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Target Zones and Forward Rates In a Model With Repeated Realignments

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Abstract

This paper studies the implications of the imperfect credibility of an exchange rate target zone on the term structure of forward premia. The relationship between spot and forward exchange rates of different maturities reflects the possibility of repeated realignments of the exchange rate band. The credibility of the commitment to the target zone implicit in forward market data can be extracted by estimating the model. Application to French/German data indicates that the model is capable of matching observed patterns of interest rate differentials during the EMS, while yielding estimates of the credibility parameters that accord with the experience of the FF/DM exchange rate during the 1980s.

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Contents

Summary

This paper examines the implications of the imperfect credibility of an exchange rate target zone for the term structure of forward premia. The relationship between spot and forward exchange rates of different maturities is developed in a model of exchange rate target zones with possible realignments. The term structure of forward rates is obtained as a function of the intervention rules and the realignment probabilities describing the credibility of the target zone.

Various sets of intervention rules at different levels of credibility are discussed. Standard target zone models are consistent with a wide variety of relationships between forward premia and spot exchange rates when allowance is made for imperfect credibility of the band. When the credibility of the target zone is low, for instance, the forward premium is a broadly increasing function of the position of the exchange rate in the band; a bimodal pattern arises when the devaluation probability at the upper boundary of the band differs from the revaluation probability at the lower boundary, a likely characterization of real-world target zones.

The credibility of the target zone implicit in forward market data can be calculated by estimating the model. Application of the model to French and German data since the start of the EMS indicates that the model is capable of matching observed patterns of interest rate differentials, while yielding estimates of the credibility parameters that accord with the experience of the FF/DM exchange rate during the 1980s, including a significant increase in the credibility of the FF/DM target zone from the end of 1986. Tests of the hypotheses of full credibility of the target zone and of linearity of the relationship between the spot exchange rate and exchange rate fundamentals point to the empirical relevance of realignment risk and the potential gains from estimating target zone models from forward market data.

I. Introduction

The inability of standard models of exchange rate bands to generate empirical patterns consistent with the evidence from the European Monetary System (EMS) has become the subject of increasing attention in the target zone literature. As discussed by Bertola and Caballero (1990), the expectation of central banks' intervention to defend the target zone implies that a decreasing interest rate differential should be induced by a weakening currency. Data from the largest EMS countries in the 1980s, however, reveal that essentially the opposite is true. Standard target zone models also predict counterfactual patterns for frequency distributions and variability of the exchange rate over different ranges of the band. Flood, Rose and Mathieson (1990) and Bodnar and Leahy (1990) have also documented the pervasive difficulty of identifying non-linearities that central banks' intervention should induce on the relationship between exchange rates and fundamentals.

While the empirical performance of standard models of target zones has been disappointing, recent studies suggest that some shortcomings of early models of exchange rate bands can be overcome by considering the possibility of realignments of the target zone. The possibility of an incipient realignment, being captured by the current level of the exchange rate even before a realignment actually takes place, has been shown by Bertola and Caballero (1990) to imply frequency distributions of spot exchange rates and patterns of instantaneous interest rate differentials more consistent with the observed evidence from the EMS. An important aspect of the analysis which has remained largely unexplored, however, is that forward exchange rates and, therefore, finite-maturity interest rate differentials should also reflect the risk of realignment of the target zone, and should incorporate information on the perceived credibility of the band.

In this paper we examine the relationship between interest rate differentials and exchange rate bands by developing a model of the term structure of forward premia in a target zone with imperfect credibility. We find that by allowing for imperfect credibility of the band, standard target zone models can be reconciled with many stylized facts from European forward markets during the EMS. When the credibility of the target zone is low, for instance, forward premia increase with the exchange rate position in the band, while bimodal patterns are induced by target zones with asymmetric credibility at the two boundaries, a likely characterization of real-world target zones. Our framework also proves useful in providing a tool for structural estimation of target zone models, a scarce commodity in the literature on exchange rate bands. 1/ To assess the empirical relevance of our framework, we estimate the model on French franc/deutsche mark (FF/DM) data. Our application confirms the pervasiveness of close-to-linear spot

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1/ To our knowledge, the only examples of structural estimation of target zone models can be found in Spencer (1991a), (1991b), and Lindberg and Söderlind (1991a), (1991b), that apply the Method of Simulated Moments to estimate full-credibility target zone models on French/German and Swedish data. Bodnar and Leahy (1990) also analyze the FF/DM market by Generalized Method of Moments.

exchange rates--fundamental relationships detected in previous studies, but indicates that the use of forward market data may represent a promising alternative for estimation and testing of target zone models. The estimation yields realistic estimates of the model's parameters, while producing patterns of interest rate differentials consistent with EMS data. The estimation also allows detection of the increased credibility of the FF/DM target zone at the end of the 1980s.

A limited number of target zone models with realignments have been studied in the literature. Bodnar (1991) considers the case in which a one-time devaluation may occur whenever the exchange rate hits the upper barrier of the band. Svensson (1991) lets devaluations be governed by a Poisson process, with realignment probability independent of the position of the exchange rate within the band. Bertola and Svensson (1991) augment standard models of target zones with an additional source of uncertainty, an exogenous process which randomly generates realignments. Rose and Svensson (1991) estimate an empirical model of realignments, incorporating the insights from Bertola and Svensson (1991), and find some support for the effects of realignments on interest rate differentials. In a spirit closer to the standard theory of target zones, Miller and Weller (1989) and Bertola and Caballero (1990) assume realignments to reflect the commitment of the central bank to the target zone when intervention to defend the band is required. The model of Bertola and Caballero (1990) suggests a flexible parameterization of realignment risk which preserves the simple structure of standard target zone models and yields analysis of spot exchange rates and instantaneous interest differentials. We follow this framework to develop a model of the term structure of forward premia. Our analysis allows unified consideration of the effect of imperfect credibility on several models of target zones--from the model with infinitesimal intervention of Krugman (1991) and Svensson (1991), to the model with discrete intervention of Flood and Garber (1989).

In the next section we outline the model of exchange rate determination that underlies our analysis. Section III develops the main analytical tool of our study, the relationship linking the current forward exchange rate to the spot rate and the level of fundamentals. Section IV discusses the implications for the term structure of forward premia of several intervention policies with varying degree of credibility of the target zone. Section V illustrates the use of the model as a framework for estimation of target zone models with an application to French/German data. Section VI concludes.

II. Target Zones With Realignments: The Spot Exchange Rate

We consider the standard log-linear model of exchange rate determination, augmented with the parametrization of central banks' intervention suggested by Miller and Weller (1989) and Bertola and Caballero (1990). Variables fundamental to the determination of the exchange rate, which include both variables with autonomous dynamics and variables under the control of the monetary authority, follow an unregulated diffusion process until they hit one of the edges of the current target zone. At that point, the central bank chooses either to defend the current band, by intervening on fundamentals while keeping the current central parity fixed, or to realign the central parity, by adjusting fundamentals and shifting to a new band.

As in all related research, the current level of the (log) exchange rate, $X(t)$,--defined as units of home currency per unit of foreign currency--is equal to the sum of the current level of fundamentals, $f(t)$, plus a linear function of its own expected change:

$$X(t) = f(t) + \gamma E_t[dX(t)/dt] \tag{1}$$

We shall omit derivation of (1), and refer the reader to Froot and Obstfeld (1991) for its interpretation in terms of a stochastic monetary model of exchange rates, and to Spencer (1991c) for its derivation from a model of intertemporal optimization. In this paper it is unnecessary to attach a specific interpretation to "fundamentals". For illustrative purposes, however, one may think of $f(t)$ as the difference between (the log of) the controllable components of domestic and foreign money supplies, plus an exogenous monetary shock.

When no intervention of the central bank is taking place, fundamentals are assumed to follow a Brownian motion:

$$df(t) = \mu dt + \sigma dw \tag{2}$$

where dw is the increment to a standard Wiener process, satisfying $E(dw)=0$ and $Var(dw)=dt$.

The no-bubble solution for the exchange rate can be written as a function of fundamentals and their current central parity, $X(t)=X(f(t),c(t))$. 1/ By applying Ito's lemma to $dX(t)$, and taking expectation as of information available at time t, one can substitute into (1) for $E_t[dX(t)]$. This gives the differential equation to be satisfied by $X(t)$ in the interior of the exchange rate band:

$$X(f,c) = f + \mu\gamma X_f(f,c) + \gamma\sigma^2 X_{ff}(f,c)/2 \tag{3}$$

1/ See Obstfeld and Rogoff (1983) for the standard argument used to exclude bubbles.

where the time-dependence of the variables has been suppressed for ease of notation. A general solution of (3) can be found by standard methods, and is given by:

$$X(f,c) = f + \mu\gamma + Ae^{\alpha(f-c)} + Be^{-\beta(f-c)} \tag{4}$$

In expression (4), A and B are integration constants to be determined by the behavior of X(t) at the boundaries of the target zone, and $\{\alpha,-\beta\}$ are the positive and negative roots of $\phi(\theta)\equiv\theta^2+2(\mu\theta-1/\gamma)/\sigma^2=0$.

The general restriction to be satisfied at the edges of the current band is that X(t) not be expected to jump, a condition necessary to prevent risk-neutral investors from expecting infinite profits as fundamentals approach the point of intervention. When the central bank intervenes infinitesimally, as in the models of Krugman (1991) and Svensson (1991), the appropriate boundary condition is a smooth-pasting condition, which requires the derivative of the spot exchange rate with respect to fundamentals to vanish at the edge of the target zone. A value-matching condition is the appropriate condition to impose when the central bank intervenes discretely as in the model of Flood and Garber (1989). A modified value-matching condition, requiring continuity of the expected value of the exchange rate across interventions, must be imposed in models with realignments at the edge of the target zone, as in Miller and Weller (1989) and Bertola and Caballero (1990). To complete the solution of the exchange rate, therefore, we need to specify the intervention policy followed by the central bank when the exchange rate reaches the boundaries of the target zone.

Figure 1 summarizes the intervention policy considered in this paper, which is parameterized as follows. Denote the central parity of the current fundamental band by c, the width of the fundamental band by $2\overline{f}$, and the probabilities that the band is realigned when fundamentals reach the upper and lower edges of their band by p and q, respectively. When the currency is devalued, the new central parity becomes (c+C) and the position of fundamentals inside the new band becomes (c+C+D). Similarly in case of revaluation: the new central parity becomes (c-C), with fundamentals repositioned in the new band at (c-C-R). If the central bank responds to devaluation pressures at the upper edge of the band by defending the central parity, fundamentals are repositioned inside the old band at (c+D*). Similarly, if defense takes place at the lower edge of the band, fundamentals are repositioned inside the old band at (c-R*). 1/

This description of the intervention policy allows for different realignment risk at the upper and lower edges of the band, as well as asymmetric rules for repositioning fundamentals inside the old and new

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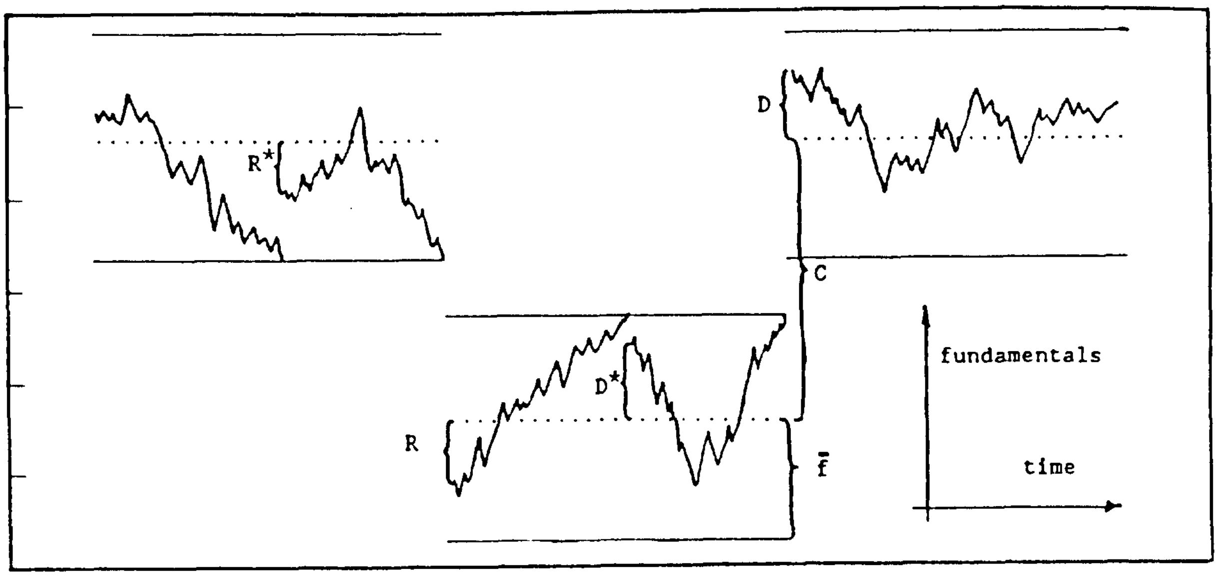
1/ Throughout the rest of this paper, we shall refer to D and R as the 'realignment' rules, to D* and R* as the 'defense' rules, and to the foursome as the 'intervention' rules. As is clear from Figure 1, $\{D,D^*,R^*,R\}\in(-\overline{f},\overline{f})$.

Figure 1

Possible Path for Fundamentals and Intervention Policy



bands. 1/ Notice that in the special case $p=q=0$, the band is fully credible, and one obtains the Flood and Garber case of discrete intervention. The Krugman-Svensson case of infinitesimal intervention, in turn, may be obtained by letting D^* and R^* go to $\bar{f}$.

Having specified the central bank's intervention policy, the required continuity (in expected value) of the exchange rate at the points of intervention yields the modified value-matching conditions to be satisfied by (4) at the upper and lower boundaries of the target zone:

$$X(c+\bar{f},c) = pX(c+C+D,c+C) + (1-p)X(c+D^*,c) \tag{5a}$$

$$X(c-\bar{f},c) = qX(c-C-R,c-C) + (1-q)X(c-R^*,c) \tag{5b}$$

Substituting the general solution (4) into (5a) and (5b) yields a system of two linear equations which readily solves for the constants A and B. These can be substituted back into (4) to complete the solution of the spot exchange rate as a function of current fundamentals and of their current central parity.

