest payment, which has the same effect on the fiscal deficit as the erosion of fiscal revenue, is likely to further limit the use of he inflation tax.

situations in Argentina, Bolivia, Brazil and Israel during the 1980s are a good example. These countries experienced high and variable inflation, which was intensified during the debt-crisis in 1982-85. While the inflation process in each country was complex and varied, a common element was the large fiscal imbalances. These were manifestations of a variety of factors, including high level of public expenditure (partly from a heavy burden of interest payments on the budget), relatively stagnant level of real tax revenue (which was already too high in Israel), varying extent and forms of restrictions on domestic and international transactions, a series of sharp depreciation of domestic currency and wage indexation. With access to foreign credit severely limited in the period, it was inevitable that the large fiscal deficits led to excessive use of inflationary finance that far exceeded the limits of sustainability. When inflation becomes ingrained, it acquires its own dynamics. This requires strong adjustment measures, containing specific remedies to deal with inertial forces, which are certain to develop as inflation rises. Although these countries launched frontal attack on inflation in 1985-86, the initial success in breaking the inflationary trend could be sustained only to the extent the adjustment of fundamental variables can be maintained. The experience of these countries in the preceding decade suggests that the task of reducing the fiscal deficits to a sustainable level was long and arduous and it has still some distance to go to regain monetary control.

The analysis and simulation results, when viewed within the perspective of the recent evidence on the effects of inflation on real fiscal revenue, have important implications for fiscal policy and tax reform. As the inflation tax is an inefficient means of resource generation and utilization, a government choosing this tax must take into account the possible effects of fiscal erosion not only on the budget deficit but also on growth. This is of particular importance when domestic rigidities or external factors contribute importantly to inflation. The use of the inflation tax not only threatens the price objective of fiscal policy, it also impedes the achievement of the growth objective with efficiency. These considerations further strengthen the case for tax reform in the adoption of a fiscal adjustment policy. Besides revenue and efficiency objectives, a tax reform should also be designed and implemented to reduce collection lags. Such a reduction can permanently raise the level of taxation. Indeed, the analysis in this paper indicates that increased reliance on fiscal revenue to accommodate an expansion in government expenditure, rather than relying on generating revenue through inflation, improves efficiency of resource utilization and enhances growth.

Effects of Inflation and Control Problem Solution

This Appendix provides a formal demonstration of the analysis contained in section II of the paper. The effects of inflation on total revenue from taxation and inflation and on the effective saving and effective consumption ratios are derived in section 1. The solution to the control problem described in equation (15), its existence and stability, as well as the effects of a change in the structural parameters on the optimal rate of monetary expansion, are demonstrated in section 2.

1. Effects of inflation

Total revenue from taxation and inflation is $R=\theta + \mu m$. Using equations (5), (8), and the assumption $\alpha_1 = \beta_1 = 1$, total revenue can be written as $R = r(\mu)f(k)$, where the total revenue ratio is:

$$r(\mu) = \phi(\mu) + \lambda(\mu). \tag{18}$$

The total revenue maximizing rate of monetary expansion, μ_r, is obtained from the solution of $r'(\mu) = (\phi)'(\mu) + \lambda'(\mu) = 0$; thus, 1/

$$\mu_r = \frac{1}{\alpha + \beta\frac{\phi(\mu_r)}{\lambda(\mu_r)}} < 1/\alpha. \tag{19}$$

The total revenue maximizing rate will be an important reference in the demonstration of the differential effects of inflation on effective saving and consumption ratios and in characterizing the existence and stability of solutions to the control problem of equation (15).

The differential effects of inflation on the effective saving and consumption ratios, $s_e(\mu)$ and $\psi(\mu)$, are given by:

$$\begin{aligned} s_e'(\mu) &= (1-s)\phi'(\mu) + \lambda'(\mu), \\ \psi'(\mu) &= -(1-s)\phi' - (1-\alpha n)\nu(\mu) \end{aligned} \tag{20}$$

where, $$\phi'(\mu) = -\beta\phi(\mu) < 0,$$

1/ The second derivative, $r''(\mu) = \phi''(\mu) + \lambda''(\mu) = \beta^2\phi(\mu) - (2/\mu - \alpha)\alpha\lambda(\mu)$, is $\begin{smallmatrix}<\\=\\>\end{smallmatrix} 0$ for all μ, such that $\mu \begin{smallmatrix}<\\=\\>\end{smallmatrix} \mu_r'' = \dfrac{2}{\alpha + \dfrac{\beta^2\phi''(\mu_r)}{\alpha\lambda(\mu_r'')}} > 2\mu_r$. Hence μ_r, is the total revenue maximizing rate of monetary expansion.

