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Distortionary Taxation and the Debt Laffer Curve

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Abstract

This paper highlights the importance of the role of the domestic tax system in determining the economic consequences of an external debt overhang. A simple taxation scheme is specified and it is shown that a country can be on the "wrong side" of its debt Laffer curve only if it is on the wrong side of its tax Laffer curve. The analysis indicates that fairly strong, and probably unrealistic, assumptions about the domestic tax system are needed to argue that the investment disincentives associated with the debt overhang are large enough to place a country on the wrong side of its debt Laffer curve.

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Summary

The rapid decline in investment observed in the heavily indebted countries has been attributed to the investment disincentive effects of an external debt overhang. The notion of a "debt Laffer curve" has been used to argue that these disincentives could potentially be so large that a reduction in the stock of debt could actually increase future repayments. Empirical work, however, has so far been unable to establish a clear relationship between debt and investment in the heavily indebted countries.

The paper links fiscal interactions within the debtor country to its debt repayment capacity and highlights the importance of the domestic tax system in determining the effect of a debt overhang on investment. As an illustrative example, a simple taxation scheme is specified, and a country is shown to be on the "wrong side" of its debt Laffer curve only if it is on the wrong side of its tax Laffer curve. The analysis indicates that strong assumptions about the domestic tax system are required to ensure that the investment disincentive effects of debt are so large that a reduction in debt increases repayment.

The model gives rise to the possibility of multiple equilibria, indicating that the debt Laffer curve may not be a simple functional relationship between the stock of debt and the expected present discounted value of future repayments. Thus, estimation of the debt Laffer curve as a functional relationship could be misleading.

As an extension of the model, the possibility of capital flight is incorporated into the framework. Multiple equilibria are again possible, and, for some levels of debt, capital flight and default obtain in one equilibrium and full repayment (with no capital flight), in another.



## I. Introduction

Since the debt crisis erupted in 1982, highly indebted countries (HICs) appear to have become trapped in a vicious circle. The burden of servicing their huge debts has squeezed investment, which has, in turn, cut their debt servicing capacity. Investment in the HICs has been weak since 1982, both by historical standards and in relation to other countries. The ratio of investment to GDP in these countries was significantly lower in 1982-88 than in the previous six years. 1/

A potentially important explanation for the fall in investment has recently begun to receive considerable attention--that the existence of a heavy debt burden reduces the incentive to invest. This effect, known as debt overhang, occurs where a country is unable to service its debt in full and so actual payments tend to depend upon a country's economic performance. If exports increase as a result of higher investment, much of the additional proceeds accrue to creditors in the form of higher debt service payments. This depresses the return to investment, and weakens the incentive to invest.

Krugman (1989) and Froot (1989) seek to explain the observed decline in investment by specifying models in which the existence of a debt overhang acts as a marginal tax on investment in the debtor country, thereby serving to depress investment. The higher is the debt burden, the higher the probability of default. The debtor, realizing that it is likely to lose its future output, is discouraged from investing and chooses to consume more of its endowment. As investment falls, expected repayment (or the amount the creditor can expect to extract) falls. This gives rise to the debt Laffer curve, which indicates that for a very large stock of debt, expected repayment to the creditor may actually decline as the level of debt increases. 2/

An important abstraction in Krugman (1989) and Froot (1989) is that the debtor country is treated as a single economic agent, and fiscal interactions within the debtor country are ignored. The purpose of this paper is to derive and analyze the debt Laffer curve in a model which features debt repayment as a fiscal problem. The advantage of this approach is that the economic consequences of a debt overhang are shown to depend on the domestic tax system.

In order to focus on the Laffer curve, the tax system specified below is kept purposely simple. The debtor country government imposes a marginal tax on the (future) output of its citizens in order to raise revenue to make the repayment, and the maximum tax rate the government can levy is  $\lambda$ . Debtor country citizens, when undertaking investment decisions today, form an expectation over what tomorrow's tax rate will be. The country's level of indebtedness influences this expectation, and a high expected tax rate,

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1/ International Monetary Fund (1989), pages 61-2.

2/ The downward-sloping portion of the debt Laffer curve has come to be known as the "wrong side".

associated with a large stock of debt, depresses investment. This gives rise to a debt Laffer curve.