The intervention policy described above, suggested by Miller and Weller (1989) and further studied by Bertola and Caballero (1990), is obviously very stylized. Central banks' are assumed to intervene only at the barrier of the target zone, as is standard in the target zone literature, and the parameters of the fundamental process are assumed to be independent of the position of the exchange rate in the band. Similarly, the idea that devaluations are more likely for weak currencies is captured through the assumption that devaluations may occur only when fundamentals reach their upper boundary, and conversely for revaluations. As we shall see, this simple framework proves flexible in generating a broad range of patterns of forward premia. Nevertheless, future work should consider extension to more realistic policies: a mean-reverting process for fundamentals could be used in place of (2) to describe intra-marginal intervention which becomes stronger as the exchange rate diverges from some long-run target. 2/ The realignment probabilities could be endogenized as a function--for instance--of the level of available reserves, 3/ or as a result of a bargain between the relevant central banks. An exogenous process for realignment risk could be overlapped with the fundamental process, as suggested by Bertola and Svensson (1991), to allow realignments from any position of the exchange rate in the band. 4/ The simple treatment outlined above, however,

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1/ For the sake simplicity, however, we restrict the analysis to the case of upward and downward realignments of the same size.

2/ Dumas and Delgado (1991) and Lindberg and Söderlind (1991b) have incorporated this assumption in fully-credible target zone models.

3/ See and Bertola and Caballero (1991).

4/ Bertola and Svensson (1991) show how their assumption can be readily incorporated in standard models of target zones, and hence also in the present model.

possesses the virtues of intuitiveness and comparability with the prevailing literature, without otherwise preventing further generalizations, and is the basis for the analysis that follows.

III. The Forward Exchange Rate

1. The forward rate in a target zone with repeated realignments

In this section we develop the relationship between fundamentals and forward exchange rates implied by the model discussed in the previous section. Define the τ-period-ahead (log) forward exchange rate by $F(f,c,\tau)$, where f is the current level of fundamentals, c is the central parity of the current fundamental band and τ is the time to the fixed maturity date T, i.e. $T \equiv t+\tau$. Calendar time t enters the analysis only through the state variables f(t) and c(t). With risk-neutral and rational investors, the current forward rate is the unbiased predictor of the future spot rate: 1/

$$F(f(t),c(t),\tau) = E_t[X(t+\tau)] \equiv E_t[X(T)] \tag{6}$$

It follows, by application of the law of iterated expectations, that the expected change in a fixed-maturity forward contract must equal zero at all times:

$$\begin{aligned} E_t[dF(f(t),c(t),\tau)] &= E_t[F(f(t+dt),c(d+dt),\tau-dt)] - F(f(t),c(t),\tau) \\ &= E_t[E_{t+dt}X(T)] - E_tX(T) \\ &= E_tX(T) - E_tX(T) = 0 \end{aligned} \tag{7}$$

The condition that the forward rate for a fixed-maturity date not be expected to change (i.e., that the forward rate is a martingale) must hold both inside the band and at the intervention points. As long as fundamentals fluctuate in the interior of their band--so that their central parity c is fixed--, Ito's lemma translates condition (7) into the following partial differential equation in f and τ,

$$F_\tau(f,c,\tau) = \mu F_f\,(f,c,\tau) + \sigma^2 F_{ff}\,(f,c,\tau)/2 \tag{8}$$

1/ Data from freely floating exchange rates traditionally provides evidence against the implication of the joint assumptions of risk neutrality and rational expectations--that uncovered interest parity should hold (see Hodrik (1987) for a survey). Svensson (1990) argues however that the uncovered premium is likely to be small for currencies regulated within a relatively small target zone. More specifically, Rose and Svensson (1991) show that the hypothesis of uncovered interest parity cannot be rejected for the FF/DM market during the EMS, the data on which our subsequent application is based.

At the intervention points, condition (7) determines two boundary conditions which follow from the specified intervention rules. As in the case of the spot exchange rate, these conditions are necessary to prevent risk-neutral agents from desiring an unbounded foreign currency position as the edge of the target zone is approached:

$$F(c+\overline{f},c,\tau) = pF(c+C+D,c+C,\tau) + (1-p)F(c+D^*,c,\tau) \tag{9a}$$

$$F(c-\overline{f},c,\tau) = qF(c-C-R,c-C,\tau) + (1-q)F(c-R^*,c,\tau) \tag{9b}$$

A final boundary condition is the initial condition that the zero-period-ahead forward rate equals the spot rate:

$$F(f,c,0) = X(f,c) \tag{10}$$

The boundary-value problem defined by (8)-(10) does not have a known closed-form solution. 1/ Therefore, in order to calculate forward rates, we use a numerical procedure. The solution algorithm is a standard one from the theory of partial differential equations, and it was used by Svensson (1991) for the case of fully credible bands. We outline it here to point out the differences arising in the case of bands' realignments.

2. Solution of the forward rate equation

First, let us define a state space for fundamentals, f, and a time step for the term of the forward contract, τ. The step-size for fundamentals is Δf, and the step-size for the term is $\Delta\tau$. There are I fundamental steps in each band, and N bands in the state space. N must be chosen sufficiently large to make the approximation involved in considering only a finite number of bands, as well as the dependence of the solution on the boundary behavior at the bottom of the lowest band and at the top of the highest band, negligible. This can be done as long as p and q are strictly smaller than one, i.e., there is some positive probability that the central bank will defend the target zone. In this case, a convenient assumption is that the top of the first band and the bottom of the last band in the state space are defended by infinitesimal intervention with probability one.

By assuming that the forward rate is a twice differentiable function of fundamentals and time to maturity, we can write F_{ff} and F_f as central differences and F_τ as a forward difference. This allows a forward solution in τ of the differential equation (8). The notation is as follows:

1/ Analytic solution of (8)-(10) is prevented by the non-standard nature of the boundary conditions (9a,b). Upon separation of variables, the infinite series of eigenfunctions that can be obtained for the time-homogeneous component of (8), neither forms an orthonormal set (except over a set of measure zero over the parameter space) nor exhibits known convergence properties.

$$F_f(f_i,\tau_j) = \frac{F^j_{i+1}-F^j_{i-1}}{2\Delta f}, \quad F_{ff}(f_i,\tau_j) = \frac{F^j_{i+1}-2F^j_i+F^j_{i-1}}{(\Delta f)^2}, \quad F_\tau(f_i,\tau_j) = \frac{F^{j+1}_i-F^j_i}{\Delta\tau} \tag{11}$$

where we define the forward rate at fundamental position i and time to maturity j as $F(f_i,\tau_j)\equiv F_i{}^j$. 1/ We substitute these expressions for F_{ff}, F_f and F_τ into (8), and define $r\equiv\sigma^2\Delta/2(\Delta f)^2$ and $s\equiv\mu\Delta\tau/2\Delta f$. This gives the difference equation corresponding to the differential equation (8) in the interior of the band. 2/

$$F_i{}^{j+1} = (r+s)F_{i-1}{}^j + (1-2r)F_i{}^j + (r-s)F_{i+1}{}^j \tag{12}$$

By indexing the fundamental space from the top of the highest band, the boundary conditions describing the jumps at the edges of each band may be written as: 3/

$$F^{j+1}_{(n-1)I+1} = pF^{j+1}_{(n-\frac{3}{2})I+\frac{1}{2}-\frac{D}{\Delta f}} + (1-p)F^{j+1}_{(n-\frac{1}{2})I+\frac{1}{2}-\frac{D^*}{\Delta f}} \qquad \text{for } n=2,\ldots,N \tag{13a}$$

$$F^{j+1}_{nI} = qF^{j+1}_{(n+\frac{1}{2})I+\frac{1}{2}+\frac{R}{\Delta f}} + (1-q)F^{j+1}_{(n-\frac{1}{2})I+\frac{1}{2}+\frac{R^*}{\Delta f}} \qquad \text{for } n=1,\ldots,N-1 \tag{13b}$$

Since the top of the first and the bottom of the last bands are always defended,

$$F_1{}^{j+1} = (1-r+s)F_1{}^j+(r-s)F_2{}^j \quad \text{and} \quad F_{NI}{}^{j+1} = (1-r-s)F_{NI}{}^j+(r+s)F_{NI-1}{}^j \tag{14}$$

Equations (12)-(14) can be written in matrix form as in:

1/ Notice that it is unnecessary to keep explicit account of the current central parity of fundamentals; the index i runs over the entire space of fundamentals, and it is defined in such a way that the same level of fundamentals receives two different indices if it belongs to two different bands (as in the case of overlapping bands). Thus c is a well defined function of i and can be omitted from the arguments of F(.).

2/ For convergence of the numerical procedure, the time step Δt must be chosen to assure $r<0.5$. See Hornbeck (1975) for details on numerical solutions of partial differential equations.

3/ The notation in (13a-b) reflects the convention for an odd number of fundamental steps in each band and rounding of $\cdot/\Delta f$ to its closest integer.

$$\begin{bmatrix} B & & & & & & & \\ & A & & & & & & \\ & & \cdot & & & & & \\ & & & \cdot & & & & \\ & & & & A & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & A & \\ & & & & & & & C \end{bmatrix} \begin{bmatrix} F_1^j \\ \cdot \\ \cdot \\ \begin{bmatrix} F_{(n-1)I+1}^j \\ \cdot \\ F_{nI}^j \end{bmatrix} \\ \cdot \\ \cdot \\ F_{NI}^j \end{bmatrix} = \begin{bmatrix} F_1^{j+1} \\ \cdot \\ \cdot \\ \begin{bmatrix} F_{(n-1)I+1}^{j+1} \\ \cdot \\ F_{nI}^{j+1} \end{bmatrix} \\ \cdot \\ \cdot \\ F_{NI}^{j+1} \end{bmatrix} \tag{15}$$

where the transition matrix A is written out in full in Figure 2, while B and C are equivalent to A with the exception of the adjusted boundary conditions at the top of B and at the bottom of C, as in (14). The solution can be written in more compact notation as:

$$F^{j+1} = HF^j \tag{16}$$

which is tied down by imposing the initial condition (10), so that:

$$F^j = (H)^j X(f,c) \tag{17}$$

Expression (17) completes the solution of the forward rate as a function of fundamentals and of their current central parity. By using (4), a fundamental state space can be created for a given exchange rate band and set of parameters. Expressions (4) and the transition equation (17), in turn, give the solution for the spot exchange rate and the forward rate with a j-week maturity.

IV. Analysis of Forward Premia with Imperfect Credibility of the Band

In this section we consider the implications of different intervention rules and various levels of credibility for the term structure of forward premia. For all cases, we consider an exchange rate band of 4.5 percent, with an adjustment parameter, γ, equal to .1, a band-shift parameter, C, equal to 4, and driftless fundamentals with annual standard deviation, σ, of 2.25 percent.

The spot exchange rate solution for the case of fully credible bands with infinitesimal control at the boundaries, studied by Krugman (1991), is presented in Figure 3. The familiar S-shape of the spot exchange rate as a function of fundamentals translates into the family of forward premia of increasing maturities shown in Figure 4. 1/ Since the current exchange rate band is fully credible, the importance of the initial position of the exchange rate decreases as the forecast horizon goes to infinity. Correspondingly, forward rates of increasing maturities converge to the unconditional mean of the spot rate, which corresponds in this case to the central parity. As a result, negative forward premia arise at the top of the exchange rate band, and positive ones at the bottom. Adding a drift to fundamentals and/or a constant devaluation risk does not alter the main qualitative features of the analysis, it essentially shifts the entire forward premium schedule upward.

In comparing Figures 3 and 4, notice that positive forward premia arise where the spot rate curve is convex and negative premia where the curve is concave. While this simple relationship holds exactly only in the case of zero-drift fundamentals and symmetric intervention policies, it does represent the essential engine of expected exchange rate dynamics in our model, following from Jensen's inequality as applied to X(f): an increase (decrease) of the exchange rate is expected over the range where X(f) is a convex (concave) function. This feature holds even when realignments are admitted, because the expected change of the exchange rate at the exact time of a realignment is zero. For this reason, the shape of the forward premium curve depends on the realignment probabilities only to the extent that these affect the curvature of the spot exchange rate-fundamental relationship. Similarly, the forward premium depends on the intervention and realignment policies, but only to the extent that the curvature of the spot rate following a defense or a realignment determines its expected change in the next interval of time. 2/

1/ This is the case studied by Svensson (1991) and--with different techniques--by Spencer (1991b). In this case an analytic solution for the forward exchange rate can be obtained, since the boundary condition for the forward rate takes the simple form of a smooth-pasting condition when the band is fully credible and intervention is infinitesimal. Even in this case, the solution must eventually be obtained numerically, as a finite-sum approximation to an infinite Fourier series (as in Svensson (1991)), or to an infinite weighted sum of truncated normals (as in Spencer (1991b)).

2/ This is also the reason why the probability of a realignment (p or q) and the size of the realignment (C+D or C-R) enter independently in the determination of the forward premia (and finite-maturity interest rate differentials) but not in the determination of the instantaneous rate of change of the spot rate (which determines the instantaneous interest differential). As noticed by Bertola and Svensson (1991), the latter depends only on the expected size of realignments. In contrast, a change in the size of a realignment compensated by an opposite change in the realignment probability will change the forward rate at all positive maturities, except in the singular case where the spot exchange rate is everywhere linear in fundamentals.