where, $$\phi'(\mu) = -\beta\phi(\mu) < 0,$$

$\lambda'(\mu) = (1-\alpha\mu)\nu(\mu) \gtreqless 0$, for all $\mu \lesseqgtr 1/\alpha$, and $\nu(\mu) = \alpha_0 e^{-\alpha(\mu-n)}$ is the real balance ratio at the rate of monetary expansion μ.

From the above, it is seen that:

$$s_e'(\mu) \gtreqless 0, \quad \text{for all } \mu \lesseqgtr \mu_{se} = \frac{1}{\alpha + (1-s)\dfrac{\beta\phi(\mu_{se})}{\lambda(\mu_{se})}},$$

and, $$\psi'(\mu) \lesseqgtr 0, \quad \text{for all } \mu \lesseqgtr \mu_\psi = \frac{1-\alpha n}{(1-s)\dfrac{\beta\phi(\mu_\psi)}{\lambda(\mu_\psi)}}. \tag{21}$$

The terms μ_{se} and υ_ψ are the effective saving maximizing and effective consumption minimizing rates of monetary expansion.

It is claimed that:
$$n \le \mu_r < \mu_{se} < \mu_\psi, \tag{22}$$

if, and only if, $\phi'(n) + \lambda'(n) \ge 0$, implying that the erosion of fiscal revenue is not large enough to offset the gain from inflation revenue when $\mu = n$.

The system of inequality in (22) can be established by considering the differential impact of inflation on total revenue, the effective saving and effective consumption ratios. Thus, utilizing $r'(\mu_r) = 0$, $s_e'(\mu_{se}) = 0$ and $\Psi'(\mu_\psi) = 0$, we have:

$$s_e'(\mu_r) - r'(\mu_r) = (1-s)\phi'(\mu_r) + \lambda'(\mu_r) - \phi'(\mu_r) - \lambda'(\mu_r)$$
$$= -s\phi'(\mu_r) > 0$$

and, $$\psi'(\mu_{se}) = -s_e'(\mu_{se}) - \alpha(\mu_{se}-n)\nu(\mu_{se})$$
$$= -\alpha(\mu_{se}-n)^{\upsilon}(\mu_{se}) < 0. \tag{23}$$

The results in equation (23) imply that $n \le \mu_r < \mu_{se}$ and $\mu_{se} < \mu_\psi$. Therefore the inequalities in (22) hold.

The signs of the second order derivatives of $s_e(\mu)$ and $\psi(\mu)$ are now verified to show that μ_{se} and μ_ψ are the turning points. Thus, we have:

$$s_e''(\mu) = (1-s)\phi''(\mu) + \lambda''(\mu)$$

$$\psi''(\mu) = -s_e''(\mu) - (1-\alpha(\mu-n))\alpha\upsilon(\mu), \tag{24}$$

where,

$$\phi''(\mu) = \beta^2\phi(\mu) > 0, \quad \text{for all } \mu, \text{ and}$$

$$\lambda''(\mu) = -(2-\alpha\mu)\alpha\upsilon(\mu)<0, \quad \text{for all } \mu<2/\alpha.$$

Thus, the signs of $s_e''(\mu)$ and $\psi''(\mu)$ are:

$$s_e''(\mu) \underset{>}{\overset{<}{=}} 0, \text{ for all } \mu \underset{>}{\overset{<}{=}} \mu_{se}'' = \frac{2}{\alpha + (1-s)\dfrac{\beta^2\phi(\mu_{se}'')}{\alpha\lambda(\mu_{se}'')}} > 2\,\mu_{se}$$

$$\psi''(\mu) \underset{<}{\overset{>}{=}} 0, \text{ for all } \mu \underset{>}{\overset{<}{=}} \mu_{\psi}'' = \frac{1-\alpha n}{(1-s)\dfrac{\beta^2\phi(\mu_{\psi}'')}{\alpha\lambda(\mu_{\psi}'')}} = \frac{\alpha}{\beta}\,\mu_{\psi} > \mu_{\psi} \tag{25}$$

because empirically the parameter α is found to be greater than the parameter β. The signs of $s_e''(\mu)$ and $\psi''(\mu)$ in equation (25) establish that μ_{se} and μ_{ψ} are the turning points.