With the simple tax system specified here, a country is shown to be on the wrong side of its debt Laffer curve only if it is on the wrong side of its tax Laffer curve. The link between the domestic tax system and the debt Laffer curve, however, has more general implications. It is shown that the creditor's extraction ability is important. The view that even if this extraction ability is small, a sufficiently large stock of debt will depress investment so much that the debtor country will be on the wrong side of the debt Laffer curve is incorrect. It turns out that in addition to high indebtedness, the extraction technology must also be quite large in order for the country to be on the wrong side. Bulow and Rogoff (1989) cite estimates of creditors' extraction ability to be between 3 percent and 15 percent of GNP. It appears, then, that even the most heavily indebted countries are unlikely to be on the wrong side of their debt Laffer curve. Thus, the debt Laffer curve and its associated investment disincentive effects do not appear to provide a satisfactory explanation for the decline in investment witnessed in the HICs since the onslaught of the debt crisis.

The fiscal interaction captured in this model gives rise to the possibility of multiple equilibria. Although a similar multiplicity of equilibria has been discussed elsewhere in the literature, 1/ its implications for the debt Laffer curve have not previously been explored. It is shown below that in the presence of multiple equilibria, the debt Laffer curve is not a simple functional relationship between the stock of debt and its expected present discounted (market) value. Several cases are discussed and the associated Laffer curves are illustrated. An important empirical message that emerges from this is that estimation of the debt Laffer curve as a functional relationship may be misleading.

The remainder of this paper is organized as follows: Section II specifies and analyzes the model. Some extensions are contained in Section III, and concluding remarks in Section IV.

## II. The Model

Consider an economy with a stock of future debt obligations  $D$  to an external creditor. The government of this country is responsible for repaying the creditor, and it does so by taxing its citizenry. For analytical convenience, the entire future is collapsed into a single period,

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1/ See, for example, Calvo (1988) and Eaton (1987).

leaving two periods--present and future. 1/ At the beginning of the first period, citizens of the debtor country decide how much of their endowment to consume, and how much to invest. Their investment bears output in the second period, which the debtor government taxes in order to repay its debt. The tax tool the government has at its disposal is a proportional tax on its citizens' second period output. Following Helpman (1989a,1989b), the government is assumed to face a tax ceiling of  $\lambda$ , where  $0 < \lambda < 1$ . 2/ Alternatively,  $\lambda$  may be interpreted as the penalty/extraction technology available to the creditor in the event of default. As long as the debt can be repaid in full with a tax rate less than  $\lambda$ , this penalty need not be exercised. If, however, full repayment requires a tax rate greater than  $\lambda$ , the debtor prefers default with the penalty to full repayment. 3/ The government turns over its tax proceeds to the creditor. If the tax proceeds are less than the amount owed, the country is in default and its repayment is a fraction  $\lambda$  of its output. Since investment decisions are undertaken by debtor citizens in the first period, and output of the second period is realized before the repayment occurs, the government may find that it does not need to tax at the rate  $\lambda$  in order to meet its debt obligation. If this is the case, the government taxes at a rate just high enough to raise  $D$  in revenue.

Second period output of an agent who invests  $I$  in period one is  $\theta f(I)$ , where  $f$  is a  $C^2$  concave function that satisfies the Inada conditions, 4/  $f(0)=0$ , and  $\theta$  is a random variable with a probability density function  $g(\theta)$ , and with mean  $\bar{\theta}$  and finite supports  $\underline{\theta}$  and  $\bar{\theta}$ .  $\theta$  is assumed to be economy-

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1/ We are cutting into the middle of a complicated dynamic game on the assumption that the past history of the game does not matter except for the current debt level. An explicitly dynamic formulation is desirable, and models in which the level of debt is determined endogenously have been studied previously. See, for example, Eaton and Gersovitz (1981) and Cohen and Sachs (1986). For the purposes of this analysis, however, the focus is exclusively on the intertemporal substitution effects of a debt overhang.

2/ The tax ceiling is a double-edged sword in the sense that the government cannot tax more than a fraction  $\lambda$  of private output, and it also cannot commit, ex-ante, to taxing any less than  $\lambda$ . Several alternate interpretations of the parameter  $\lambda$  have been offered in the literature. Froot (1989), for example, considers  $\lambda$  to be the creditor's ability to extract resources from the debtor country in the event of a default. Bulow and Rogoff (1989), on the other hand, derive  $\lambda$  as the result of a game between the debtor and the creditor in which both parties bargain over the amount of the repayment.

3/ This interpretation is consistent with Corden (1989), Froot (1989), and Krugman (1989), who also assume  $\lambda = 1$ .