Figure 2

The Transition matrix **A**

$$
\mathbf{A} = \left[\begin{array}{c|c|c}
\ldots, 0, p(r+s), p(1-2r), p(r-s), 0, \ldots & \ldots, 0, (1-p)(r+s), (1-p)(1-2r), (1-p)(r-s), 0, \ldots & 0, \ldots \quad \ldots, 0 \\
0, \ldots \quad \ldots, 0 & (r+s), (1-2r), (r-s), 0, \ldots & \cdot \quad \cdot \\
\cdot \quad \cdot & 0, (r+s), (1-2r), (r-s), 0, \ldots & \cdot \quad \cdot \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\vdots & \ldots, 0, \bullet \quad \bullet \quad \bullet, 0, \ldots & \vdots \\
\cdot \quad \cdot & \ldots, 0, (r+s), (1-2r), (r-s), 0 & \cdot \quad \cdot \\
\cdot \quad \cdot & \ldots, 0, (r+s), (1-2r), (r-s) & 0, \ldots \quad \ldots, 0 \\
0, \ldots \quad \ldots, 0 & \ldots, 0, (1-q)(r+s), (1-q)(1-2r), (1-q)(r-s), 0, \ldots & \ldots, 0, q(r+s), q(1-2r), q(r-s), 0, \ldots
\end{array}\right]
$$

IxI   IxI   IxI

where : p = devaluation probability
r = revaluation probability
r = $\sigma^2\Delta/2(\Delta f)^2$
s = $\mu\Delta\tau/2\Delta f$

Figure 3

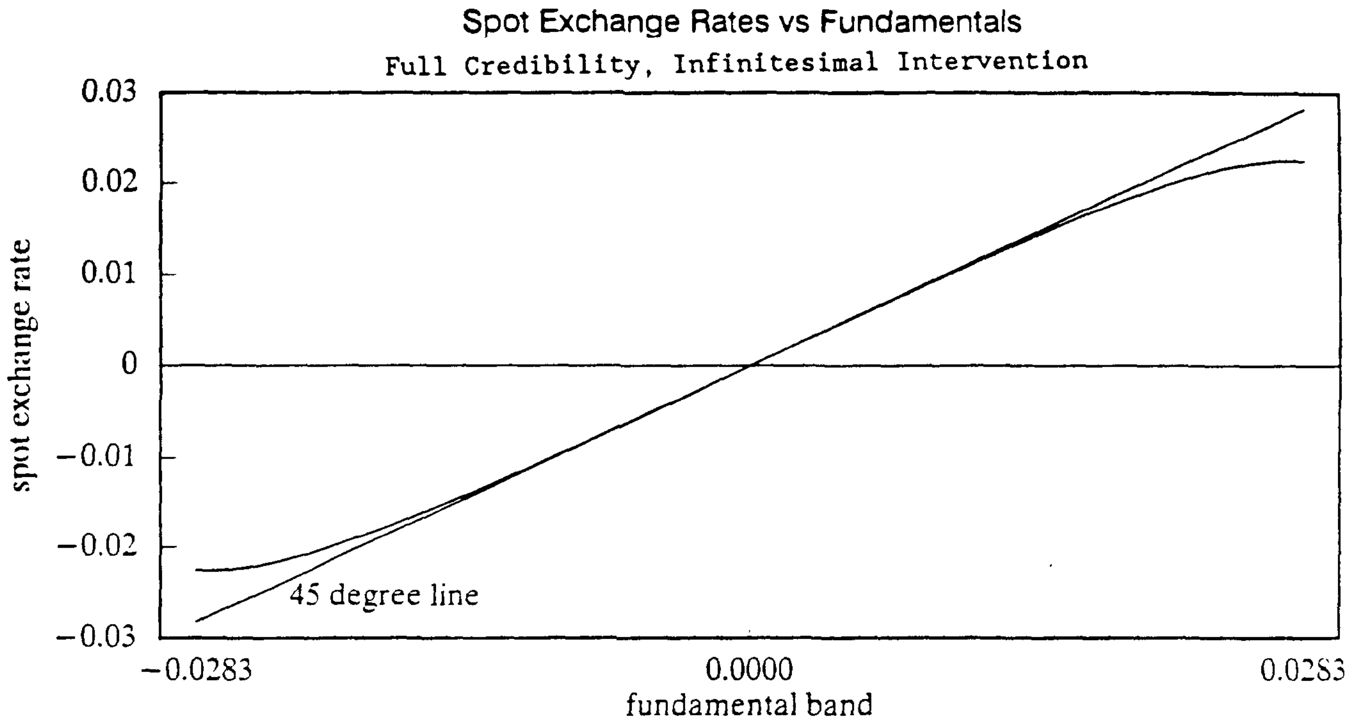


Figure 4

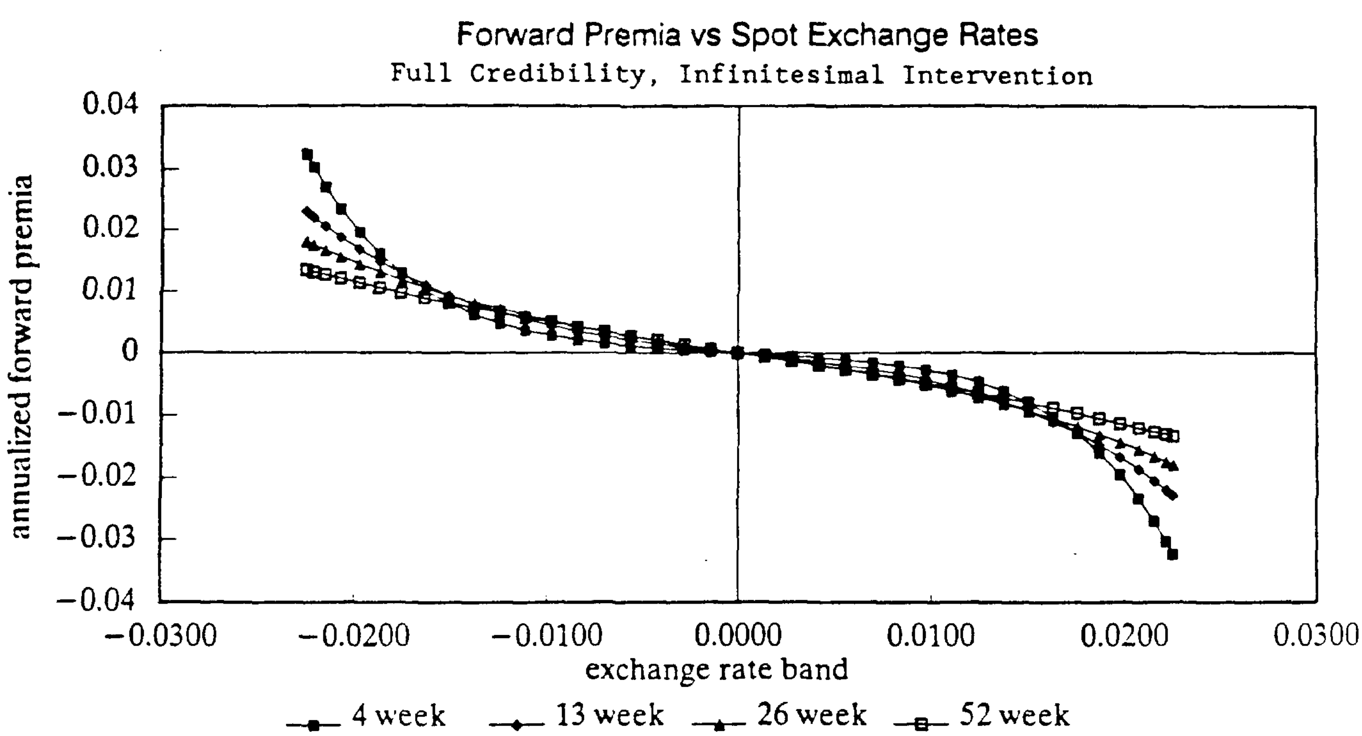


Figure 5 presents the spot exchange rate solution for the full-credibility case with discrete intervention discussed by Flood and Garber (1989). In this case, the spot exchange rate is constrained to remain strictly inside its own band at the boundary of the fundamental band because of the required equality between the values immediately before and immediately after intervention. This implies a more pronounced S-shape for the exchange rate and the somewhat peculiar loops for the forward premia displayed in Figure 6. As in the case of infinitesimal intervention, full credibility of the band determines expected reversion of the exchange rate to its unconditional mean, thus yielding a pattern of forward premia qualitatively similar to that examined in the previous case.

Combinations of the previous two cases can also be considered. However, the essential implications of a fully credible target zone remain: a currency near the edges of a credible band tends to revert towards the middle of the band in the long run, thus generating forward premia broadly decreasing in the position of the exchange rate within the target zone.

The next examples we consider are those with imperfect credibility of the band; these allow for the added feature of realignments. In the next several examples that follow we assume that intervention results in movements of fundamentals to the center of the bands ($D=D^*=R^*=R=0$). This assumption underlies the zero forward premia induced at the edges of the band, a feature which need not arise with the more general intervention rules discussed later in this section.

Figure 7 shows the spot exchange rate solution for the case of a target zone with relatively high credibility, $p=q=.2$. In this case the curve of the spot exchange rate resembles the curve obtained in the fully-credible case of infinitesimal intervention. When the probability of realignment is sufficiently high, however, the spot rate curve no longer smooth-pastes the edges of the target zone, and its slope is everywhere positive. As we see in Figure 8, the first-convex then-concave relationship between the spot exchange rate and fundamentals translates in this case into a hump-shaped profile of the forward premium. Nonetheless, high credibility of the target zone generates a pattern of forward premia broadly similar to the previous two cases: positive forward premia are generated over the lower range of the band and negative ones over the upper range.

Next we consider the case of an exchange rate authority with relatively low credibility at both barriers, $p=q=.4$. Figure 9 displays the spot exchange rate solution that features the "inverted S-shape" discussed in Bertola and Caballero (1990). Figure 10 gives the corresponding solution for the term structure of the forward premium. Notice the striking difference with all the cases previously considered: weak currencies are now found to be associated with positive forward premia. Formally, positive forward premia arise over the upper half of the band because of the convexity of the spot exchange rate curve over that range of fundamentals. The economics underlying this feature is investors' forecast of a further depreciation of a weak currency over the next τ-interval of time (and conversely for a strong currency). In this case, intervention of the central bank, which is expected to renege on its commitment to the current band as fundamentals approach either edge of the band, has a destabilizing effect on the future level of the exchange rate.

Figures 11 and 12 present the mixed case of a central bank with higher credibility at the lower edge of the band than at the upper edge, with $p=.4$ and $q=.2$. In this case the spot-rate curve is convex throughout its entire range, a feature that prompts positive forward premia throughout the entire target zone. The intervention policy is expected to induce devaluations of both strong and weak currencies, in the first case following a likely defense of the band, in the second case following a likely realignment. These features are reflected in the bimodal shape of the forward premia function in Figure 12. This results from the sharper convexity of the spot exchange function near the edges of the band, which contrasts with the flatter segment over the middle range of the target zone.

Finally, Figures 13 gives the spot exchange rate solution when changing the intervention rules with respect to the base-line case in which intervention always repositions fundamentals to the center of the band, for a target zone with low credibility ($p=q=.4$). Changing the intervention rules will generally affect the shape of both the spot rate and the forward premia throughout the entire band. The most apparent effect, however, will be that of removing the constraint on the forward premium to equal zero at the edges of the band, as we see in Figure 14 for the case $D^*=R^*=.8$, $D=R=0$. Here the intervention policy contemplates a small fundamental shift in the case of a successful defense, and a large fundamental shift in the case of a realignment. As a result, the exchange rate is expected to further depreciate as the upper edge of the band is approached, thus inducing positive premia at the top of the band, and conversely at the opposite end.

The examples in this section highlight the effects that different degrees of credibility of the target zone and intervention policies have on the shape of the forward premium curve. The main implication of the analysis is that by allowing for imperfect band credibility, standard target zone models can be made consistent with a richer set of patterns of forward premia as a function of the position of the exchange rate in the band. We have discussed in the previous section how modifications of the basic model could be considered to make the stylized framework discussed here more consistent with observed intervention and realignment policies. In the next section we try to assess the empirical relevance of our simple model of the term structure of forward premia by considering an application to FF/DM data.

Before proceeding with estimation of the model, a remark is in order on the interpretation of the credibility parameters p and q. Notice that the probabilities p and q, while representing structural model parameters, provide only limited information on the likelihood of observing a realignment over any given time interval when considered in isolation from the other parameters of the model. This is because their role is solely