2. Control problem solution

The current value Hamiltonian, H, of the control problem in equation (15) is

$$H = e^{-\delta t}[u(k,\mu) + p(f(k)s_e(\mu) - nk)], \tag{26}$$

where p is the shadow price of capital. The maximum conditions are:

$$\frac{\partial H}{\partial \mu} = f(k)\,[\psi'(\mu) + ps_e'(\mu)] = 0$$

$$\dot{p} = \delta p - \frac{\partial H}{\partial k} = (\delta + n)p - f'(k)[\psi(\mu) + ps_e(\mu)] \tag{27}$$

$$\dot{k} = \frac{\partial H}{\partial p} = f(k)\; s_e(\mu) - nk$$

Setting $\dot{k} = \dot{p} = 0$ and solving the maximum conditions yields the solution relationships shown in equation (28). The first equation determines the optimal rate of monetary expansion and the other two determine the corresponding values of the capital-labor ratio and the shadow price of capital, respectively:

$$-\frac{\psi'(\mu)}{s_e'(\mu)} = \frac{\psi(\mu)}{(1/\sigma + \delta/n\sigma - 1)s_e(\mu)} ,$$

$$k = [\frac{\sigma_o s_e(\mu)}{n}]^{1/1-\sigma} , \tag{28}$$

$$p = -\frac{\psi'(\mu)}{s_e'(\mu)} .$$

The rate of monetary expansion, μ^*, that satisfies the first relationship in equation (28) is optimal. Given this rate, the steady-state values of k and p are determined. Substituting the second right-hand side expression for $\psi'(\mu)$ from equation (20) into the first equation in (28), the rate μ^* can be expressed as:

$$\mu^* = n + \frac{s_e'(\mu^*)}{\alpha\nu(\mu^*)} [\frac{\psi(\mu^*)}{(1/\sigma + \delta/n\sigma - 1)s_e(\mu^*)} - 1] . \tag{29}$$

The existence of a non-negative optimal rate of inflation, $\pi^* = \mu^* - n$, is satisfied if and only if: (i) the marginal effective saving ratio $s_e'(\mu^*) \geq 0$; and (ii) $\psi(\mu^*)/s_e(\mu^*) > 1/\sigma + \delta/n\sigma - 1$. Assuming that both (i) and (ii) hold, it follows from (i) that: 1/

$$\mu^* \leq \mu_r \tag{30}$$

If μ^* is greater than μ_r, then the steady-state total revenue, $R(\mu^*)$ will be less than maximum, implying that lowering the rate of monetary expansion to μ_r would result in a higher total revenue and, therefore, more government investment and, in turn, a higher capital-labor ratio, output and consumption. However, if the initial fiscal deficit requires more forced saving than is available at the rate of monetary expansion, μ_r, then total expenditure outstrips the available maximum total revenue. Attempts to close the fiscal gap by stepping up monetary expansion would only increase the fiscal deficit further and would reduce government investment. In such a situation, a non-negative optimal rate of inflation does not exist.

The local stability of non-negative inflation rate solutions to the differential equations in (27) can be analyzed by a transformation of the

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1/ If the extent of fiscal erosion is sufficiently large, it is possible that a positive inflation rate at which total revenue is maximum does not exist, implying $\mu_r < n$. Such a solution is not considered here.

$\dot{p}$ equation into $\dot{\mu}$ equation, which can be obtained by equating the $\dot{p}$ equation in (27) with the time derivative of the optimal path of $p = -\frac{\psi'(\mu)}{s_e'(\mu)}$ from equation (28). Thus, we have the two differential equations in the (k,μ) space:

$$\dot{k} = f(k)s_e(\mu) - nk\ , \tag{31}$$

$$\dot{\mu} = \frac{s_e'(\mu)}{a(\mu)}[-(\delta + n)\psi'(\mu) - f'(k)\{\psi(\mu)s_e'(\mu) - \psi'(\mu)s_e(\mu)\}],$$

where, $$a(\mu) = \psi'(\mu)s_e''(\mu) - \psi''(\mu)s_e'(\mu) > 0.$$ 1/

The local stability of solutions to the differential equations in (31) can be determined from the characteristic roots of the matrix of coefficients obtained by a linear expansion of these equations at the point (k^*, μ^*). Thus, we can write:

$$\begin{aligned} \dot{k} &= a_{11}dk^* + a_{12}d\mu^* \\ \dot{\mu} &= a_{21}dk^* + a_{22}d\mu^*, \end{aligned} \tag{32}$$

where $dx_i^* = x_i - x_i^*$, $x = (k, \mu)$ and using the signs of the first and second derivatives of $f(k)$, $s_e(\mu)$, and $\psi(\mu)$ (and omitting the arguments from these functions):

$$a_{11} = f's_e - n = -n(1 - \sigma) < 0,$$

$$a_{12} = f(k)s_e'(\mu) > 0,$$

$$a_{21} = \frac{f''s_e'(\psi s_e' - \psi' s_e)}{a(\mu)} > 0, \text{ and}$$

$$a_{22} = \frac{(\delta + n)\ \psi s_e'}{\psi s_e' - \psi' s_e} > 0.$$

Therefore, the characteristic roots, r_1 and r_2, of the matrix in equation (32), are given by:

1/ See footnote one on page 11.

$$(r_1, r_2) = \frac{(a_{11} + a_{22}) \pm [(a_{11} - a_{22})^2 + 4a_{12}\, a_{21}]^{1/2}}{2}. \tag{33}$$

Simplifying the above further, these roots can be rewritten as:

$$r_1 = [a_{11}(1 + \rho) + a_{22}(1 - \rho)]/2 \text{ , and}$$

$$r_2 = [a_{11}(1 - \rho) + a_{22}(1 + \rho)]/2 \text{ ,}$$

$$\text{where,} \quad \rho = [1 + \frac{4a_{12}\, a_{21}}{(a_{11} - a_{22})^2}]^{1/2}.$$

Since $a_{11} < 0$ and $\rho > 1$, it follows that the roots are real and of opposite sign. Hence, the equilibrium point (k^*, μ^*) is a saddle point, implying the existence of a stable branch of points $(k^*(t), \mu^*(t))$ that eventually reach the steady-state. 1/

The effect on the optimal rate of monetary expansion of changes in the structural parameters can be determined by differentiating both sides of equation (29) with respect to each of the parameters, κ_i. Thus, collecting and rearranging the terms, and omitting the arguments from the functions and their derivatives, the effect of a change in κ_i on μ^* is:

$$D \cdot \frac{d\mu^*}{d\kappa_i} = \frac{1}{s_e'} \frac{ds_e'}{d\kappa_i} + \frac{s_e \frac{d\psi}{d\kappa_i} - \psi \frac{ds_e}{d\kappa_i} - \frac{\psi s_e}{a} \cdot \frac{da}{d\kappa_i}}{a(s_e)^2 \left(\frac{\psi}{a s_e} - 1\right)} \tag{34}$$

$$- \left(\frac{1}{\alpha} - \pi^*\right) \frac{d\alpha}{d\kappa_i} - \frac{1}{\alpha_o} \frac{d\alpha_o}{d\kappa_i},$$

where $(\kappa_i) = (\beta, \alpha, \beta_o, \alpha_o, c_1, \delta, s)$,

1/ Notice that the saddle point exists as long as $\mu^* \leq \mu_r$.

$$a = 1/\sigma + \delta/n\sigma - 1 ,$$

$$D = (\frac{1}{\pi^*} - \alpha) + B > 0 \text{ since } B = - \frac{s_e''}{s_e'} + \frac{s_e'\psi - s_e\psi'}{s_e(\psi - as_e)} > 0 .$$

From equation (34), it can be shown that:

(i) $\frac{d\mu^*}{d\beta} < 0$, when $\mu^* \leq \mu_r$,

(ii) $\frac{d\mu^*}{d\alpha} < 0$,

(iii) $\frac{d\mu^*}{d\beta_o} < 0$,

(iv) $\frac{d\mu^*}{d\alpha_o} < 0$, when α_o is less than a certain level,

(v) $\frac{d\mu^*}{dc_1} > 0$,

(vi) $\frac{d\mu^*}{d\delta} < 0$, and

(vii) $\frac{d\mu^*}{ds} < 0$, provided β_o is less than a critical value.

It should be noted that the signs of (i) through (vii) are all within a small range of the optimal rate of μ^*, thereby ensuring the existence of a new optimal rate of monetary expansion. Given this, it follows that the steady state capital-labor ratio of k^* and the shadow price of capital of p^* also moves in the same direction as the rate of μ^*.

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