4/ Each agent possesses the production technology  $f(\cdot)$ , which is identical across agents.

wide, and may be interpreted as a terms of trade shock. The tax rate the debtor government levies in order to make the repayment, then, is 1/

$$t = \begin{cases} \frac{D}{\theta \cdot f(I)} & \text{if } D \leq \lambda \theta \cdot f(I) \\ \lambda & \text{otherwise,} \end{cases} \quad (1)$$

### 1. Investment behavior and equilibrium

A debtor citizen's investment decision is influenced by his expectation of next period's tax rate. Assume that the debtor country consists of a large number of identical agents, each with an initial endowment of  $Q$ . The representative agent's optimization problem is

$$\max_I u(Q-I) + \beta f(I) E\{(1-t)\theta\} , \quad (2)$$

where  $u$  is a  $C^2$  concave utility function that satisfies the Inada conditions,  $\beta$  is the agent's intertemporal discount factor, and  $E$  is the expectations operator. The agent consumes  $(Q-I)$  in the first period, and  $(1-t)\theta f(I)$  in the second. Note that both  $t$  and  $\theta$  are uncertain from the ex-ante point of view of a debtor country agent making an investment decision. Since the debtor country is populated with a large number of agents, the investment decision of one agent has no effect upon the total level of investment in the economy. Thus, the representative citizen takes next period's tax rate as given. 2/ The first order condition of the agent's problem is

$$u'(Q-I) = \beta f'(I) [\theta - E\{t\theta\}] . \quad (3)$$

Observe that the optimal investment choice,  $I$ , falls as the expected tax distortion,  $E$ , rises. 3/

1/ We could, in addition, allow for both an output tax and a consumption tax in the initial period. The former is simply a lump-sum tax and may be ignored. The latter is a distortionary tax which increases investment. In the single tax formulation considered here, the second period consumption tax may be loosely interpreted as a measure of the net tax distortion. An explicit consideration of alternate tax systems is taken up in Husain (1990a).

2/ The assumption that the government cannot commit to a tax rate less than  $\lambda$  ex-ante is very important here. If such a commitment were possible, the country could never be on the wrong side of the tax Laffer curve, because the government would just commit to the revenue-maximizing tax rate.

3/ For notational convenience, the symbol  $E$  is used interchangeably with  $E(t\theta)$  for the remainder of the paper.

Equilibrium in the debtor economy requires that the expected tax distortion on which the representative citizen bases his optimal investment decision be the same as the distortion that would be expected given that choice. Defining  $J$  such that

$$J(E, \bar{I}(E), D) = \int_{\underline{\theta}}^{\frac{D}{\lambda \bar{I}}} \lambda \theta g(\theta) d\theta + \int_{\frac{D}{\lambda \bar{I}}}^{\bar{\theta}} \frac{D}{f(\bar{I})} g(\theta) d\theta - E, \quad (4)$$

$J(E, \bar{I}(E), D) = 0$  in equilibrium. Observe that

$$\lim_{E \rightarrow 0} J(E, \bar{I}(E), D) \geq 0 \quad \text{and} \quad \lim_{E \rightarrow \lambda \bar{\theta}} J(E, \bar{I}(E), D) \leq 0. \quad (5)$$

Thus, for all  $D$ , there exists an equilibrium  $E^*$  such that  $J(E^*, \bar{I}(E^*), D) = 0$ . Some levels of debt, however, may support more than one equilibrium. 1/ If, for example, agents anticipate a high tax, they invest very little and the government is forced to levy a high tax rate in order to make the repayment. On the other hand, if agents expect a low tax rate and accordingly invest a lot, the government need only tax at that low rate.

## 2. The tax Laffer curve

The expected tax distortion faced by the representative agent is  $E(t\theta)$ . The tax Laffer curve simply associates a level of expected revenue with every level of expected tax distortion, and the expected revenue maximizing distortion is labelled  $E(t\theta)$ . This is illustrated in Figure 1.

Using expression (1),  $E(t\theta)$  may be written as

$$E(t\theta) = \int_{\underline{\theta}}^{\frac{D}{\lambda \bar{I}}} \lambda \theta g(\theta) d\theta + \int_{\frac{D}{\lambda \bar{I}}}^{\bar{\theta}} \frac{D}{f(\bar{I})} g(\theta) d\theta. \quad (6)$$

If the debt is to be fully repaid in every state of nature (full repayment),  $D \leq \lambda \bar{\theta} f(\underline{I})$  and  $E(t\theta) \leq \lambda \underline{\theta}$ . If default occurs in every state (certain default),  $E(t\theta) = \lambda \bar{\theta}$ . If  $\lambda \underline{\theta} < E(t\theta) < \lambda \bar{\theta}$ , default occurs in some states (partial default). Turning back to Figure 1, note that the familiar hump-shape of the domestic Laffer curve is preserved if and only if the expected revenue-maximizing tax distortion,  $E(t\theta)$ , is less than the maximum expected distortion,  $\lambda \bar{\theta}$ . If  $\lambda \bar{\theta} < E(t\theta)$ , the wrong side of the domestic tax Laffer curve does not exist.