Figure 5

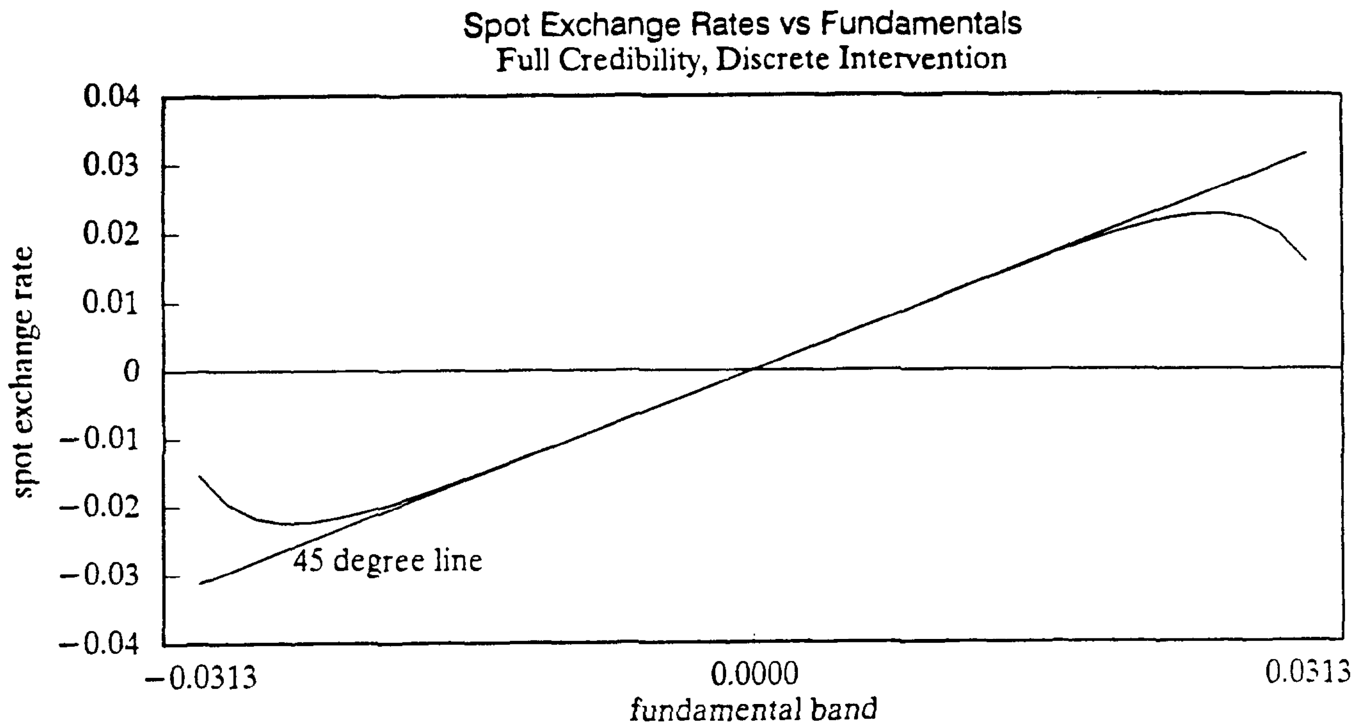


Figure 6

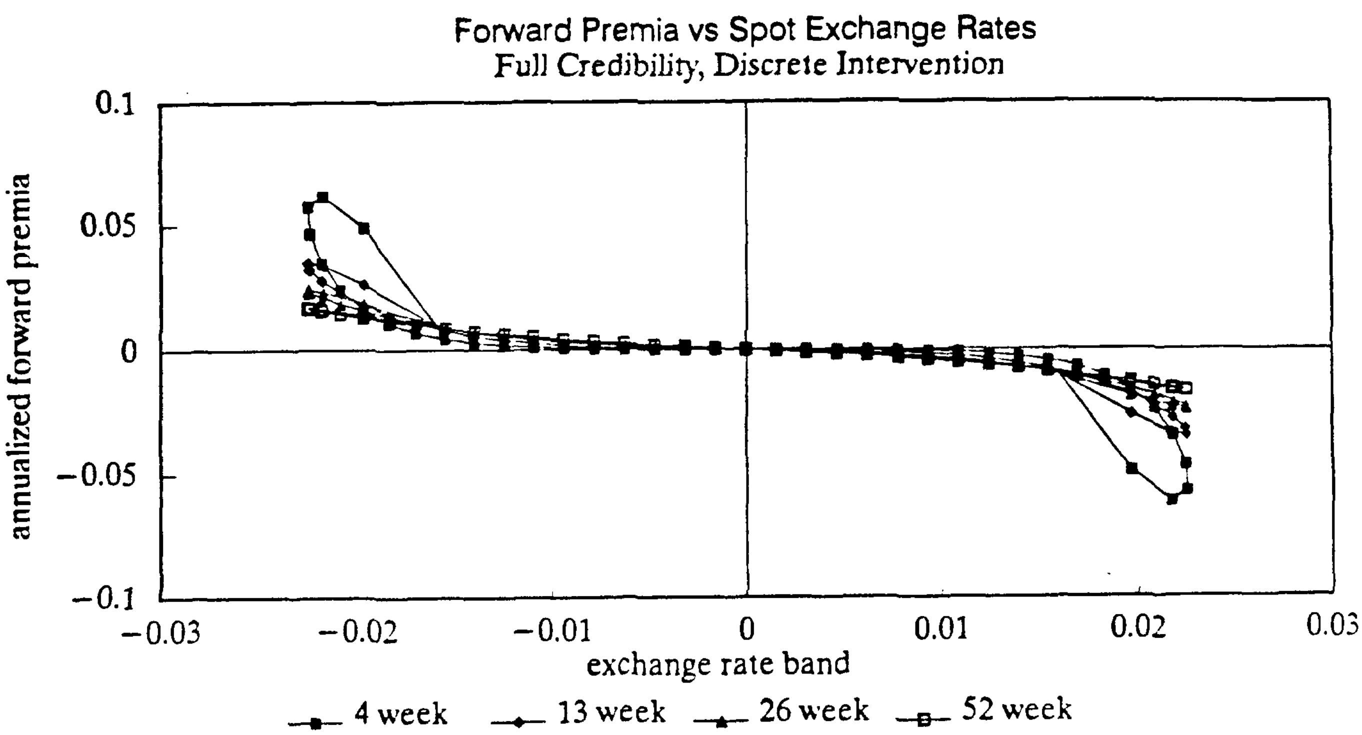


Figure 7

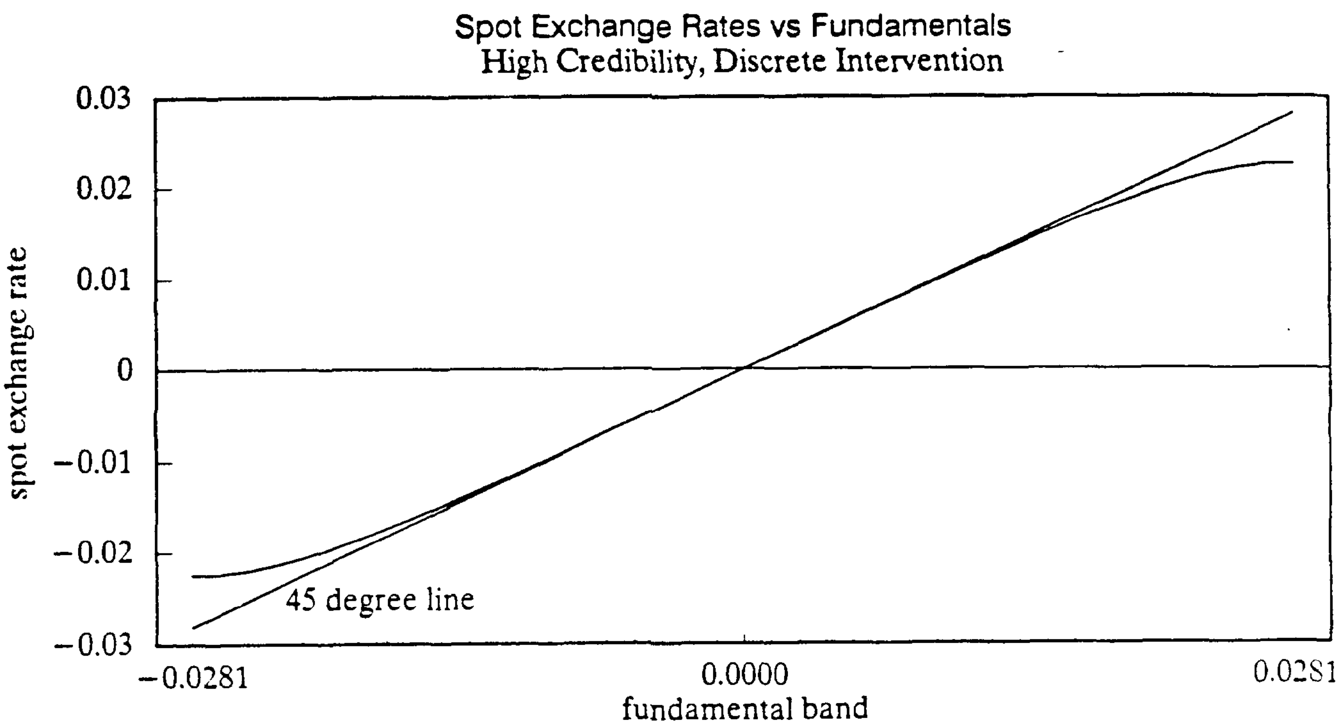


Figure 8

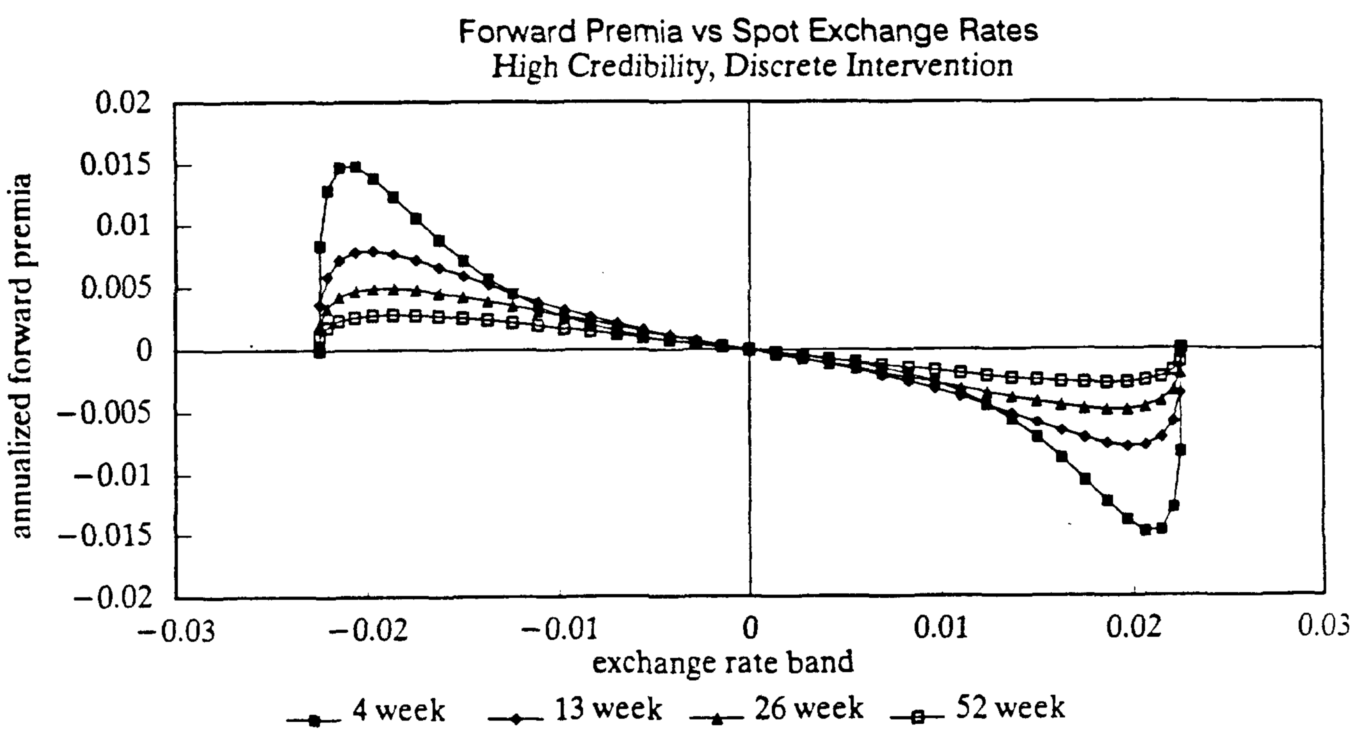


Figure 9

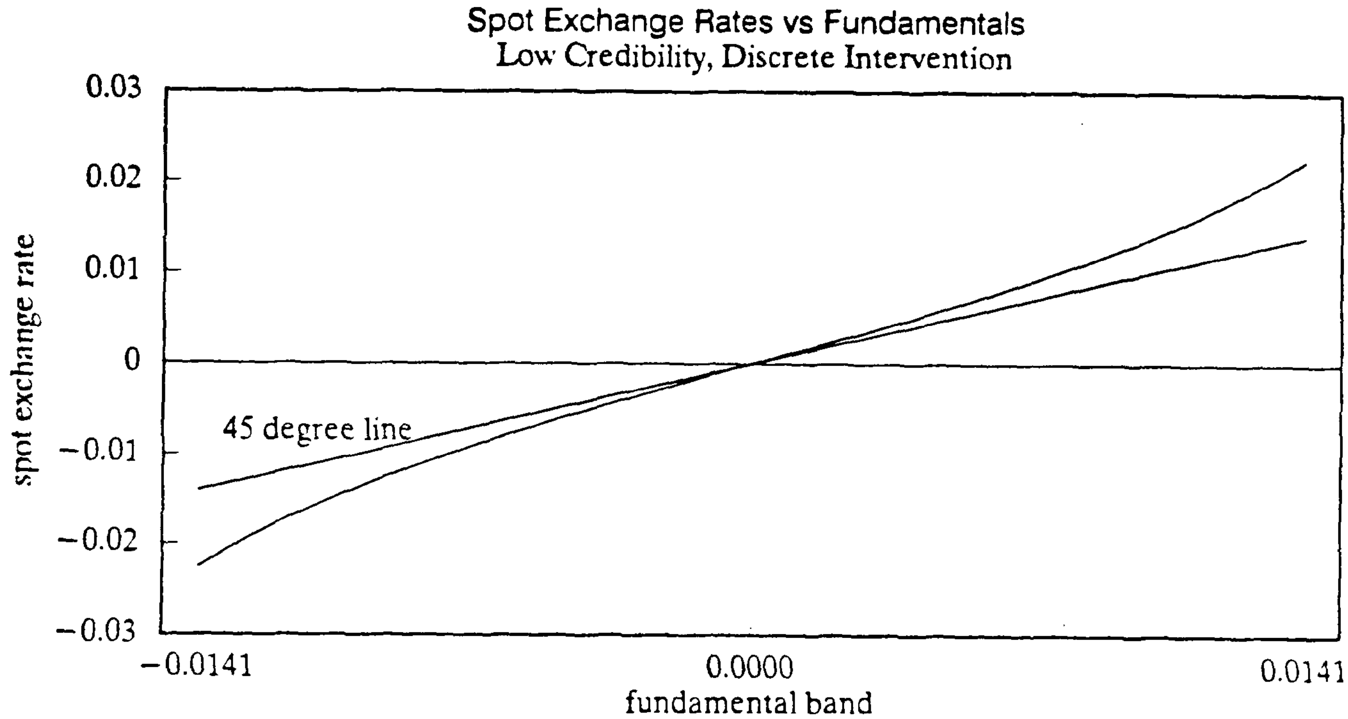


Figure 10

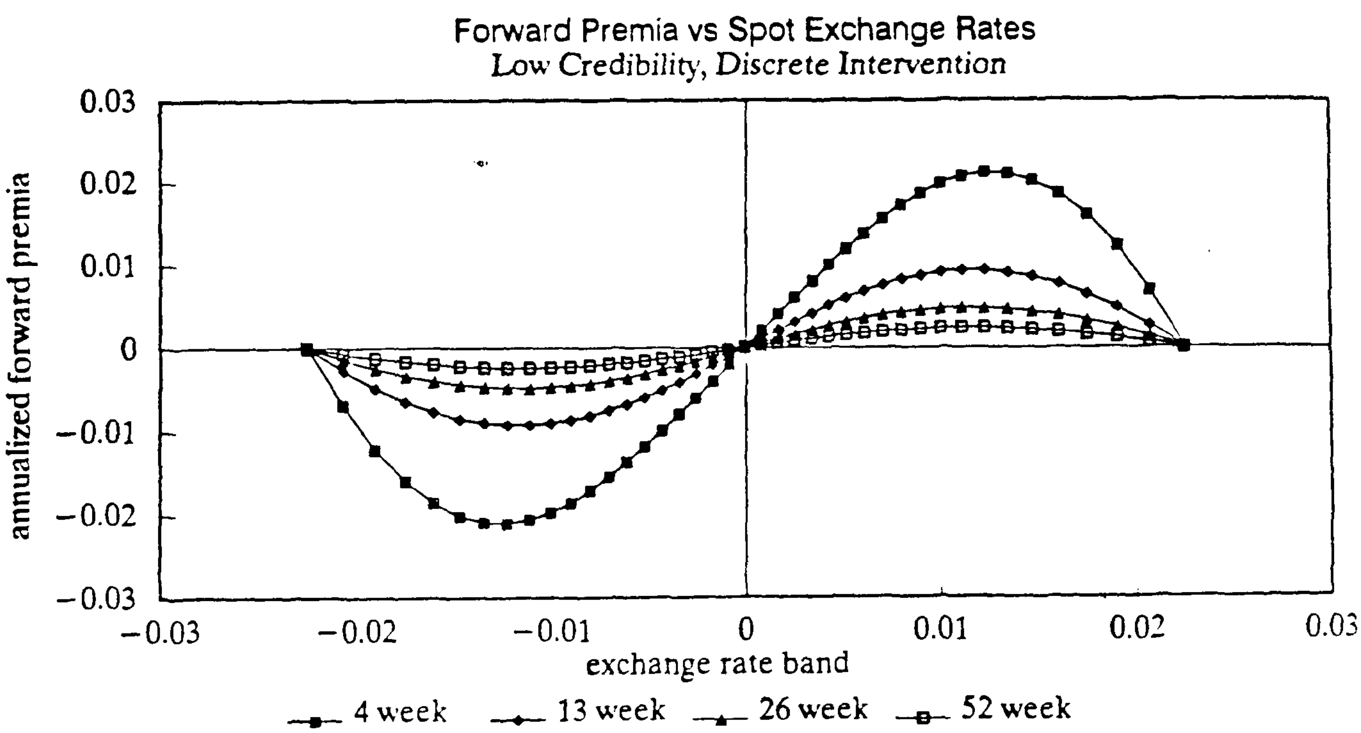


Figure 11

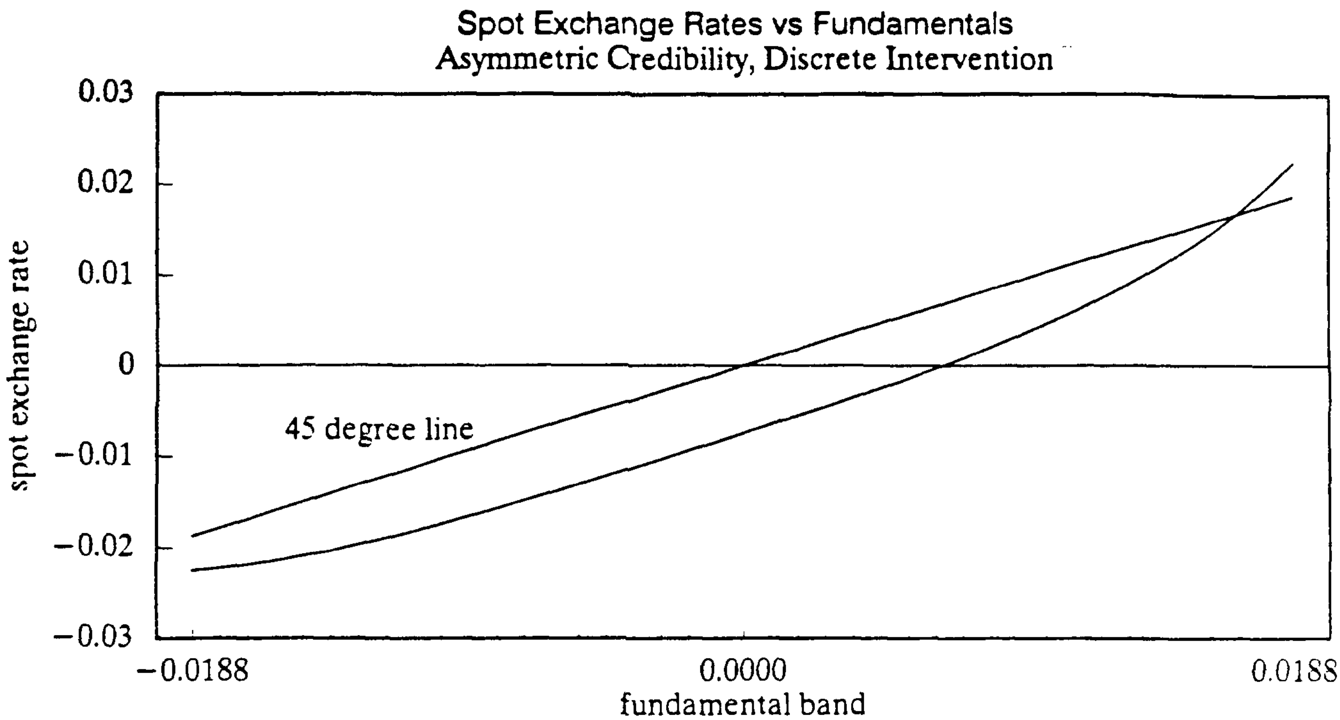


Figure 12

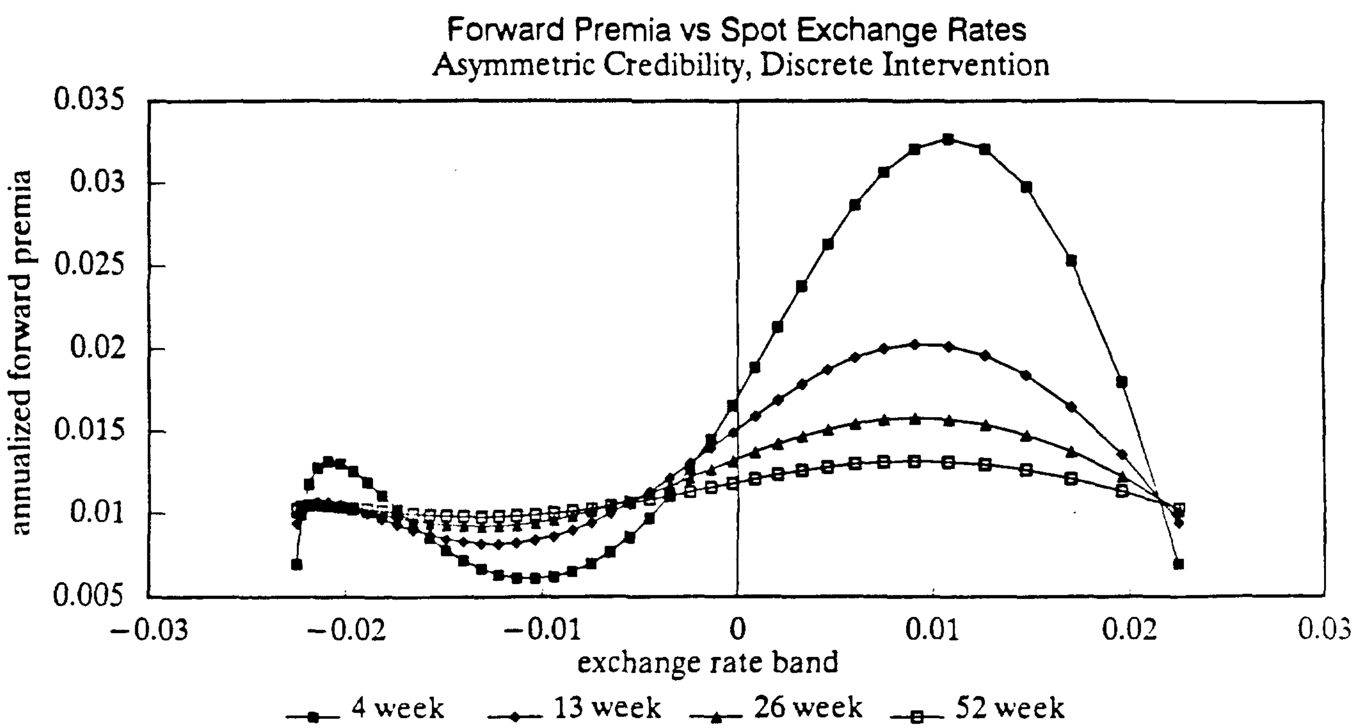


Figure 13

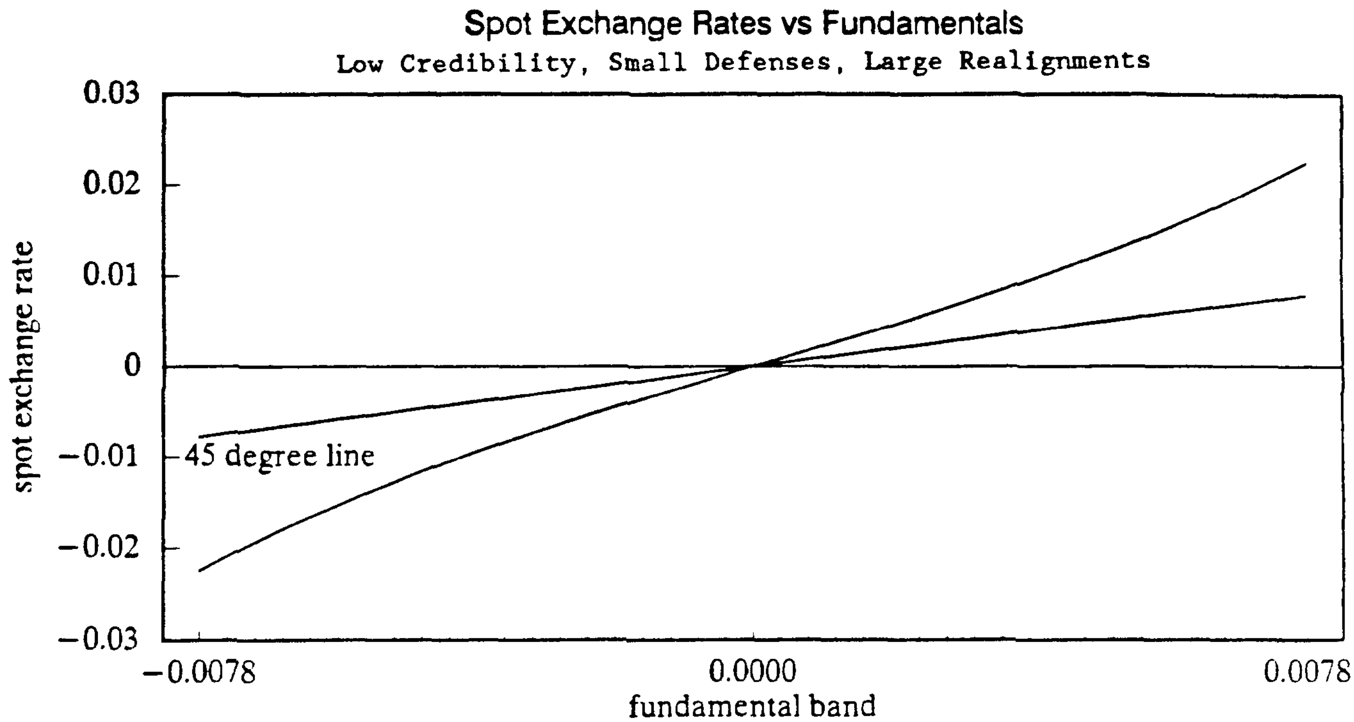
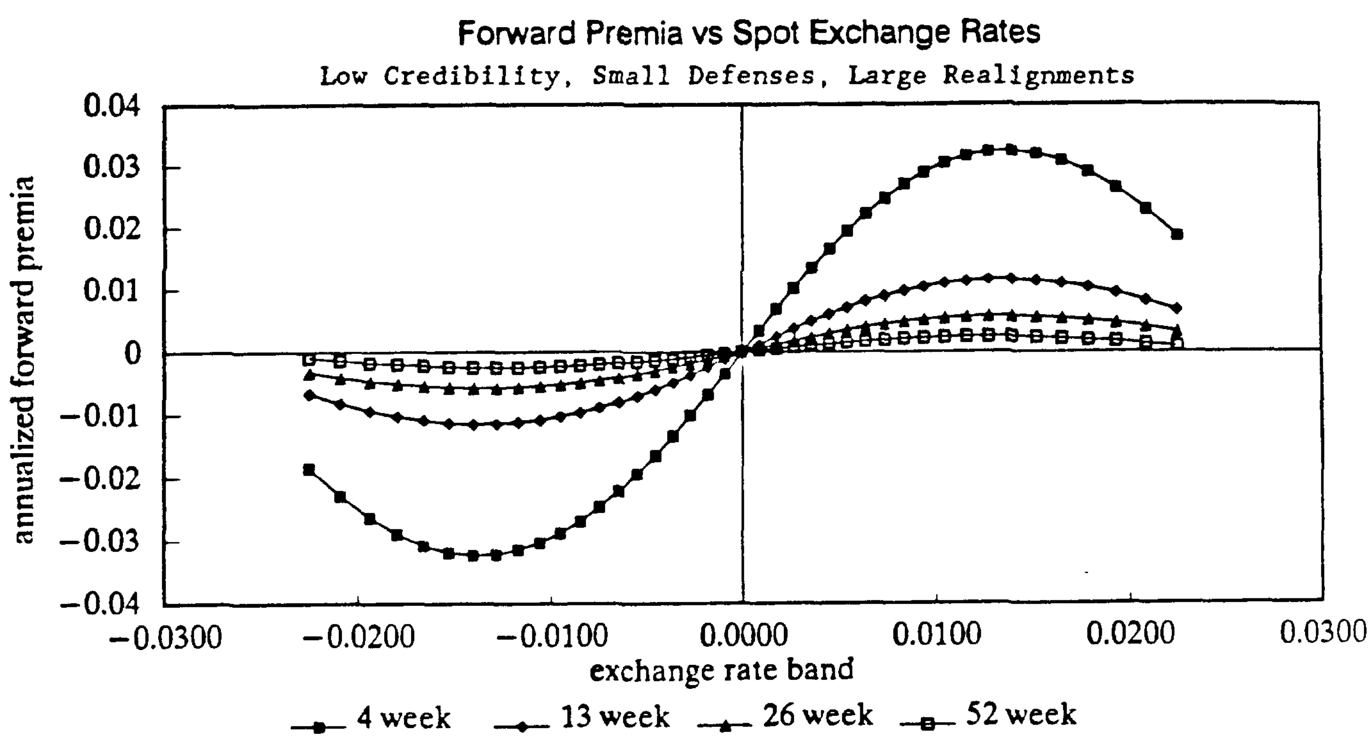


Figure 14



that of defining the credibility of the central bank at the barrier, i.e., the probability of an instantaneous realignment, given that one of the band's edges has been reached. Since the frequency with which the edges of the target zone are reached depends on the policy rules and on the fundamental parameters, these must be suitably combined to obtain an economically significant index of realignment risk. To this purpose, one should compute the probability of observing devaluations and revaluations within a given time interval, conditional on the initial position of fundamentals inside their band. We show in the Appendix how this can be done for a time horizon T and an initial position of fundamentals at the top of their band. Realignment probabilities over a three-month horizon are computed for the parameters estimated in the next section, and reported as P(D) and P(R) in Table 1. 1/

V. Application to FF/DM Data

1. Estimation

In this section we estimate the model of forward premia of the previous sections using data from the French franc-deutsche mark (FF/DM) market during the EMS. We should underscore that the simple one-factor model we estimate is not intended to provide an exhaustive characterization of the expected future behavior of the exchange rate within a target zone. Our analysis maintains the simple structure of standard target zone models, from which we deviate only by explicitly allowing for imperfect credibility of the band. Nevertheless, our framework proves useful in explaining some important stylized facts from French/German interest differentials during the last decade, and in generating estimates of the model's parameters implied by the given set of forward market data.