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1/ Conditions that guarantee a unique equilibrium are derived and discussed in Appendix B. It turns out that if there exists more than one equilibrium, there will generally exist an odd number of equilibria.

### 3. Debt forgiveness

The model can be used to derive the following propositions related to debt forgiveness: 1/

Proposition 1: *If a small forgiveness of debt causes the expected tax distortion to rise, the debtor country must be on the wrong side of its tax Laffer curve. If the country is on the upward-sloping portion of its tax Laffer curve, a small debt forgiveness will reduce the expected distortion.*

Investment and the expected tax distortion are inversely related. If the distortion rises, investment will fall. The proposition indicates that debt forgiveness will cause investment to rise as long as the debtor country is not on the wrong side of its tax Laffer curve. If, however, investment falls in response to debt forgiveness, the country must be on the wrong side of its tax Laffer curve.

Proposition 2: *If the debtor country is in a full repayment equilibrium, a small reduction in debt will cause the expected tax distortion to rise if and only if the country is on the wrong side of its tax Laffer curve.*

### 4. The debt Laffer curve

The debt Laffer curve associates levels of expected repayment with the stock of debt. Expected repayment to the creditor is equal to expected tax revenue. The slope of the debt Laffer curve, then, is

$$\frac{dE(R)}{dD} = \frac{dE(R)}{dE(t\theta)} \frac{dE(t\theta)}{dD}, \quad (7)$$

where  $dE(R)/dE(t\theta)$  is the slope of the domestic tax Laffer curve.

Proposition 3: *The debtor country is on the wrong side of its debt Laffer curve only if it is on the wrong side of its tax Laffer curve.*

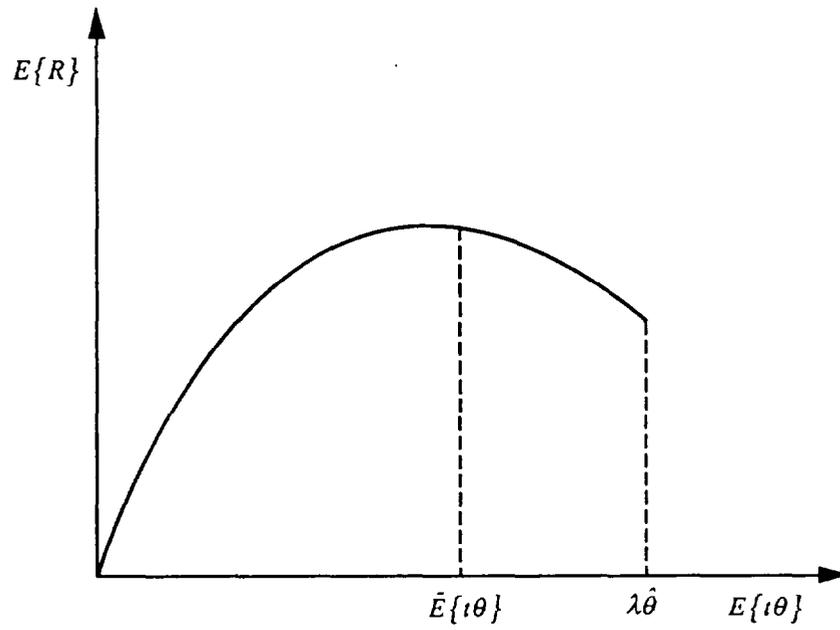
Proof: Suppose the country is on the wrong side of its debt Laffer curve but on the upward-sloping portion of its tax Laffer curve. Thus,  $dE(R)/dE(t\theta) > 0$ . From Proposition 1, then,  $dE(t\theta)/dD > 0$ . This implies  $dE(R)/dD > 0$ , which is a contradiction. Hence,  $dE(R)/dD < 0$  implies  $dE(R)/dE(t\theta) < 0$ . ■

In order to sketch the debt Laffer curve, several possible cases must be considered. First, the maximum expected tax distortion,  $\lambda\theta$ , may be less than the revenue-maximizing distortion,  $E$  (case 1). Second,  $\lambda\theta$  may be

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1/ Proofs of Propositions 1 and 2 are contained in Section 1 of the appendix.