The most direct way to estimate the model is to assume that each observation of the forward rate corresponds to a randomly disturbed realization of the theoretical level of the forward rate, at each given level of the spot exchange rate, or

$$F_t^{\tau} = F(X_t, \tau, \Omega) + \epsilon_t \qquad (18)$$

In equation (18), $\Omega \equiv (p, q, \mu, \sigma, \gamma, C, R, R^*, D, D^*)$, *while* X_t *and* F_t^{τ} *are (the* log of) the spot exchange rate and forward exchange rate with maturity τ at time t. $F(X_t, \tau, \Omega)$ is the theoretical level of the (log) forward rate with

---

1/ The procedure can be easily adapted to computing realignment probabilities for fundamentals starting from any position in the band and for any time horizon. However, the approximation involved in our procedure--that uses the one-side hitting probability density--may become non-trivial for long horizons and initial fundamentals far away from the relevant edge. A numerical evaluation of the two-side hitting probability should be used in that case.

Table 1. Parameter Estimates for the FF/DM Target Zone Model

	Full Sample 01/04/80-01/02/91	Early Sample 01/04/80-08/08/86	Late Sample 08/15/86-01/02/91
p	.052 (.013)	.070 (.016)	.044 (.005)
q	.019 (.014)	.029 (.021)	.007 (.035)
μ	.041 (.009)	.031 (.010)	.015 (.011)
σ	.036 (.024)	.038 (.008)	.047 (.012)
γ	.0006 (.089)	.0005 (.010)	.0015 (.051)
C	4.2 (.59)	5.5 (.70)	4.0 (.26)
D	-0.9	-0.9	-0.8
D*	0.9	0.9	0.9
R*	0.9	0.8	0.8
R	0.5	0.5	0.4
P(D)	.394	.512	.310
P(R)	.088	.090	.025
$\bar{f}$	.0224	.0213	.0247
ρ	.120	.532	.329
N	570	345	225

Table Notes: The coefficients are defined as follows: p is the probability of devaluation at the upper edge of the band, q is the probability of revaluation at the lower edge, μ and σ are the annualized drift and standard deviation of the fundamental process, γ is the annualized parameter linking the exchange rate to its expected devaluation, C is the central parity shift-parameter, measured in units of $\bar{f}$; D is the devaluation parameter, R is the revaluation parameter, D* is the intervention-against-depreciation parameter, and R* is the intervention-against-appreciation parameter; D,D*,R* and R are measured as decile fractions of $\bar{f}$, the band radius, and a positive sign indicates an adjustment with respect to the central parity in the direction indicated in Figure 1; P(D) and P(R) are the probabilities of devaluation (revaluation, respectively) over a three-month horizon, for a fundamental currently at the upper boundary (lower boundary) of its band; $\bar{f}$ is the estimated size of the fundamental half-band; ρ is the Pearson correlation coefficient between actual and predicted interest differentials; N is the number of observations in the sample. Asymptotic standard errors, based on numerically computed derivatives of the sum of square errors and a Newey-West covariance matrix, are reported in parentheses. Standard errors are not computed for the discretely defined variables D,D*,R*,R.

maturity τ, corresponding to a current spot rate X_t and parameter set Ω. We shall assume the error term to be a mean-zero disturbance, uncorrelated with the exchange rate X_t, and possibly heteroskedastic and serially correlated. Since $F(X_t,\tau,\Omega)$ has been obtained in Section III as the expected future spot rate, we are in fact assuming that the forward rate is an unbiased predictor of the spot rate. This is a standard assumption in the target zone literature, which is known to be problematic for freely-floating currencies, but is a reasonable hypothesis for currencies regulated within a relatively narrow target zone (see also footnote 8). 1/ Under the assumption of competitive and frictionless capital markets, the condition of covered interest parity implies that the estimate of Ω from (18) maps into a curve for interest rate differentials of maturity τ. We shall therefore conduct the discussion in the remainder of this section in terms of the implied curve of interest rate differentials.

The model is estimated as follows. 2/ Each position in the ten-dimensional grid over the space with coordinates $\Omega=(p,q,\mu,\sigma,\gamma,C,R,R^*,D,D^*)$ defines the relationship between X(f) and the entries of the transition matrix H. The observed spot exchange rate X(f) can be inverted into a fundamental f, and the theoretical level of the forward rate is then obtained from (17). 3/ Formally, the estimation procedure for Ω consists of applying a Nonlinear Least Squares technique to (18). We use weekly data for spot exchange rates and three-month Eurocurrency interest rates for France and Germany. The data are from the Harris Bank Data Tape, and consist of end-of-week Chicago-noon quotations, running from January 4, 1980, to January 2, 1991.

1/ Although somewhat extraneous to the tradition of target zone models, one may introduce a constant risk premium in equation (18). The main effect of this strategy, however, is to make the estimate of μ insignificantly different from zero. This outcome is not surprising, since μ acts essentially as a vertical shift parameter on the forward premium schedule.

2/ The Fortran program used for this estimation is available from the authors upon request.

3/ In theory, non-monotonicity of X(f) -- that may arise for large interventions and very credible bands (the Flood-Garber case discussed in Section 4) -- does not hinder inversion of X(f). This is because the applicable solution of f=f(X) can be obtained with probability one by keeping continuous track of the path followed by fundamentals from the last time realignment has occurred. In practice, however, only observations at discrete points in time are available, and data must be discarded over the non-monotonic range of X(f). Besides the reasons discussed later in the text, the restriction to realistic defense rules helps in minimizing the loss of information due to this problem. As a matter of fact, the exchange rate was monotone in fundamentals for all our estimates except at the very bottom of the band in the late EMS subsample. No data had to be discarded, however, because no observations were available over this range, given the persistent weakness of the FF/DM exchange rate during this period.

The highly non-linear nature of our objective function makes the use of grid-searches on the parameter space preferable to explicit optimization algorithms, that proved sensitive to the initial guess. Our grid-search is carried out initially using large grids, covering the relevant range of the parameter space, followed by progressively smaller grids, to estimate the parameters to an arbitrary degree of precision. A wide variety of grids were tried, to minimize the possible dependence of the parameter estimates on the sequence of grids. 1/ Notice that the dimension of the transition matrix H increases with the square of the number of steps in the fundamental band, while the size of each grid search increases with the fourth power (with four intervention parameters) of the number of fundamental steps. Thus, the dimension of the estimation increases with the eighth power of the number of fundamental steps. After verifying the robustness of the qualitative results of our application to changes in the fundamental step, we have decided that a fundamental band with 21 steps represented a reasonable compromise between computability and flexibility. 2/ With this specification, the intervention rules R,R*,D and D* are expressed as decile fractions of $\bar{f}$ (which is endogenously determined given the size of the exchange rate band), with D,D*,R*,R ranging over $\{-1,...,1\}$. We use a state space with 21 bands, which proved sufficient to make the approximation involved in using a finite state space negligible, and a time interval Δt equal to one week.

In the model the intervention rules are free to vary over the entire range of the band. However, observation of actual FF/DM realignments and defenses suggests realistic ranges for these rules. We make use of these observations to maintain realism in the analysis, to reduce computation and to allow more powerful estimation of the parameters that we feel are more insightful--the realignment probabilities and the parameters of the fundamental process. Specifically, we impose that following a devaluation, the franc/mark exchange rate be positioned near the bottom of the new band--a fact that occurred with regularity over the last decade. 3/ Also, we impose defense rules of moderate size to account for the fact that the exchange rate has never been shifted away from the edge of the target zone

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1/ Given the highly non-linear structure of the regression equation, identification of the model's parameters cannot be verified analytically. For this reason we have verified the extremum properties of our estimators numerically in the neighborhood of the solution. See Gallant (1987) for a discussion of identification in non-linear models.

2/ An additional reason for considering a rather coarse grid for the policy rules is given by the noise that typically characterizes exchange rate data, combined with the indirect way in which the intervention rules enter into the determination of the forward premium (see the discussion in Section 4). Under these conditions, the estimation is bound to have little power in detecting small differences in the policy rules.

3/ This restriction is removed when the model is estimated in the late EMS sample, since the only observed realignment in this period did not occur to the bottom of the new band.

by more than a third of the band radius within a week. Since the French franc never revalued with respect to the deutsche mark during the EMS period, we are unable to place a priori restrictions on the policy that would follow a revaluation of the currency. 1/

Since our sample includes weekly observations of three-month data, the forward contracts that we consider overlap over a range of thirteen observations. This well known problem of overlapping observations typically generates highly serially correlated errors (see Hodrik (1987)). For this reason, we compute standard errors of the estimates using a Newey-West heteroskedasticity and serial correlation consistent covariance matrix, with a 13-lag truncation. 2/ We should note that, given the non-linear nature of our problem, the derivatives used in the computation of the covariance matrix are evaluated numerically. Therefore, the particular values of the estimated standard errors and test statistics must be interpreted with caution. We have verified that rejection/acceptance decisions at standard significance levels of the hypotheses that we consider were robust to changes of the step sizes used in calculating the derivatives, but standard errors were sensitive to this specification when very small step sizes were taken.

2. Results

It is common wisdom that the credibility of the FF/DM exchange rate agreement has drastically improved towards the end of the 1980s. 3/ Since pooling the data from the entire sample period into a single sample would lead to model misspecification if a structural break has occurred and to test the ability of our model to detect the changes in policy taking place in the EMS at the end of the 1980s, we allow for the possibility of a structural break in our sample. Realignments, however, are endogenous in our model. The choice of a particular realignment as a break-point--as well as any other break-point suggested by examination of the data--is therefore unsuitable, for reasons of data-mining. The selection of the break-point is based upon a recent test of structural change with unknown break-point. We

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1/ The restrictions that we impose can be parametrized as follows: $D\in\{-.9,-.8\}$ in the Full Sample and in the Early-EMS Sample; and D^* and $R^*\in\{.7,.8,.9\}$ for all samples. These restrictions are placed in fundamental space, and map approximately in the corresponding restrictions in exchange rate space.

2/ We have verified robustness of our tests to changes in the lag truncation. For instance, by applying no Newey-West correction on the test of structural change considered in the next subsection, the optimal break-point changed by only three weeks, although the test statistic increased substantially. Similarly on the other side, the covariance distinctively flattened after inclusion of about 15 lags, and the test statistic remained broadly stable thereafter.

3/ See, for instance, Ungerer et al. (1990), Weber (1991), and Kenen and Dominguez (1991).

use a SupLM(t) test based on a serial correlation and heteroskedasticity consistent covariance matrix of a partial sum of estimated errors. 1/ While this test is designed as a test for a single structural break, Andrews (1990) reports that it has power against all alternatives for which the parameters are non-constant. We have specified our test as a test of partial structural break, with respect to the realignment probabilities p and q, to increase the power against the alternative hypothesis that a change in credibility has occurred. 2/

Figure 15 displays the results of the test. The full sample runs from January 4, 1980, to January 2, 1991. The endogenously selected break-point, significant at the one percent level, is the middle of August 1986, four months before the last realignment of the FF/DM exchange rate. This break-point results in two subsamples: January 4, 1980-August 8, 1986 and August 15, 1986-January 2, 1991. Given that the results of the test strongly support the structural break, we focus our discussion only on the two subsamples resulting from the split. Parameter estimates and the actual and fitted values of the interest rate differential for the full sample are reported in Table 1 and Figure 16, respectively.

Table 1 reports estimates of the model's parameters from the non-linear regression equation (18). The implied curves for interest rate differentials, plotted as a function of the position of the exchange rate within its band, are displayed in Figures 17 and 18 for the two subsamples.