Figure 1: The Tax Laffer Curve





greater than  $E$ , but the highest possible distortion associated with full repayment,  $\lambda\theta$ , may be less than  $E$  (case 2). Finally, both  $\lambda\hat{\theta}$  and  $\lambda\theta$  may be greater than  $E$  (case 3). In addition, the possibility of multiple equilibria must be accounted for.

First assume a unique equilibrium and consider cases 1 and 2. 1/ If  $\lambda\hat{\theta} < E$  (case 1), the equilibrium expected tax distortion will be less than  $\lambda\theta$  for sufficiently small  $D$ , and there will be full repayment. The slope of the debt Laffer curve in the full repayment region is unity since  $E(R)=D$  when there is full repayment. Defining  $D' = \lambda\theta f(I)$ , any level of debt less than or equal to  $D'$  involves full repayment. As debt is increased past  $D'$ , a full repayment equilibrium can no longer be supported. The expected distortion will be greater than  $\lambda\theta$ , and default will occur in some states. Thus, the economy will be in a partial default equilibrium. As  $D$  becomes very large, the equilibrium will involve default in every state. Define  $\underline{D} = \lambda\hat{\theta} f(I)$ . If  $D \geq \underline{D}$ ,  $dE(\tau\theta)/dD$  is equal to zero, and a further increase in debt has no impact either on the expected distortion or on investment. This curve is illustrated in case 1 of Figure 2. Observe that there is no wrong side of either the tax or the debt Laffer curve in this case.

Now suppose  $\lambda\theta < E < \lambda\hat{\theta}$  (case 2). Observe first that if the economy is in a full repayment equilibrium, it must be on the upward-sloping portion of its tax Laffer curve. Thus, the slope of the debt Laffer curve is unity for all  $D \leq D'$ . For levels of debt between  $D'$  and  $\underline{D}$ , the economy will be in a partial default equilibrium. If the expected distortion associated with this equilibrium is less than  $E$ ,  $dE(R)/dE(\tau\theta) > 0$ , and the slope of the debt Laffer curve will be positive but less than unity. For  $E > E$ ,  $dE(R)/dE(\tau\theta) < 0$ , and the slope of the debt Laffer curve will be negative. Finally, for  $D \geq \underline{D}$ , the slope will, as before, be zero. This curve is sketched in case 2a in Figure 2.

Turning to the case where there are multiple equilibria, Lemma 2 in Section 2 of the appendix indicates that there will, in general, be an odd number of equilibria. Proposition 5, stated and proved in Section 3 of the appendix, indicates that there exists a unique equilibrium for all levels of debt when case 1 holds. Conversely, case 1 cannot hold if there are multiple equilibria. Cases 2b and 3 are illustrated in Figure 3. 2/ These curves are derived under the assumption that there exist at most three equilibria for any debt level. Although the debt Laffer curves will become more complicated if more than three equilibria exist for some levels of debt, they will be similar in spirit to the ones illustrated in Figure 3.

## 5. Welfare

Froot (1989) shows that debt reduction is Pareto-improving if and only if the debtor country is on the downward-sloping portion of its debt Laffer

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1/ It turns out that case 3 cannot hold if there is a unique equilibrium. A proof of this is relegated to Section 3 of the appendix.

2/ A detailed discussion on the construction of these curves is contained in Husain (1990b).

curve. A reduction in the stock of debt reduces the expected future tax (improving the welfare of the debtor, who responds by investing more) and increases expected repayment (improving the welfare of the creditor). Assuming that the creditor is interested in maximizing expected repayment in the model presented above, Froot's result is preserved, but with some qualifications. First, the creditor's extraction technology  $\lambda$  must satisfy the inequality  $\lambda \hat{\theta} > E$  for a downward-sloping portion of the debt Laffer curve even to exist. Froot assumes  $\lambda = 1$  in his model, but claims that his result holds for any  $\lambda$  between zero and unity. That is not true here.

The possibility of multiple equilibria also complicates the welfare analysis of debt reduction. If such a reduction changes tax expectations significantly, the economy could "jump" to a new equilibrium. For example, in the case illustrated in Case 2b in Figure 3, if the original equilibrium is at point A, a reduction in the stock of debt could, for some reason, give rise to fears that the tax distortion will rise. The economy could, in that event, settle at point C. In this case, debt reduction when the economy is on the downward-sloping portion of the debt Laffer curve is Pareto-worsening. Conversely, debt reduction could take the economy from point B to point A by giving rise to optimistic expectations about the tax distortion. In this case debt reduction is Pareto-improving even when the economy is on an upward-sloping portion of its debt Laffer curve.

With the domestic tax system specified above, debt reduction is Pareto-improving only if the country is on the wrong side of its tax Laffer curve. This simple formulation serves to highlight the importance of the fiscal situation in the debtor country in determining the economic consequences of a debt overhang. In addition, the analysis indicates that even with a more general tax system, strong assumptions regarding the nature and magnitude of domestic distortions need to be made in order to generate the sort of debt Laffer curve results that would explain the severe decline in investment in the HICs.

### III. Extensions

Two interesting extensions of the model are explored next. The first allows for a more general utility function for the debtor country citizen. The second incorporates capital flight. The importance of the domestic tax Laffer curve in generating the debt Laffer curve is preserved in both cases.

#### 1. Risk-averse agents

Consider a representative agent with an investment problem that can be expressed as





$$\max_I u(Q-I) + \beta E\{v[(1-t(\theta))\theta f(I)]\} , \quad (8)$$

where  $v$  is a  $C^2$  concave utility function that satisfies the Inada conditions. The agent's optimal investment choice, then, satisfies the first order condition

$$u'(Q-I) = \beta E\{v'[(1-t(\theta))\theta f(I)](1-t(\theta))\theta f'(I)\} . \quad (9)$$

Investment responsiveness to a change in the level of debt is

$$\frac{d\tilde{I}}{dD} = \frac{\beta \int_0^{\bar{\theta}} \left[ v''(1-t)\theta f' + \frac{v'f'}{f} \right] g(\theta) d\theta}{u'' + \beta E\{v''(1-t)^2\theta^2 f'^2 + v'(1-t)\theta f''\}} . \quad (10)$$

The first term in the numerator, which measures the rate of return effect, indicates that the expected return on investment falls as debt increases, inducing the agent to invest less. The second term, which is a wealth effect, is positive. An increase in debt, given a level of investment, increases the marginal utility of consumption in the future period. This causes the investor to invest more. The net effect of an increase in debt on investment can be shown to be negative if and only if the Arrow-Pratt measure of relative risk aversion is less than unity.

This result is consistent with that obtained by Helpman (1989b), and has interesting implications for the tax and debt Laffer curves. If the degree of relative risk aversion is greater than or equal to unity, the debt Laffer curve is upward-sloping throughout and there exists a unique equilibrium. The economy can never be on the wrong side of its tax Laffer curve in this case. If the degree of relative risk aversion is less than unity, the debt Laffer curve will slope downward after a point. Multiple equilibria are indeed possible, but the country must still be on the wrong side of its tax Laffer curve in order to be on the wrong side of the debt Laffer curve.

## 2. Capital Flight

Turning to a certainty version of the model in Section II, agents are now allowed to evade taxation by engaging in capital flight. A unit of capital repatriated abroad earns the riskless interest rate  $(1+r)$ , but the agent also bears a cost  $\alpha$  in sending each unit of capital out of the country. The net return to a unit of flight capital, then, is

$$R = (1+r)(1+\alpha) . \quad (11)$$

The agent's investment problem can be expressed as

$$\max_{I, K} u(Q-I-K) + \beta [(1-t)f(I) + RK] , \quad (12)$$

where  $K$  is the amount of capital repatriated abroad. 1/ The first order conditions are

$$u'(Q-I-K) = \beta(1-t)f'(I) \quad (13)$$

and

$$u'(Q-I-K) \geq \beta R , \quad (14)$$

where (14) holds with equality whenever  $K > 0$ .

The agent's optimal choice of domestic investment,  $I^*$ , must satisfy (13). Defining  $t^*$ , given  $Q$ ,  $R$ ,  $u$ , and  $f$ , as the tax rate at which the agent is indifferent between holding his last unit of investment domestically or abroad, 2/  $t^*$  must either satisfy

$$u'(Q-I^*(t^*)) = \beta R \quad (15)$$

or it must equal zero. As long as  $0 < t^* \leq \lambda$ , investment responsiveness to a change in the tax rate is

$$\left. \frac{dI^*}{dt} \right|_{t < t^*} = \frac{\beta f'}{\beta(1-t)f'' + u''} \quad (16)$$

$$\left. \frac{dI^*}{dt} \right|_{t > t^*} = \frac{\beta f'}{\beta(1-t)f''} . \quad (17)$$

Furthermore,

$$\lim_{t \rightarrow t^*} \left. \frac{dI^*}{dt} \right|_{t < t^*} > \lim_{t \rightarrow t^*} \left. \frac{dI^*}{dt} \right|_{t > t^*} . \quad (18)$$

The slope of the tax Laffer curve is

1/ As in Section 2, it is assumed that the agent cannot borrow from abroad. Thus,  $K \geq 0$ .