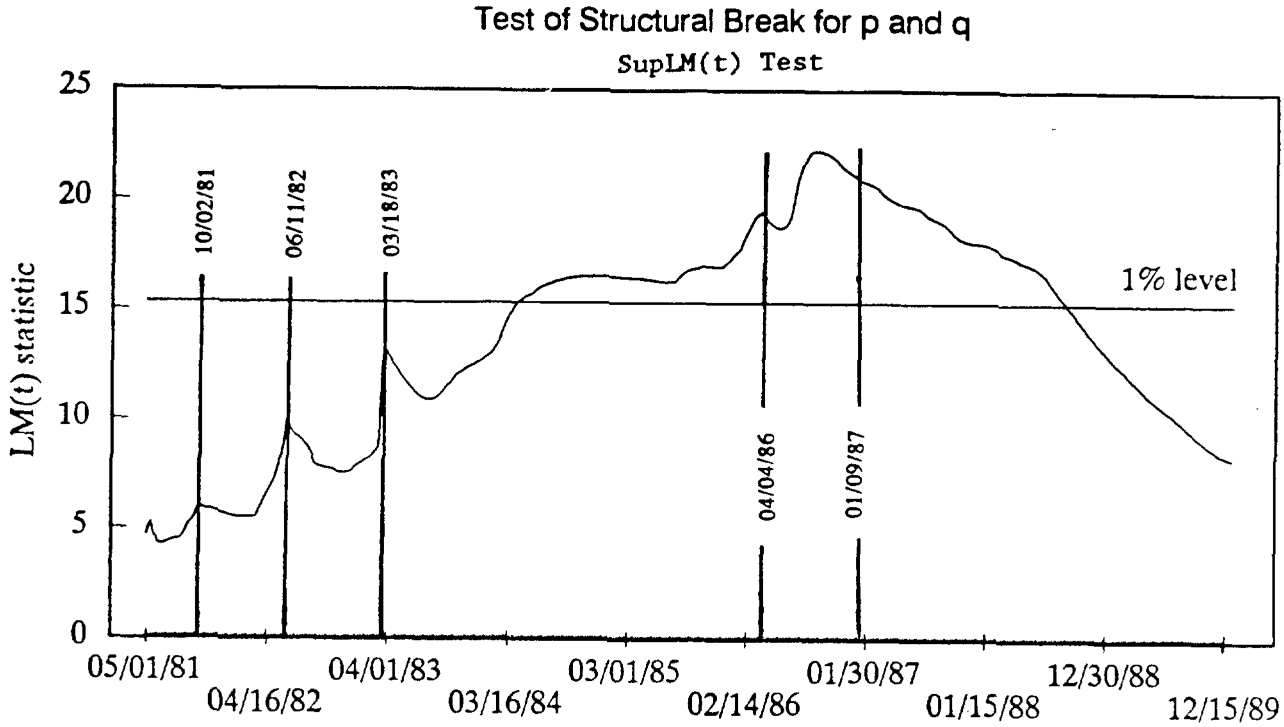
Comparison of the parameter estimates of the early and the late subsamples suggest a pattern of decreasing within-the-band fundamental drift, of somewhat increasing fundamental volatility and of growing credibility of the target zone. This development is also visually clear in Figures 17 and 18, where the slope of the predicted curve decreases from the first to the second sample (in correspondence of the decreased estimates of p and q), and the curve shifts downward (in accord with the reduced estimate of μ).

Additional evidence on the increased stability of the FF/DM exchange rate at the end of the 1980's emerges from the implied estimates of the realignment probabilities over a three-month horizon (see the Appendix for the computation), which are also reported in Table 1. The probability of devaluation within three months, conditional on an initial position of fundamentals at the upper edge of their band, decreases from 51 percent in the early-EMS sample, to 31 percent in the late-EMS sample. The corresponding revaluation probabilities P(R) are 9 percent and 2 percent.

1/ See Hansen (1990) and Andrews (1990). The distribution of the SupLM(t) test statistic is known in the literature as a Squared Standardized Tied-Down Bessel Process. A tabulated distribution can be found in Andrews (1990).

2/ Inclusion of the other parameters in the test changed the endogenously selected break-point by at most eight weeks.

Figure 15



vertical lines mark FF/DM realignment dates

Figure 16

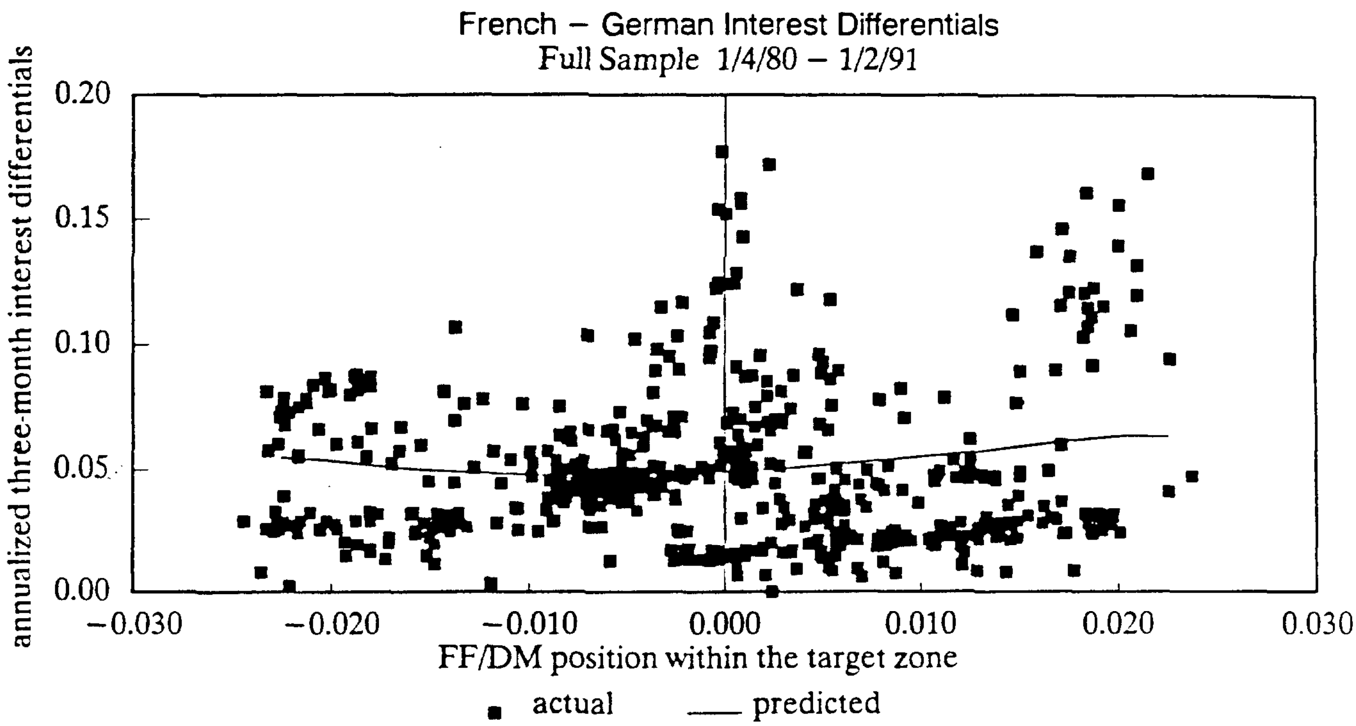


Figure 17

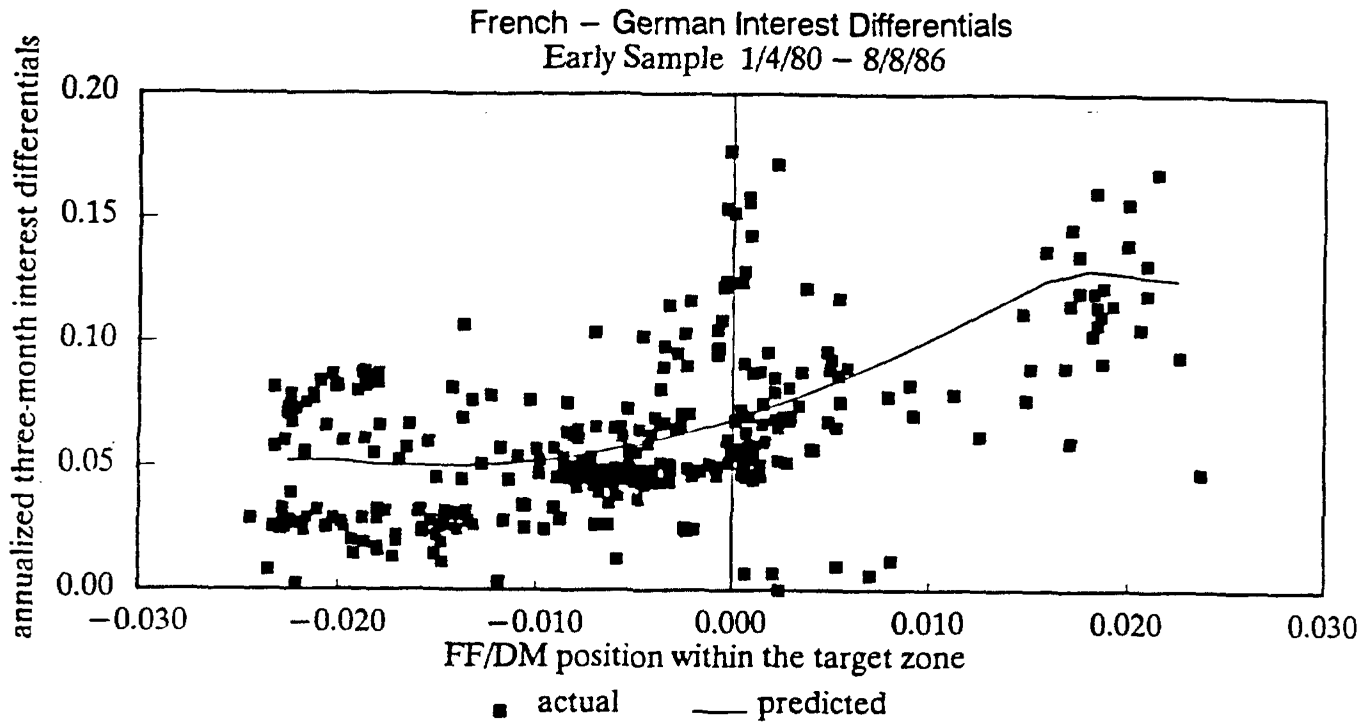
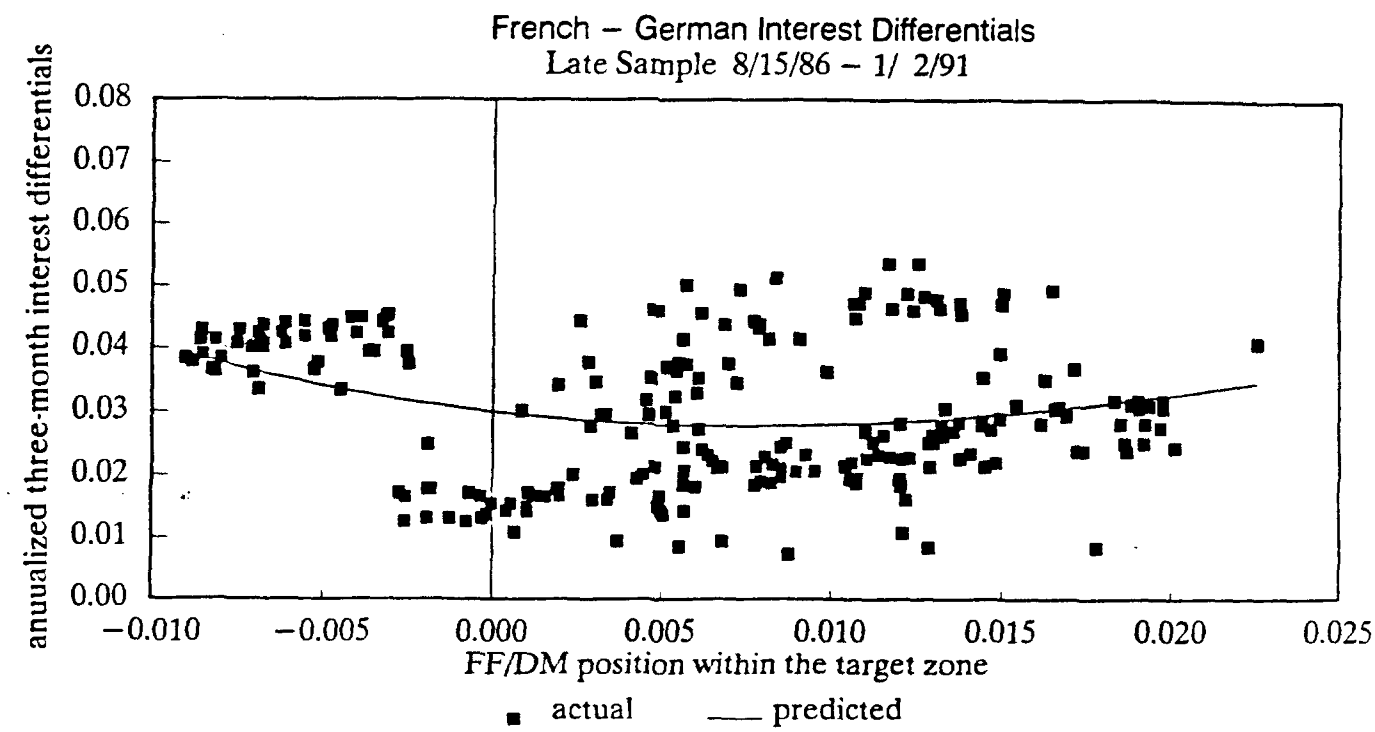


Figure 18



Finally, also the increase from the early sample to the late sample of the estimate of $\bar{f}$, the radius of the fundamental band corresponding to a two-and-a-quarter percent exchange rate half-band, provides additional evidence of exchange rate stabilization (a wider fluctuation of fundamentals is consistent with a given exchange rate band in the later sample).

Thus, the application of the model to FF/DM data confirms the marked improvement in the credibility of the franc/marc target zone towards the end of the 1980s, as well as the convergence of macroeconomic performance between France and Germany from the early to the late part of the 1980s. Our estimates indicate, however, the continued existence of non-trivial realignment risk, as perceived by investors on the FF/DM market after August 1986, which is revealed by the positively sloped curve for interest differentials in the upper range of Figure 18.

Table 1 also reports the correlation between the predicted and the actual interest rate differentials. While the unexplained variability of interest differentials at each position in the band remains substantial, the extent to which our predicted curves explain actual data appears reasonable, especially considering the simplicity of the one-factor model on which our application is based. The correlation coefficients for the full, early and late samples are .120, .532, .329, statistically significant at the .006, .000, and .002 levels, respectively.