2/  $t^*$  may be interpreted as the lowest tax rate at which there will be some capital flight. If  $R$  or  $Q$  is high, or if  $f'$  is low,  $t^*$  will be low-- there will be some capital flight even when domestic taxes are low.

$$T'(t) = f(I^*) + t f'(I^*) \frac{dI^*}{dt} , \quad (19)$$

and there is a discontinuity in the slope of this curve at  $t^*$ . This is illustrated in Figure 4. If  $t^* = 0$  or  $t^* \geq \lambda$ , this discontinuity in the tax Laffer curve will not exist. If  $t^* \geq \lambda$ , the possibility of capital flight has no bearing on the debt Laffer curve--agents never find it profitable to hold any capital abroad, regardless of the level of debt. If  $t^* = 0$ , there is always some capital flight. As in Section II, the country must be on the wrong side of its tax Laffer curve in order to be on the wrong side of its debt Laffer curve.

If  $\tau < \lambda$ , there are multiple equilibria. If  $T(\lambda) < T(t^*)$ , then for some levels of debt at least one of these equilibria involves capital flight and at least one does not. In this case, debt forgiveness may eliminate the equilibrium with capital flight and result in a Pareto-improvement. This is consistent with Eaton (1987), and provides an appealing explanation for capital flight coexisting with default. The additional insight, however, is that capital flight and default can coexist only if  $\tau < \lambda$  and if the country is also on the wrong side of its tax Laffer curve.

#### IV. Conclusion

Previous work has sought to explain the rapid decline in investment witnessed in the heavily indebted countries as a consequence of the investment disincentive effects associated with the debt overhang itself. The debt Laffer curve has been used to argue that these disincentives could potentially be so large that debt reduction could result in an improvement in the welfare of both the debtor and the creditor.

The model presented above serves to illustrate the importance of the debtor country's domestic tax system in assessing the investment impact of an external debt overhang--an aspect which has so far been ignored. The analysis indicates that the debt Laffer curve results implicitly rely on strong assumptions about the domestic tax system. The share of the debtor's output that the creditor may extract through direct sanctions or bargaining must exceed the revenue maximizing tax rate in order to even get the debt Laffer curve. The evidence cited by Bulow and Rogoff (1989)--that creditors' have been able to extract 3 to 15 percent of debtor country resources--suggests that the assumptions needed to generate the debt Laffer curve are unrealistic. Thus, severe investment disincentives associated with the wrong side of the debt Laffer curve appear to be an inadequate explanation for the observed decline in investment in the HICs and do not provide a justification for debt reduction on efficiency grounds alone.

Analytcs of the Model

This Appendix provides proofs of the propositions stated in the text, and states and proves some additional propositions.

1. Proofs of Propositions 1 and 2

*Proof* (Proposition 1): Using expression (6),

$$\frac{dE\{t\theta\}}{dD} = \frac{\frac{1}{f(\bar{I})} \int_{\frac{D}{\lambda \bar{I}}}^{\bar{v}} g(\theta) d\theta}{1 + \frac{D}{f(\bar{I})} \frac{f'(\bar{I})}{f(\bar{I})} \frac{d\bar{I}}{dE} \int_{\frac{D}{\lambda \bar{I}}}^{\bar{v}} g(\theta) d\theta} \quad (20)$$

Thus,

$$\frac{dE}{dD} < 0 \Leftrightarrow -\frac{D}{f(\bar{I})} \frac{f'(\bar{I})}{f(\bar{I})} \frac{d\bar{I}}{dE} \int_{\frac{D}{\lambda \bar{I}}}^{\bar{v}} g(\theta) d\theta > 1. \quad (21)$$

The debtor country is defined to be on the wrong side of its tax Laffer curve when  $dE(R)/dE\{t\theta\} < 0$ . This is true if and only if

$$-\frac{D}{f(\bar{I})} \frac{f'(\bar{I})}{f(\bar{I})} \frac{d\bar{I}}{dE} \int_{\frac{D}{\lambda \bar{I}}}^{\bar{v}} g(\theta) d\theta - \frac{f'(\bar{I})}{f(\bar{I})} \frac{d\bar{I}}{dE} \int_{\frac{D}{\lambda \bar{I}}}^{\bar{v}} \lambda \theta g(\theta) d\theta > 1. \quad (22)$$

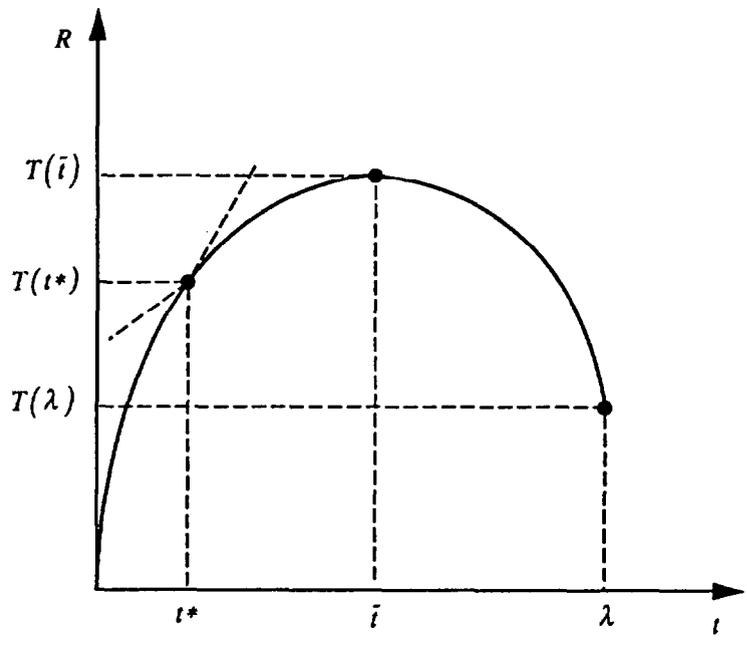
Thus,

$$\frac{dE}{dD} < 0 \rightarrow \frac{dE\{R\}}{dE\{t\theta\}} < 0 \quad \text{and} \quad \frac{dE\{R\}}{dE\{t\theta\}} > 0 \rightarrow \frac{dE}{dD} > 0 \quad \blacksquare \quad (23)$$

*Proof* (Proposition 2): Recall that in a full repayment equilibrium  $D \leq \lambda \theta f(I)$ . Inequality (22) is then exactly equivalent to the inequality in expression (21). Thus,

$$\frac{dE\{t\theta\}}{dD} < 0 \rightarrow \frac{dE\{R\}}{dE\{t\theta\}} < 0 \quad \blacksquare \quad (24)$$

Figure 4: The Tax Laffer Curve with Capital Flight





2. Lemmas 1 and 2

Lemma 1: If  $J(E, I(E), D)$  is strictly monotonic in the distortion space, there exists a unique equilibrium for all nonnegative debt levels.

Proof: The strict monotonicity assumption implies that  $dJ/dE < 0$ ,  $\forall E$ .  
Since

$$\lim_{E \rightarrow 0} J(\cdot) \geq 0 \quad \text{and} \quad \lim_{E \rightarrow \lambda\theta} J(\cdot) \leq 0, \quad (25)$$

$\exists! E^* \in [0, \lambda\theta] \ni J(E^*, I(E^*), D) = 0, \forall D. \blacksquare$

Lemma 2: If, for some level of debt, there exists more than one equilibrium, there must, in general, be an odd number of equilibria. 1/

Proof: The value of  $J(\cdot)$  at the boundaries of the E-space is contained in the proof of Lemma 1. Thus,  $J(\cdot) = 0$  at an odd number of points within this space.  $\blacksquare$

3. Propositions 4 and 5

Proposition 4: If the maximum expected tax associated with a full repayment equilibrium is greater than the expected revenue-maximizing distortion,  $J(\cdot)$  cannot be strictly monotonic.

Proof: If  $E < \lambda\theta$ , then  $\forall E \in (E, \lambda\theta) \ni D^* \ni J(E, I(E), D^*) = 0$ . These are full repayment equilibria that are on the wrong side of the tax Laffer curve. From Proposition 2, an increase in debt causes  $E$  to rise. This, however, is not possible if  $J(\cdot)$  is strictly monotonic.  $\blacksquare$

Proposition 5: If the maximum expected tax distortion is less than the expected revenue-maximizing distortion, there exists only one equilibrium for any level of debt.

Proof: From Lemma 2, if there exists more than one equilibrium, there must be an odd number. At least one of these must have the property that  $dE/dD < 0$ . From Proposition 1, this equilibrium must be on the wrong side of the tax Laffer curve. Since  $\lambda\theta < E$ , however, the wrong side does not exist. Thus,  $\exists!$  equilibrium for any level of debt when  $\lambda\theta < E$ .  $\blacksquare$

1/ Debt must be in the interval  $[0, \infty]$ . At a finite number of points within this interval, there will indeed be an even number of equilibria.

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