One unsatisfactory aspect of our application to FF/DM data is the dispersion and small magnitude of the estimates of the parameter γ. The estimated standard errors for this parameter are in the order of fifteen-to-thirty times larger than the point estimates. The point estimates are insignificantly different from each other in the two sub-samples, and correspond to a degree of dependence of the current exchange rate on its expected future changes which seems unrealistically small. It should be mentioned that the difficulty of estimating the adjustment parameter γ from target zone models appears pervasive. Other studies that have examined FF/DM data, such as Flood, Rose and Mathieson (1990) and Bodnar and Leahy (1990), have also documented the small magnitude and variability of estimates of γ, while a flat loss function in terms of γ is reported in Spencer (1991a) and (1991b). 1/

The application of the model on FF/DM data also provides additional insight regarding the relationship between spot exchange rates and fundamentals. Figure 19 plots the exchange rate and the forward premium as a function of fundamentals, and--on the same panel--the forward premium as a function of the spot exchange rate. The curves are parametrized by the

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1/ Flood, Rose and Mathieson (1990) estimate γ by direct integration of the arbitrage equation (1). They split the sample of FF/DM data during the EMS in 13 subsamples. In only one subsample the estimate of γ is significantly different from zero. In only five subsamples the point estimate is larger than .05 and in three subsamples it is in fact negative.

estimates obtained over the early sample. Notice that the estimated spot exchange rate curve is essentially indistinguishable from a straight line, for all but the last steps of the band (consistent with the representation given by the asset pricing equation (1) for small values of γ). This confirms the difficulty, documented by Flood, Rose and Mathieson (1990) on a large set of spot exchange rate data, of identifying significant deviations from linearity in the exchange rate fundamental relationship.

Despite the negative outlook on the estimation of target zone models provided by the examination of spot-market data, our findings suggest that shifting the focus to forward market data and exploiting the sensitivity of forward premia to small deviations from linearity in spot rates may be a fruitful strategy to circumvent the problem of the little information content of spot data. The implication of a perfectly linear spot exchange rate--fundamental relationship is that the forward premium should be a constant function of both fundamental and exchange rate positions in the band. Figure 19, however, shows that despite the small deviation from linearity of the spot exchange rate schedule (corresponding to the very small value of γ which parameterizes the curve), the estimated curve of the forward premium displays a noticeable non-linear, upward-sloping pattern. This feature is even more evident when the forward premium is plotted as a function of the spot exchange rate, which is the relevant state variable in our estimation. A test of the significance of the non-linearity of the spot exchange rate fundamental relationship may be constructed by assessing the significance of the deviation of the estimated forward premium curve from a constant. The likelihood ratio statistics for this hypothesis (formally expressed by $H_0:A(\Omega)=B(\Omega)=0$ in equation (4)), which are reported in Table 2, indicate strong rejection for all our samples. 1/ The high significance of the test in the samples resulting after the split makes it seem unlikely that the rejection decision may be explained by the sensitivity of the likelihood ratio test to serial correlation. 2/ Our results thus confirm that only a large number of spot exchange rate observations near the edges of the target zone may allow detection of the non-linearities induced by central banks' intervention on the spot rate fundamental schedule; they also indicate, however, that the impact of the small non-linearities in the spot rate may be more apparent in forward market data.

We also use Wald statistics to test two additional hypothesis. First, an important test for the relevance of the model is a test of the null hypothesis of no realignment risk, formally expressed by $H_0:p=q=0$. Under this condition, our model collapses into the Flood-Garber model with perfect

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1/ Notice that unlike models with perfect credibility, where the exchange rate curve must smooth-paste the edges of the target zone, the present model allows for an *exactly linear* schedule of the spot exchange rate.

2/ A serial correlation-consistent Wald test could not be constructed from our estimates, given the discrete treatment of the policy rules.

Figure 19

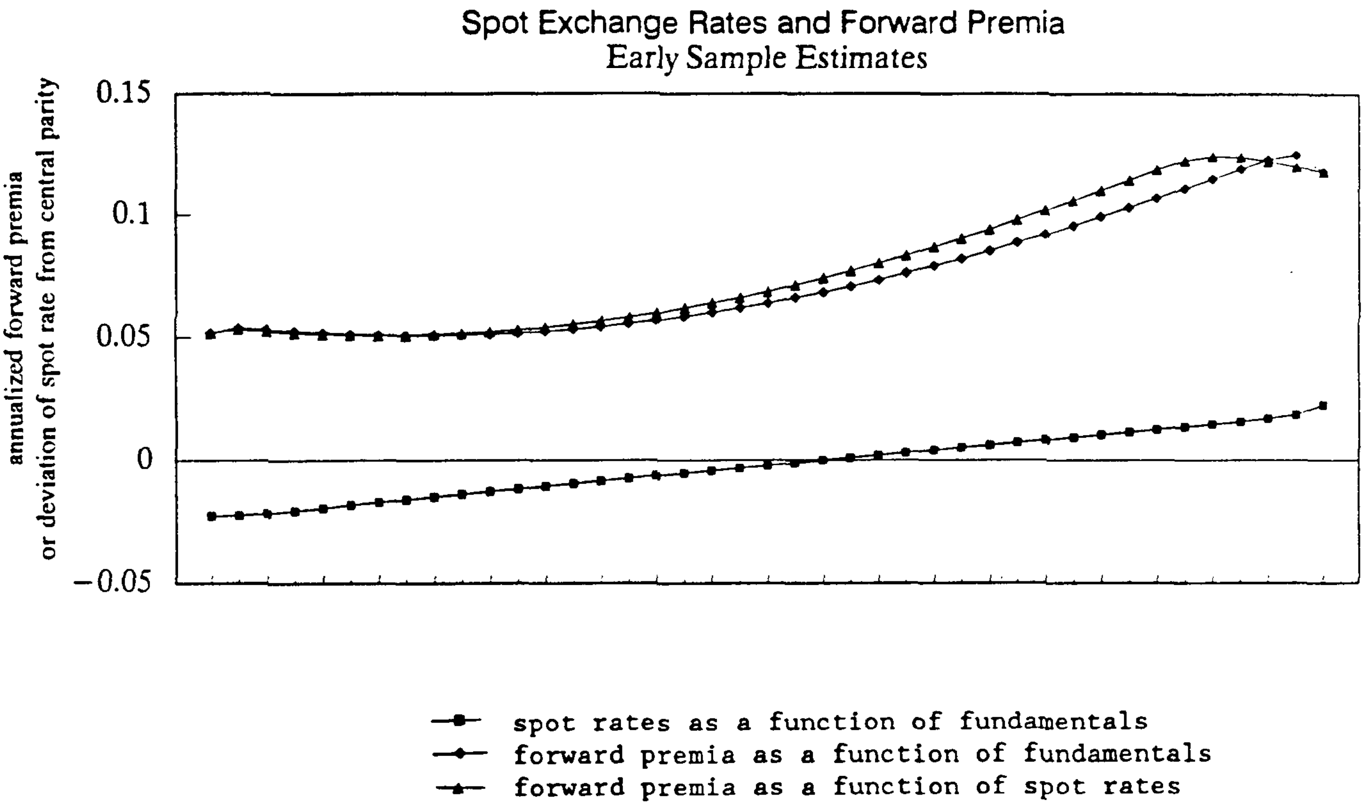
Spot Exchange Rates and Forward Premia
Early Sample Estimates
annualized forward premia
or deviation of spot rate from central parity
0.15
0.1
0.05
0
−0.05
spot rates as a function of fundamentals
forward premia as a function of fundamentals
forward premia as a function of spot rates

Table 2. Hypotheses Tests

	Full Sample 01/04/80-01/02/91	Early Sample 01/04/80-08/08/86	Late Sample 08/15/86-01/02/91
SupLM(t)	22.2 (.001)		
$A(\Omega)=0$, $B(\Omega)=0$	10.8 (.005)	115.7 (.000)	26.3 (.000)
$p=q=0$	37.1 (.000)	55.4 (.000)	16.6 (.002)
$p=q$	1.5 (.231)	1.3 (.250)	0.7 (.429)

Table Notes: p-values are reported in parentheses. The coefficients are defined as follows: p is the probability of devaluation at the upper edge of the band, q is the probability of revaluation at the lower edge. $A(\Omega)$ and $B(\Omega)$ are the integration constants from the general solution of the exchange rate equation (4), and their solution in terms of the parameter vector Ω is obtained by substitution of (4) into (5a) and (5b). The distribution of the SupLM(t) statistic is tabulated in Andrews (1990). The order of the test statistic is two, under the null hypothesis that the probabilities p and q remain constant throughout the sample. The hypothesis $\{A(\Omega)=0, B(\Omega)=0\}$ is tested by a likelihood ratio test. The hypotheses $\{p=q=0\}$ and $\{p=q\}$ are tested by Newey-West-corrected Wald tests.

credibility of the band (or, in the limiting case of infinitesimal intervention, into the Krugman model). Table 2 indicates strong rejection of this hypothesis for all samples, with p-values virtually indistinguishable from zero. This result suggests that consideration of realignment risk in target zone model represents a significant improvement from an empirical as well as analytical standpoint over earlier models with full credibility. A second test that we consider is against the hypothesis that the two boundaries of the target zone were equally credible. The point estimates of p and q suggest higher credibility of the lower barrier in all samples, a likely feature of the FF/DM target zone during the EMS. The estimates are, however, insufficiently precise for this difference to be statistically significant (H_0:p=q in Table 2).

Overall, our application to French/German data indicates that the model of forward premia presented in this paper is capable of matching the patterns observed in the FF/DM market during the EMS. The analysis yields estimates of the credibility parameters and summary statistics for realignment risk that accord well with the common wisdom on the historical development of this market. Although individual parameters are estimated imprecisely, particularly the estimates of the adjustment parameter γ, our application has sufficient power to detect the change in the credibility of the FF/DM target zone at the end of the 1980s. Specification tests considered against the hypotheses of full credibility of the target zone and of linearity of the spot exchange rate - fundamental relationship, point to the empirical relevance of realignment risk and at the potential gains from estimating target zone models from forward market data.

VI. Concluding Remarks

This paper has developed an analysis of forward premia in a model of exchange rate target zones with possible realignments. The condition that forward premia (and therefore interest rate differentials) must reflect agents' perception of the credibility of the band is exploited by deriving the relationship linking a given set of forward market data to their implicit realignment probabilities. We show that by allowing for imperfect credibility of the band, standard models of exchange rate target zones may be made consistent with a wide variety of forward premia--spot exchange rate relationships. Real-world target zones may be characterized by low-credibility or asymmetric credibility at the two edges of the band, and we have seen that these imply patterns of interest rate differentials sharply different from the standard case studied in the literature. Our framework also provides a tool for direct estimation of a large class of target zone models using forward market data. An application of the model to FF/DM data during the EMS provides results consistent with the perceived evolution of the FF/DM market during the EMS, including a significant increase of the credibility of the FF/DM target zone from the end of 1986.

As discussed in Section II, this paper can be extended along several directions. Future research should focus, in particular, on a more realistic description of central banks' behavior inside the band, although this strategy would run against the appealing simplicity of early target zone models, which focus on the infrequent boundary intervention of central banks. In this respect, two extensions of our research may be promising. One possibility is to account for inframarginal intervention to defend the band by assuming a fundamental process that explicitly displays mean-reversion. For fully credible target zones, Delgado and Dumas (1991) develop the relevant theory, while Lindberg and Söderlind (1991) outline a possible estimation strategy. A second possibility is to regard the residual forward premium from our model as reflecting "inside-the-band" realignment risk, as suggested by Bertola and Svensson (1991) and Rose and Svensson (1991). Our model provides structural estimates of these residuals, net of the credibility effects already captured by the position of the exchange rate in the band. The task would be to relate these estimates to observable macroeconomic variables, thus providing additional insight on the determinants of realignment risk.

Finite Horizon Realignment Probabilities

We outline the procedure for estimation of the probability of devaluation before a given date T, denoted by P(D), for an initial fundamental position at the upper edge of the band. The computation of the revaluation probability, P(R), is symmetric.

The probability that a devaluation occurs before time T is the probability of an immediate devaluation, p, plus the probability of a defense-devaluation sequence, p(1-p), multiplied by the one-time hitting probability over the horizon T, plus the probability of a defense-defense-devaluation sequence, $p(1-p)^2$, multiplied by the two-time hitting probability over the horizon T, etc. The process may be thought as being absorbed at the upper edge as soon as a devaluation occurs.

Denote by F(i) the probability that the fundamental path hits the barrier at least i times before t=T, conditional on being reset at distance $(D^*\bar{f})$ from the barrier every time this is hit. We have:

$$P(D) = \sum_{i=0}^{\infty} p(1-p)^i F(i) \tag{19}$$

Since our path starts from the edge of the target zone, F(0)=1. F(1) is given by:

$$F(1) = \int_0^T f(t_1)\,dt_1 \tag{20}$$

where f(t), $t=[0,\infty]$, is the hitting time probability density function.

Given that the barrier has been hit at time t_1, the two-time hitting probability before T is the integral of $f(t_2)$ over the remaining interval $(T-t_1)$, multiplied by the probability of observing that particular t_1, given by $f(t_1)$, and integrated over all possible t_1s. Similarly for F(3),...,F(i),... The generic term F(i) is therefore given by:

$$F(i) = \int_0^T f(t_1) \int_0^{T-t_1} f(t_2) \;\ldots\; \int_0^{T-t_1-\ldots-t_{i-1}} f(t_i)\,dt_i \ldots dt_2\,dt_1 \tag{21}$$

The computation would be exact if we were using the two-side hitting time distribution for f(t), and the defense policy rules were symmetrical. Since no closed-form solution is available for this probability density, and the estimated rules are almost identical, we use as an approximation the one-side probability distribution. This involves only a small approximation for our estimated parameters and three-month horizon (in our samples it is bounded above by .005 for both P(D) and P(R)). We therefore have:

$$f(t) = \left[\frac{D^{*}\overline{f}}{\sigma(2\pi t^{3})^{1/2}}\right] \exp\left[\frac{-(D^{*}\overline{f}-\mu t)^{2}}{2\sigma^{2}t}\right] \tag{22}$$

which is evaluated at each given set of μ, σ, D^* and $\overline{f}$.

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